

**QUANTUM CONNECTIONS  
PART 2**

**Artur Ekert**

**The ultimate limits of privacy ...  
...for the paranoid ones**



**Artur Ekert**

# Outline

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- **Is there a perfect cipher?**
- **Key distribution – the holy grail of cryptography**
- **Privacy amplification / randomness extraction**
- **For whom the Bell tolls**
- **Less reality more security**
- **Device independent cryptography**

# We all have secrets...



**Alice**

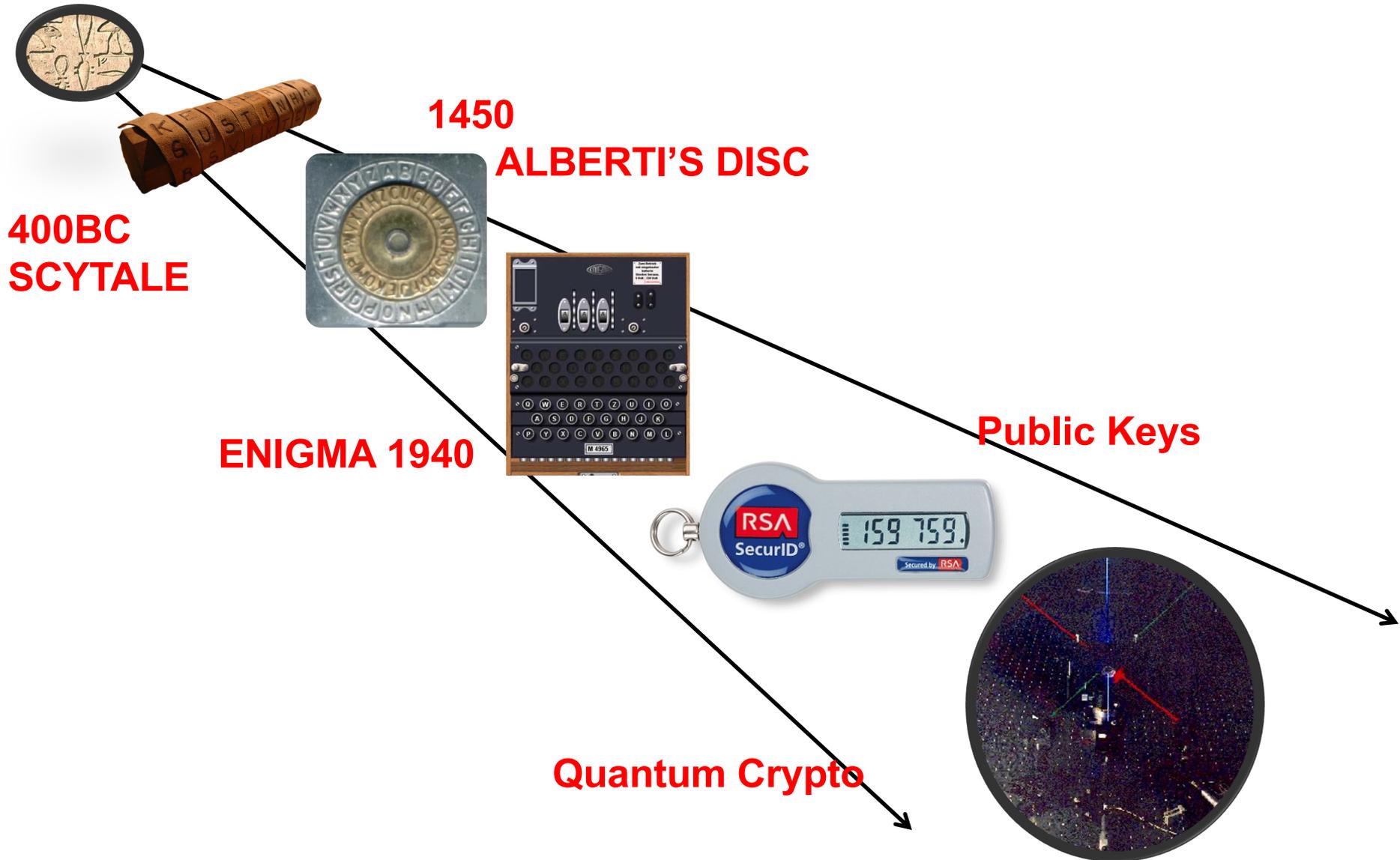


**Bob**



**Eavesdropper**

# Quest for a perfect cipher



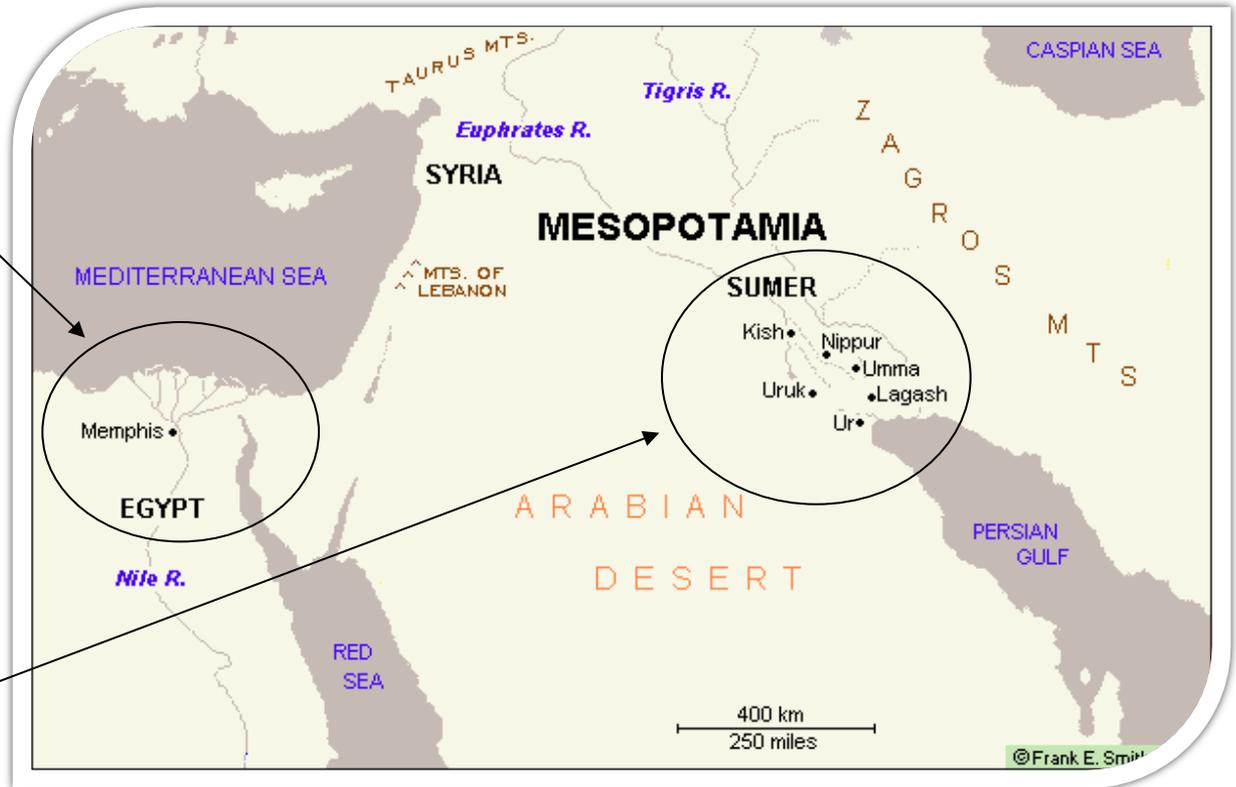
# It starts with writing...



Hieroglyphs  
Egypt, circa 3300 BC



Cuneiform  
Sumer, circa 3300 BC



# Basic techniques

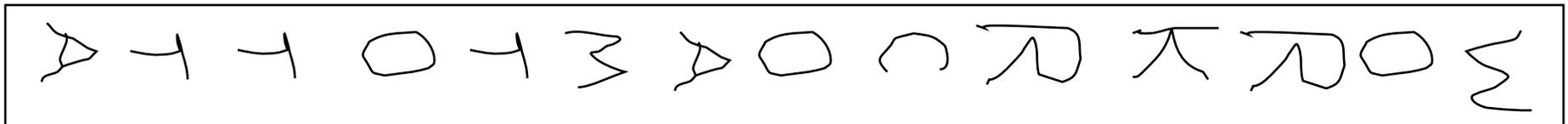
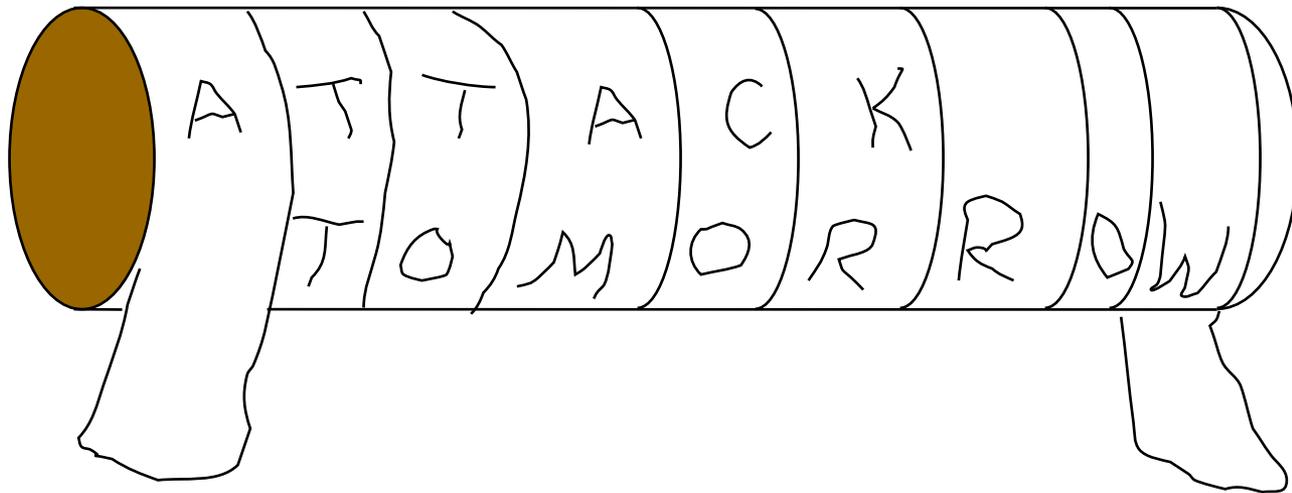
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- **PERMUTATIONS**
  - SCYTALE (400 BC)
- **SUBSTITUTIONS**
  - CAESAR SIPHER (50 BC)
- **PERMUTATIONS + SUBSTITUTIONS**

