Quantum Field Theory on the Lattice

Z. Fodor

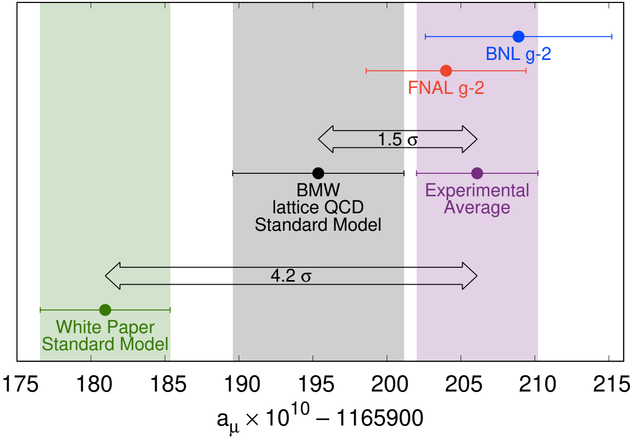
Two sets of lattice field theory talks

Michael Creutz: three talks
Zoltan Fodor: four talks

”computational details ... might be better for Zoltan to cover, i.e. things like hybrid monte carlo, the hadron spectrum ... g-2” and QCD thermodynamics.

- Scalar theory, Higgs bound & Monte Carlo
- QCD and hadron spectrum (Wilson)
- QCD thermodynamics (staggered & overlap)
- g-2 of the muon (staggered/Wilson/domain wall/overlap)

Tensions in $(g-2)_\mu$: take-home message



[Muon g-2 Theory Initiative, Phys.Rept. 887 (2020) 1-166]

[Budapest–Marseille–Wuppertal-coll., Nature (2021)]

[Muon g-2 coll., Phys. Rev. Lett. 126, 141801 (2021)]

Outline

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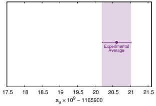
1. 
Experimental Average
17.5
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$a_\mu \times 10^9 - 1165900$

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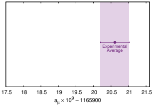
1. 
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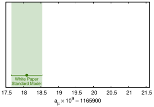
2. 
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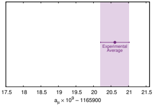
1. 
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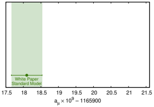
2. 
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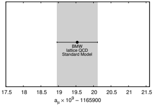
3. 
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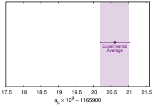
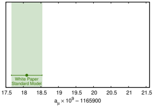
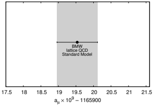
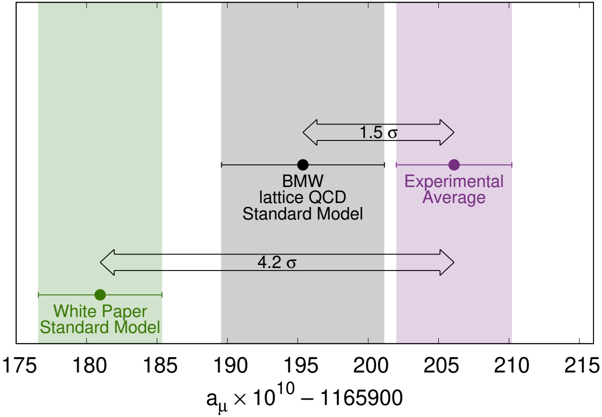
1. 

2. 

3. 

