Eigenstate Thermalization, Interlinking and the Emergence of Classical Physics in Quantum Theory

mostly based on:

Lecture 1:

Tobias Helbig, Tobias Hofmann, Ronny Thomale, and MG,

Theory of Eigenstate Thermalization, in preparation.

Lecture 2:

MG, *Interlinking and the Emergence of Classical Physics in Quantum Theory,*

submitted to SciPost, arXiv:2112.07040

Lecture 1

Theory of Eigenstate Thermalization

Part I

1. Eigenstate thermalization hypothesis (ETH)
2. Consistency requirements and universal eigenvalue distribution for locally interacting quantum systems
3. Statistical mechanics from the microcanonical ensemble

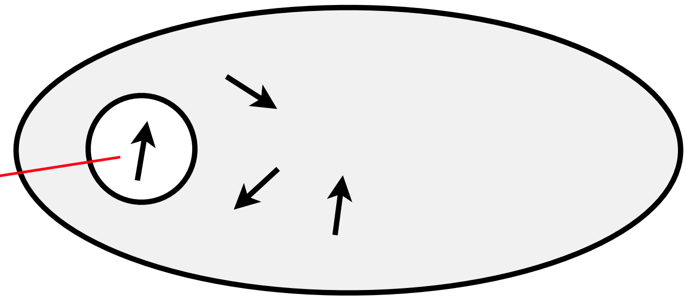
Part II

4. Numerical decomposition of eigenstates: system S & bath B
5. Results form Dyson-Brownian motion random matrix theory
6. Remarks and summary

1. Eigenstate thermalization hypothesis

Josh Deutsch 1991, Marc Sredenicki 1994

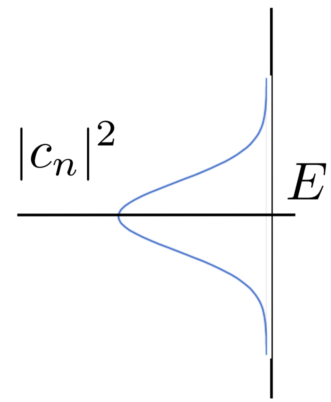
closed quantum system with energy $\approx E$



local perturbation of $\hat{A}$ at $t = 0$

at $t = 0$ the system is in state $|\psi(t)\rangle = \sum_n c_n |n\rangle$

$$\hat{H}|n\rangle = E_n |n\rangle$$



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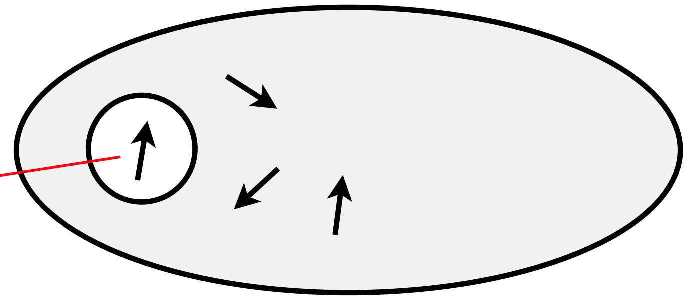

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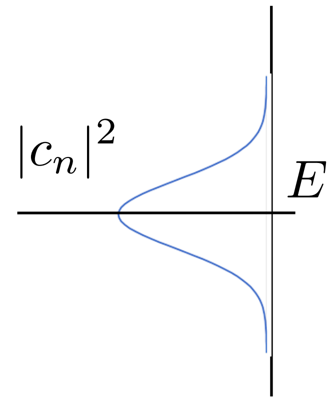

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How is this possible (given the deterministic evolution)?

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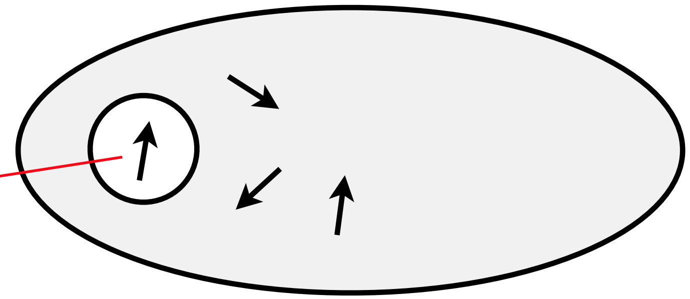

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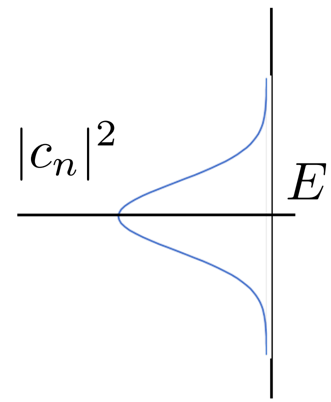

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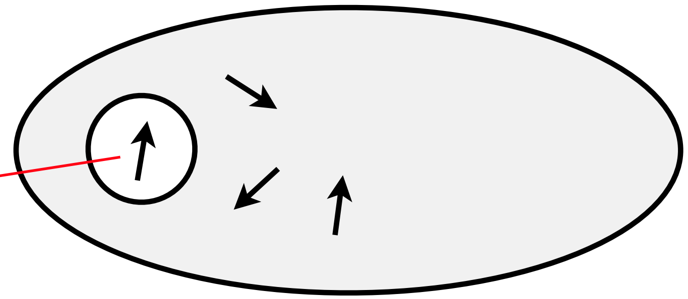

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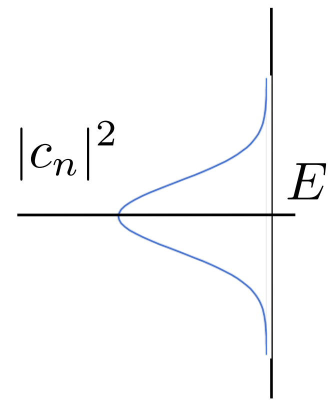

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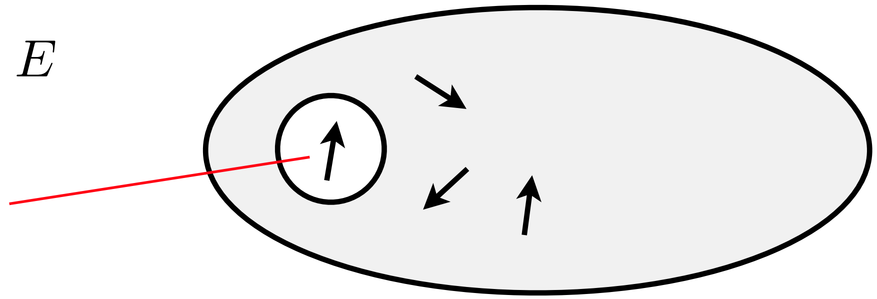

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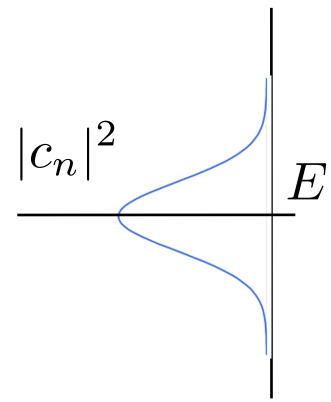

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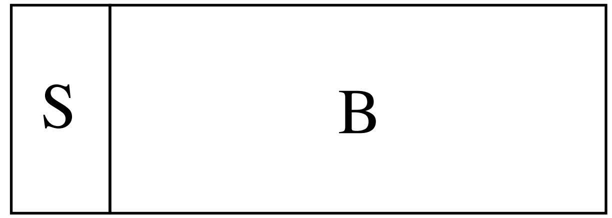
$$\approx \sum_n |c_n|^2 \cdot \langle m| \hat{A} |m\rangle \quad \text{with } E_m \approx E$$

ETH: independent of n for $E_n \approx E$

More concise statement of the ETH:

entire system in eigenstate $|\psi_n\rangle,\ \hat{H}\,|\psi_n\rangle = \lambda_n\,|\psi_n\rangle$



(sub-)system   bath

$$\hat{\rho}_{\mathrm{S}}^{n} \equiv \mathrm{Tr}_{\mathrm{B}}\,|\psi_n\rangle\,\langle\psi_n|$$

ETH $\|$

$$\hat{\rho}_{\mathrm{S}}(\beta) \equiv \mathrm{Tr}_{\mathrm{B}}\,\hat{\rho}(\beta) \quad \text{with} \quad \hat{\rho}(\beta) = \frac{1}{Z}\,e^{-\beta\hat{H}}$$

thermal density matrix

inverse temperature β fixed by $$\langle\hat{H}\rangle_\beta = -\frac{\partial}{\partial\beta}\ln Z = \lambda_n,\ Z = \mathrm{Tr}\,e^{-\beta\hat{H}} \qquad (*)$$

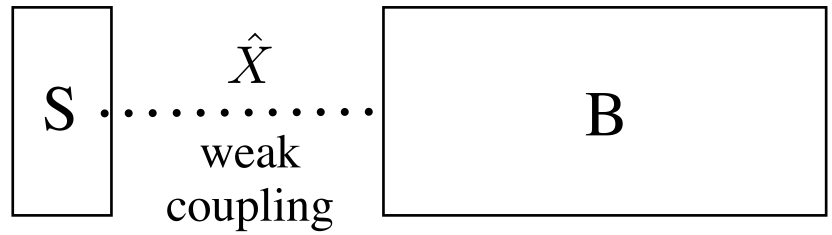
thermal energy $=$ energy of eigenstate

The local properties of eigenstates are indistinguishable from those of a thermal state!

