Utrecht University

Theoretical Physics Department

Can standard quantum field theories have an ontological origin?

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Summer School on
Quantum Connections
Stockholm
Wednesday June 21, 2023

Gerard 't Hooft

At large time and distance scales the laws of nature appear to be entirely deterministic.

But at the atomic scale, indeterminism seems to emerge:

quantum mechanics.

Whence this mysterious fact? Why are we unable to follow atoms and molecules more precisely when they evolve?

Copenhagen: *do not ask that question, just follow the rules and you get the best predictions that are possible.*

Alas, the predictions come in the form of probabilities.

Like weather predictions

As in the case of the weather,

we wish to explain where the statistical fluctuations come from.
Is there an underlying, deterministic set of laws? How can we find them?

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But, for *understanding* the laws of nature, it may well be necessary
to *assume* complete determinism.

Consider this clue:

an unstable particle, regardless whether it decays
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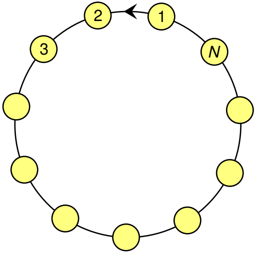
Can one construct models along such lines?

[see later in these talks]



Step 1. The periodic chain.

Ontological states:

$$|0\rangle,\ |1\rangle,\ \dots\ |N-1\rangle$$

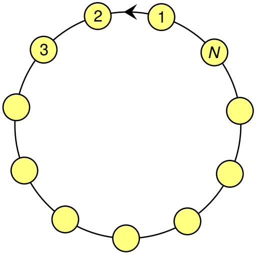
Evolution law:

$$|k\rangle_{t+\delta t} = U(\delta t)\,|k\rangle_t$$

$$U(\delta t)|k\rangle = |k+1 \mod N\rangle$$

$$U(\delta t) = e^{-iH\,\delta t}\ , \quad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle \quad \dots$$

Basic Ingredient for Models



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$$|n\rangle^E \stackrel{\text{def}}{=} \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} e^{2\pi ikn/N}|k\rangle^{\text{ont}} \ ,$$

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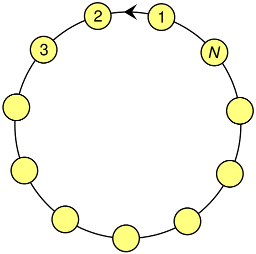

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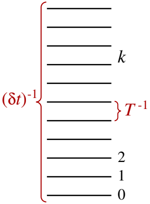
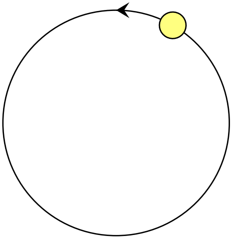


$$H = \frac{2\pi}{N\,\delta t}\,n = \omega n$$



Step 1a. The continuum limit.

Ontological states: $|\phi\rangle$

Evolution law:

$$\frac{\mathrm{d}}{\mathrm{d}t}|\phi\rangle_t = \omega$$

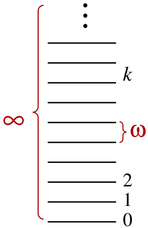
$$U(\delta t)|\phi\rangle = |\phi + \omega\delta t\rangle$$

$$U(\delta t) = e^{-iH\,\delta t}\ , \qquad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

$$|n\rangle^E \ \stackrel{\text{def}}{=}\ \frac{1}{\sqrt{2\pi}}\oint e^{i\phi n/N}|\phi\rangle^{\text{ont}}\ , \qquad 0\le\phi<2\pi\,;\quad n=0,\cdots,\ \infty\ .$$

$$|\phi\rangle^{\text{ont}} \ = \ \frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty} e^{-i\phi n/N}|n\rangle^E\ .$$

