Lecture 2:

Randomized Measurements

Exp: T. Brydges, A. Elben et al., Science (2019),
Probing Renyi Entanglement Entropy via Randomized Measurements

Theory: A Elben, B Vermersch, CF Roos, and PZ, PRA (2019),
Randomized Measurements: A Toolbox …

A Elben
→ Caltech

B Vermersch
→ Grenoble

C Kokail

R van Bijnen

Classical Shadows

H.-Y. Huang, R. Kueng, and J. Preskill, Nat. Phys. 16, 1050 (2020)
Predicting Many Properties of a Quantum System from Very Few Measurements

Review article Nature Reviews Physics 1 (2022).

The randomized measurement toolbox

Andreas Elben[1,2,3,4], Steven T. Flammia[1,5], Hsin-Yuan Huang[1,6], Richard Kueng[7], John Preskill[1,2,5,6], Benoît Vermersch[3,4,8] & Peter Zoller[3,4]

Measure first, ask questions later

Lecture 2:

Randomized Measurements

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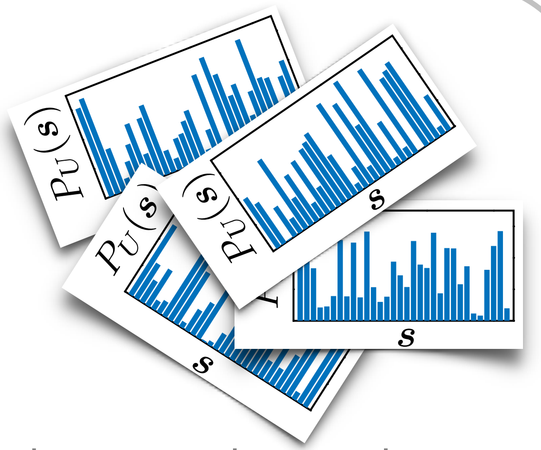


data reused many times …

Classical Shadows

H.-Y. Huang, R. Kueng, and J. Preskill, Nat. Phys. 16, 1050 (2020)
Predicting Many Properties of a Quantum System from Very Few Measurements

Rigorous error bounds:

$$M \propto \log(L) 4^w / \epsilon^2$$

independent randomized measurements suffice to …

Review article Nature Reviews Physics 1 (2022).

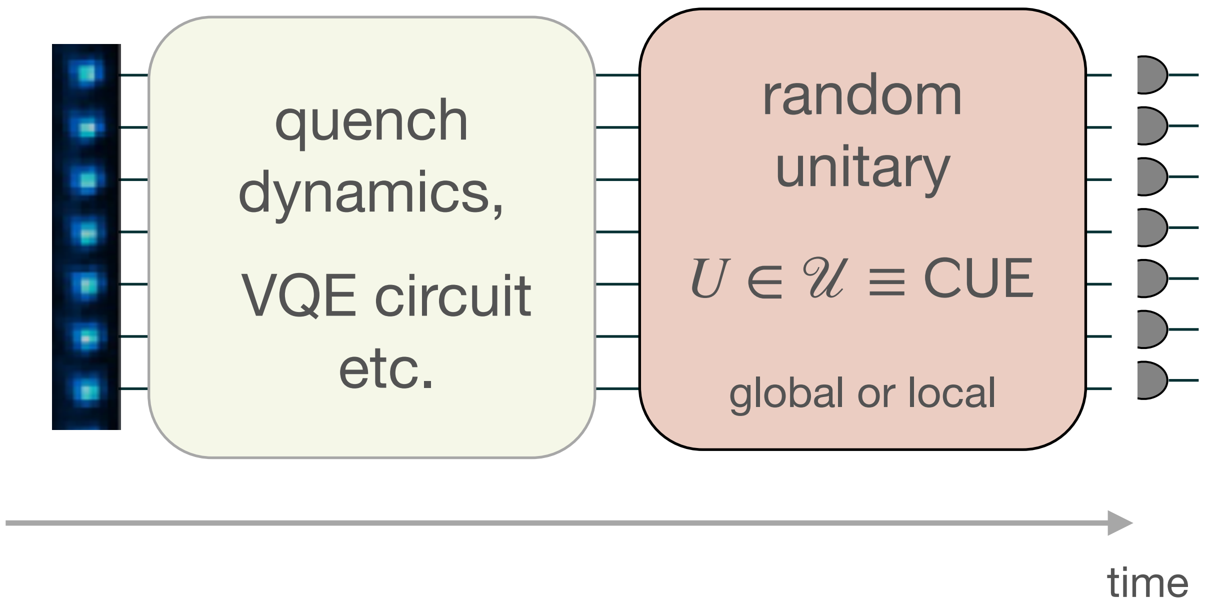
The randomized measurement toolbox

Andreas Elben[1,2,3,4], Steven T. Flammia[1,5], Hsin-Yuan Huang[1,6], Richard Kueng[7], John Preskill[1,2,5,6], Benoît Vermersch[3,4,8] & Peter Zoller[3,4]

Measure first, ask questions later

Randomized Measurements

Measurement post-processing



Repeat

Probability for bitstring s

$$P_U(\mathbf{s})$$

is a random variable depending on U

(Cross-) Correlation of probabilities

$$\mathbb{E}_{U\sim\mathcal{U}}[P_U(\mathbf{s})P_U(\mathbf{s}')]$$

'Noise' or ensemble average (e.g. CUE)

experiment `day 1, lab 1'

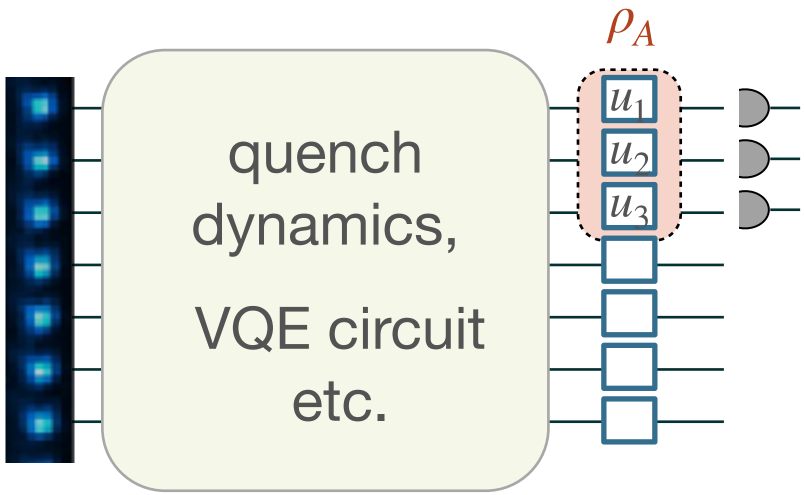
experiment `day 2 lab 2'

2-design, …

… hybrid classical-quantum protocols

Randomized Measurements

Measurement post-processing



(Cross-) Correlation of probabilities

$$\mathbb{E}_{U\sim\mathcal{U}}[P_U(\mathrm{s})P_U(\mathrm{s}')]$$

experiment `day 1, lab 1'

experiment `day 2 lab 2'

- **OTOCS**

  theory - B. Vermersch et al., Phys. Rev. X **9**, 021061 (2019).
  exp - M. K. Joshi et al., Phys. Rev. Lett. **124**, 240505 (2020).

- **topological invariants**

  theory - A. Elben et al., Science Advances **6**, eaaz3666 (2020).

- **Partially transposed density matrix**

  theory [+ exp] - A. Elben et al., Phys. Rev. Lett. 125, 200501 (2020).
  theory [+ exp] - A. Neven et al., Npj Quantum Inf. **7**, (2021).

- **Entanglement Hamiltonian Tomography**

  theory - C Kokail et al., Nat. Phys. **17**, 936 (2021).
  theory + exp - MK Joshi, C Kokail, R van Bijnen et al., arXiv 2023

- **Spectral form factor & quantum chaos**

  theory - L. K. Joshi et al., Phys. Rev. X. **12**, (2022).
  exp - L. K. Joshi et al. [Monroe group], unpublished

- **observation of quantum Mpemba effect**

  theory + exp - MK Joshi et al. unpublished

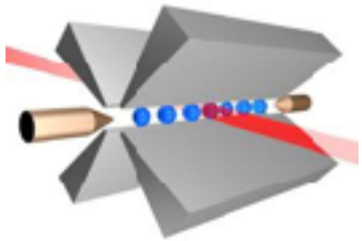

C Kokail


R van Bijnen


M Joshi

Exploring Large-Scale Entanglement in Quantum Simulation

Trapped ions



UIBK & IQOQI

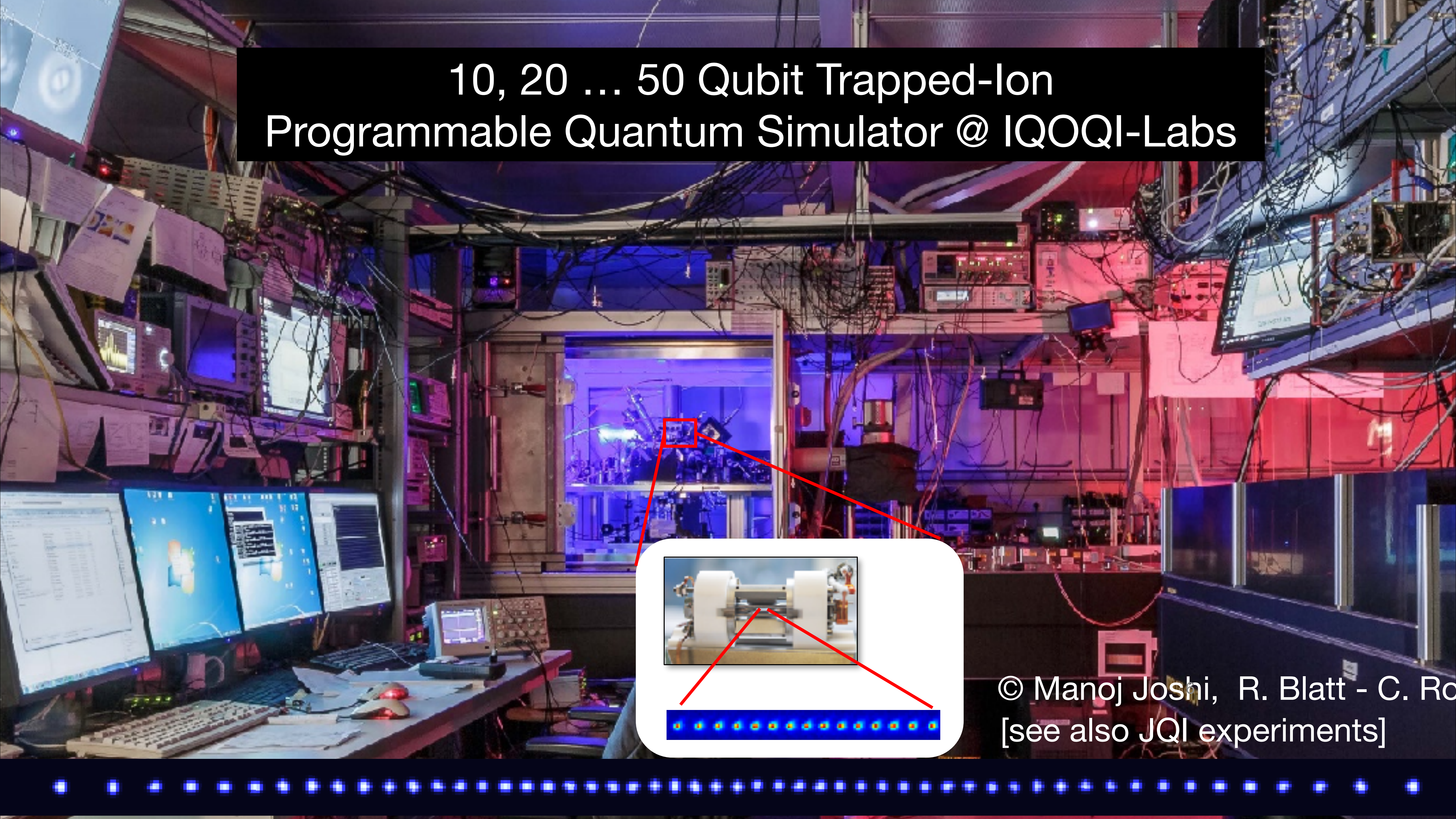
Theory: C Kokail, R van Bijnen, TV Zache, and P.Z.

Experiment: ML Joshi, F Kranzl, R Blatt, CF Roos

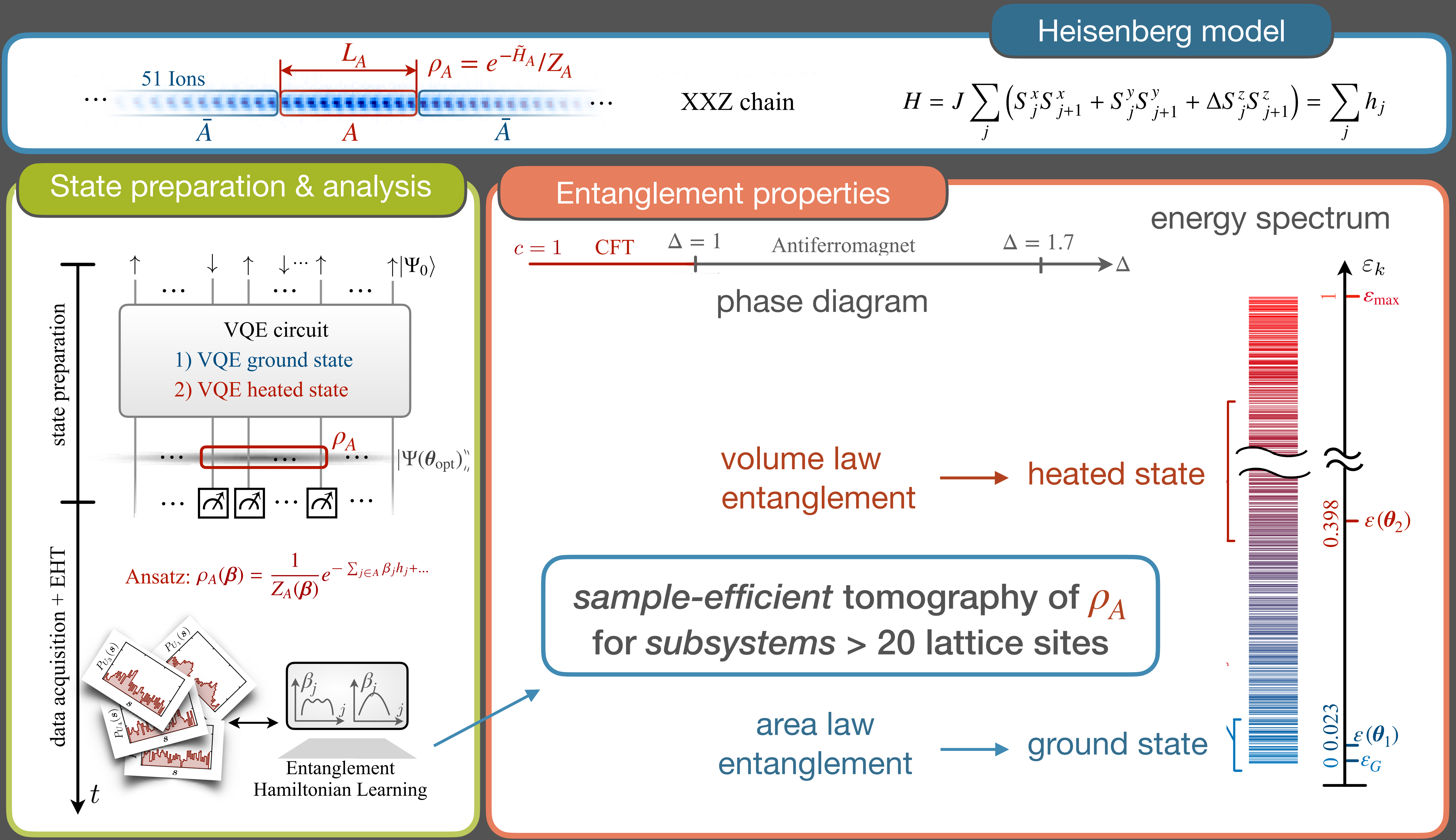
arXiv:2306.00057

Early collaborations: M Dalmonte (→ ICTP), B Vermersch (→ Grenoble)

10, 20 … 50 Qubit Trapped-Ion Programmable Quantum Simulator @ IQOQI-Labs



© Manoj Joshi, R. Blatt - C. Ro
[see also JQI experiments]

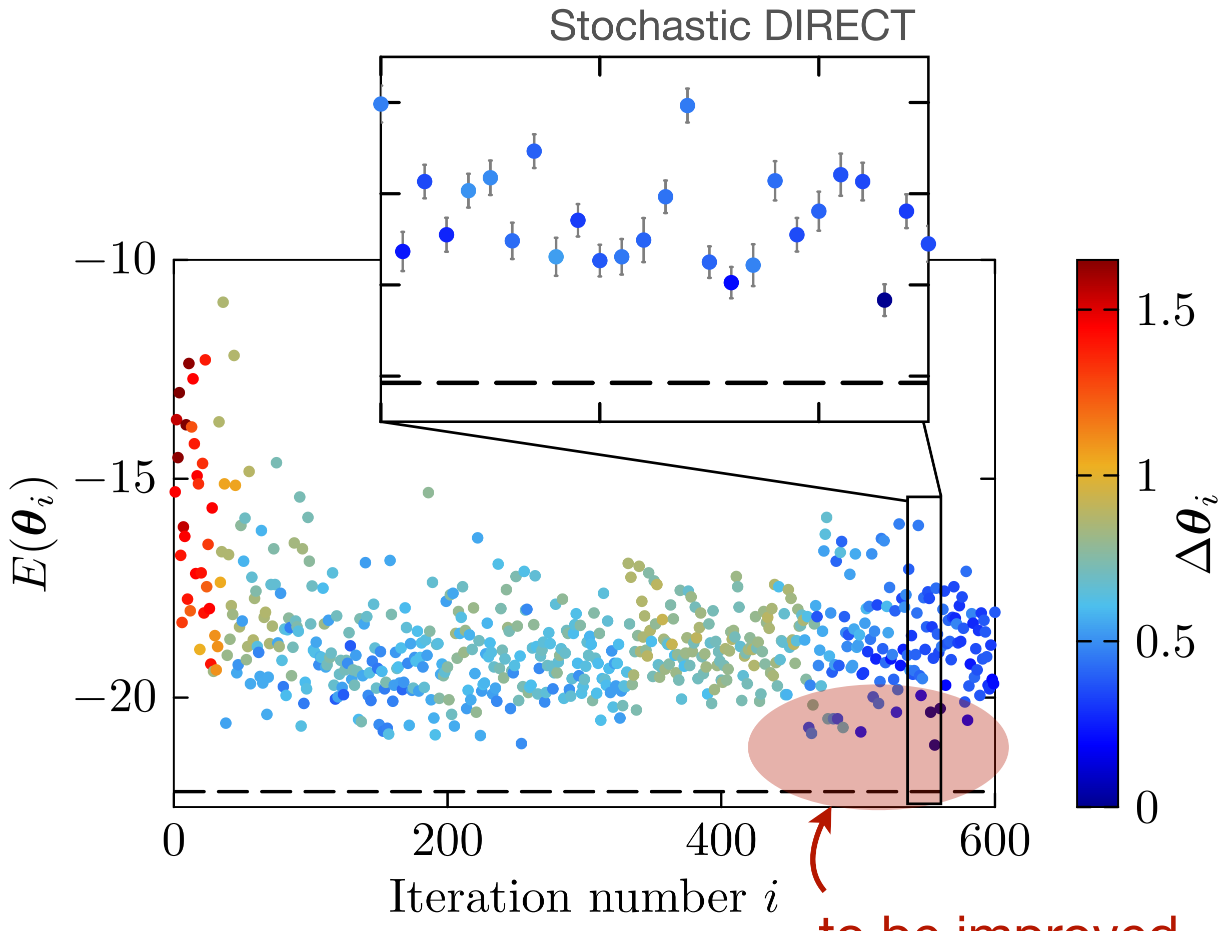


Heisenberg model
51 Ions
L_A
$\rho_A = e^{-\tilde{H}_A}/Z_A$
$\bar{A}$
A
$\bar{A}$
XXZ chain
$H = J \sum_j \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z\right) = \sum_j h_j$
State preparation & analysis
state preparation
data acquisition + EHT
$|\Psi_0\rangle$
VQE circuit
1) VQE ground state
2) VQE heated state
ρ_A
$|\Psi(\theta_{opt})\rangle$
t
Ansatz: $\rho_A(\beta) = \frac{1}{Z_A(\beta)} e^{-\sum_{j\in A} \beta_j h_j + \dots}$
β_j
Entanglement
Hamiltonian Learning
Entanglement properties
$c = 1$
CFT
$\Delta = 1$
Antiferromagnet
$\Delta = 1.7$
Δ
phase diagram
energy spectrum
ε_k
ε_{max}
$\varepsilon(\theta_2)$
$\varepsilon(\theta_1)$
ε_G
1
0.398
0.023
0
volume law
entanglement
heated state
sample-efficient tomography of ρ_A
for *subsystems* > 20 lattice sites
area law
entanglement
ground state

