T Symmetry and Its Violation

Part 3: Applications in Physics

(1) Forces from Exchange

A “Clean” Warm-up

Very light, very feebly interacting particles are an intriguing frontier for fundamental physics.

Axions are one example.

Exchange of such particles could mediate new macroscopic forces. (“Fifth forces”)

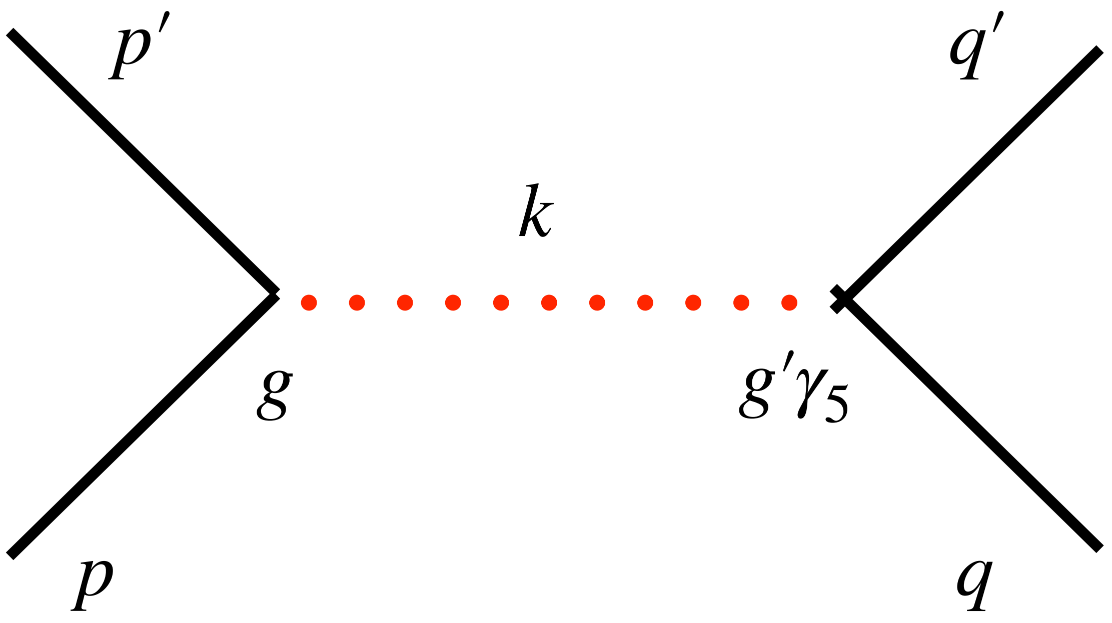
It is interesting and instructive to calculate the form of such forces.

If such forces bring in T violation, that helps to distinguish them from backgrounds!

It is convenient to calculate scattering at very small momentum, and interpret it as potential scattering (Fourier transform).

$\bar{\psi}\psi$ is natural under T, while $\bar{\psi}\gamma_5\psi$ is unnatural.

Thus exchange of a spin-0 particle with both types of couplings will give us a T violating force. It will also violate P.



Non-relativistic limit spinor: $(s, \sigma \cdot p/2m\, s)^t$

$$\frac{gg'}{2m}\frac{\sigma \cdot k}{k^2 + m^2}$$

$$V(r) = \frac{gg'}{2m}\int \frac{d^3k}{(2\pi)^3} e^{ikr} \frac{\sigma \cdot k}{k^2 + m^2}$$

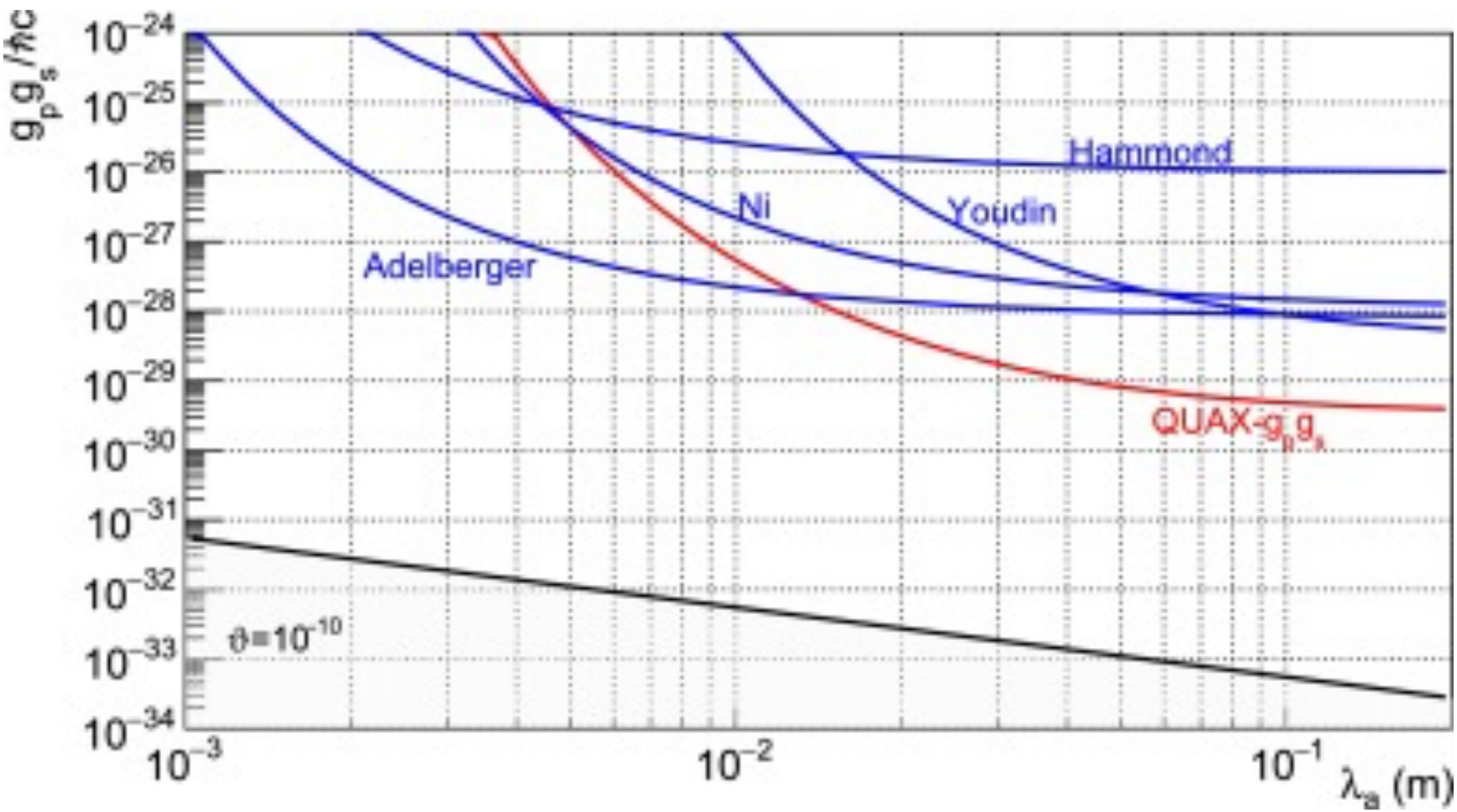
$$= \frac{gg'}{4\pi}\frac{\sigma \cdot \nabla}{2m}\frac{e^{-mr}}{r}$$

