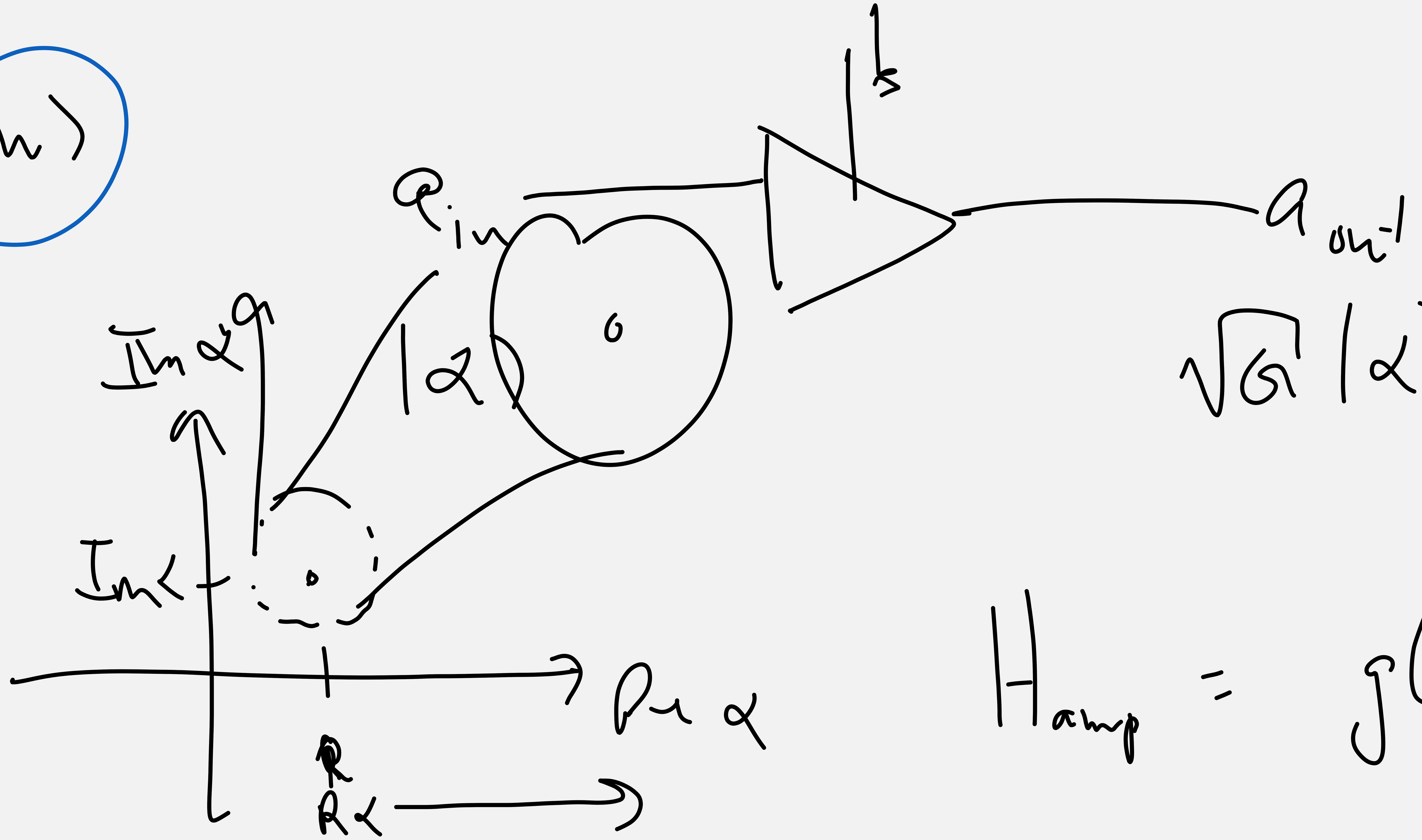
Lecture 4 - Measurement related phenomena + applications.