Experimental Energy Optimization Trajectory for Ground State (VQE)

Theory: C Kokail, R van Bijnen et al., unpublished
Experiment: M Joshi et al., unpublished

N = 51 ions



to be improved

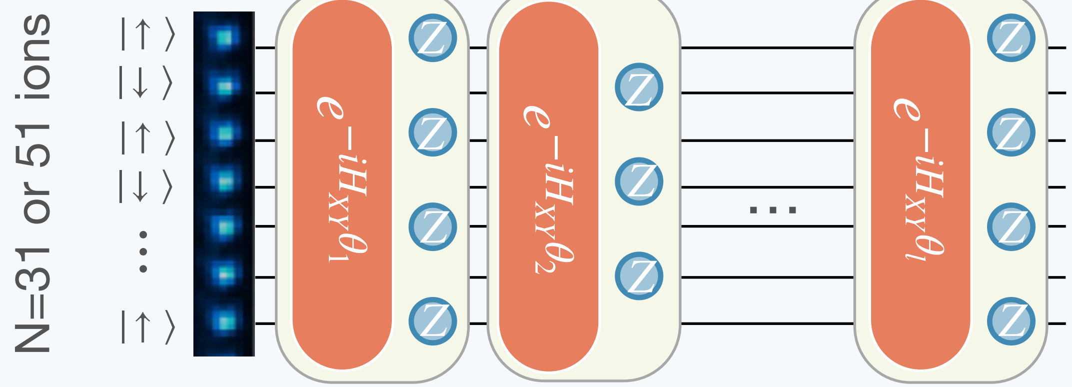
~ low temperature state T ~ few J

Heisenberg Model (spin-½)

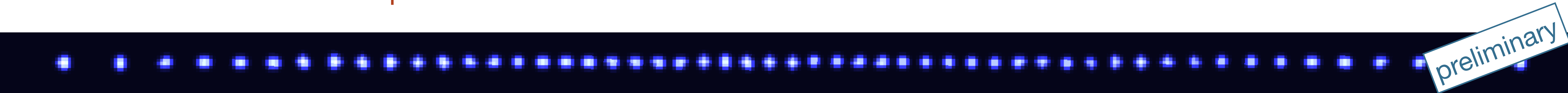
$$\hat{H} = J \sum_{i=1}^{N-1} \left(\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y \right) + \Delta \sum_{i=1}^{N-1} \hat{S}_i^z \hat{S}_{i+1}^z + h \sum_{i=1}^{N} \hat{S}_i^z$$

$J = 1 \quad \Delta = 1 \quad h = 0.5$

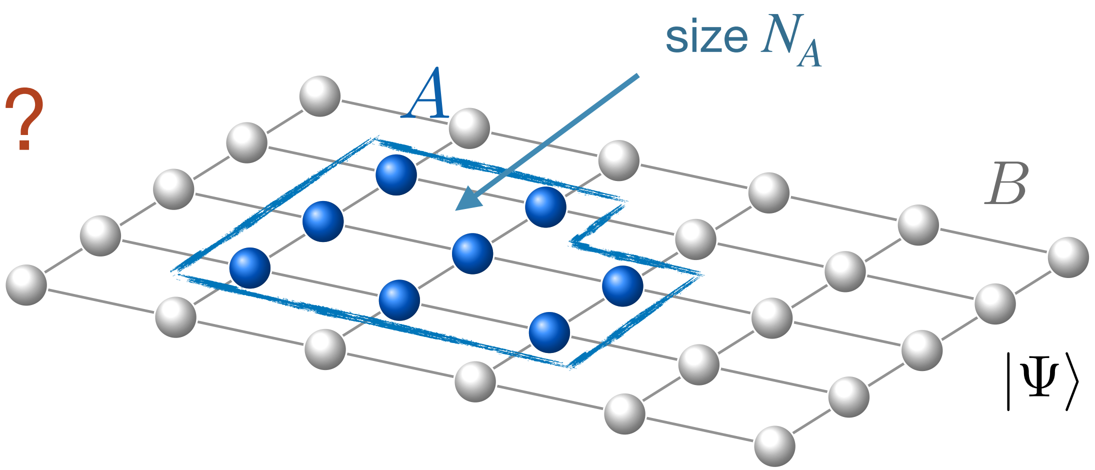
VQE Circuit with Trapped Ion Resources



5 layers and 10 parameters



Learning the Entanglement Hamiltonian?



Protocol 0: Quantum state tomography

measurement data ⟶ $\rho_A = \exp\left(-\tilde{H}_A\right)$

✓ expensive *

exponential in subsystem size N_A

?

* except: for small system sizes, or if we know something about the quantum state

Do we know something about the *structure of* $\tilde{H}_A$ to make tomography *efficient*?

A. Anshu et al., *Sample-Efficient Learning of Interacting Quantum Systems*, Nat. Phys. **17**, 931 (2021).

Entanglement Hamiltonian in QFT: *Bisognano-Wichmann Theorem*

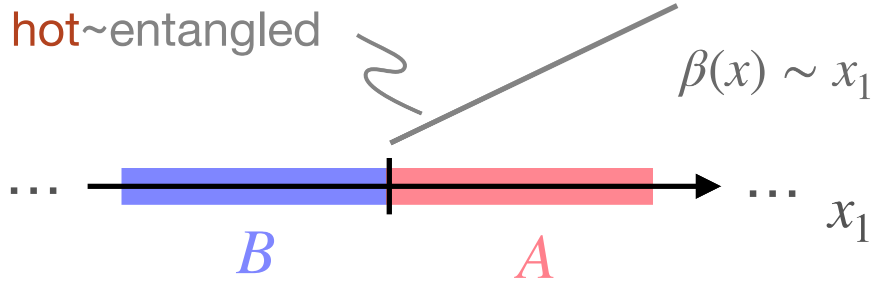
Relativistic Quantum Field Theory

Lorentz invariance

$$H = \int_{A \cup B} d^d x \, \mathcal{H}(x)$$

vacuum state $|\Omega\rangle$ ⟶

Entanglement Hamiltonian



$$\rho_A \equiv \mathrm{Tr}_B |\Omega\rangle\langle\Omega| = \exp\left[- \int_A d^d x \, \beta(x) \mathcal{H}(x) \right]$$

$\tilde{H}_A$

Gibbs state with *local* temperature $\beta(x) \sim x_1$

EH $\tilde{H}_A$ as *deformed* system Hamiltonian

Bisognano and Wichmann, J. Math. Phys. (1976)   Casini, Huerta & Myers, Journal of HEP (2011)

Review: M Dalmonte, V Eisler, M Falconi, B Vermersch, *Entanglement Hamiltonians - from field theory to lattice models & experiments*, Ann Phys 2022,534, 2200064

Entanglement Hamiltonian in QFT: *Bisognano-Wichmann Theorem*

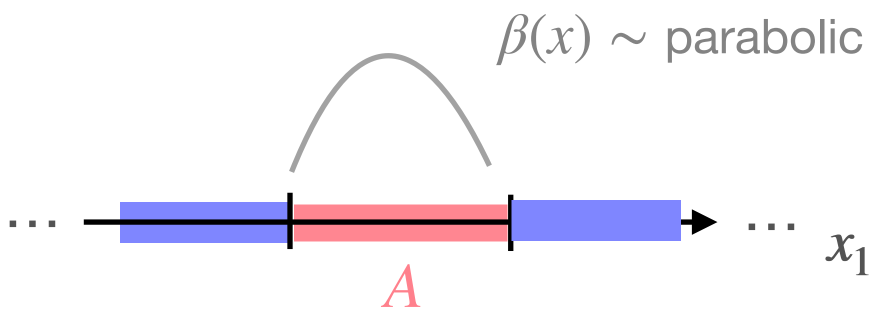
Conformal Field Theory

scale invariance

$$H = \int_{A \cup B} d^d x \, \mathscr{H}(x)$$

vacuum state $|\Omega\rangle$ ⟶

Entanglement Hamiltonian



$$\rho_A \equiv \mathrm{Tr}_B |\Omega\rangle\langle\Omega| = \exp\left[- \int_A d^d x \, \beta(x) \mathscr{H}(x) \right]$$

$\tilde{H}_A$

Gibbs state with *local* temperature $\beta(x) \sim x_1$

Entanglement Hamiltonian as *deformed* system Hamiltonian

Casini, Huerta & Myers, Journal of HEP (2011)

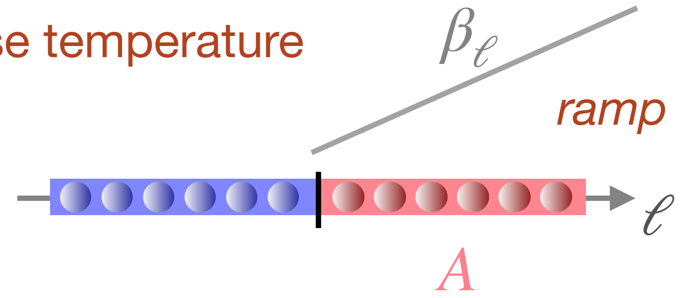
1+1 CFT: Hislop, Longo, Cardy, Calabrese, Tonni, Wen, Ryu, Ludwig, … (ground state, thermal & quench)

Review: M Dalmonte, V Eisler, M Falconi, B Vermersch, *Entanglement Hamiltonians - from field theory to lattice models & experiments*, Ann Phys 2022,534, 2200064

Lattice Bisognano-Wichmann & beyond

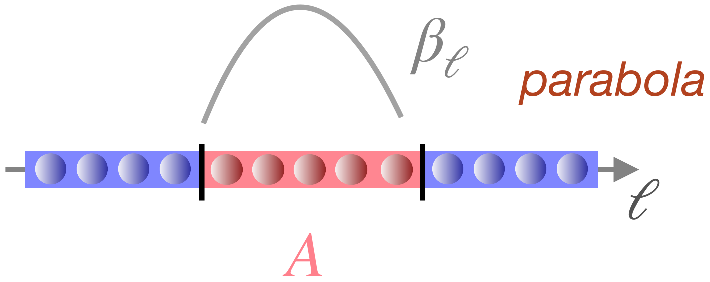
ground state of many-body lattice model

Gibbs state with local inverse temperature



$$\rho_A = e^{-\tilde{H}_A}$$

entanglement Hamiltonian



BW recipe

$$\hat{H} = \sum_{\ell} \hat{h}_\ell$$ k-local Hamiltonian

↓

$$\tilde{H}_A = \sum_{\ell \in A} \beta_\ell \hat{h}_\ell + \ldots$$ EH as *local deformation* of system Hamiltonian

1. Validity of BW-like EH, non-local corrections often sub-leading

Analytical results: non-interacting, non-critical chains

Numerical evidence: lattice models, quench dynamics

Review: M Dalmonte, V Eisler, M Falconi, B Vermersch, *Entanglement Hamiltonians - from field theory to lattice models & experiments*, Ann Phys 2022,534, 2200064

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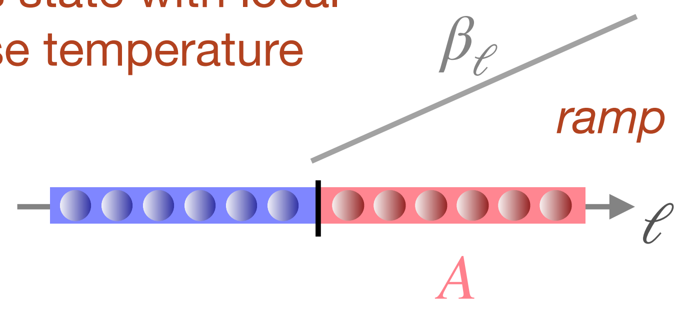

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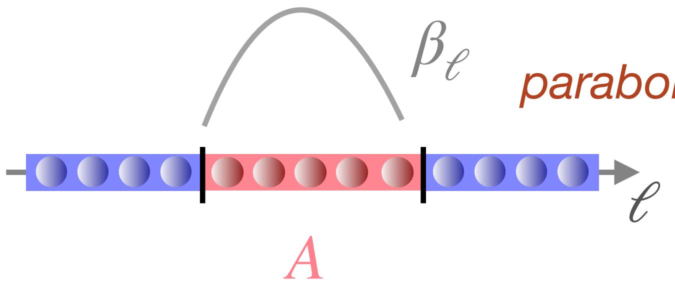
entanglement Hamiltonian



BW recipe

$$\hat{H} = \sum_\ell \hat{h}_\ell$$ k-local Hamiltonian

$$\downarrow$$

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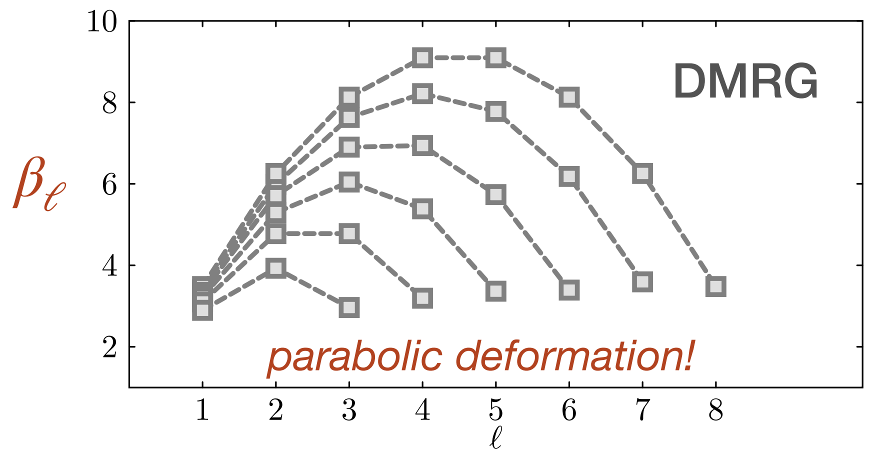
Numerical example: Heisenberg model 1D



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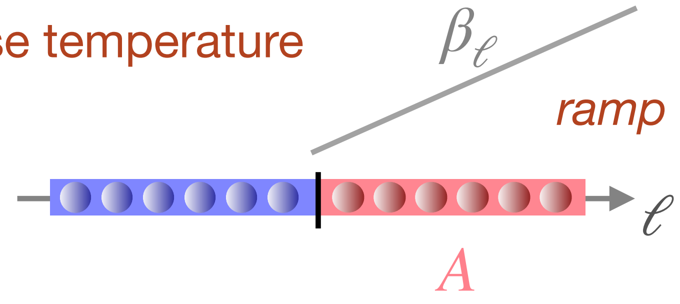
Lattice Bisognano-Wichmann & beyond

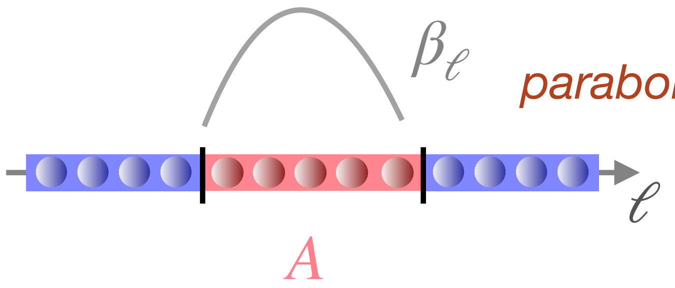
ground state of many-body lattice model

Gibbs state with local inverse temperature



$$\rho_A = e^{-\tilde{H}_A}$$

entanglement Hamiltonian



BW recipe

$$\hat{H} = \sum_\ell \hat{h}_\ell$$ k-local Hamiltonian

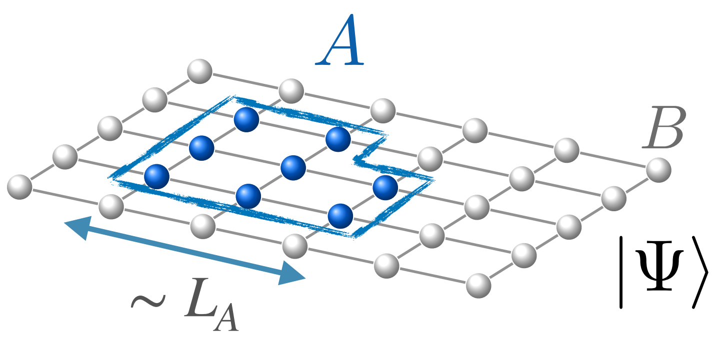
$$\downarrow$$

$$\tilde{H}_A = \sum_{\ell \in A} \beta_\ell \hat{h}_\ell + \ldots$$ EH as *local deformation* of system Hamiltonian

2. suggests an efficient ansatz to `learn' Entanglement Hamiltonian

Review: M Dalmonte, V Eisler, M Falconi, B Vermersch, *Entanglement Hamiltonians - from field theory to lattice models & experiments*, Ann Phys 2022,534, 2200064

Entanglement in Many-Body Quantum Systems



Many-Body Problem

energy spectrum

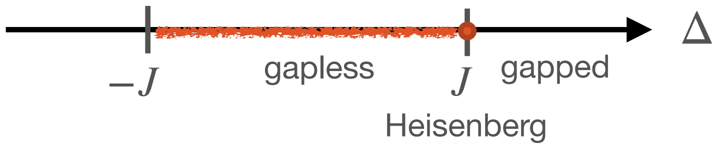
Hamiltonian

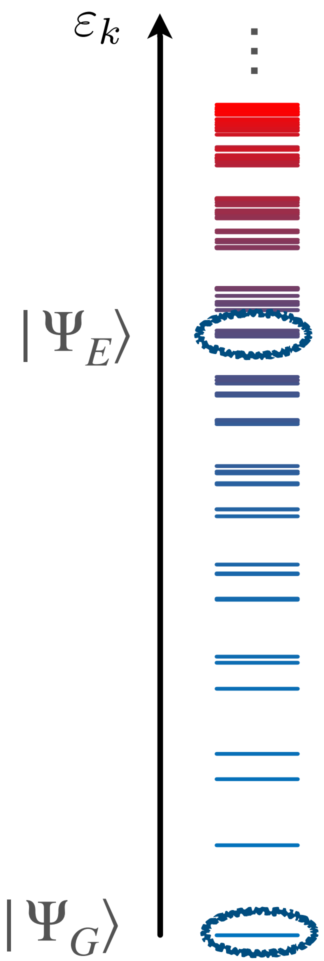
$$\hat{H} = \sum_j \hat{h}_j$$

k-local

Example: XXZ / Heisenberg model (1D)

$$\hat{H}_T = J \sum_{i=1}^{N-1} \left(\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y \right) + \Delta \sum_{i=1}^{N-1} \hat{S}_i^z \hat{S}_{i+1}^z$$