# Scytale

400 BC

SPARTA



**Permutation of characters**

# Caesar ciphers

50 BC

ROME

ABCDEFGHIJKLMNOPQRSTUVWXYZ  
ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ  
ABCDEFGHIJKLMNOPQRSTUVWXYZABC

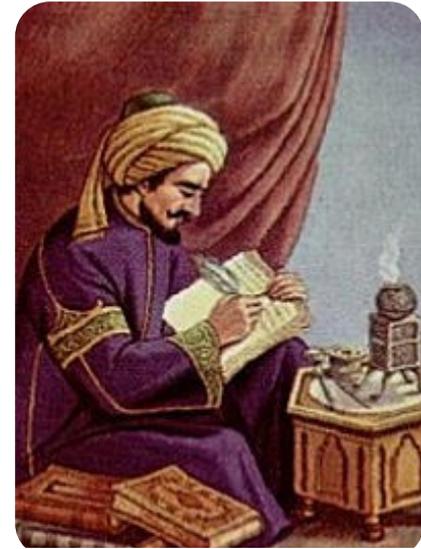
A T T A C K T O M O R R O W  
D W W D F N W R P R U U R Z

# Code-makers versus code-breakers

**Julius Caesar  
(100-44 BC)**



**Al Kindi  
(800-873)**

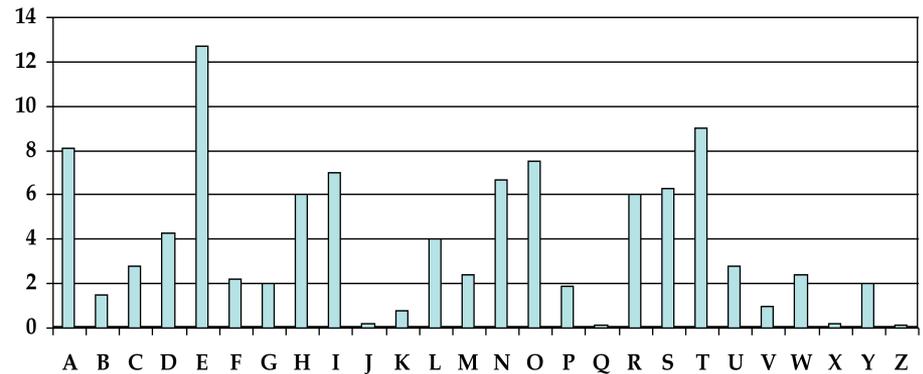


ABCDEFGHIJKLMNOPQRSTUVWXYZ



NWDEAPYFGTIJUKLMOZQRSBVCXH

$\approx 4 \times 10^{26}$  SUBSTITUTIONS



# Counterexamples - Lipograms

That's right - this is a lipogram - a book, paragraph or similar thing in writing that fails to contain a symbol, particularly that symbol fifth in rank out of 26 (amidst 'd' and 'f') and which stands for a vocalic sound such as that in 'kiwi'. I won't bring it up right now, to avoid spoiling it...

**The most famous lipogram: Georges Perec, *La Disparition* (1969) 85000 words without the letter e:**

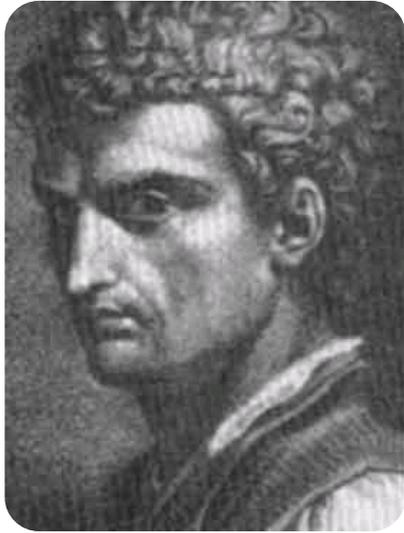
Tout avait l'air normal, mais tout s'affirmait faux. Tout avait l'air normal, d'abord, puis surgissait l'inhumain, l'affolant. Il aurait voulu savoir où s'articulait l'association qui l'unissait au roman : sur son tapis, assaillant à tout instant son imagination, ...

**English translator, Gilbert Adair, in *A Void*, succeeded in avoiding the letter e as well**

**Gottlob Burmann (1737-1805) R-LESS POETRY.** An obsessive dislike for the letter r; wrote 130 poems without using that letter, he also omitted the letter r from his daily conversation for 17 years...

# Polyalphabetic ciphers

## CODEMAKERS



**Leone Battista Alberti**  
(1404-1472)

**Johannes Trithemius**  
(1462-1516)

**Blaise de Vigenere**  
(1523-1596)



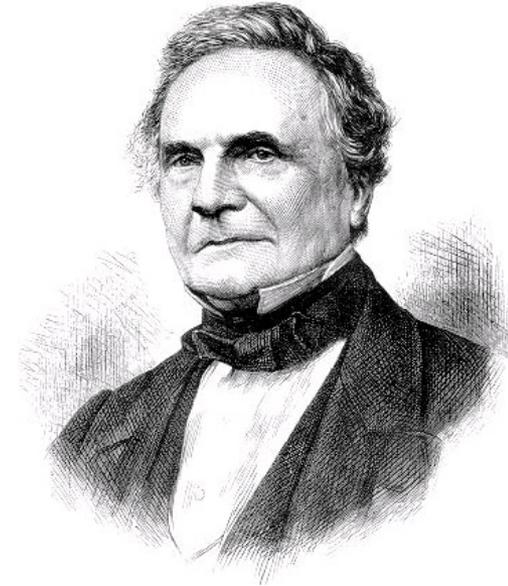
**Alberti's encryption disk**

**Sequence of substitutions e.g.**  
7, 14, 19

Plaintext: **S E L L**

Cryptogram: **Z S E S**

## CODEBREAKERS



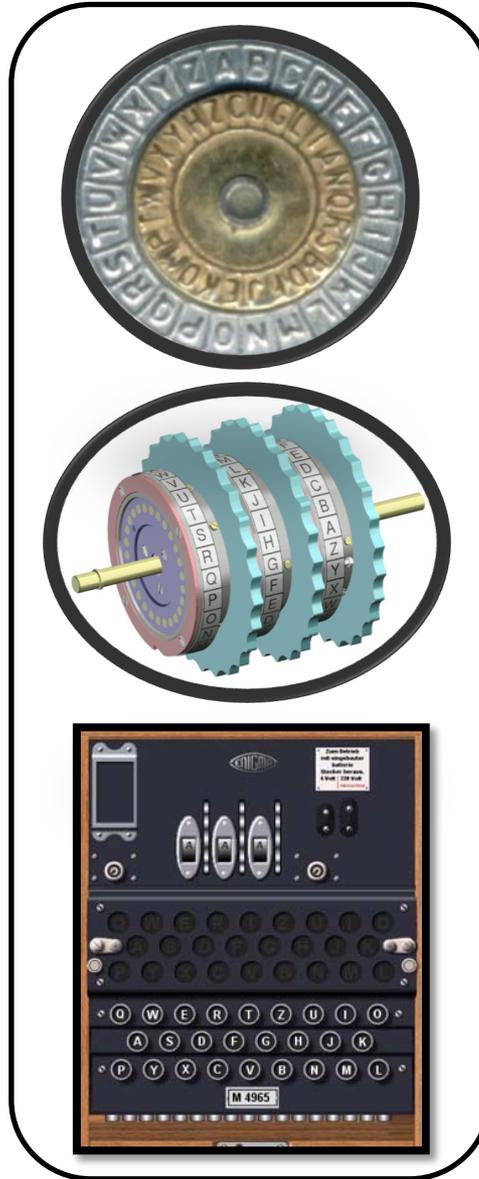
**Charles Babbage**  
(1791-1871)

# From Alberti's disk to rotor machines

## CODEMAKERS



**Arthur Scherbius  
(1878-1929)**



## CODEBREAKERS



**Marian Rejewski  
(1905-1980)**

# The Poles who broke Enigma

(BS-4 Section)



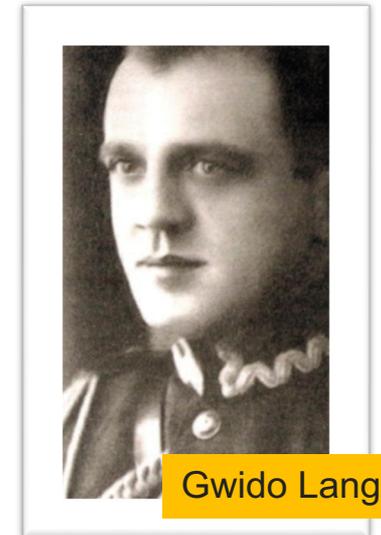
Henryk Zygałski.

Jerzy Różycki

Marian Rejewski



Maksymilian Ciężki



Gwido Langer

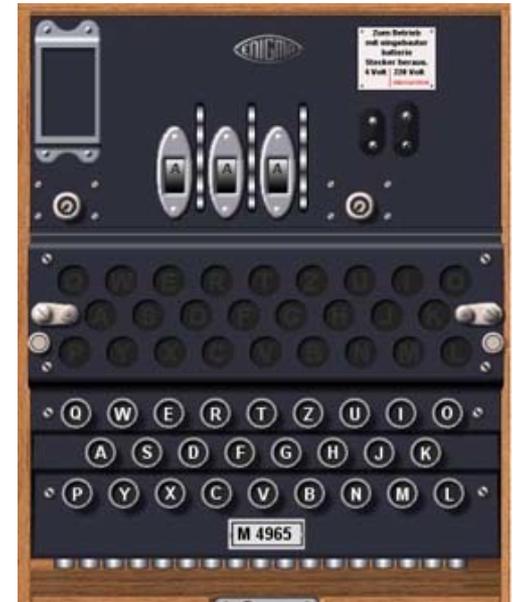
# Is there a perfect cipher ?



