T Symmetry and Its Violation

Part 1: Fundamentals

Time-reversal T is a very poor symmetry of the experienced world, but it is a **near**-perfect symmetry of physical law.

This raises two big questions:

Question 1: Why is T routinely trashed, in experience?

- Thermodynamics
- Radiation
- Psychology
- …

Answer 1: Cosmology

Really?

Why??

Question 2: Why $\approx$ T, in the first place?

Answer 2: As an “accidental” consequence of deeper principles

Really?

Axions! (?)

(1) Classical Physics and Elementary Quantum Mechanics

Newton, Maxwell+, Schrödinger, Pauli

$$m^j \frac{d^2 x^j}{dt^2} \;=\; F(x^k)$$

If, given

$$x^k(0), \frac{dx^k}{dt}(0)$$

$$\Rightarrow \quad x^k(t) \;=\; x_S^k(t)$$

$$m^j \frac{d^2 x^j}{dt^2} = F(x^k)$$

Then, given

$$x^k(0), -\frac{dx^k}{dt}(0)$$

$$\Rightarrow \quad x^k(t) = x_S^k(-t)$$

$$\frac{dx^k}{dt}(t) = -\frac{dx_S^k(-t)}{dt}(t)$$

To describe this situation, we say that Newtonian mechanics has T symmetry …

… and that positions are even under T, while velocities are odd.

(Going forward, I will be more telegraphic.)

$$\nabla \cdot E = -\kappa \nabla a \cdot B$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot B = 0$$

$$\nabla \times B = \frac{\partial E}{\partial t} + \kappa(\dot{a}B + \nabla a \times E)$$

$$\frac{\partial^2 a}{\partial t^2} - \nabla^2 a + m^2 a \propto \kappa E \cdot B$$

T: *E* even, *B* odd, *a* odd

$$E = -\frac{\partial A}{\partial t} - \nabla A_0$$

$$B = \nabla \times A$$

T: A_0 even, A odd

We say (A_0, A) is “unnatural”.

$$i\frac{\partial\psi}{\partial t} = H\psi$$

T is implemented with an *anti*unitary transformation, with

$$(U\phi, U\psi) = (\psi, \phi) \ ; \ U(\alpha\psi) = \alpha^*\psi$$

This gives $i\frac{\partial(U\psi)}{\partial(-t)} = UHU^{-1}(U\psi)$

T works if $UHU^{-1} = H$

For the ordinary (spinless) Schrödinger equation, with a real potential, we can take

$$U\psi \;=\; \psi^*.$$

Note that electromagnetic covariant derivatives $i\frac{\partial}{\partial x^\mu} + qA_\mu$ transform homogeneously, because both terms are unnatural.

As an instructive exercise, let's consider a multi-component Schrödinger equation

$$i\frac{\partial \psi_k}{\partial t} \;=\; \big(-K_{kl}\nabla^2 + V_{kl}(x)\big)\psi_l$$

For unitarity (Hermitian *H*) we require that *K* and *V* are Hermitian.

By a unitary re-definition $\psi \rightarrow \Omega\psi$ we can diagonalize $K \rightarrow \Omega K \Omega^{-1}$. Positivity of energy requires that the entries of the diagonal *K* are positive. They define effective masses.

(This re-definition of ψ also redefines

$$V \ \rightarrow \ \Omega V \Omega^{-1}.)$$

We would like to implement T as complex conjugation. But that will leave the equation invariant only if *V* is not merely hermitian, but also real.

If we assume all the masses are unequal, our remaining freedom is to multiply the ψ_l by independent phase factors.

If we have two components, we can exploit that freedom to make *V* real - so T is always valid!

But with three or more components opportunities for T violation appear.

Now let us bring in spin.

To transform spin in a way compatible with orbital angular momentum, we want to have

$$U\vec{\sigma}U^{-1} = -\vec{\sigma}.$$

This motivates choosing $U = \sigma_2 K$, where K is complex conjugation.

