On non-Riemannian geometries and singularities in Double Field Theory

Non-Relativistic Strings and Beyond

Kevin Morand

Sogang University – Seoul

Based on work in collaboration with (Miok $+$ Jeong-Hyuck) $\times$ Park

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Singularities in General Relativity

- The notion of singularities in GR can be unfolded according to the following layers:
  1. coordinate singularity
  2. curvature singularity
  3. geodesic incompleteness

Example (Schwarzschild metric):

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

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The Schwarzschild metric:

1. admits a coordinate singularity at $r = 2M$.
2. admits a curvature singularity at $r = 0$.
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The Schwarzschild metric:

1. admits a coordinate singularity at $r = 2M$.

   Changing the time coordinate allows to reexpress it in the ingoing Eddington–Finkelstein coordinates:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2\, dv dr + r^2 \sin^2\theta \, d\phi^2$$

2. admits a curvature singularity at $r = 0$.
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1. admits a coordinate singularity at $r = 2M$.
2. admits a curvature singularity at $r = 0$.

   Although the Ricci scalar vanishes $(R = 0)$, the Kretschmann scalar:

   $$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{48M^2}{r^6} \quad \text{diverges at} \quad r \to 0.$$

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The Schwarzschild metric:

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   There are geodesics which have bounded proper time only.

   They reach the singularity at $r = 0$ after finite proper time.

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The determinant $\det g = -r^4 \sin^2\theta$ vanishes at $r = 0$, so that the metric is not invertible there.

Question: *Is there a meaningful sense in which (pseudo)-Riemannian geometry becomes non-Riemannian at singularities?*

Double Field Theory (DFT)

Stringy geometry

- Spacetime is doubled $x^A = (\tilde{x}_\mu, x^\mu)$ and $\partial_A = (\tilde{\partial}^\mu, \partial_\mu)$ where $A \in \{1, \ldots, 2D\}$ and $\mu \in \{1, \ldots, D\}$
- Spacetime is endowed with a canonical $\mathbf{O}(D, D)$ metric $\mathcal{J}_{AB} = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}$.
- Section condition $\quad \partial^A \partial_A \sim 0$
- Doubled diffeomorphisms $\quad (\hat{\mathcal{L}}_X Y)^A = X^B \partial_B Y^A + (\partial^A X_C - \partial_C X^A) Y^C$

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Fundamental fields

- Fundamental objects of the theory are the generalized metric $\mathcal{H}_{AB}$ and the dilaton d.

$$\underbrace{\mathcal{H}_{AB} = \mathcal{H}_{BA}}_{\text{Symmetric}} \quad , \quad \underbrace{\mathcal{H}_A{}^C \mathcal{H}_B{}^D \mathcal{J}_{CD} = \mathcal{J}_{AB}}_{\mathbf{O(D,D)}}$$

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Curvature

- The fundamental $\mathbf{O}(D,D)$ variables can be used to build $\mathbf{O}(D,D)$-curvature tensors generalising the Ricci calculus of General Relativity:
  - A generalised Ricci scalar $\mathcal{R}(\mathcal{H}, d)$
  - A generalised Ricci tensor $\mathcal{R}_{AB}(\mathcal{H}, d)$

Double Field Theory (DFT)

- The DFT action reads

$$S_{\mathsf{DFT}} = \int d^{2D}X \, e^{-2d} \, \mathcal{R}(\mathcal{H}, d)$$

where the generalised Ricci scalar $\mathcal{R}(\mathcal{H}, d)$ is the unique $\mathbf{O}(D, D)$ scalar built in terms of second derivatives of the fundamental $\mathbf{O}(D, D)$ variables $\mathcal{H}$ and d.