**SCYTALE 400BC**



**ALBERTI'S DISC 1450**



**ENIGMA 1940**

# One-time pad

message	0	1	1	1	0	1	0	0	1	1
key	0	1	0	1	1	1	0	0	1	0
cryptogram	0	0	1	0	1	0	0	0	0	1



0	0	1	0	1	0	0	0	0	0	1
---	---	---	---	---	---	---	---	---	---	---



0	0	1	0	1	0	0	0	0	0	1
---	---	---	---	---	---	---	---	---	---	---



0	0	1	0	1	0	0	0	0	0	1
0	1	0	1	1	1	0	0	1	0	
0	1	1	1	0	1	0	0	1	1	

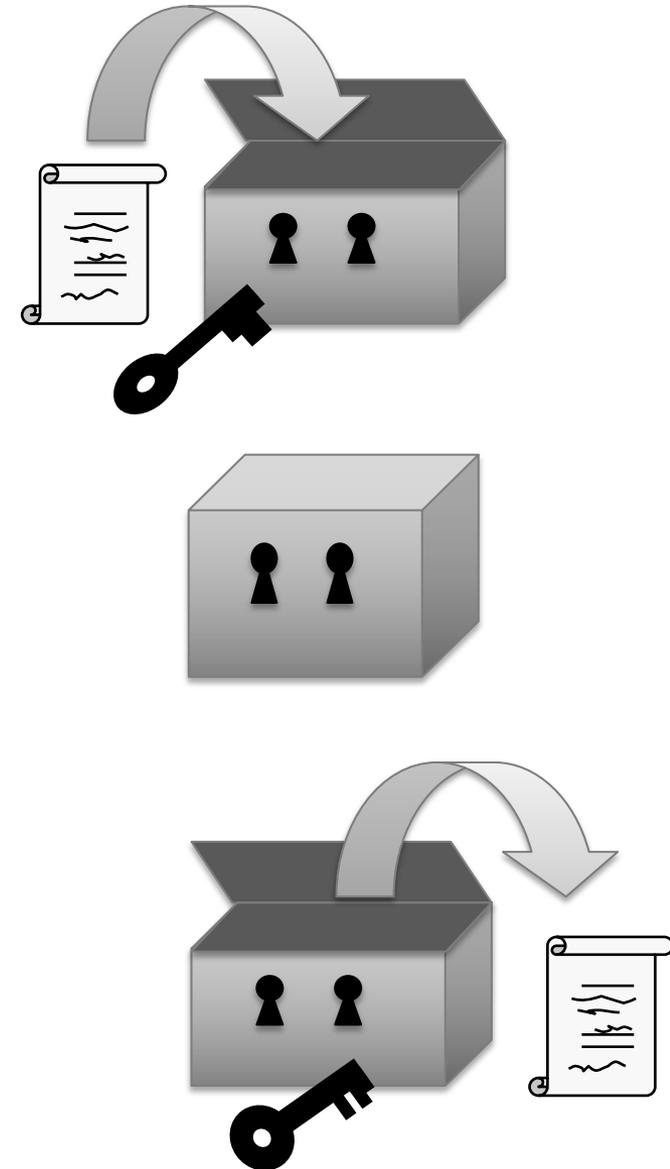
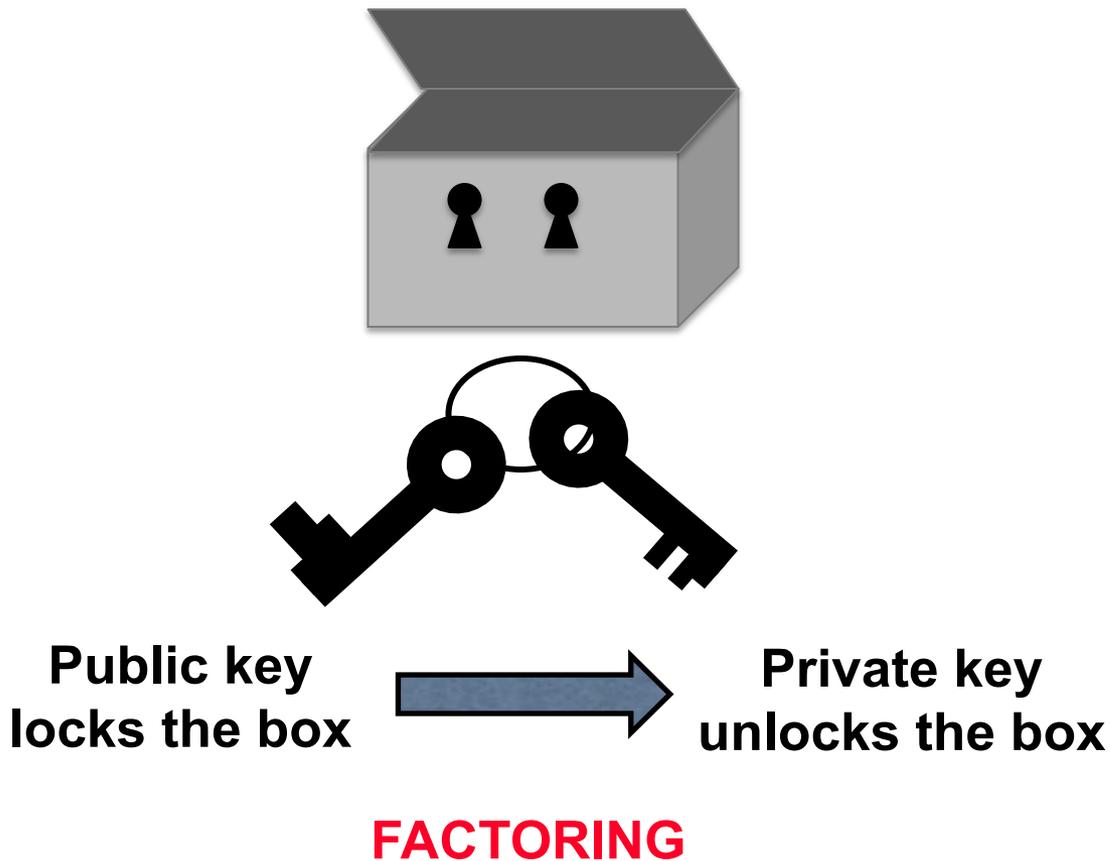
cryptogram

key

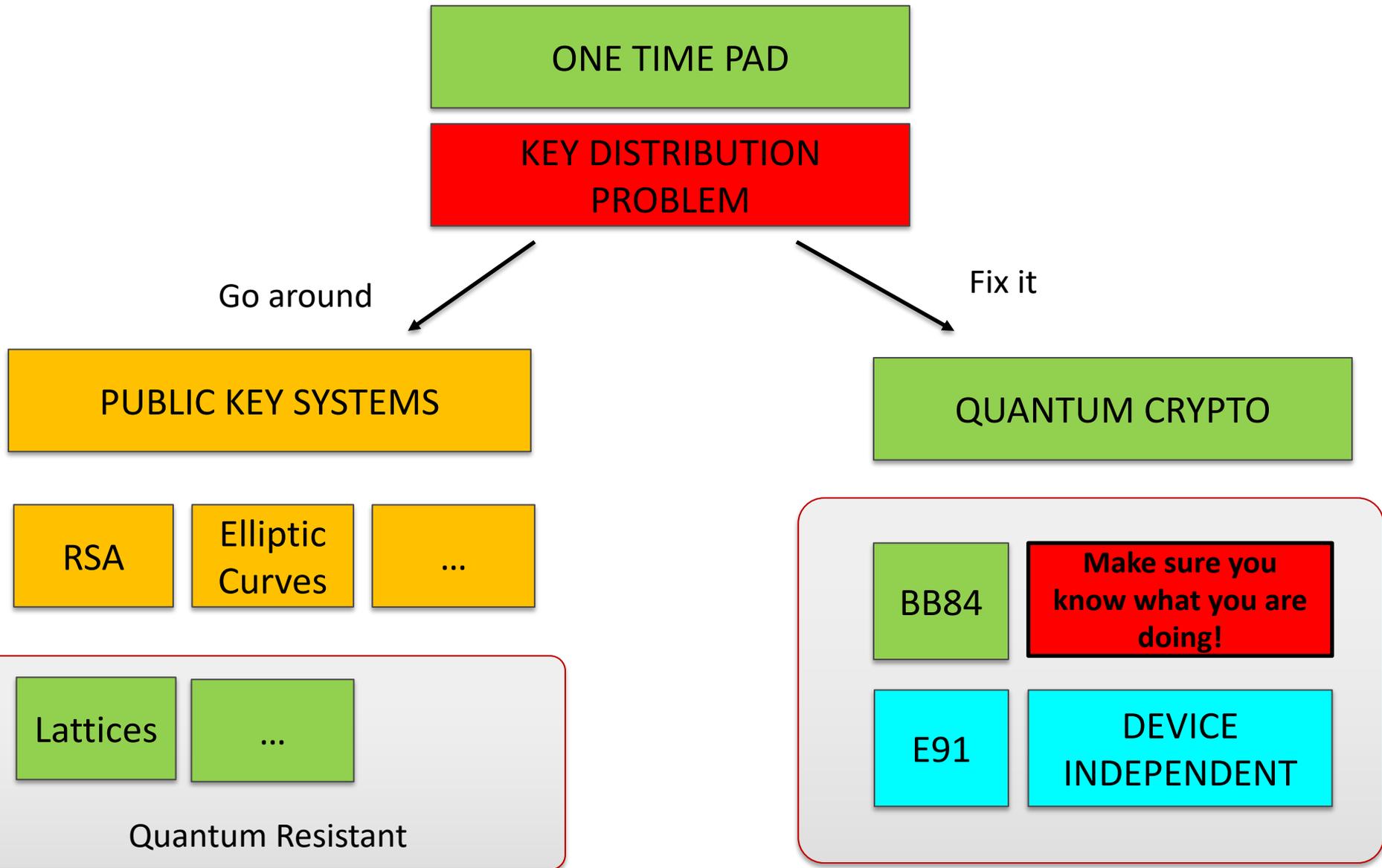
message

## KEY DISTRIBUTION PROBLEM

# Public Key Cryptosystems



# Quest for perfect secrecy



# Post-quantum: there is still room for improvement

## Report on the Security of LWE: Improved Dual Lattice Attack

The Center of Encryption and Information Security – MATZOV\*†  
IDF

### Abstract

Many of the leading post-quantum key exchange and signature schemes rely on the conjectured hardness of the Learning With Errors (LWE) and Learning With Rounding (LWR) problems and their algebraic variants, including 3 of the 6 finalists in NIST's PQC process. The best known cryptanalysis techniques against these problems are primal and dual lattice attacks, where dual attacks are generally considered less practical.

In this report, we present several algorithmic improvements to the dual lattice attack, which allow it to exceed the efficiency of primal attacks. In the improved attack, we enumerate over more coordinates of the secret and use an improved distinguisher based on FFT. In addition, we incorporate improvements to the estimates of the cost of performing a lattice sieve in the RAM model, reducing the extra count of random product

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## SOLILOQUY: A CAUTIONARY TALE

PETER CAMPBELL, MICHAEL GROVES AND DAN SHEPHERD

*CESG, Cheltenham, UK*

### 1. INTRODUCTION

The SOLILOQUY primitive, first proposed by the third author in 2007, based on cyclic lattices. It has very good efficiency properties, both terms of public key size and the speed of encryption and decryption. There are straightforward techniques for turning SOLILOQUY into a key exchange or other public-key protocols. Despite these properties, we abandoned our search on SOLILOQUY after developing (2010 to 2013) a reasonably efficient quantum attack on the primitive. A similar quantum algorithm has been



Cryptology ePrint Archive

Paper 2022/214  
**Breaking Rainbow Takes a Weekend on a Laptop**  
Ward Beullens , IBM Research – Zurich

**Abstract**  
This work introduces new key recovery attacks against the Rainbow signature scheme, which is one of the three finalist signature schemes still in the NIST Post-Quantum Cryptography standardization project. The new attacks outperform previously known attacks for all the parameter sets submitted to NIST and make a key-recovery practical for the SL 1 parameters. Concretely, given a Rainbow public key for the SL 1 parameters of the second-round submission, our attack returns the corresponding secret key after on average 53 hours (one weekend) of computation time on a standard laptop.