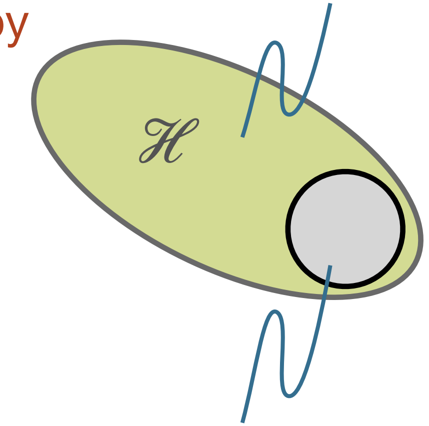


excited state

$$S_A \propto V = L_A^d$$

thermal entropy

volume law entanglement

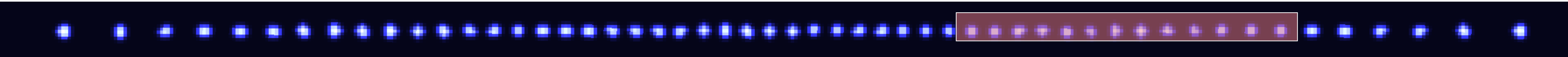


ground state

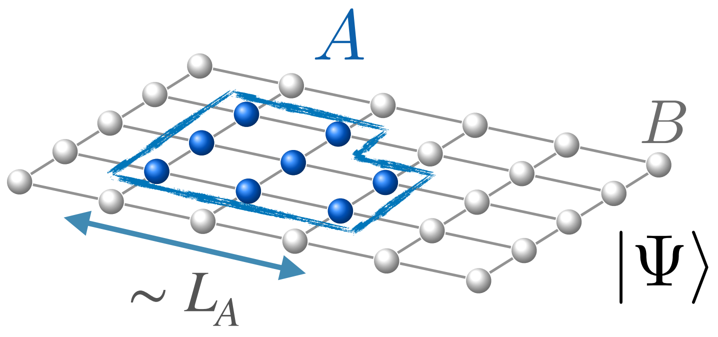
$$S_A \propto L_A^{d-1}$$

area law entanglement

$\sim c \log L_A$ CFT $d = 1$



Area Law vs. Volume Law Entanglement



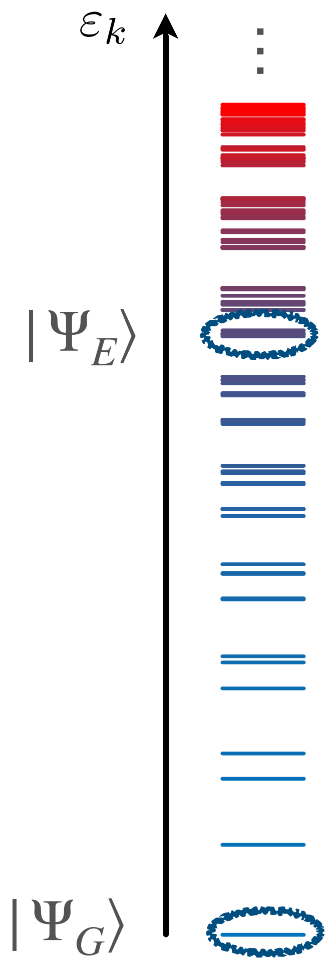
Many-Body Problem

Hamiltonian

$$\hat{H} = \sum_j \hat{h}_j$$

k-local

energy spectrum



q-simulators can represent volume law states → quantum advantage

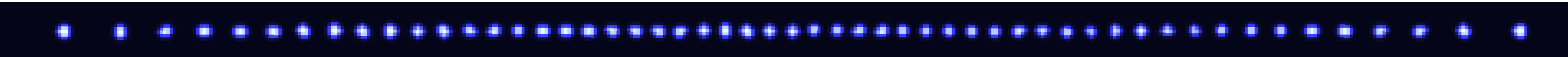
excited state $S_A \propto V = L_A^d$ volume law entanglement

thermal entropy

classical simulations with tensor networks

ground state $S_A \propto L_A^{d-1}$ area law entanglement

$\sim c \log L_A$ CFT $d = 1$



Area Law vs. Volume Law Entanglement

Eigenstate Thermalization Hypothesis (ETH)

Garrison et.al. PRX 2018

Reduced density operator in A

$$\rho_A = \mathrm{Tr}_B |\Psi_E\rangle\langle\Psi_E|$$

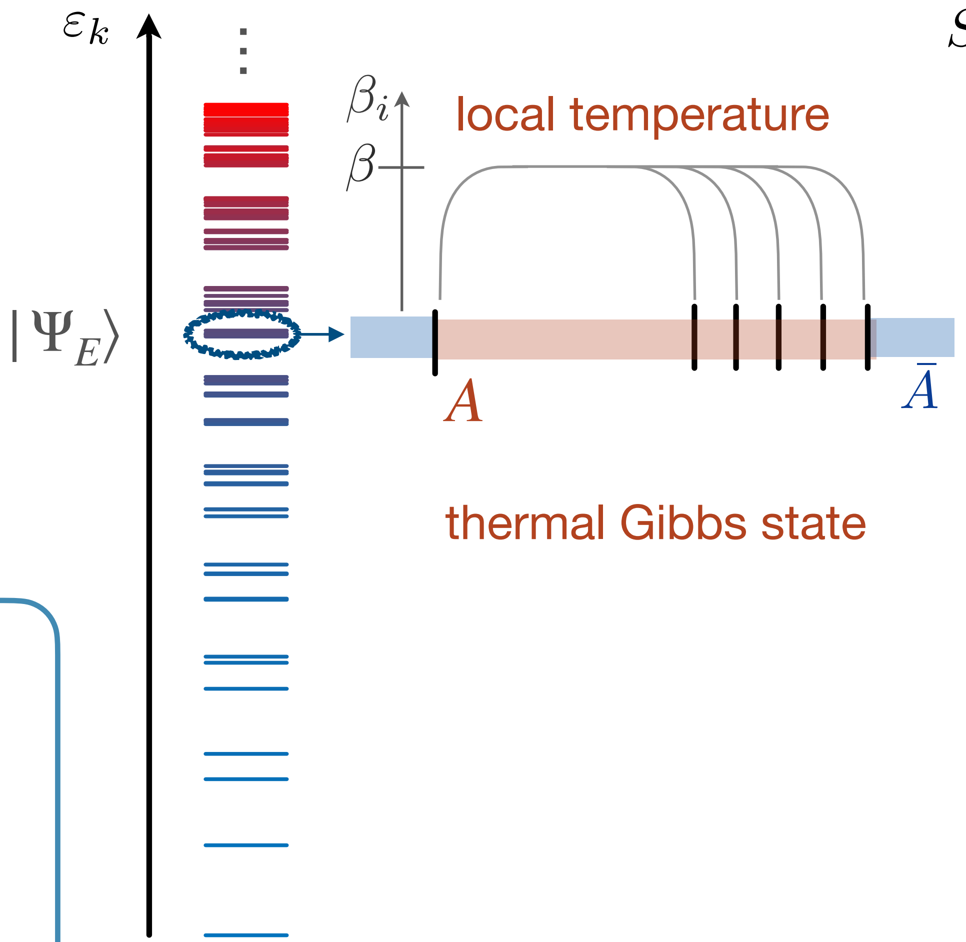
$$\sim e^{-\beta \hat{H}_A} \equiv e^{-\beta \sum_{i\in A} \hat{h}_i}$$

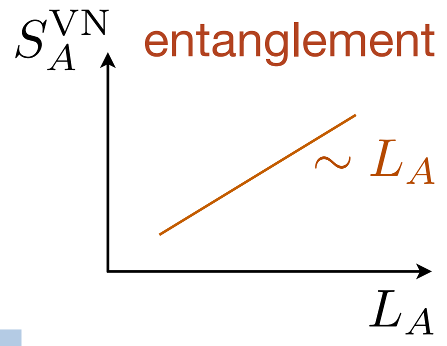
thermal Gibbs state

Parametrization as Gibbs state with *local inverse temperature*

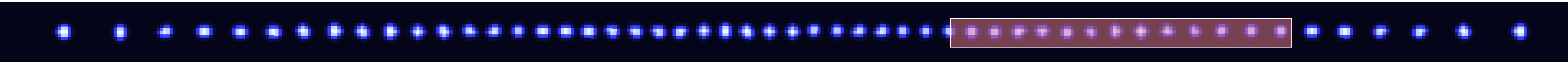
$$\rho_A \sim \mathrm{e}^{-\sum_{i\in A} \beta_i \hat{h}_i}$$

local temperature





subsystem A



Area Law vs. Volume Law Entanglement

Eigenstate Thermalization Hypothesis (ETH)

Garrison et.al. PRX 2018

Reduced density operator in A

$$\rho_A = \mathrm{Tr}_{\bar{A}} |\Psi_E\rangle\langle\Psi_E|$$

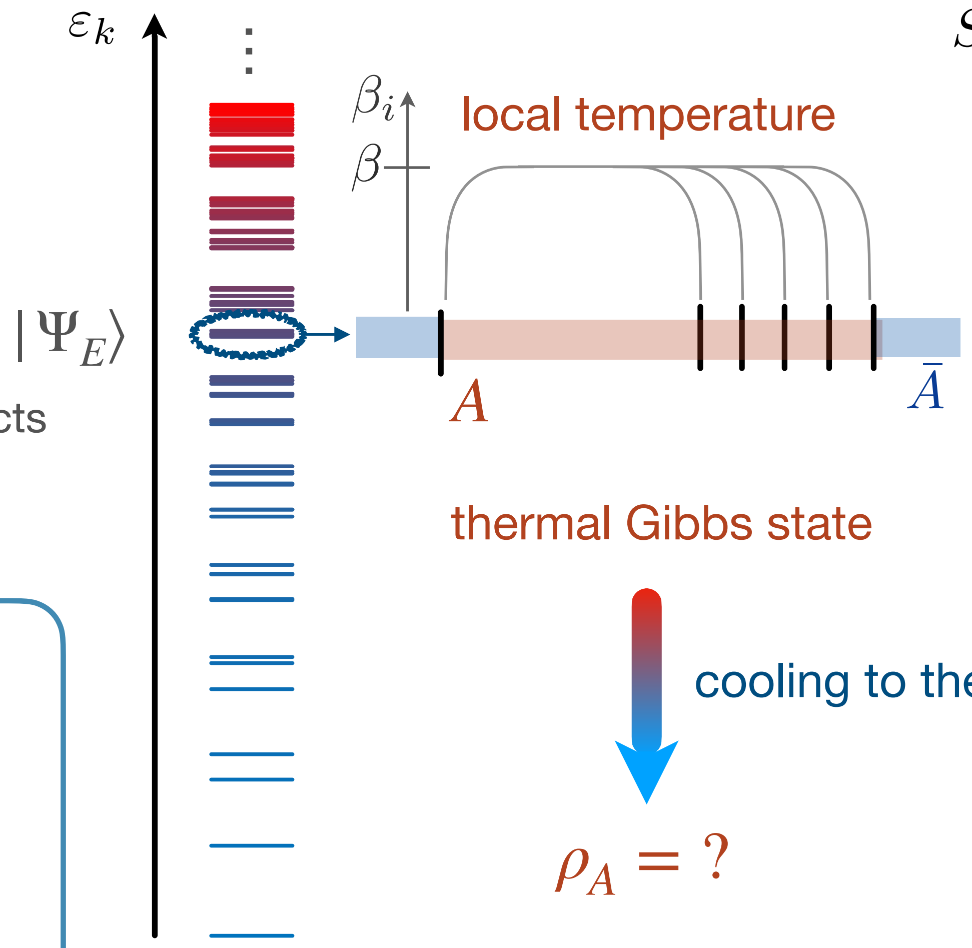
$$\sim e^{-\beta \hat{H}_A} \equiv e^{-\beta \sum_{i\in A} \hat{h}_i} \quad + \text{ boundary effects}$$

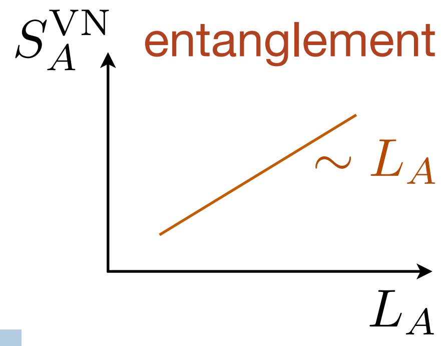
thermal Gibbs state

Parametrization as Gibbs state with *local inverse temperature*

$$\rho_A \sim \mathrm{e}^{-\sum_{i\in A} \beta_i \hat{h}_i}$$

local temperature



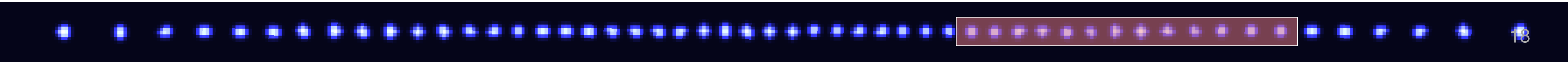


thermal Gibbs state

cooling to the ground state

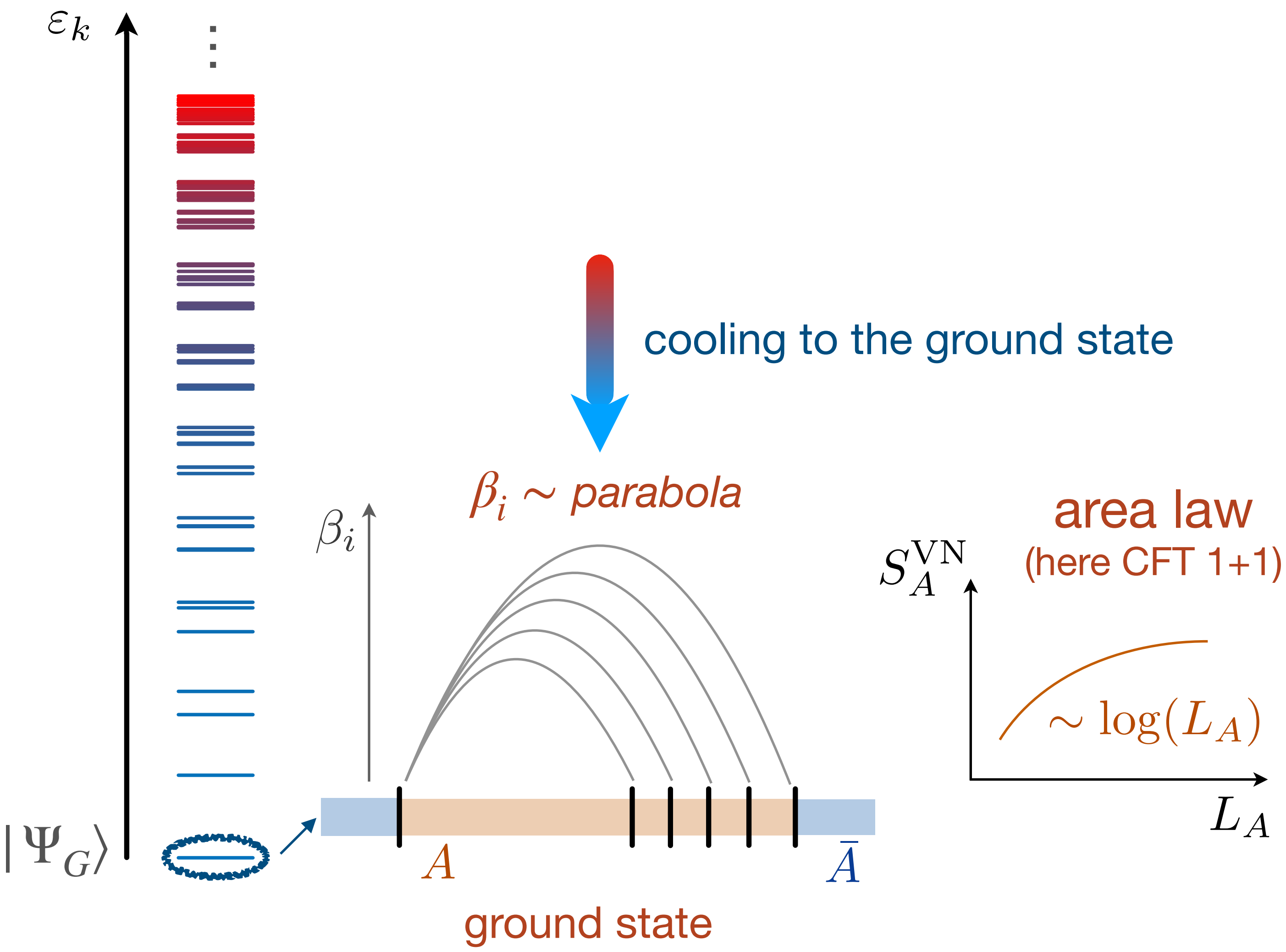
$$\rho_A = ?$$

subsystem A



Area Law vs. Volume Law Entanglement

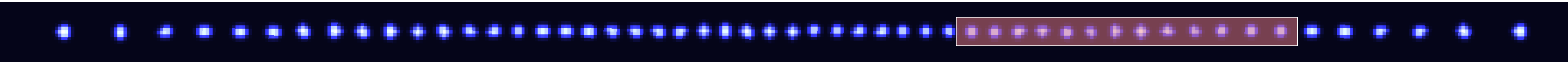
EH for ground states at critical point



Parametrization as Gibbs state with *local temperature*

$$\rho_A \sim \mathrm{e}^{-\sum_{i\in A} \beta_i \hat{h}_i}$$

local inverse temperature



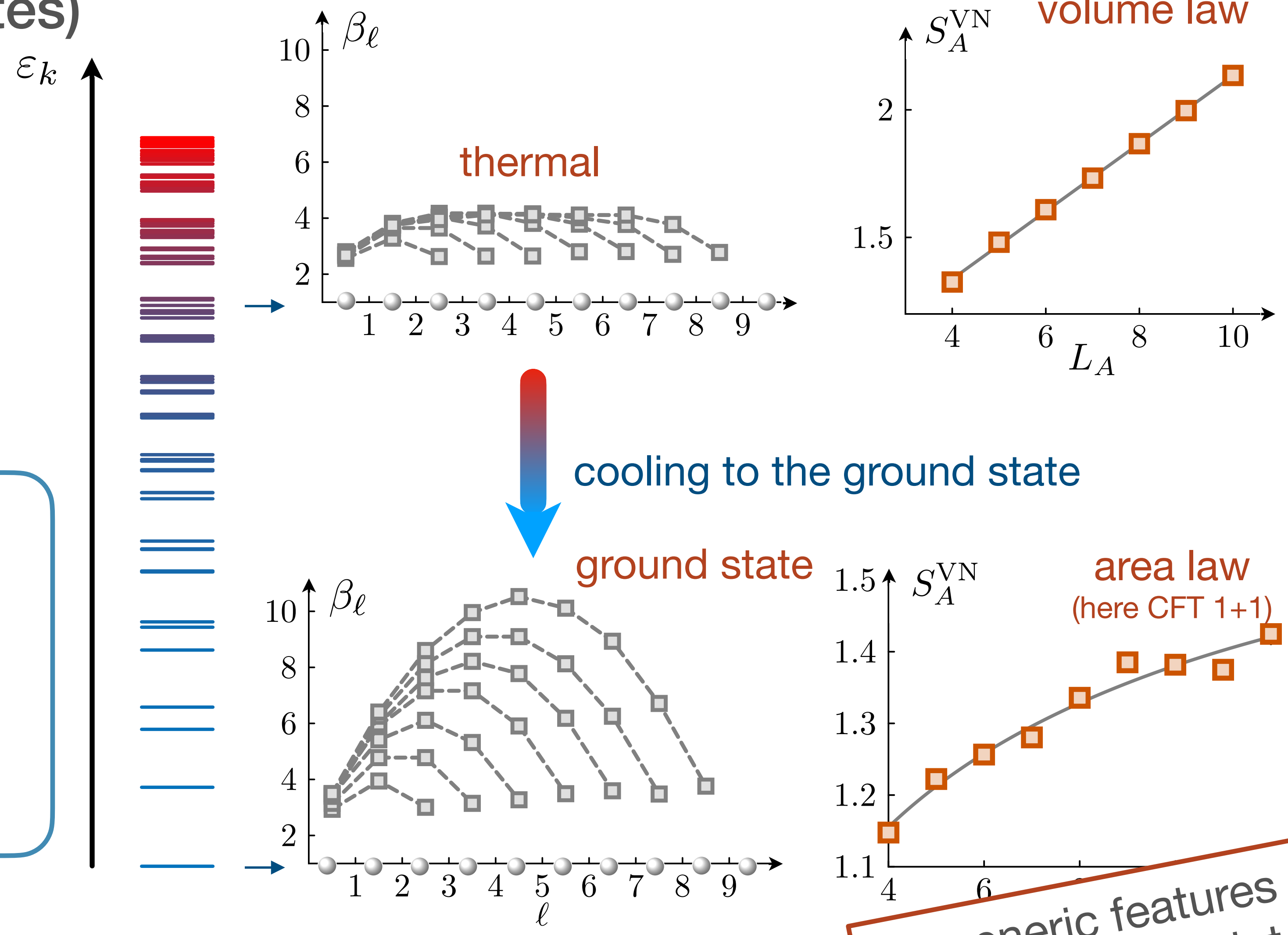
Entanglement of many-body wavefunction: area to volume law

Theory

Numerical example: XXZ model (51 sites)

$$\hat{H} = \frac{1}{2}\sum_i \left(\hat{S}_i^+ \hat{S}_{i+1}^- + \mathrm{H.c}\right) + \Delta \sum_i \hat{S}_i^z \hat{S}_{i+1}^z$$

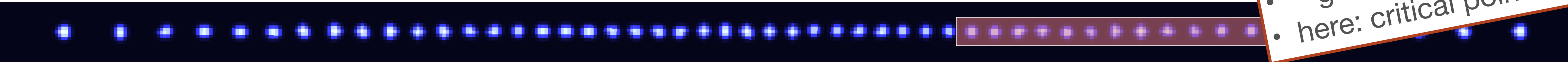
$$\Delta = 1$$



Can we see this in experiment?