Application 1: Kramers degeneracy

$$U \;=\; \sigma_2 K \;\Rightarrow\; U^2 = -1$$

and with n spins 1/2

$$U \;=\; \sigma_2 \otimes \sigma_2 \otimes \ldots \otimes \sigma_2 K \;\Rightarrow\; U^2 = (-1)^n$$

For n odd, therefore

$$(\psi, U\psi) \;=\; (U\psi, U^2\psi)^* = -(U\psi, \psi)^* = -(\psi, U\psi)$$

- i.e., ψ and $U\psi$ are orthogonal.

If T is valid, then ψ and $U\psi$ are also degenerate.

This doubling of the spectrum is known as *Kramers' degeneracy,* and we say that the states come in *Kramers doublets*.

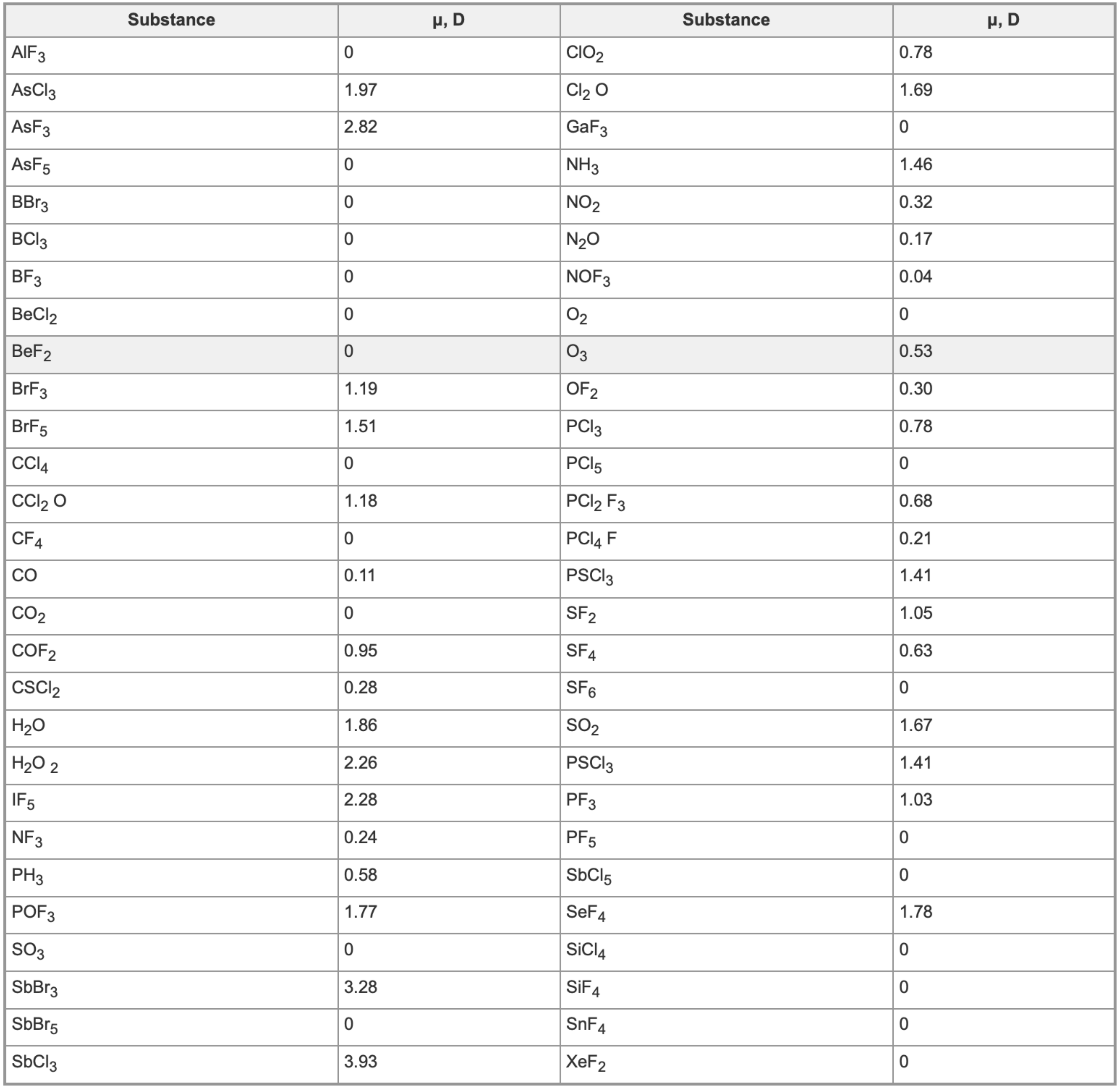
Application 2: Dipole moments

$\Delta H \propto \sigma \cdot B$ is T invariant, since both factors are T odd.

$\Delta H \propto \sigma \cdot E$ is T violating.

Electric Dipole Moments

D ≡ Debye ≈ .2 e-Å



Substance	μ, D	Substance	μ, D
AlF_3	0	ClO_2	0.78
$AsCl_3$	1.97	Cl_2O	1.69
AsF_3	2.82	GaF_3	0
AsF_5	0	NH_3	1.46
BBr_3	0	NO_2	0.32
BCl_3	0	N_2O	0.17
BF_3	0	NOF_3	0.04
$BeCl_2$	0	O_2	0
BeF_2	0	O_3	0.53
BrF_3	1.19	OF_2	0.30
BrF_5	1.51	PCl_3	0.78
CCl_4	0	PCl_5	0
CCl_2O	1.18	PCl_2F_3	0.68
CF_4	0	PCl_4F	0.21
CO	0.11	$PSCl_3$	1.41
CO_2	0	SF_2	1.05
COF_2	0.95	SF_4	0.63
