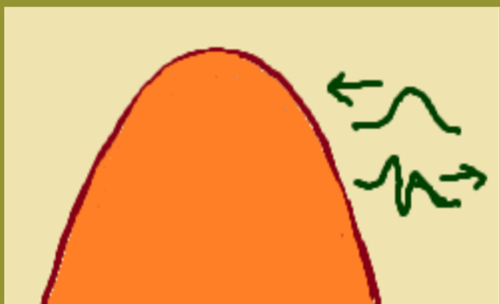
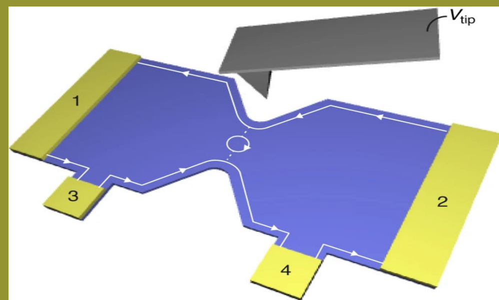
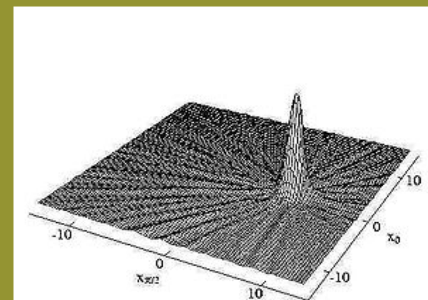
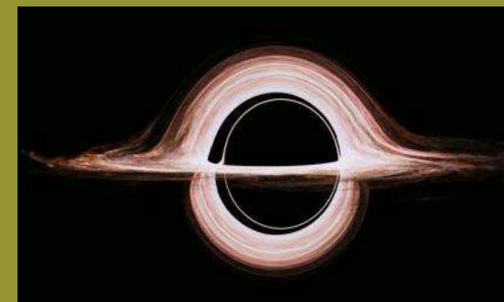






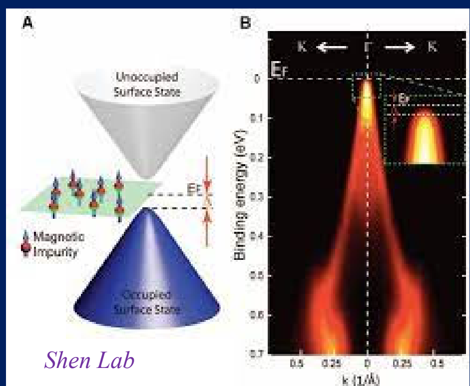


Quantum Connections: From Quantum Hall Physics to Optics To Blackholes

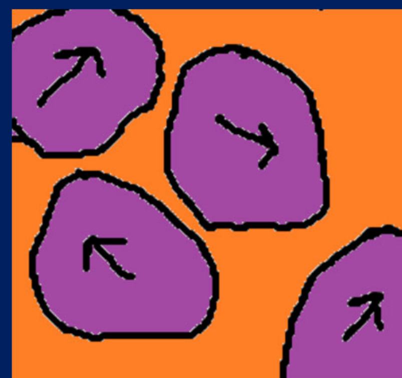
Smitha Vishveshwara, University of Illinois at Urbana Champaign

Quantum Connections Series, Summer School In Sweden, 22 June 2023

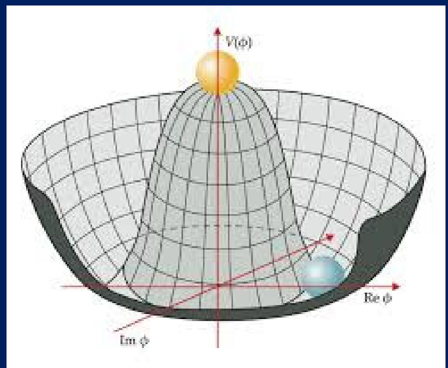
Universal Physics Across Scales



Dirac and Majorana Fermions
Quantum Materials



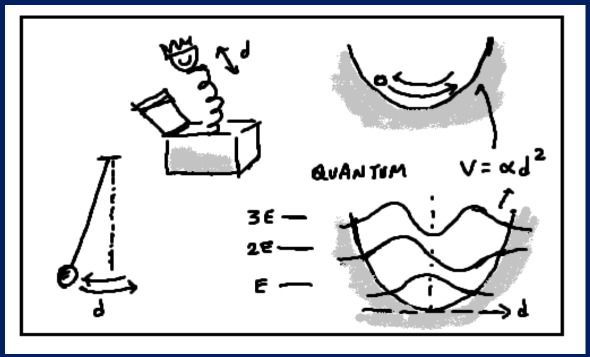
Kibble-Zurek Physics
Cosmic structure formation
Vortices in Helium



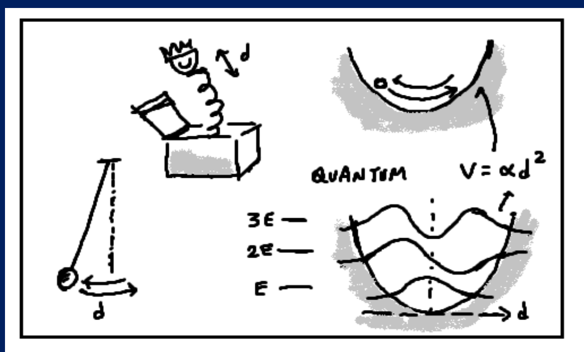
Higgs-Anderson mechanism
Elementary Particles
Superconductors

Powerful Symmetry Considerations; E.g. Noether's Theorem

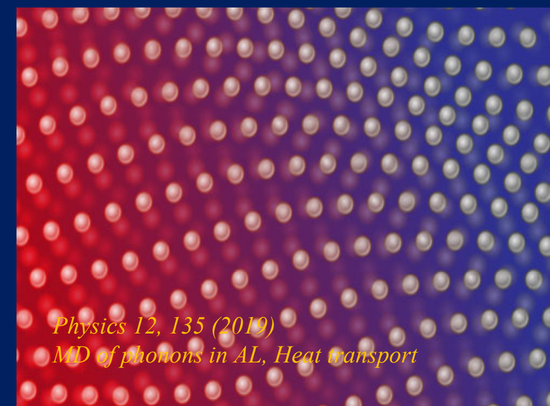
Simple Models---Simple Harmonic Oscillator



Simple Models---Simple Harmonic Oscillator

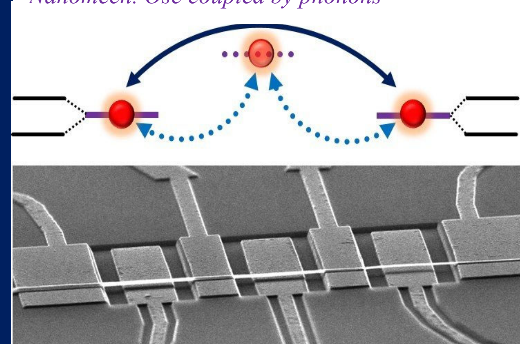


Trapped particles, Nanomechanics,
Phonons, plasmons, other excitations..
Quantum Hall physics, and more….

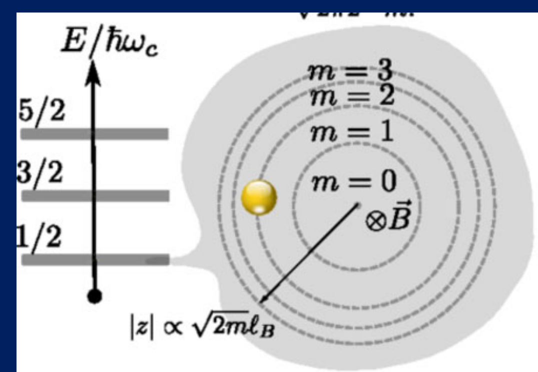


Physics 12, 135 (2019)
MD of phonons in AL, Heat transport

classicalmpr.org

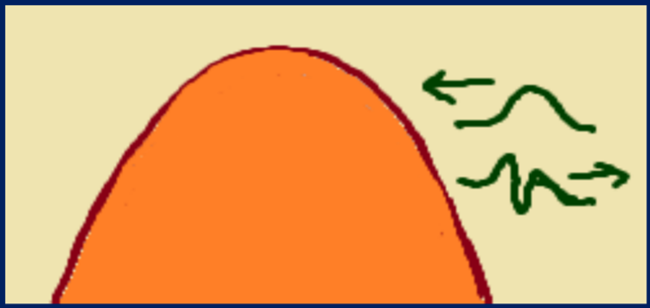


Luo et al, Nat. Comm (2018)
Nanomech. Osc coupled by phonons

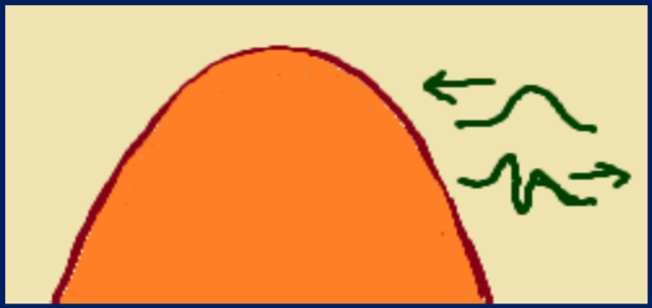


Lee, Papic, Thomale, PRX(2015)

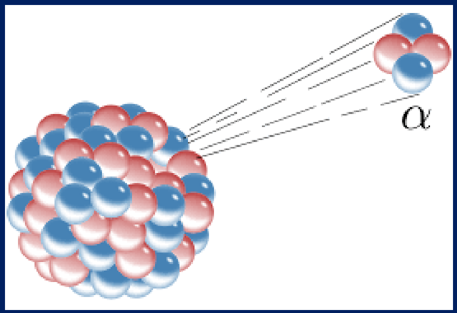
Inverted Harmonic Oscillator (IHO)

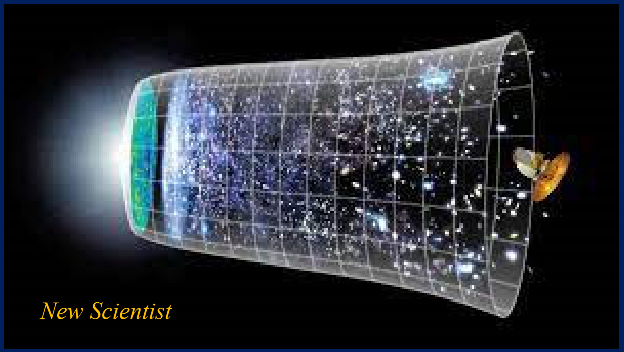


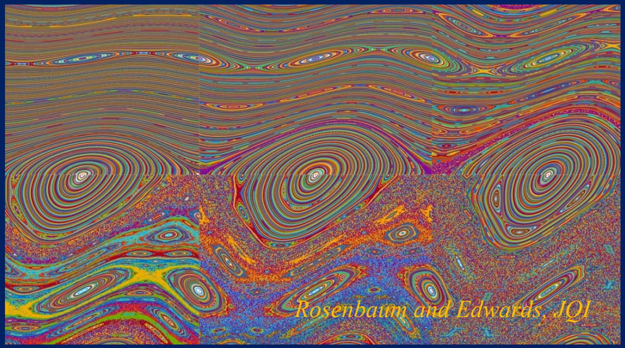
Inverted Harmonic Oscillator (IHO)



"Quantum Escapades" (since early 1900's)
Nuclear decay, Scattering, Cosmology
Activation, Atomic cooling, Quantum Chaos….





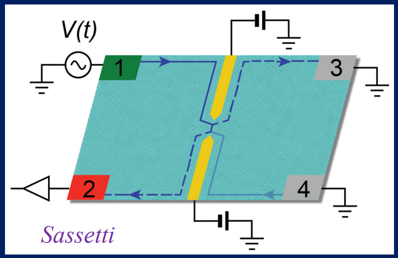


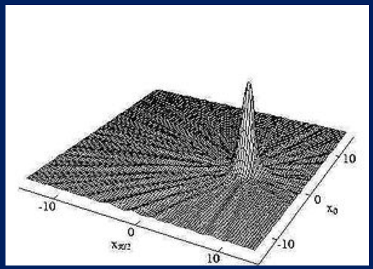
Today's Explorations

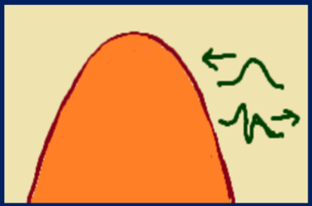
Quantum Hall Point Contact

Coherent States
Squeezing

Quantum Optics



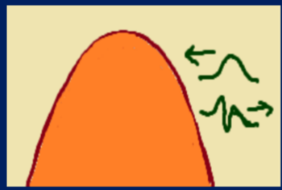




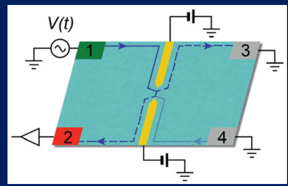
Hawking-Unruh radiation
Quasinormal Modes

Black Hole Dynamics

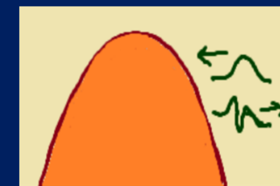
In what follows—Part I



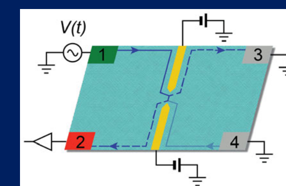
- Quantum Hall Physics
  - Basics, Point Contact, Saddle Potential, Tunneling,



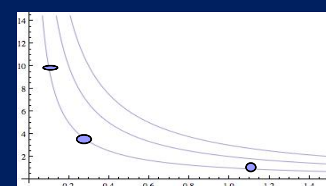
In what follows—Part I

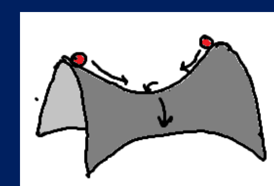


- Quantum Hall Physics
  Basics, Point Contact, Saddle Potential, Tunneling,

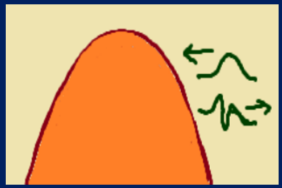


- Quantum Optics Parallels
  Coherent States, Squeezing

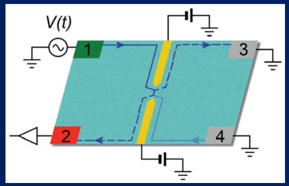




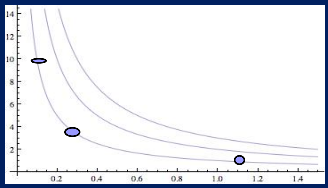
In what follows—Part I

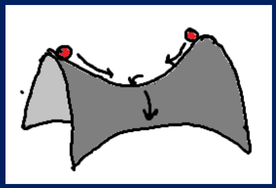


- Quantum Hall Physics
  Basics, Point Contact, Saddle Potential, Tunneling

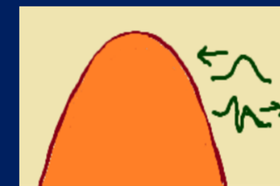


- Quantum Optics Parallels
  Coherent States, Squeezing

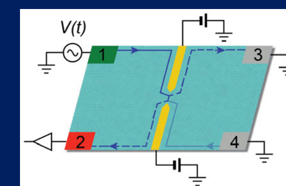




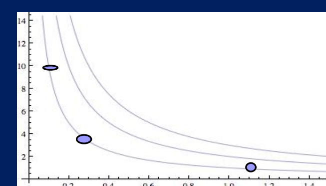
In what follows—Part II

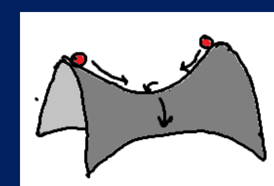


- Quantum Hall Physics
  Basics, Point Contact, Saddle Potential, Tunneling,
  ANYONS

