The Gravitational Double Copy and Spin

Refs
- 1909.01358 "BCJ review"
- 2203.13013 "SAGEX review"
- 2204.06547 "Snowmass White paper"

Double copy:
- Ampl's in GR can be obtained from YM ampl's
- $GR \sim (YM)^2$

Classical solu's version

- Schwarzschild $\sim$ (Coulomb)^2

  $\downarrow$ spin $\qquad\qquad$ $\downarrow$ spin

- Kerr solution $\sim$ ("root-Kerr")

YM

$$\frac{1}{4 g_{YM}} \mathrm{Tr}\,(F_{\mu\nu})^2$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\,[A_\mu, A_\nu]$$

EoM: $\quad D_\mu F^{\mu\nu} = 0$

GR

$$\left(\kappa^2 = 32\pi G_N \right)$$

$$\frac{2}{\kappa^2} \sqrt{-g}\, R$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

Weak Coupling

$$A_\mu^a = c^a \varepsilon_\mu e^{ip\cdot x}$$

$$h_{\mu\nu} = \varepsilon_\mu \varepsilon_\nu e^{ip\cdot x}$$

Polarizations $\quad \varepsilon\cdot p = 0\,, \quad \varepsilon\cdot\varepsilon = 0$

$$\varepsilon^\mu \sim \varepsilon^\mu + p^\mu \quad \text{(gauge inv.)}$$

Two states

$\varepsilon_\mu^+,\ \varepsilon_\mu^-$ gluons

$\varepsilon_\mu^+\varepsilon_\nu^+,\ \varepsilon_\mu^-\varepsilon_\nu^-$ gravitons

3pt amplitudes

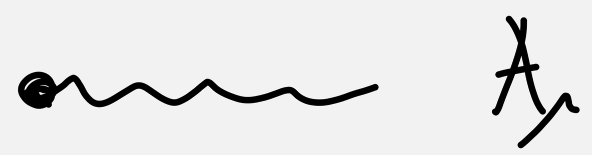
$$A(123) = g_{YM} f^{a_1 a_2 a_3} \overbrace{\left(\eta_{\mu\nu} (p_1 - p_2)_\rho + \text{cyclic} \underset{123}{(\mu\nu\rho)} \right)}^{V_{123}} \varepsilon_1^\mu \varepsilon_2^\nu \varepsilon_3^\rho$$

$$M(123) = \frac{\kappa}{2} \left[\left(\eta_{\mu\nu} (p_1 - p_2)_\rho + \text{cyclic} \underset{123}{(\mu\nu\rho)} \right) \varepsilon_1^\mu \varepsilon_2^\nu \varepsilon_3^\rho \right]^2$$

helicity basis

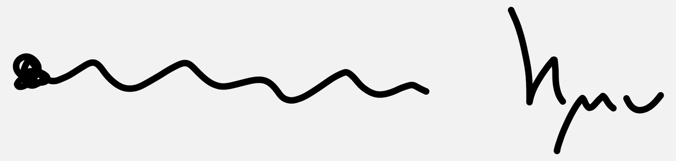
$$\Longrightarrow \quad A(1^- 2^- 3^+) = \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle}, \quad M(1^{--}, 2^{--}, 3^{++}) = \left(\frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle} \right)^2$$