4. Summary



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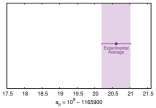
1. 
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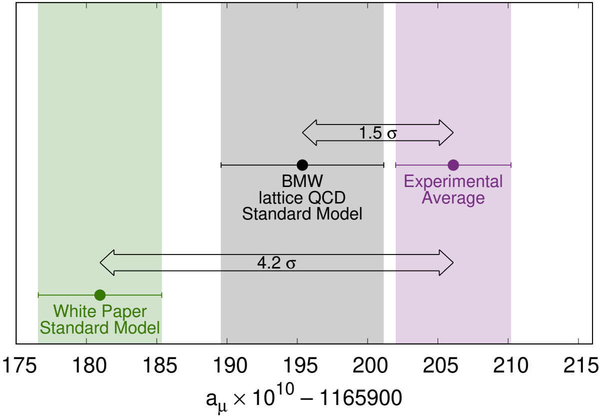
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- Newly announced result at Fermilab

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- Target uncertainty: (1.6)

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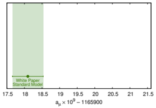
2. 
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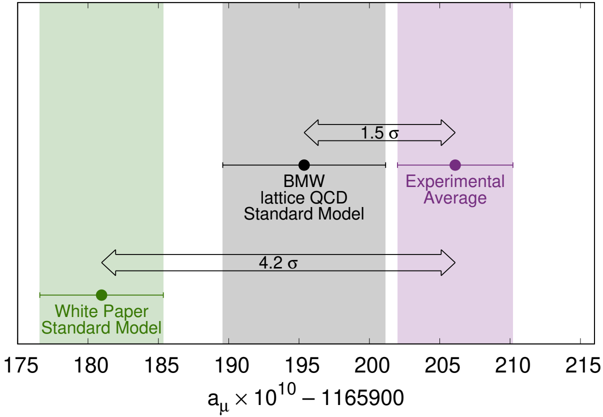
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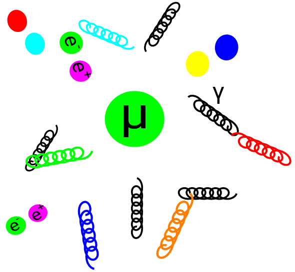
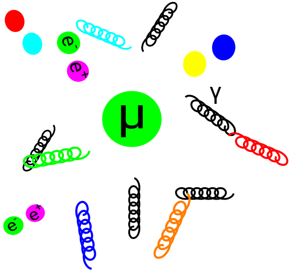
Theory: Standard Model



Sum over all known physics:

Theory: Standard Model



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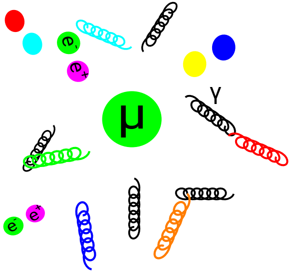

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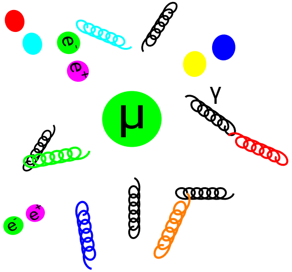

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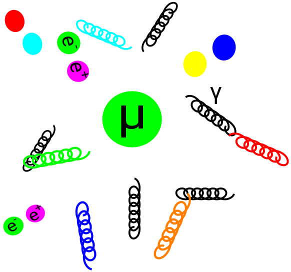

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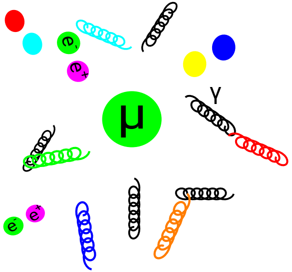

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	$a_\mu \times 10^{-10}$
QED	11658471.9(0.1)
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total	11659181.0(4.3)

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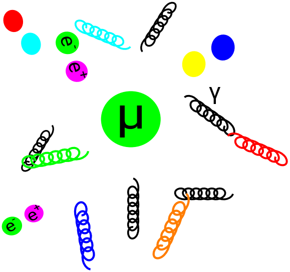

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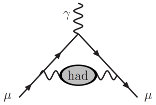
4.0 out of the 4.3 error comes from LO hadron vacuum polarisation

Hadronic contributions

- LO hadron vacuum polarization (LO-HVP, $(\frac{\alpha}{\pi})^2$)



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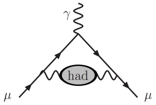

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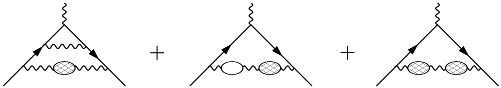