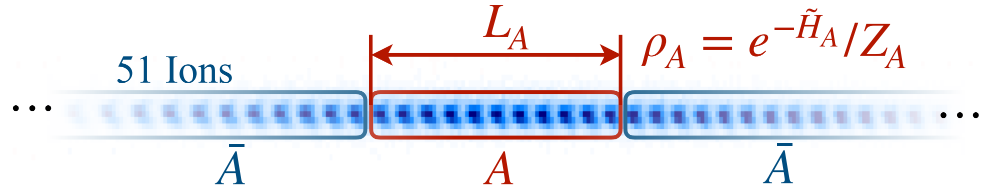
- prepare approx. ground & thermal state
- Entanglement Hamiltonian Tomography

- ~ generic features
- here: critical point



Results: Theory and Experiment

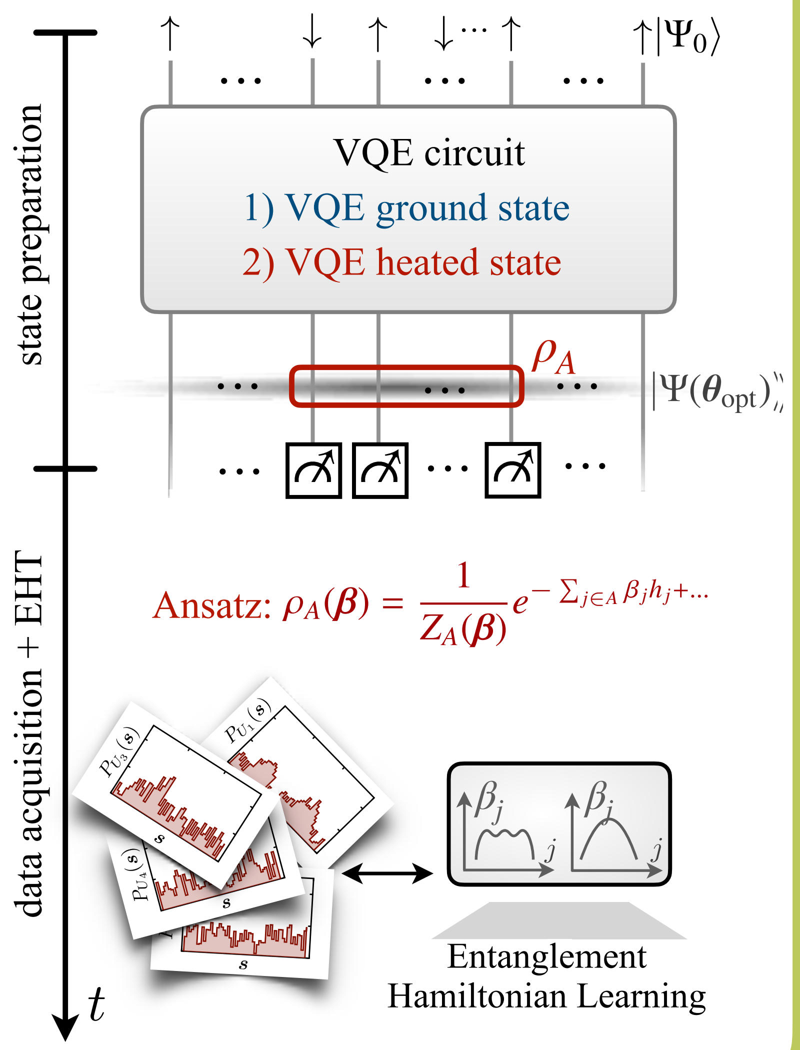
Heisenberg model



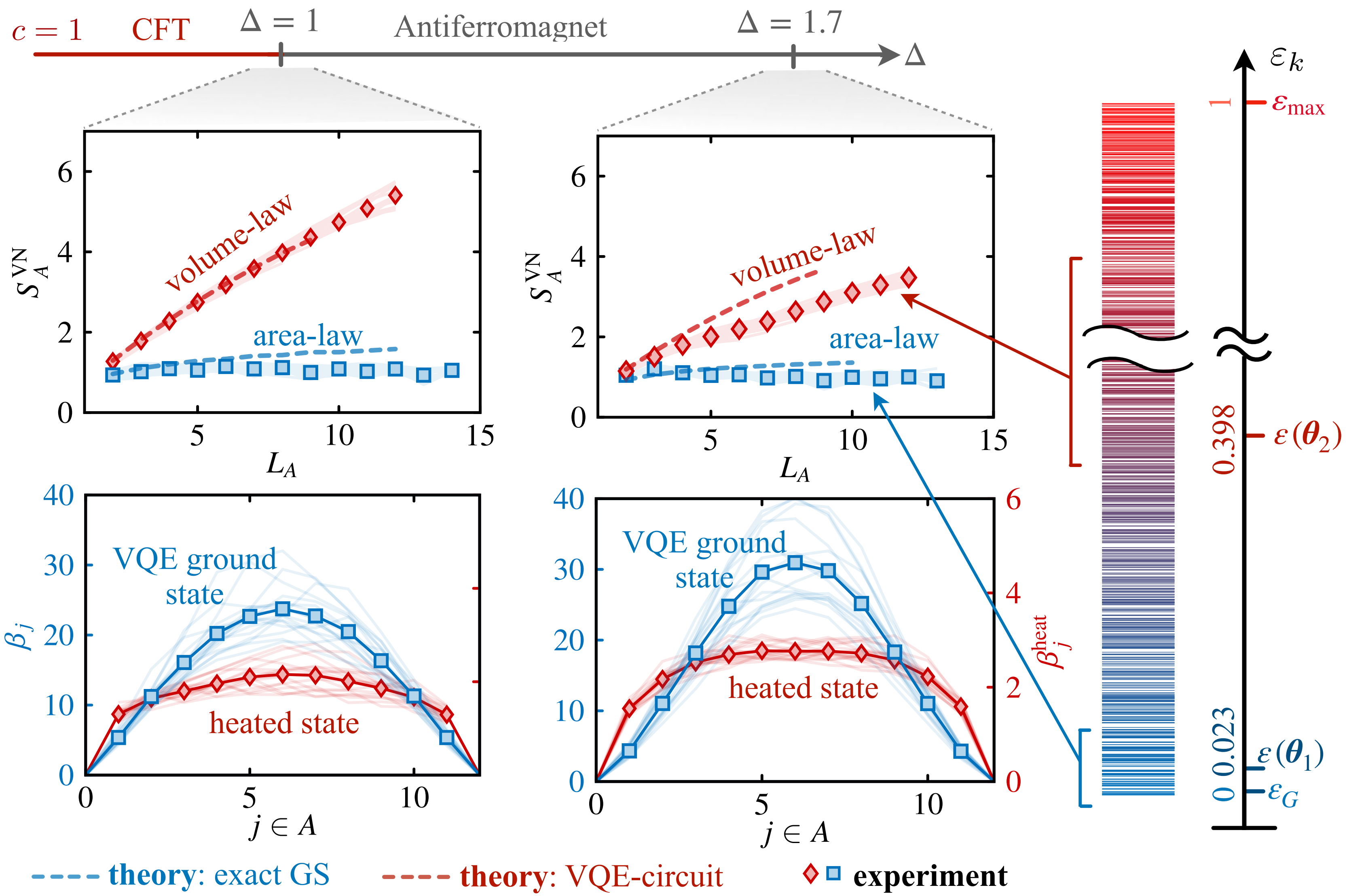
XXZ chain

$$H = J \sum_j \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \right) = \sum_j h_j$$

State preparation & analysis



Entanglement properties

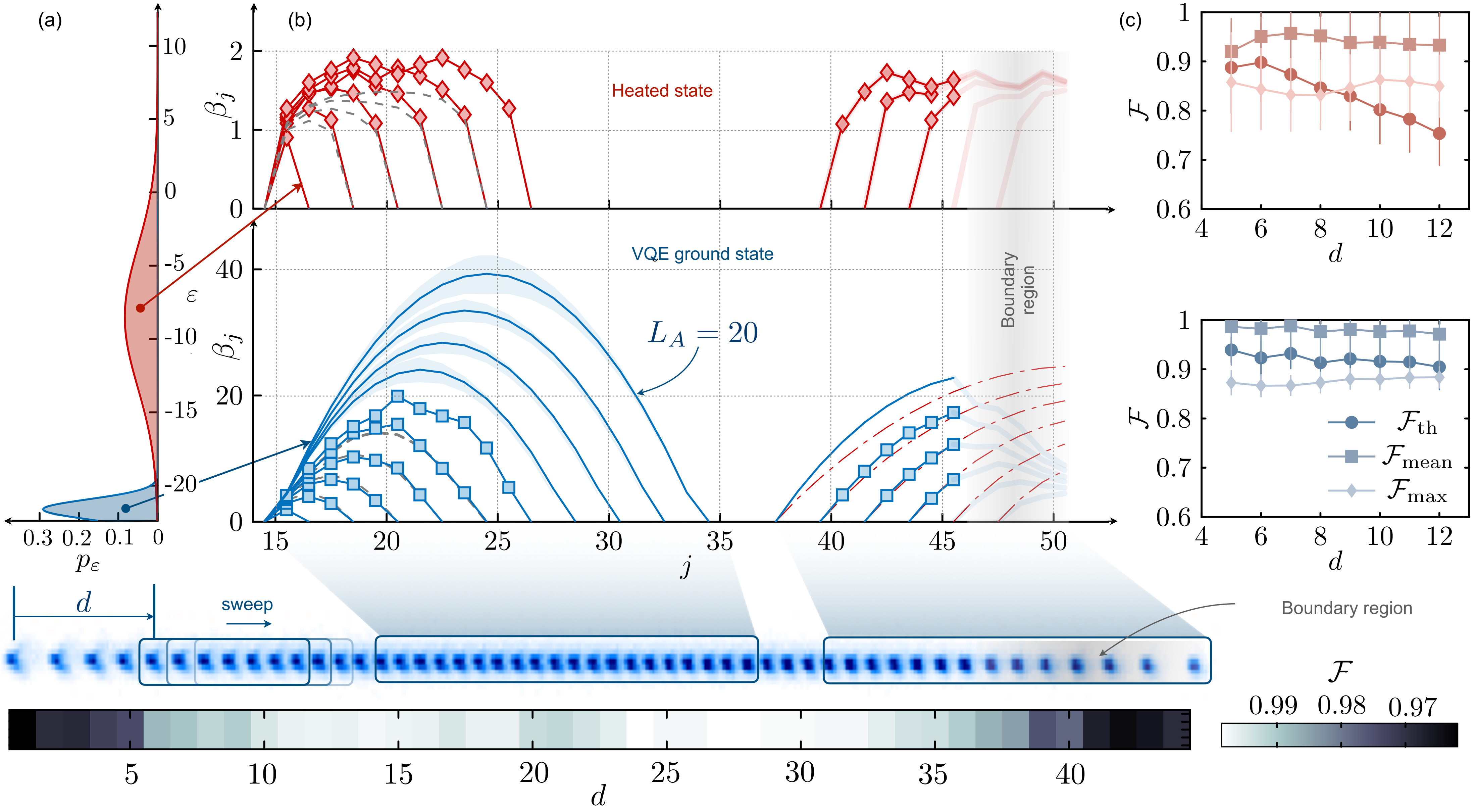


Results: Experiment + Theory


C Kokail

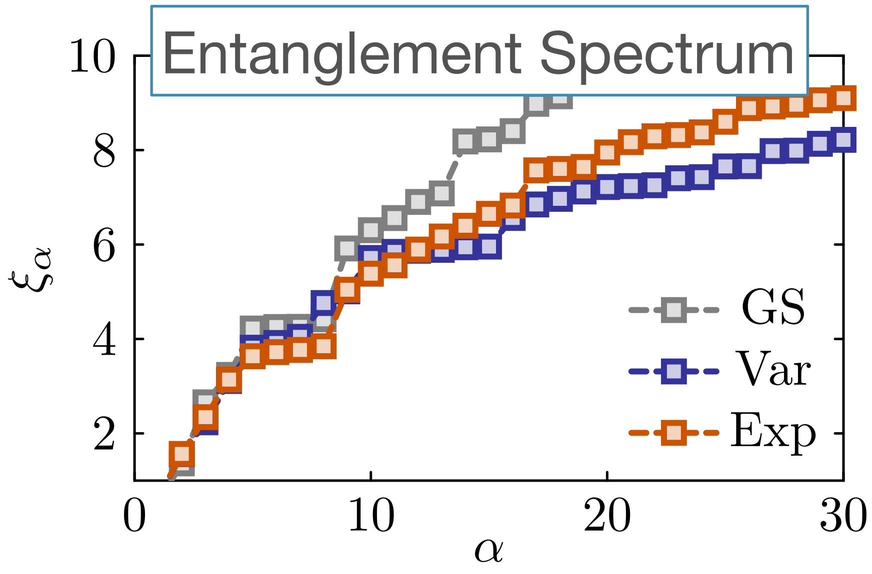

R van Bijnen


M Joshi



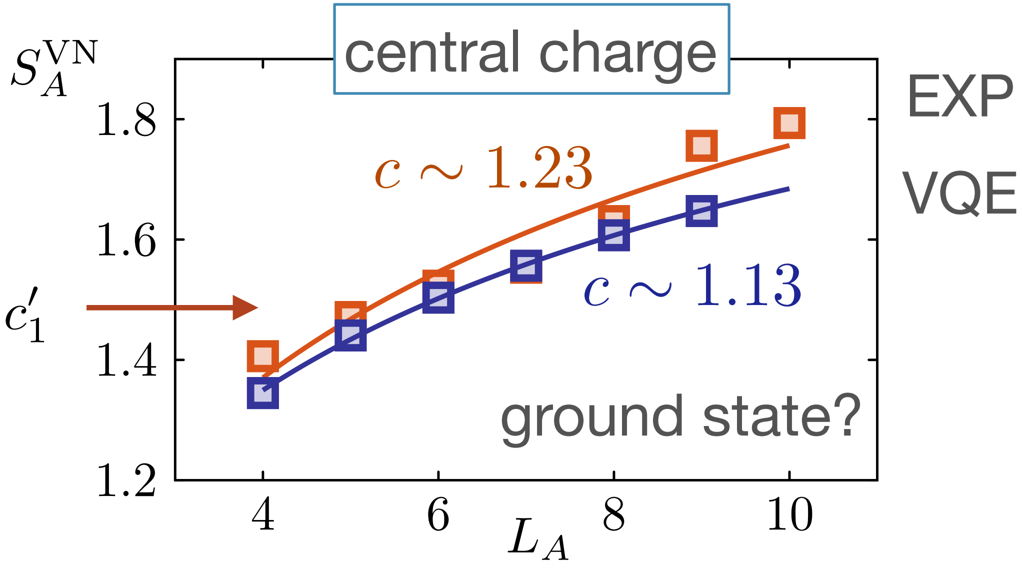
Theory vs. Experiment

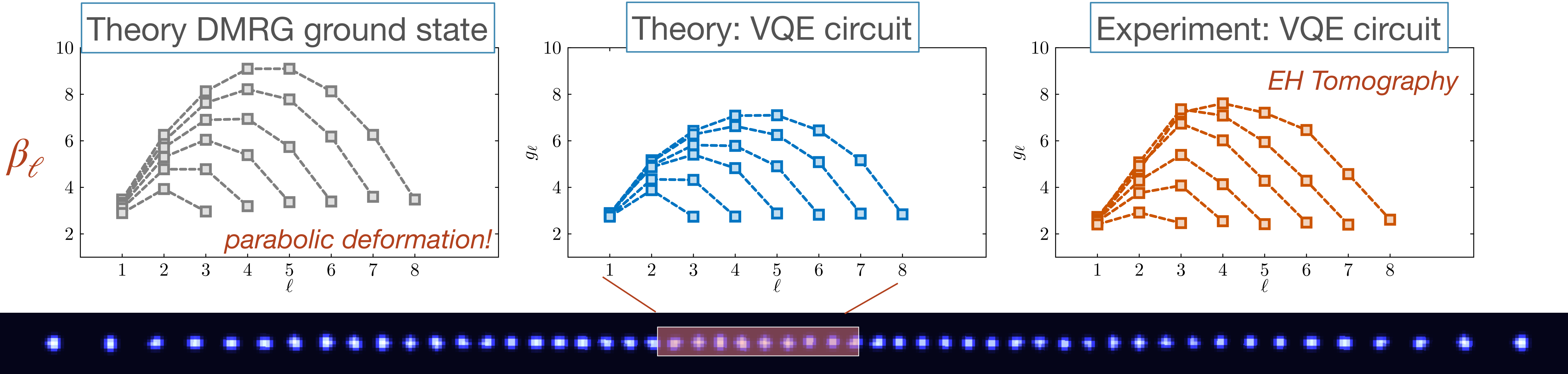
N = 51 ions



ground state entropy from CFT:

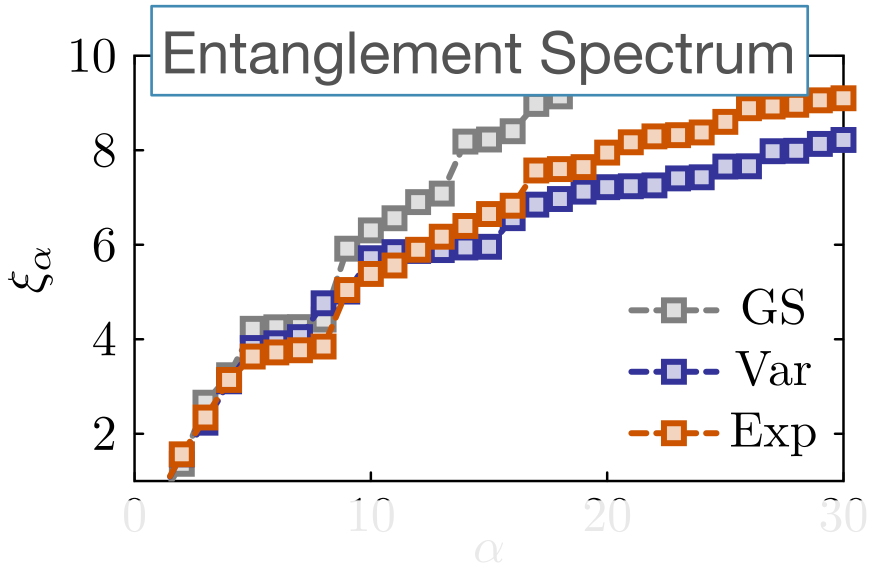
$$S_A^{\mathrm{VN}} = \frac{c}{3} \log \left[\frac{L}{\pi} \sin \left(\frac{\pi L_A}{L} \right) \right] + c_1'$$





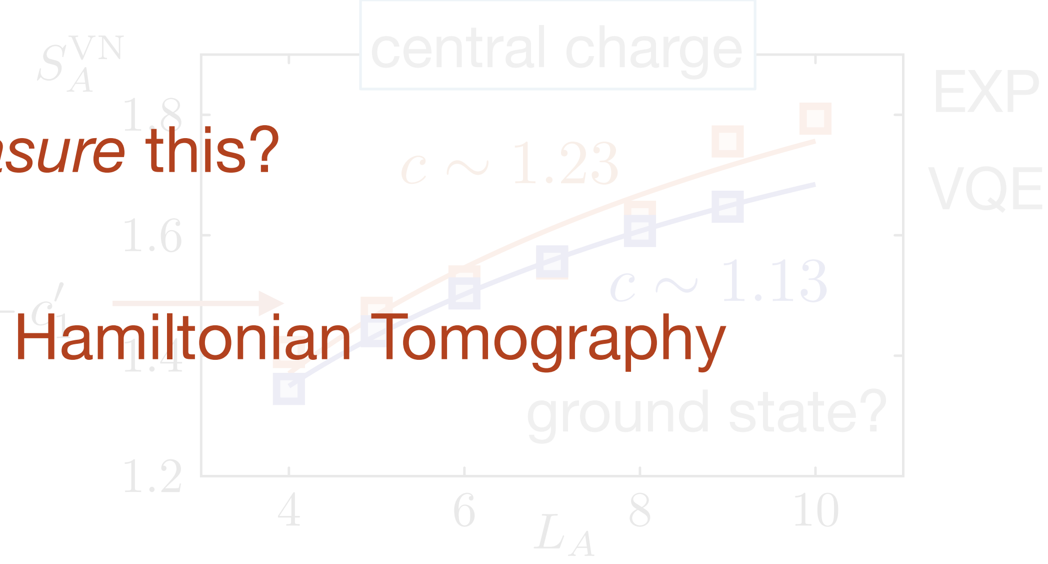
Theory vs. Experiment

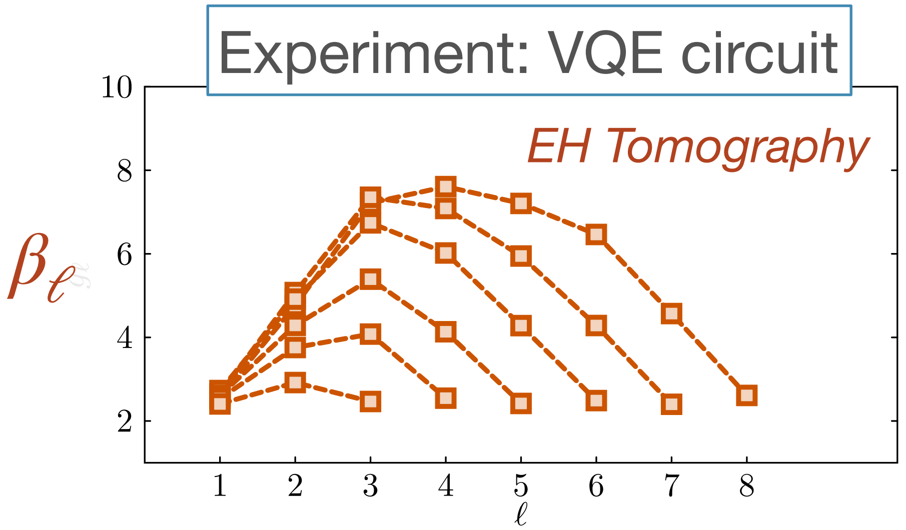
N = 51 ions

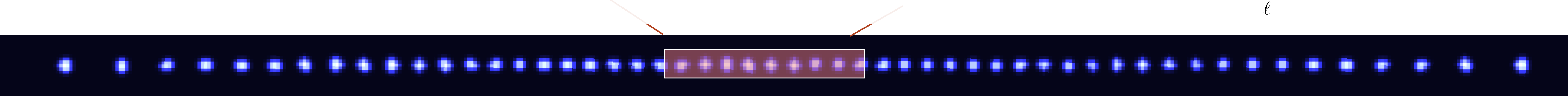


How did we *measure* this?