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Paper 2022/975  
**An efficient key recovery attack on SIDH (preliminary version)**  
Wouter Castryck, KU Leuven  
Thomas Decru, KU Leuven

**Abstract**  
We present an efficient key recovery attack on the Supersingular Isogeny Diffie-Hellman protocol (SIDH), based on a "glue-and-split" theorem due to Kani. Our attack exploits the existence of a small non-scalar endomorphism on the starting curve, and it also relies on the auxiliary torsion point information that Alice and Bob share during the protocol. Our Magma implementation breaks the instantiation SIKEp434, which aims at security level 1 of the Post-Quantum Cryptography standardization process currently ran by NIST, in about one hour on a single core. This is a preliminary version of a longer article in preparation.

# Key distribution problem



The key should be random, sufficiently long and secret (known only to Alice and Bob)



0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

X



0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

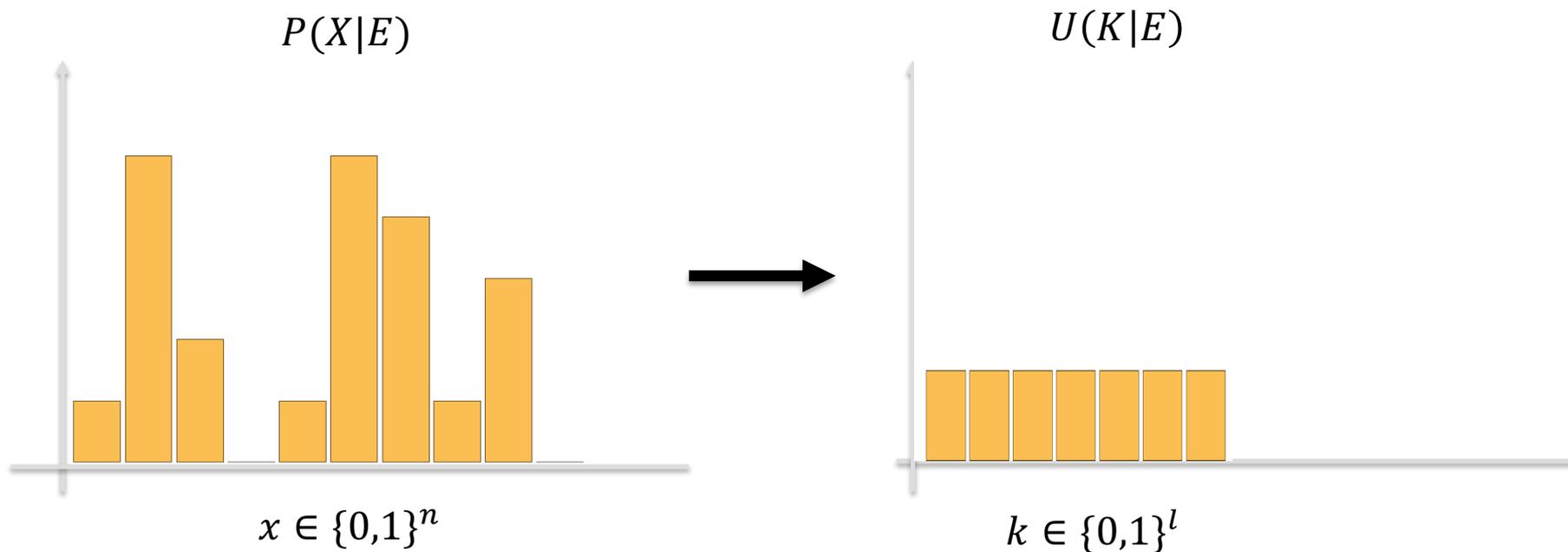
X

0	?	?	1	?	0	0	?	?	?
---	---	---	---	---	---	---	---	---	---

E

Probability of Eve guessing the key correctly should be very close to  $\frac{1}{2^n}$

# Privacy amplification



For independent bits try parity

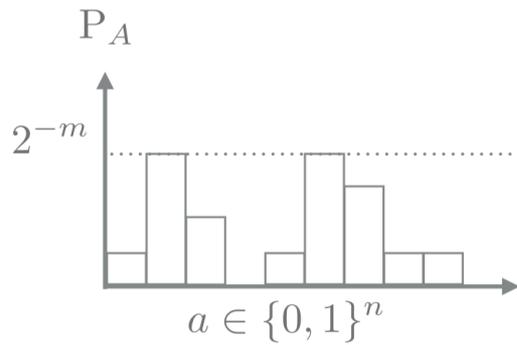
$$x_1 \oplus x_2 \oplus x_3 \dots$$

Suppose Eve knows one of the two bits,  
but Alice and Bob are not sure which one

$$k = x_1 \oplus x_2$$

# Randomness extraction

Weak source of randomness



Min-entropy:

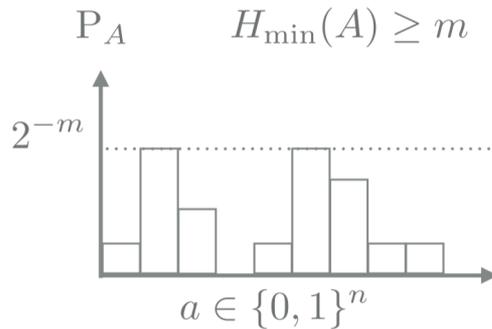
$$H_{\min}(A) = -\log \left( \max_a \Pr[a] \right)$$

$p_{\text{guess}}(A)$

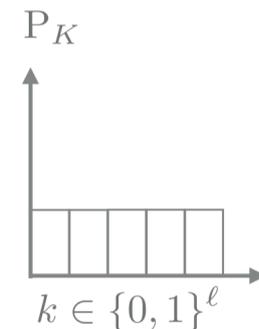
$$H_{\min}(A) \geq m :$$

$$\forall a \in \{0, 1\}^n, \quad \Pr[a] \leq 2^{-m}$$

Weak source of randomness



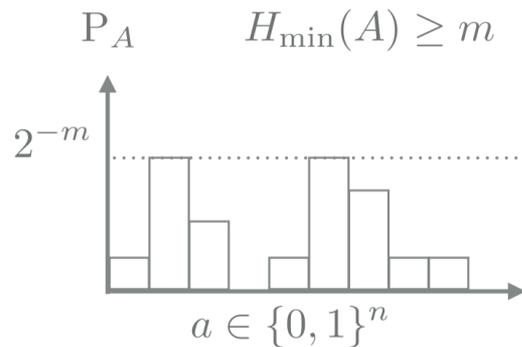
Uniform distribution



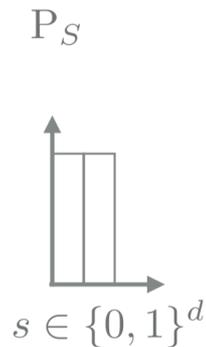
# Randomness extraction

- ▶ Impossible to achieve deterministically
- ▶ Possible with an additional short random seed

Weak source of randomness

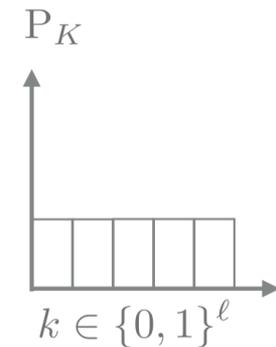


×



$\xrightarrow{\text{Ext}(A, S)}$

Uniform distribution



# Randomness extraction

► Def. [Randomness extractor]: A function  $\text{Ext}(A, S) : \{0, 1\}^n \times \{0, 1\}^d \rightarrow \{0, 1\}^\ell$  is called a **strong**  $(m, \varepsilon)$ -randomness extractor if for

1.  $S = U_d$

2. any  $P_A$  with  $H_{\min}(A) \geq m$

we have

$$\|\text{Ext}(A, S)S - U_\ell \times S\| \leq \varepsilon$$

The diagram shows the equation  $\|\text{Ext}(A, S)S - U_\ell \times S\| \leq \varepsilon$ . An arrow points from the text 'Output of the extractor' to the  $\text{Ext}(A, S)S$  term. Another arrow points from the text 'Uniform key' to the  $U_\ell \times S$  term.

Eve?

(Strong extractor: the seed is made public during the QKD protocol)

# Privacy amplification



Alice and Bob can turn their partially secure key into a secure key as long as they can estimate how much Eve knows about the raw key.



Probability of Eve guessing the key correctly should be very close to  $\frac{1}{2^n}$

$$H_{\min}(X|E) = -\log p_{\text{guess}}(X|E)$$

$$l = H_{\min}(X|E) - 2 \log \frac{1}{2\delta}$$

# Extractors

Raw key



$$H_{\min}(X|E)$$



Eve's uncertainty

Eve knowledge

$K$

$$2 \log \frac{1}{2\delta}$$



$\delta$  secure key

$$l = H_{\min}(X|E) - 2 \log \frac{1}{2\delta}$$

# How to find out how much Eve knows?



0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

X

$$H_{\min}(X|E)$$



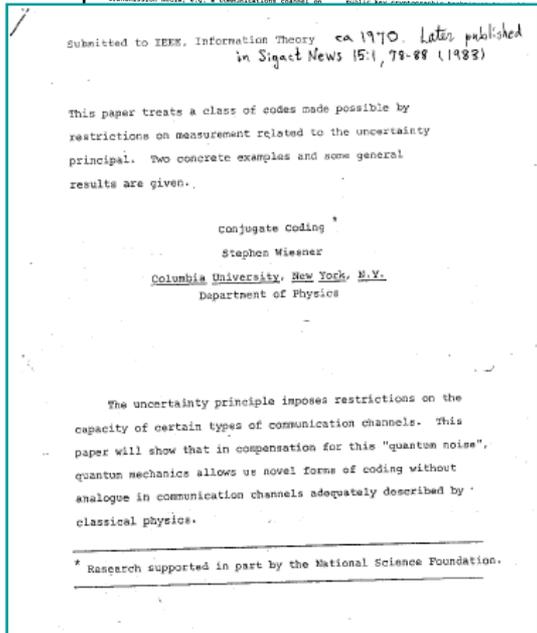
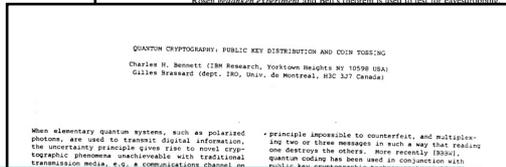
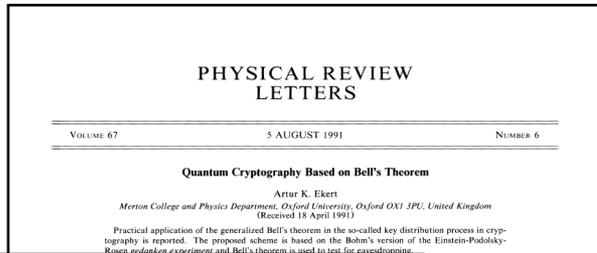
0	1	0	1	1	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