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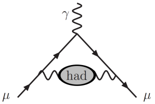

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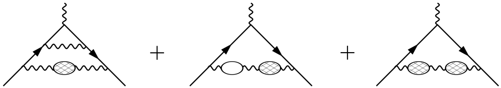
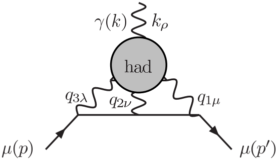


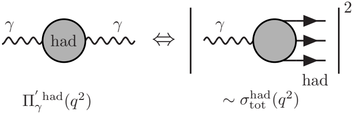
- Hadronic light-by-light (HLbL, $(\frac{\alpha}{\pi})^3$)



- pheno $a_\mu^{\text{HLbL}} = 9.2(1.9)$
  [Colangelo, Hoferichter, Kubis, Stoffer et al '15–'20]
- lattice a_μ^{HLbL}=7.9(3.1)(1.8) or 10.7(1.5)
  [RBC/UKQCD '19 and Mainz '21]

HVP from R-ratio

- Optical theorem



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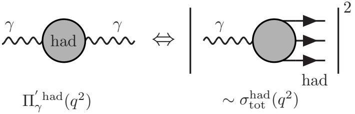

Use $e^+e^- \rightarrow$ had data of CMD, SND, BES, KLOE, BABAR, ...
systematics limited

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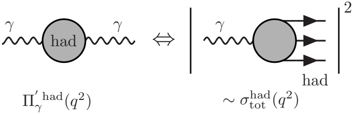

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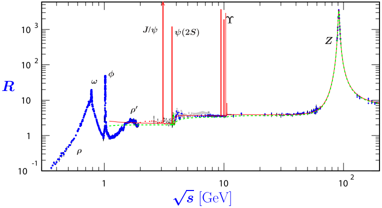

$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{ds}{s^2} K_\mu(s) R(s)$$

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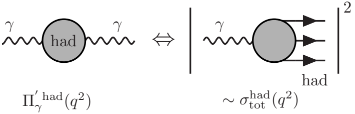

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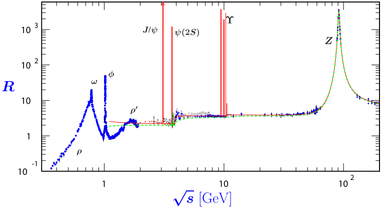

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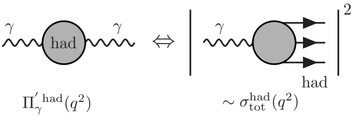

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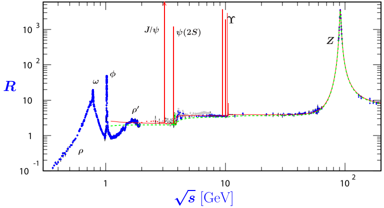

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Systematic uncertainty: ≈4 times larger than the statistical error (e.g. Davier et al.)

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CMD3 [2302.08834] $e^+e^- \to \pi^+\pi^-$ for $\sqrt{s}$: 0.60–0.88 GeV

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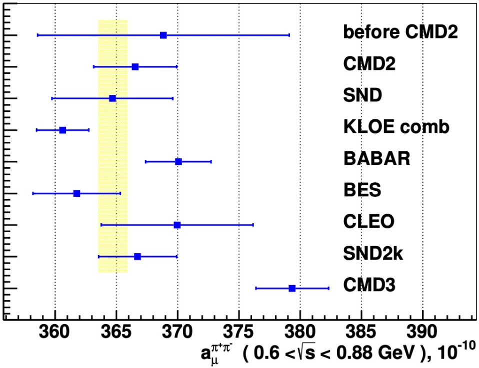