...Entanglement Hamiltonian Tomography

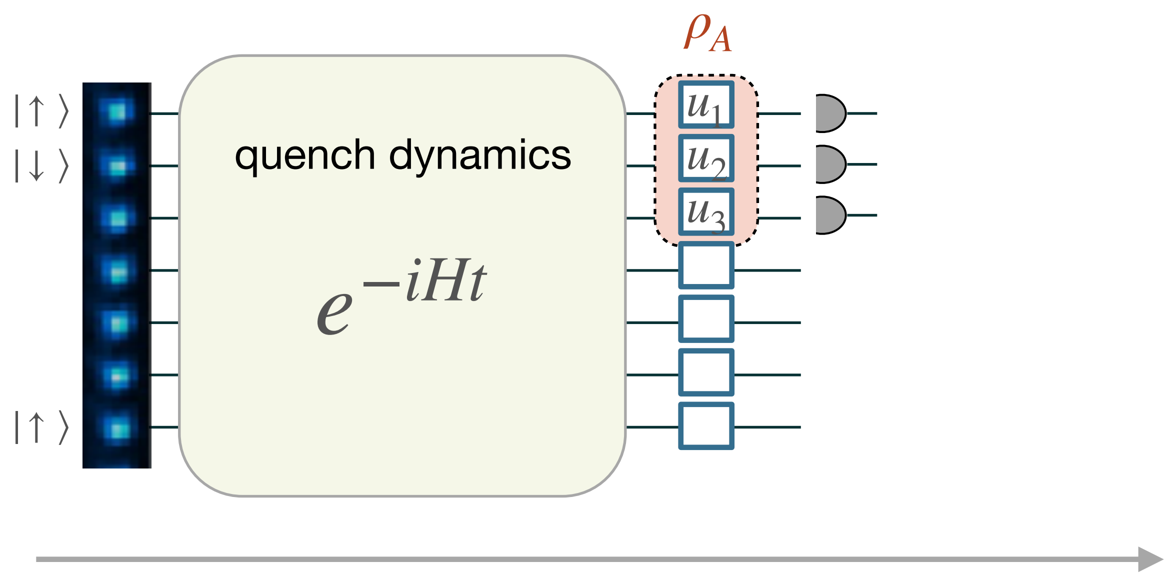






Randomized Measurements: Tomography

Quench dynamics with analog quantum simulator



time

product state → entangled state

time

Randomized Tomography

$$\rho_A = \mathbb{E}_{U\sim\mathrm{CUE}}[\hat{\rho}_A]$$

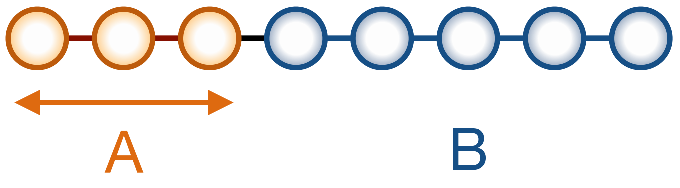
exponentially expensive

$$\hat{\rho}_A = \sum_{\mathbf{s},\mathbf{s}'} \sum_{U} P_U(\mathbf{s})(-2)^{-D[\mathbf{s},\mathbf{s}']} U|\mathbf{s}'\rangle\langle\mathbf{s}'|U^\dagger$$

tomographically complete

Measuring (Large-Scale) Entanglement

Protocol 0: Quantum State Tomography



data → ρ_A ✓expensive* $\sim$ rank$(\rho_A)\ 2^{N_A}$ (scales exponentially)

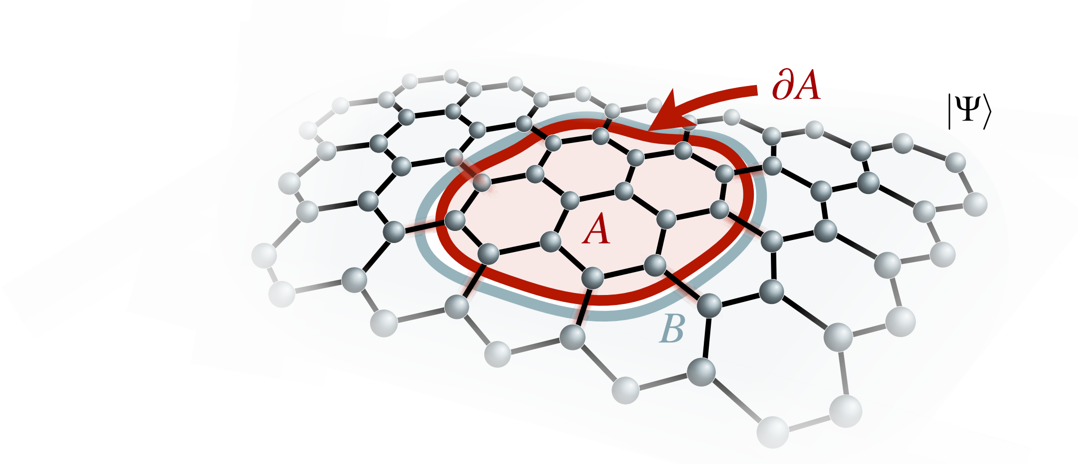
sample-efficient entanglement
Hamiltonian tomography

Classical Shadows

* tomography can be made `more efficient' if we *know* something about the quantum state: MPS, low rank, neural network, …

* or, we are only interested in certain functionals of ρ_A, e.g. expectation values $\langle O \rangle = \mathrm{Tr}\, O_A \rho_A$

Sample-Efficient Learning of the Entanglement Hamiltonian (EH)

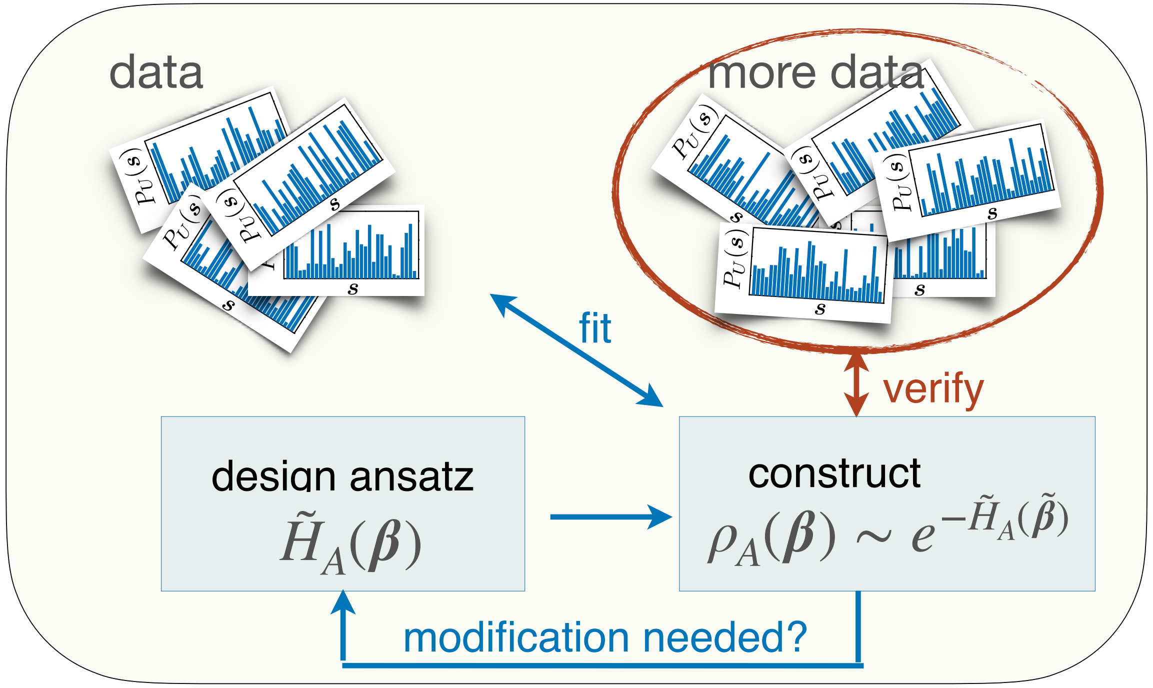


Protocol: *efficient* parametrization of EH

data ⟶ $\rho_A = e^{-\tilde{H}_A(\boldsymbol{\beta})}$ with $\tilde{H}_A(\boldsymbol{\beta}) = \sum_{i \in A} \beta_i \hat{h}_\ell + \ldots$ polynomial # β_i ?

Gibbs state

simple operator structure



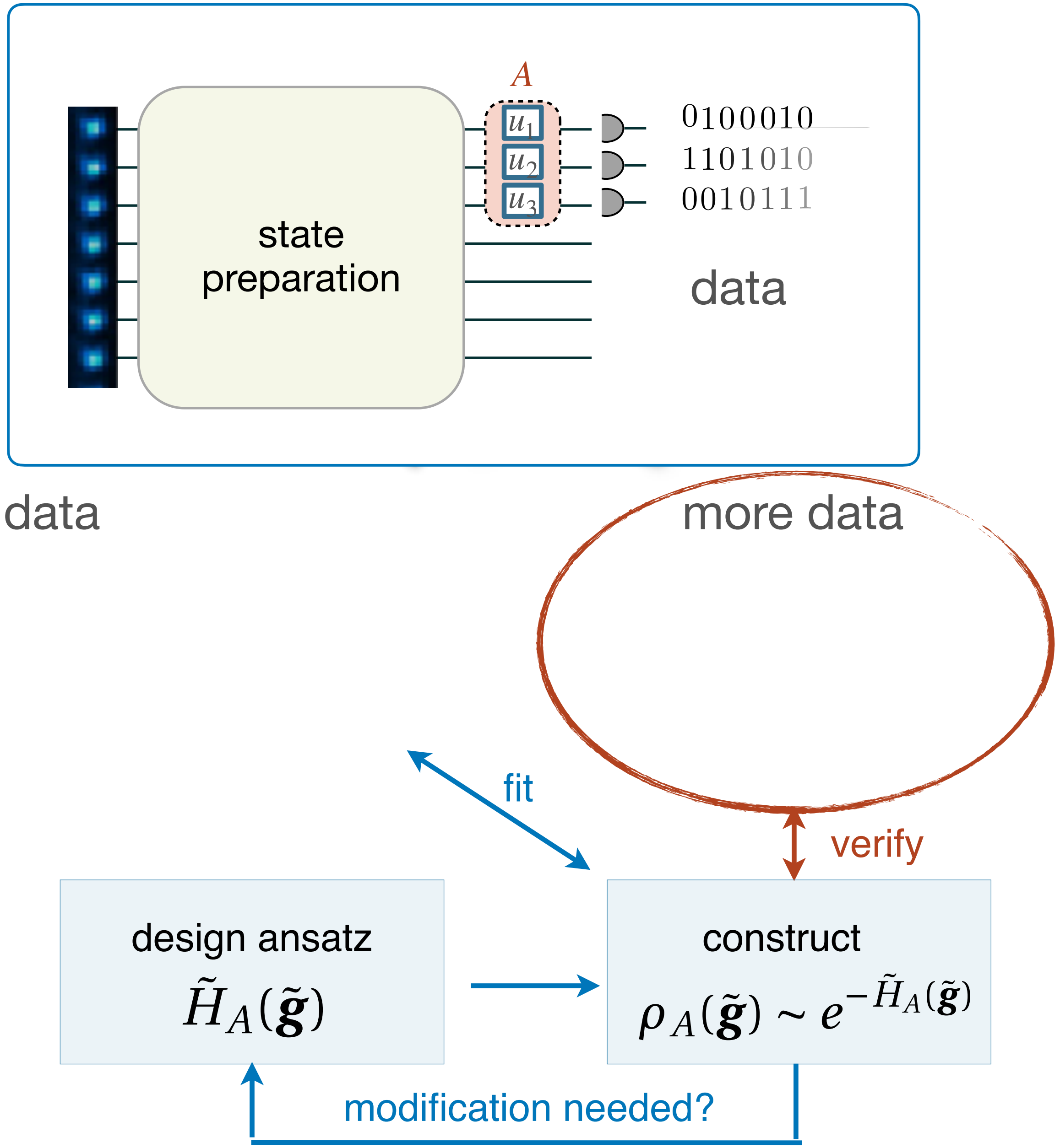
measurement protocol

- sample complexity
- time efficiency

C. Kokail, R. van Bijnen, A. Elben, B. Vermersch, & PZ, *Entanglement Hamiltonian Tomography in Quantum Simulation*, Nat. Phys. (2021).

A. Anshu, S. Arunachalam, T. Kuwahara, and M. Soleimanifar, *Sample-Efficient Learning of Interacting Quantum Systems*, Nat. Phys. (2021).

Learning the Entanglement Hamiltonian



Learning Protocol

- **Measure** experimental frequencies:

$$P_U(\mathbf{s}) = \mathrm{Tr}\left(U\rho_A U^\dagger |\mathbf{s}\rangle\langle\mathbf{s}|\right)$$

- **Ansatz** for Entanglement Hamiltonian:

$\tilde{H}_A(\tilde{\mathbf{g}})$
  - e.g. deformation of system Hamiltonian plus corrections

- **Fit** optimal parameters $\tilde{\mathbf{g}}$ by minimizing the distance to the frequencies

$$\chi^2(\tilde{\mathbf{g}}) = \sum_{U,\mathbf{s}}\left[\mathrm{Tr}\left(U|\mathbf{s}\rangle\langle\mathbf{s}|U^\dagger \frac{e^{-\tilde{H}_A(\tilde{\mathbf{g}})}}{Z(\tilde{\mathbf{g}})}\right) - P_U(\mathbf{s})\right]^2$$

- **Verify** by measuring Hilbert-Schmidt fidelities

$$\mathcal{F} \sim \mathrm{Tr}\left[\rho_A^{\text{data}}\rho_A^{\text{more data}}\right] = \ldots$$

A. Elben et. al. PRL (2020)

EHT for the Ground state of a long-range Ising chain (Theory)

System Hamiltonian

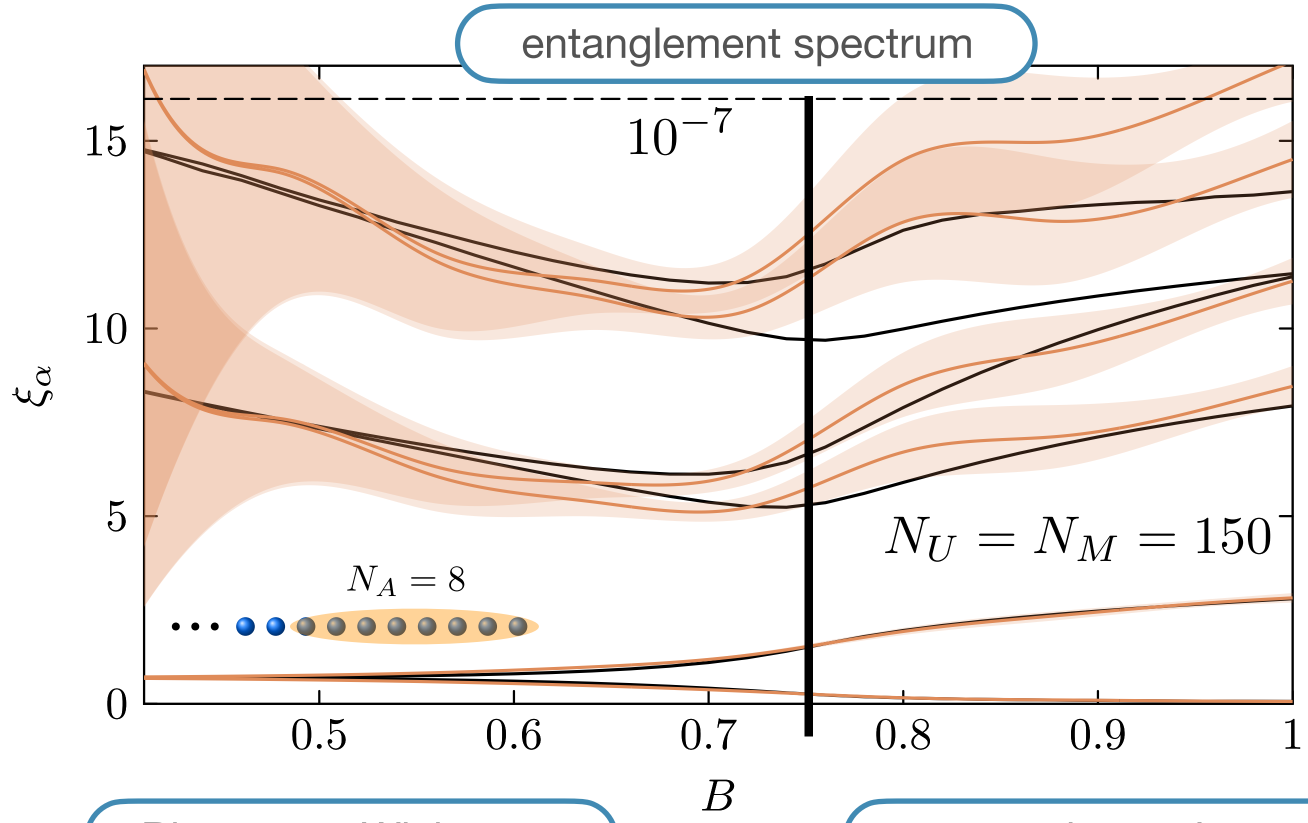
$$H = \sum_{i,j>i} \frac{J_0}{|i-j|^\alpha} \sigma_i^x \sigma_j^x + B \sum_i \sigma_i^z$$