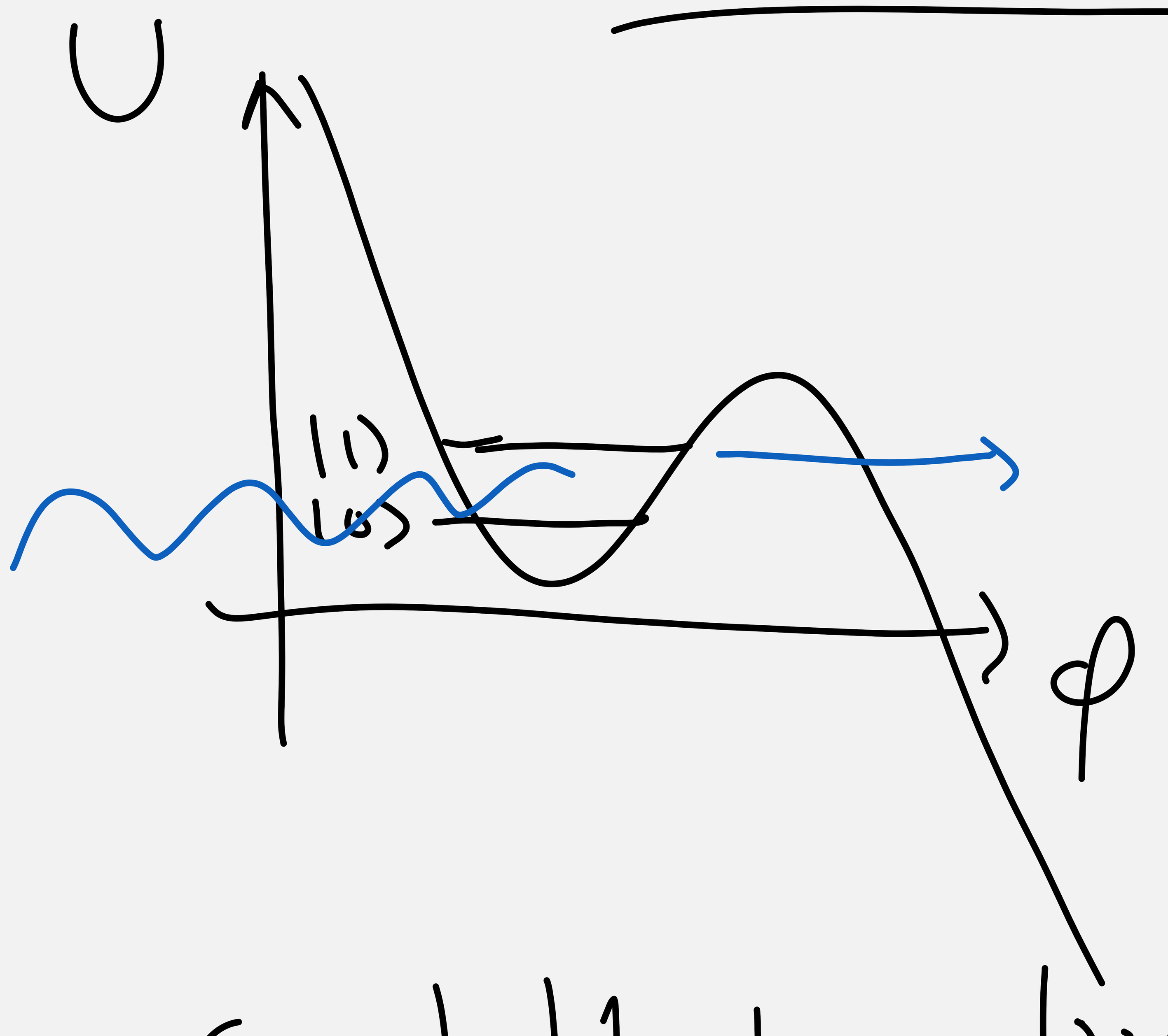
$$\boxed{|\alpha\rangle} = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