This is the T violating monopole-dipole force.

Improved constraints on monopole–dipole interaction mediated by pseudo-scalar bosons

N. Crescini [a,b,*], C. Braggio [c], G. Carugno [c], P. Falferi [d], A. Ortolan [b], G. Ruoso [b]

Physics Letters B 773 (2017) 677–680



Other (NR) Spin-0 Exchanges

	P	T	(C)
1	+	+	+
γ_5	-	-	+
γ_0	+	+	-
$\gamma_0\gamma_5$	-	+	+

Especially interesting are exchanges with vertices $1, \gamma_0\gamma_5$ or $\gamma_0, \gamma_0\gamma_5$.

They give velocity-dependent forces

$$\propto \ \sigma \cdot v \, \frac{e^{-mr}}{r}$$

that violate P but not T.

(2) Electric and Magnetic Moments

Nuclei, Atoms, and Molecules

When does it make sense to interpret measurements of electric dipole moments as evidence about fundamental T violation?

If the system is spin 0 and there is a gap, there is no possibility of energy splitting $\propto E$, for small E :

$$H = \begin{pmatrix} \Delta & (\sum e_j x^j) \cdot E \\ (\sum e_j x^j) \cdot E & 0 \end{pmatrix}$$

Here, for accurate work, we should use the appropriate terms in the standard model!

When the off-diagonal terms are much smaller than Δ, the response is quadratic.

When the ground state is spin 1/2, the $s_z = \pm 1/2$ states are degenerate in vacuum.

Rotational symmetry allows $E \propto \hat{z}$ to connect them:

$$H = \begin{pmatrix} 0 & \kappa E \\ \kappa E & 0 \end{pmatrix}$$

Here we see the possibility of linear response, i.e. an electric dipole moment, for $\kappa \neq 0$.

But according to Kramers' theorem, if T is valid there can be no splitting!

Thus, we arrive at a clean and powerful way of testing fundamental T symmetry.

It can be applied to neutrons, electrons protons, … and to appropriate molecules

Another perspective: if the only degree of freedom in play is a spin 1/2, then the candidate Hamiltonian linear in electric field is $\propto \boldsymbol{\sigma} \cdot \boldsymbol{E}$, which violates T.

Neutron: $(0.0\pm1.1)\times10^{-26}$ $e\cdot$cm.

Electron: $< 4.1\times10^{-30}$ $e\cdot$cm

These should be compared with the Compton wavelengths $l_n \sim 10^{-18}$ cm., $l_e \sim 10^{-14}$ cm.

Intuitively: rotation does not lead to charge separation.

The limits are obtained by looking for effects of E on spin precession or on spectral splittings.

When our system has many low-energy states, and the splittings are sub-thermal or the electric field is not tiny, it can be appropriate to ignore the splittings.

Instead of treating the interaction introduced by E as a perturbation, it becomes appropriate (within the manifold of low-energy states) to treat it as the main Hamiltonian, and diagonalize it!

Then the interaction energy is manifestly proportional to E. The field self-organizes a responsive state, that has an effective dipole moment.

Going back to the 2-state system:

$$H = \begin{pmatrix} \Delta & (\sum e_j x^j) \cdot E \\ (\sum e_j x^j) \cdot E & 0 \end{pmatrix}$$

When the off-diagonal terms are much larger than Δ the energetically favored state is not the ($E = 0$) ground state, and the energy splitting is linear in E.

A version of this idea is implicit the treatment of electric dipole moments you find in basic chemistry texts. By fixing the position of nuclei, one in effect neglects “thermal scale” energy splittings associated with nuclear motion, i.e. molecular rotation and vibration.

The “interaction states” picked out be diagonalizing the E interaction are also those that have simple dynamical interactions with ordinary matter.

In general, effective electric and magnetic moments should be discussed within the framework of a description of the low-energy states.

In short, there’s more to life than ground states characterized by angular momentum alone.

Big molecules with rings can have low-energy states with non-trivial, dynamical distributions of charge and spin, including currents of both, that are selected by ambient interactions as the preferred basis.

Similar remarks apply to magnetic moments, with the important difference that natural environments will still tend to pick out the low-lying states of the *electric* interaction, and the magnetic interaction will be a perturbation on those.

[recollection: resolving the “paradox” of electric dipole moments: chemistry (and biology) $d \neq 0$ versus fundamental physics $d = 0$]

In general, effective electric and magnetic moments should be discussed within the framework of a description of the low-energy states.

In short, there's more to life than ground states characterized by angular momentum alone.