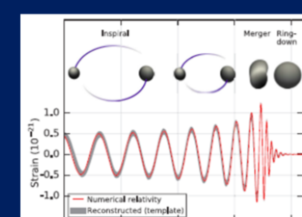


- Quantum Optics Parallels
  Coherent States, Squeezing

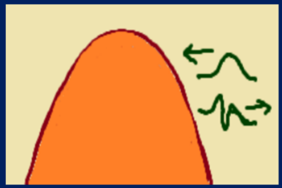




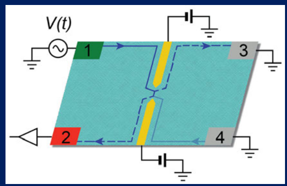
- Black hole Dynamics
  Hawking radiation, Quasinormal Modes



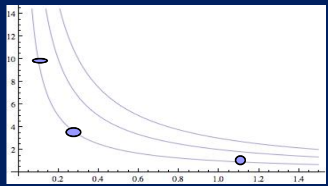
In what follows—Part II

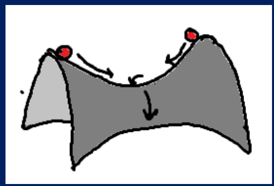


- Quantum Hall Physics
  - Basics, Point Contact, Saddle Potential, Tunneling,
  - ANYONS

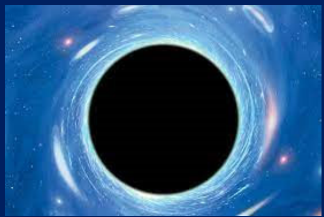


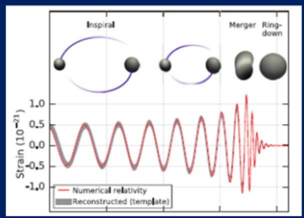
- Quantum Optics Parallels
  - Coherent States, Squeezing





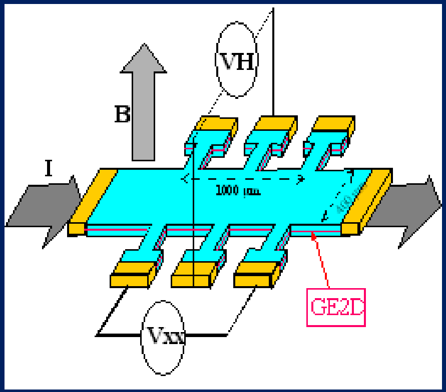
- Black hole Dynamics
  - Hawking radiation, Quasinormal Modes

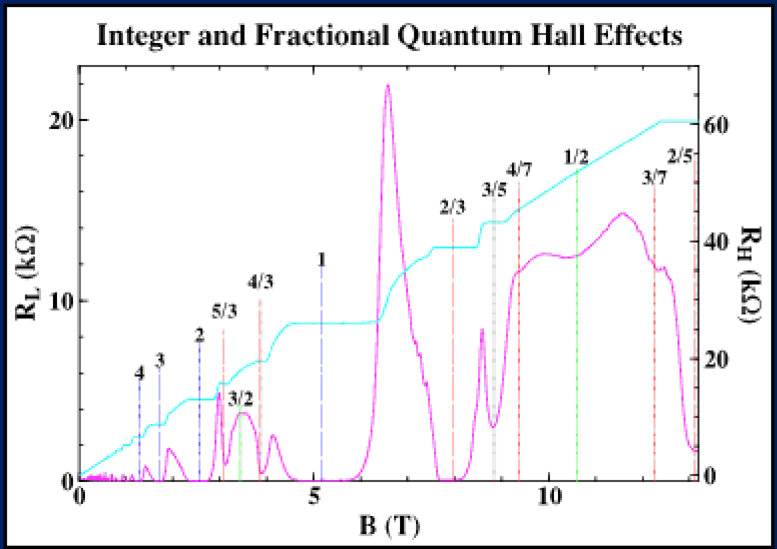




Quantum Hall system

Charged particles in 2D subject to high magnetic fields



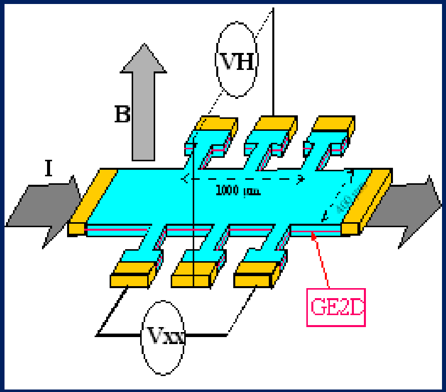


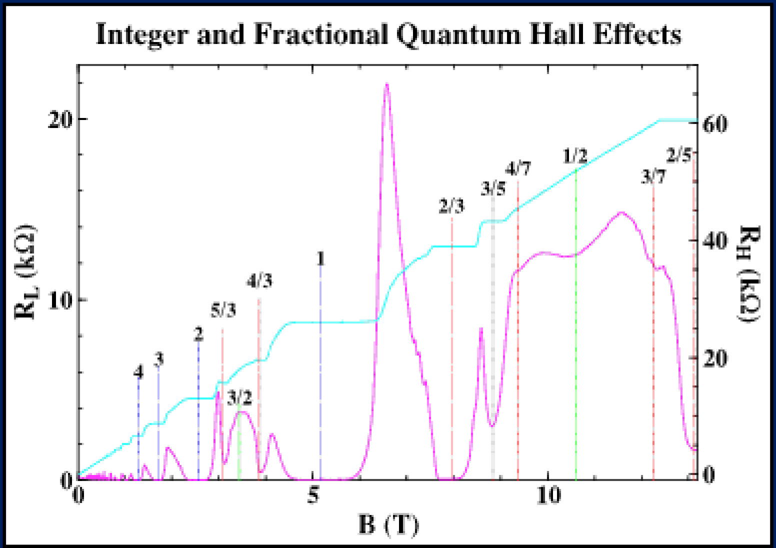
For e.g.,Tsui, Stormer, Gossard PRL 48 (1982)

Quantized Hall Conductance – $\nu e^2/h$
Precise measure related to universal constants

Quantum Hall system

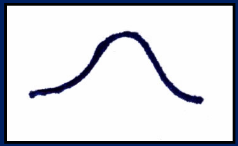
Charged particles in 2D subject to high magnetic fields



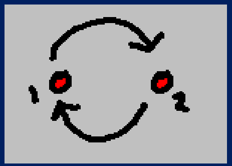


For e.g.,Tsui, Stormer, Gossard PRL 48 (1982)

Fractional quasiparticles



$e^* = \nu e$

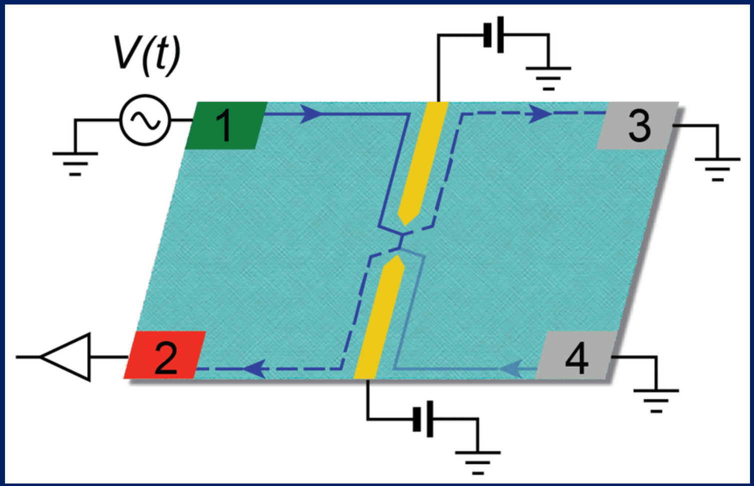


$e^{\pm i\pi\nu}$

Quantized Hall Conductance – $\nu\, e^2/h$
Precise measure related to universal constants

Quantum Hall Point Contacts (QPC)

Constrictions which connect opposite edges of the Hall bar



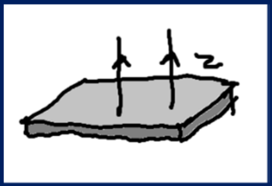
- **Quantum Tunneling across QPC**
- **Probe of fractional charge:** Shot noise
- **Probe of fractional statistics (anyons)**: Two-particle correlations

Fractional Charge: E.g. Kane and Fisher, (1994); Saminadayar et al., (1997), R de-Picciotto et al., (1997)
Fractional Statistics: Bartolomei at al, Science (2020); Nakamura et al, Nature Physics (2020)

Quantum Hall Basics

Charged particles in 2D subject to high magnetic fields

$$H = \frac{1}{2m}\left(p_x + \frac{qB}{c}y\right)^2 + \frac{1}{2m}\left(p_y - \frac{qB}{c}x\right)^2$$



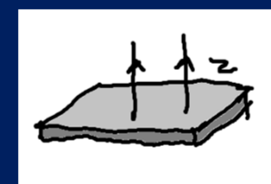
Symmetric gauge

Magnetic length $\ell_B = \sqrt{\hbar c/(qB)}$

Quantum Hall Basics

Charged particles in 2D subject to high magnetic fields

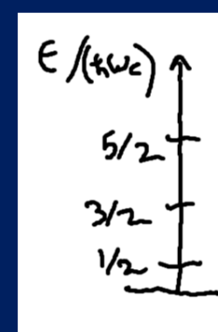
$$H = \frac{1}{2m}\left(p_x + \frac{qB}{c}y\right)^2 + \frac{1}{2m}\left(p_y - \frac{qB}{c}x\right)^2$$



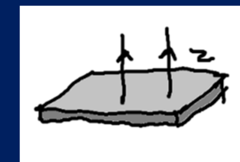
Symmetric gauge

Magnetic length $\ell_B = \sqrt{\hbar c/(qB)}$

- *Choose appropriate conjugate variables*
- *Map to 1D simple harmonic oscillator, Landau levels*
- *Each Landau level—infinite degeneracy*



Quantum Hall Basics



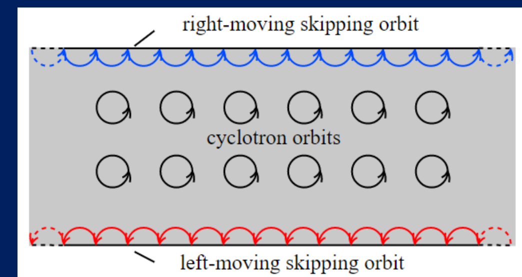
Charged particles in 2D subject to high magnetic fields

$$\mathcal{H} = \frac{1}{2m}\Big(\underbrace{p_x + \frac{qB}{c}y}_{\Pi_x}\Big)^2 + \frac{1}{2m}\Big(\underbrace{p_y - \frac{qB}{c}x}_{\Pi_y}\Big)^2$$

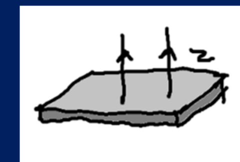
Symmetric gauge

Magnetic length

$$\ell_B = \sqrt{\hbar c/(qB)}$$



Quantum Hall Basics



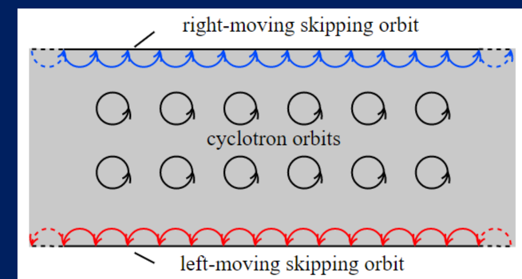
Charged particles in 2D subject to high magnetic fields

$$H = \frac{1}{2m}\Big(\underbrace{p_x + \frac{qB}{c}y}_{\pi_x}\Big)^2 + \frac{1}{2m}\Big(\underbrace{p_y - \frac{qB}{c}x}_{\pi_y}\Big)^2$$

Symmetric gauge

Magnetic length

$$\ell_B = \sqrt{\hbar c/(qB)}$$



$$a = \frac{\ell_B}{\sqrt{2\hbar}}(\pi_x - i\pi_y), \quad a^\dagger = \frac{\ell_B}{\sqrt{2\hbar}}(\pi_x + i\pi_y)$$

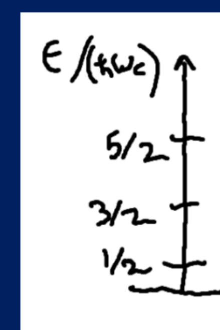
$$[a, a^\dagger] = 1$$

$$H = \hbar\omega_c\left(a^\dagger a + \frac{1}{2}\right), \quad \omega_c = \frac{eB}{m}$$

Compare with 1D SHO

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$$

$$H_{SHO} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 = \hbar\omega\left(a_- a_+ - \frac{1}{2}\right)$$

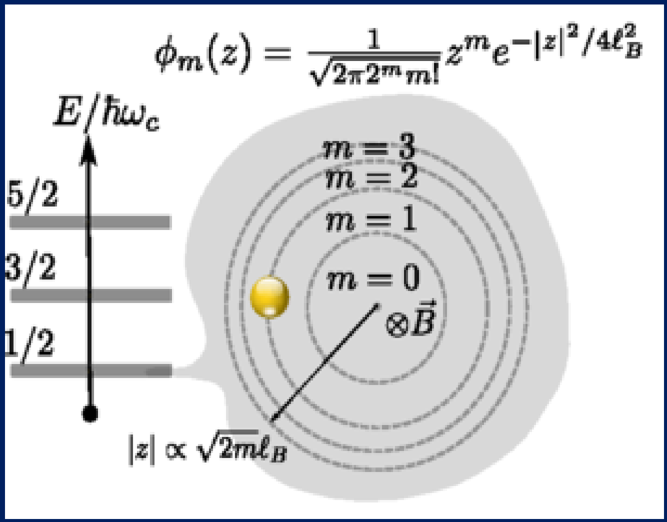


Lowest Landau Level (LLL)

$$H = \frac{(p_i - eA_i)^2}{2m} = \frac{\vec{\Pi}^2}{2m} = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

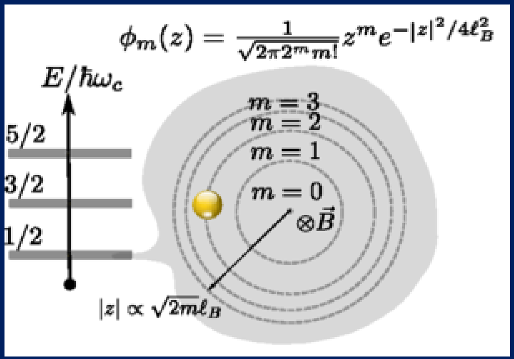
Projecting to the lowest Landau level: $\Pi_i \to 0, B \to \infty, n \to 0$ ...