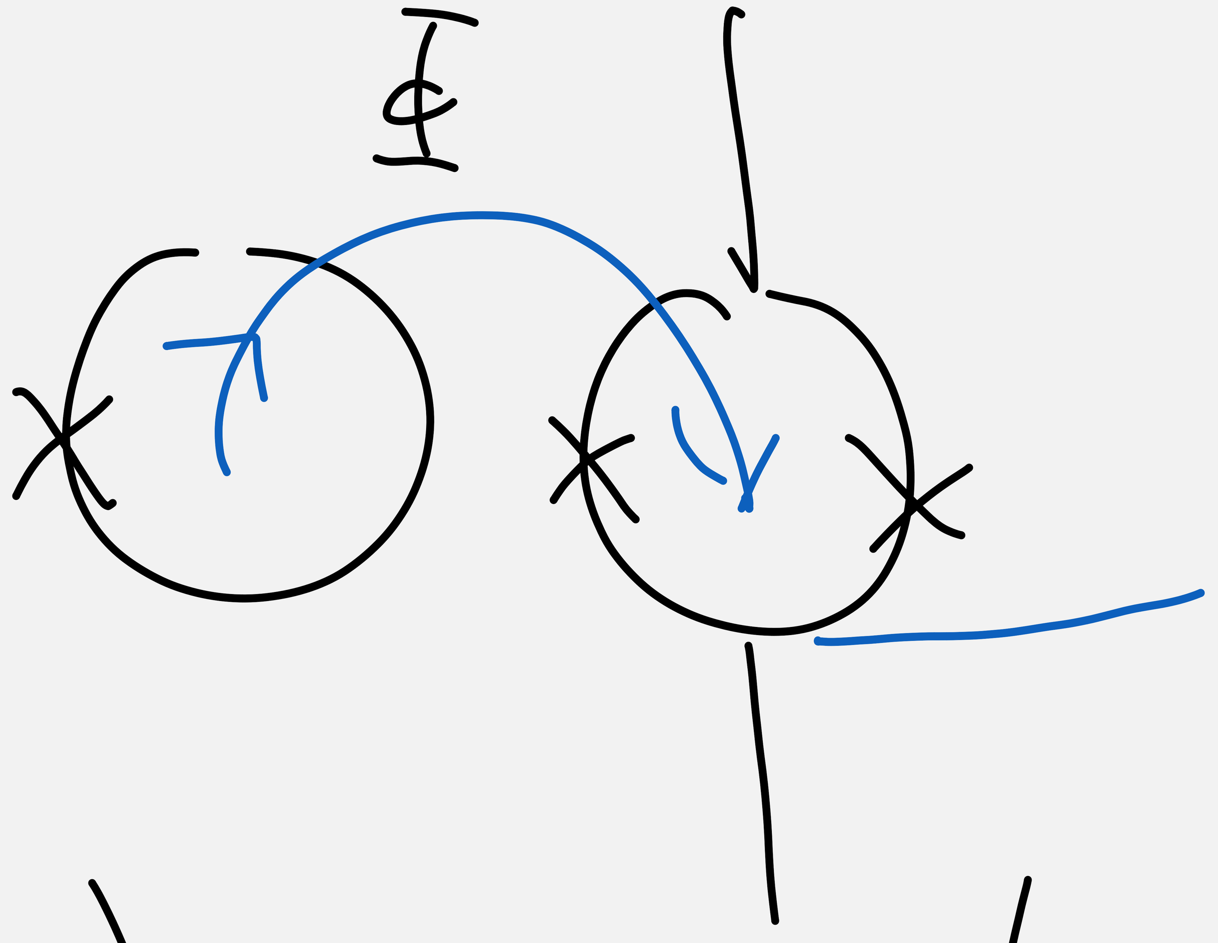


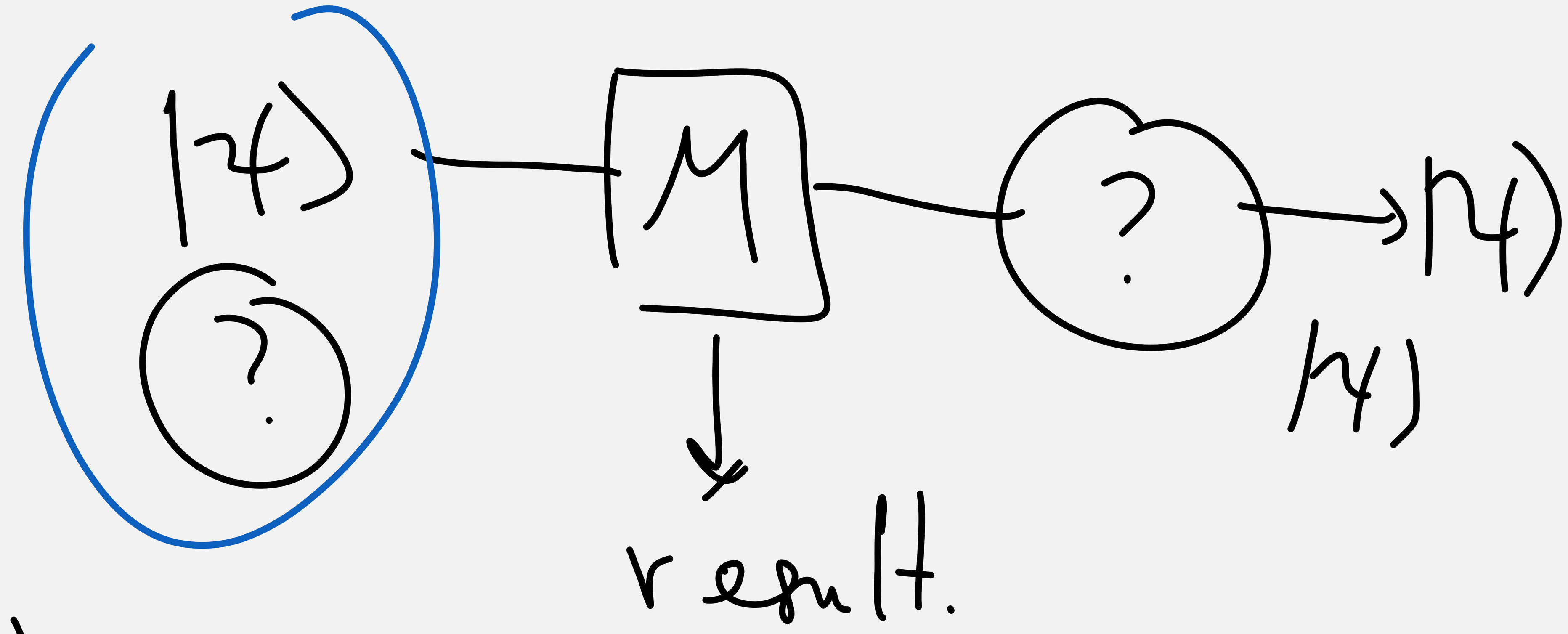
$\sqrt{G}\,|\alpha\rangle$ + noise

$$H_{amp} = g(a^2 + a^{\dagger 2})$$

Irreversibility $\longmapsto$ Can we undo a measurement?







$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \xmapsto[\text{no-click}]{t} \begin{pmatrix} \alpha e^{-\frac{\Gamma t}{2}} \\ \beta \end{pmatrix} \frac{1}{\sqrt{\;}} \longrightarrow \begin{pmatrix} \beta e^{-\frac{\Gamma t}{2}} \\ \alpha e^{-\frac{\Gamma t}{2}} \end{pmatrix}$$

1. tilt + no click
2. apply π-pulse.
   Exchanges amplitudes
3. tilt for same amount of time $\longrightarrow$ post-select on no-click.
4. final π-pulse.

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \longrightarrow \begin{pmatrix} \alpha e^{-\Gamma t/2} \\ \beta \end{pmatrix}$$

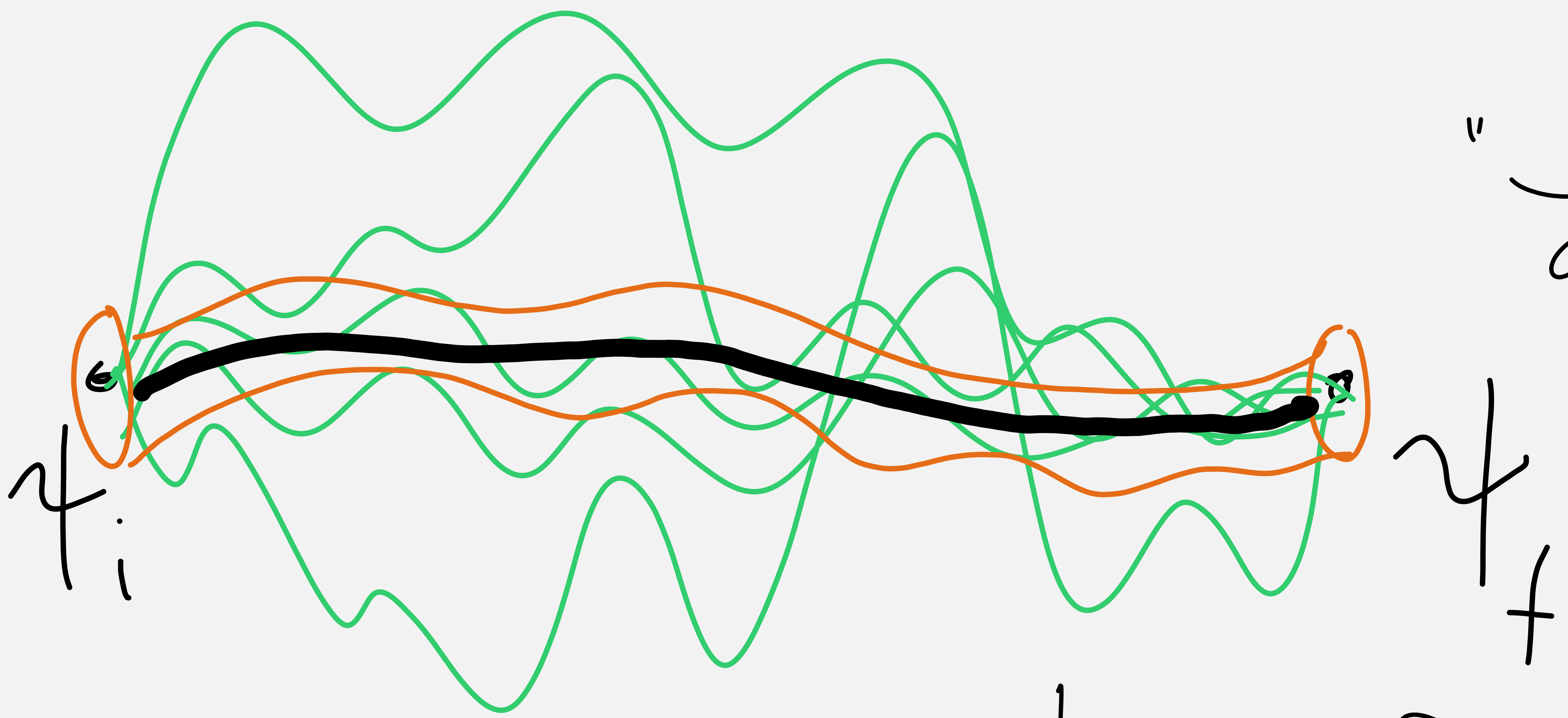
$$\xrightarrow{\pi} \begin{pmatrix} \beta \\ \alpha e^{-\Gamma t/2} \end{pmatrix} \xrightarrow[\text{2nd } M]{} \begin{pmatrix} \beta e^{-\Gamma t/2} \\ \alpha e^{-\Gamma t/2} \end{pmatrix} \longrightarrow \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