Big molecules with rings can have low-energy states with non-trivial, dynamical distributions of charge and spin, including currents of both, that are selected by ambient interactions as the preferred basis.

(3) Signatures of T Breaking in Matter

A Smörgåsbord

Potential T violating* effects and signatures:

correlation of moments - $\langle d \cdot \mu \rangle \neq 0$

static response

propagation of light through solutions and crystals

scattering at interfaces

violation of Onsager reciprocal relations

failure of detailed balance

(*Here we are speaking of non-dissipative T violation. Of course, exploitation of entropy gradients - “burning”, in a general sense - is an extremely common way to accomplish things in time.)

(3a) Correlation of Moments

A (the?) Basic Phenomenon

correlation of moments - $\langle d \cdot \mu \rangle \neq 0$

Such correlations, if present with a consistent sign, manifestly violate T (and P).

They can be accessed through spectroscopy or through spin resonance.

Objects with such correlation can be useful for magnetic field sensing - specifically, for fixing the origin. They provide *labelled* compass needles.

Multiferroic condensation (see below) is a real and well-studied phenomenon.

(3b) Static Response

E verus B

static response

$\Delta L \propto E \cdot B \Rightarrow$ applied E yields some B,
And vice versa

In particular, we have "B-induced ferroelectricity".
There is an elegant experimental protocol for it:

** *The operational significance of ferroelectricity*

A spontaneous electric field In bulk tends to be neutralized by surface charges. But if the material involved is connected to a capacitor, one can collect the charges as the ferroelectric phase forms or changes:

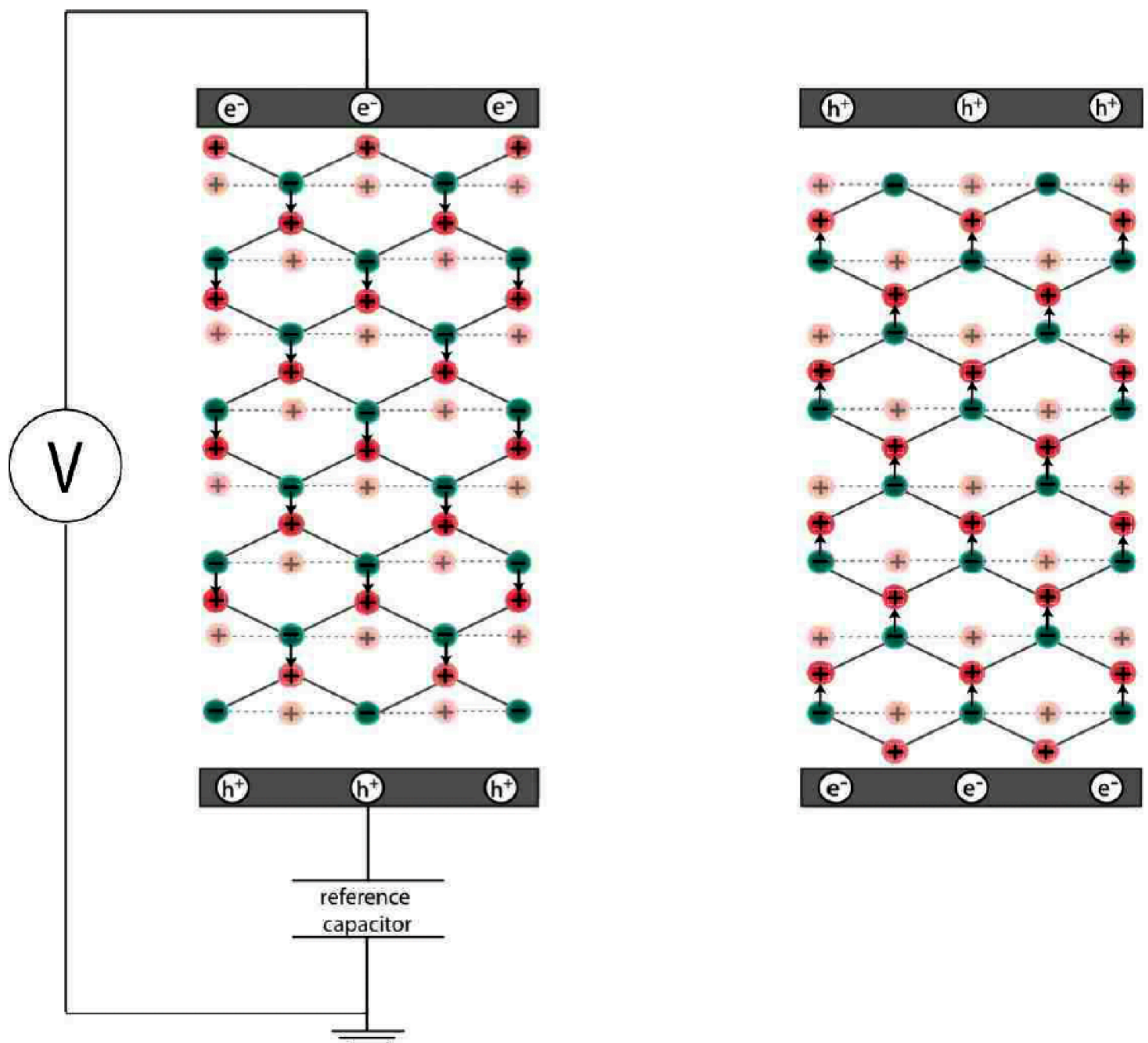
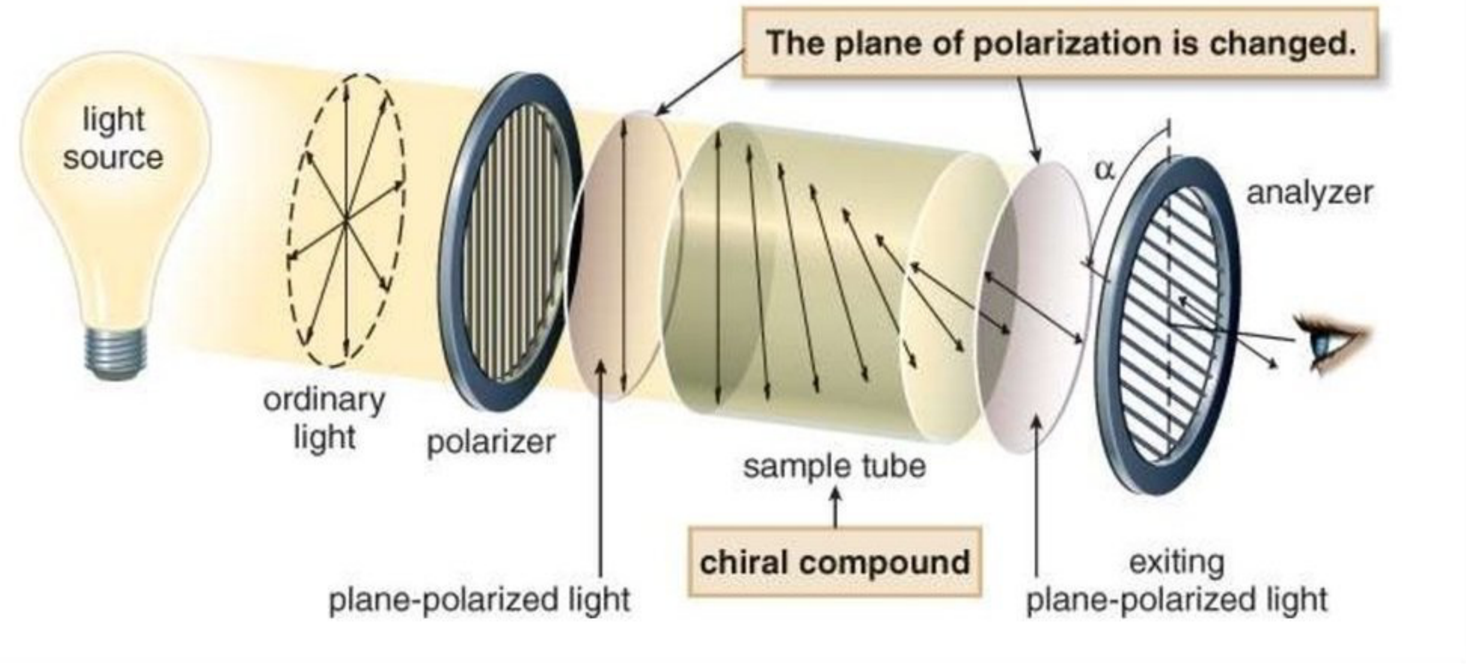