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- Substituting into the $\mathbf{O}(D,D)$ variables the Riemannian parameterisation in terms of $(g_{\mu\nu}, B_{\mu\nu}, \phi)$:

$\mathbf{O}(D,D)$ dilaton $\quad e^{-2d} = \sqrt{-g}e^{-2\phi}$

Generalized metric $\quad \mathcal{H}_{AB} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ B & \mathbf{1} \end{pmatrix} \begin{pmatrix} g^{-1} & \mathbf{0} \\ \mathbf{0} & g \end{pmatrix} \begin{pmatrix} \mathbf{1} & -B \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

  yields the universal spacetime low-energy action for the closed string massless (NS-NS) sector ubiquitous in all string theories:

$$\int \mathrm{d}^D x\, \sqrt{-g}\, e^{-2\phi}\, \left(R_g + 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12} H_{\lambda\mu\nu}H^{\lambda\mu\nu}\right) \quad \text{where} \quad H = \mathrm{d}B$$

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- The action is invariant under the doubled diffeomorphisms:
  - (Undoubled) Diffeomorphisms: $\quad \delta_\xi g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu} \quad , \quad \delta_\xi B_{\mu\nu} = \mathcal{L}_\xi B_{\mu\nu} \quad , \quad \delta_\xi \phi = \mathcal{L}_\xi \phi$
  - B-gauge transformations: $\quad \delta_\Lambda B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$

Summary of this talk

- **Background**
  The stringy geometry of DFT can accommodate not only Riemannian geometries but also non-Riemannian ones.

- **Main result**
  What appear as genuine singularities in conventional GR geometry can be recast as regular DFT geometry with a non-Riemannian sector.

- **Applications**
  We identify among this class physically relevant examples and show their geodesic completeness.

Summary of this talk

- **Background**

  The stringy geometry of DFT can accommodate not only Riemannian geometries but also non-Riemannian ones.

  - Solutions to the defining equations of the DFT generalised metric $\mathcal{H}$ are classified by two non-negative integers $(n, \bar{n})$ such that $0 \leq n + \bar{n} \leq D$.

    Example: supergravity $(0, 0)$, Gomis–Ooguri NR string $(1, 1)$

  - The geometry of the (undoubled) D-dimensional spacetime is generically characterised by:
    - $n + \bar{n}$ longitudinal directions
    - $D - (n + \bar{n})$ transverse directions.
  - Particles freeze along the $n + \bar{n}$ directions and strings become (anti)-chiral along n (resp. $\bar{n}$) directions.

- **Main result**

  What appear as genuine singularities in conventional GR geometry can be recast as regular DFT geometry with a non-Riemannian sector.

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- **Background**
  The stringy geometry of DFT can accommodate not only Riemannian geometries but also non-Riemannian ones.

- **Main result**
  What appear as genuine singularities in conventional GR geometry can be recast as regular DFT geometry with a non-Riemannian sector.

- Specifically, we exhibit a class of supergravity spacetimes featuring genuine curvature singularities in Riemannian geometry, for which we prove that:
  - The corresponding DFT generalised metric can be made regular via a suitable use of doubled diffeomorphisms.
  - The corresponding $\mathbf{O}(D,D)$-covariant curvature tensors are all regular, in contradistinction to their Riemannian counterparts.

- **Applications**
  We identify among this class physically relevant examples and show their geodesic completeness.

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  What appear as genuine singularities in conventional GR geometry can be recast as regular DFT geometry with a non-Riemannian sector.

- **Applications**
  We identify among this class physically relevant examples and show their geodesic completeness.
  - $D=2$ Black hole solution Witten 1991
  - $D=4$ Spherical solution Burgess, Myers, Quevedo 1994
  - $D=10$ Black 5-brane Horowitz, Strominger 1991

Main ansatz

- We focus on the following supergravity ansatz, with $x^\mu = (t, y, z^i)$:

$$\text{Metric} \quad \mathrm{d}s^2 = \frac{1}{F(x)}\left(-\mathrm{d}t^2 + \mathrm{d}y^2\right) + G_{ij}(x)\mathrm{d}z^i\mathrm{d}z^j$$

$$\text{Kalb–Ramond field} \quad B = \pm\frac{1}{F(x)}\,\mathrm{d}t \wedge \mathrm{d}y + \frac{1}{2}\,\beta_{\mu\nu}(x)\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$$

$$\text{Dilaton scalar} \quad e^{-2\phi} = F(x)\Psi(x)$$

  where G_{ij}, $\beta_{\mu\nu}$ and Ψ are assumed to be regular.