X

0	?	?	1	?	0	0	?	?	?
---	---	---	---	---	---	---	---	---	---

E

# Quantum cryptography



... Before I proceed any further, some basic notions of cryptography of a cryptotext depended on the encrypting and decrypting process use ciphers for which the algorithm could be revealed compromising the security of a paragraph ciphers a set of specific supplied together with the plain-crypting algorithm, and together in input to the decrypting algorithm and decrypting algorithms are security of the cryptogram depends on the secrecy of the key, and this key, may consist of any randomly string of bits. Once the key is communication involves sending a channel which is vulnerable to e.g., public announcement in order to establish the key, two information initially, must at a station use a reliable and a very interception is a set of measurements might be from a technological any classical channel can also, without the legitimate users eavesdropping has taken place. channels [3]. In the following and which distributes the key

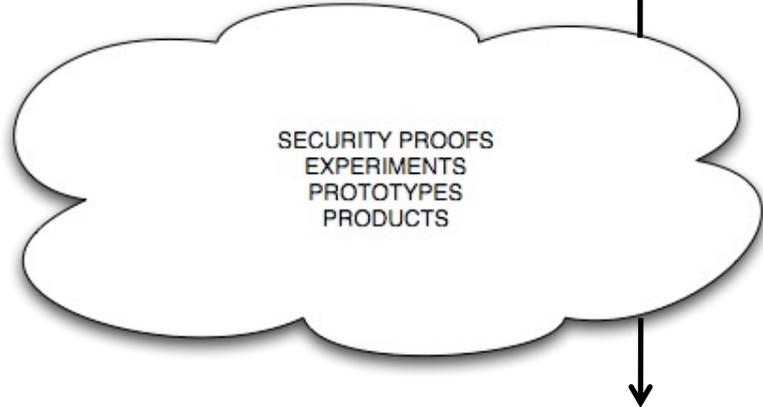
**STEVEN WIESNER**  
1970

**CHARLES H. BENNETT  
GILLES BRASSARD**  
1984

**ARTUR EKERT**  
1991

PREPARE &  
MEASURE

ENTANGLEMENT  
BASED



Device independence etc

# The story of worry

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

## Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in

quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

### 1.

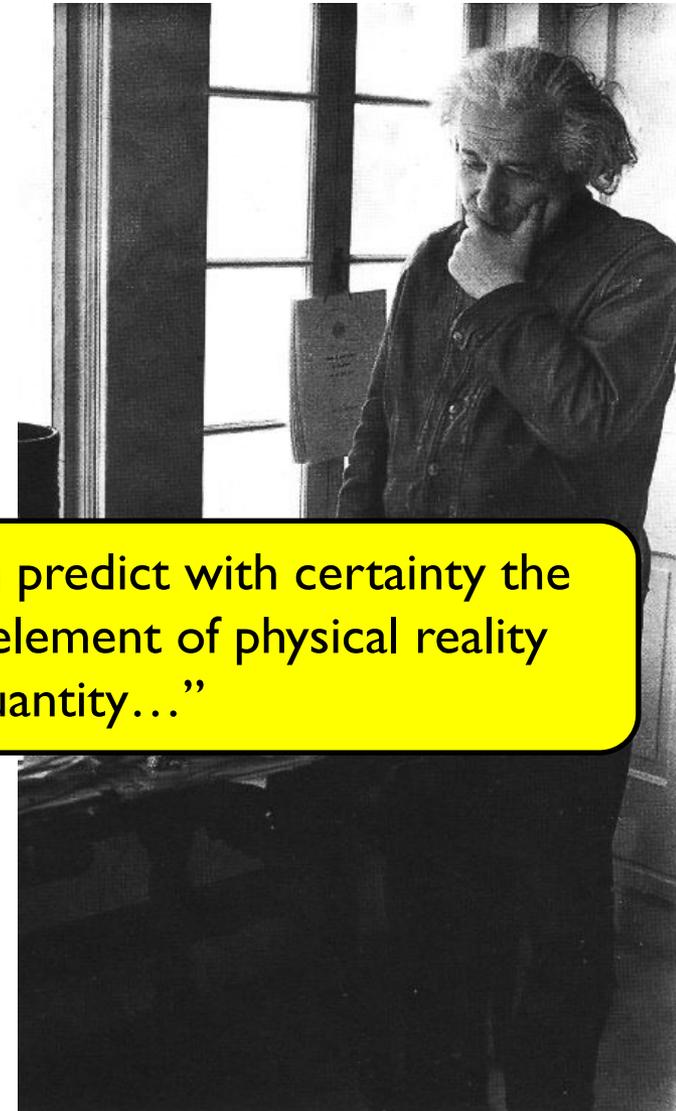
ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the

“...If without any way disturbing a system, we can predict with certainty the value of a physical quantity then there exists an element of physical reality corresponding to this physical quantity...”

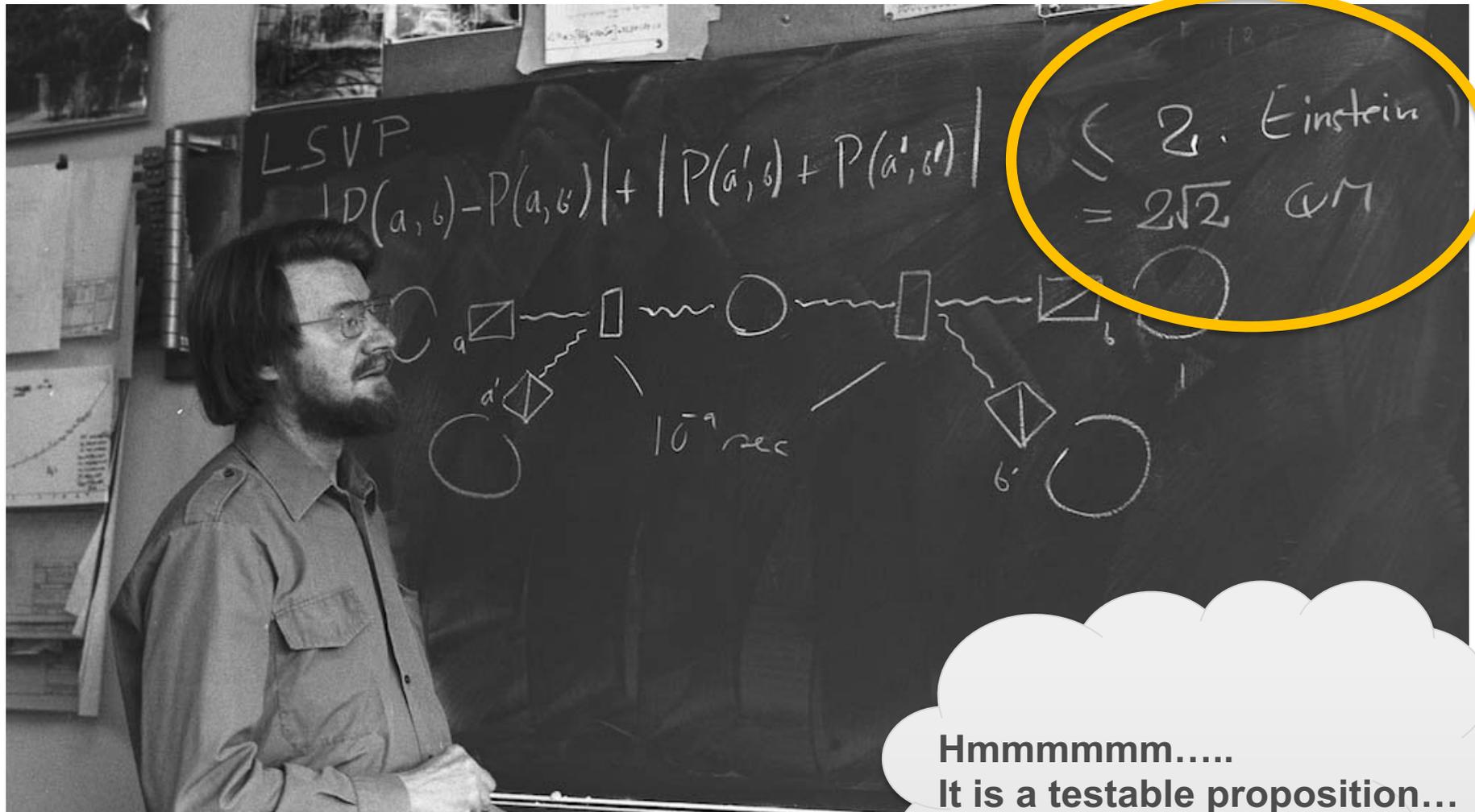
It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one