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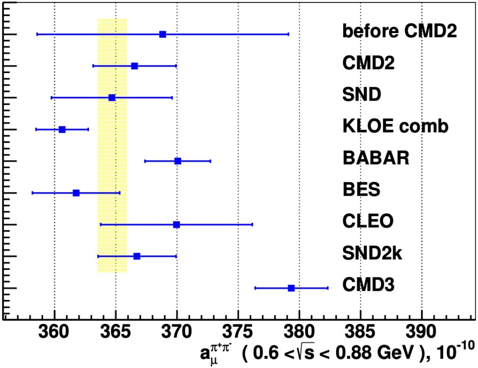

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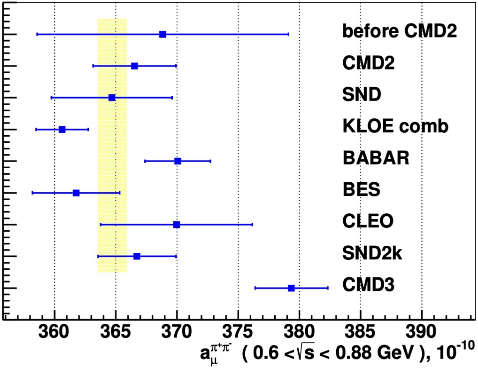

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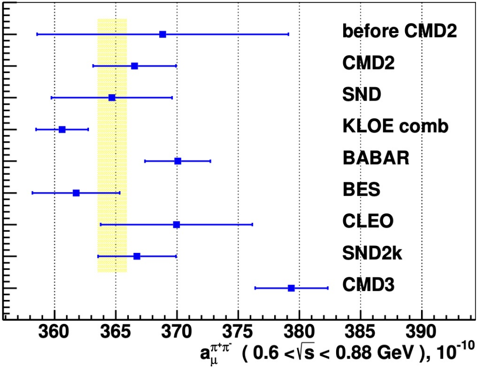

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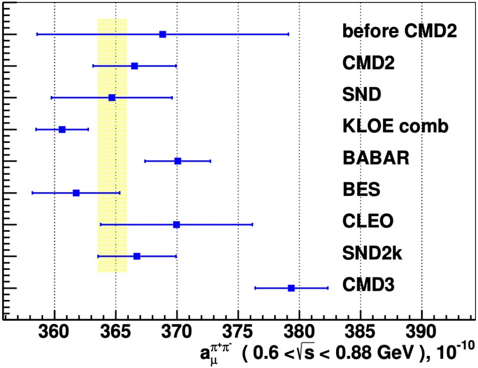

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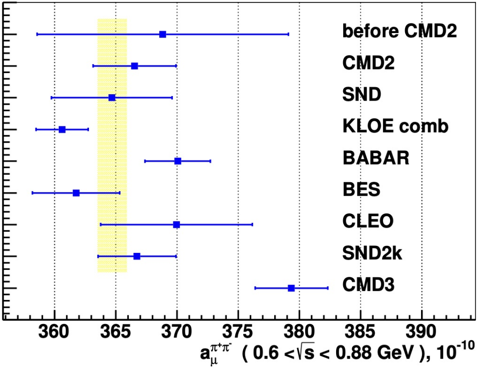

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White Paper must further inflate errors: less chance for new physics?

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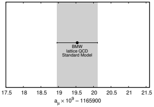
3. 
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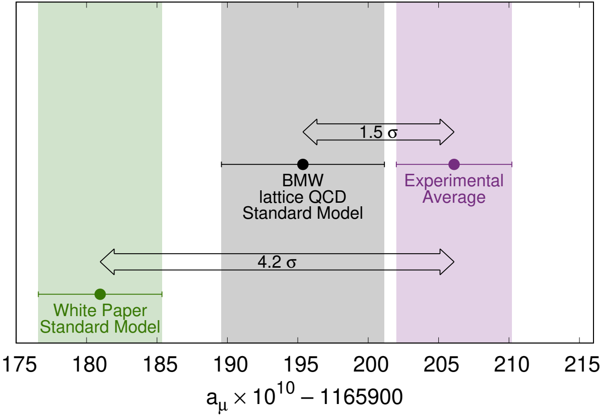
1.5 σ
BMW
lattice QCD
Standard Model
Experimental
Average
4.2 σ
White Paper
Standard Model
175 180 185 190 195 200 205 210 215
$a_\mu \times 10^{10} - 1165900$