The ETH does not (fully) apply to systems with extensive symmetries
(integrable systems, many-body localization, quantum scars).

2. Consistency requirements

If we place the entire system in a thermal state



$$\hat{\rho}(\beta) = \frac{1}{Z(\beta)} \sum_n e^{-\beta E_n} \left|\psi_n\right\rangle \left\langle\psi_n\right|$$

and apply the ETH to each of them, we get a weighted sum of canonical distributions with different temperatures β_n for the subsystem S:

$$\mathrm{Tr}_{\mathrm{B}} \left|\psi_n\right\rangle \left\langle\psi_n\right| = \frac{1}{Z_{\mathrm{S}}(\beta_n)} \sum_\mu e^{-\beta_n \varepsilon_\mu} \left|\phi_\mu^{\mathrm{S}}\right\rangle \left\langle\phi_\mu^{\mathrm{S}}\right| \qquad \hat{H}^{\mathrm{S}} \left|\phi_\mu^S\right\rangle = \varepsilon_\mu \left|\phi_\mu^S\right\rangle$$

$$\mathrm{Tr}_{\mathrm{B}} \hat{\rho}(\beta) = \sum_\mu \underline{\frac{1}{Z(\beta)} \sum_n e^{-\beta E_n} \frac{1}{Z_{\mathrm{S}}(\beta_n)} e^{-\beta_n \varepsilon_\mu}} \left|\phi_\mu^{\mathrm{S}}\right\rangle \left\langle\phi_\mu^{\mathrm{S}}\right|$$

$$? \;\; = \frac{1}{Z_{\mathrm{S}}(\beta)} e^{-\beta \varepsilon_\mu} \;\; ?$$

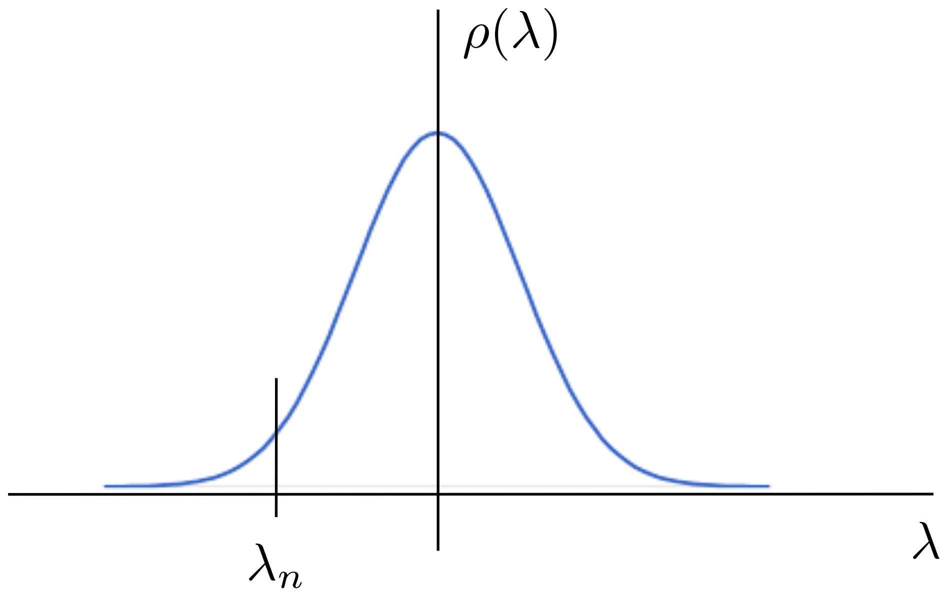
A sum of exponential functions usually does not give a single exponential!

However, it works out if the eigenvalue density of the entire system is given by a Gaussian, and the bath B >> subsystem S

Theorem (Hartmann, Mahler, Hess (2005), Hofmann, Helbig, Thomale, MG, to appear):
The eigenvalue density of a locally interacting quantum system approaches a Gaussian in the limit of large systems

$$\rho(\lambda) = \sqrt{\frac{\alpha}{2\pi}}\, e^{-\frac{1}{2}\alpha\lambda^2}$$

where $\frac{1}{\alpha} = \sigma^2_{\rm tot}$ is the (matrix) variance of the entire system



Expansion around λ_n yields:

$$\rho(\lambda) \propto e^{\beta_n \lambda} \quad \text{with} \quad \beta_n = \frac{\partial \ln \rho(\lambda)}{\partial \lambda}\bigg|_{\lambda=\lambda_n} = -\alpha\lambda_n \qquad (**)$$

why this is the (inverse) temperature β_n will become clear below!

another consistency condition:

According to the ETH, β_n will also be connected to λ_n via $(*)$:

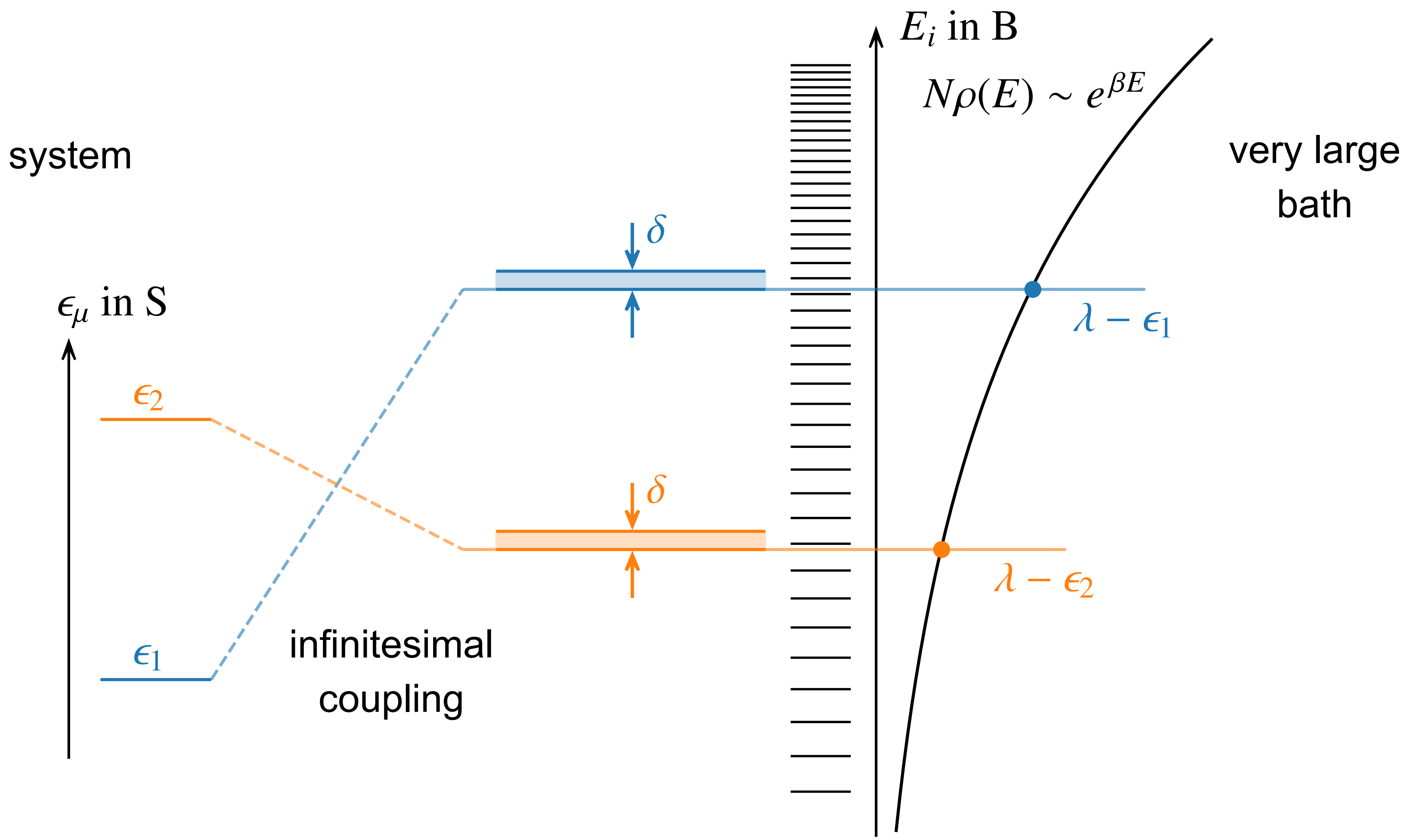
$$Z(\beta) = N \int \mathrm{d}\lambda \ \rho(\lambda) \, e^{-\beta\lambda} = N \exp\left(\frac{\beta^2}{2\alpha}\right)$$

$$\lambda_n = \langle \hat{H} \rangle_{\beta_n} = -\frac{\partial}{\partial\beta} \ln Z(\beta) \Big|_{\beta=\beta_n} = -\frac{\beta_n}{\alpha}$$ is equivalent to $(**)$!