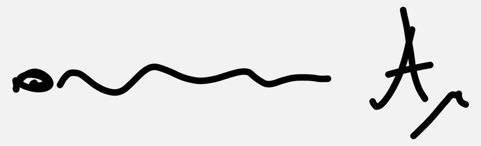
Diagrammatic double copy



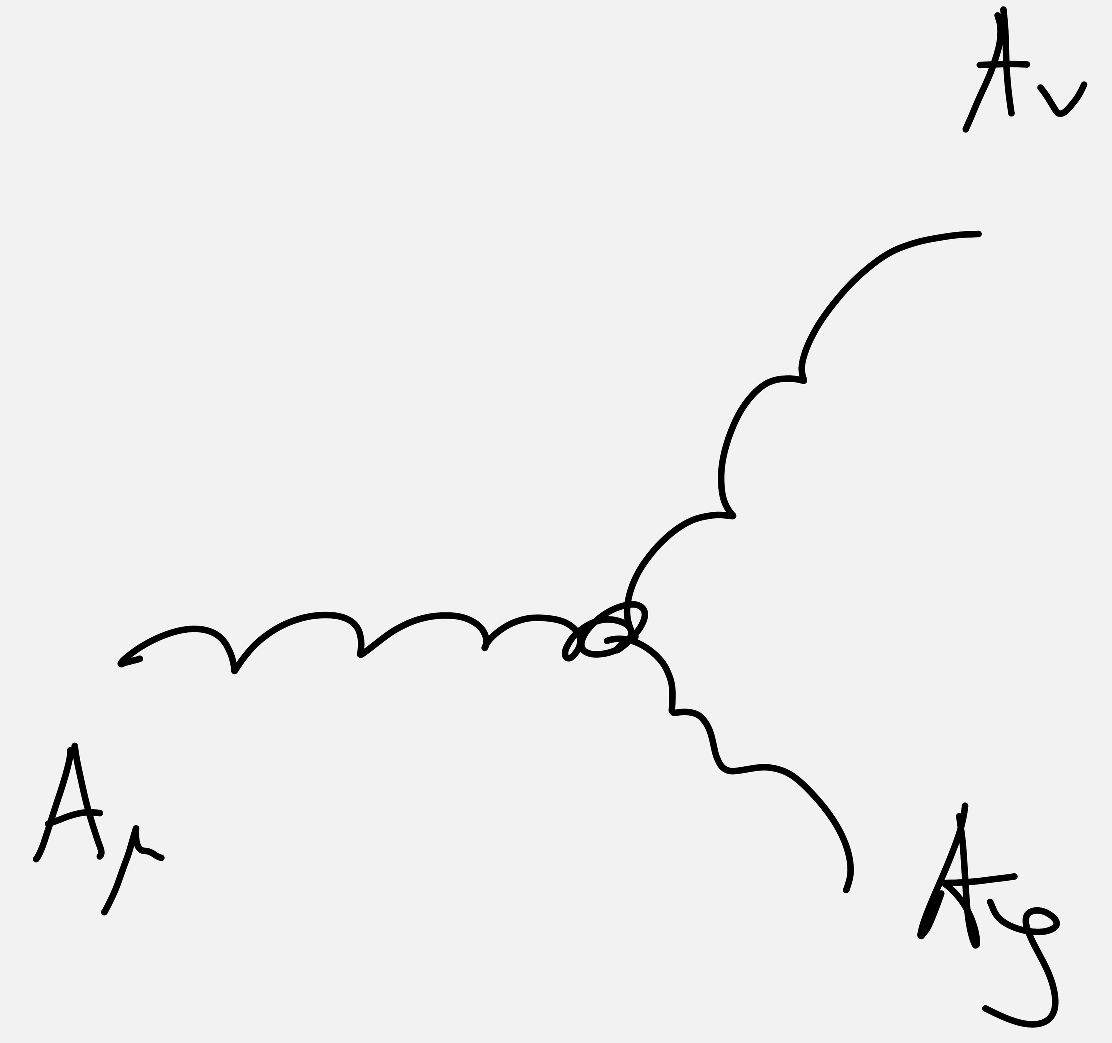
$\epsilon_\mu e^{ip\cdot x}$

$\Longleftrightarrow$

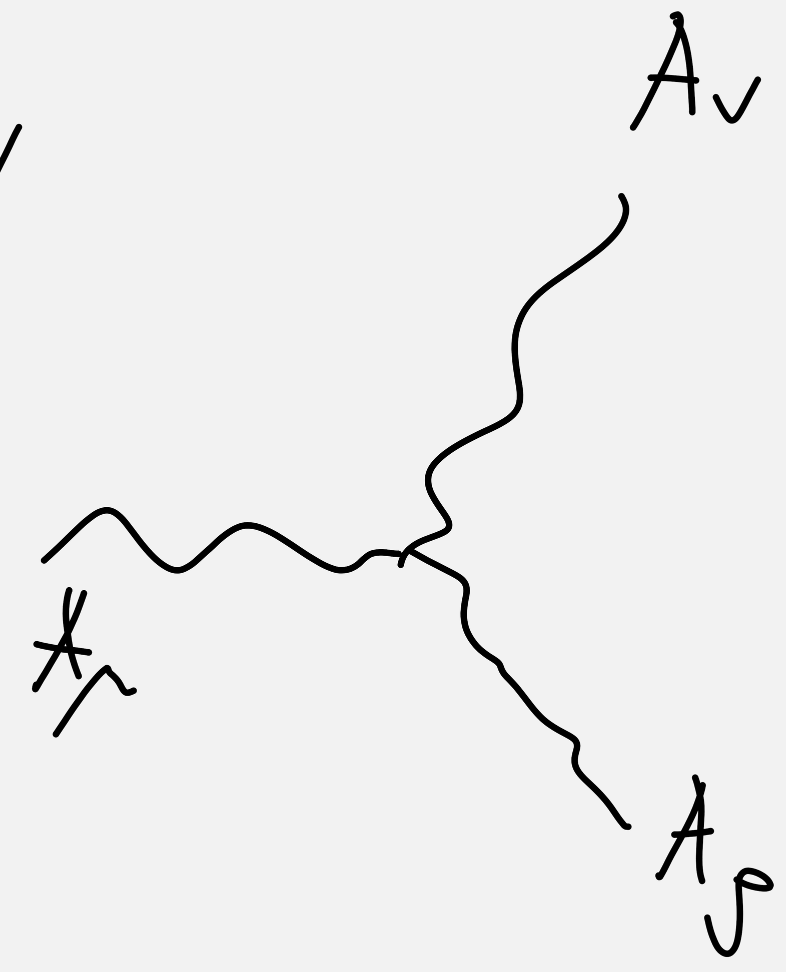


$= \Big($  $\Big)^2$

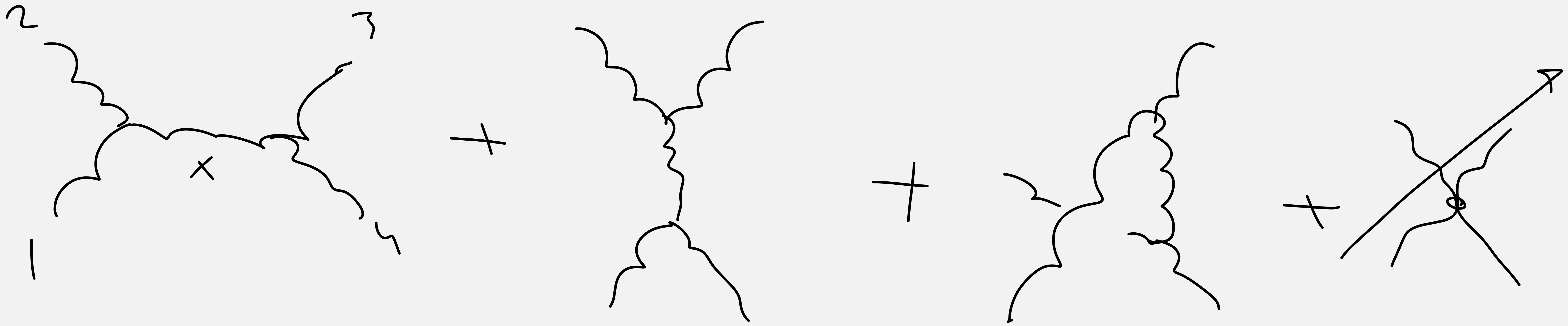
$\epsilon_\mu \epsilon_\nu e^{ip\cdot x}$



$\Longleftrightarrow$

$= \Bigg($  $\Bigg)^2$

4pt Tree amplitudes

$A_4^{YM} =$ 

0 absorbed into n_s, n_u, n_t

$$= \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

$$s = (p_1+p_2)^2$$
$$+$$
$$t = (p_1+p_4)^2$$
$$+$$
$$u = (p_1+p_3)^2$$
$$\parallel$$
$$0$$

$$n_s = V_{12x}^{\mu} V_{\mu x34} + s\left[(\varepsilon_1\cdot\varepsilon_3)(\varepsilon_2\cdot\varepsilon_4) - (\varepsilon_1\cdot\varepsilon_4)(\varepsilon_2\cdot\varepsilon_3)\right]$$