$a_\mu^{\text{LO-HVP}}$ from lattice QCD Nature 593 (2021) 7857, 51

- Compute electromagnetic current-current correlator

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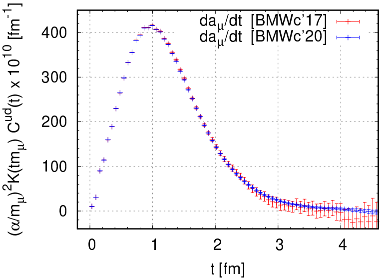
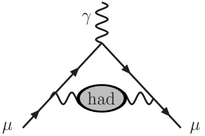
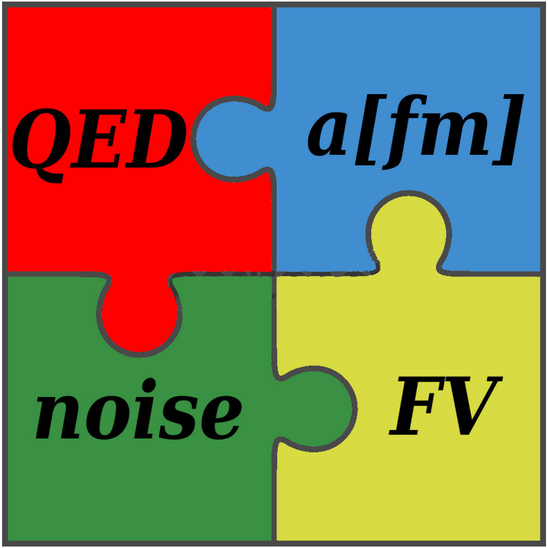
$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty dt\, K(t)\, C(t)$$



$K(t)$ describes the leptonic part of diagram



New challenges



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2. For separation of isospin breaking effects: w_0 scale setting
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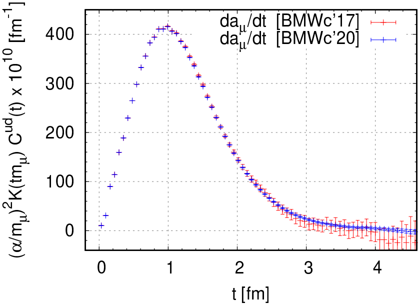
   - No experimental value
     $\longrightarrow$ Determine value of w_0 from $M_\Omega \cdot w_0$

$$w_0 = 0.17236(29)(63)[70] \text{ fm}$$

Noise reduction

- noise/signal in $C(t) = \langle J(t)J(0) \rangle$ grows for large distances



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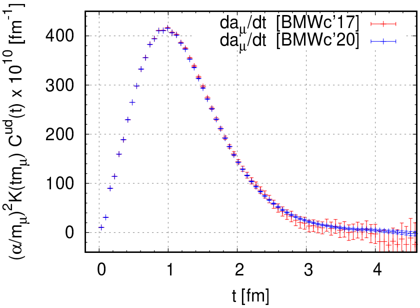

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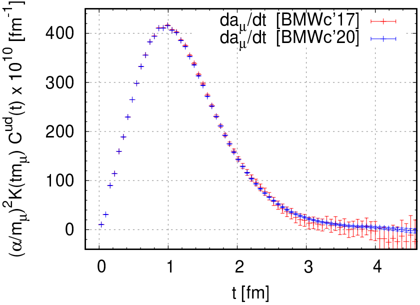

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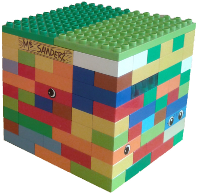

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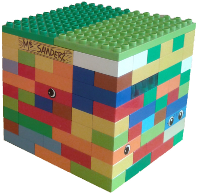

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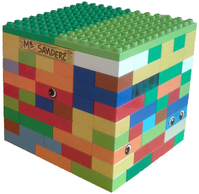