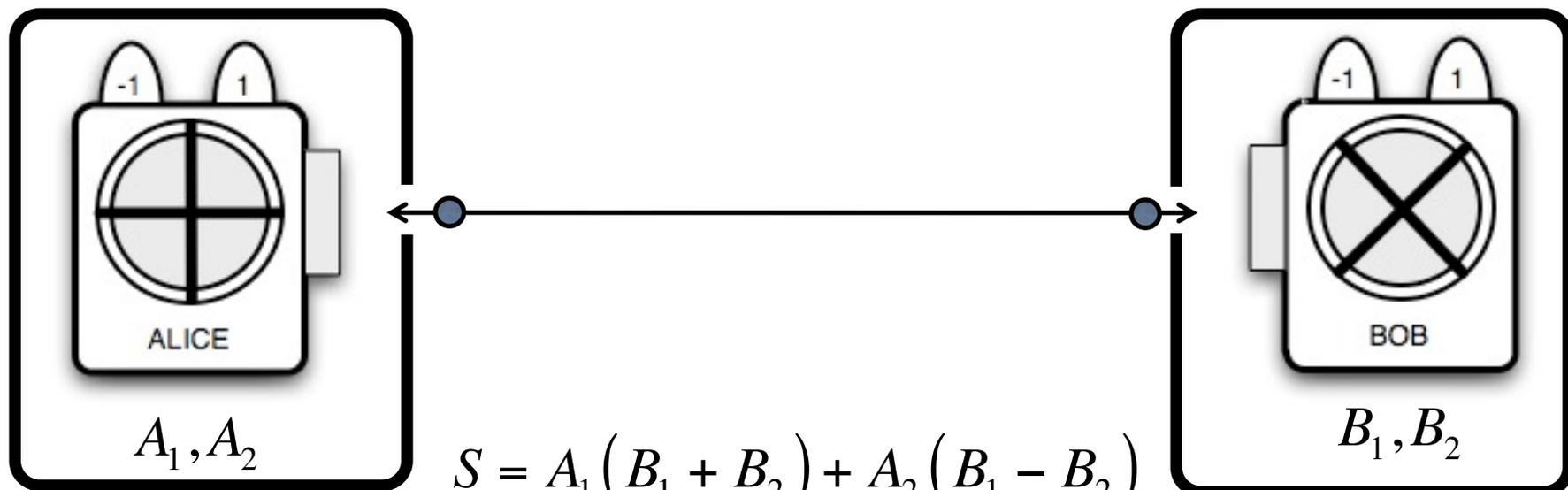
DEFINITION OF EAVESDROPPING

# Enter John Bell



year 1964

# Bell's inequalities...



$$S = A_1(B_1 + B_2) + A_2(B_1 - B_2)$$

One of these terms is 0 and the other is  $\pm 2$

$S = \pm 2$  hence  $-2 \leq \langle S \rangle \leq 2$

# John Clauser – postdocs have ideas...

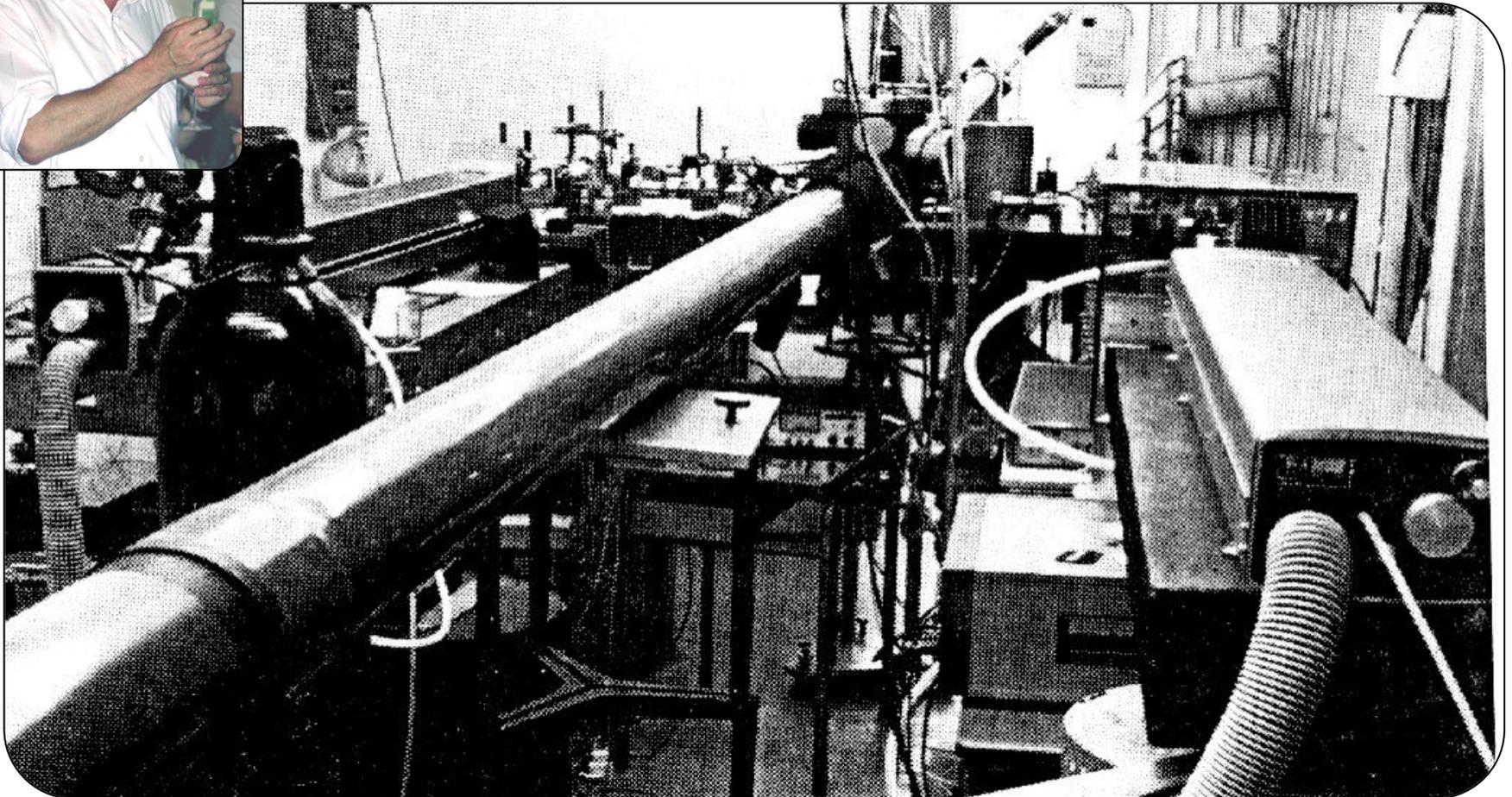


# Alain Aspect and his quantum magic

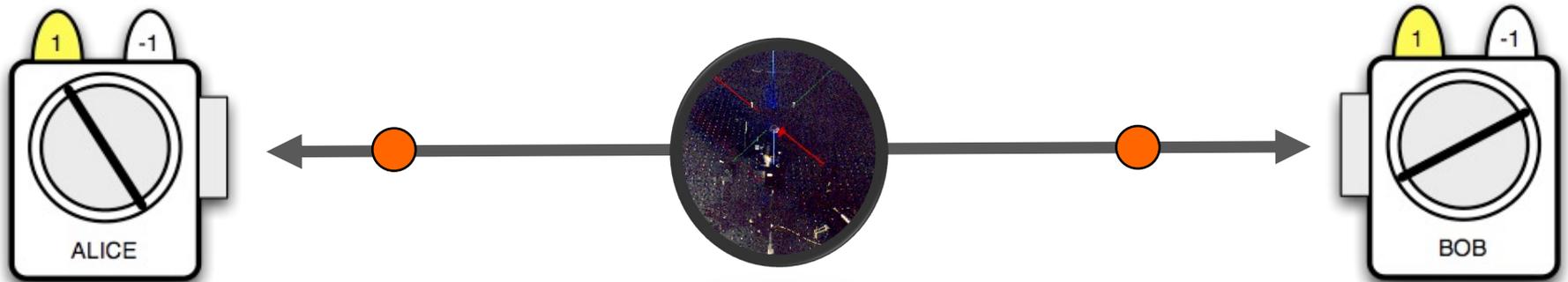


$$S > 2$$

Et voilà!



# Less reality more security

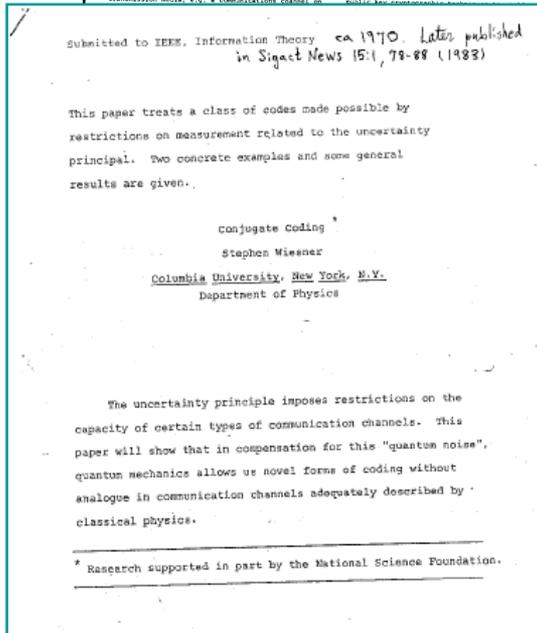
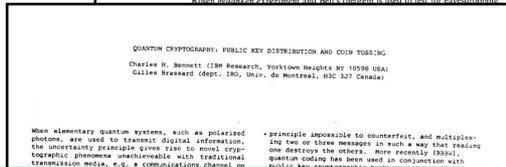
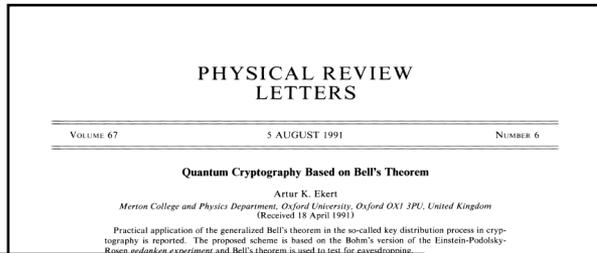


**PHOTONS DO NOT CARRY PREDETERMINED VALUES OF POLARIZATIONS**

**IF THE VALUES DID NOT EXIST PRIOR TO MEASUREMENTS THEY WERE NOT AVAILABLE TO ANYBODY INCLUDING EAVESDROPPERS**

**TESTING FOR THE VIOLATION OF BELL'S INEQUALITIES = TESTING FOR EAVESDROPPING**

# Quantum cryptography



... Before I proceed any further, some basic notions of cryptography of a cryptotext depended on the decrypting and decrypting process use ciphers for which the algorithm could be revealed compromising the security of a paragraph ciphers a set of specific supplied together with the plain-crypting algorithm, and together in input to the decrypting algorithm and decrypting algorithms are security of the cryptogram depends on the secrecy of the key, and this key, may consist of any randomly string of bits. Once the key is communication involves sending a channel which is vulnerable to e.g., public announcement in order to establish the key, two information initially, must at a station use a reliable and a very interception is a set of measurements might be from a technological any classical channel can also, without the legitimate users eavesdropping has taken place. channels [3]. In the following and which distributes the key