This condition relates the derivative of $\rho(\lambda)$ to a weighted integral over $\rho(\lambda)$.

It is hard to see how any eigenvalue distribution different from the Gaussian $\rho(\lambda)$ {with arbitrary width and normalization) can satisfy this.

3. Statistical mechanics from the microcanonical ensemble

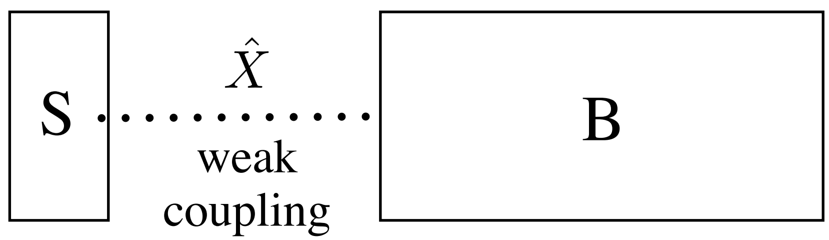


same probability for all states with energy in $[\lambda, \lambda + \delta]$ → probability of level μ: $w_\mu \propto e^{-\beta \varepsilon_\mu}$

4. Numerical decomposition of eigenstates

entire system $\hat{H} = \hat{H}^{\mathrm{S}} + \hat{H}^{\mathrm{B}} + \hat{X}$ basis: $\hat{H}\,|\psi_n\rangle = E_n\,|\psi_n\rangle$

(sub-)system bath



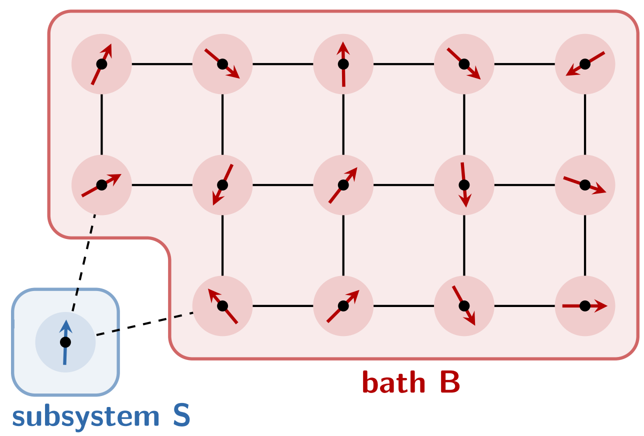
$\hat{H}^{\mathrm{S}}\,|\phi_\mu^S\rangle = \varepsilon_\mu\,|\phi_\mu^S\rangle$ $\hat{H}^{\mathrm{B}}\,|\phi_i^B\rangle = E_i\,|\phi_i^B\rangle$ $|\phi_{\mu i}\rangle \equiv |\phi_\mu^{\mathrm{S}}\rangle \otimes |\phi_i^{\mathrm{B}}\rangle$

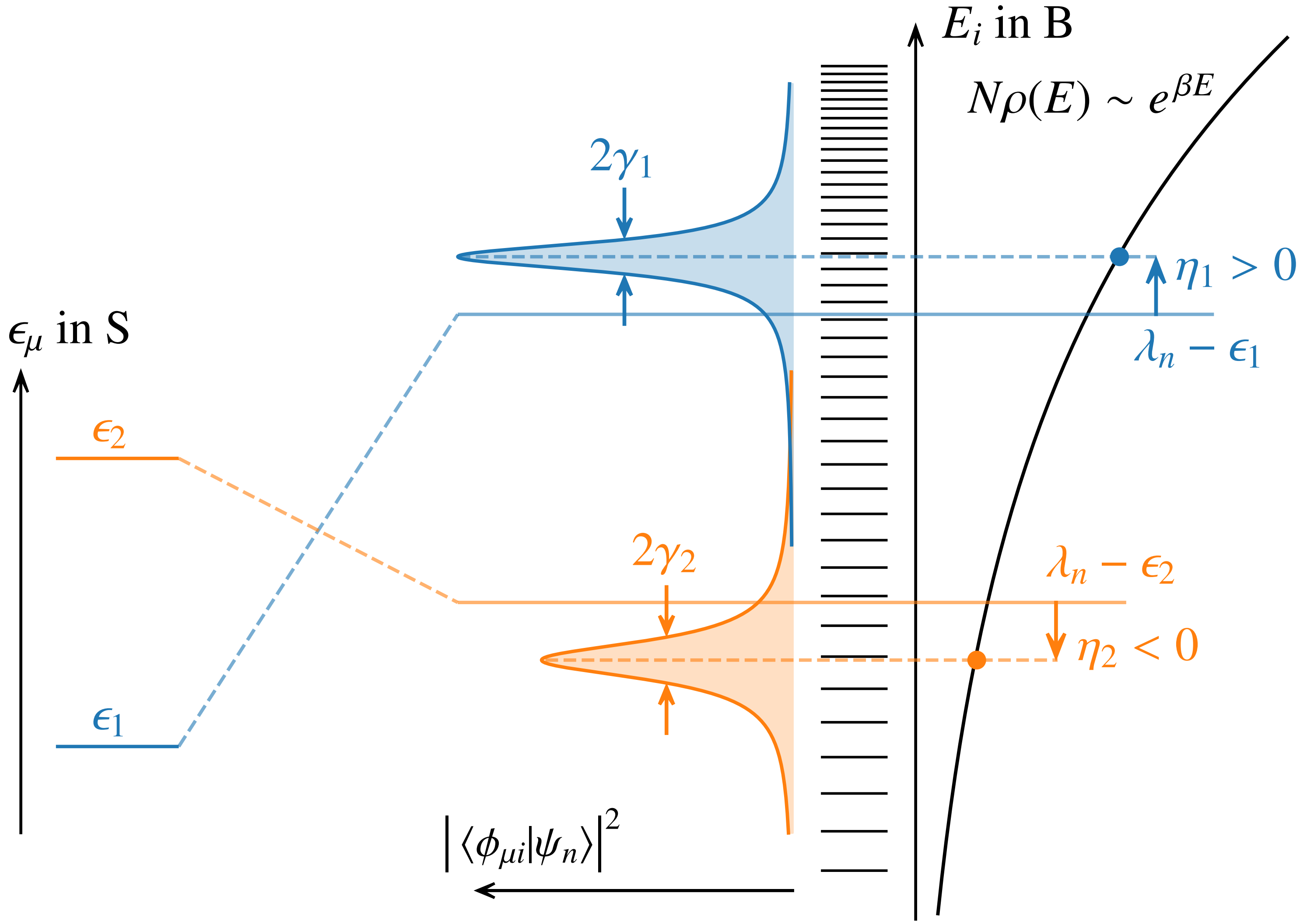
$\mu = 1, \ldots, N_{\mathrm{S}}$ $i = 1, \ldots, N_{\mathrm{B}}$

calculate the overlap

$$|\langle \phi_{\mu i}|\psi_n\rangle|^2$$

for 2000 states for 2 x 50 different random couplings for a single spin coupled to a bath of 15 interacting spins to obtain the statistical expectations values





We find perfect Lorentzians for the ensemble averages of the overlap curves, which differ in width and height, but enclose exactly the same area, and are shifted relative to $\lambda_n - \varepsilon_\mu$.

Shifts are due to (1) "level repulsion" (2) $\frac{1}{\alpha} = \sigma^2_{\mathrm{tot}}$ increases due to the coupling.

This implies that we obtain canonical distributions for the (sub-)system S, but with probabilities according to shifted levels.