We generate exactly the spectrum of the harmonic oscillator: $H=\omega\, n$



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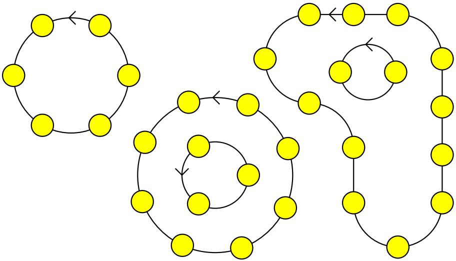
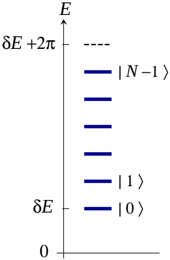


Generic, finite, deterministic, time reversible models

are mixtures of different oscillators.



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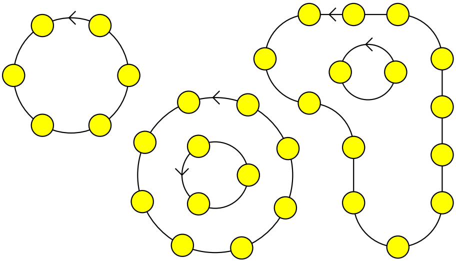

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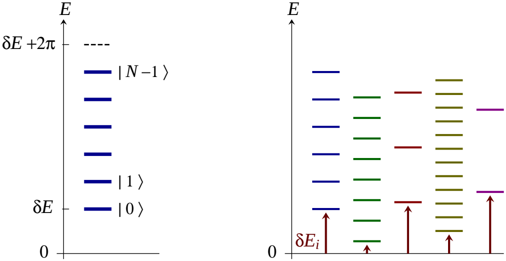

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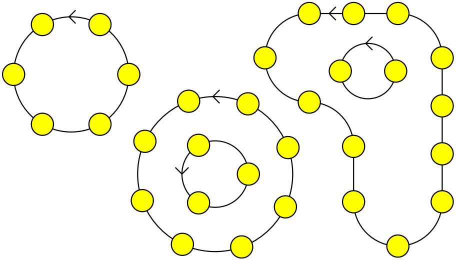

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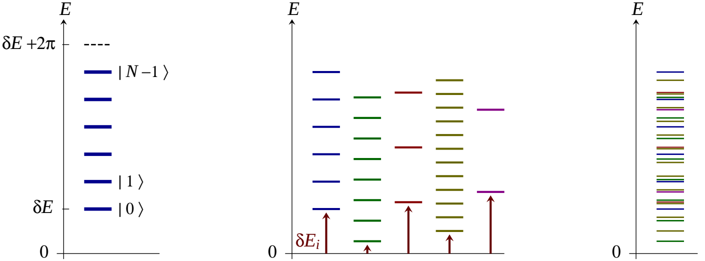

The time steps δt are discrete, ...

In the discrete case,
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possibly important in black hole physics,
where the time coordinate flips across the horizon.

These models are too simple to generate "real" quantum mechanics. But even classical systems may be fundamentally complicated. Can one arrange things such that "genuine" QM can be mimicked?

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These models are too simple to generate “real” quantum mechanics. But even classical systems may be fundamentally complicated. Can one arrange things such that “genuine” QM can be mimicked?