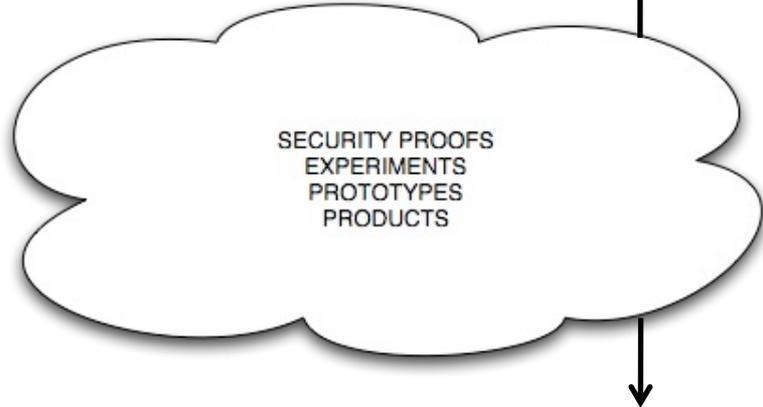
**STEVEN WIESNER**  
1970

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PREPARE & MEASURE

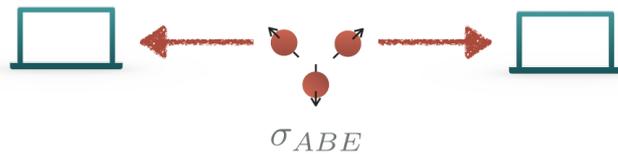
ENTANGLEMENT BASED



Device independence etc

# You need some mathematical gymnastics

Eve uses the same strategy in each round, independently of all other rounds

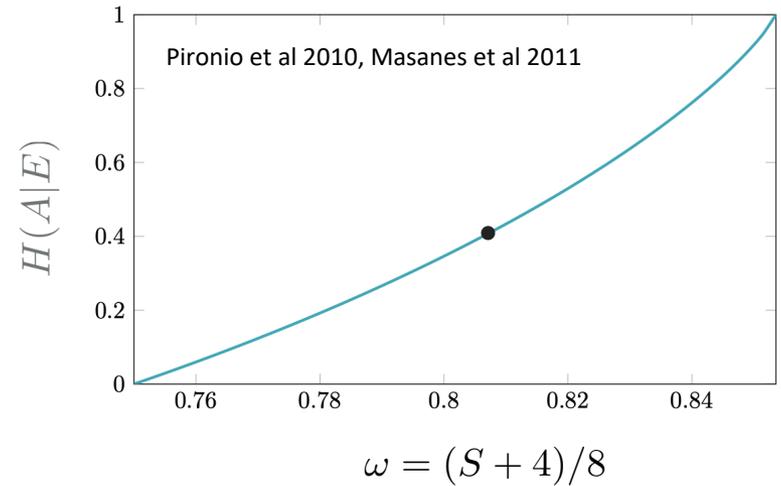


$S$

$$H_{\min}^{\epsilon}(\mathbf{A}|\mathbf{E})_{\rho} \geq nH(A|E)_{\sigma} - c_{\epsilon}\sqrt{n}$$

Extractors

Secret key



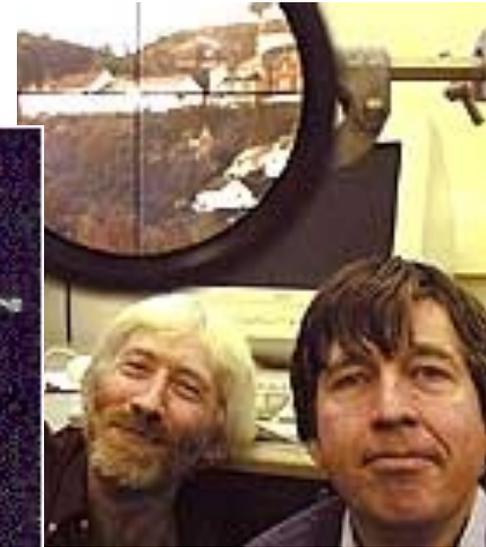
Quantum Asymptotic Equipartition Property  
M. Tomamichel et al (2009) IDD CASE

**Eve distributes the key!**

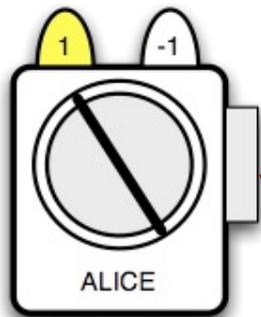
# And all this can be demonstrated...

## Parametric down conversion

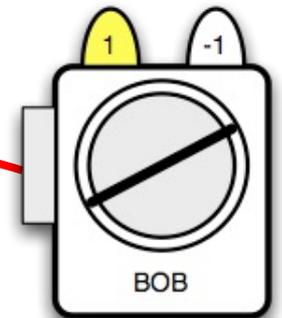
Entangled photons



Optical fibers



Polarizing filters  
& photodetectors

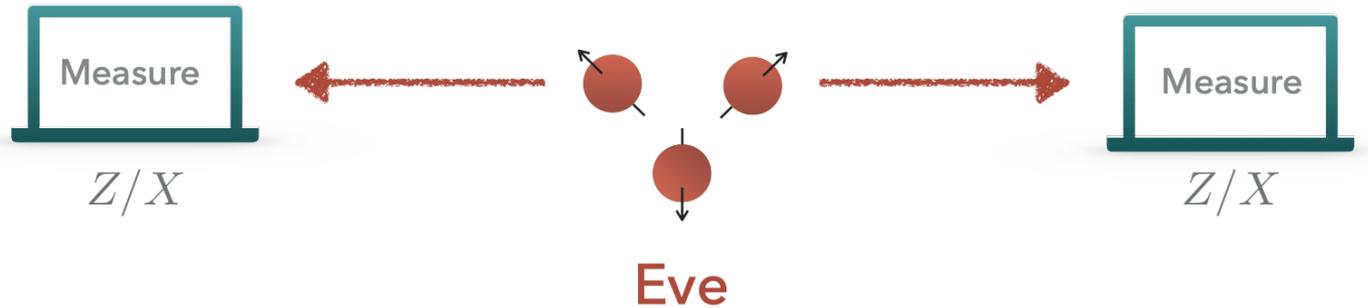


Polarizing filters  
& photodetectors

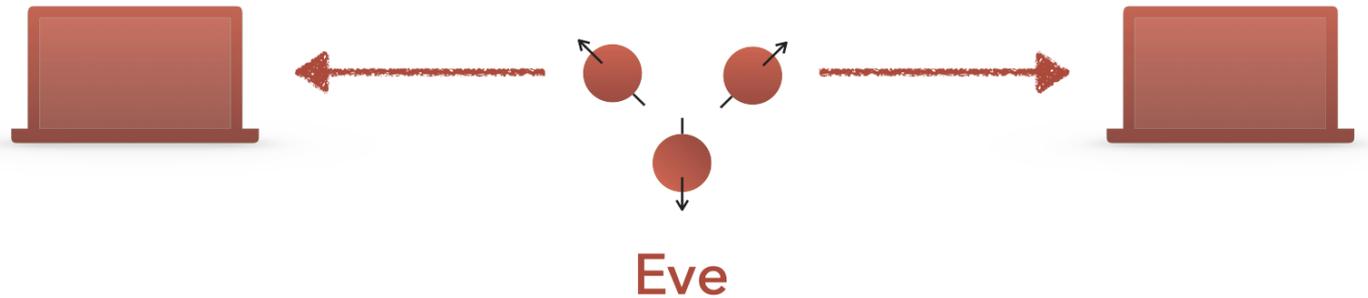
**DRA MALVERN – OXFORD 1991**

# At the mercy of Eve

Ekert 91



Device-independent



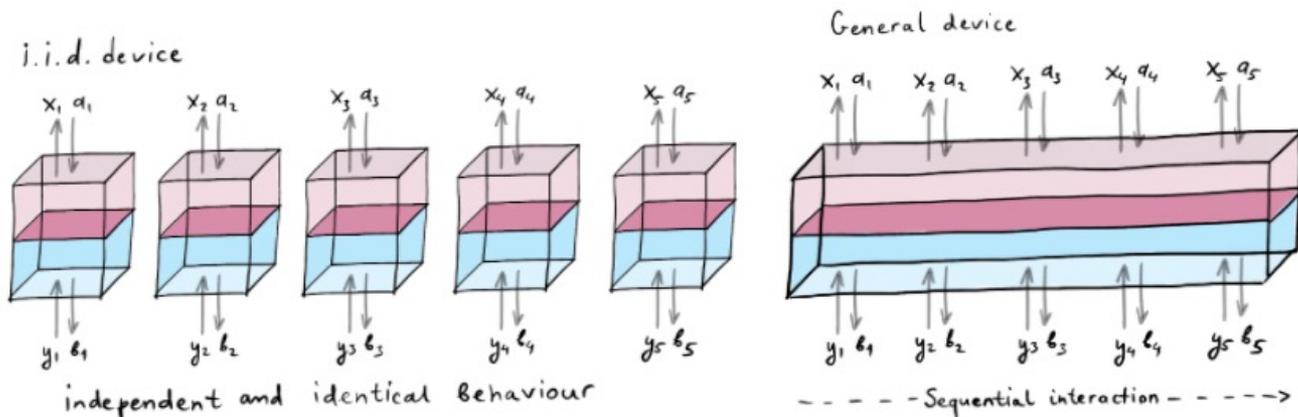
# Towards device-independent crypto

A. Acin, N. Brunner, N. Gisin, S. Massar, V. Scarani



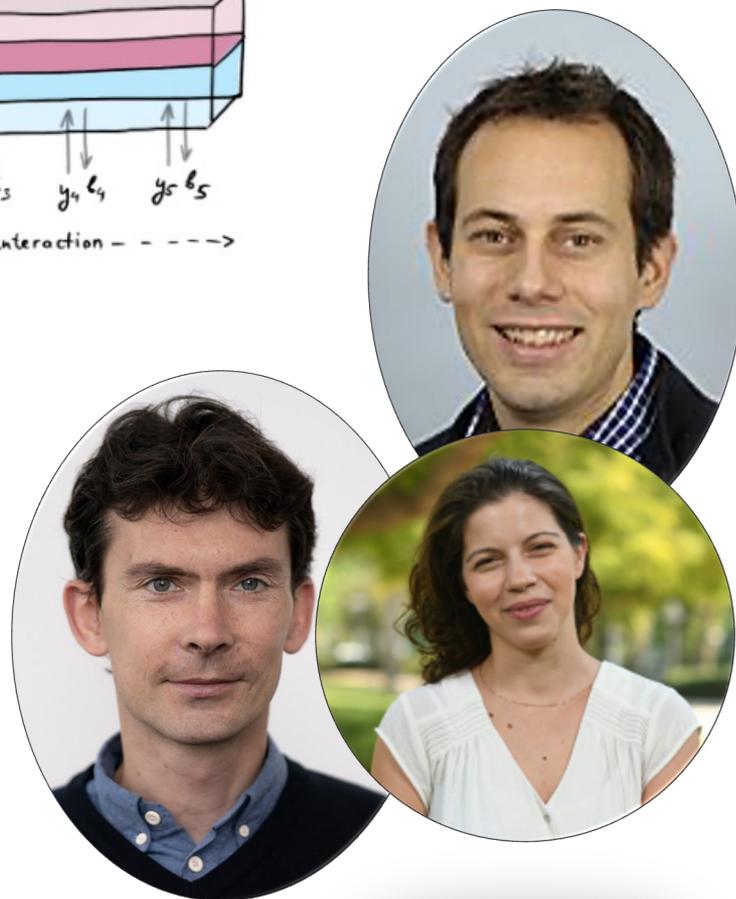
Courtesy Rotem Arnon-Friedman

# EAT...



**Entropy Accumulation Theorem (EAT) allows us to reduce arbitrary strategies to i.i.d. strategies and enables simple device-independent security proofs.**

Rotem Arnon-Friedman, Renato Renner and Thomas Vidick.  
Simple and tight device-independent security proofs.  
*SIAM J. Comput.* **48**, 181 (2019). doi: [10.1137/18M1174726](https://doi.org/10.1137/18M1174726)



# You can have your key and EAT it

1. Winning a non-local game

$$H(A|E) \geq f(\text{win prob.})$$



2. Entropy accumulation  
(Reduction to IID)

$$H_{\min}^{\varepsilon}(\mathbf{A}|\mathbf{E})_{\rho} \geq nH(A|E)_{\sigma} - c_{\varepsilon}\sqrt{n}$$



3. Quantum-proof extractors

$$\|\rho_{\text{Ext}(A,S)SE} - \rho_{U_{\ell}} \otimes \rho_{SE}\| \leq \varepsilon$$



4. Secrecy

$$(1 - \Pr(\text{abort})) \|\rho_{K_A E} - \rho_{U_{\ell}} \otimes \rho_E\| \leq \varepsilon_{\text{sec}}$$

# And this is for real

Article 95884 secret bits in 8 hours

## Experimental quantum key distribution certified by Bell's theorem

<https://doi.org/10.1038/s41586-022-04941-5>

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 Check for updates

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Cryptographic key exchange protocols traditionally rely on computational

It is because of quantum crypto we still keep testing Bell inequalities...