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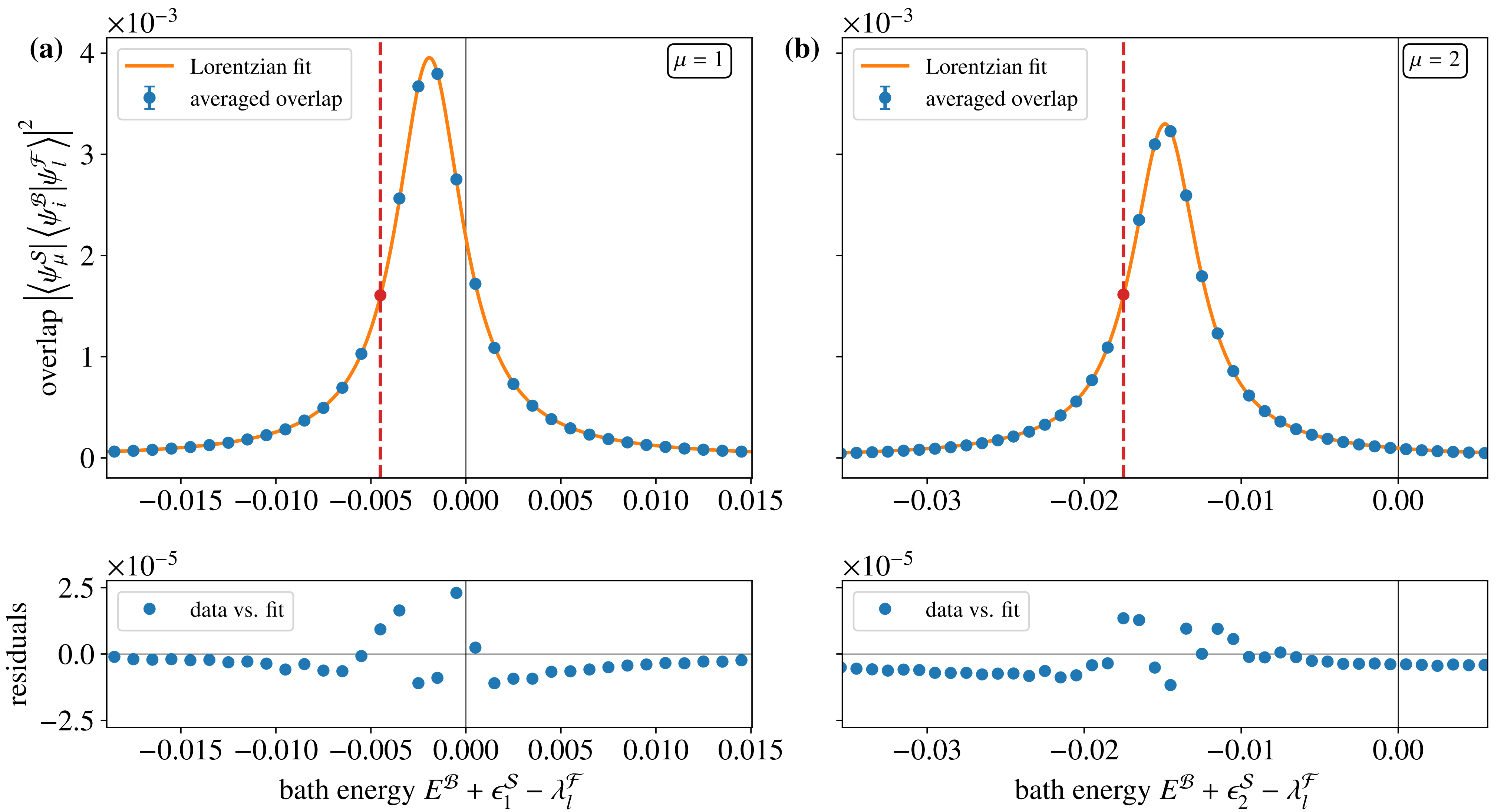
sample of actual data:



5. Results form Dyson-Brownian motion random matrix theory

overlaps $\chi_{\mu i,n} = \mathbb{E}\big[|\langle\phi_{\mu i}|\psi_n\rangle|^2\big]$ are given by Cauchy-Lorentz distributions

$$\chi_{\mu i,n} \propto \frac{\gamma_\mu}{\big(a_{\mu i} - \lambda_n - \eta_\mu\big)^2 + \gamma_\mu^2} \qquad \text{where } a_{\mu i} \equiv \varepsilon_\mu + E_i$$

To first order in the matrix variance t of the perturbation $\hat{X}$

$$t = \frac{1}{N}\,\mathbb{E}[\,\mathrm{Tr}(\hat{X}^2)\,]$$

we find:

1. The shifts are given by $\eta_\mu = -t\frac{\sqrt{\pi}}{N_\mathrm{S}\Delta}\sum_\nu e^{-\tilde{\varepsilon}_{\mu\nu}^2}\,\mathrm{erfi}(\tilde{\varepsilon}_{\mu\nu})$

   where: $\mathrm{erfi}(z) \equiv -\mathrm{i}\,\mathrm{erf}(\mathrm{i}z)$ is the imaginary error function

   $$\tilde{\varepsilon}_{\mu\nu} \equiv \tfrac{1}{\Delta}\varepsilon_{\mu\nu} - b,\ \varepsilon_{\mu\nu} \equiv \varepsilon_\mu - \varepsilon_\nu,\ b \equiv \tfrac{1}{4}\beta\Delta,$$

   $\Delta \approx$ bandwidth of the part of the bath $\hat{X}$ couples to.

2. The half-widths of the Lorentzians are given by

$$\gamma_\mu = -t \frac{\sqrt{\pi}}{N_\mathrm{S}\Delta} \sum_\nu e^{-\tilde{\varepsilon}_{\mu\nu}^2}$$

3. While the widths and heights of the Lorentzians depend on the index μ, the integrated areas beneath them do not.

   Since expansion of the Gaussian eigenvalue density yields $\rho_\mathrm{B}(E) \propto e^{\beta E}$, the reduced density matrix of the system S has diagonal entries

$$\langle \phi_\mu^\mathrm{S} | \, \hat{\rho}_\mathrm{S} \, | \phi_\mu^\mathrm{S} \rangle \propto e^{-\beta\,(\varepsilon_\mu - \eta_\mu)}$$

   The off-diagonal entries vanish as $\frac{1}{\sqrt{N_\mathrm{B}}}$, and hence exponentially, with the size of the bath B.

4. When we spectroscopically observe level spacings in S, we do not observe the bare levels ε_μ, but the shifted levels $\varepsilon_\mu - \eta_\mu$. Therefore it is appropriate that they, and not the bare levels, enter the distribution.

6. Remarks and Summary

1. Eigenstate thermalization

   (a) requires a Gaussian eigenstate density, which is always the case for locally interacting quantum systems.

   (b) can be derived using Dyson-Brownian motion random matrix theory.

2. Our analysis provides a derivation of statistical mechanics which requires neither the concept of ergodicity or typicality, nor that of entropy. Thermodynamic behaviour follows solely from the applicability of quantum mechanics to large systems, locality, and the absence of integrability.

Lecture 2

Interlinking and the Emergence of Classical Physics in Quantum Theory

Part I

1. Review of Quantum Mechanics
2. Problems with Copenhagen
3. Many Worlds Interpretations

Part II

4. Interlinking and the ensemble of macroscopic objects (EMO)
5. Measurements, Schrödinger's cat, and EPR
6. Is there a collapse?

1. Review of Quantum Mechanics

(a) At any fixed time, the state of a system is described by a (normalized) vector in Hilbert space, $|\psi\rangle$.

(b) The state vector evolves according to Schrödinger's equation,

$$\mathrm{i}\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle\,, \tag{1}$$

where the Hamilton H of the system is a linear, self-adjoint operator.

(c) Observables are likewise described by linear, self-adjoint operators. If $|\psi\rangle$ is in an eigenstate of an observable A, $A|\psi\rangle = a|\psi\rangle$, the observed value of A is a.

(d) Every measurement of A with a device described by classical physics yields one of the eigenvalues of A. The probability of finding a particular eigenvalue a when measuring $|\psi\rangle$ is $\|P(A,a)|\psi\rangle\|^2$, where $P(A,a)$ is the projection operator on the subspace of states with eigenvalue a (Born's rule). If A is measured again thereafter, the observed value will be a again.

According to the Copenhagen interpretation of 1927, (d) implies that the state of the system after the first measurement is given by

$$\frac{P(A,a)\,|\psi\rangle}{\|P(A,a)\,|\psi\rangle\|}. \tag{3}$$

This is referred to as the "collapse" of the wave function. The probability in the process is assumed to be frequentist, i.e., due to the occurence of random events, as opposed to a Bayesian probability, which is a subjective probability an observer assigns due to inaccessibility of information.

2. Problems with Copenhagen

1. Schrödingers equation is deterministic, does not describe a collapse

2. When should a collapse occur? Schrödinger’s cat

... or more generally: Where is the quantum-classical boundary?