YES !! By assuming a

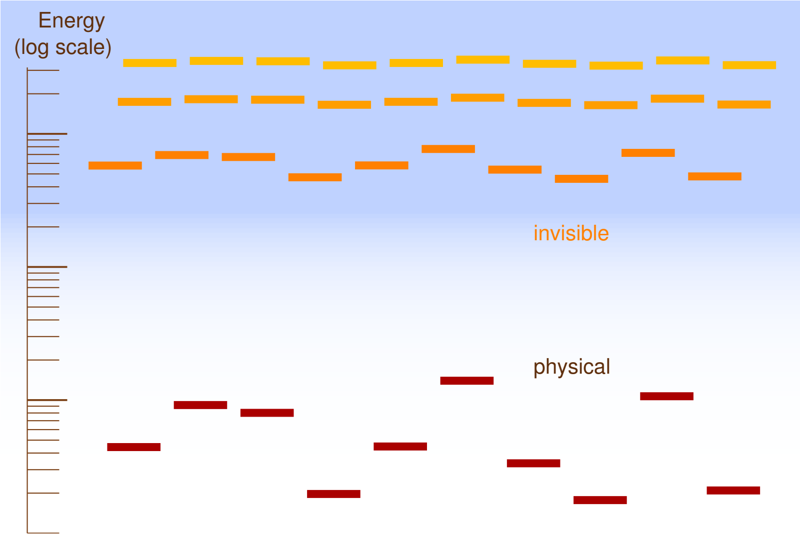
fast fluctuating background.

This is our
third step of sophitication: Split the universe up in small sectors, each sector containing only a finite number of states.
Each sector can only consist of pieces that are periodic.

In first approximation there is no interaction between the sectors.
Then introduce interaction exactly as in QFT: get the effects due to the steps used in the usual perturbation theory.

Now put the sectors with very high oscillation frequences in their energy eigen states, Now the energies do not commute with the ontological operators in the slow states. This is why any interaction between fast and slow, turns our theory into a quantum theory.

Think of at least one variable, with ultra short periodicity. at every $\vec{x}$, Think of at least one variable, with ultra short periodicity. at every $\vec{x}$,



The total Hamiltonian $H = H^{\text{fast}} + H^{\text{int}}$ acts exactly according to the rules of QM.

We cannot follow the fast variables!

Therefore, we *have to project* out the lowest N energy states.

This does not affect the equations when H^{int} is small.
Now H^{int} generates a H^{slow}, where $[H^{\text{fast}}, H^{\text{slow}}] \neq 0$,
and, since the states used in H^{fast} are energy eigen modes, the perturbative steps are real quantum mechanics,

whereas the theory in total is still ontological.

This creates a new – and interesting – situation,
which can indeed occur in ordinary classical theories.

Quantum mechanics is
Nature's punishment for doing perturbation theory !

See arxiv:2010.02019 .

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Outline:
Let there be given a quantum system with a finite dimensional 'Hilbert' space of states,
and we ask for an arbitrary Hamiltonian: a $K \times K$ hermitian matrix.

We can construct a model that will generate this matrix as an 'effective' or 'emergent' quantum Hamiltonian.

Simple example:

A particle, decaying by an exponential decay law.

$K = 2$. The particle ("atom") can be in two states: 1) one entire particle, 2) it has decayed.

The fast variables i each live on a circle with discrete period $L_i = N_i \delta t$, like in our elementary unit model.

Take the different N_i to be relative primes.

All periods N_i are much shorter than the inverse energies of the decaying atom or particle.

e.o.m.: $x_i(t+1) = x_i(t) + 1 \mod N_i$.

This is driven by the hamiltonian:

$$H = \sum_i p_i \ , \quad p_i = \frac{\partial}{\partial x_i} = \frac{2\pi n_i}{N_i} \ , \qquad n_i = 0, 1, \cdots, N_i - 1 \ .$$

Assume an *even distribution* of these variables. This means that, in our *formal* quantum language, they are all in their energy ground states (only if the distribution is not even, we need the excited states).

Their excitation energies, are at least $2\pi/N_i\delta t$, which we take to be much larger than the energies of our decaying atom.

According to thermodynamics they are rarely in an excited state. In the ground state, the probability distribution is completely flat.

Now consider our two-state atom.

We begin with

$$H_{\rm class} = 0.$$

Now consider its two classical states, 1 and 2. Assume that I want to add δH_{ij} to my Hamiltonian. Our classical atom would only allow three possible forms:

$$H = \alpha_1 \sigma_1 \,,$$

$$\text{with} \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

If we impose $\alpha_1 = \frac{1}{2}\pi \,, \quad \alpha_2 = \alpha_3 = 0 \,,$ then

$$e^{-iH} = -\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} ; \qquad \text{is a classical flip-flop} \ldots$$

taking place with frequency 1 in the given time unit.

