Nonrelativistic IIB Supergravity

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Non-relativistic Strings and Beyond
Nordita

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Introduction

- Extension of nonrelativistic NS-NS supergravity.
- Dualities in nonrelativistic string theory.
- $SL(2\,,\mathbb{R})$ realization in NR IIB theory.
- Nonrelativistic holography.

IIB Supergravity

- The IIB action:

$$\hat{S}_{IIB} = \frac{1}{2\kappa^2}\int d^{10}x\,\hat{E}\left(\hat{R} + \frac{1}{4}\,Tr(\partial_\mu \hat{\mathcal{M}}\,\partial^\mu \hat{\mathcal{M}}^{-1}) - \frac{1}{12}\hat{\mathcal{H}}^{\intercal}_{\mu\nu\rho}\,\hat{\mathcal{M}}\,\hat{\mathcal{H}}^{\mu\nu\rho}\right)$$

$$-\frac{1}{8\kappa^2}\int\left(\hat{F}^{(5)}\wedge\star\hat{F}^{(5)} + \hat{\mathcal{C}}^{(4)}\wedge\hat{\mathcal{H}}^{\intercal}\wedge\epsilon\,\hat{\mathcal{H}}\right),$$

where,

$$\hat{\mathcal{M}} = e^{\hat{\Phi}}\begin{pmatrix}(\hat{C}^{(0)})^2 + e^{-2\hat{\Phi}} & \hat{C}^{(0)}\\ \hat{C}^{(0)} & 1\end{pmatrix}, \qquad \epsilon = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, \quad \hat{\mathcal{H}} = \begin{pmatrix}d\hat{B}^{(2)}\\ d\hat{C}^{(2)}\end{pmatrix}$$

$$\hat{F}^{(q+1)} = d\,\hat{C}^{(q)} + \hat{C}^{(q-2)}\wedge\hat{H}\,, \quad \hat{\mathcal{C}}^{(4)} = \hat{C}^{(4)} + \frac{1}{2}\hat{B}^{(2)}\wedge\hat{C}^{(2)}\,.$$

- Self-duality Condition:

$$\hat{F}^{(5)} = \star\hat{F}^{(5)}\,.$$

SL(2,ℝ) Symmetry

- IIB action is invariant under an SL$(2\,,\mathbb{R})$ transformation generated by the matrix

$$\Lambda = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}, \qquad \alpha\,,\beta\,,\gamma\,,\delta \in \mathbb{R}\,, \qquad \alpha\,\delta - \beta\,\gamma = 1\,.$$

- The various fields transform as follows

$$\hat{E}^{\hat{A}} \to \hat{E}^{\hat{A}}\,, \qquad \hat{\mathcal{C}}^{(4)} \to \hat{\mathcal{C}}^{(4)}\,, \qquad \hat{F}^{(5)} \to \hat{F}^{(5)}\,,$$
$$\hat{\mathcal{H}}^{(3)} \to (\Lambda^{-1})^{\mathsf{T}}\,\hat{\mathcal{H}}^{(3)}\,, \qquad \hat{\mathcal{M}} \to \Lambda\,\hat{\mathcal{M}}\,\Lambda^{\mathsf{T}}\,.$$

Nonrelativistic limit

- Foliation and reparameterization of metric [1]:

$$\hat{E}^{\hat{A}} = \begin{cases} \hat{E}^{A} = \omega^{3/4}\,\tau^{A} & \{0,1\} \in A \quad \text{longitudinal} \\ \hat{E}^{A'} = \omega^{-1/4}\,E^{A'} & \{2,9\} \in A' \quad \text{transverse} \end{cases}$$

- Reparameterization:

$$\hat{\Phi} = \Phi + \ln\omega\,,$$
$$\hat{B}^{(2)} = -\omega^{2}\,e^{\Phi/2}\,\ell^{(2)} + B^{(2)}\,,$$
$$\hat{C}^{(q)} = \omega^{2}\,e^{\Phi/2}\,\ell^{(2)} \wedge C^{(q-2)} + C^{(q)}\,.$$

- Here we defined worldsheet volumeform as $\ell^{(2)} = \frac{1}{2}\,\tau^{A} \wedge \tau^{B}\epsilon_{AB}$.

[1] Bergshoeff, Ebert, Lahnsteiner, Romano, Rosseel, Şimşek, Yan.