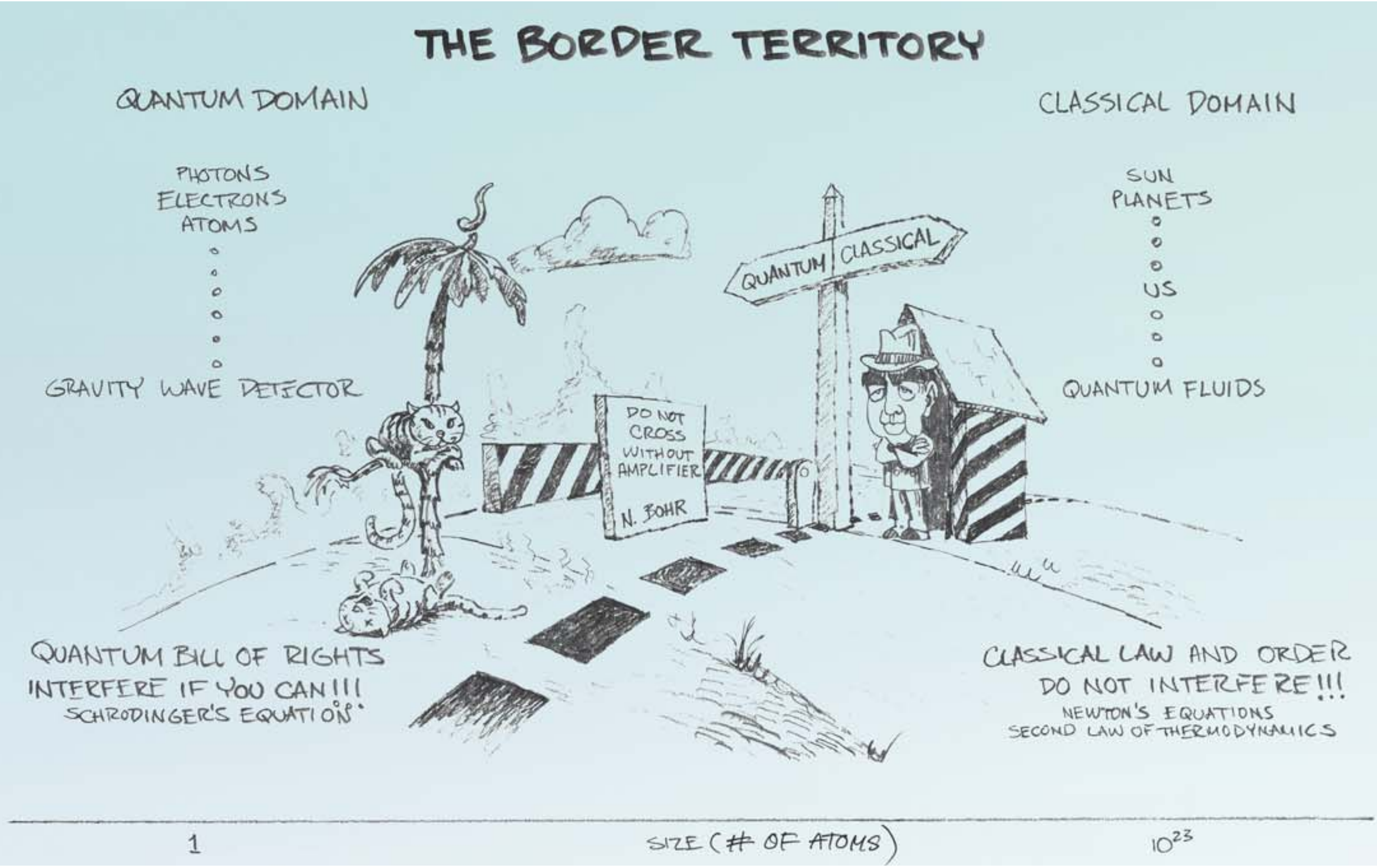
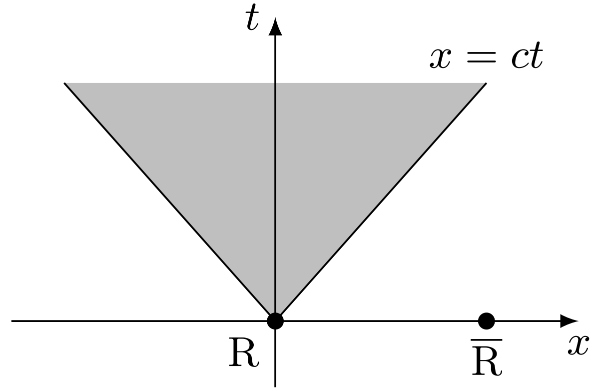


image from: Zurek, W. H., 2002 [1991], Los Alamos Science **27**, 1.

3. Einstein-Podolsky-Rosen (EPR) Paradox

prepare a spin singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\big(|{\uparrow_1}\rangle \otimes |{\downarrow_2}\rangle - |{\downarrow_1}\rangle \otimes |{\uparrow_2}\rangle\big),$$



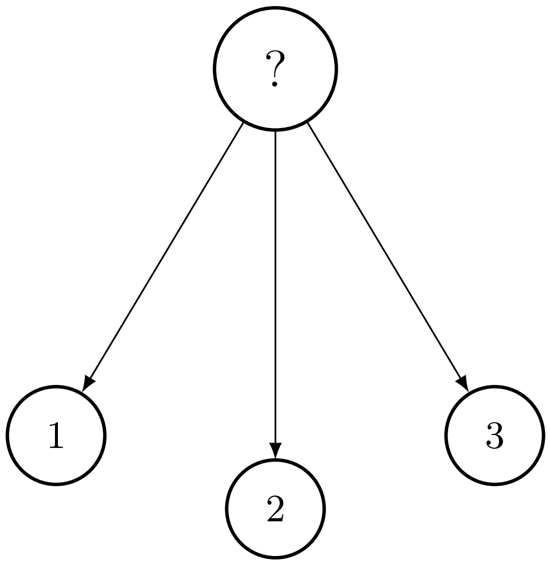
measure $\sigma^{z}_{(1)}$ and $\sigma^{z}_{(2)}$ in space-like separated regions R and $\overline{\mathrm{R}}$.

$\rightarrow$ product of eigenvalues will always be -1 when compared later on.

$\rightarrow$ probabilities cannot be both frequentist.

This led EPR to believe in hidden variables, a possibility ruled out since through tests of Bell's inequality.

A pedagogically outstanding way how this can be done in principle is due to Greenberger, Horne, and Zeilinger (GHZ, 1989) send out samples to three space-like separated stations, where we measure either A or B:



We obtain records like

$$
\begin{array}{lll}
A_1 = 1 & B_2 = -1 & B_3 = -1 \\
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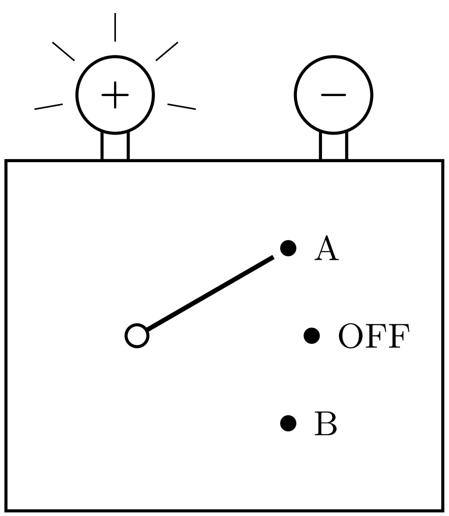
B_1 = 1 & B_2 = 1 & A_3 = 1 \\
& \dots &
\end{array}
$$



We find whenever we have measured $A_1B_2B_3$ it is $+1$. Likewise for $B_1A_2B_3$ and $B_1B_2A_3$.

If the outcomes were predetermined, this would imply

$$A_1A_2A_3 = A_1B_2B_3 \cdot B_1A_2B_3 \cdot B_1B_2A_3 = +1$$

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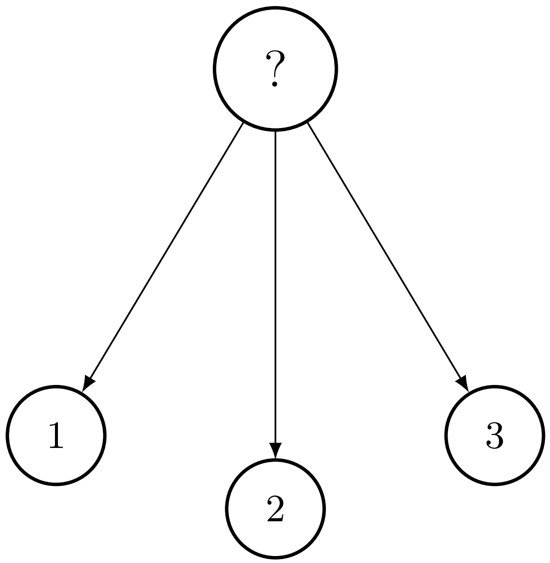

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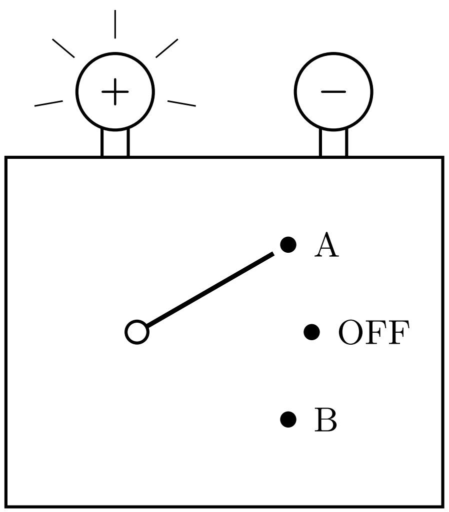

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for GHZ states, however, we measure $A_1 A_2 A_3 = -1$

How does this work?

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle\right]$$

$$A_1 = \sigma_x^{(1)} \quad B_1 = \sigma_y^{(1)} \quad \text{etc.}$$

$$A_1 B_2 B_3 |\psi\rangle = \sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)} |\psi\rangle = |\psi\rangle$$

etc. for $B_1 A_2 B_3$ and $B_1 B_2 A_3$.