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Editors' Suggestion

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### Toward a Photonic Demonstration of Device-Independent Quantum Key Distribution

Wen-Zhao Liu, Yu-Zhe Zhang, Yi-Zheng Zhen, Jian-Wei Pan

Phys. Rev. Lett. **129**, 050502 – Published 27 J

 See Research News: [Hiding Secrets Usin](#)

Article

## A device-independent quantum key distribution system for distant users

<https://doi.org/10.1038/s41586-022-04891-y>

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Open access

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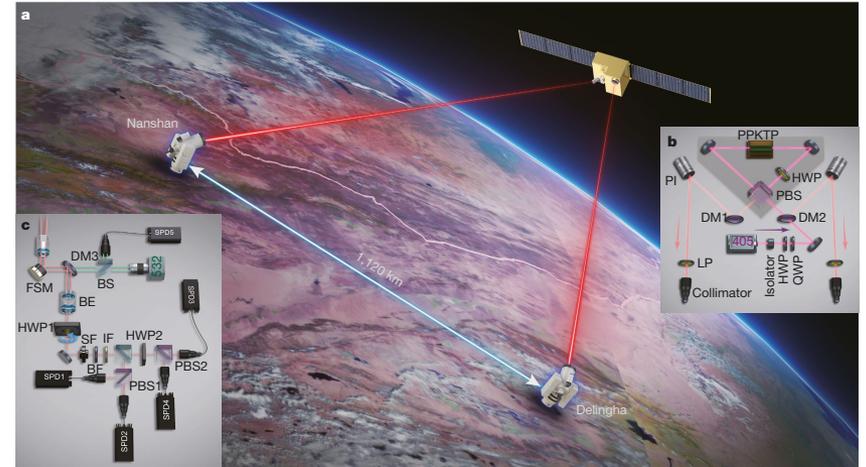
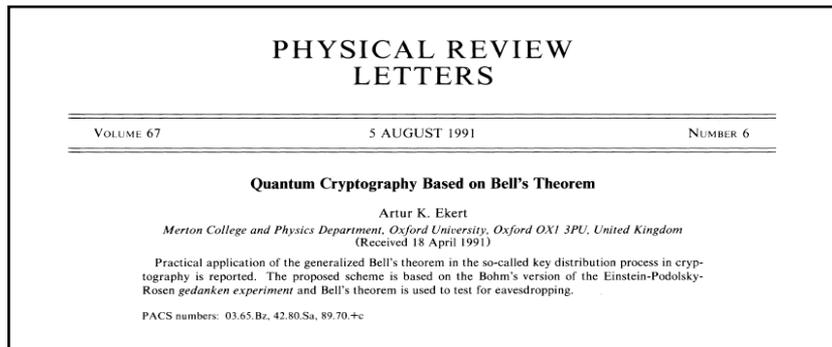
Wei Zhang<sup>1,2,9</sup>, Tim van Leent<sup>1,2,9</sup>, Kai Redeker<sup>1,2,9</sup>, Robert Garthoff<sup>1,2,9</sup>, René Schwonnek<sup>3,4</sup>, Florian Fertig<sup>1,2</sup>, Sebastian Eppelt<sup>1,2</sup>, Wenjamin Rosenfeld<sup>1,2</sup>, Valerio Scarani<sup>5,6</sup>, Charles C.-W. Lim<sup>4,5,8,10</sup> & Harald Weinfurter<sup>1,2,7,11</sup>

Device-independent quantum key distribution (DIQKD) enables the generation of secret keys over an untrusted channel using uncharacterized and potentially untrusted devices<sup>1–9</sup>. The proper and secure functioning of the devices can be certified by a statistical test using a Bell inequality<sup>10–12</sup>. This test originates from the foundations of quantum physics and also ensures robustness against implementation

# Thirty years ago...



From Oxford in 1991...



...to China in 2019

## Article

# Entanglement-based secure quantum cryptography over 1,120 kilometres

<https://doi.org/10.1038/s41586-020-2401-y>

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Check for updates

Juan Yin<sup>1,2,3</sup>, Yu-Huai Li<sup>1,2,3</sup>, Sheng-Kai Liao<sup>1,2,3</sup>, Meng Yang<sup>1,2,3</sup>, Yuan Cao<sup>1,2,3</sup>, Liang Zhang<sup>2,3,4</sup>, Ji-Gang Ren<sup>1,2,3</sup>, Wen-Qi Cai<sup>1,2,3</sup>, Wei-Yue Liu<sup>1,2,3</sup>, Shuang-Lin Li<sup>1,2,3</sup>, Rong Shu<sup>2,3,4</sup>, Yong-Mei Huang<sup>5</sup>, Lei Deng<sup>6</sup>, Li Li<sup>1,2,3</sup>, Qiang Zhang<sup>1,2,3</sup>, Nai-Le Liu<sup>1,2,3</sup>, Yu-Ao Chen<sup>1,2,3</sup>, Chao-Yang Lu<sup>1,2,3</sup>, Xiang-Bin Wang<sup>7</sup>, Feihu Xu<sup>1,2,3</sup>, Jian-Yu Wang<sup>2,3,4</sup>, Cheng-Zhi Peng<sup>1,2,3,8,9</sup>, Artur K. Ekert<sup>10</sup> & Jian-Wei Pan<sup>1,2,3,10</sup>

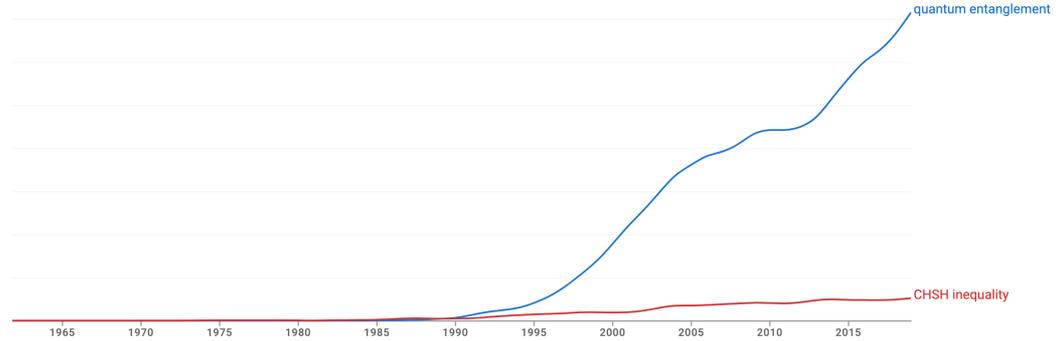
# Crypto helps quantum foundations



1935



1972

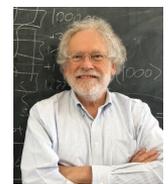


curiosity

1964



1982



E91

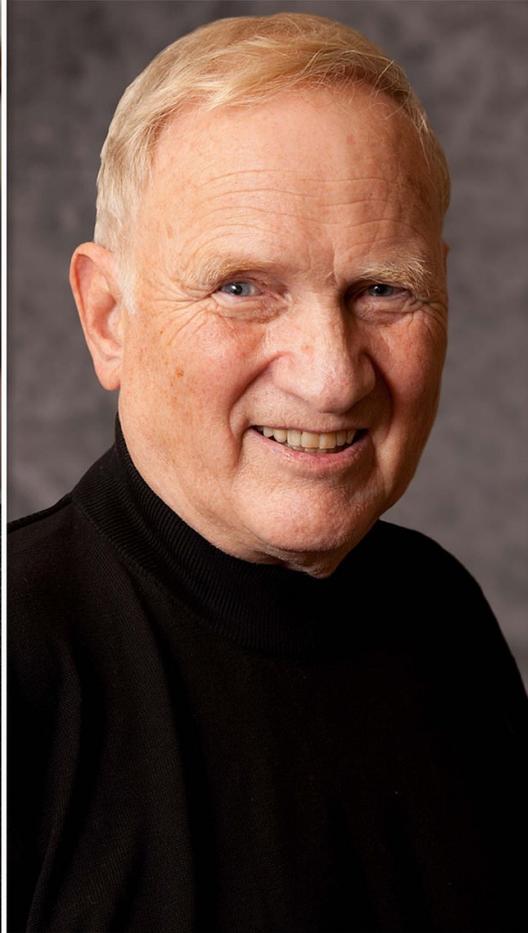
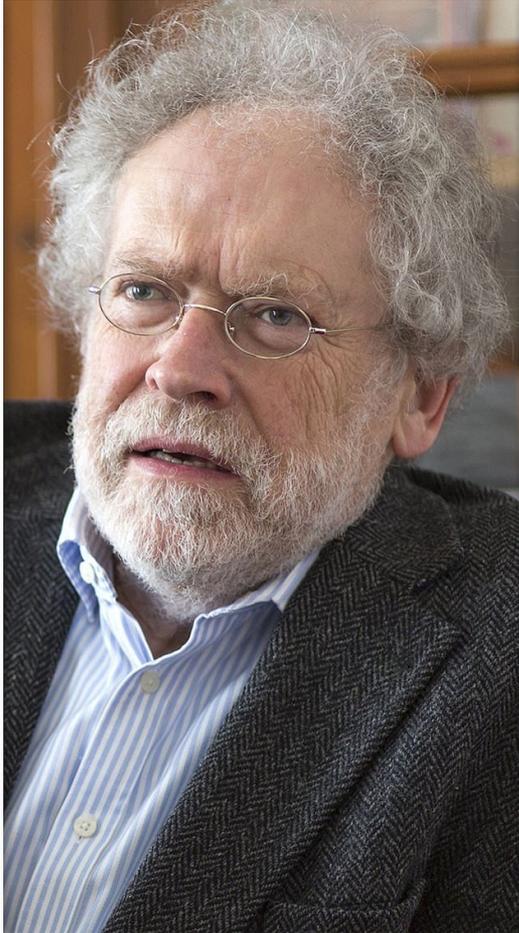


security

BB84



# Nobel 2022



# End of worries?



**You need perfect randomness, right ?**