$$\xrightarrow{\pi \text{ pulse}} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$P_{2^{nd}\,null} = \frac{e^{-\Gamma t}}{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}} \xrightarrow{\Gamma t \gg 1} 0.$$

$$P_{rev} = P_{1^{st}} \cdot P_{2^{nd}} = e^{-\Gamma t}$$

Most likely path.



Extremum of action $\delta S = 0$

$$S = \int dt' \left[-i \tilde{p} \cdot \dot{q} + H \right]$$

"$\tilde{q}$" $\to (x, y, z)$

(q, p)

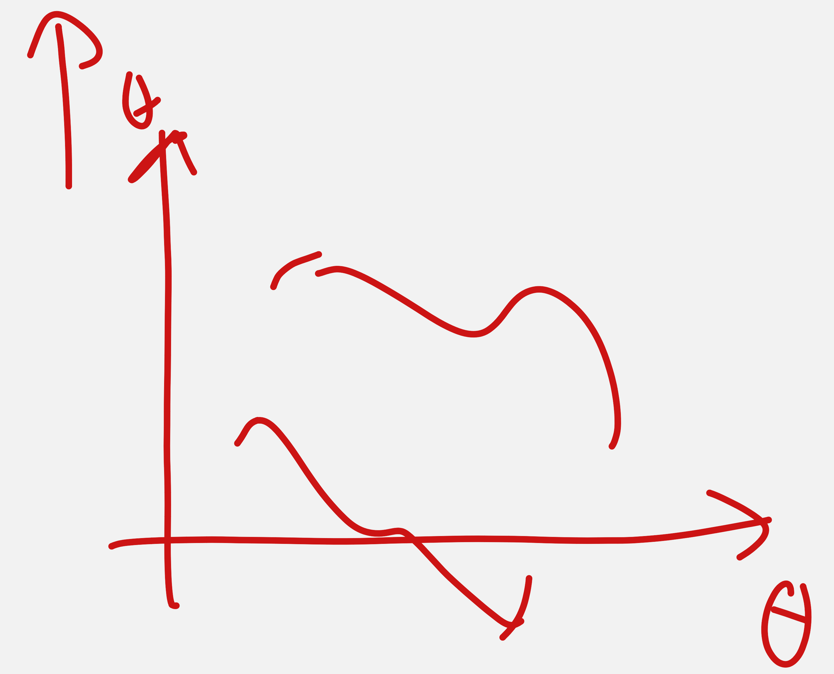
Quantum degrees of freedom

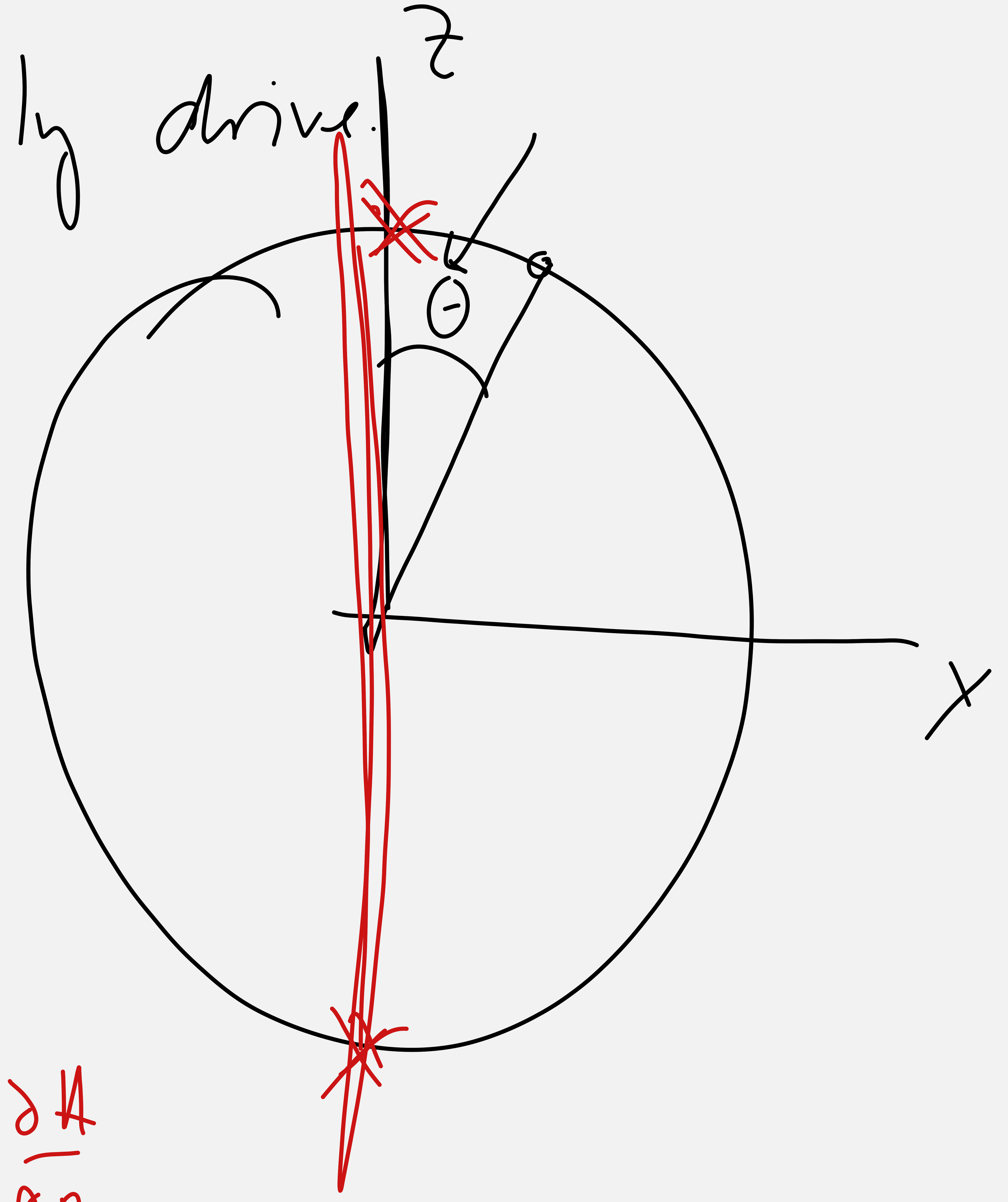
$$\dot{q} = \frac{\partial H}{\partial \tilde{p}} \; ; \quad \dot{\tilde{p}} = -\frac{\partial H}{\partial q} \; ; \quad \frac{\partial H}{\partial r} = 0$$

Example

continuous z-measurement ; apply drive

$$H = \Delta \sigma_x$$





$$S = \int dt' \left[-p_\theta \dot{\theta} + \mathcal{H} \right]$$

$$\mathcal{H} = a(\theta)\left[p_\theta^2 - 1\right] + b(\theta) p_\theta$$

$$a(\theta) = \frac{\sin^2\theta}{2\tau} \quad ; \quad b(\theta) = \Delta - \frac{\sin\theta\cos\theta}{\tau}$$