Figure 2: Schematic of the Sawyer-Tower method of measuring ferroelectric polarization. The material on the left is polarized in the up direction and its surface charge is screened by electrons in the upper electrode (grey) and holes in the lower electrode. When the polarization is switched (right), electrons and holes flow through the external circuit to screen the new opposite surface charges, and are counted by comparing the voltage across the material with that across a reference capacitor.

(3c) Propagation

The Optical Activity Complex



Optical Activity: A Signature of P Violation

Optical Activity as an Effective Lagrangian

(Methodological note: When we treat rapidly oscillatory phenomena using effective Lagrangians, we should allow for arbitrary powers of time derivatives. This leads to the appearance of frequency-dependent coupling parameters, … Here I will leave that implicit.)

$$\Delta L = \kappa B \cdot (\nabla \times B)$$

P odd, T even

$$\Delta L = \kappa E \cdot (\nabla \times E)$$

is a valid alternative

$$\rho^{\text{eff}} = 0$$

$$j_\alpha^{\text{eff}} = 2\kappa \nabla^2 B_\alpha$$

$$\nabla \cdot B \;=\; 0 \qquad\qquad \nabla \cdot E \;=\; 0$$

$$\nabla \times E \;=\; -\,\partial_t B \qquad\qquad \nabla \times B \;=\; \partial_t E + 2\kappa \nabla^2 B$$

$$A \;=\; \varepsilon\, e^{i(kx-\omega t)}$$

$$B \;=\; ik \times \varepsilon\, e^{i(kx-\omega t)} \qquad\qquad E \;=\; i\omega\varepsilon\, e^{i(kx-\omega t)}$$

$$k \cdot \varepsilon \;=\; 0$$

$$-k^2\varepsilon \;=\; -\,\omega^2\varepsilon \;-\; 2i\kappa\, k^2\, k \times \varepsilon$$

$$\mathbf{k}\cdot\boldsymbol{\varepsilon} = 0 \qquad -k^2\boldsymbol{\varepsilon} = -\omega^2\boldsymbol{\varepsilon} - 2i\kappa k^2\,\mathbf{k}\times\boldsymbol{\varepsilon}$$

$$\mathbf{k} = (0, 0, k) \qquad \boldsymbol{\varepsilon} \propto (1, \pm i, 0)$$ (Circular Polarizations)