Symmetric gauge:
Degenerate states---
eigenstates of angular momentum



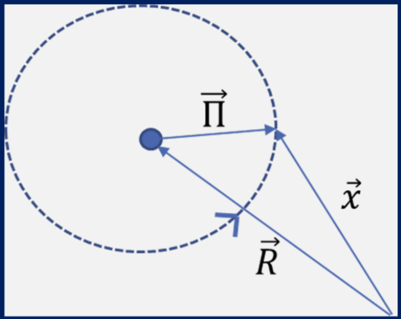
Lowest Landau Level (LLL)

Symmetric gauge:
Degenerate states---
eigenstates of angular momentum



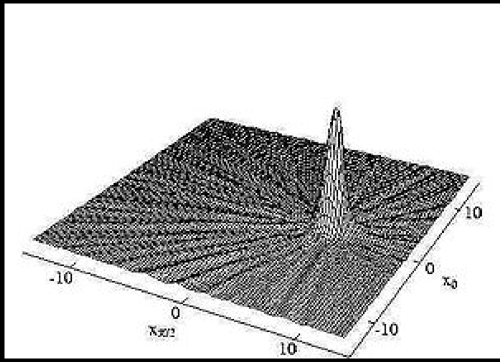
X and Y coordinates do not commute

$$[R_y, R_x] = [y, x]_{LLL} = i l_B^2$$



Projecting to the lowest Landau level: $\Pi_i \to 0, B \to \infty, n \to 0$ ...

Parallels with Quantum Optics

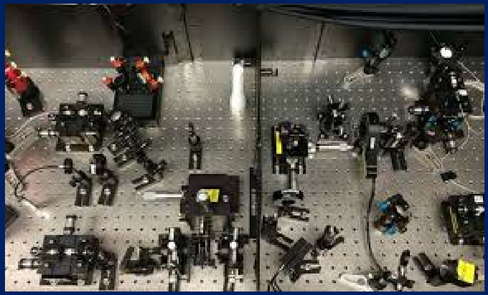


Quantum Optics Parallels– Quantum Uncertainty

Uncertainty in conjugate quantities:

- Position-Momentum (also SHO)
- Number-Phase (single-mode photons)

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$



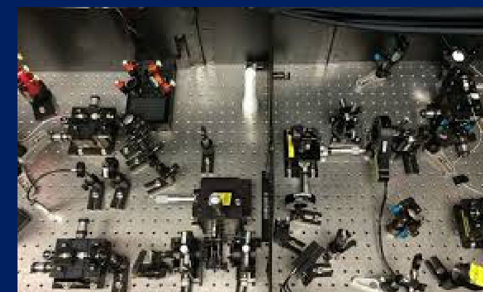
Quantum Hall:

- X-Y
- Angular Momentum-Phase

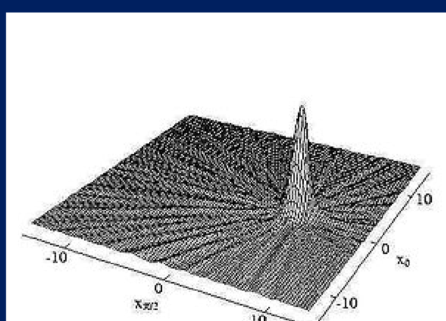
Quantum Optics Parallels– Quantum Uncertainty

Uncertainty in conjugate quantities:
- Position-Momentum (also SHO)
- Number-Phase (single-mode photons)

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$



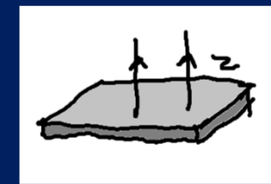
Quantum Hall:
- X-Y
- Angular Momentum-Phase



Coherent States:
Respect Minimum uncertainty
Superposition of fixed number states

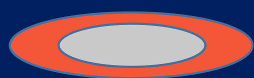
$$b|\alpha\rangle = \alpha|\alpha\rangle$$

Quantum Hall LLL Coherent States



Angular momentum eigenstates

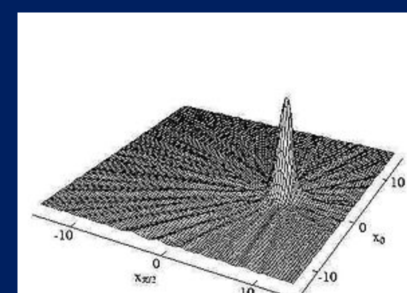
$$\hat{L}\,|n\rangle \;= n\hbar\,|n\rangle,$$



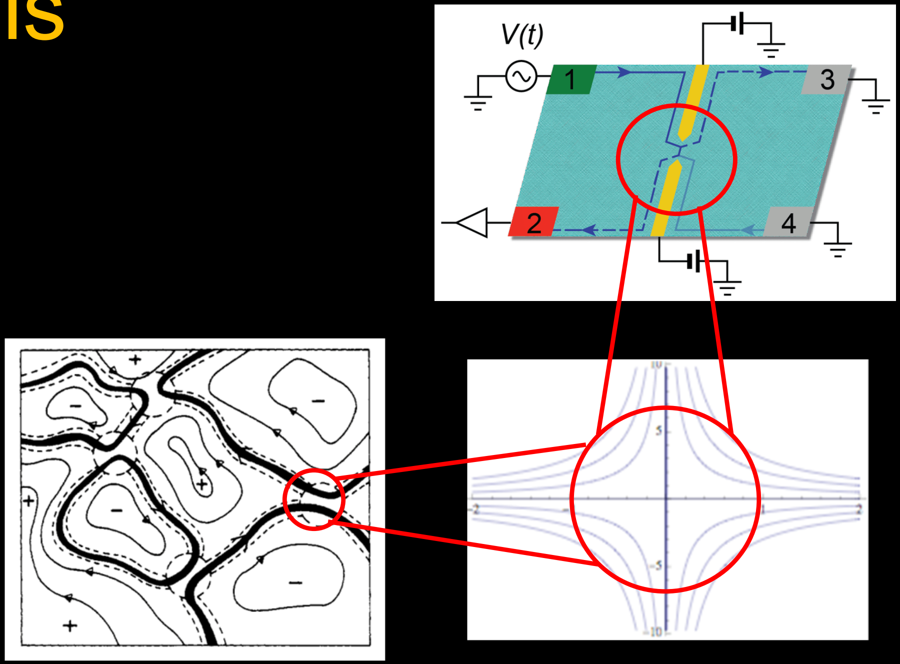
Coherent States—localized superpositions

$$|Z\rangle \;= e^{-|Z|^2/2}\sum_{n=0}^{\infty}\frac{(Z^*)^n}{\sqrt{n!}}|n\rangle$$

Coherent States:
Respect Minimum uncertainty
Here, in real space---centered around Z;
Also, no dynamics (unlike SHO and photons)



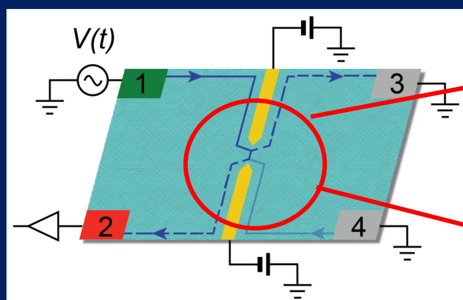
Quantum Hall Tunneling and Saddle Potentials

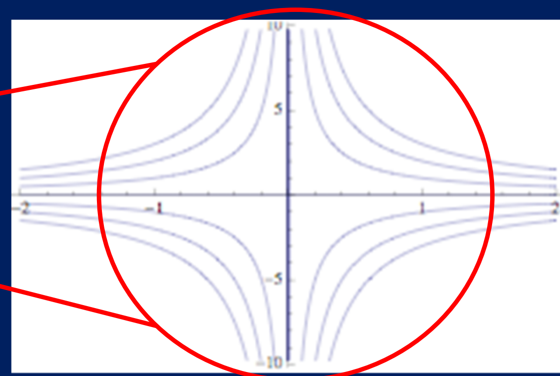


QPC

Disorder

Point contact, saddle potential, beam-splitter





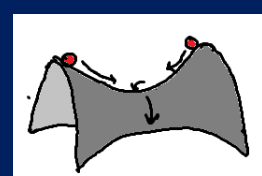
Sassetti

Quantum Hall (QH)
Point contact

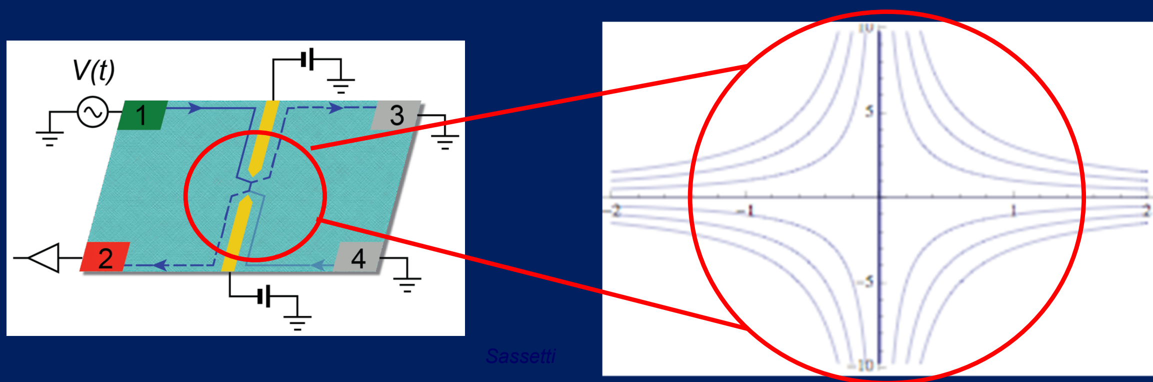
$$H_s = U x y$$

$$H_s' = U'(y^2 - x^2)$$

Saddle potential



Point contact, saddle potential, beam-splitter

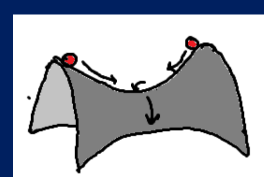


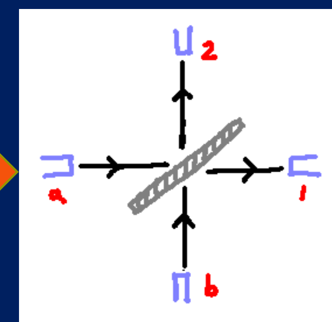
Quantum Hall (QH)
Point contact

$$H_s = Uxy$$

$$H_s' = U'(y^2 - x^2)$$

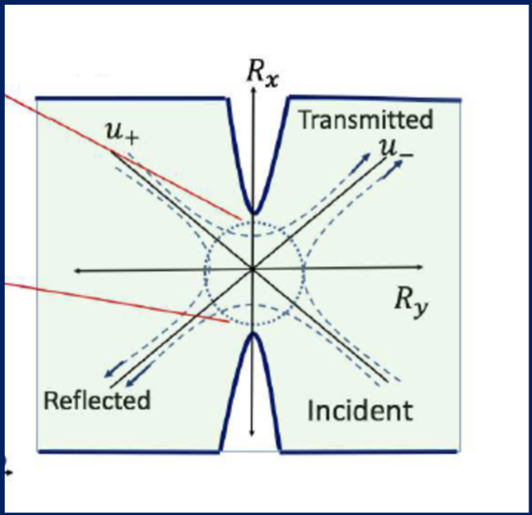
Saddle potential





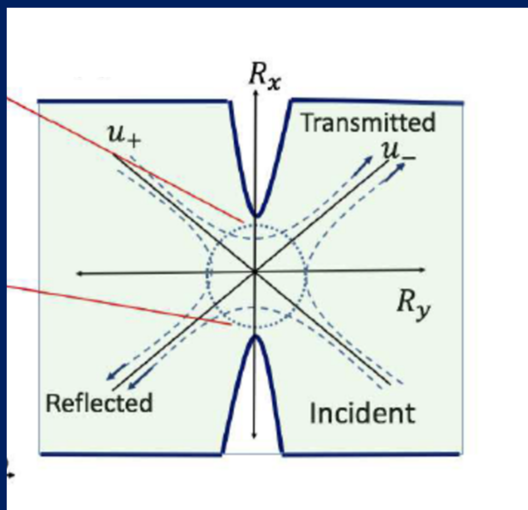
Beam splitter
Quantum optics, QH, and more

Tunneling and Inverted Harmonic Oscillator



Tunneling across saddle
In-coming and out-going related by
Bogoliubov Transforms (cosh, sinh)

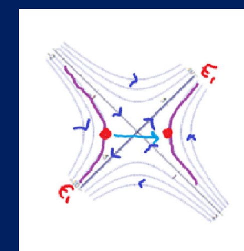
Saddle potential and Inverted Harmonic Oscillator



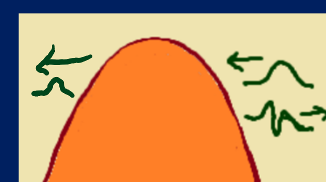
Tunneling across saddle
In-coming and out-going related by
Bogoliubov Transforms (cosh, sinh)

X and Y coordinates do not commute

$$H_s' = U'(y^2 - x^2)$$



$$H_{IHO} = 2\hbar_{eff} U' (\tilde{p}^2 - \tilde{x}^2/4)$$



Saddle potential—Ubiquitous—disorder landscape; area-preserving shear deformations

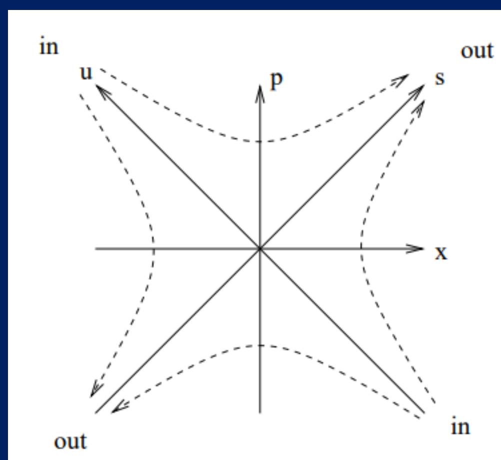
Fertig & Halperin, (1987); Subramanyan et al (2021)

Properties of the Inverted Harmonic Oscillator

$$\hat{H} = \frac{\hat{p}^2}{2} - \frac{\hat{x}^2}{2}$$

1) Continuous Real Spectrum
2) Discrete Imaginary Spectrum

Scattering States
Continuous Real Spectrum

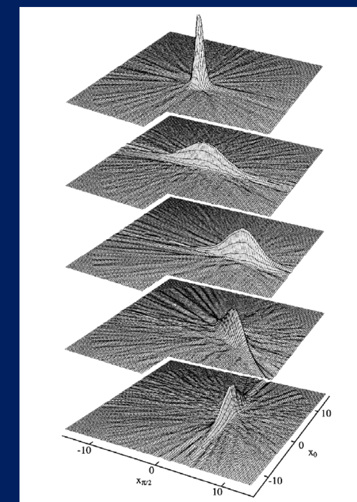


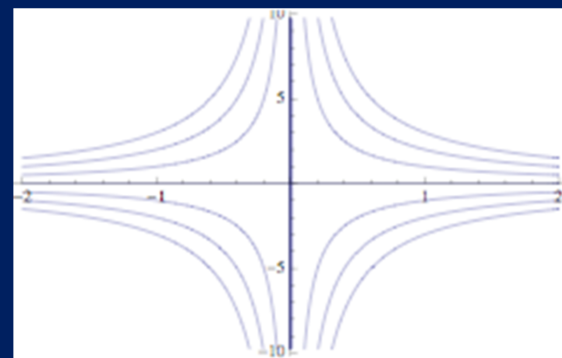
$$H = \frac{p^2}{2} - \frac{x^2}{2} = \frac{1}{2}(su + us) = i\left(u\partial_u + \frac{1}{2}\right) = -i\left(s\partial_s + \frac{1}{2}\right)$$

$$E_n^{\pm} = \mp i(n + \frac{1}{2})$$

Resonant/Quasinormal Modes
Discrete Imaginary Spectrum

Saddle potential and squeezing (optics)