Nonrelativistic limit

- Plugging these expansion anzatze into the action, we have

$$\hat{S} = \omega^2 \overset{(2)}{S} + \overset{(0)}{S} + \omega^{-2} \overset{(-2)}{S} + \cdots .$$

- The ω^2 terms are given by

$$\overset{(2)}{S}_{\rm EH} = -\frac{1}{2\kappa^2} \int d^{10}x\, E\, \tau_{A'B'A}\tau^{A'B'A} ,$$

$$\overset{(2)}{S}_{\hat{\mathcal{H}}} = \frac{1}{2\kappa^2} \int d^{10}x\, E \left(\tau_{A'B'A}\tau^{A'B'A} + \frac{1}{2} e^{2\Phi} F_{A'} F^{A'} - \frac{1}{2\cdot 3!} e^{\Phi} F_{A'B'C'} F^{A'B'C'} \right) ,$$

$$\overset{(2)}{S}_{\hat{\mathcal{M}}} = \frac{1}{2\kappa^2} \int d^{10}x\, E \left(-\frac{1}{2} e^{2\Phi} F_{A'} F^{A'} \right) ,$$

$$\overset{(2)}{S}_{\hat{F}_5} = \frac{1}{2\kappa^2} \int d^{10}x\, E \left(\frac{1}{4!} e^{\Phi} F_{A'B'C'} F^{A'B'C'} - \frac{1}{4\cdot 5!} F_{A'_1\cdots A'_5} F^{A'_1\cdots A'_5} \right) ,$$

$$\overset{(2)}{S}_{\rm CS} = -\frac{1}{2\kappa^2} \int \frac{1}{2} e^{\Phi/2} F^{(5)} \wedge F^{(3)} \wedge \ell^{(2)} .$$

Nonrelativistic limit

- Collect:

$$\omega^2 \overset{(2)}{S} = \frac{\omega^2}{2\kappa^2} \int \left(\Omega^{(3)} \wedge \ell^{(2)}\right) \wedge \star\left(\Omega^{(3)} \wedge \ell^{(2)}\right) . \tag{2}$$

- Here we have defined the three-form:

$$\Omega^{(3)} = \frac{1}{2}\left[\star\left(F^{(5)} \wedge \ell^{(2)}\right) - e^{\Phi/2} F^{(3)}\right] .$$

- Hubbard-Stratonovich transformation and auxiliary 5-form field $\mathcal{A}^{(5)}$:

$$\omega^2 \overset{(2)}{S} \to \frac{1}{2\kappa^2} \int \left[\mathcal{A}^{(5)} \wedge \star\left(\Omega^{(3)} \wedge \ell^{(2)}\right) - \frac{1}{4\omega^2} \mathcal{A}^{(5)} \wedge \star\mathcal{A}^{(5)}\right] . \tag{3}$$

- The equation of motion for $\mathcal{A}^{(5)}$ is $\mathcal{A}^{(5)} = 2\,\omega^2\,\Omega^{(3)} \wedge \ell^{(2)}$ (*)

Nonrelativistic limit

- After introducing auxilary field $\mathcal{A}^{(5)}$, the expansion of action becomes

$$\hat{S} = \overset{(0)}{S} + \omega^{-2} \overset{(-2)}{S} + \cdots .$$

- Take the limit $\omega \to \infty$, auxilary field $\mathcal{A}^{(5)}$ becomes Lagrange multiplier imposing the following constraint

$$\Omega^{(3)} \wedge \ell^{(2)} = 0 .$$

- In component form this is given by

$$F_{A'_1 \cdots A'_5} = -\frac{e^{\Phi/2}}{3!} \epsilon_{A'_1 \cdots A'_8} F^{A'_6 A'_7 A'_8} .$$

- This is one of the two conditions originating from the non-relativistic limit of the self-duality condition $\hat{F}^{(5)} = \star \hat{F}^{(5)}$. The other one is

$$F_{AA'_1 \cdots A'_4} = \frac{1}{4!} \epsilon_{AB} \epsilon_{A'_1 \cdots A'_4 B'_1 \cdots B'_4} F^{BB'_1 \cdots B'_4} .$$

Nonrelativistic IIB action

- Now, we collect the $O(\omega^0)$ terms in the action $\overset{(0)}{S} = S_{IIB}$:

$$S_{IIB} = S_{\rm NS} + S_{\rm R} + S_{\mathcal{A}} + S_{\rm CS} \ .$$

- Explicitly (NS-NS[2])

$$\begin{aligned} S_{\rm IIB} = \frac{1}{2\kappa^2} \int d^{10}x\, E \Big(& R - \frac{3}{8} \partial_{A'}\Phi\, \partial^{A'}\Phi - \frac{1}{2\cdot 3!} e^{-\Phi} H_{A'B'C'} H^{A'B'C'} \\ & + 2\, \tau_{A'A}{}^{A}\, \tau^{A'B}{}_{B} + (\partial_{A'}\Phi)\, \tau^{A'A}{}_{A} + e^{-\Phi/2}\, \epsilon_{AB}\, \tau_{A'B'}{}^{A}\, H^{A'B'B} \\ & - \frac{1}{2} e^{2\Phi} F_A F^A - \frac{1}{4} e^{\Phi} F_{A'B'A} F^{A'B'A} - \frac{1}{2} e^{3\Phi/2} F^{A'} F_{A'AB}\, \epsilon^{AB} \\ & - \frac{1}{4\cdot 4!} F_{A'_1\cdots A'_4 A} F^{A'_1\cdots A'_4 A} - \frac{1}{4!} F^{A'B'C'} F_{A'B'C'AB} \epsilon^{AB} \Big) \\ & + \frac{1}{2\kappa^2} \int \Big(\mathcal{A}^{(5)} \wedge \star (\Omega^{(3)} \wedge \ell^{(2)}) - \frac{1}{2} \mathcal{C}^{(4)} \wedge H \wedge dC^{(2)} \Big) \, . \end{aligned}$$