$CSCl_2$	0.28	SF_6	0
H_2O	1.86	SO_2	1.67
H_2O_2	2.26	$PSCl_3$	1.41
IF_5	2.28	PF_3	1.03
NF_3	0.24	PF_5	0
PH_3	0.58	$SbCl_5$	0
POF_3	1.77	SeF_4	1.78
SO_3	0	$SiCl_4$	0
$SbBr_3$	3.28	SiF_4	0
$SbBr_5$	0	SnF_4	0
$SbCl_3$	3.93	XeF_2	0

?

(2) Dirac Equation Basics

T and Chirality

$$(i\gamma^\mu \partial_\mu + m)\,\psi \;=\; 0$$

$\Rightarrow$ We want γ^μ to be unnatural.

$$\gamma^0 \;\equiv\; \sigma_3 \otimes 1 \qquad \gamma^j \;\equiv -\, i\sigma_2 \otimes \sigma_j$$

satisfy $\{\gamma^\mu, \gamma^\nu\} \;=\; 2\eta^{\mu\nu}$

With this basis choice, $U \;=\; i\gamma^1\gamma^3 K$ does the job.

(Note $i\gamma^1\gamma^3 \;=\; 1 \otimes \sigma_2$.)

$$\gamma^5 \;=\; i\gamma^0\gamma^1\gamma^2\gamma^3 \;\propto\; i\epsilon_{\rho\sigma\tau\mu}\gamma^\rho\gamma^\sigma\gamma^\tau\gamma^\mu$$

$$(\gamma^5)^\dagger = \gamma^5\,;\;(\gamma^5)^2 = 1\,;\;\{\gamma^5,\gamma^\mu\} = 0$$

"left-handed" "right-handed"

⇒ Projections $P_L = \dfrac{1-\gamma^5}{2}, P_R = \dfrac{1+\gamma^5}{2}$

For $m = 0$ (only) we can use the projected Dirac equation (≡ Weyl equation)

$i\gamma^\mu\partial_\mu\psi_L \;=\; 0$ - or, of course, the right-handed version.