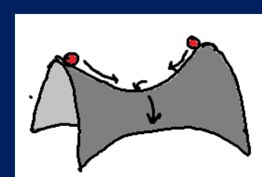




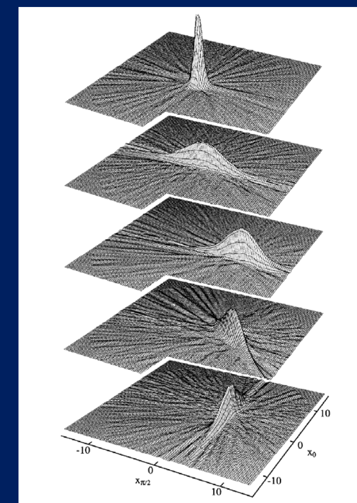
$$H_s = Uxy$$

$$H_s' = U'(y^2 - x^2)$$

Saddle potential

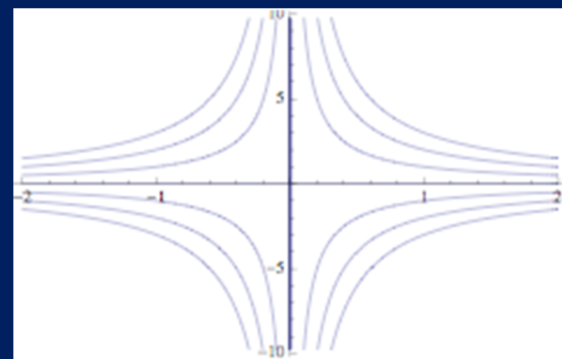


Saddle potential and squeezing (optics)



$$U x y = \frac{i}{2} U \ell^2 \left[b^2 - b^{\dagger 2} \right]$$

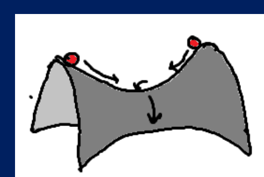
$$x = \frac{\ell}{2}(b + b^\dagger), \quad y = \frac{\ell}{2} i (b - b^\dagger)$$



$$H_s = U x y$$

$$H_s' = U'(y^2 - x^2)$$

Saddle potential

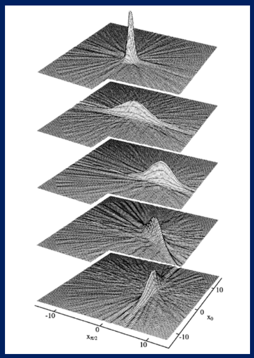


Time Evolution and Squeeze Operator

$$S(\xi) = \exp\left(\xi \frac{b^{\dagger 2}}{2} - \xi^* \frac{b^2}{2} \right)$$

$$\xi = -\frac{U t \ell^2}{\hbar} \equiv r e^{i\varphi}$$

Saddle potential and squeezing (optics)



$$U_{xy} = \frac{i}{2} V \ell^2 \left[b^2 - b^{\dagger 2} \right]$$

$$x = \frac{\ell}{2}(b + b^\dagger), \quad y = \frac{\ell}{2} i (b - b^\dagger)$$

Squeeze Operator properties

$$S b S^{-1} = b \cosh r - b^\dagger e^{i\varphi} \sinh r$$

Time Evolution and Squeeze Operator

$$S(\xi) = \exp\left(\xi \frac{b^{\dagger 2}}{2} - \xi^* \frac{b^2}{2} \right)$$

$$\xi = -\frac{V t \ell^2}{\hbar} \equiv r e^{i\varphi}$$

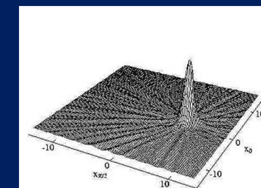
Coherent States and Squeezing

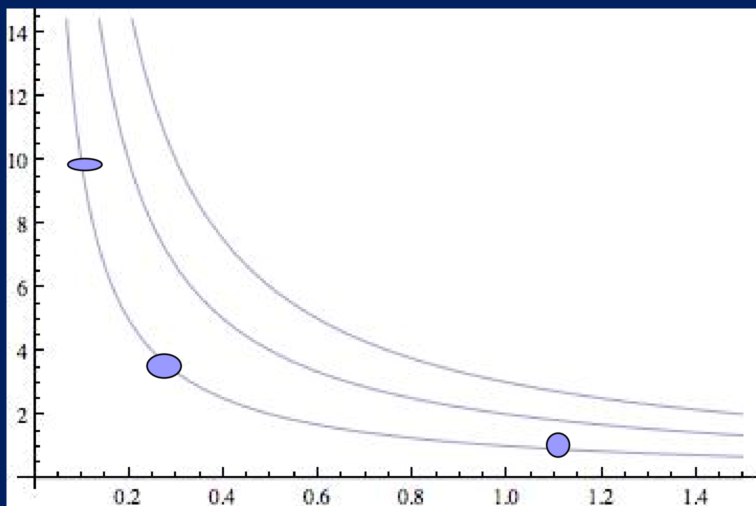
$$D(\alpha) = \exp(\alpha b^\dagger - \alpha^* b)$$

Coherent State: $D(\alpha)|0\rangle$

$$S D S^{-1} = D(\alpha \cosh r + \alpha^* e^{i\varphi} \sinh r)$$

Saddle potential coherent state dynamics





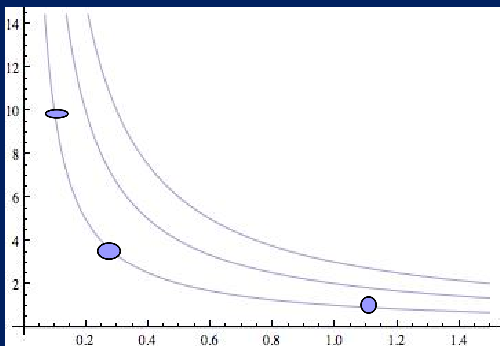
$H_s = Uxy$

- Analogy with quantum optics
- Follows equipotential contours
- Saddle acts as squeezing operator

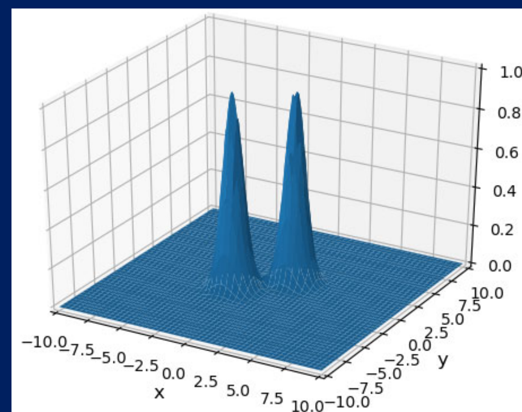
  $re^{i\phi} \equiv Utl^2/\hbar$

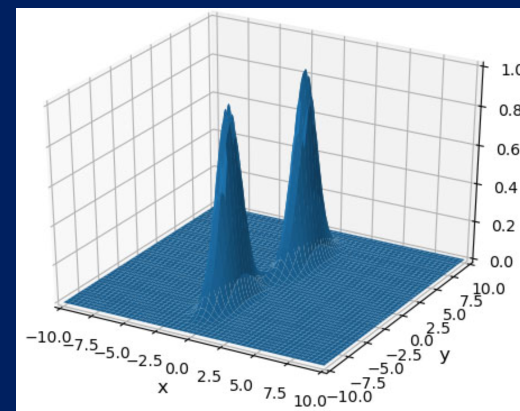
- Path: $(Xe^{-Utl^2/\hbar}, Ye^{Utl^2/\hbar})$

Saddle potentials as beam splitters

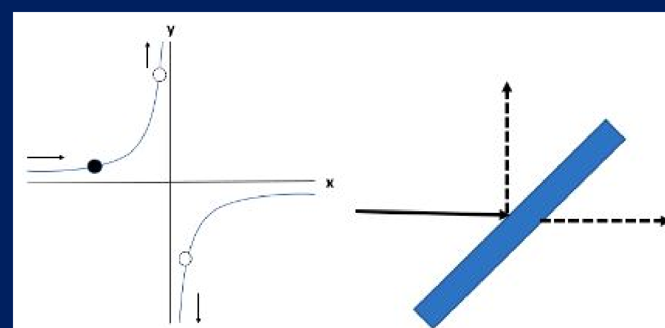


$$H_s = Uxy$$

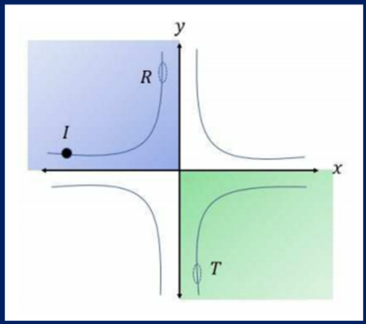


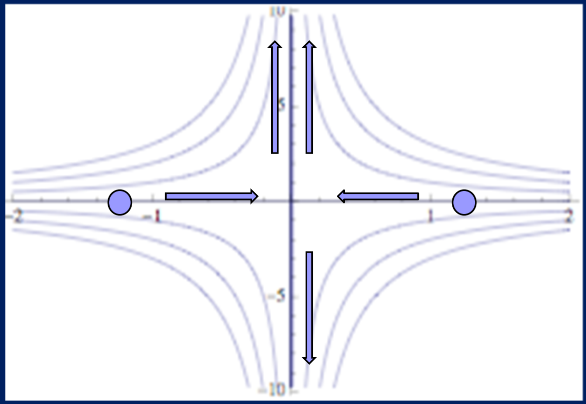


Beam Splitter

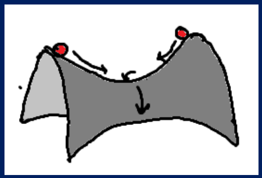


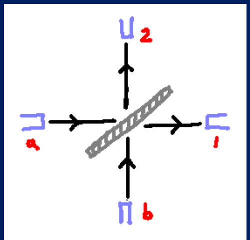
Tunneling, beam splitter, two-particles





Did the two particles go in
the same direction or different ones?

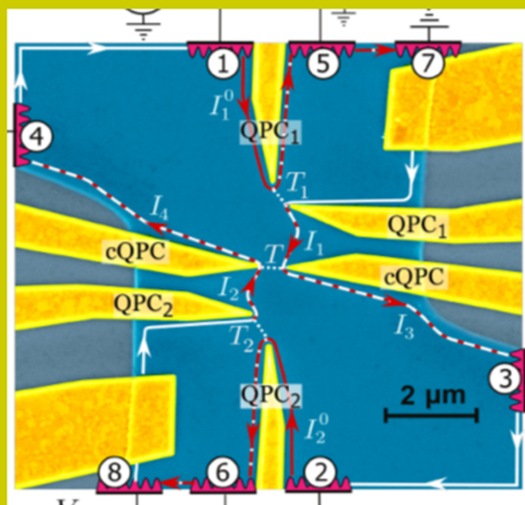




E.g. Hong, Ou and Mandel (1987)

Fertig & Halperin, (1987); S.V.& N. R. Cooper, (2010); Subramanyan & S.V, (2019)

Quantum Hall Anyon beam-splitter

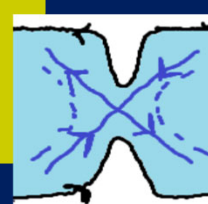


MESOSCOPIC PHYSICS

Bartolomei et al., Science 368, 173–177 (2020) 10 April 2020

Fractional statistics in anyon collisions

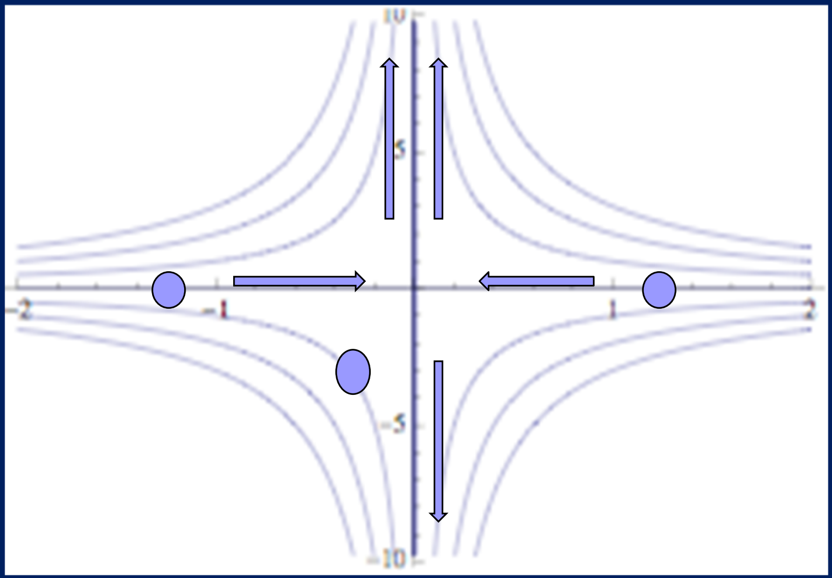
H. Bartolomei[1*], M. Kumar[1*†], R. Bisognin[1], A. Marguerite[1‡], J.-M. Berroir[1], E. Bocquillon[1], B. Plaçais[1], A. Cavanna[2], Q. Dong[2], U. Gennser[2], Y. Jin[2], G. Fève[1§]



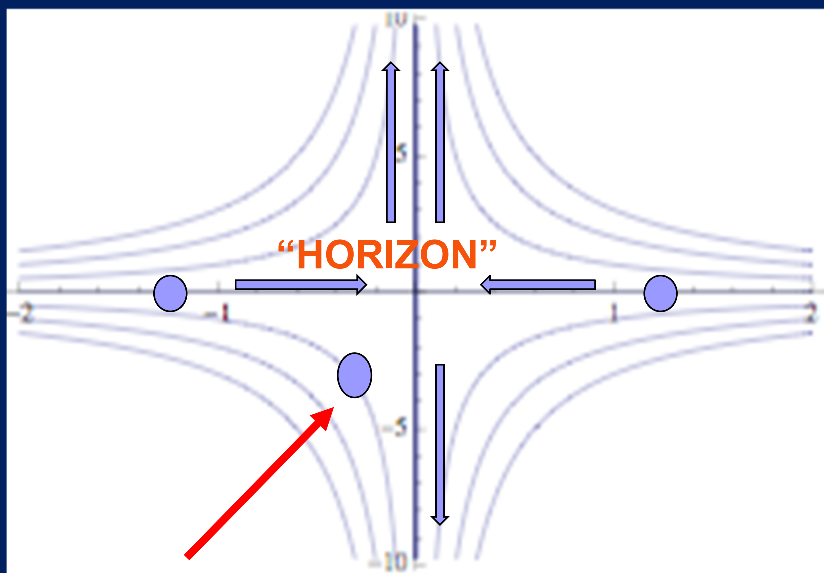
Anyon signatures observed in experiments!