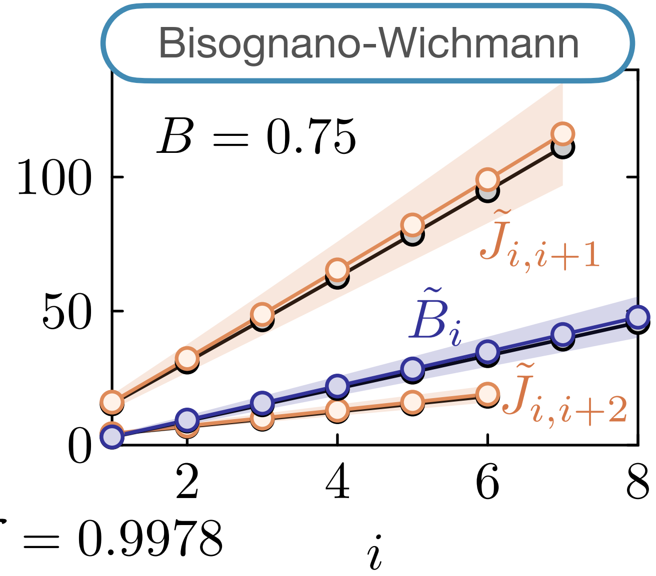
$\alpha = 2.5$

Ansatz for Entanglement Hamiltonian

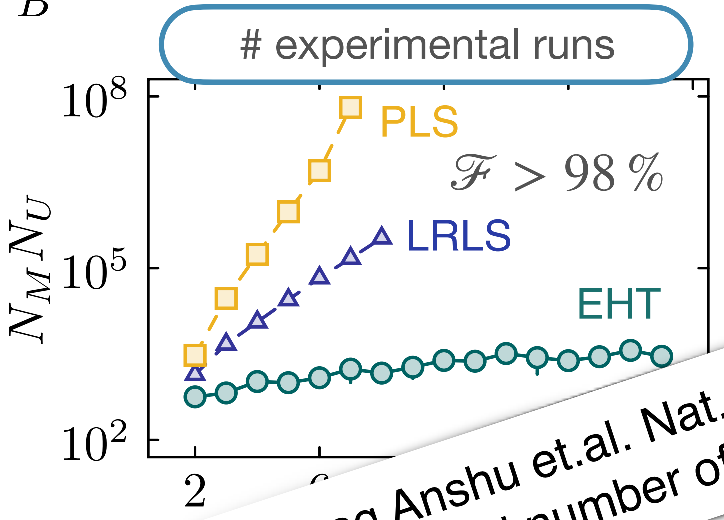
We choose a simple deformation of the system Hamiltonian

$$\tilde{H}_A = \sum_{i,j \in A} \tilde{J}_{ij} \sigma_i^x \sigma_j^x + \sum_{i \in A} \tilde{B}_i \sigma_i^z$$



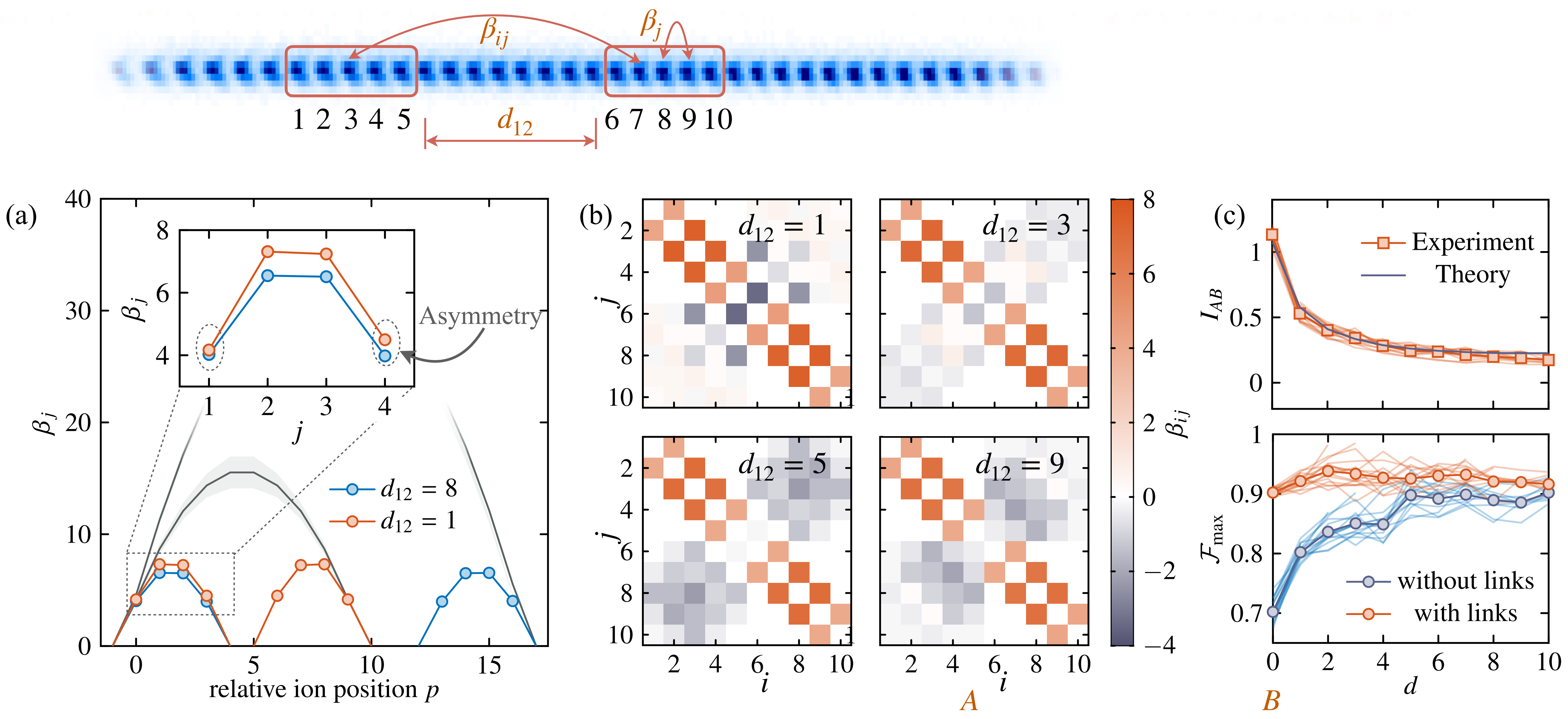


$\mathcal{F} = 0.9978$



Anurag Anshu et.al. Nat.Phys 20
(polynomial number of sample

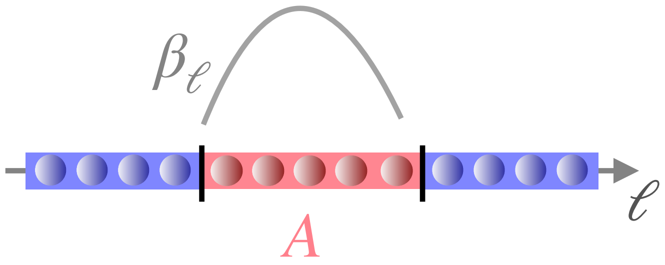
EHT for spatially separated regions



Future: *Seeing* RG Flow, and ”Quantum Gravity in the Lab"

Quantum Simulator

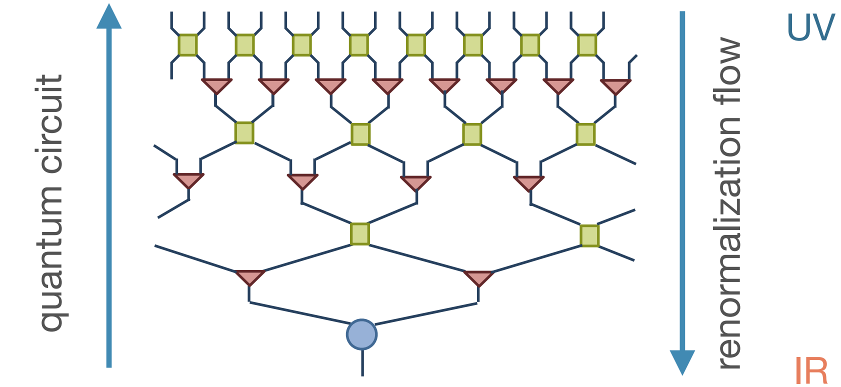
$$\rho_A = e^{-\tilde{H}_A}$$ entanglement Hamiltonian



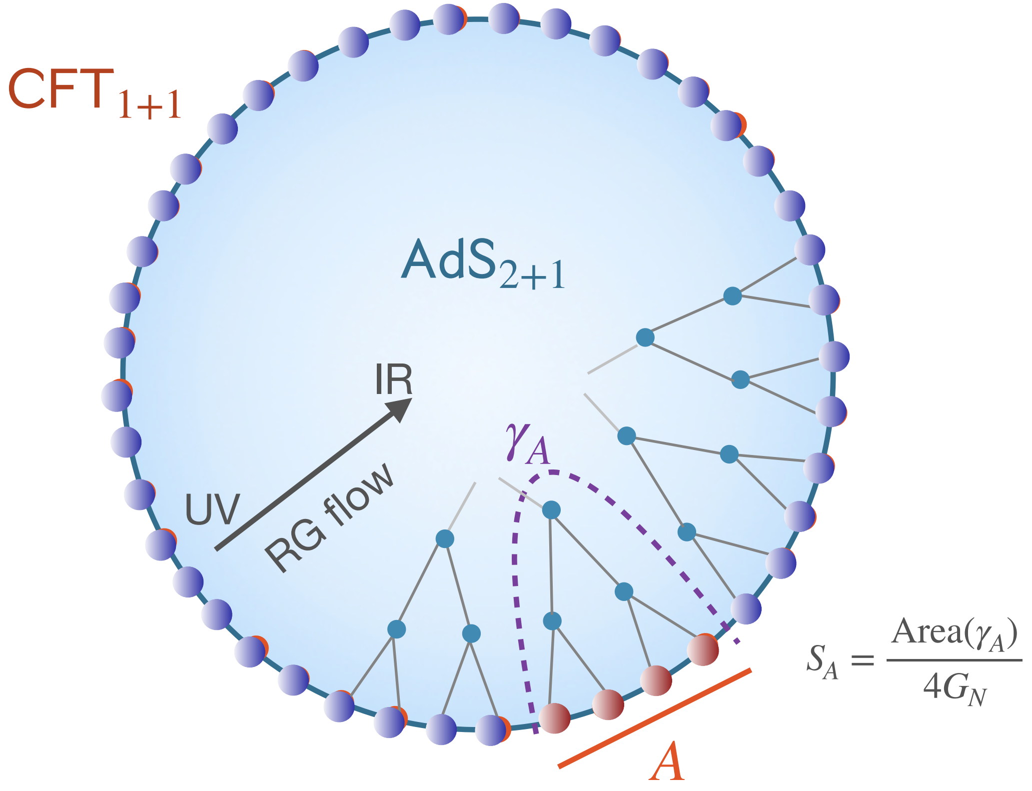
Tensor Network: MERA

G Vidal, PRL (2007).

$$|\Psi\rangle = \sum_{\sigma_1,\sigma_2,\ldots,\sigma_n} c(\sigma_1, \sigma_2, \ldots, \sigma_n)\,|\sigma_1, \sigma_2, \ldots, \sigma_n\rangle$$



Holographic gauge/ gravity duality



entanglement ↔ geometry

A Periwal, M Schleier-Smith et al, Nature (2021);
N Yao, C Monroe et al., Nature (2019)
B Swingle, PRD (2012); M Nozaki, S... ...EP (2011);
A. R. Brown et al., arXiv:1911.06314 ... al., arXiv:2102.01064

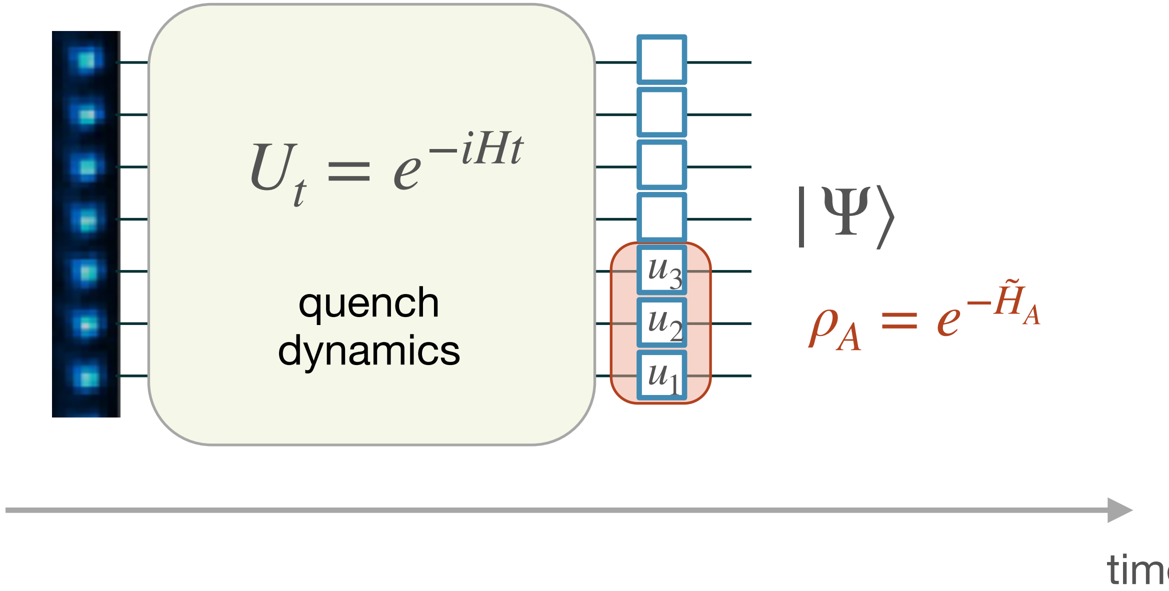
We measure entanglement on q-simulator

Learning the Entanglement Hamiltonian in Quench Dynamics

C. Kokail, R. van Bijnen, A. Elben, B. Vermersch, and P. Zoller, Nat. Phys. **17**, 936 (2021).

Entanglement Spectrum & Quench Dynamics

Quench dynamics with analog quantum simulator



Q.: Can we *measure* Entanglement Spectrum in quench dynamics?

$$|\Psi\rangle = \sum_{\alpha=1}^{\chi} e^{-\xi_\alpha/2} |\Phi_\alpha^A\rangle \otimes |\Phi_\alpha^B\rangle$$

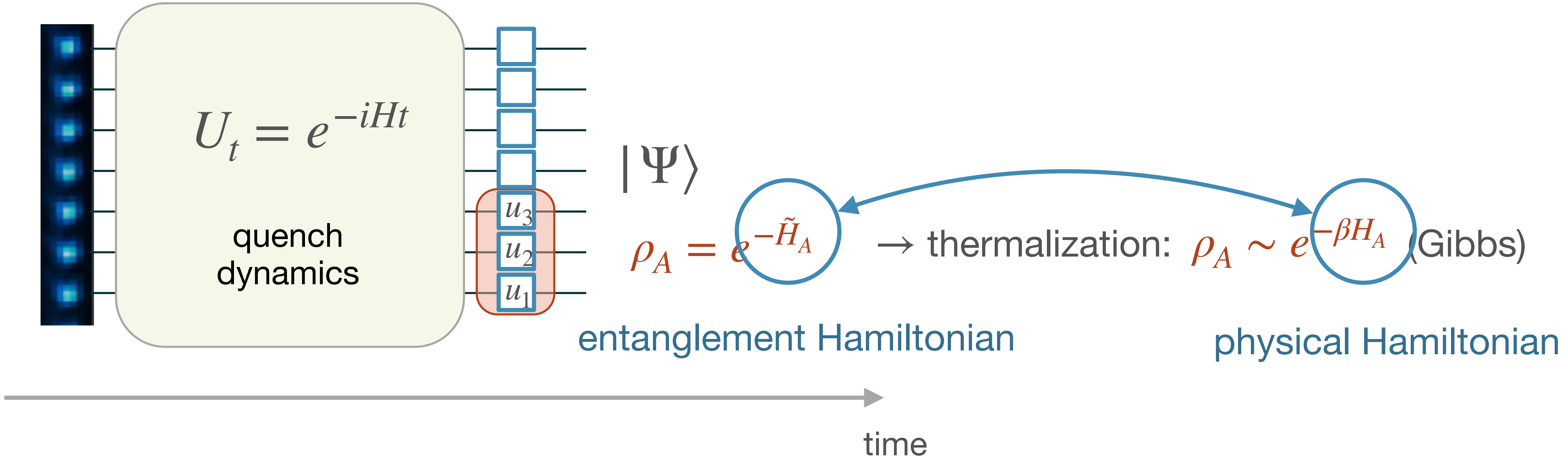
Schmidt values as function of time

product → entangled state

… more *efficiently* than tomography?

Entanglement Spectrum & Quench Dynamics

Quench dynamics with analog quantum simulator



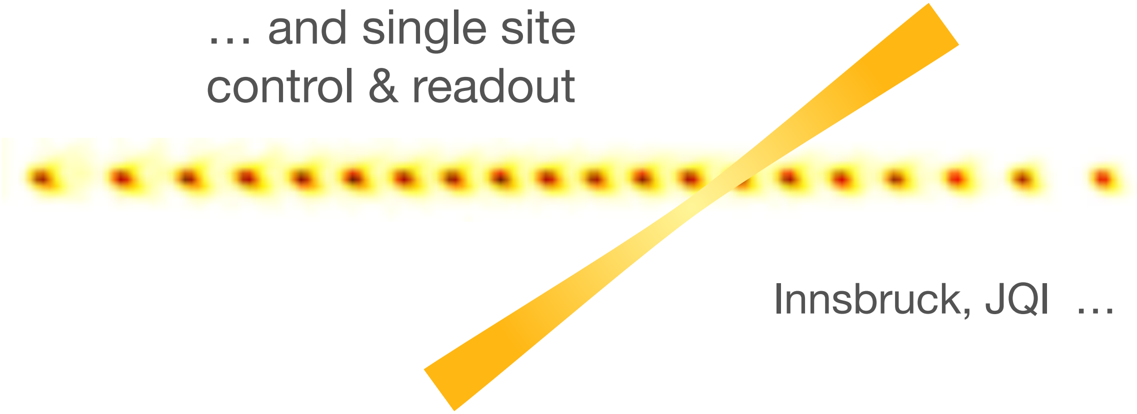
Local operator structure of $\tilde{H}_A$?
with *few* parameters

see also Bisognano-Wichmann theorem, Conformal Field Theory

10, 20 … 50 Qubit Trapped-Ion Programmable Quantum Simulator @ IQOQI-Labs

Programmable Quantum Simulator with Trapped Ions

… and single site control & readout



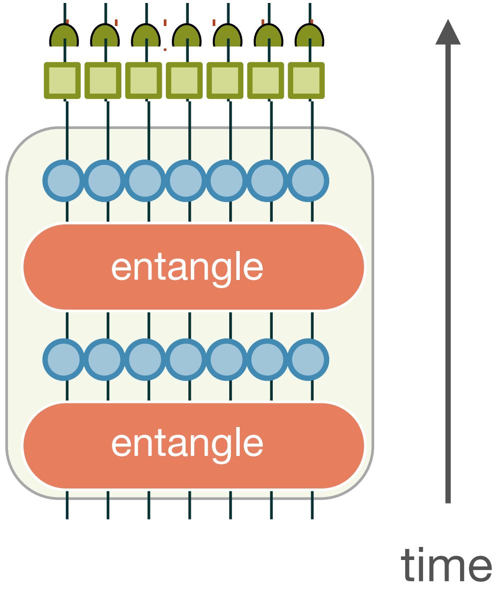
Innsbruck, JQI …

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij}\hat{\sigma}_i^x\hat{\sigma}_j^x + B\sum_i \hat{\sigma}_i^z$$