[2] NR NS-NS action: Bergshoeff, Lahnsteiner, Romano, Rosseel, Şimşek, 2021

Galilean and dilatational symmetries

- *Galilei boosts* act on the NS-NS and the Ramond-Ramond fields as

$$\delta_{\mathrm{G}}\tau_\mu{}^A = 0\,, \qquad \delta_{\mathrm{G}} E_\mu{}^{A'} = -\lambda_A{}^{A'}\,\tau_\mu{}^A\,, \qquad \delta_{\mathrm{G}} B^{(2)} = \epsilon_{AB}\,\lambda^B{}_{A'}\,e^{\Phi/2}\,\tau^A \wedge E^{A'}\,,$$
$$\delta_{\mathrm{G}} C^{(q)} = -\epsilon_{AB}\,\lambda^B{}_{A'}\,e^{\Phi/2}\,\tau^A \wedge E^{A'} \wedge C^{(q-2)}\,,$$
$$\delta_G\,\mathcal{A}^{(5)} = 2\,\epsilon_{AB}\,\lambda^A{}_{A'}\,E^{A'} \wedge \tau^B \wedge \Omega^{(3)}\,.$$

- *Dilatational symmetry* [3]: $\mathcal{O} \to e^{\Delta(\mathcal{O})\,\lambda_{\mathrm{D}}}\,\mathcal{O}$

$$\Delta\big(\tau_\mu{}^A\big) = \frac{3}{4}\,, \qquad \Delta\big(B^{(2)}\big) = \Delta\big(C^{(q)}\big) = 0\,,$$
$$\Delta\big(E_\mu{}^{A'}\big) = -\frac{1}{4}\,, \qquad \Delta\big(\mathcal{A}^{(5)}\big) = 0\,, \qquad \Delta\big(e^\Phi\big) = 1\,.$$

[3] Bergshoeff, Lahnsteiner, Rosseel, Şimşek, Romano.

SL(2 , ℝ) Symmetry

Let us define some new objects

$$\kappa = \frac{\gamma\, e^{-\Phi}}{\gamma\, C^{(0)} + \delta}\,, \qquad \mathcal{V} = e^{\Phi/2} \begin{pmatrix} C^{(0)} \\ 1 \end{pmatrix}\,,$$

$$\mathcal{B}^{(2)} = e^{-\Phi/2} B^{(2)}\,, \qquad \mathcal{C}^{(2)} = e^{\Phi/2} \left(C^{(2)} + C^{(0)}\, B^{(2)} \right)\,,$$

The SL(2 , ℝ) tranformations are [4] :

$$\mathcal{V} \to \Lambda\, \mathcal{V}\,,$$

$$\mathcal{B}^{(2)} \to \mathcal{B}^{(2)} - \kappa\, \mathcal{C}^{(2)} + \frac{1}{2}\, \kappa^2\, \ell^{(2)}\,, \qquad \mathcal{C}^{(2)} \to \mathcal{C}^{(2)} - \kappa\, \ell^{(2)}\,,$$

$$\mathcal{C}^{(4)} \to \mathcal{C}^{(4)} - \frac{1}{2}\, \kappa\, \mathcal{B}^{(2)} \wedge \ell^{(2)} + \frac{1}{4}\, \kappa^2\, \mathcal{C}^{(2)} \wedge \ell^{(2)}\,,$$

$$\mathcal{A}^{(5)} \to \mathcal{A}^{(5)} - \left(\kappa\, e^{-\frac{\Phi}{2}}\, H^{(3)} - \tfrac{1}{2}\, \kappa^2\, e^{\frac{\Phi}{2}}\, F^{(3)} \right) \wedge \ell^{(2)}\,.$$

[4] Bergshoeff, Lahnsteiner, Grosvenor, Yan, UZ

Conclusion

Summary:

- Nonrelativistic IIB action and symmetries.
- Polynomial realization of $SL(2, \mathbb{R})$ symmetry.

Future directions:

- Realization of $SL(2, \mathbb{R})$ on the equation of motions.
- Solutions.

Thanks!