- The latter ansatz encompasses the previously mentioned examples and (hopefully) more.
- The only source of singularity is therefore $F \to 0$, which clearly features a coordinate singularity.
- Generically, the metric features a curvature singularity:

$$R \to \infty \quad , \quad R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \to \infty \quad \text{whenever} \quad F \to 0.$$

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- Substituting into the $\mathbf{O}(D,D)$ variables in Riemannian parameterisation:

$$\mathbf{O}(D,D)\text{ dilaton} \quad e^{-2d} = \sqrt{-g}e^{-2\phi}$$

$$\text{Generalized metric} \quad \mathcal{H}_{AB} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ B & \mathbf{1} \end{pmatrix}\begin{pmatrix} g^{-1} & \mathbf{0} \\ \mathbf{0} & g \end{pmatrix}\begin{pmatrix} \mathbf{1} & -B \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

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$$\mathbf{O}(D,D)\text{ dilaton} \quad e^{-2d} = \Psi\sqrt{G}$$

$$\text{Generalized metric} \quad \mathcal{H}_{AB} = \begin{pmatrix}\mathbf{1} & \mathbf{0}\\ \beta & \mathbf{1}\end{pmatrix}\mathring{\mathcal{H}}\begin{pmatrix}\mathbf{1} & -\beta\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$$

where $$\mathring{\mathcal{H}}_{AB} = \begin{pmatrix}-F\sigma_3 & 0 & \pm\sigma_1 & 0\\ 0 & G^{-1} & 0 & 0\\ \pm\sigma_1 & 0 & 0 & 0\\ 0 & 0 & 0 & G\end{pmatrix}$$

with Pauli matrices:

$$\sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\quad,\quad \sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Main observation

- Substituting into the $\mathbf{O}(D,D)$ variables in Riemannian parameterisation:

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$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- The coordinate singularity is absent from the $\mathbf{O}(D,D)$ fundamental variables *i.e.* no negative power of F appears (*c.f.* Lee, Park 13', Blair 15', Berman, Blair and Otsuki 19', Blair 19' for earlier examples)

- In the limit $F \to 0$, the generalised metric $\mathcal{H}_{AB}$ becomes non-Riemannian of type $(1,1)$:

$$H^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G^{ij} \end{pmatrix} \quad , \quad K_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G_{ij} \end{pmatrix} \quad , \quad \beta = \beta_{\mu\nu} dx^\mu \wedge dx^\nu$$

$$X_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 & 1 & 0 \end{pmatrix} \quad , \quad \bar{X}_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \mp 1 & 1 & 0 \end{pmatrix} \quad , \quad Y^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 \\ 1 \\ 0 \end{pmatrix} \quad , \quad \bar{Y}^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \mp 1 \\ 1 \\ 0 \end{pmatrix}$$

Main observation

- Substituting into the $\mathbf{O}(D,D)$ variables in Riemannian parameterisation:

$$\begin{aligned} \mathbf{O}(D,D)\text{ dilaton} \quad e^{-2d} &= \Psi\sqrt{G} \\ \text{Generalized metric} \quad \mathcal{H}_{AB} &= \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \beta & \mathbf{1} \end{pmatrix} \mathring{\mathcal{H}} \begin{pmatrix} \mathbf{1} & -\beta \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \end{aligned}$$

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- The parameterisation of the generalised metric $\mathcal{H}$ depends of the spacetime point:
  - Riemannian at $F \neq 0$.
  - non-Riemannian at $F = 0$.
- The GR limit Regular $\rightarrow$ Singular is traded for the DFT limit Riemannian $\rightarrow$ non-Riemannian.
- The Kalb–Ramond field $B = \pm\frac{1}{F}\,\mathrm{d}t \wedge \mathrm{d}y + \cdots$ plays a crucial rôle in regularising $\mathcal{H}$.
- In particular, whenever $\pm\frac{1}{F}\,\mathrm{d}t \wedge \mathrm{d}y$ is pure gauge, the curvature singularity of the GR metric is eliminated through doubled diffeomorphisms, hence is a coordinate singularity in DFT.