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In our atom system of 2 atomic states, we want much *lower* energies, *larger* time units, and, if $K > 2$ different, non-commuting elements in the Hamiltonian.

$$\boxed{|\alpha_i| \neq 0\,,\ \text{but}\ \ \ll 1\ \ \text{would give real QM.}}$$

To obtain such behaviour in our model, we now use the fast fluctuating variables x_i :

Only if x_i all take on some special value, say $x_i = x_0$, our system makes its classical flip-over.

Thus, we add to the total Hamiltonian, a term

$$\delta H_{12} = \tfrac{1}{2}\pi\sigma_1^{12}\prod_i \delta(x_i - x_0)\,. \tag{1}$$

Here, σ_1^{ij} is the flipping operator σ_1 acting on the two state system $|1\,\rangle,\ |2\rangle$.

Our fast variables are all in their lowest energy state, which takes the same value at all points $|x_i\rangle$.
Thus we use the *expectation value* of Hamiltonian (1) as the new effective Hamiltonian:

$$\delta H_{ij}^{\mathrm{eff}} \;\rightarrow\; \langle\delta H_{ij}\rangle \;=\; \tfrac{1}{2}\pi\sigma_1^{ij}\cdot\tfrac{1}{\prod_i N_i} \quad ; \qquad\qquad \delta t = 1$$

Indeed, classically, the flip-flop takes place after time $\prod_i N_i$.

One can also use σ_2^{ij} and σ_3^{ij}, the same way.

And now, we can repeat this for all other flipflops, to obtain a Hamiltonian

$$H^{\mathrm{eff}} = \sum_{i<j,\,a} \tfrac{1}{2}\pi\sigma_a^{ij}\cdot\tfrac{n_{ij,a}}{\prod N_i}\ .$$

This way, for N_i and n_{ij} large enough, we can mimic almost any Hamiltonian for the slow system.

But how can this model be quantum mechanical and classical at the same time?

We are only using quantum terminology. This is QM , using perturbation theory w.r.t. the induced H^{eff}.

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However, There is a catch. All usual quantum paradoxes arise when you allow an observer or detector, to change the ontological basis at will. This freedom is not granted in our models. From the Big Bang onwards, we must assume that we have real, ontological variables that stay that way. Rotating your detector towards a non ontological basis is impossible.

Only ontological states evolve into other ontological states. The Ontology conservation law.

It holds for completely deterministic models, and it holds for QM. Only when ignoring this rule you get into paradoxes.

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Determinism all the way

Bob and Alice have no free will

Our proposal is now to take our models and perform the $1/N_i$ expansions.

If N_i are large enough, these expansions converge exactly as in quantum field theories.
They do not converge precisely, and nobody cares about that.

And there is a bonus: *The physical values of $1/N_i$ are rational.* This suggests that some values for the coupling strengths in the Standard Model (SM) will be preferred:

The N_i must be sufficiently small !!

Quantum gravity puts limits on the density of quantum states (black holes)

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- rotations
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- Local gauge symmetries
- BEH mechanism,
- General coordinate transformations ???

I think this question can be studied and understood. The answers might shed new lights on the >20 freely adjustable parameters of the SM.

Imagine what happens when you discretise physics:

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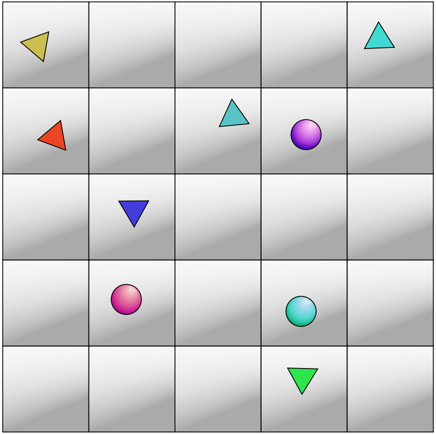
The Cellular Automaton: Only *classical* evolution equations.