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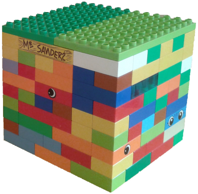
$L_{\text{ref}} = 6.272\,\text{fm}$   $L_{\text{big}} = 10.752\,\text{fm}$

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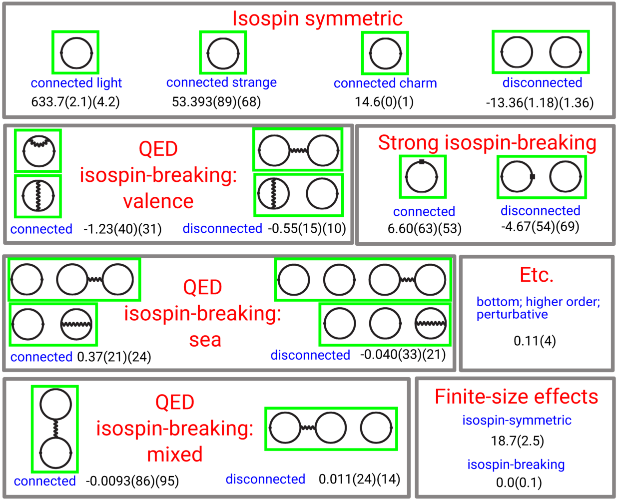
2. $a_\mu(\infty) - a_\mu(\text{big})$
   - use models for remnant finite-size effect of “big” $\sim 0.1\%$

Isospin breaking effects

- Include leading order IB effects: $\mathcal{O}(e^2)$, $\mathcal{O}(\delta m)$



$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

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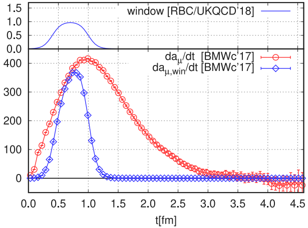
[RBC/UKQCD'18]



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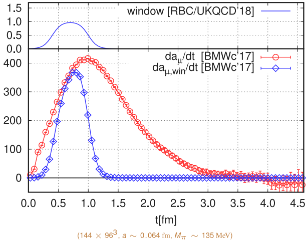

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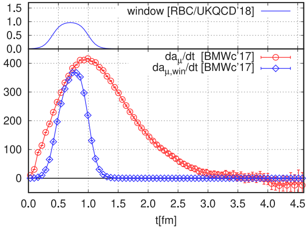

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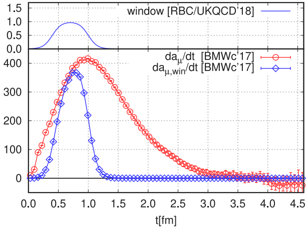

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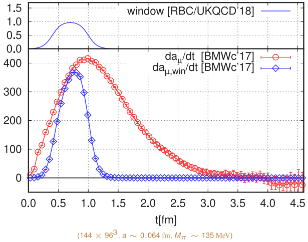

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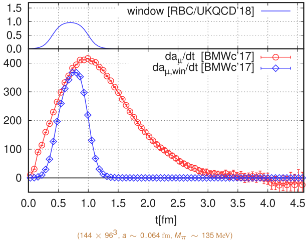

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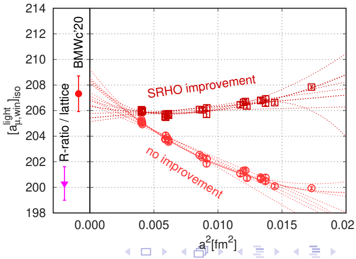
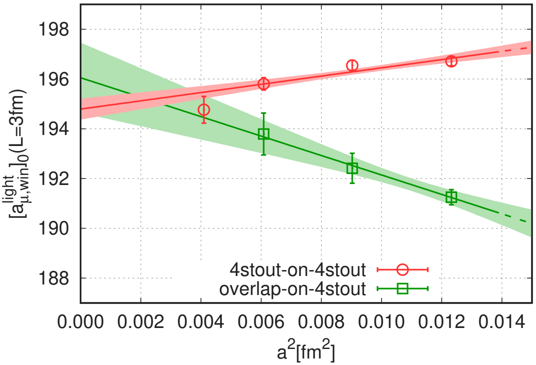