$$J_{ij} \sim \frac{1}{|i-j|^\alpha} \quad \alpha = 0\ldots3$$

long range

Quantum Circuits as quench dynamics



T Brydges et al., Science 2019; C Kokail et al., Nature 2019

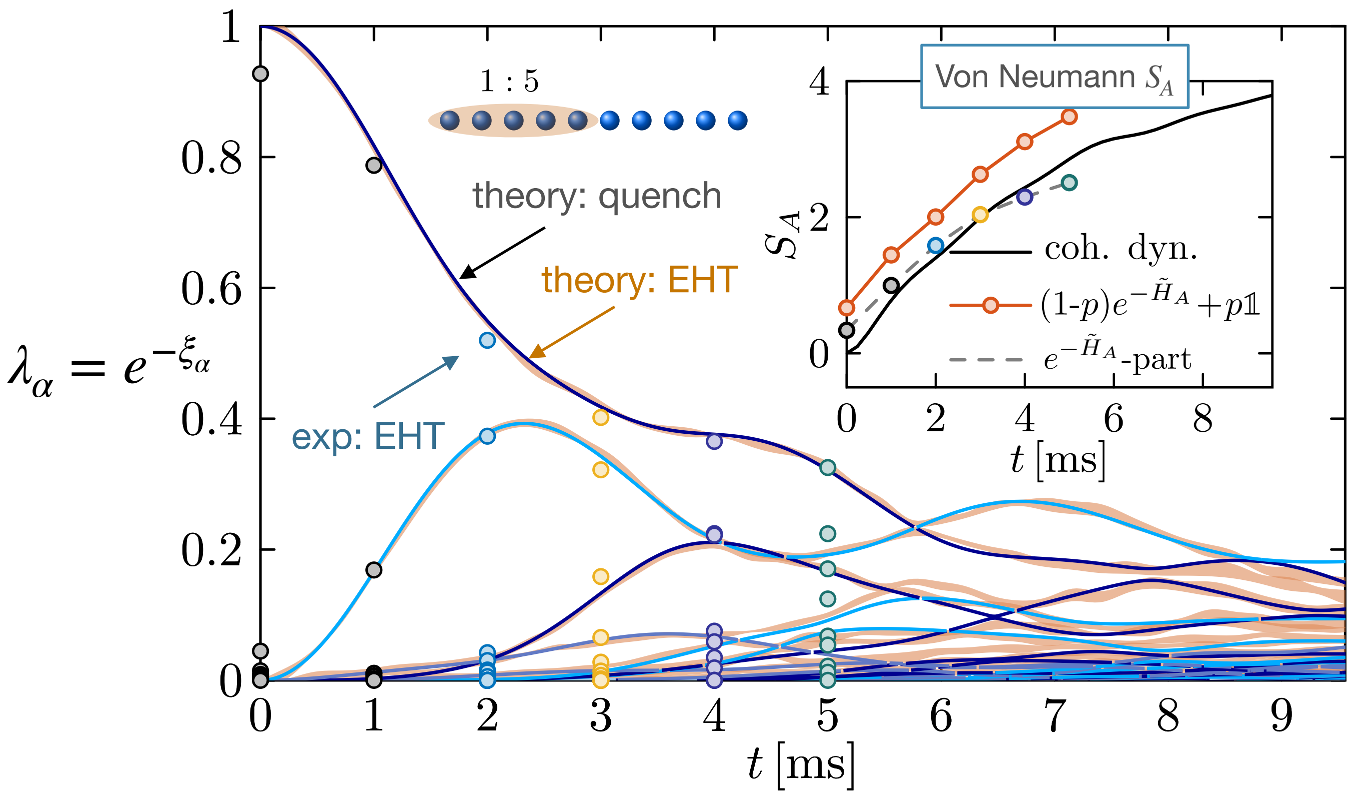
Blatt - C. Ro

eriments]

Entanglement Spectrum in Quench Dynamics: Exp. vs. Theory

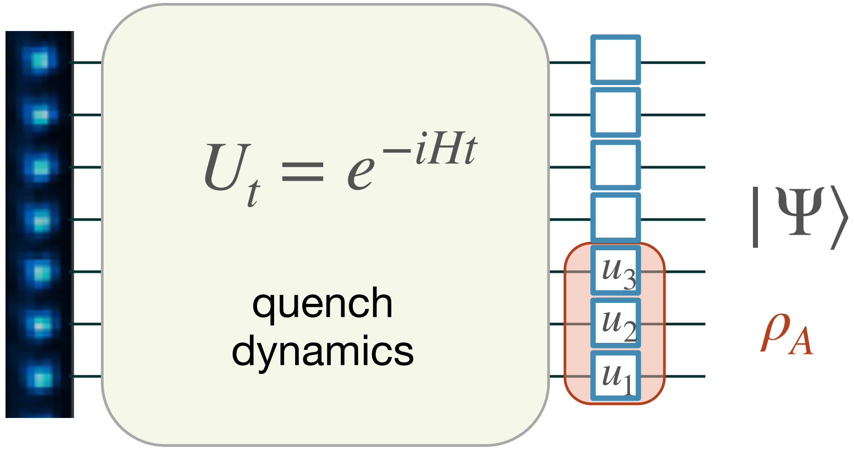
C Kokail

Sub-system [1:5] of 10 ions [similar data for 20 ions and subsystem [8:14]]



$$H = \sum_{i<j} \left(J_{ij} \sigma_i^+ \sigma_j^- + \text{ h.c.} \right) + B \sum_i \sigma_i^z \quad \text{for } B \gg J$$

C Kokail, R van Bijnen, A Elben, B Vermersch, & P.Z, *arXiv:* 2009.09000.

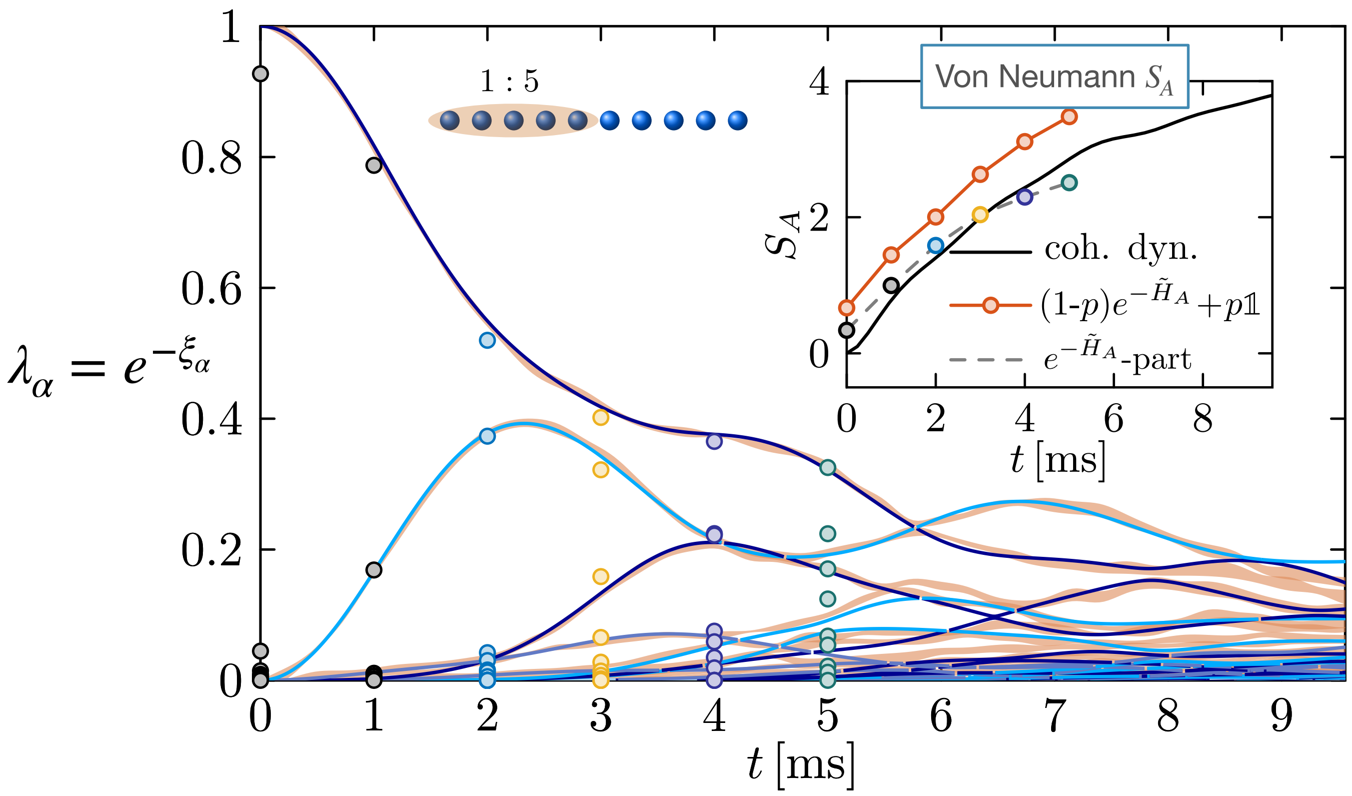


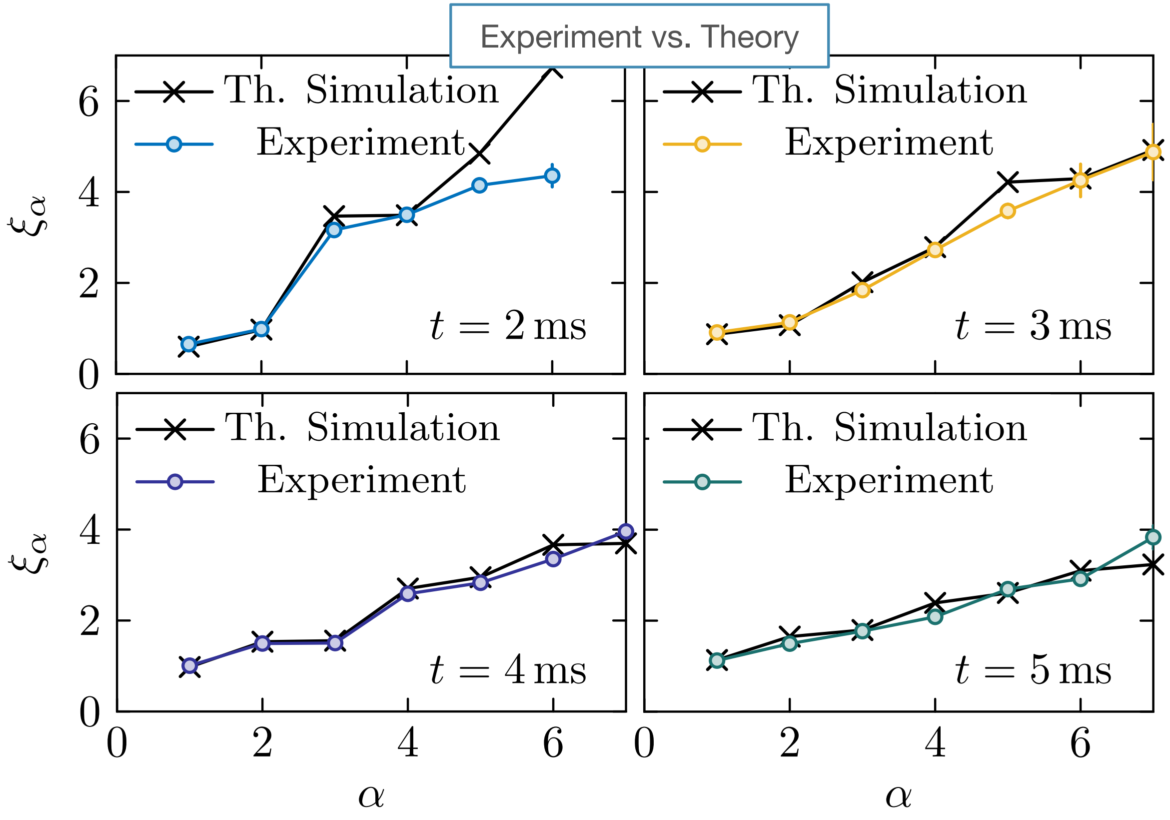
$$|\Psi\rangle = \sum_{\alpha=1}^{\chi} e^{-\xi_\alpha/2} |\Phi_\alpha^A\rangle \otimes |\Phi_\alpha^B\rangle$$

We wish to `see' the Schmidt values $\lambda_\alpha = e^{-\xi_\alpha}$ in quench dynamics in time

Entanglement Spectrum in Quench Dynamics: Exp. vs. Theory

Sub-system [1:5] of 10 ions



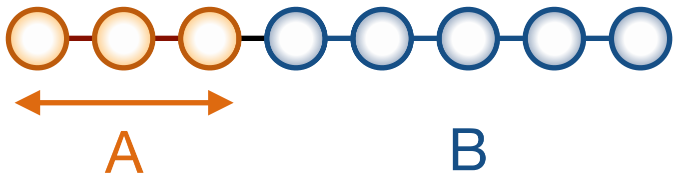


$$H = \sum_{i<j} \left(J_{ij}\sigma_i^+\sigma_j^- + \text{ h.c.} \right) + B\sum_i \sigma_i^z \quad \text{for } B \gg J$$

Q.: How to extract the ES (and EH) from experimental data?

Technical Slide: Measuring ES and Tomography of EH

Efficient Entanglement Hamiltonian Tomography



$$H = H_A + H_B + H_{AB}$$

ρ_A ⟷ $\rho_A(\tilde{\boldsymbol{g}}) = \exp\left(-\tilde{H}_A(\tilde{\boldsymbol{g}})\right)$

experiment

Ansatz for $\tilde{H}_A(\tilde{\boldsymbol{g}})$ as few k-local terms, typically as deformation of H_A + corrections.

Protocol: (systematic = success or fail)

classical postprocessing

Find best $\rho_A(\tilde{\boldsymbol{g}})$ from *fit to data* → *Verify* from x-platform fidelity → fidelity threshold → expand ansatz EH … new physics (?) →

few parameters & measurements

Step 1: Ansatz for Entanglement Hamiltonian — The Recipe

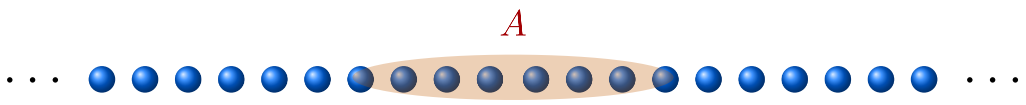
Hamiltonians H of physical systems: as sum of quasi-local few-body terms

Example:

$$H = \sum_{i<j} \left(J_{ij} \sigma_i^+ \sigma_j^- + \text{ h.c.} \right) + B \sum_i \sigma_i^z$$

few parameters

for $B \gg J$



Structure of Entanglement Hamiltonian $\tilde{H}_A$: quasi-local few-body terms?

Ansatz:

$$\tilde{H}_A(\tilde{\boldsymbol{J}}, \tilde{\boldsymbol{B}}) = \sum_{ij} \left(\tilde{J}_{ij} \sigma_i^+ \sigma_j^- + \text{ h.c.} \right) + \sum_i \tilde{B}_i \sigma_i^z \quad + \quad \ldots$$

few variational parameters

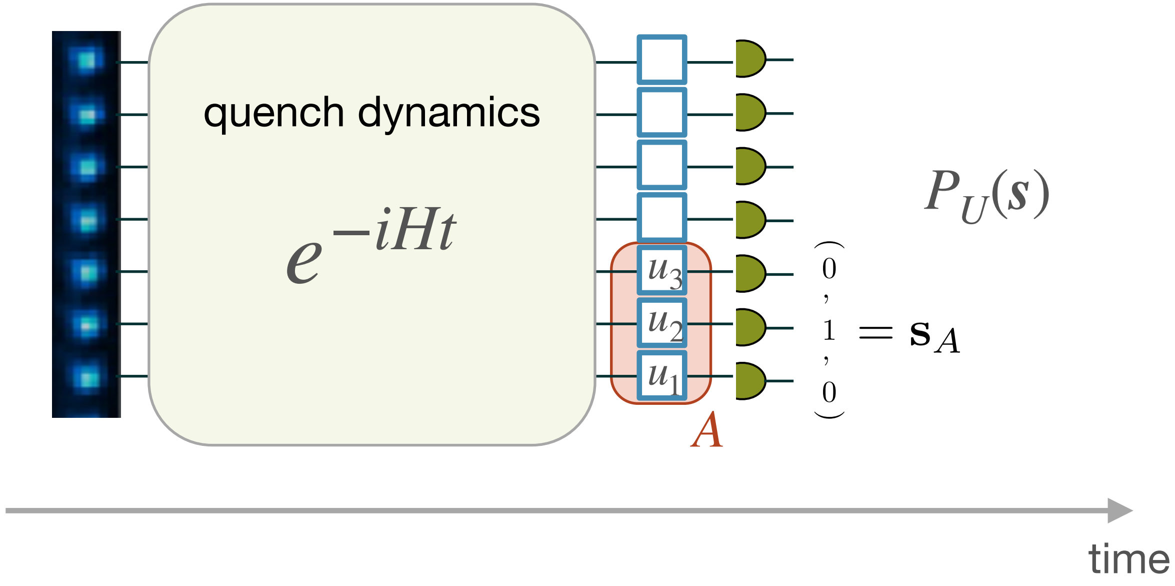
deformed system Hamiltonian (base level)

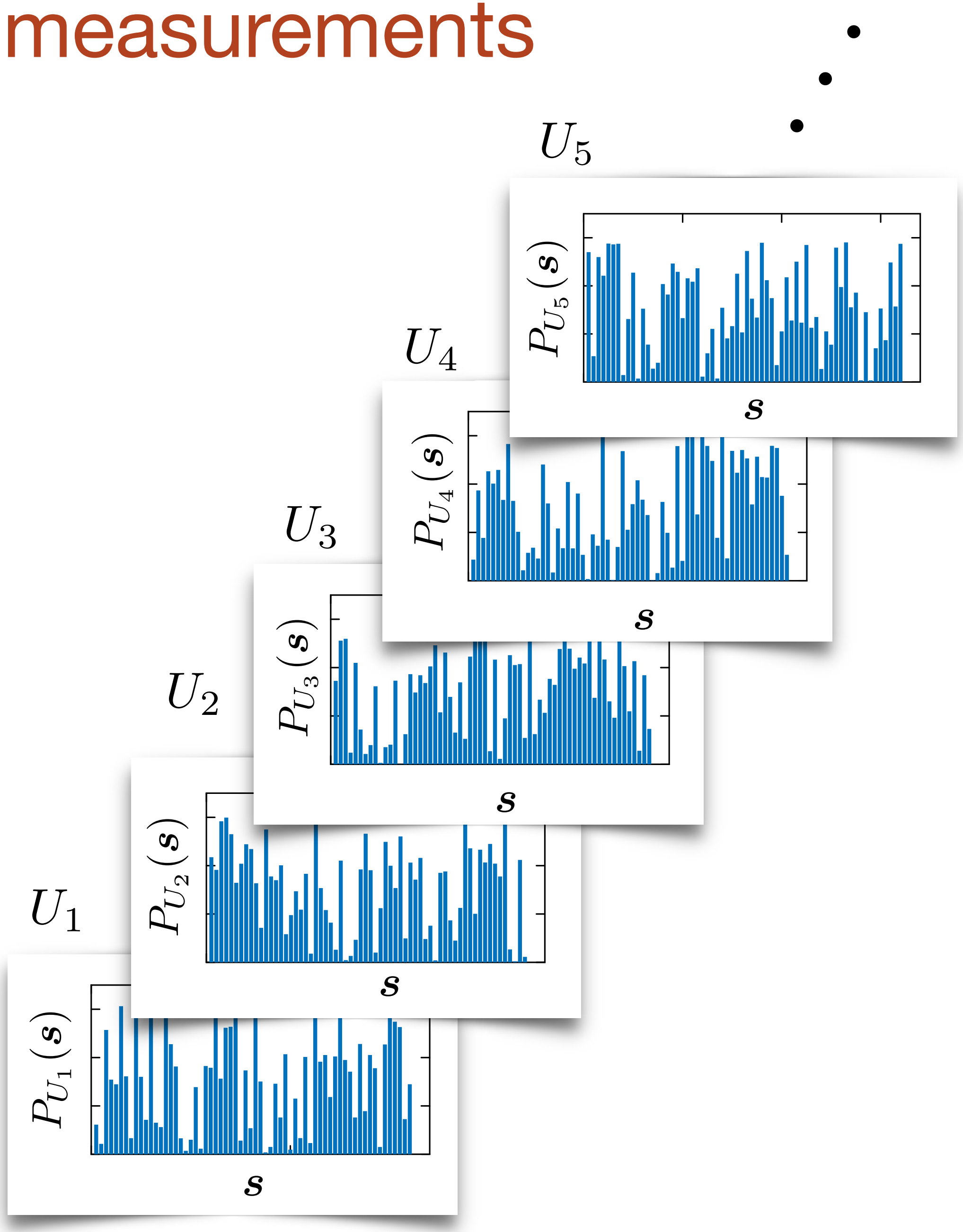
corrections (level 1)