Claim:

- A cellular automaton is mathematically equivalent to a genuine quantum field theory on a lattice.
- ? Every lattice quantum field theory can be accurately approximated by a *classical* cellular automaton.

Are there ways to link discrete systems to the Standard Model?

Suppose we want to construct the most direct classical model that would be linked to

free bosons in $D+1$ dimensions? For instance $D = 3$

http://arxiv.org/abs/2306.09885

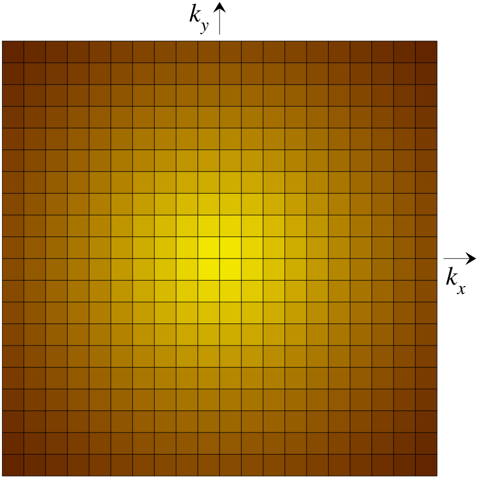
Bosons are harmonic oscillators.
The only complication is that they are *harmonically coupled* oscillators:

$$H = \frac{1}{2} \sum_i p_i^2 + \omega_{i\,j}\, q_i\, q_j \ .$$

This should be easy to handle: *diagonalise* $\omega_{i\,j}$.

Put the bosons inside a box with periodic boundary conditions, and Fourier transform. In this case, momenta $\vec{k}$ are discrete:

$$k_i L_i = 2\pi n_i \ , \qquad n_i = 0, \ \pm 1 \, , \ \pm 2 \, , \cdots$$



The function $\omega(\vec{k})$ in Fourier space is diagonal:

$$\omega = \sqrt{\vec{k}^2 + M^2}$$

This model has the same energy spectrum as a *h*armonic oscillator: $E_n = \omega\, n$

Therefore we can map these Hilbert spaces onto one another in the energy basis.

Our rotating variable on a circle *is* the quantum harmonic oscillator.

In the circular model, define the annihilation operator a and creation $a^\dagger$ as

$$\langle n-1|\, a\, |n\rangle = \sqrt{n}\,; \qquad \langle n|\, a^\dagger\, |n-1\rangle = \sqrt{n}\ . \quad \text{and}$$

$$p = \sqrt{\frac{\omega}{2}}(a + a^\dagger) \quad \text{and} \quad q = \frac{1}{i\sqrt{2\omega}}(a^\dagger - a)\ .$$

Then: $[a,\, a^\dagger] = 1\ ,\qquad E = \omega n = \omega a^\dagger a = \frac{1}{2}(p^2 + \omega^2 q^2 - 1)\ .$

Time dependence of a and $a^\dagger$:

$$a(t) = a(0)e^{-i\omega t}\ , \qquad a^\dagger(t) = a^\dagger(0)e^{i\omega t}\ .$$

Therefore: $$[a(t),\, a(0)] = 0\ , \qquad [a^\dagger(t),\, a^\dagger(0)] = 0\ .$$

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Can we use $a(t)$ as "ontological variable" for the harmonic oscillator ?
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I tried, but there are two problems:

1) We can only use either $a(t)$ or $a^\dagger(t)$, but not both ...
2) $a(t)$ is complex $\rightarrow$ its eigen states are all over the complex plane.

There is a better way.