Bartolomei at al, Science (2020);

Saddle potential beam-splitter properties

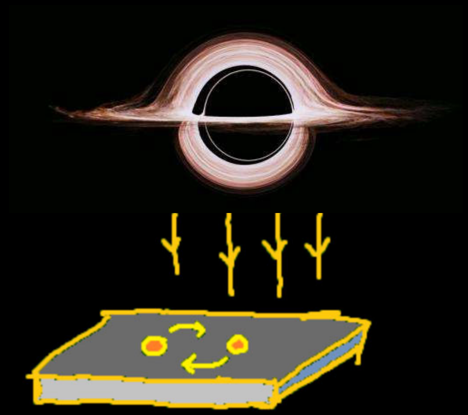


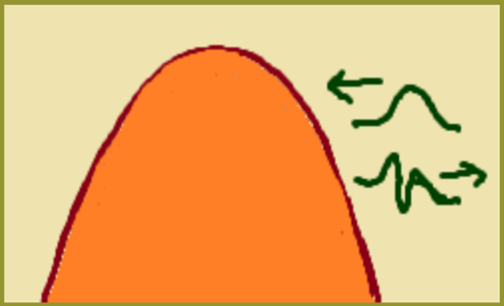
Saddle potential beam-splitter properties

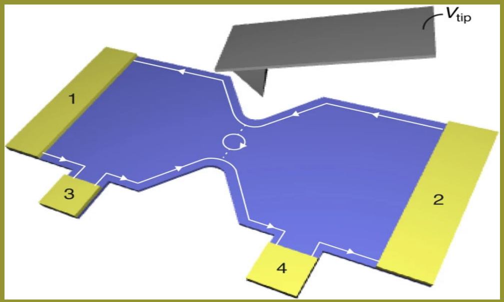


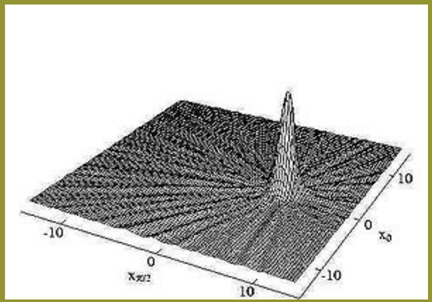
Saddle-defined horizon – Black Hole physics!

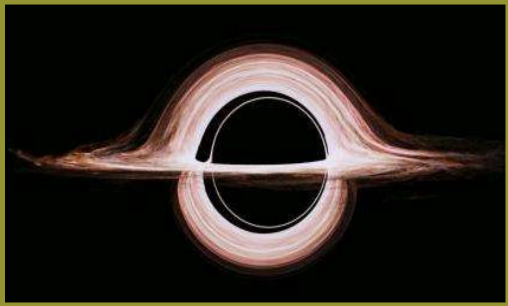
Black Hole dynamics and Quantum Hall parallels











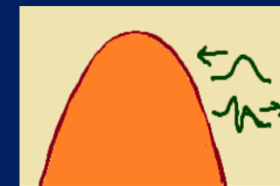
Quantum Connections: From Quantum Hall Physics to Optics To Blackholes

Smitha Vishveshwara, University of Illinois at Urbana Champaign

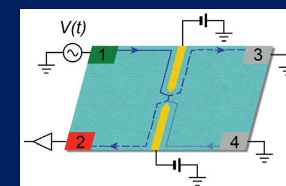
Quantum Connections Series, Summer School In Sweden, 22 June 2023

Quantum Connections

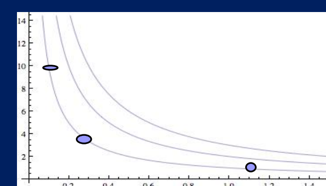
In what follows—Part II

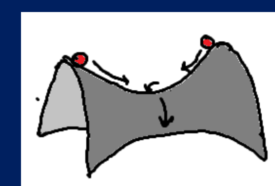


- Quantum Hall Physics
  Basics, Point Contact, Saddle Potential, Tunneling,
  ANYONS

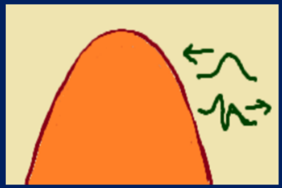


- Quantum Optics Parallels
  Coherent States, Squeezing





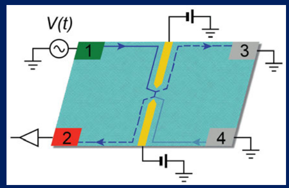
In what follows—Part II



- Quantum Hall Physics

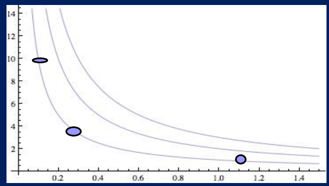
  Basics, Point Contact, Saddle Potential, Tunneling,

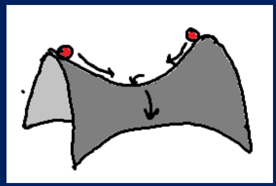
  ANYONS



- Quantum Optics Parallels

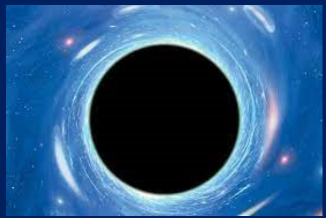
  Coherent States, Squeezing

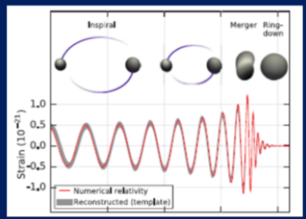




- Black hole Dynamics

  Hawking radiation, Quasinormal Modes





Collaborators

Barry Bradlyn
UIUC

Nigel Cooper
Cambridge Univ.

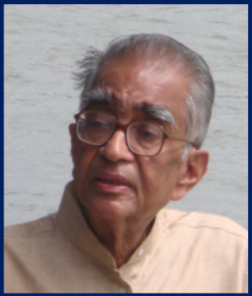
Suraj Hegde,
MPI Dresden

Hans Hansson
Stockholm Univ.

Diptiman Sen
IISc

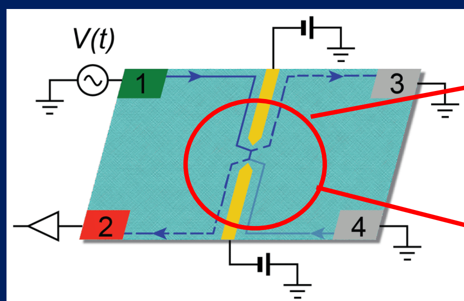
Michael Stone
UIUC

Varsha Subramanyam
UIUC



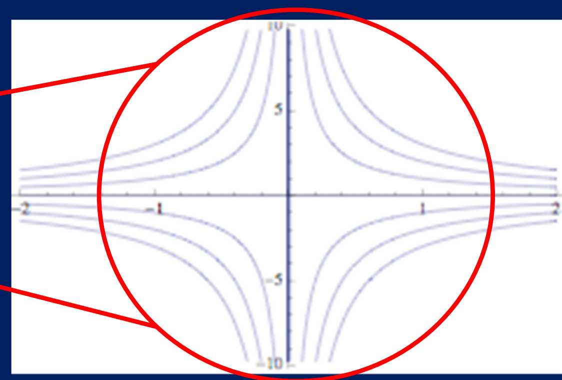
C. V. Vishveshwara

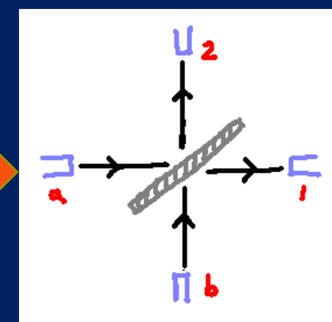
Point contact, saddle potential, beam-splitter



Sassetti

Quantum Hall (QH)
Point contact



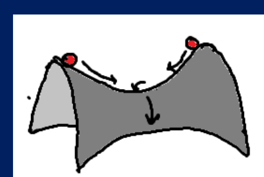


Beam splitter
Quantum optics, QH, and more

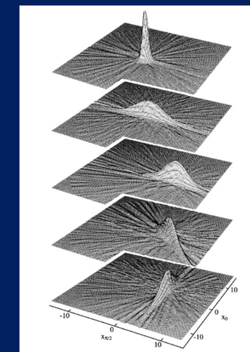
$$H_s = Uxy$$

$$H_s' = U'(y^2 - x^2)$$

Saddle potential



Saddle potential and squeezing (optics)



$$U_{xy} = \frac{i}{2} U \ell^2 \left[b^2 - b^{\dagger 2} \right]$$

$$x = \frac{\ell}{2}(b + b^\dagger), \quad y = \frac{\ell}{2} i (b - b^\dagger)$$

Squeeze Operator properties

$$S b S^{-1} = b \cosh r - b^\dagger e^{i\varphi} \sinh r$$

Coherent States and Squeezing

$$D(\alpha) = \exp(\alpha b^\dagger - \alpha^* b)$$

Coherent State: $D(\alpha)|0\rangle$

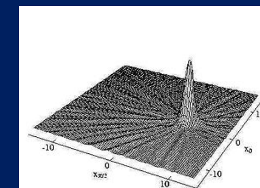
$$S D S^{-1} = D(\alpha \cosh r + \alpha^* e^{i\varphi} \sinh r)$$

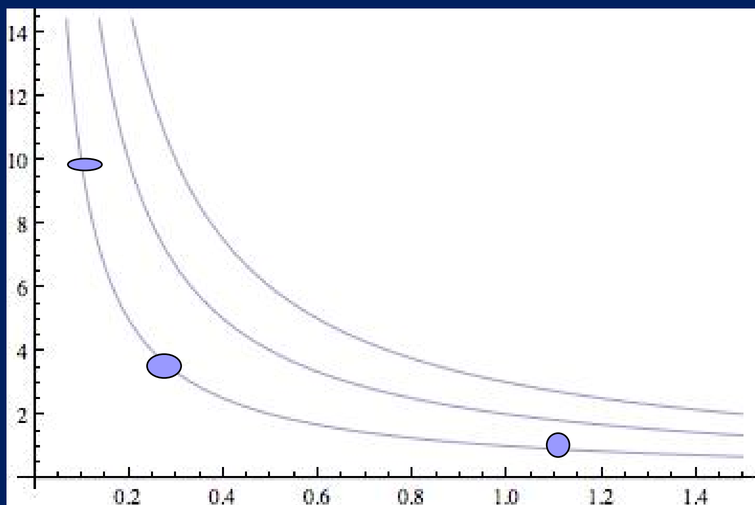
Time Evolution and Squeeze Operator

$$S(\xi) = \exp\left(\xi \frac{b^{\dagger 2}}{2} - \xi^* \frac{b^2}{2}\right)$$

$$\xi = -\frac{U t \ell^2}{\hbar} \equiv r e^{i\varphi}$$

Saddle potential coherent state dynamics





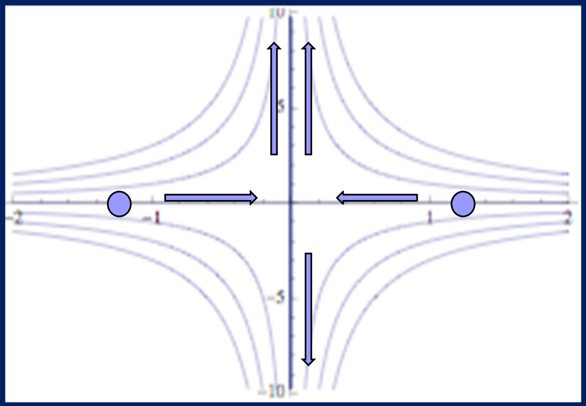
$H_s = Uxy$

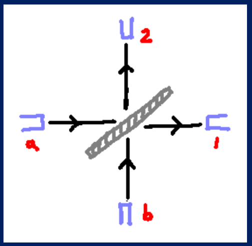
- Analogy with quantum optics
- Follows equipotential contours
- Saddle acts as squeezing operator

$$re^{i\phi} \equiv Utl^2/\hbar$$

- Path: $\left(Xe^{-Utl^2/\hbar}, Ye^{Utl^2/\hbar}\right)$

Tunneling, beam splitter, two-particles

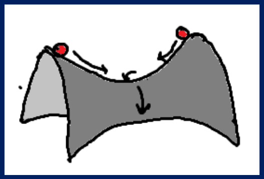


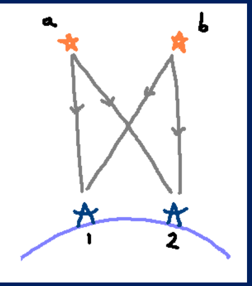


E.g. Hong, Ou and Mandel (1987)

Did the two particles go in
the same direction or different ones?

Reminiscent of Hanbury-Brown Twiss experiments
For stellar bodies

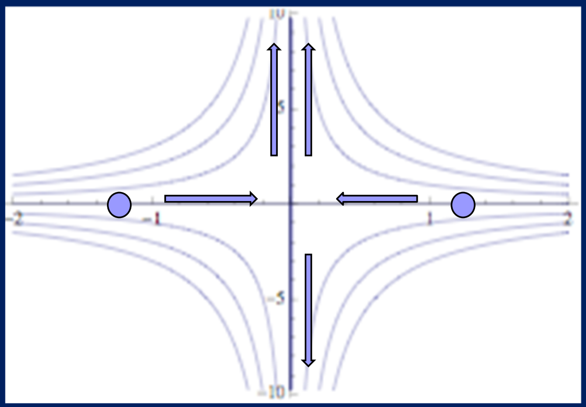


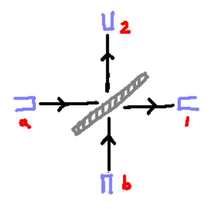


HBT (1954)

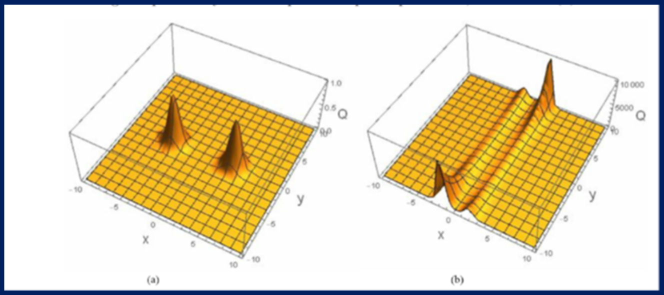
Fertig & Halperin, (1987); S.V.& N. R. Cooper, (2010); Subramanyan & S.V, (2019)

Tunneling, beam splitter, two-particles

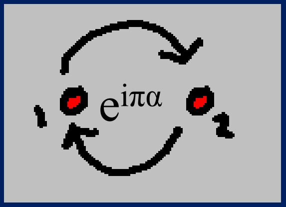




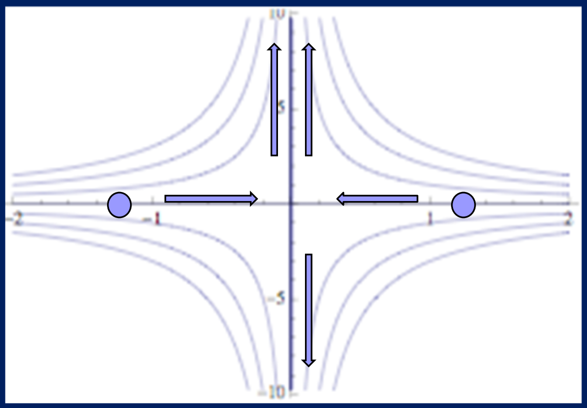
Did the two particles go in
the same direction or different ones?

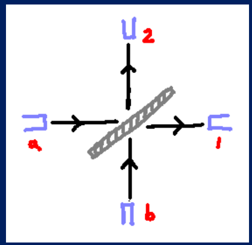


Quantum Statistics:
Bosons: Same
Fermions: Different
Anyons: Depends on initial conditions



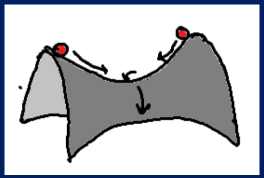
Tunneling, beam splitter, two-particles





E.g. Hong, Ou and Mandel (1987)

Did the two particles go in
the same direction or different ones?