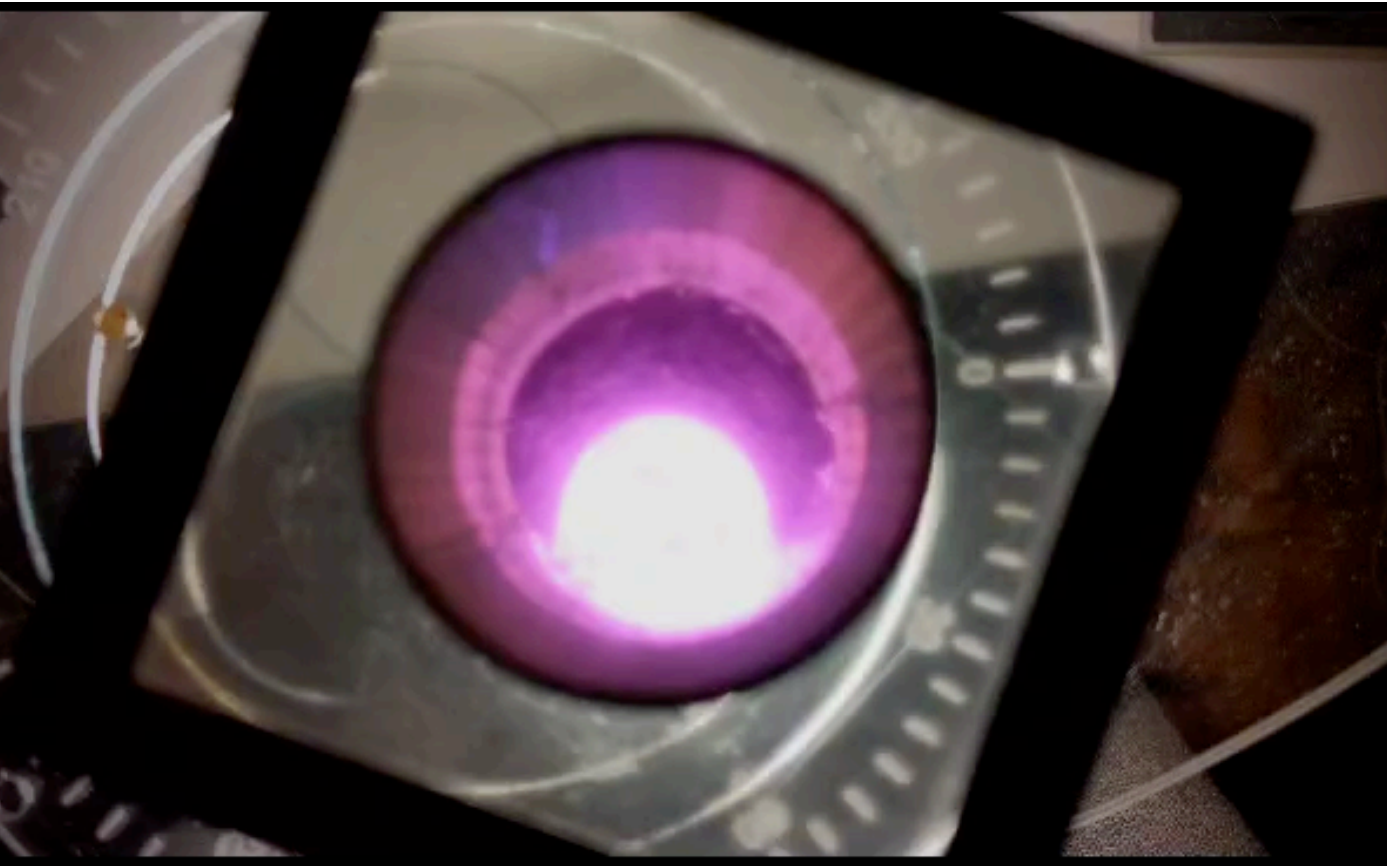
$$\omega^2 - k^2 \mp 2\kappa k^3 = 0$$

⇒ Different circular polarizations travel at different speeds

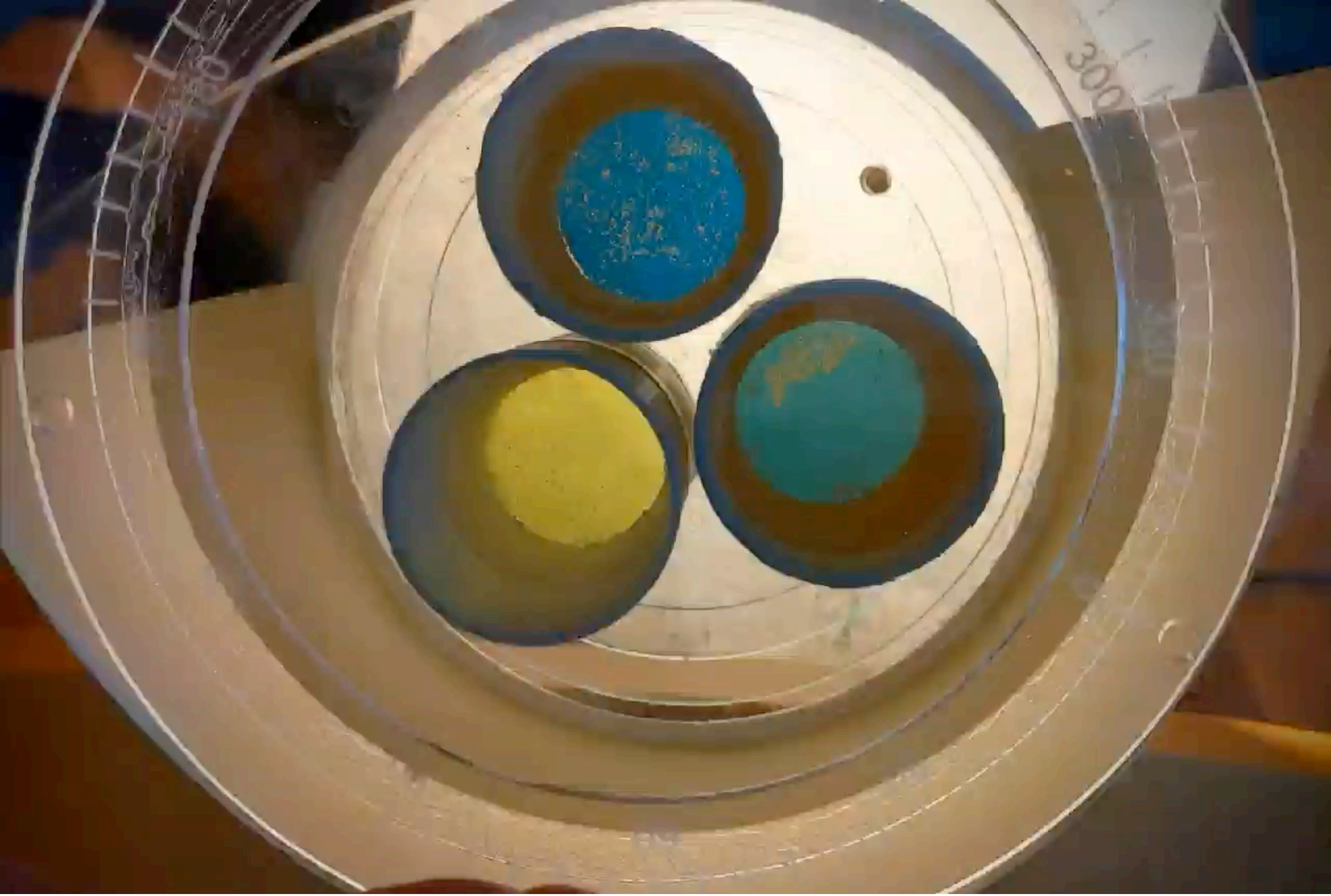
⇔ Optical activity

Because we can express optical activity as an effective interaction, i.e. a contribution to the energy, we can calculate it in second-order perturbation theory for atoms, molecules, … .

The effective Lagrangian technique packages the microscopics in a theoretically convenient way that is user-friendly for drawing macroscopic consequences.



Glucose



Three Sugars

(Magnetic) Faraday Effect

$$\Delta L = \kappa \left(E \times (\nabla \times B)\right) \cdot B$$

P even, T even

$$\Delta L = \kappa \left(E \times (\nabla \times B)\right) \cdot B^0$$

$$\rho^{\text{eff}} = 0$$

$$j_\alpha^{\text{eff}} = 2\kappa B_p^0 \partial_0 \partial_p B_\alpha$$

Analysis similar to optical activity.

Different circular polarizations propagate at different speeds.

Linear polarization rotates proportional to distance.