Construct an operator $b(t)$ obeying:

$$\langle n-1 \bmod N\,|b|n\rangle = 1\ , \qquad \langle n|b^\dagger|n-1 \bmod N\,\rangle = 1\ .$$

Also here, $\quad b(t) = b(0)e^{-i\omega t}\ , \qquad b^\dagger(t) = b^\dagger(0)e^{i\omega t}\ .$

but now, $b(t)$ and $b^\dagger(t')$ all commute !

b and $b^\dagger$ are the ontological parameters of the harmonic oscillator.

We can write: $\;b \;=\; (1+a^\dagger a)^{-\frac{1}{2}}\,a \;=\; a(a^\dagger a)^{\frac{1}{2}}\ .$

We do have to write $\;n-1 \bmod N\;$,

but we can easily send $N\to\infty$ at the end.

In $\vec{k}$ space, all ontological states have indefinite numbers of bosons at every $\vec{k}$. Indeed he occupation numbers can be anything and all configurations have the same probability.

'in quantum field theory we take it for obvious that there are only very fe bosons in the entire space. So we must assume the initial state to have almost zero bosons. Then this means that all ontological states are equally probable.

Quantum field theories (QFT) are just harmonically coupled oscillators. Should that not be easy? This would prove that (in the absence of interactions) QFT can also be mapped onto ontological theories.

But first a possible complication:

Annihilation: $a = \dfrac{1}{\sqrt{2\omega}} p - i\sqrt{\tfrac{1}{2}\omega}\, q \ ,$ $\qquad [q, p] = i \ ;$

Creation: $a^\dagger = \dfrac{1}{\sqrt{2\omega}} p + i\sqrt{\tfrac{1}{2}\omega}\, q \ ,$ $\qquad [a, a^\dagger] = 1 \ .$

after which we write $\quad H = \omega\, a^\dagger a = \tfrac{1}{2}(p^2 + \omega q^2 - \omega) \, ; \qquad [a, H] = \omega H$

From these derive matrix elements: $\quad a^\dagger a = n;$

$$\langle n-1|a(t)|n\rangle = \sqrt{n}\, e^{-i\omega t} \ , \quad \langle n|a^\dagger(t)|n-1\rangle = \sqrt{n}\, e^{i\omega t} \ .$$

Notice time dependence.

The a operator could be used as ontological operator, since it commutes with itself at different times t. But a does not commute with $a^\dagger$.

We can do better. Introduce operators $b\,,\ b^\dagger$ with matrix elements

$$\langle n-1 \mod N\,|b(t)\,|n\rangle = e^{-i\omega t}\ , \quad \langle n|b^\dagger(t)\,|n-1 \mod N\ \rangle = e^{i\omega t}\ .$$

We also have $[b,\,H]=\omega H$, but now:

$[b(t),\,b^\dagger(0)]=0$; *and* $[b(t),\,b(0)]=0$.

In terms of the variables b and $b^\dagger$, the entire evolution process keeps them diagonalised. These must be the ontological variables we looked for.

Does this give us ontological bosons ?

If we know the $b(t)$ and $b^\dagger(t)$, all other operators $p\,,\ q\,,$ and H follow.

b only rotates. Its amplitude stays constant. Therefore, write

$b = e^{-i\varphi}$. Since $\varphi(t) = \omega t$, we can write

$$H = i\frac{\partial}{\partial t} = \omega\frac{\partial}{\partial\varphi} = -i\omega\frac{b\,\partial}{\partial b} .$$

The states $\varphi = n\omega t$ describe the rotation of b with time.

From there, define a and $a^\dagger$:

$$a = (1+H)^{\frac{1}{2}}b = bH^{\frac{1}{2}} ; \qquad a^\dagger = b^\dagger(1+H)^{\frac{1}{2}} = H^{\frac{1}{2}}b^\dagger .$$

Then fill in:

$$p = \sqrt{\frac{\omega}{2}}(a + a^\dagger) \quad \text{and} \quad q = \frac{1}{i\sqrt{2\omega}}(a^\dagger - a) .$$

Any element of Hilbert space can be written as a superposition of collapsed wave functions: Dirac delta peaks, in any basis you like .