(3) Understanding $\approx$ T

(3a) Standard Model Census

Symmetries and Particles

$$SU(3) \times SU(2) \times U(1)$$

$\Rightarrow$ Gluons, W/Z, Photons

$(U, D)_L^{1/6}$ $U_R^{2/3}$ $D_R^{-1/3}$

$(N, E)_L^{-1/2}$ E_R^{-1}

$$H \equiv (\phi, \lambda)^{-1/2}$$

(3b) Standard Model Operating System

Building Blocks

Kinetic Terms

$$\nabla_\mu = \partial_\mu + iA_\mu$$

$$[\nabla_\mu, \nabla_\nu] \cdot = iF_{\mu\nu} \cdot$$

$$\frac{1}{g^2}\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} \quad (\,*\,)$$

* For perturbation theory, use $A/g \to A$.

$$\theta\ \epsilon^{\mu\nu\alpha\beta}\ \mathrm{Tr}\ F_{\mu\nu}\,F_{\alpha\beta} \quad (\,*\,)$$

* T odd. [Why?]. Much more later.

$$\bar{\psi}\, i\gamma^{\mu}\, \nabla_{\mu}\, \psi \quad (\,*\,,\,*\,*\,)$$

$$*\;\; \bar{\psi} \;\equiv\; \psi^{\dagger}\, \gamma^{0}$$

** Chiral projection factors through:
these terms connect L to L and R to R

$$\eta^{\mu\nu}(i\nabla_{\mu}H)^{\dagger}(i\nabla_{\nu}H)$$

Einstein (Wheeler):

Matter tells space-time how to curve, space-time tells matter how to move.

Here:

Charges tell gauge fields how to curve, gauge fields tell charges how to move.

Or:

Charges tell internal spaces how to curve, internal space curvature tells charges how to move.

“Potential” Terms

$$\bar{\psi}\psi' H + \mathrm{h.c.} \quad (*,**,***)$$

* Connects L to R. Names and faces later.

** H is required to get gauge invariant terms.

*** Upon condensation $H \rightarrow (\mathrm{v}, 0)$, this becomes a (complex) mass matrix.

$$\mu^2 H^\dagger H,\ \lambda\,(H^\dagger H)^2 \quad (\,*\,)$$

* Used to construct a potential that encourages condensation.

The potential terms are not as beautiful as the kinetic terms.

(They are the analogue of the cosmological term in general relativity.)

(3c) Effective Theory and Renormalization

The Yoga of Restraint

We want to assemble our building blocks into an action $S = \int d^4x\, L$

The kinetic terms dictate that the mass dimension of the fermion fields is 3/2, while the mass dimension of the scalar fields and gauge potentials is 1.

Our building blocks include the templates for all gauge invariant terms with mass dimension ≤ 4.

Terms with larger dimension would occur in $\Delta S = \int d^4x\, \Delta L$ with coefficients that involve inverse powers of mass.