But ...

$$A_1 A_2 A_3 |\psi\rangle = \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)} |\psi\rangle = -|\psi\rangle$$

What spooky action-at-a-distance?

3. Many Worlds Interpretations

The idea is simply that there is only quantum mechanics, and only deterministic evolution according to Schrödinger's equation. Measurements do not entail a projection or collaps of the wave function. The definiteness of our daily life experience is subjective only. All probabilities are Bayesian.

consider a spin half particle $\quad |\psi\rangle = u\,|\uparrow\rangle + v\,|\downarrow\rangle$

measuring device $\quad \mathrm{M} = \{|\mathrm{M}_0\rangle\,, |\mathrm{M}_\uparrow\rangle\,, |\mathrm{M}_\downarrow\rangle\}$

observer consciousness $\quad \mathrm{C} = \{|\mathrm{C}_0\rangle\,, |\mathrm{C}_\uparrow\rangle\,, |\mathrm{C}_\downarrow\rangle\}$

world $\quad \mathrm{W} = \{|\mathrm{W}_0\rangle\,, |\mathrm{W}_\uparrow\rangle\,, |\mathrm{W}_\downarrow\rangle\}$

Measurements in Many Worlds

A measurement evolves the initial state

$$|\psi_{\mathrm{i}}\rangle = \big(\underline{u\,|{\uparrow}\rangle} + \underline{v\,|{\downarrow}\rangle}\big) \otimes |\mathrm{M}_0\rangle \otimes |\mathrm{C}_0\rangle$$

following the von Neumann chain via the intermediate states

$$|\psi_{\mathrm{M}}\rangle = \big(u\,|{\uparrow}\rangle \otimes |\mathrm{M}_{\uparrow}\rangle + v\,|{\downarrow}\rangle \otimes |\mathrm{M}_{\downarrow}\rangle\big) \otimes |\mathrm{C}_0\rangle$$

$$|\psi_{\mathrm{M,C}}\rangle = u\,|{\uparrow}\rangle \otimes |\mathrm{M}_{\uparrow}\rangle \otimes |\mathrm{C}_{\uparrow}\rangle + v\,|{\downarrow}\rangle \otimes |\mathrm{M}_{\downarrow}\rangle \otimes |\mathrm{C}_{\downarrow}\rangle$$

into the final state

$$|\psi_{\mathrm{f}}\rangle = \underline{u|\uparrow\rangle \otimes |\mathrm{M}_{\uparrow}\rangle \otimes |\mathrm{C}_{\uparrow}\rangle \otimes |\mathrm{W}_{\uparrow}\rangle} + \underline{v|\downarrow\rangle \otimes |\mathrm{M}_{\downarrow}\rangle \otimes |\mathrm{C}_{\downarrow}\rangle \otimes |\mathrm{W}_{\downarrow}\rangle}.$$

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following the von Neumann chain via the intermediate states

$$|\psi_{\mathrm{M}}\rangle = \big(u\,|{\uparrow}\rangle \otimes |\mathrm{M}_{\uparrow}\rangle + v\,|{\downarrow}\rangle \otimes |\mathrm{M}_{\downarrow}\rangle\big) \otimes |\mathrm{C}_0\rangle$$

$$|\psi_{\mathrm{M,C}}\rangle = u\,|{\uparrow}\rangle \otimes |\mathrm{M}_{\uparrow}\rangle \otimes |\mathrm{C}_{\uparrow}\rangle + v\,|{\downarrow}\rangle \otimes |\mathrm{M}_{\downarrow}\rangle \otimes |\mathrm{C}_{\downarrow}\rangle$$

into the final state

$$|\psi_{\mathrm{f}}\rangle = \underline{u|\uparrow\rangle \otimes |\mathrm{M}_{\uparrow}\rangle \otimes |\mathrm{C}_{\uparrow}\rangle \otimes |\mathrm{W}_{\uparrow}\rangle} + \underline{v|\downarrow\rangle \otimes |\mathrm{M}_{\downarrow}\rangle \otimes |\mathrm{C}_{\downarrow}\rangle \otimes |\mathrm{W}_{\downarrow}\rangle}.$$

Definiteness operator *D*

has eigenvalue 1 if C perceives a definite outcome, eigenvalue 0 otherwise:

$$D\,|\mathrm{C}_{\uparrow}\rangle = |\mathrm{C}_{\uparrow}\rangle\,, \;\; D\,|\mathrm{C}_{\downarrow}\rangle = |\mathrm{C}_{\downarrow}\rangle$$

due to David Albert

EPR in Many Worlds

measuring devices M and $\overline{\mathrm{M}}$

observer consciousnesses C and $\overline{\mathrm{C}}$

in spacetime regions R and $\overline{\mathrm{R}}$

Hilbert space of the world $\mathrm{W} = \{|\mathrm{W}_0\rangle, |\mathrm{W}_{\uparrow_1\uparrow_2}\rangle, |\mathrm{W}_{\uparrow_1\downarrow_2}\rangle, |\mathrm{W}_{\downarrow_1\uparrow_2}\rangle, |\mathrm{W}_{\downarrow_1\downarrow_2}\rangle\}$

initial state

$$|\psi_\mathrm{i}\rangle = \frac{1}{\sqrt{2}}\Big(\underline{|\uparrow_1\rangle \otimes |\downarrow_2\rangle} - \underline{|\downarrow_1\rangle \otimes |\uparrow_2\rangle}\Big) \otimes |\mathrm{M}_0\rangle \otimes |\mathrm{C}_0\rangle \otimes |\overline{\mathrm{M}}_0\rangle \otimes |\overline{\mathrm{C}}_0\rangle \otimes |\mathrm{W}_0\rangle$$

will evolve following the von Neumann chain into

$$|\psi_\mathrm{f}\rangle = \frac{1}{\sqrt{2}}\Big(\underline{|\uparrow_1\rangle \otimes |\downarrow_2\rangle \otimes |\mathrm{M}_\uparrow\rangle \otimes |\mathrm{C}_\uparrow\rangle \otimes \big|\overline{\mathrm{M}}_\downarrow\big\rangle \otimes \big|\overline{\mathrm{C}}_\downarrow\big\rangle \otimes |W_{\uparrow_1\downarrow_2}\rangle}$$

$$- \underline{\text{same term with} \uparrow \leftrightarrow \downarrow}\Big)$$

regardless which world we find ourselves in, $\sigma^z_{(1)}\sigma^z_{(2)} = -1$

Problems with Many Worlds interpretations

1. Which branch of the multiverse do I find myself in?

   $\rightarrow$ collapse is pushed back to the level of consciousness

2. All attempts to derive the Born rule (probabilities) have failed.

3. There is a universe for every possible outcome of measurements, no matter how unlikely they are.

   e.g. measure σ^z for 10^{20} spin which were aligned to a precision of 10^{-3} in $\sigma^z = +1$ direction. There is a universe in which all these 10^{20} measurements yield $\sigma^z = -1$, even though the probability for this is

$$\left(10^{-3}\right)^{10^{20}}$$

cf. age of universe $= 4 \cdot 10^{20}$ secs., size of universe $= 1.3 \cdot 10^{26}$ m

4. Interlinking and the ensemble of macroscopic objects (EMO)

Assumptions:

(i) The fundamental theory is a quantum theory. The entire universe can be described by a solution of this quantum theory, which for simplicity we call wave function Ψ.

(ii) The evolution of Ψ is, to an approximation we have not been able to challenge, given by the linear regime of the quantum theory. For simplicity, let us assume time is fundamental (as opposed to emerging) and let us refer to the theory describing this evolution as the Schrödinger equation. For time to be meaningful, Ψ must not be an eigenstate of the time evolution operator.

(iii) The universe started with the big bang, and at that time, many degrees of freedom of the universe were entangled with their environments. We expect that there is still significant entanglement.

(iv) For simplicity, we further assume that at a time we refer to as the present, there is only one "world". Since all the branchings into other "worlds", should they have occurred in the past, have no influence on our perception of the present, any consistent theory based on this assumption will be sufficient.

Our daily life experience, however, is mostly described by either classical dynamics or statistical physics.

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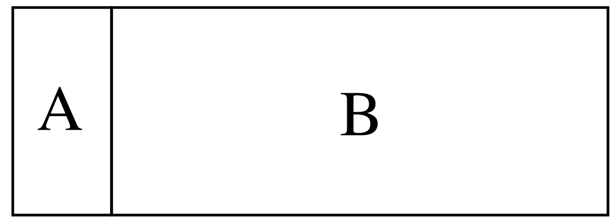
statistical behavior can be explained by the eigenstate thermalization hypotheses of Deutsch and Srednicki:



partition a system in a pure state $|\psi(t)\rangle$ into a subsystem A and an environment B.