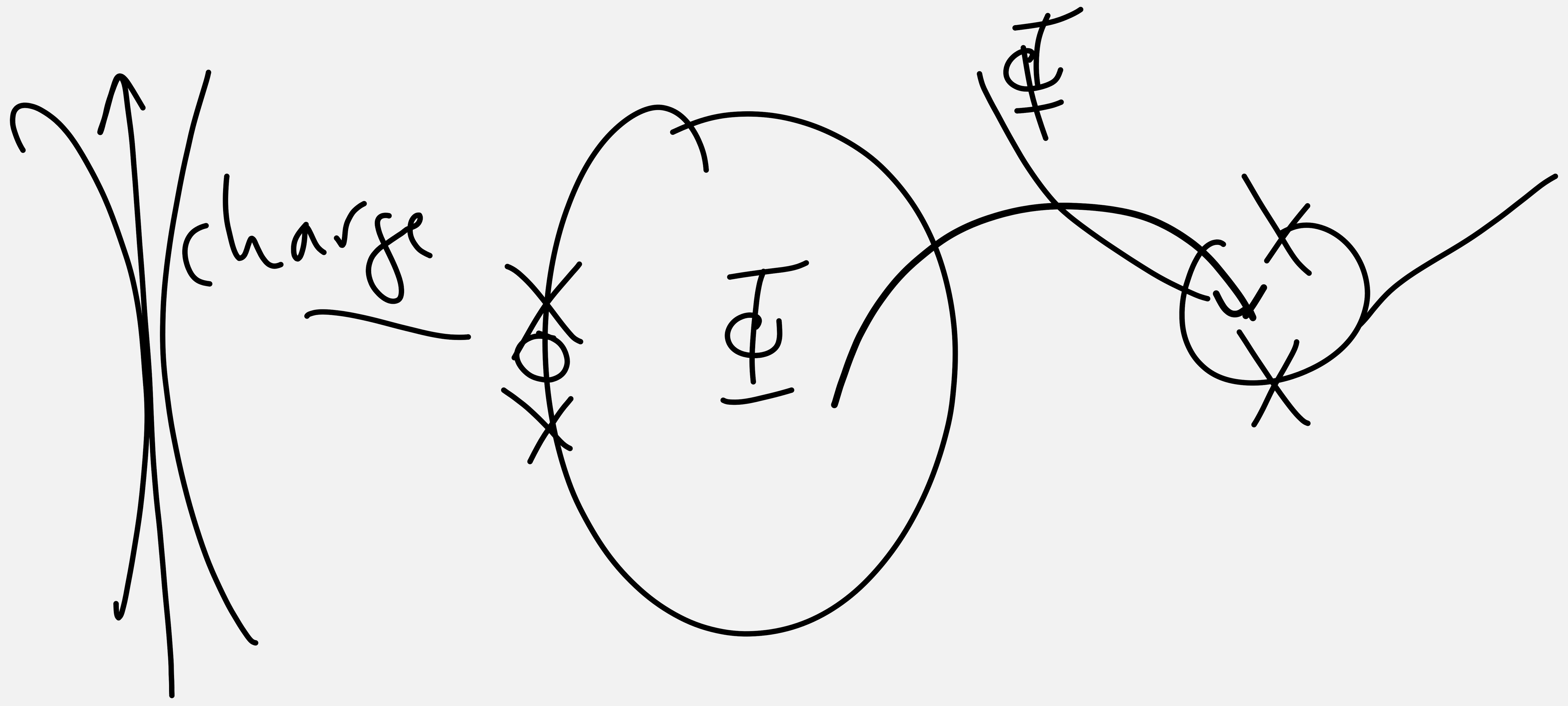
$$\dot{\theta} = \frac{\partial H}{\partial p}$$

$$\dot{p} = -\frac{\partial H}{\partial \theta}$$

$\Delta\tau \ll 1 \longrightarrow$ quantum Zeno $\longrightarrow$ jumps

$\Delta\tau \gg 1 \longrightarrow$ diffusive Rabi oscillations

$$[\hat{\varphi}, \hat{\Phi}] = i\hbar$$



Yes

2 handels on it!