But if an operator commutes with itself at all times, then delta peaks evolve as delta peaks.

The b operator is such an operator. Ergo,

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Let's try.

For quantum bosons in $D+1$ dimensions:

$$H = \int_V \mathrm{d}^3\vec{x}\big(\tfrac{1}{2}(\Pi^2(\vec{x}) + \vec{\nabla}\Phi(\vec{x})^2 + M^2\Phi^2(\vec{x})\big) \ ,$$

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$$H = \int_V \mathrm{d}^3\vec{x}\left(\tfrac{1}{2}(\underset{\substack{\uparrow\\ p^2}}{\Pi^2(\vec{x})} + \underset{\substack{\uparrow\\ (\vec{k}^{\,2}+M^2)q^2}}{\vec{\nabla}\Phi(\vec{x})^2 + M^2\Phi^2(\vec{x})}\right) ,$$

Put them in a box with periodic boundary conditions.
lengths $L_x,\ L_y,\ L_z$. Fourier transform. Then the coupled oscillators diagonalise. And the vectors $\vec{k}$ discretise:

$$H = \sum_{\vec{k}} \left(\tfrac{1}{2}(\Pi^2(\vec{k}\,) + (\vec{k}^{\,2} + M^2)\Phi^2(\vec{k}\,)\right),$$

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$$p^2 \qquad\qquad (\vec{k}^{\,2} + M^2)q^2$$

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Remember notation: $\Phi^2(\vec{k}) \equiv \Phi^\dagger(-\vec{k})\Phi(\vec{k})$.

Then do the same things as in previous slides,

The energy density operator $\mathcal{H}(\vec{k})$ diagonalises in $\vec{k}$ space. Use the energy density, write $\omega(\vec{k})$ and construct $b(\vec{k})$, which will also evolve by rotating in circles.

And then, Fourier back to get this operator in position space.

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SHOULDN'T IT ???

Things are not so simple. The field $b(\vec{k}\,)$ runs in circles. But $b(\vec{x}\,)$ is a superposition of many such fields. It does not seem to go in circles.

When Fourier transforming an arbitrary function back to $\vec{k}$ space, it won't go in circles.

The equations in slide number 26 do not seem to obey locality,
How do we guarantee the the constraint

$$|b(\vec{k}\,)|^2 = b^*(-\vec{k}\,)\,b(\vec{k}\,) \;\stackrel{?}{=}\; 1$$

It goes as follows.

But we can prove something else. In $\vec{x}$ space the field $b(\vec{x},t)$ field obeys:

$$\begin{aligned} \frac{\partial}{\partial t} b(\vec{x},t) &= -i \int \mathrm{d}\vec{y}\, F(\vec{y}-\vec{x}) b(\vec{y},t); \\ F(\vec{z}) &= \frac{1}{(2\pi)^3} \int \mathrm{d}^3\vec{k}\, \omega(\vec{k}) e^{i\vec{k}\cdot\vec{z}} \ , \\ \text{where} \qquad \omega(\vec{k}) &= +\sqrt{\vec{k}^{\,2}+M^2} \ . \end{aligned}$$

Squaring this gives: $\qquad (\partial_t^2 - \vec{\partial}_x^2 + M^2)\, b(\vec{x},t) = 0$, $\qquad$ a local equation!

And moreover this is the standard classical equation. We proved that not only the quantum theory is equivalent to a classical theory, it **is** <u>the</u> classical theory.

But then, this could also hold for the interactions.

Compare the quantum theory

$$H = \left(\tfrac{1}{2}(\Pi^2(\vec{k}\,) + (\vec{k}^{\,2} + M^2)\Phi^2(\vec{k}\,)\right) + \tfrac{\lambda}{4!}\Phi(\vec{x})^4,$$

with the corresponding classical theory. Can't there still be a mapping?