Then the reduced density matrix of A: $\rho_A(t) \equiv \mathrm{tr}_B\big(|\psi(t)\rangle\langle\psi(t)|\big)$

will converge towards a Boltzmann distribution

$$\rho_A(\beta) \equiv \mathrm{tr}_B\big(\rho(\beta)\big) \quad \text{with} \quad \rho(\beta) = \frac{1}{Z}\exp\left(-\beta H\right)$$

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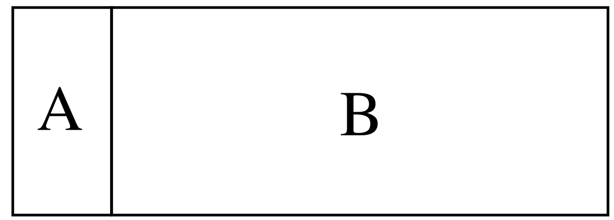

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$\rightarrow$ thermal entropy = entanglement entropy assumption (i) “All is Ψ“ $\rightarrow$ all of entropy is entanglement entropy

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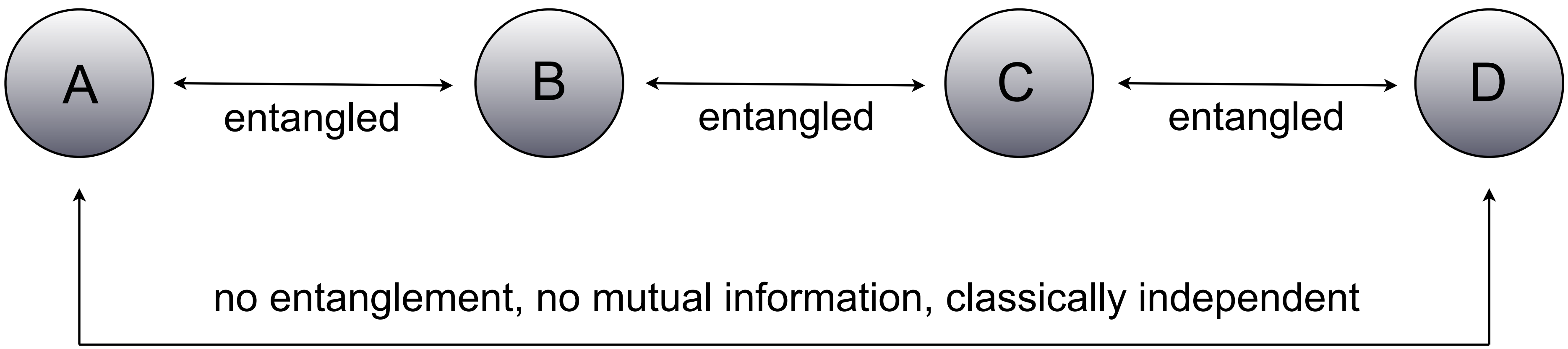

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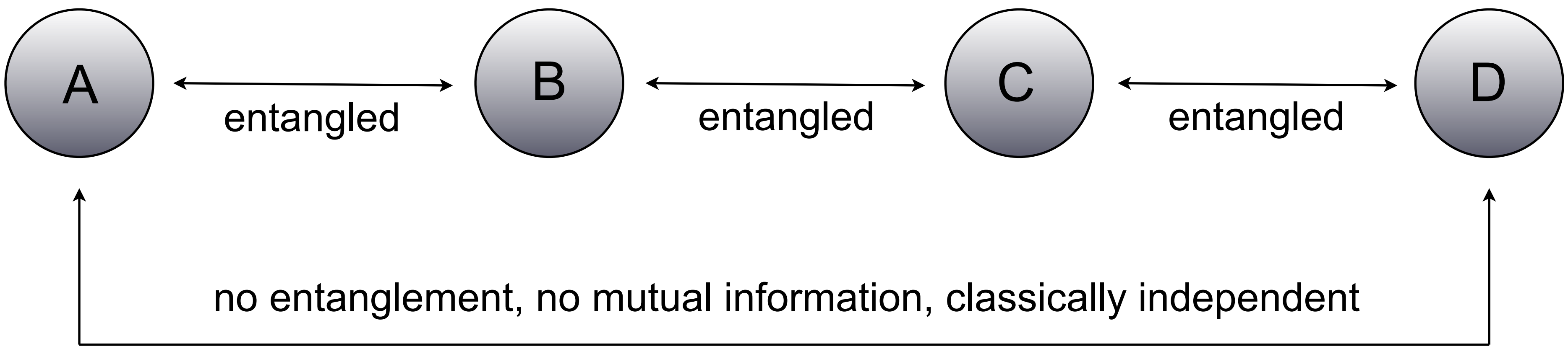

interlinked as it is not possible to factorize Ψ

classical reality = ensemble of macroscopic objects (EMO)

rule of thumb: objects which have a temperature belong to the EMO

Example for interlinking: four qubits A, B, C, D with states

$$|n_1; n_2; n_3; n_4\rangle \equiv |n_1\rangle_{\mathrm{A}} \otimes |n_2\rangle_{\mathrm{B}} \otimes |n_3\rangle_{\mathrm{C}} \otimes |n_4\rangle_{\mathrm{D}}$$

$$\begin{aligned} |\psi\rangle &= \frac{1}{2} \sum_{i=0}^{1} \sum_{k=0}^{1} |i; (i+k) \mod 2; k; k\rangle \\ &= \frac{1}{2}\Big(|0;0;0;0\rangle + |0;1;1;1\rangle + |1;1;0;0\rangle + |1;0;1;1\rangle \Big) \end{aligned}$$

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entropy of A: $S(\mathrm{A}) = -\mathrm{tr}_{\mathrm{A}}(\rho_{\mathrm{A}} \ln \rho_{\mathrm{A}})$ with $\rho_{\mathrm{A}} = \mathrm{tr}_{\mathrm{BCD}}\big(|\psi\rangle\langle\psi|\big)$

joint entropy of A and D: $S(\mathrm{A}, \mathrm{D}) = -\mathrm{tr}_{\mathrm{AD}}(\rho_{\mathrm{AD}} \ln \rho_{\mathrm{AD}})$

→ $S(\mathrm{A}) = S(\mathrm{D}) = \ln 2, \quad S(\mathrm{A}, \mathrm{D}) = 2 \ln 2$

mutual information between A and D:

$$S(\mathrm{A}\!:\!\mathrm{D}) \equiv S(\mathrm{A}) + S(\mathrm{D}) - S(\mathrm{A}, \mathrm{D}) = 0$$

5. Measurements, Schrödinger's cat, and EPR

A measurement occurs when one or several microscopic degrees of freedom, which were previously disentangled from the EMO, become entangled with degrees of freedom belonging to the EMO, and hence interlinked with all of them.

(This is when the collapse occurs in the Copenhagen interpretation.)

The process, however, is not as envisioned by von Neumann in 1932:

The correct initial state is $$|\psi_{\rm i}\rangle = \big(u\,|{\uparrow}\rangle + v\,|{\downarrow}\rangle\big) \otimes |{\rm M}_0, {\rm C}_0, {\rm W}_0\rangle$$

and not $$|\psi_{\rm i}\rangle = \big(u\,|{\uparrow}\rangle + v\,|{\downarrow}\rangle\big) \otimes |{\rm M}_0\rangle \otimes |{\rm C}_0\rangle \otimes |{\rm W}_0\rangle$$

An initial state

$$|\psi_{\mathrm{i}}\rangle = \big(u\,|{\uparrow}\rangle + v\,|{\downarrow}\rangle\big) \otimes |\mathrm{M}_0, \mathrm{C}_0, \mathrm{W}_0\rangle$$

evolves via

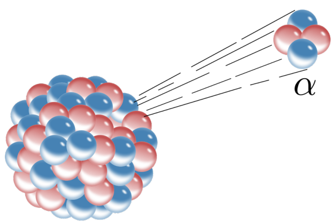
$$|\psi_{\mathrm{M}}\rangle = u\,|{\uparrow}\rangle \otimes |\mathrm{M}_{\uparrow}, \mathrm{C}_0, \mathrm{W}_{\uparrow}\rangle + v\,|{\downarrow}\rangle \otimes |\mathrm{M}_{\downarrow}, \mathrm{C}_0, \mathrm{W}_{\downarrow}\rangle$$

into the final state

$$|\psi_{\mathrm{f}}\rangle = u\,|{\uparrow}\rangle \otimes |\mathrm{M}_{\uparrow}, \mathrm{C}_{\uparrow}, \mathrm{W}_{\uparrow}\rangle + v\,|{\downarrow}\rangle \otimes |\mathrm{M}_{\downarrow}, \mathrm{C}_{\downarrow}, \mathrm{W}_{\downarrow}\rangle.$$

The ”bifurcation into two worlds” occurs when the spin becomes entangled with the measuring device M, and hence interlinked with a world.

Schrödinger's cat: The measurement occurs when the alpha ray becomes entangled with the detector, and hence interlinked with the cat, the box, and the observer outside.