Crosscheck – overlap



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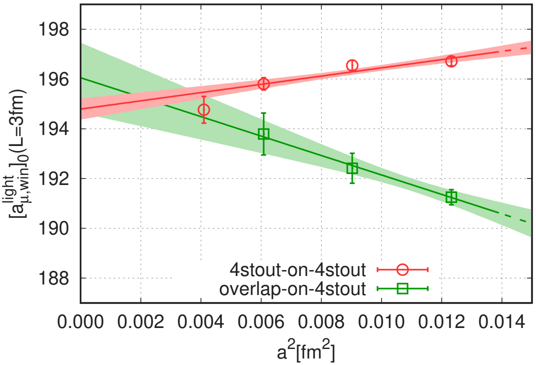
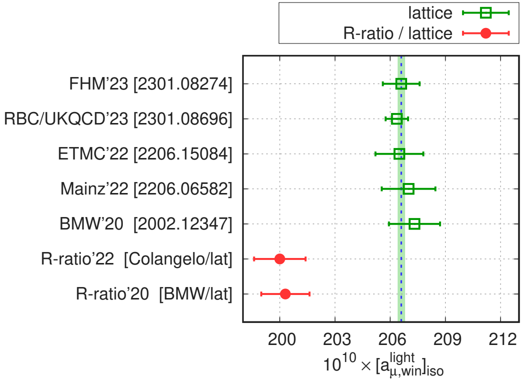


- compute $a_{\mu,\text{win}}$ with overlap valence
- local current instead of conserved $\longrightarrow$ had to compute Z_V
- cont. limit in $L = 3\,\text{fm}$ box consistent with staggered valence

Tension in the window observables



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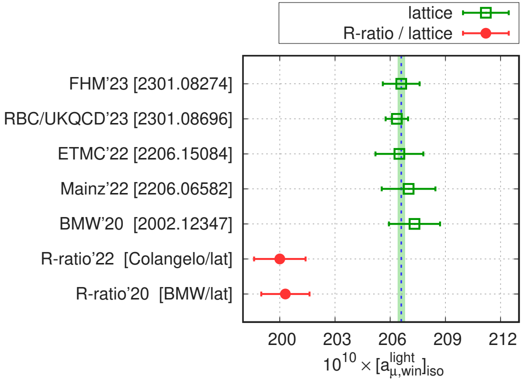

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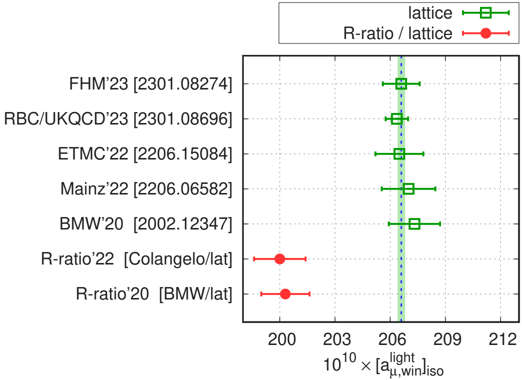

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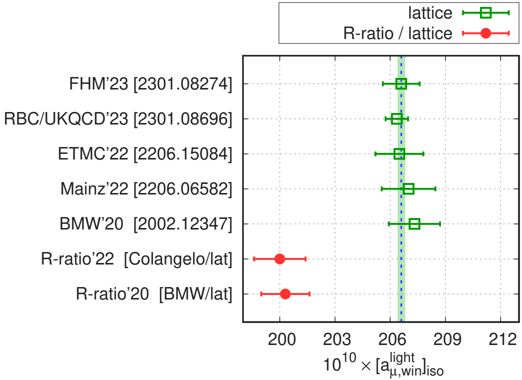