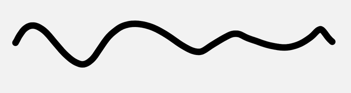
$$n_s + n_t + n_u = 0$$

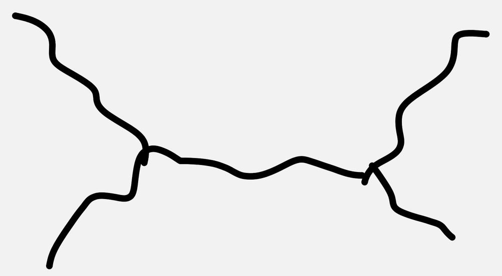
$c_s = f^{a_1 a_2 x} f^{x a_3 a_4}$

$c_t =$ perm

$c_u =$ perm

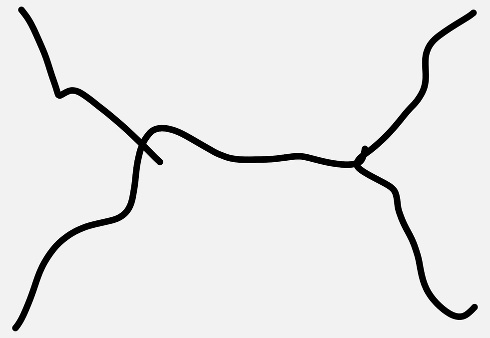
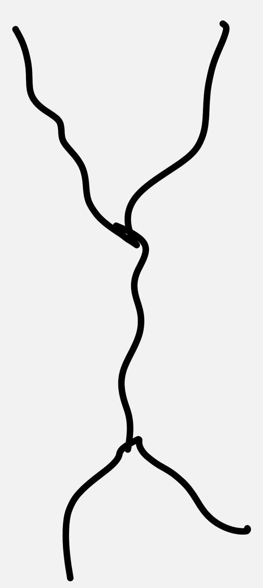
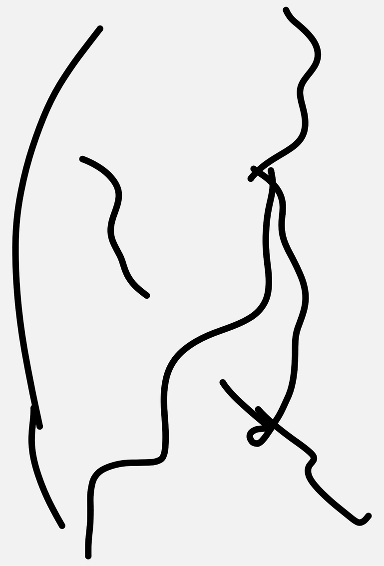
SU(N_c) Lie algebra

GR  ~ 100 ,  ~ 3

 $= 100^2 \times 3 = 30000$

$$f^{a_1 a_2 x} f^{x a_3 a_4} + \text{cyclic}(123) = c_s + c_t + c_u = 0$$

Kinematic Lie algebra ((S → Volume-pres Diffeos)

$n_s + n_t + n_u = 0 =$ n () $+ n$ () $+ n$ ()

Gravitational double copy

$$\mathcal{M}_4^{GR} = A_4^{YM}\Big|_{c_i \to n_i} = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

Why is this GR?

Claim: tree-level GR uniquely fixed by

1) Unitarity $\Rightarrow$ factorization & physical states h^{++}, h^{--}

2) 2-derivative interactions

3) Diffeo inv.

Check Diffeo inv.

$$\delta g_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$\delta A_\mu = \partial_\mu \xi$$

gauge parameters

Consider only variation of ε_4, $\delta \varepsilon_4 = p_4$, $\delta \varepsilon_1, \varepsilon_2, \varepsilon_3 = 0$

Lie algebra

$$\delta A_4^{YM} = \frac{c_s\, \delta n_s}{s} + \frac{c_t\, \delta n_t}{t} + \frac{c_u\, \delta n_u}{u} = \alpha\,(c_s + c_t + c_u) = 0$$

$$\left(HW: \quad \frac{\delta n_s}{s} = \frac{\delta n_u}{u} = \frac{\delta n_t}{t} = \alpha(\varepsilon_i, p_i) \right)$$

Kinematic algebra

$$\delta M_4^{GR} = 2\left(\frac{n_s\, \delta n_s}{s} + \frac{n_t\, \delta n_t}{t} + \frac{n_u\, \delta n_u}{u} \right) = \alpha\,(n_s + n_t + n_u) = 0$$