Two particles in the lowest Landau level!

Fertig & Halperin, (1987); S.V.& N. R. Cooper, (2010); Subramanyan & S.V, (2019)

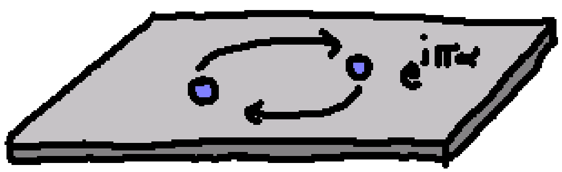
Two-anyon model

$$H = \frac{1}{4\mu}\left(P_x + \frac{qB}{c}Y\right)^2 + \frac{1}{4\mu}\left(P_y - \frac{qB}{c}X\right)^2$$
$$+ \frac{1}{\mu}\left(p_x + \frac{qB}{4c}y\right)^2 + \frac{1}{\mu}\left(p_y - \frac{qB}{4c}x\right)^2$$

Center of mass: $(\vec{R}, \vec{P})$

Relative co-ordinates: $(\vec{r}, \vec{p})$

Magnetic field B perpendicular to plane



Anyonic feature

$$\psi(-\vec{r}) = e^{\pm i\pi\alpha}\psi(\vec{r})$$

E.g. Leinaas and Myrheim, 1977; Wilczek 1982
Halperin, 1984; Arovas, Schrieffer, Wilczek, 1984

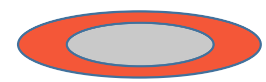
Two-anyon LLL Hilbert space

Center of mass | **Relative coordinates**

Angular momentum eigenstates

$$\hat{L}\,|n\rangle_c = n\hbar\,|n\rangle_c$$

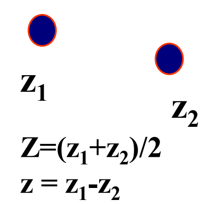
$$\hat{L}|k,\alpha\rangle_r = (2k+\alpha)\hbar|k,\alpha\rangle_r$$



Localized (coherent) states

$$|Z\rangle_c = e^{-|Z|^2/2}\sum_{n=0}^{\infty}\frac{(Z^*)^n}{\sqrt{n!}}|n\rangle_c$$

$$|z\rangle_\alpha = N_{\alpha,z}\sum_{k=0}^{\infty}\frac{(z^*/2)^{2k+\alpha}}{\sqrt{\Gamma(2k+\alpha+1)}}|k,\alpha\rangle_r$$



T. H. Hansson, J. M. Leinaas, J. Myrheim ,1992 ; H. Kjonsberg and J. M. Leinaas, 1997

Two-anyon LLL Hilbert space

Relative coordinates:

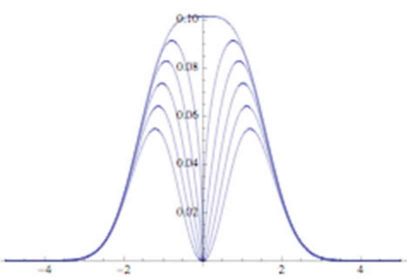
Guiding center coordinates

$$\hat{a} \equiv (\hat{x}^2 + \hat{y}^2)/8l^2, \hat{b} \equiv (\hat{x}^2 - \hat{y}^2)/8l^2$$
$$\hat{c} \equiv (\hat{x}\hat{y} + \hat{y}\hat{x})/8l^2, \hat{L} = \hbar(2\hat{a} - 1/2)$$

Respect sp(1,R) algebra

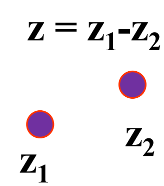
Angular momentum eigenstates

$$\hat{L}|k,\alpha\rangle_r = (2k+\alpha)\hbar|k,\alpha\rangle_r$$
$$\psi_k(\vec{r}) \propto \frac{z^{2k+\alpha}}{\sqrt{\Gamma(2k+\alpha+1)}}$$



Localized states
(not coherent states)

$$|z\rangle_\alpha = N_{\alpha,z} \sum_{k=0}^{\infty} \frac{(z^*/2)^{2k+\alpha}}{\sqrt{\Gamma(2k+\alpha+1)}} |k,\alpha\rangle_r$$



Fermions/
Bosons

$$|z\rangle_{1/0} = e^{|z|^2/8} N_{1/0,z}[|z\rangle_d \mp |-\ z\rangle_d],$$

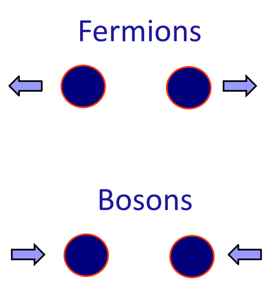
HLM92; KL97

Bunching properties

<u>Bunching parameter</u>

$$\chi(|z|,\alpha) \equiv \frac{1}{4l^2}[\langle z|\hat{r}^2|z\rangle_\alpha - \langle z|\hat{r}^2|z\rangle_d]$$

$$\langle \hat{r}^2 \rangle \equiv \langle \hat{x}^2 + \hat{y}^2 \rangle$$

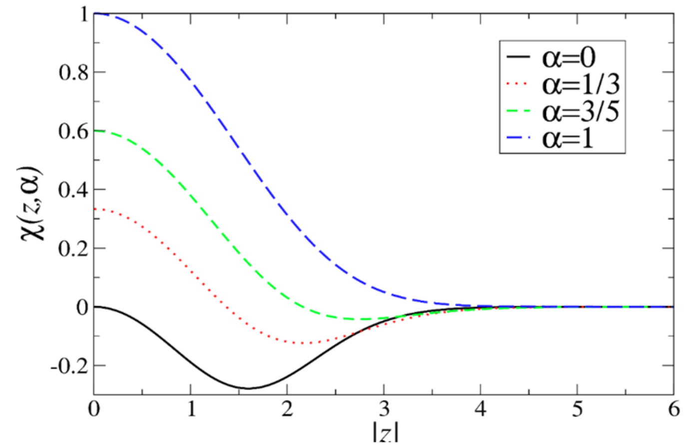


Bunching properties

<u>Bunching parameter</u>

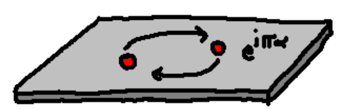
$$\chi(|z|,\alpha) \equiv \frac{1}{4l^2}\left[\langle z|\hat{r}^2|z\rangle_\alpha - \langle z|\hat{r}^2|z\rangle_d\right]$$

$$\langle \hat{r}^2\rangle \equiv \langle \hat{x}^2 + \hat{y}^2\rangle$$

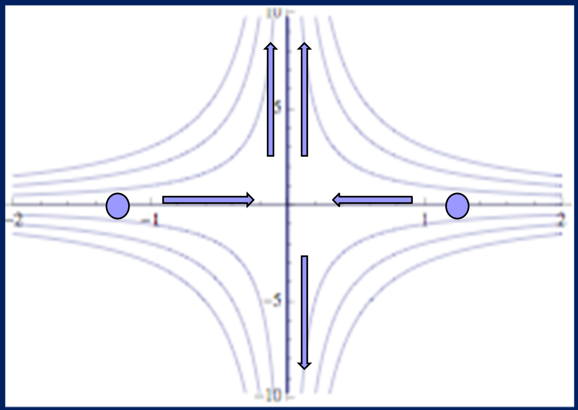


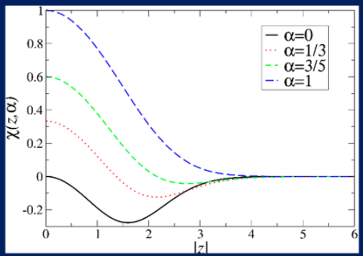
Fermions

Bosons



Saddle potential beam splitter properties



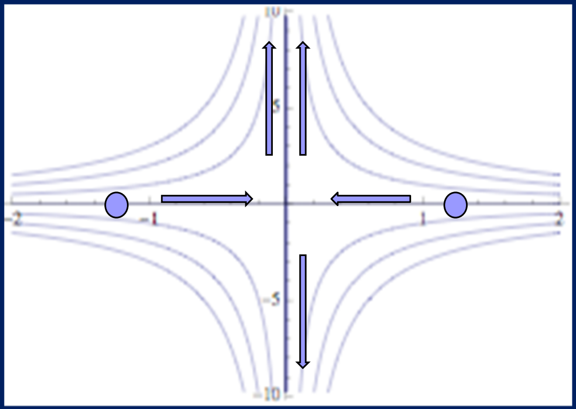


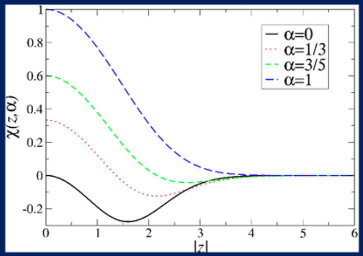
Did the particles go in the same direction or different ones?

Behavior of <$y_1 y_2$>

S.V. & N. R. Cooper, 2010
V. Subramanyan and S. V. 2019

Saddle potential beam splitter properties





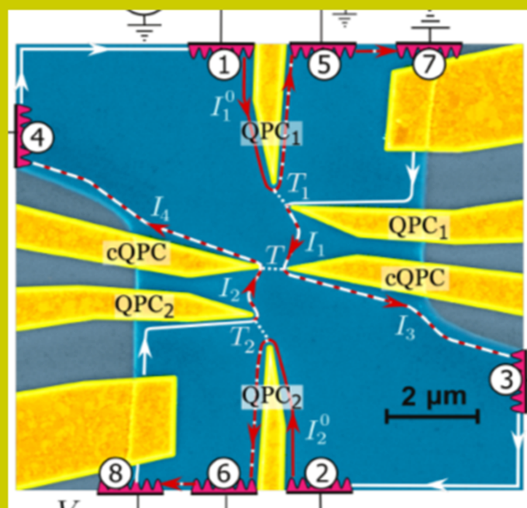
Did the particles go in the same direction or different ones?

Depends on statistics and bunching parameter- "DUAL NATURE"

Behavior of <$y_1 y_2$>

$$\langle \hat{y}_1 \hat{y}_2 \rangle = l^2 e^{2Utl^2/\hbar} \left[\mathrm{Im}[Z]^2 - \frac{1}{4}\mathrm{Im}[z]^2 - \frac{1}{2}\chi + \delta \right]$$

Quantum Hall Anyon beam-splitter

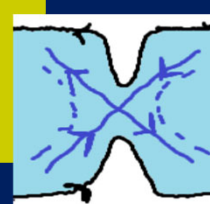


MESOSCOPIC PHYSICS

Bartolomei et al., Science 368, 173–177 (2020) 10 April 2020

Fractional statistics in anyon collisions

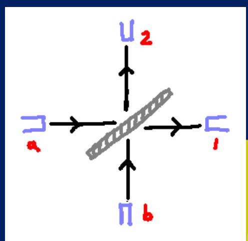
H. Bartolomei[1*], M. Kumar[1*†], R. Bisognin[1], A. Marguerite[1‡], J.-M. Berroir[1], E. Bocquillon[1], B. Plaçais[1], A. Cavanna[2], Q. Dong[2], U. Gennser[2], Y. Jin[2], G. Fève[1§]

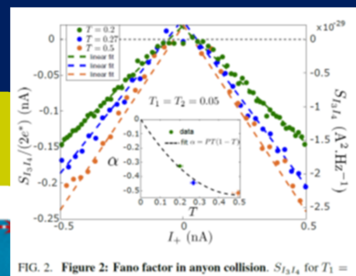


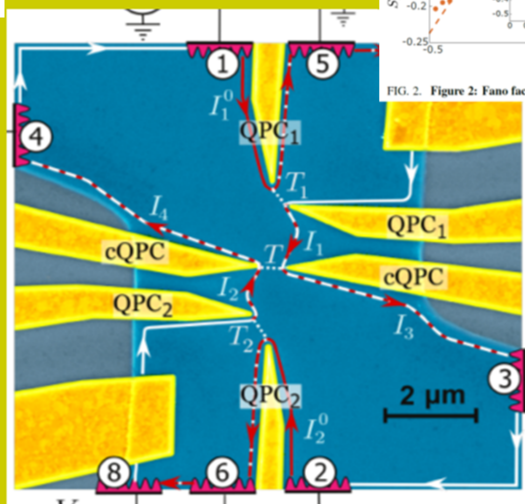
Anyon signatures observed in experiments!

Bartolomei at al, Science (2020);

Anyon signatures observed in experiment





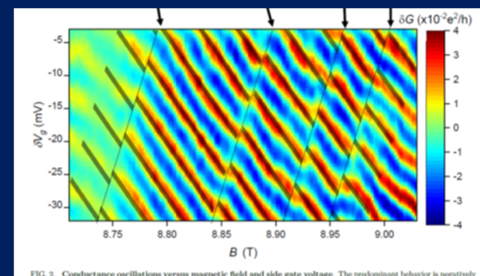


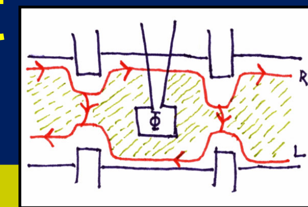
MESOSCOPIC PHYSICS Bartolomei et al., Science 368, 173–177 (2020) 10 April 2020

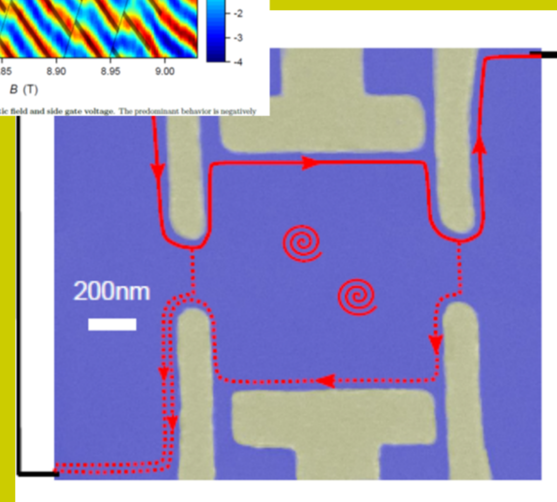
Fractional statistics in anyon collisions

H. Bartolomei[1*], M. Kumar[1*†], R. Bisognin[1], A. Marguerite[1‡], J.-M. Berroir[1], E. Bocquillon[1], B. Plaçais[1], A. Cavanna[2], Q. Dong[2], U. Gennser[2], Y. Jin[2], G. Fève[1§]

Bartolomei at al, Science (2020)







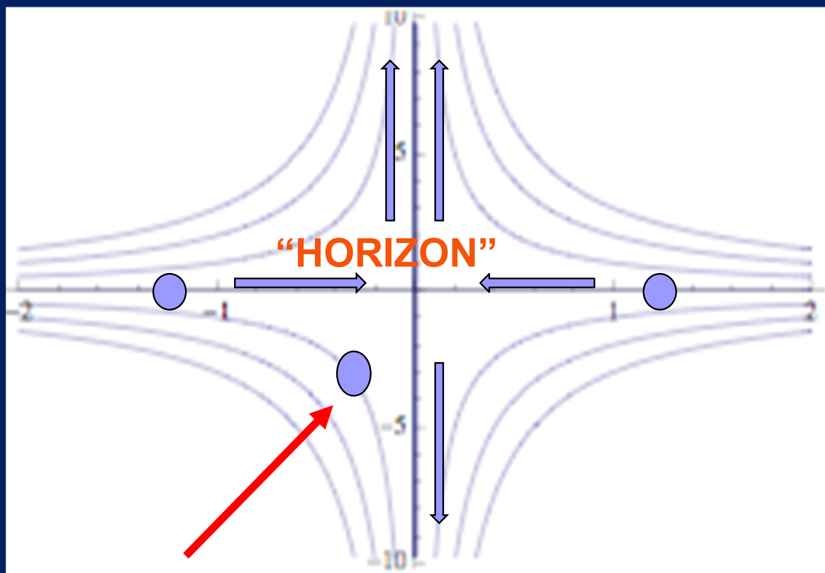
NATURE PHYSICS | VOL 16 | SEPTEMBER 2020 | 931–936