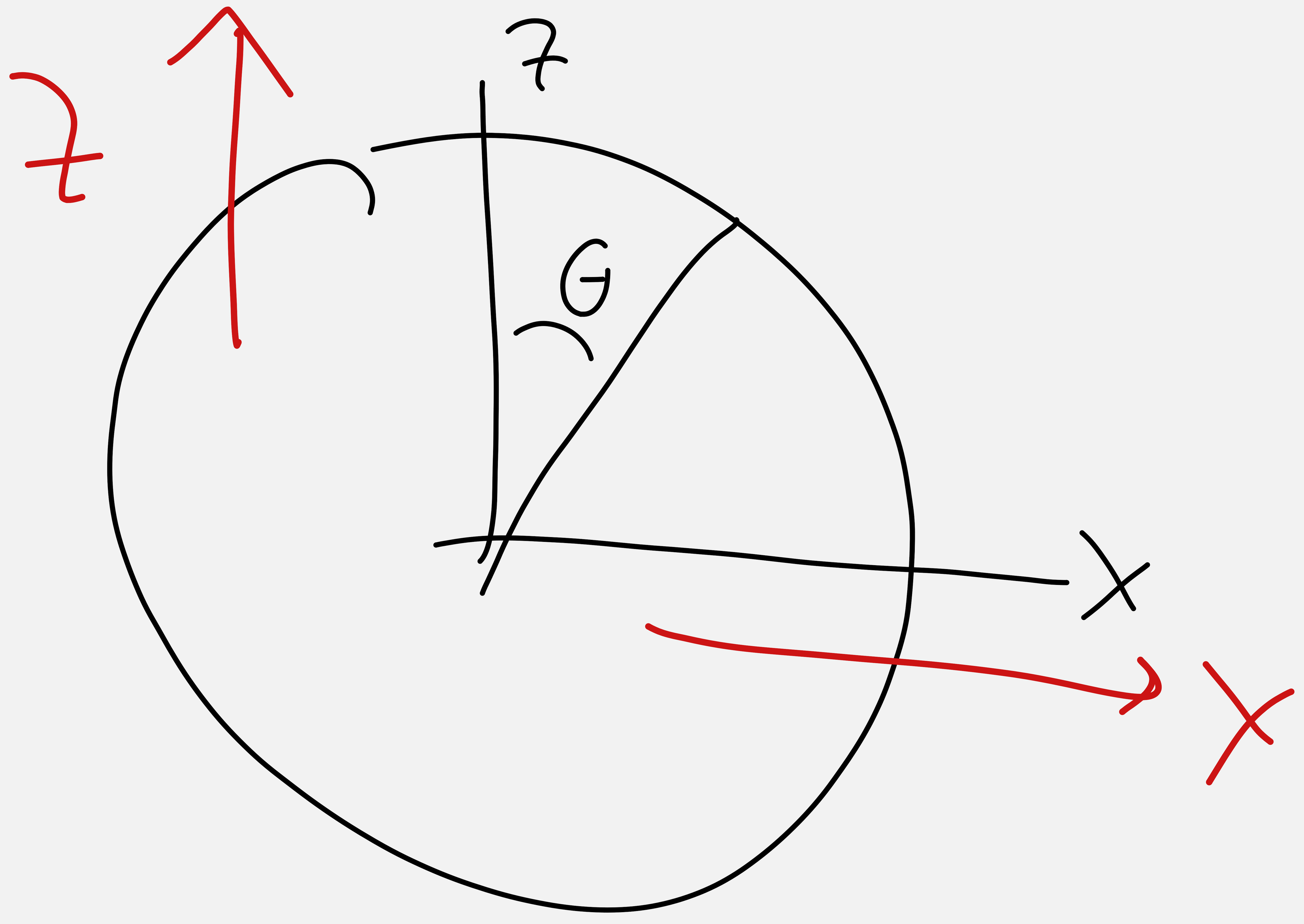
Can we measure non-commuting observables jointly?

weak measurements!

Simplest case.

$$\dot{\theta} = \sin\theta\cos\theta\left(\frac{1}{2\tau_x} - \frac{1}{2\tau_z}\right)$$

$$- \frac{\sin\theta}{\sqrt{\tau_z}}\,\xi_z + \frac{\cos\theta}{\sqrt{\tau_x}}\,\xi_x$$



(Itô)

Limits

$\tau_z \gg \tau_x$ — x measurement wins

$\tau_z \ll \tau_x$ — z " "

Let $\tau_x = \tau_z$

Define new noise variable

$$\xi_\theta = -\sin\theta\, \xi_z + \cos\theta\, \xi_x \qquad \langle \xi_x \xi_z \rangle = 0$$

$$\langle \xi_\theta \rangle = 0$$

$$\langle \xi_\theta(t)\, \xi(0) \rangle = \underbrace{\left(\sin^2\theta + \cos^2\theta\right)}_{1} \delta(t)$$

Super simple.

$$\dot{\Theta} = \frac{\xi_\theta}{\sqrt{\tau}}$$

Simple diffusion on a circle.

Joint measurements DO NOT lead to collapse.

Entanglement by measurement

start w/ seperable state of 2 qubits.

$$|\psi_{in}\rangle = \left(|0\rangle_A + |1\rangle_A\right)\left(|0\rangle_B + |1\rangle_B\right)/2$$

$$= \left(|00\rangle + |\cancel{10}\rangle + |\cancel{01}\rangle + |11\rangle\right)/2$$

Measure in subspace.

$$\begin{cases} \Pi_e = |00\rangle\langle 00| + |11\rangle\langle 11| \\ \Pi_o = |01\rangle\langle 01| + |10\rangle\langle 10| \end{cases}$$

$$\hat{\Pi}_e|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Bell state.

$$\Pi_o|\psi\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}};$$

Entangling 'ments — Direct
— Indirect.

<u>Direct</u>

Measure. $(\hat{\sigma}_z^A + \hat{\sigma}_z^B) \longrightarrow$ 'partial parity'

$(\hat{\sigma}_z^A \otimes \hat{\sigma}_z^B) \longrightarrow$ 'full parity'.