Gravitational double copy @ n points

n-point YM : $\mathcal{A}_n^{YM} = \sum_{i \in \{\text{cubic diagrams}\}} \frac{c_i \, n_i}{D_i}$

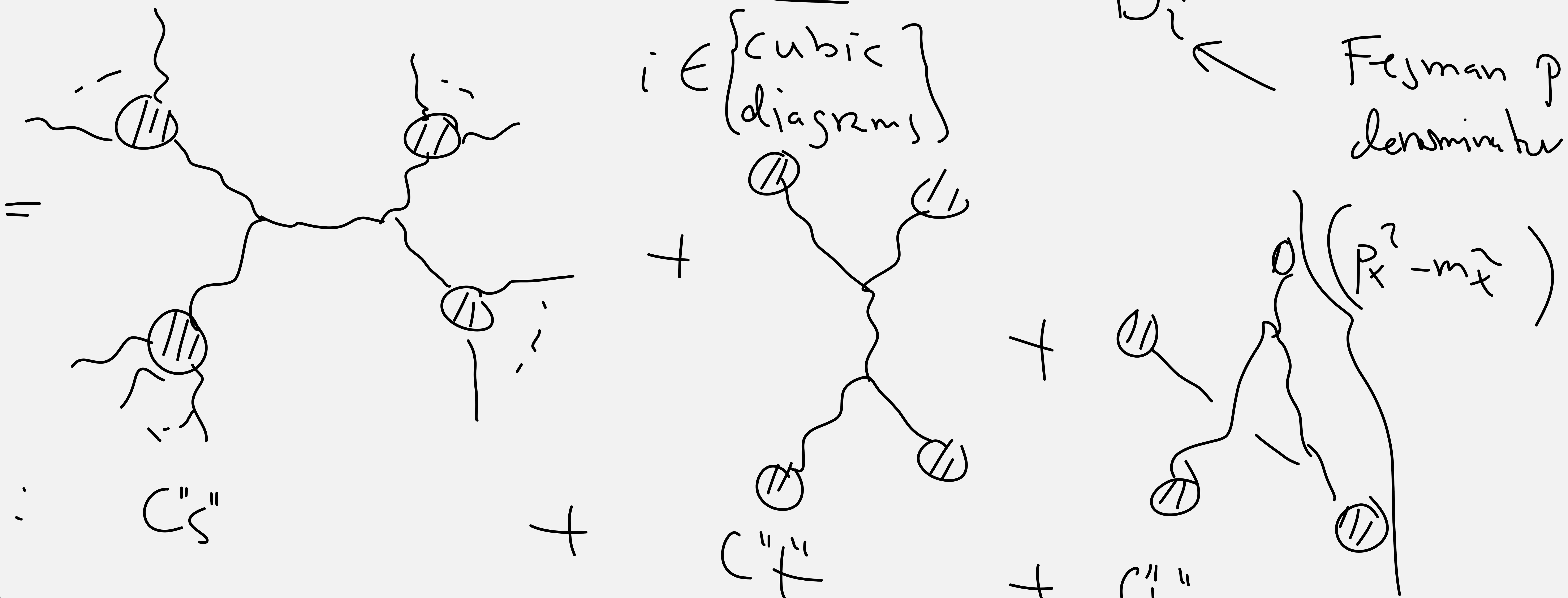
color factors from f^{abc}'s

kinematic numerators

Feynman propagator denominator

Consider all 3-term Jacobi Id

$0 =$



$(p_x^2 - m_x^2)$

Color-Lie alg : $c_{\text{"}s\text{"}} + c_{\text{"}t\text{"}} + c_{\text{"}u\text{"}} = 0$

Kinematic Lie alg $n_s + n_t + n_u = 0$

n-pt GR : $M_n^{GR} = \mathcal{A}_n^{YM}\Big|_{c_i \to n_i} = \sum_i \frac{n_i^2}{D_i}$

Comments

1) Note numerators n_i are gauge dependent
hence non-trivial task to find $n_{s\cdots} + n_{t\cdots} + n_{u\cdots} = 0$

2) Double copy generalizes to loop level
numerators unknown (in general) $\Longleftrightarrow$ kinematical algebra unknown

3) When including matter $\Longrightarrow$ Web of theories that double copy