In contrast to optical activity, here the sense of rotation depends on the direction of propagation.

“Fore and back” optical rotation cancels for optical activity, but adds for the Faraday effect.

To calculate the Faraday effect we need second order perturbation theory in the presence of a magnetic field …

Electric Faraday Effect

$$\Delta L = \kappa \Big(B \times (\nabla \times E) \Big) \cdot E$$ P odd, **T odd**

$$\Delta L = \kappa \Big(B \times (\nabla \times E) \Big) \cdot E^0$$

$\Rightarrow$ Same equations as magnetic Faraday effect.

E B Terms in Crystals

$$\Delta L \;=\; \kappa_{lm} E_l B_m$$

P odd, **T odd**

$$\rho^{\text{eff}} = -\,\kappa_{lm}\,\partial_l B_m$$

$$j_\alpha^{\text{eff}} = \kappa_{lm}\,\epsilon_{\alpha pm}\,\partial_p E_l + \kappa_{\alpha m}\,\partial_0 B_m$$

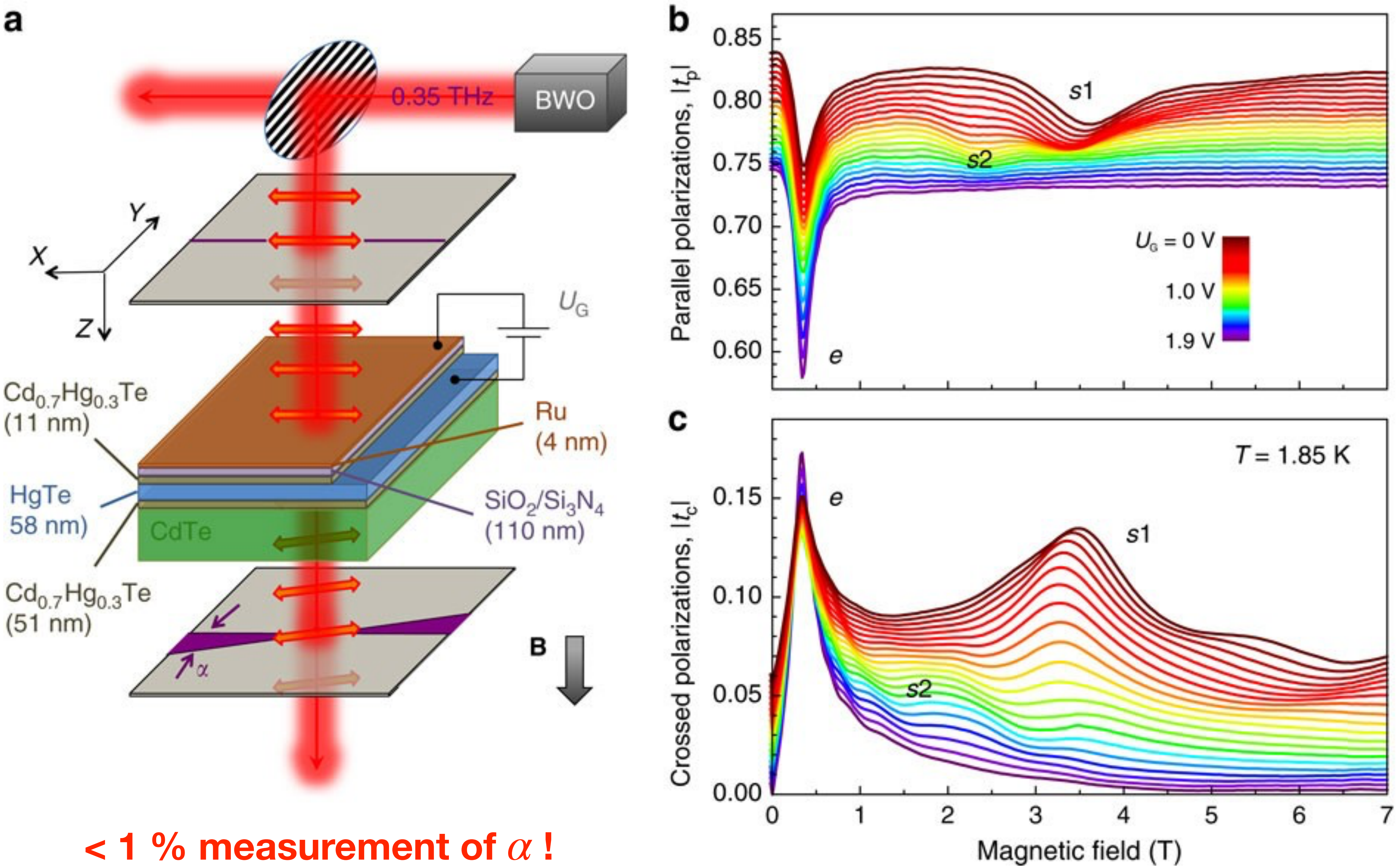
Vanishes for $\kappa_{\alpha\beta} \;\propto\; \delta_{\alpha\beta}$; seems complicated to analyze in general (but could be fruitful …)