Direct observation of anyonic braiding statistics

J. Nakamura[1,2], S. Liang[1,2], G. C. Gardner[2,3] and M. J. Manfra[1,2,3,4,5]

Nakamura et al, Nature Physics (2020)

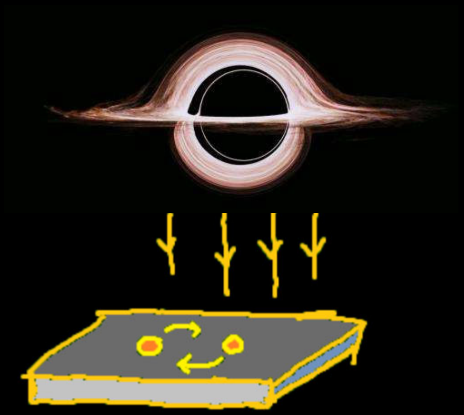
Saddle potential beam-splitter properties



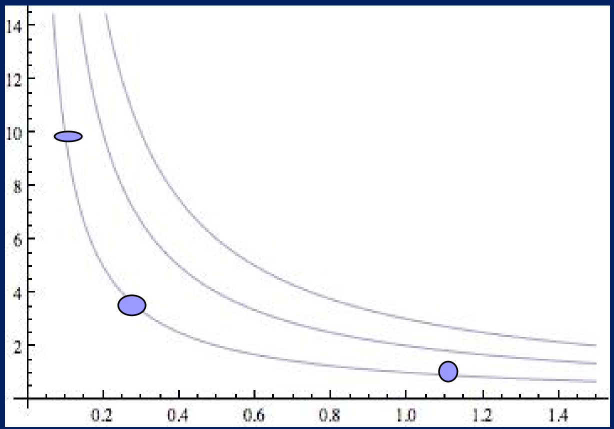
Saddle-defined horizon – Black Hole physics!

Black Hole dynamics and Quantum Hall parallels:

- Hawking-Unruh Radiation
- Black hole ringdown



Saddle potential and relativistic dynamics



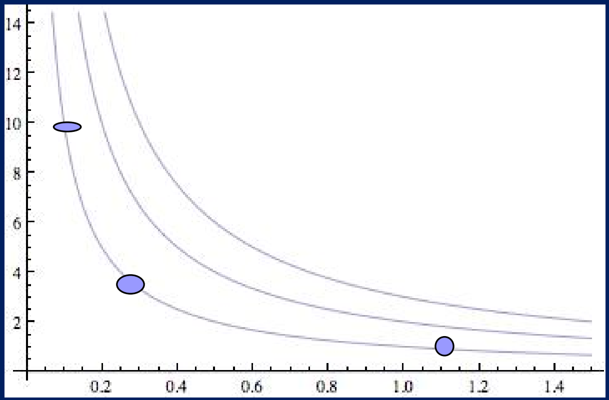
Time evolution

- Saddle acts as squeezing operator

$$re^{i\phi} \equiv Utl^2/\hbar$$

Parallels--commonality in symmetry and algebra; Squeezing and dilation; Lorentz boosts and Bogoliubov transforms

Saddle potential and relativistic dynamics



Time evolution

- Saddle acts as squeezing operator

$$re^{i\phi} \equiv Utl^2/\hbar$$

Parallels motion of accelerating observer

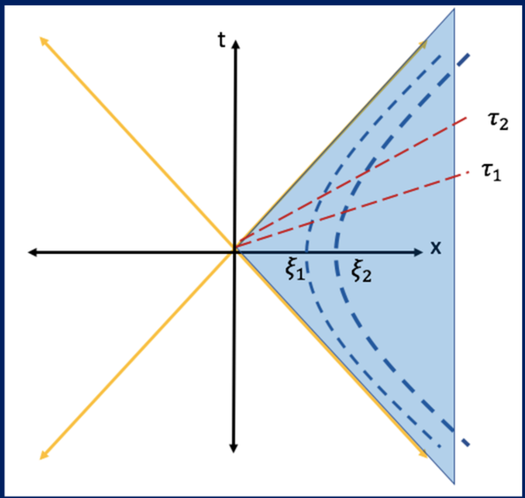
Rindler coordinates ---accelerating frame as seen by inertial observer

$$x = e^{a\xi} \cosh a\tau \, , t = e^{a\xi} \sinh a\tau$$

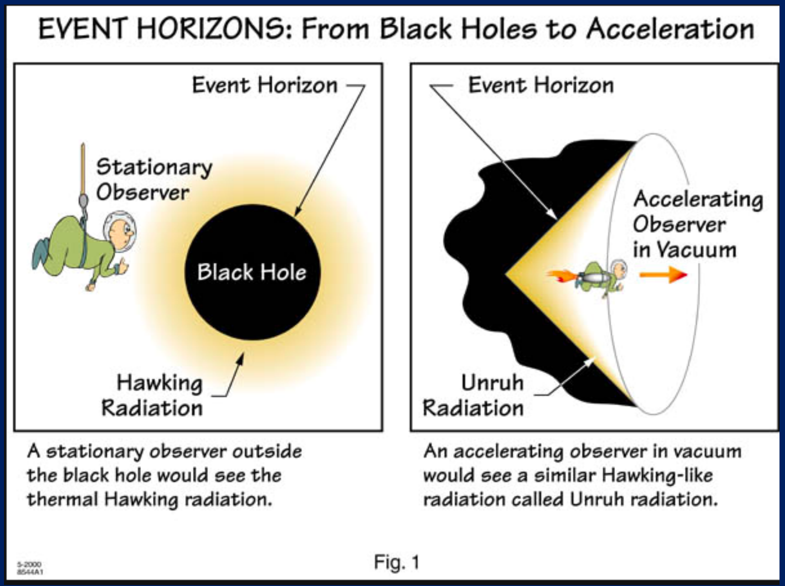
$$u, v = x \pm t = e^{a(\xi \pm \tau)}$$

"Light cone Coordinates"

Rindler Hamiltonian → Time translation generator in Rindler spacetime

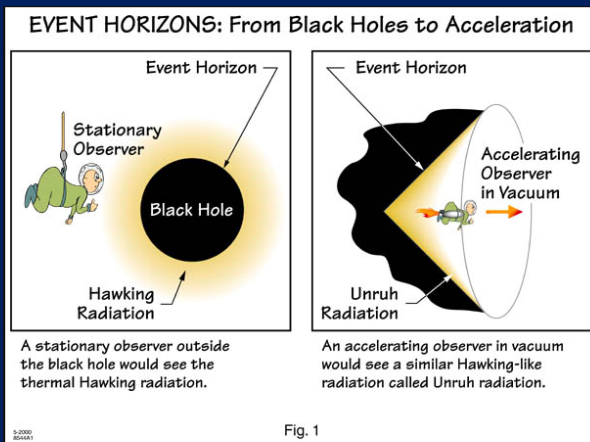


Hawking-Unruh Radiation



Found in Physicsnapkins blog

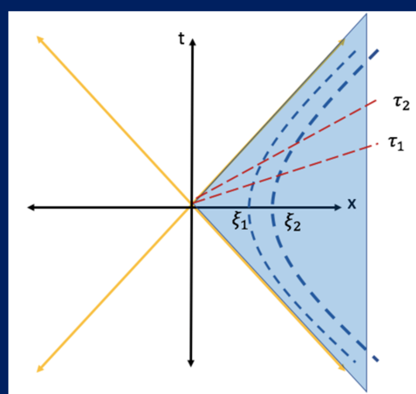
Hawking-Unruh Radiation



Φ

Minkowski: $a_\omega |0_M\rangle = 0$ → $\langle 0_M | a_\omega^\dagger a_\omega | 0_M \rangle = 0$

Rindler: $b_\Omega |0_R\rangle = 0$ → $\langle 0_R | b_\Omega^\dagger b_\Omega | 0_R \rangle = 0$

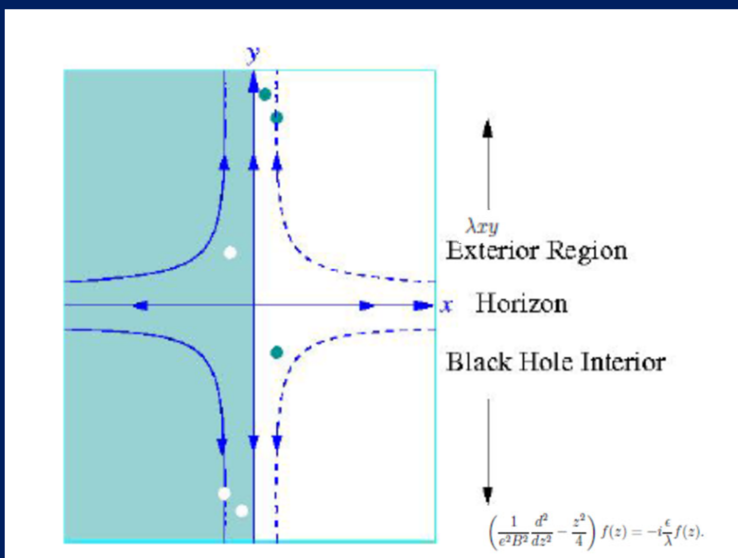


Bogoliubov Transformation/Squeezing

$$\frac{1}{V} \langle 0_M | b_\Omega^\dagger b_\Omega | 0_M \rangle = \frac{1}{e^{2\pi\Omega} - 1}$$

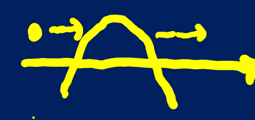
An analogue of Hawking radiation in the quantum Hall effect

Stone, Class. Qtm. Grav. (2013)



Probability of out-going particle

$$P(\epsilon) = \frac{1}{1 + \exp\left(\frac{2\pi eB}{\hbar\lambda}\epsilon\right)}$$



Hawking radiation analog

$$k_B T_{\text{Hawking}} = \frac{\hbar\kappa}{2\pi} \quad \longleftrightarrow \quad \kappa = \frac{\lambda}{eB} = \left.\frac{dv_{\text{edge}}}{dy}\right|_{\text{horizon}}.$$

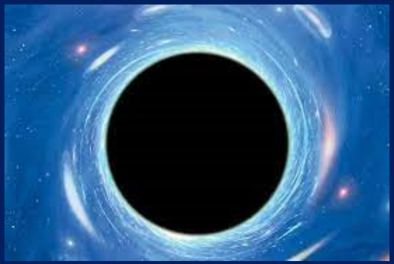
Surface gravity κ

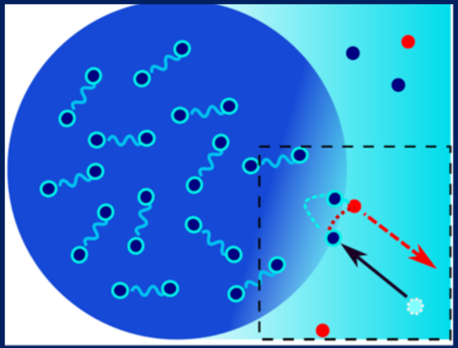
Edge-velocity; saddle strength

In-going and out-going states and quantum fields in two frames each related by a Bogoliubov transformation

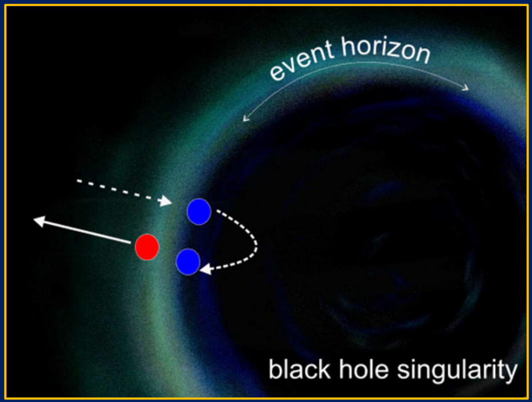
Rindler (1969); Hawking, (1974); Unruh, (1976); Fertig & Halperin, (1987)

Black holes, information paradox, superconductor analogues





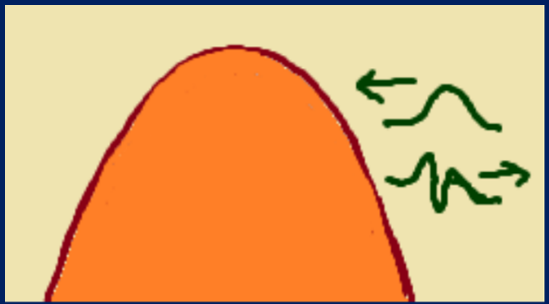
Andreev reflection



Black hole, while accepting particles, reflects quantum information in the outgoing modes.