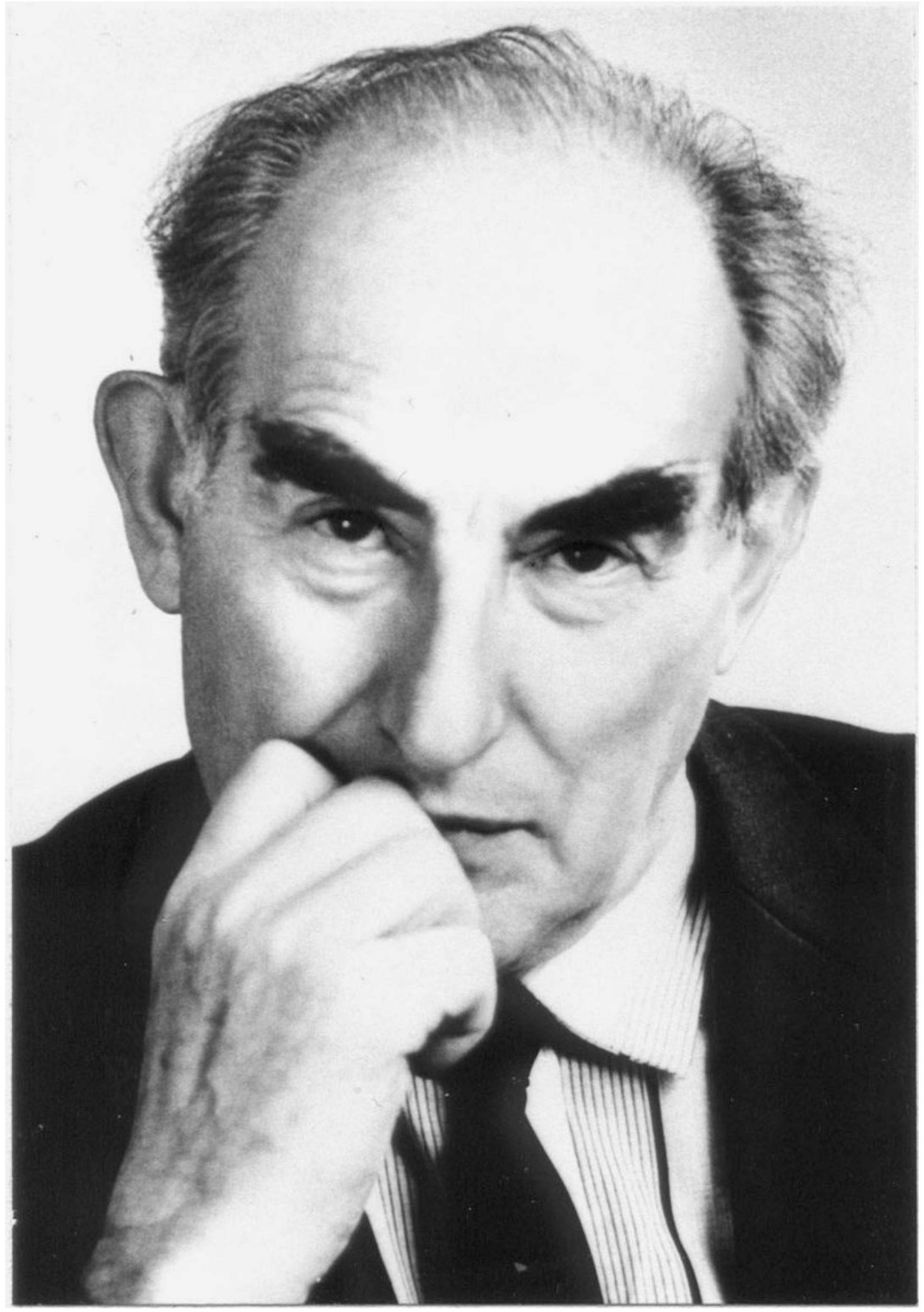
“All things being equal, the simplest solution tends to be the best one.”

William of Ockham

Anticipating the Yoga of Restraint


Gauge Symmetry


Symmetry and Conservation



Systematic Symmetry, Simplicity and Locality

Bringing it All Together

(3d) Standard Model Standardization

Fundamental Theorem

In more detail, and keeping track of a three-valued “family” index, on the face of it we must accommodate quite complicated structures:

$$Z_{L}{}^{a}{}_{b}\,(\bar{U}_L,\bar{D}_L)_a\; i\gamma^{\mu}\nabla_{\mu}\,(U_L,D_L)^b$$

$$Z_{R}^{U}{}^{c}{}_{d}\;\bar{U}_{R\,c}\; i\gamma^{\mu}\nabla_{\mu}\; U_R{}^{d}$$

$$Z_{R}^{D}{}^{e}{}_{f}\;\bar{D}_{R\,e}\; i\gamma^{\mu}\nabla_{\mu}\; D_R{}^{f}$$

$$M^{U}{}^{a}{}_{b}\,(\bar{U}_L,\bar{D}_L)_a\; U_R{}^{b}\; H \qquad + \mathrm{h.c.}$$

$$M^{D}{}^{c}{}_{d}\,(\bar{U}_L,\bar{D}_L)_c\; D_R{}^{d}\; \tilde{H} \qquad + \mathrm{h.c.}$$

(the Zs are Hermitian)

By unitary transformations on the family indices of $(U,D)_L^a$, U_R^c , D_R^e we can diagonalize the Zs.