Even for $\boldsymbol{E} \cdot \boldsymbol{B}$, one has surface effects:

Quantized Faraday and Kerr rotation and axion electrodynamics of a 3D topological insulator

Liang Wu[1,*,†], M. Salehi[2], N. Koirala[3], J. Moon[3], S. Oh[3], N. P. Armitage[1,*]

Science 02 Dec 2016:
Vol. 354, Issue 6316, pp. 1124-1127



< 1 % measurement of α !

Nice problem: Analyze how this is consistent with / implied by T.

One can analyze $E_1 B_1$ $(\kappa_{lm} = \delta_{l1}\delta_{m1})$ in a similar style to optical activity and the others …

Of course, one should also bring in asymmetric $\epsilon_{\alpha\beta} E_\alpha E_\beta$ and $\mu^{-1}_{\alpha\beta} B_\alpha B_\beta$, so things get complicated.

The direction as well as the speed for different polarizations differ.

There should be clean tests of T based on reversing the beam(s) …

(4) Mechanisms of Symmetry Breaking in Matter

Spontaneous, Induced, Competitive

We say that a symmetry of physical laws is broken in a system when the material does not remain the same under the symmetry operation.

Symmetry breaking can occur for many different reasons, including trivial ones (e.g. complex constructions, finiteness, presence of defects, …).

The interesting cases are when the reason for symmetry breaking is not trivial or obvious.

It is useful to distinguish three kinds of symmetry breaking: spontaneous, induced, competitive.

Spontaneous symmetry breaking is the most developed.

Examples: Crystal structure, ferromagnetism and other magnetic structures, ferroelectric structure**, superconductivity, …

General characteristics:
Possible in equilibrium****
Collective Field (order parameter)*
Infinite volume idealization
Stable against *local* perturbations, sensitive to appropriate *global* perturbations***

**The paradigmatic "microscopic" calculation of why spontaneous symmetry breaking occurs, in ferromagnetism*

$$H = J \sum_{i,\mathrm{neigbhbors}} \sigma_i \cdot \sigma_n$$ *Effective* interaction

1. Assume $\langle \sum_{\mathrm{neighbors}} \sigma_n \rangle = F$

2. Induce $\langle \sigma_i \rangle = m$

3. Calculate $F(F)$ using $\langle \sigma_n(F) \rangle = m$

4. Impose consistency $F(F) = F$

5. Check for energetic favorability (or metastability)

This is a theme that supports *many* variations …

*** *The yoga of the magnetic compass*

It is ***stable*** against all kinds of ***local*** perturbations, but especially ***sensitive*** to ***global*** symmetry-restoring perturbations.

(This is a slightly vague but more generally applicable version of Nambu-Goldstone phenomenology.)

**** *Dissipative structures (?)*

Many interesting cases of spontaneous symmetry breaking occur in non-equilibrium systems.

Some but not all of the ideas from equilibrium SSB carry over.

Induced symmetry breaking occurs when a system's environment violates the symmetry.

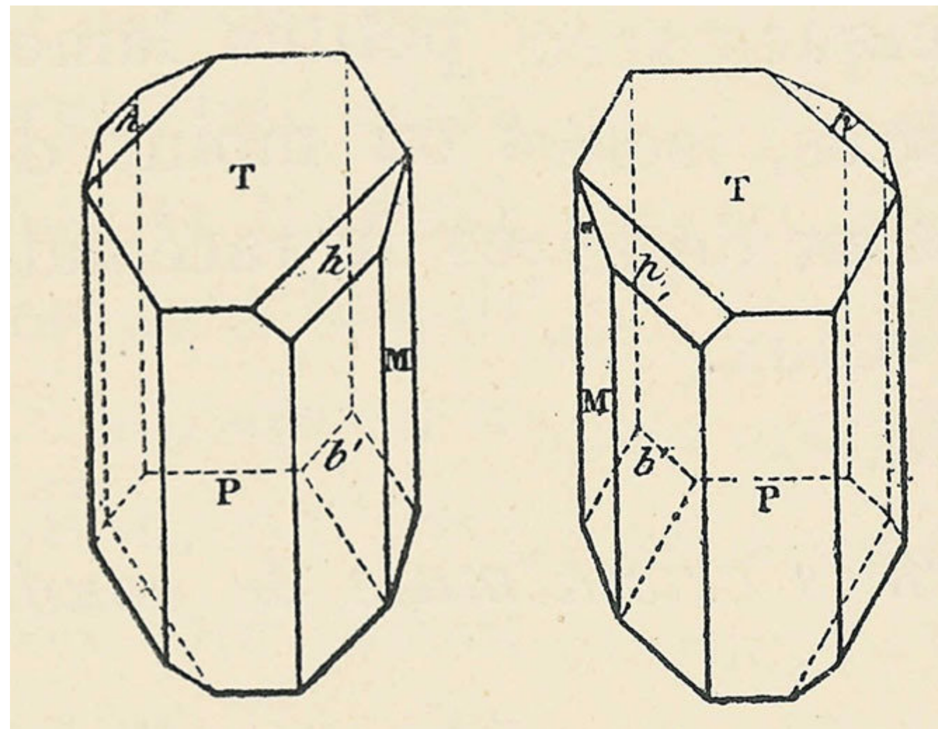
This shades into "no symmetry at all", but if the effect of the environment is structured one can still derive many consequences from the broken symmetry.