Geodesics

- The notion of DFT geodesics for Riemannian generalised metrics agree with the conventional GR one computed in the string frame. Since the DFT geometry is regular, we expect the null and timelike geodesics associated with our supergravity ansatz to be complete.
- Focusing on particular supergravity solutions, we verify that this is indeed the case:
  - $D = 2$ Black hole solution Witten 1991
  - $D = 4$ Spherical solution Burgess, Myers, Quevedo 1994
  - $D = 10$ Black 5-brane Horowitz, Strominger 1991
- Additionally, it can be checked that the corresponding geodesic deviation remains regular:

$$\frac{\mathrm{D}^2\xi^\mu}{\mathrm{d}\lambda^2} = R^\mu{}_{\nu\rho\sigma}\,\dot{x}^\nu\,\dot{x}^\rho\,\xi^\sigma$$

- Hence, despite featuring a curvature singularity, the physically measurable quantities of these solutions remain finite.
- From the general behavior of particles and strings on non-Riemannian backgrounds, we expect that:
  - geodesics freeze on non-Riemannian points $F = 0$
  - strings become chiral at $F = 0$

Example ($D = 2$ black hole)

- The $D = 2$ black hole solution from Witten 1991 reads:

$$\mathrm{d}s^2 = \frac{\mathrm{d}y^+\mathrm{d}y^-}{F(y^+, y^-)} \qquad \text{and} \qquad H = 0 \qquad \text{with} \qquad F = -1 + \frac{y^+y^-}{l^2} = \frac{F}{|F|}e^{-2\phi}.$$

- The latter solves the supergravity field equations with cosmological constant $\Lambda_{\mathrm{DFT}} = -\frac{2}{l^2}$.
- The Ricci scalar reads $R = -\frac{4}{l^2 F}$ so that the hyperbola $y^+y^- = l^2$ is a curvature singularity.
- Although the H-flux is trivial, we introduce a pure gauge B-field as $B = \pm\frac{1}{F}\,\mathrm{d}y^+ \wedge \mathrm{d}y^-$.
- The resulting generalised metric is non-Riemannian regular on the hyperbola.
- Timelike geodesics will never reach the non-Riemannian hyperbola while null ones may approach only at past or future infinity (freezing).
- Although certain components of the Riemann tensor diverge, the contraction with $\dot{x}$ remain finite so that the geodesic deviation $\frac{\mathrm{D}^2\xi^\mu}{\mathrm{d}\lambda^2} = R^\mu{}_{\nu\rho\sigma}\dot{x}^\nu\dot{x}^\rho\xi^\sigma$ is regular, with vanishing norm $\left|\frac{\mathrm{D}^2\xi}{\mathrm{d}\lambda^2}\right|^2 = 0$.
- One of $\{y^+, y^-\}$ is chiral and the other anti-chiral on the non-Riemannian hyperbola.

Summary

- We identify a class of singular supergravity spacetimes as regular DFT geometries by re-analysing the three layers of singularities from a DFT perspective:

  1. coordinate singularity: The curvature singularity of Riemannian geometry appears as a coordinate singularity within DFT which can be removed by doubled diffeomorphisms.
  2. curvature singularity: All DFT curvature tensors are regular, as a consequence of the regularity of the generalised metric and dilaton field.
  3. geodesic incompleteness: Focusing on particular known supergravity solutions, it is shown that the non-Riemannian points $F = 0$ form an impenetrable sphere where particles freeze and strings become chiral. Computed in the string frame, geodesics outside the non-Riemannian sphere are complete with no singular deviation.

- Relying on the geometry of DFT allows to address the singularity problem for this class already at the classical level (no α'-expansion required).

- Exploring the non-Riemannian sector of DFT allows to go beyond supergravity and to accommodate nonrelativistic physical theories (*e.g.* Gomis–Ooguri string, see *e.g.* Ko, Melby-Thompson, Meyer and Park 15', Berman, Blair and Otsuki 19', Cho and Park 19', Blair 20', Park and Sugimoto 20', Gallegos, Gürsoy, Verma and Zinnato 20', Blair, Oling, and Park 20', *etc.*) as well as to shed new light on issues within GR.

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