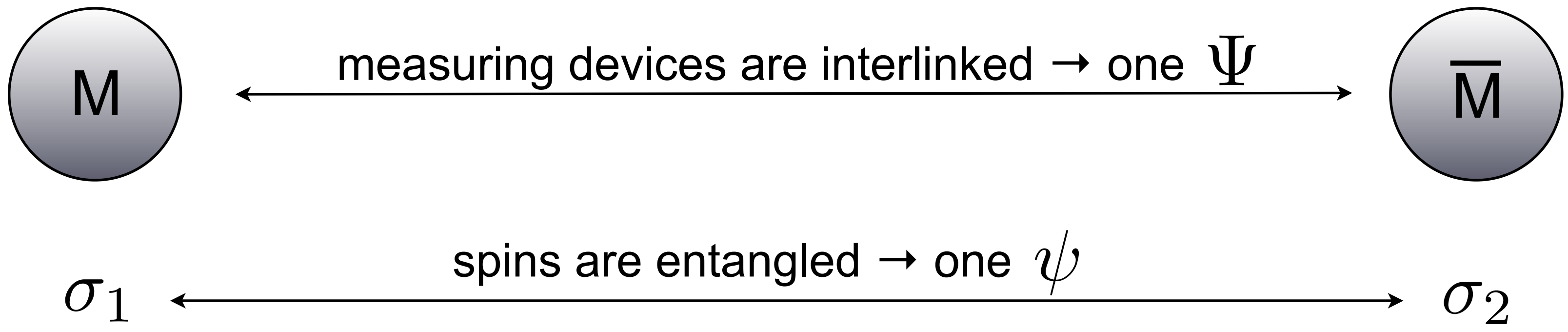


$\otimes$ det. $\longleftrightarrow$ cat $\longleftrightarrow$ box $\longleftrightarrow$ me

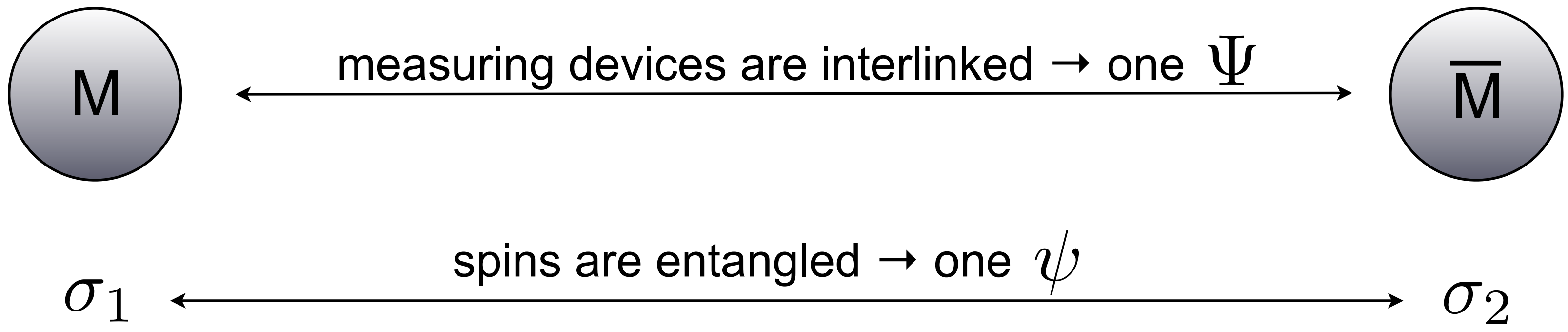
all macroscopic objects are interlinked

$$\psi_{\mathrm{atom}} \otimes \Psi_{\mathrm{EMO}} \to \Psi'_{\mathrm{EMO}}$$

EPR:


M
measuring devices are interlinked → one Ψ
$\overline{\mathrm{M}}$
σ_1
spins are entangled → one ψ
σ_2

EPR:



after the “first” measurement, everything is described by a single $\Psi = \Psi_{\mathrm{EMO}}$

$\rightarrow$ when we measure one spin, we have measured both!

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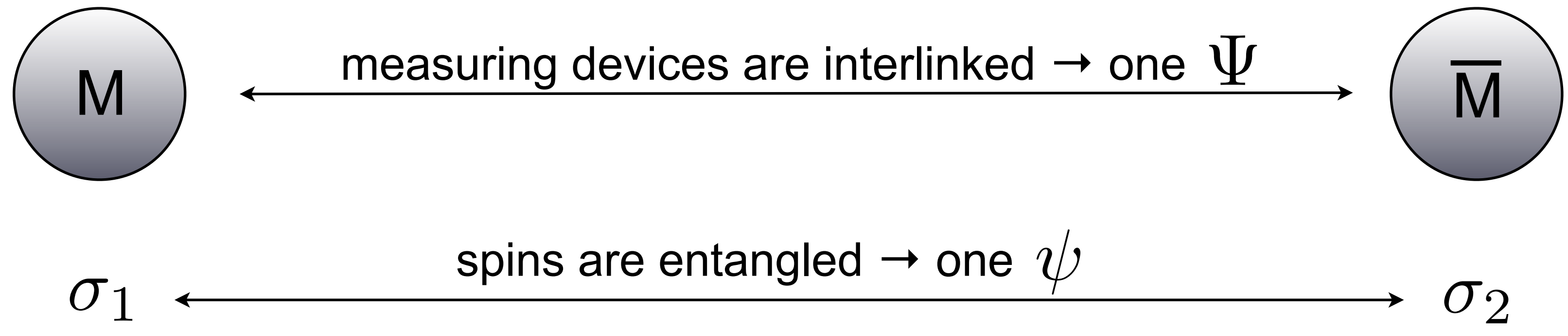

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$$|\psi_\text{i}\rangle = \frac{1}{\sqrt{2}}\Big(|{\uparrow_1}\rangle \otimes |{\downarrow_2}\rangle - |{\downarrow_1}\rangle \otimes |{\uparrow_2}\rangle\Big) \otimes |\text{M}_0, \text{C}_0, \overline{\text{M}}_0, \overline{\text{C}}_0, \text{W}_0\rangle$$

evolves after measurement of spin 1 into

$$\begin{aligned}|\psi_\text{f}\rangle = \frac{1}{\sqrt{2}}\Big(&|{\uparrow_1}\rangle \otimes |{\downarrow_2}\rangle \otimes |\text{M}_\uparrow, \text{C}_0, \overline{\text{M}}_0, \overline{\text{C}}_0, W_{\uparrow_1\downarrow_2}\rangle \\ &- |{\downarrow_1}\rangle \otimes |{\uparrow_2}\rangle \otimes |\text{M}_\downarrow, \text{C}_0, \overline{\text{M}}_0, \overline{\text{C}}_0, W_{\downarrow_1\uparrow_2}\rangle\Big)\end{aligned}$$

6. Is there a collapse?

A collapse requires a non-linearity in the time evolution of QM.

Models in the literature, like Ghirardi, Rimini, and Weber (1986), or Diósi (1987) and Penrose (1996), assume that a collapse takes place locally.

There is no experimental evidence for this - the systems we are able to probe become larger and larger, but linear QM works just fine!

Interlinking, however, changes the scale. Recall our measurement process:

$$|\psi_{\mathrm{i}}\rangle = \Big(u\,|{\uparrow}\rangle + v\,|{\downarrow}\rangle\Big) \otimes |W_0\rangle \quad \rightarrow \; |\psi_{\mathrm{f}}\rangle = u\,|{\uparrow}\rangle \otimes |\mathrm{W}_{\uparrow}\rangle + v\,|{\downarrow}\rangle \otimes |\mathrm{W}_{\downarrow}\rangle$$

is a superposition of two states of the universe, with 10^{80} baryons

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So we may assume a collapse takes place, due to non-linearities of QM at length scales we are unable to access.

It is very possible that linear QM works just fine for every system we will ever be able to access in a laboratory (be it 10^{30} or 10^{50} baryons), but non-linearities appear at much larger scales.

These non-linearities might be related to gravity.

Remarks

1. From our assumptions, only the “All is Ψ“ assumption (i) is required.

2. If a collapse takes place, the spin we measured will be in a disentangled state $|\uparrow\rangle$ or $|\downarrow\rangle$ again. The evolution is linear before and after the measurement.

   In the process, the spin first gains and then looses (entanglement) entropy. If it was entangled with other degrees of freedom beforehand, the measurement reduces its entropy.

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   Interlinking happens in Hilbert space, while relativity applies to spacetime.

4. If the non-linearities in QM are due to gravity, there cannot be a canonical quantization of gravity, as canonical quantization is inherently linear.

Summary (1)

1. We assume that the fundamental theory is a quantum theory.

2. The classical reality we perceive is given by the ensemble a macroscopic objects (EMO). Due to interlinking, the wave function of all these objects cannot be factorized.

3. Therefore, we observe quantum behavior only for (microscopic) degrees of freedom disentangled from the EMO.

4. A measurement occurs whenever a microscopic degrees of freedom becomes entangled with its environment and thereby interlinked with the EMO, which includes the visible universe.

5. Even though we lack a microscopic understanding how a collapse occurs, it is reasonable to assume it does.

Summary (2)

The fundamental assumptions resemble MWIs. The difference is that we introduce interlinking, abandon the von Neumann chain, and take into account that the scales relevant for non-linearities in the quantum evolution are currently inaccessible to us.

The phenomenology resembles Copenhagen. The difference is that we do not embed the quantum theory in a classical domain, but find the classical domain within the quantum theory.

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