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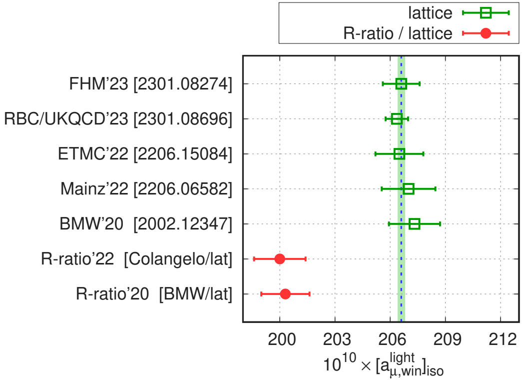

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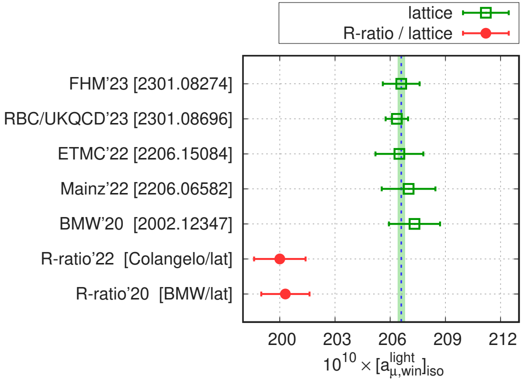

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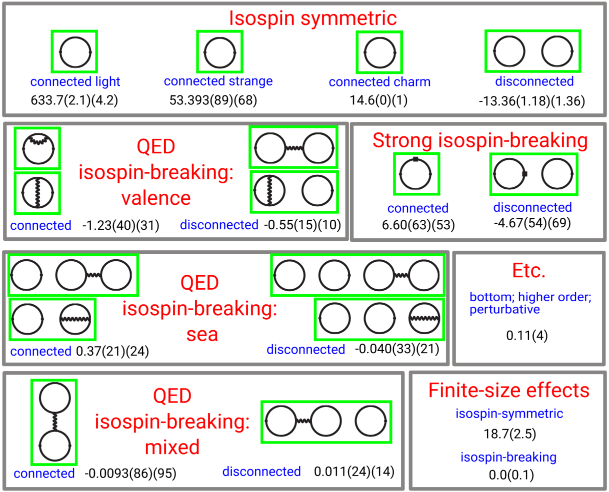
either new physics
or underestimated errors

Outline

Final result



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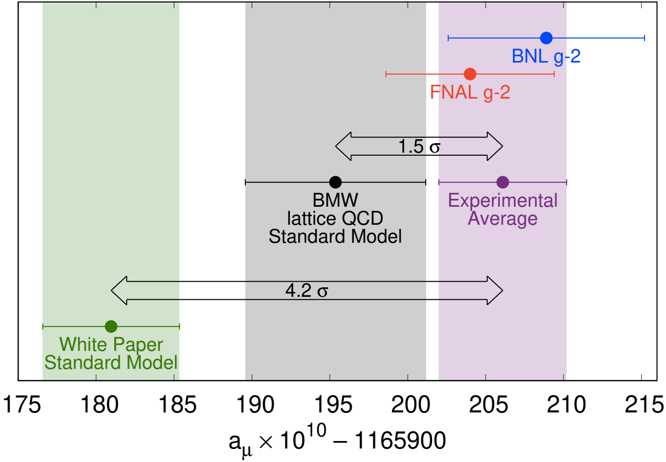
Tension: take-home message #1 full g-2

Systematic/statistical error ratios: lattice ≈2; R-ratio ≈4



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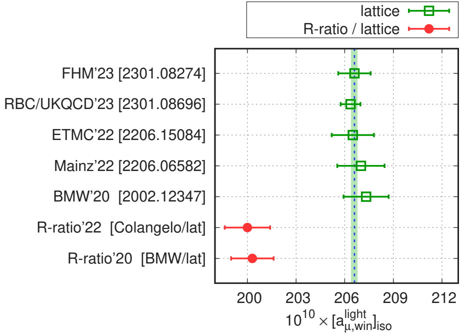

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