The quantum theory would just be the classical system in disguise.
Before shouting: *"Impossible !"* consider the following:

Assume we consider only that part of the theory where we allow only physical particles that have energies $E(\vec{k}) = \sqrt{\vec{k}^2 + M^2} < \Lambda$, a cutoff.

What is new in our formalism is that, even in the classical theory, one may define *energy* as in a quantum system. This means that the ontological fields $b(\vec{x}\,)$ do *not commute* with the energies.

Postulating that at large $\vec{k}$ values the energies are zero means in practice that these states include *all* possible values for the ontological states, in a statistical distribution that is completely flat

We can't diagonalise the energies and the ontological variables at the same time. The interaction terms $\frac{1}{4!}\lambda\Phi^4$ will couple things that are not all diagonalised.

That is where quantum mechanics comes from!

And now imagine what happens when you discretise physics:

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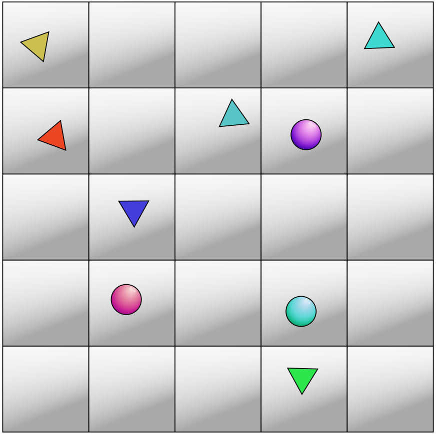
The Cellular Automaton: Only *classical* evolution equations.



Claim:

- A cellular automaton is mathematically equivalent to a genuine quantum field theory on a lattice.

•? Every lattice quantum field theory can be accurately approximated by a *classical* cellular automaton.

A cellular automaton is the prototype of a *deterministic* system. The evolution law is straightforward and requires no Hilbert space.
Yet, the mathematics of quantum operators is indispensable. It does not change the theory, but enables us to perform statistical calculations that otherwise would be impossible.

This is the cause of much confusion.

Consider the bosonic particles in our theory. They are harmonic oscillators and as such deterministic. Of course the evolution law defines an operator that *generates the evolution in time.* In all respects, this operator plays the role of a quantum Hamiltonian. It enables us to do the statistics for all events in such a model.

In a world of non-interacting bosons all this is obvious and straightforward.

The Hamiltonian generates super fast and super slow oscillations. This defines *energy*. At low energy (long time scales) it describes everything infinitely precisely.

At the highest energies, smallest time scales, also everything is infinitely precisely defined, but things happen much faster than we can register. Here, QM is not a theory but a mathematical tool. We introduce the energy eigen states for describing the fastest events. This requires very little memory space so it is very efficient.
It works even though *energy does not commute with time.* Frequencies close to Planck scale are all put in their ground states. This generates formidable amounts of efficiency, but alas also uncertainties. The zero-energy ground state, the vacuum, is a man-made quantum superposition of all states. It's a completely flat probability distribution. But the first excited state is a highly energetic, and in general a highly improbable, superheavy particle.

Interactions, in principle, don't change this situation. We can't do this exactly, because interactions would generate much more mixture and longer time scales. Strictly speaking, if you want infinite precision, you can't handle interactions, even if you know what they are.

But in practice, we can do perturbation expansion.

In a cellular automaton, the interactions between the bosons are "rare" events. They occur infrequently, when unlikely coincidences occur in the fast particles. We can use $1/N$ expansions.

Possibly all interaction coefficients ("fine-structure constants") originate this way

The problem is to guess what kinds of cellular automaton rules will produce the complex symmetry groups of the SM. The calculation presented here, may well provide for a trail to follow.

THANK YOU