Examples:

- Rotation
- Magnetic field
- Irradiation
- Infiltration

Our example of electric dipole moments, and more generally the practical success of the idea that molecules have shapes, is a profound example of induced symmetry breaking.

Competitive symmetry breaking can occur in conditions of growth, for instance when establishing phase separation.



Given competition for common resources, one form prevails.

In repeated instances, either is equally likely.

Another nice example is driving on the right (or left) side of the road.

(5) Example of Scalar T Breaking in Matter

Multiferroics

Magnetism in general violates T, but there are often modified “locked” versions of T that are preserved.

In classic anti-ferromagnetism, we preserve symmetry under T combined with translation by one lattice spacing.

In classic ferromagnetism, we preserve symmetry under T combined with rotation by π around an orthogonal axis.

Landau-Ginzburg theory is basically an abstraction and codification of the molecular field concept we discussed earlier.

In that framework, let us suppose that we have electric and magnetic polarization fields P, M and an effective Hamiltonian that has a term

$$\propto -(P \cdot M)^2$$

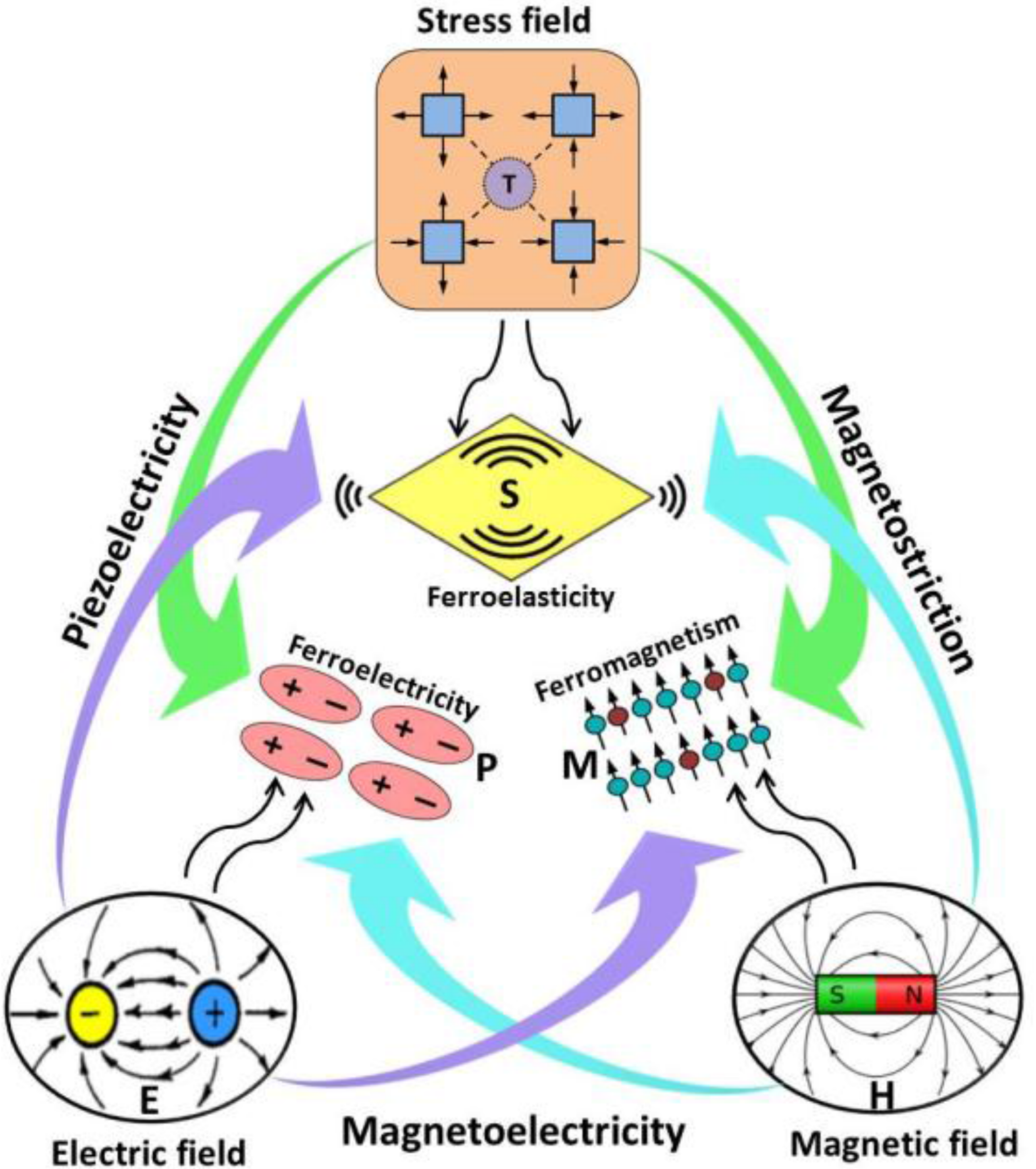
(which breaks no symmetry)

The $-(P \cdot M)^2$ term does not break any symmetry, in itself, but it will encourage the formation of correlations $P \parallel M$.

And the choice of one sign or the other breaks P and T.

To stabilize things we can include terms $\propto M^4, P^4$, or $(M \cdot P)^4$

Materials with both $\langle M \rangle, \langle P \rangle \neq 0$ are called multiferroics. Like magnetic ordering - but even more so - multiferroic ordering comes in many varieties.


Stress field
T
S
Ferroelasticity
Piezoelectricity
Magnetostriction
Ferroelectricity
Ferromagnetism
P
M
E
Electric field
H
Magnetic field
S
N
Magnetoelectricity