Assuming that the (real) entries are positive, we can rescale to make them all $=1$.

So now we have

$$(\bar{U}_L , \bar{D}_L)_a \quad i\gamma^\mu \nabla_\mu \quad (U_L , D_L)^a$$

$$\bar{U}_{R\,c} \quad i\gamma^\mu \nabla_\mu \quad U_R{}^c$$

$$\bar{D}_{R\,e} \quad i\gamma^\mu \nabla_\mu \quad D_R{}^e$$

$$M^{U\,a}{}_{b} (\bar{U}_L , \bar{D}_L)_a \quad U_R{}^b \quad H \quad + \mathrm{h.c.}$$

$$M^{D\,c}{}_{d} (\bar{U}_L , \bar{D}_L)_c \quad D_R{}^d \quad \tilde{H} \quad + \mathrm{h.c.}$$

(with redefined fields and different values of the Ms).

After condensation

$$(\bar{U}_L, \bar{D}_L)_a \quad i\gamma^\mu \nabla_\mu \quad (U_L, D_L)^a$$

$$\bar{U}_{R\,c} \quad i\gamma^\mu \nabla_\mu \quad U_R{}^c$$

$$\bar{D}_{R\,e} \quad i\gamma^\mu \nabla_\mu \quad D_R{}^e$$

$$M^{U\,a}{}_b \quad \bar{U}_{L\,a} \quad U_R{}^b \quad + \mathrm{h.c.}$$

$$M^{D\,c}{}_d \quad \bar{D}_{L\,c} \quad D_R{}^d \quad + \mathrm{h.c.}$$

We want to simplify this description of free propagation.

We can render general matrices positive and diagonal by making unitary transformations on both sides:

$$V^{-1}MW \rightarrow m$$

To do this for both M^U and M^D, we need to make unitary transformations on U_L, D_L, U_R, D_R independently.

It is almost, but not quite, possible to make unitary transformations on U_L, D_L, U_R, D_R independently without messing up the work we did earlier on the Zs:

The relative rotation between U_L, D_L interferes with the previously diagonalized doublet structure.

Up to that issue, all the terms in our standardized action (other than θ terms) admit T symmetry, using the transformations we discussed earlier.

(3e) Kobayashi-Maskawa Theory

Constrained T Violation

The perturbed doublet structure gets accessed in terms that connect the two parts of the doublet, i.e. in $W^{\pm}$ vertices.

We have $\Delta L \;=\; \bar{U}_{L\,a}\gamma^{\mu}W_{\mu}^{+}D_{L}^{b}\;S_{b}^{a}+\mathrm{h\,.\,c\,.}$

Our straightforward implementation of T transforms $S \rightarrow S^{*}$ (using the h.c.).

We still have one more card to play.

Without disturbing the canonical kinetic terms or the reality of the (diagonal) mass terms, we can make simultaneous phase rotations on the left- and right-handed version of any quark field.

Using that freedom, we can re-define the fields so that S takes the form:

$$S = \begin{pmatrix} r_1 & r_2 & r_3 \\ r_4 & * & * \\ r_5 & * & * \end{pmatrix}$$

where the r s are real numbers.

Given this form, the unitarity of S allows us to introduce (only) a single complex number phase parameter.

Thus, we arrive at a one-parameter theory of T violation.

This line of thought has proved extraordinarily successful, empirically.