$\tilde{H}_A$ as deformation of system Hamiltonian $H_A \equiv H|_A$ + corrections

Step 2: Fit to data, e.g. from randomized measurements

Experimental data





Protocol:

- Ansatz for $\tilde{H}_A(\tilde{g})$, which is physically motivated
- Best fit to experimental observations: $\tilde{g}$

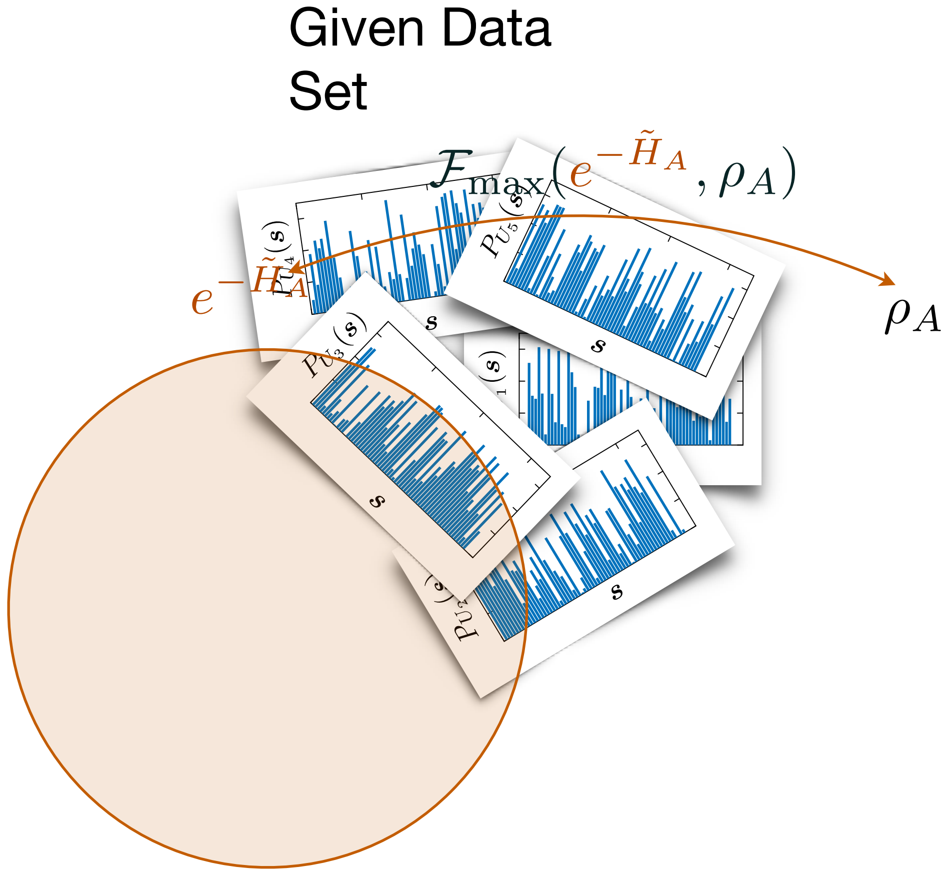
$$\chi^2(\tilde{g}) = \sum_{U,s} \left[\mathrm{Tr}\left(U|s\rangle\langle s|U^\dagger \frac{\exp\left(-\tilde{H}_A(\tilde{g})\right)}{Z(\tilde{g})} \right) - P_U(s) \right]^2$$

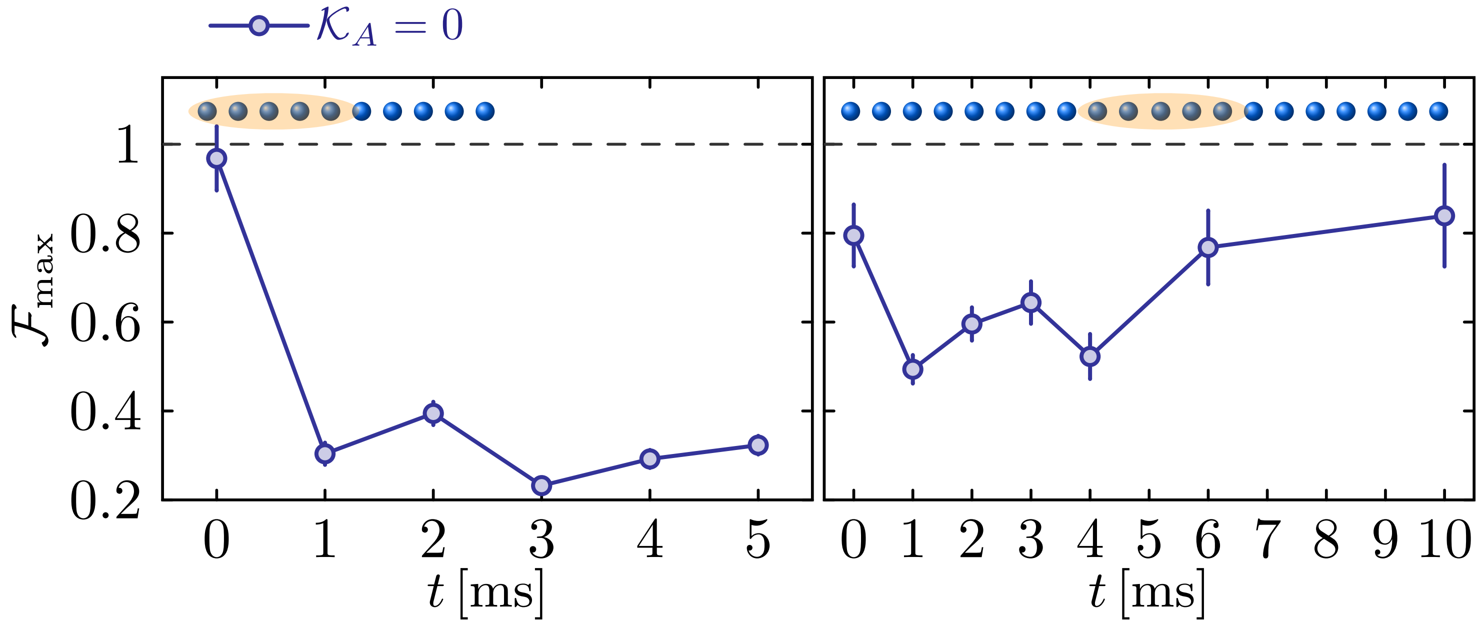
Step 3: Testing, and Improving Ansatz via Fidelity Estimation

$$\mathcal{F}_{\max}(\rho_1, \rho_2) = \frac{\mathrm{Tr}[\rho_1\rho_2]}{\max\{\mathrm{Tr}[\rho_1^2], \mathrm{Tr}[\rho_2^2]\}}$$

see cross-platform verification of NISQ

A.Elben et.al.
Phys. Rev. Lett. 124, 010504





$$\tilde{H}_A = \sum_{i,j<i} \tilde{J}_{ij}\left(\sigma_i^+\sigma_j^- + \mathrm{H.c.}\right) + \sum_i \tilde{B}_i\sigma_i^z$$

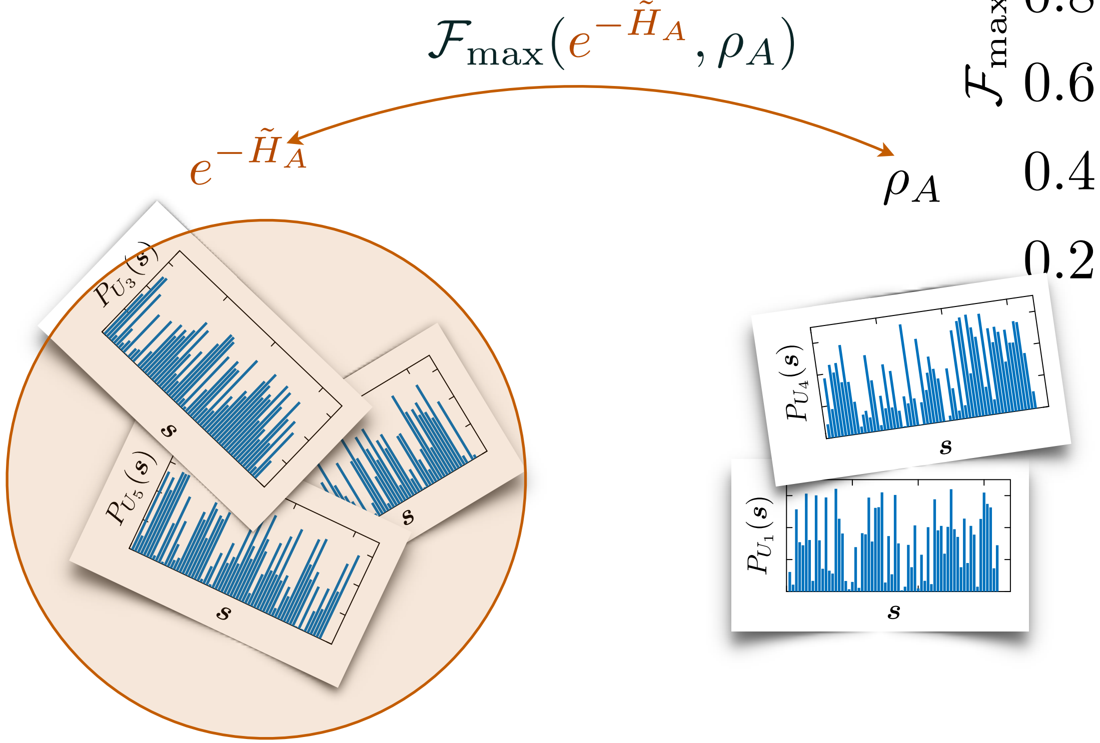
Step 3: Testing, and Improving Ansatz via Fidelity Estimation

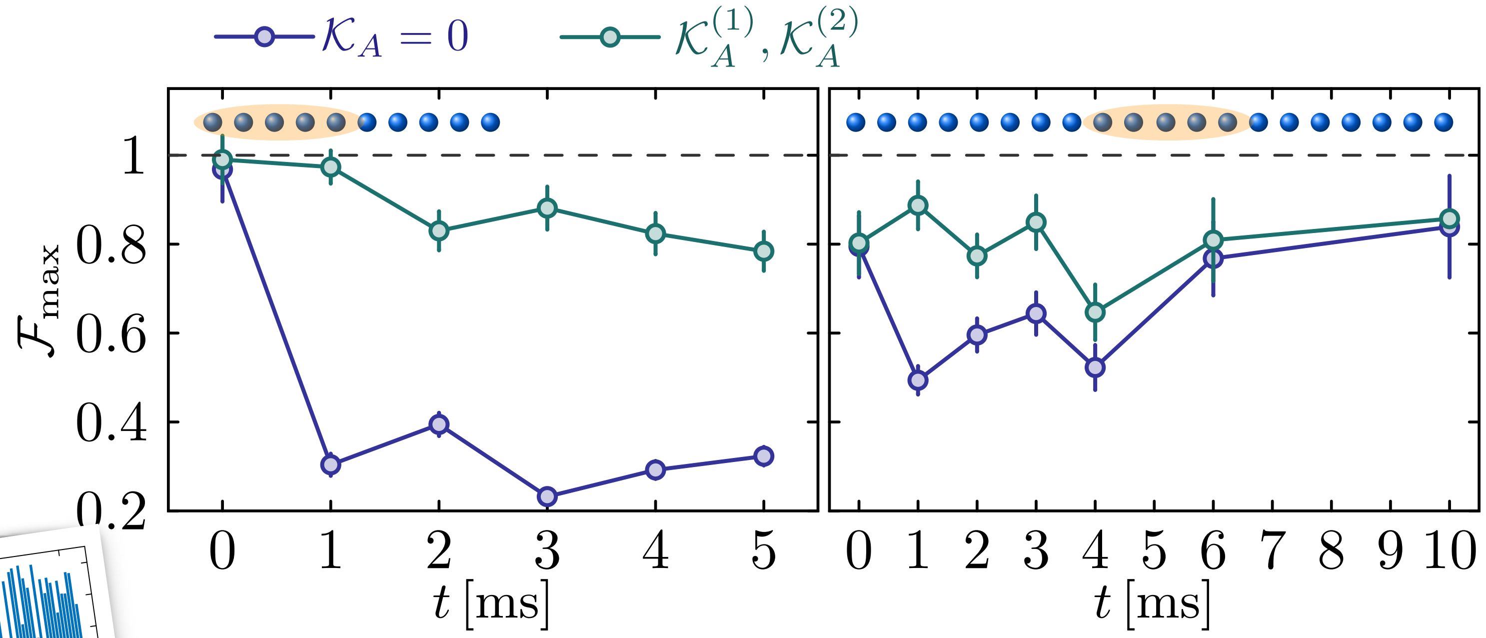
$$\mathcal{F}_{\max}(\rho_1, \rho_2) = \frac{\mathrm{Tr}[\rho_1 \rho_2]}{\max\{\mathrm{Tr}[\rho_1^2], \mathrm{Tr}[\rho_2^2]\}}$$

see cross-platform verification of NISQ

A.Elben et.al.
Phys. Rev. Lett. 124, 010504

Given Data Set





$$\mathcal{K}_A^{(1)} = \sum_{k<l\in A} \tilde{J}_{kl}^{XY} (\sigma_k^x \sigma_l^y - \sigma_k^y \sigma_l^x)$$

$$\mathcal{K}_A^{(2)} = \sum_{k<l} \sum_{m\neq k,l} \tilde{J}_{klm}^{XYZ} (\sigma_k^x \sigma_l^y \sigma_m^z - \sigma_k^y \sigma_l^x \sigma_m^z)$$

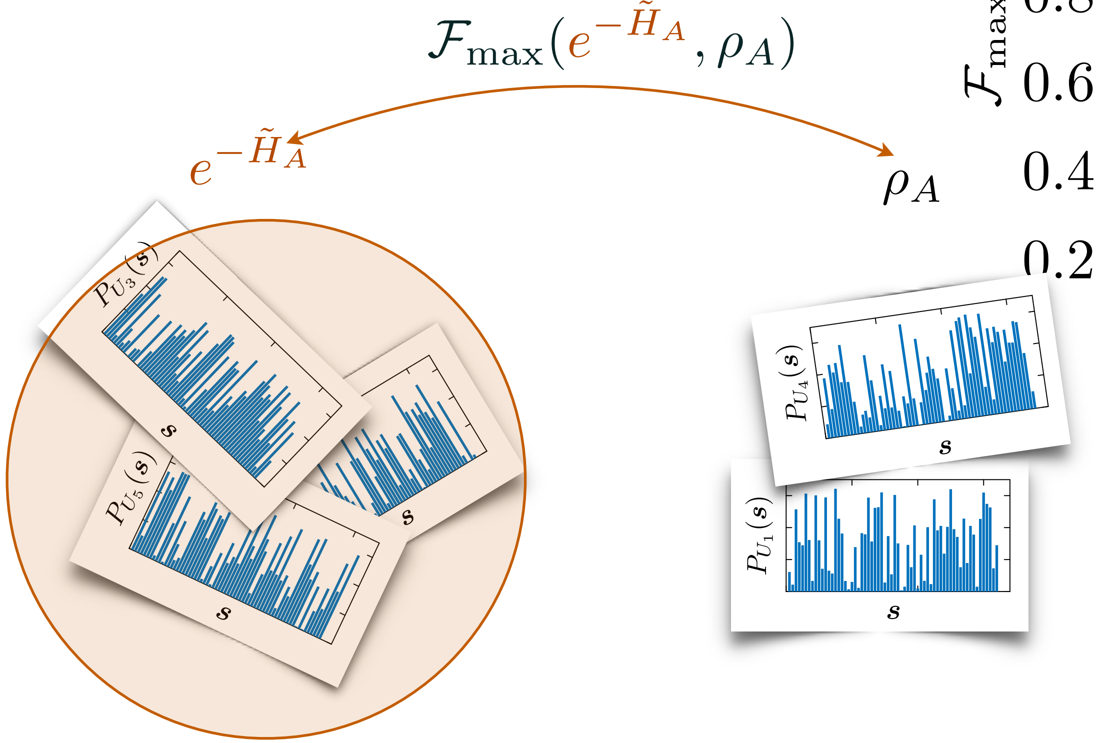
Step 3: Testing, and Improving Ansatz via Fidelity Estimation

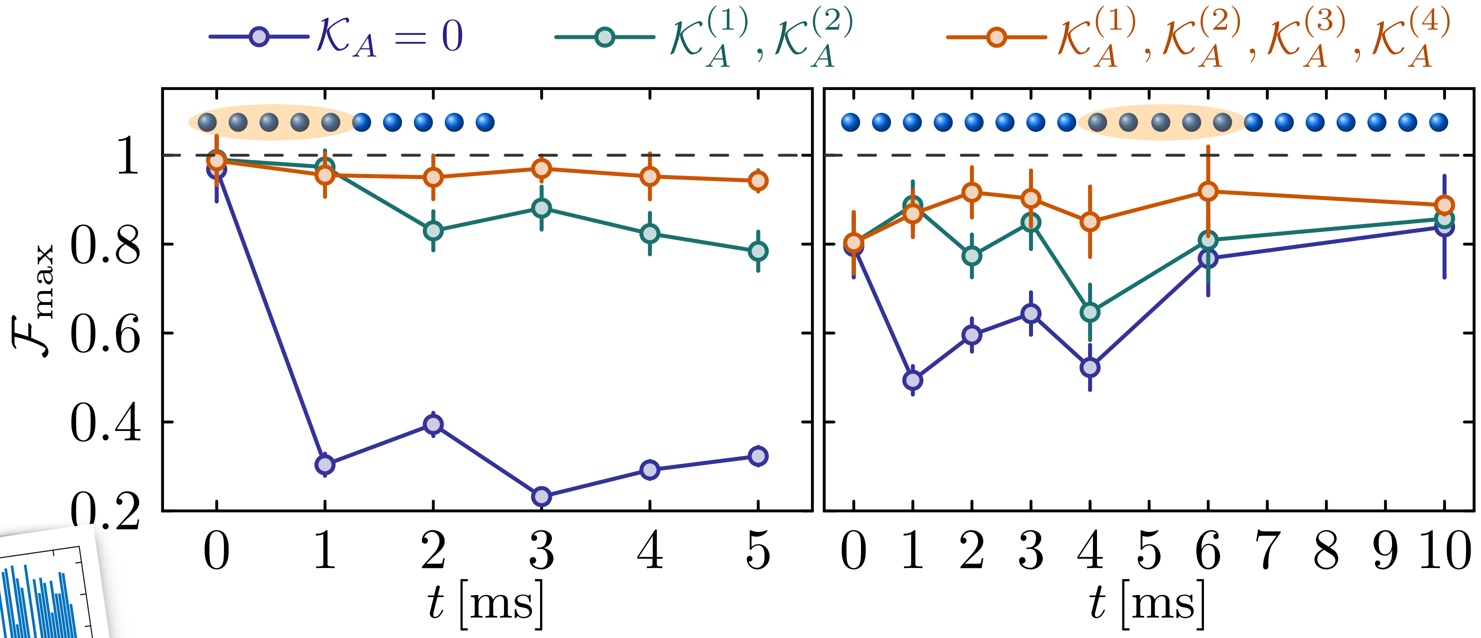
$$\mathcal{F}_{\max}(\rho_1,\rho_2) = \frac{\mathrm{Tr}[\rho_1\rho_2]}{\max\{\mathrm{Tr}[\rho_1^2],\mathrm{Tr}[\rho_2^2]\}}$$

see cross-platform verification of NISQ

A.Elben et.al.
Phys. Rev. Lett. 124, 010504

Given Data Set





$$\mathcal{K}_A^{(3)} = \sum_{k<l\in A} \tilde{J}_{kl}^{ZZ}\sigma_k^z\sigma_l^z + \sum_{k<l<m\in A} \tilde{J}_{klm}^{ZZZ}\sigma_k^z\sigma_l^z\sigma_m^z$$

$$\mathcal{K}_A^{(4)} = \sum_{k<l}\sum_{m\neq k,l} \tilde{J}_{klm}^{XXZ}(\sigma_k^x\sigma_l^x\sigma_m^z + \sigma_k^y\sigma_l^y\sigma_m^z)$$

here: heuristic ansatz for EH