S. Manikandan and A. Jordan, Phys Rev. D (2020)

IHO and quasinormal modes



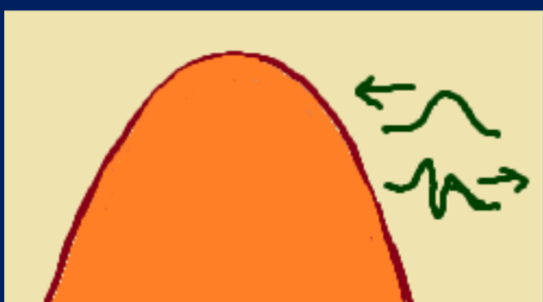
Decaying modes intrinsic to system
Related to black hole ringdown

Properties of the Inverted Harmonic Oscillator

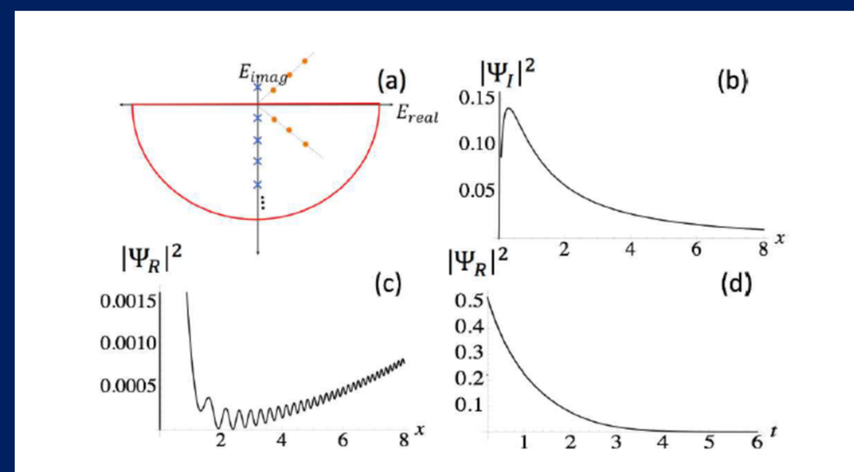
$$\hat{H} = \frac{\hat{p}^2}{2} - \frac{\hat{x}^2}{2}$$

1) Continuous Real Spectrum (scattering)
2) Discrete Imaginary Spectrum

S-Matrix poles and Wavepackets

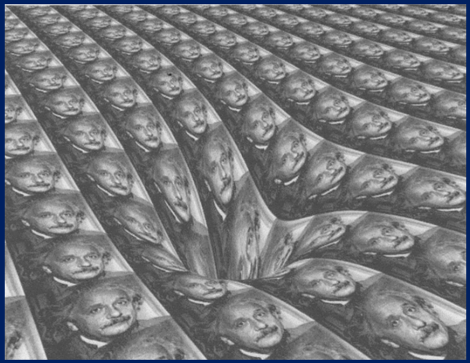


$$E_n^{\pm} = \mp i(n + \frac{1}{2})$$



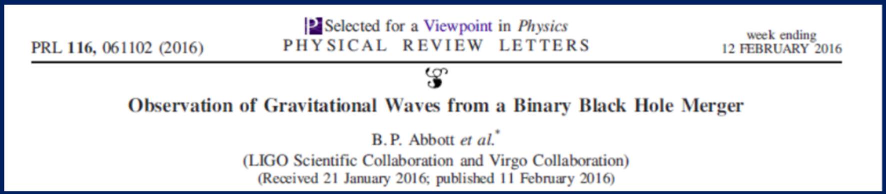
Resonant/Quasinormal Modes
Discrete Imaginary Spectrum

Discovery of Gravitational Waves



Black hole merger

Feb 2016:
First Announcement



Selected for a Viewpoint in *Physics*
PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

Discovery of Gravitational Waves

Ringdown and Quasinormal Modes

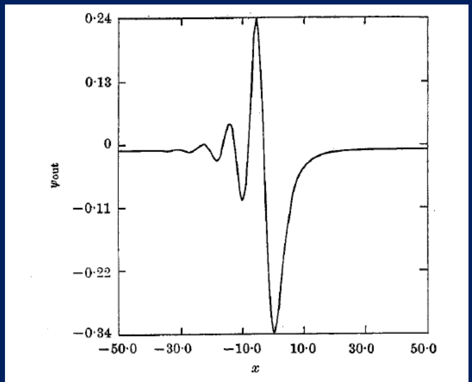
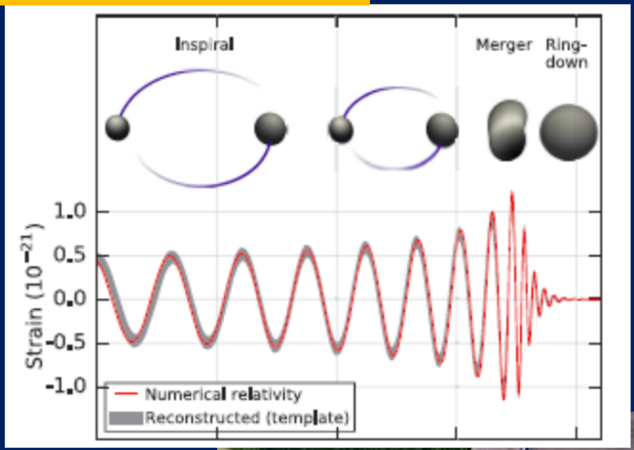


Fig. 3. The outgoing wave packet $\psi_{out}(x)$ at spatial infinity corresponding to the incident Gaussian wave packet $\psi_{in}(x) = e^{-ax^2}$ with $a = 1$.

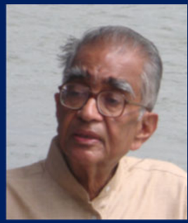
[8] C. V. Vishveshwara, Nature (London) 227, 936 (1970).

Black hole merger



Extensive numerical simulations
Black Hole QNMs in diverse contexts

Original derivation of black hole QNMs



C. V. Vishveshwara

Scattering of Gravitational Radiation by a Schwarzschild Black-hole

NATURE VOL. 227 AUGUST 29 1970

Wave packet scattering off Schwarzschild metric

$$ds^2 = -(1-2m/r)dt^2 + (1-2m/r)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2) + (h_0(r)\, dt\, d\varphi + h_1(r)\, dr\, d\varphi) \exp(-i\omega t) \sin\theta \frac{dP}{d\theta} l(\cos\theta)$$

Wave equation In 'tortoise coordinates'

$$\frac{\partial^2 \Psi}{\partial r_*^2} + V(\ell, r_*)\Psi = 0.$$

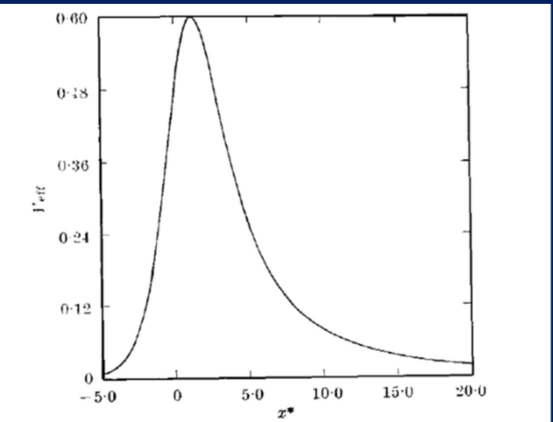
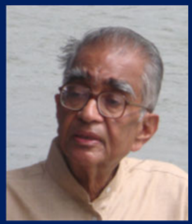


Fig. 1. The effective potential V_{eff} for the odd-parity gravitational waves of the lowest mode $l=2$ plotted against r^*.

Original derivation of black hole QNMs



C. V. Vishveshwara

Scattering of Gravitational Radiation by a Schwarzschild Black-hole

NATURE VOL. 227 AUGUST 29 1970

Wave packet scattering off Schwarzschild metric

$$ds^2 = -(1-2m/r)dt^2 + (1-2m/r)^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + (h_0(r)\,dt\,d\varphi + h_1(r)\,dr\,d\varphi)\exp(-i\omega t)\sin\theta\,\frac{dP}{d\theta}\,l(\cos\theta)$$

Wave equation
In 'tortoise coordinates'

$$\frac{\partial^2\Psi}{\partial r_*^2} + V(\ell, r_*)\Psi = 0.$$

Although the scattering of monochromatic waves did not show obvious characteristics of the black hole, I felt that scattering of wave packets might reveal the imprint of the black hole. So, I started pelting the black hole with Gaussian wave packets. If the wave packet was spatially wide, the scattered one was affected very little. It was like a big wave washing over a small pebble. But when the Gaussian became sharper, maxima and minima started emerging, finally levelling off to a set pattern when the width of the Gaussian became comparable to or less than the size of the black hole. The final outcome was a very characteristic decaying mode, to be christened later as the quasinormal mode. The whole experiment was extraordinarily exciting.

Black hole quasinormal modes in QH point contacts

Wave packet scattering off Schwarzschild metric

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta\, d\phi^2).$$

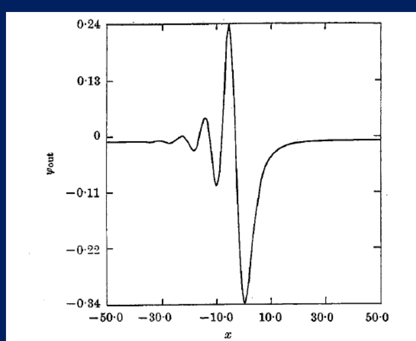
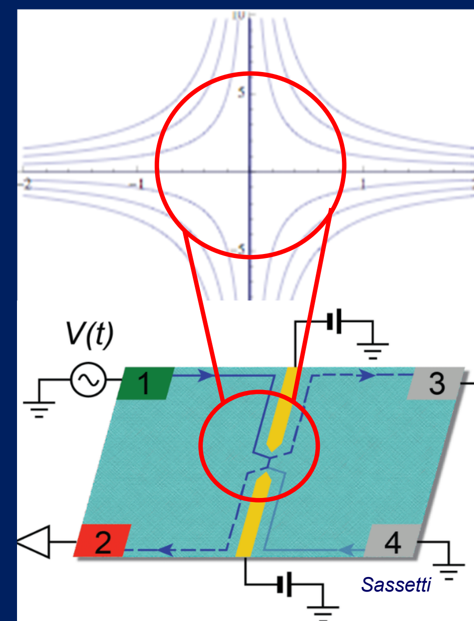


Fig. 3. The outgoing wave packet $\psi_{out}(x)$ at spatial infinity corresponding to the incident Gaussian wave packet $\psi_{in}(x) = e^{-ax^2}$ with $a = 1$.

Tunneling across saddle potential

$$\left(\frac{1}{e^2B^2}\frac{d^2}{dz^2} - \frac{z^2}{4}\right) f(z) = -i\frac{\epsilon}{\lambda} f(z).$$



Black hole quasinormal modes in QH point contacts

Wave packet scattering off Schwarzschild metric

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\phi^2).$$

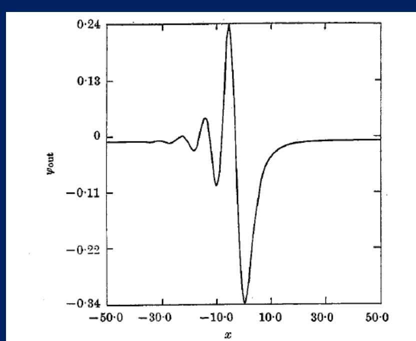


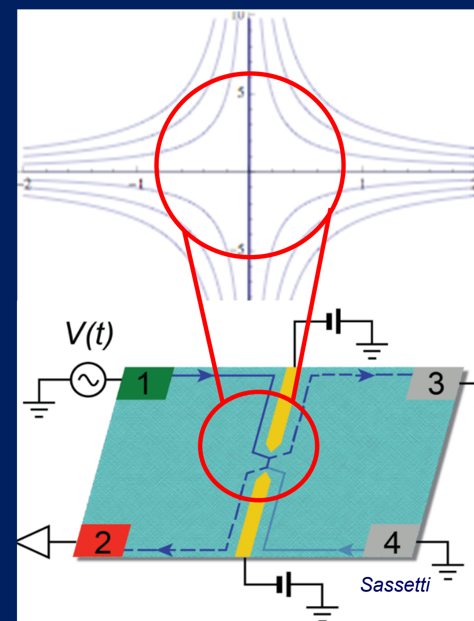
Fig. 3. The outgoing wave packet $\psi_{out}(x)$ at spatial infinity corresponding to the incident Gaussian wave packet $\psi_{in}(x) = e^{-ax^2}$ with $a = 1$.

1D Inverted SHO potential scattering

Quasinormal Mode Spectrum!

Tunneling across saddle potential

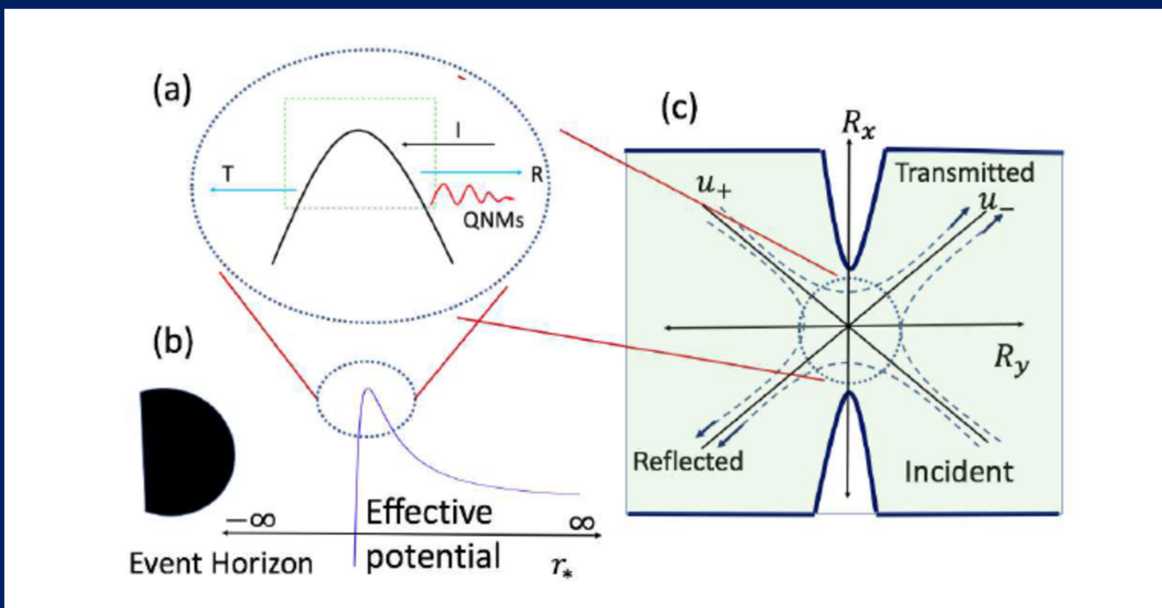
$$\left(\frac{1}{e^2 B^2}\frac{d^2}{dz^2} - \frac{z^2}{4}\right) f(z) = -i\frac{\epsilon}{\lambda} f(z).$$



Black hole quasinormal modes in QH point contacts

Characteristic scales for ringdown:
Black hole– One Solar Mass: 0.35ms
Quantum Hall—Potential Strength (micron spread)
Energy k*125mK: Nanosecond scale

Common IHO model, black hole, QPC



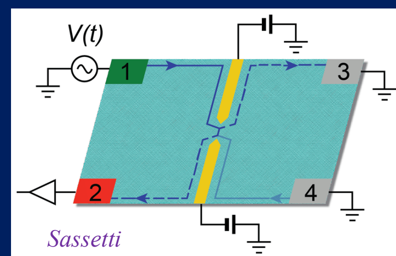
S. Hegde, V. Subramanyan, B. Bradlyn, S.V.;PRL (2019), Ann Phys P. W.A Issue, (2021)

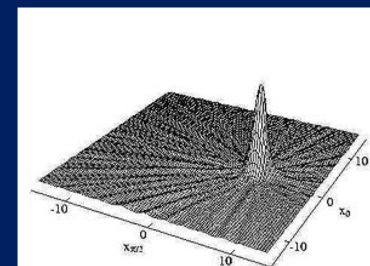
In Summary,

Quantum Hall Point Contact

Coherent States
Squeezing

Quantum Optics





Hawking-Unruh radiation
Quasinormal Modes

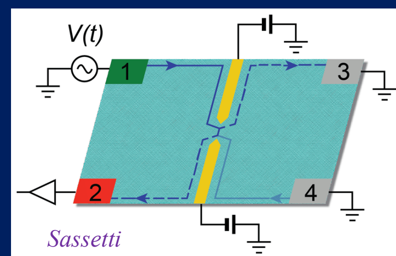
Black Hole Dynamics

In Summary,

Quantum Connections

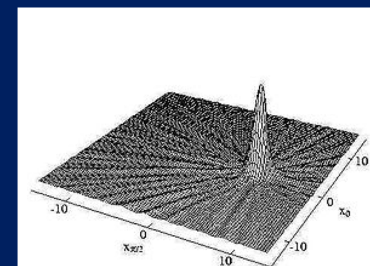
From Quantum Hall Physics
To Quantum Optics
To Blackholes

Quantum Hall Point Contact

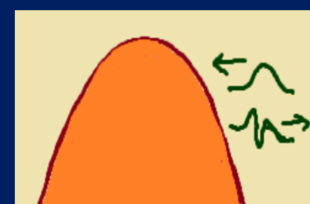


Coherent States
Squeezing

Quantum Optics



Hawking-Unruh radiation
Quasinormal Modes



Black Hole Dynamics

Thanks to
Organizers,
Collaborators

AND TO YOU ALL!