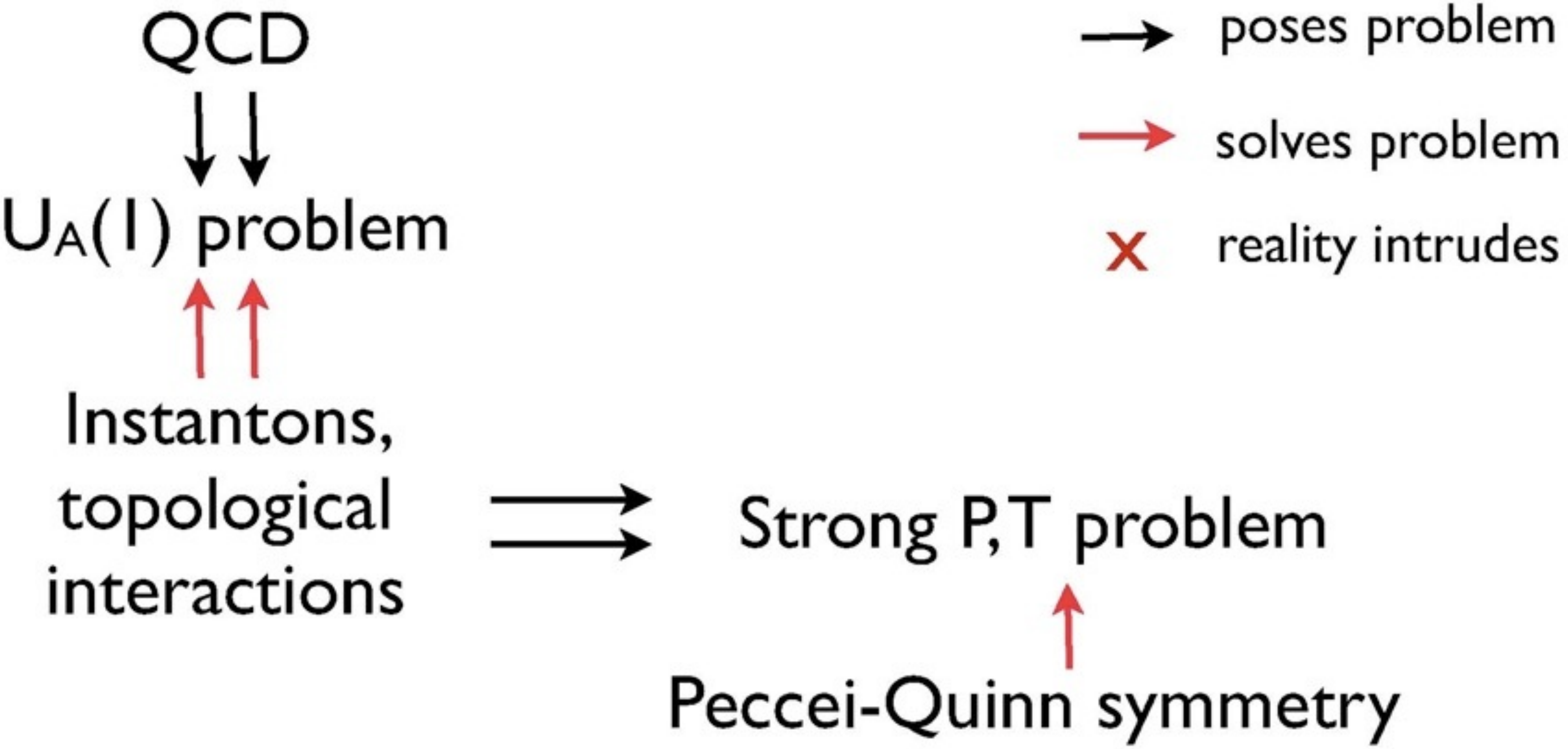
(4) θ Term

Deep Theory and A Deep Puzzle

Formally, the θ terms are total derivatives.

This implies that they have no effect classically*, and to all orders in perturbation theory.

Fortunately, the full story is more complicated.



→ poses problem

→ solves problem

✗ reality intrudes

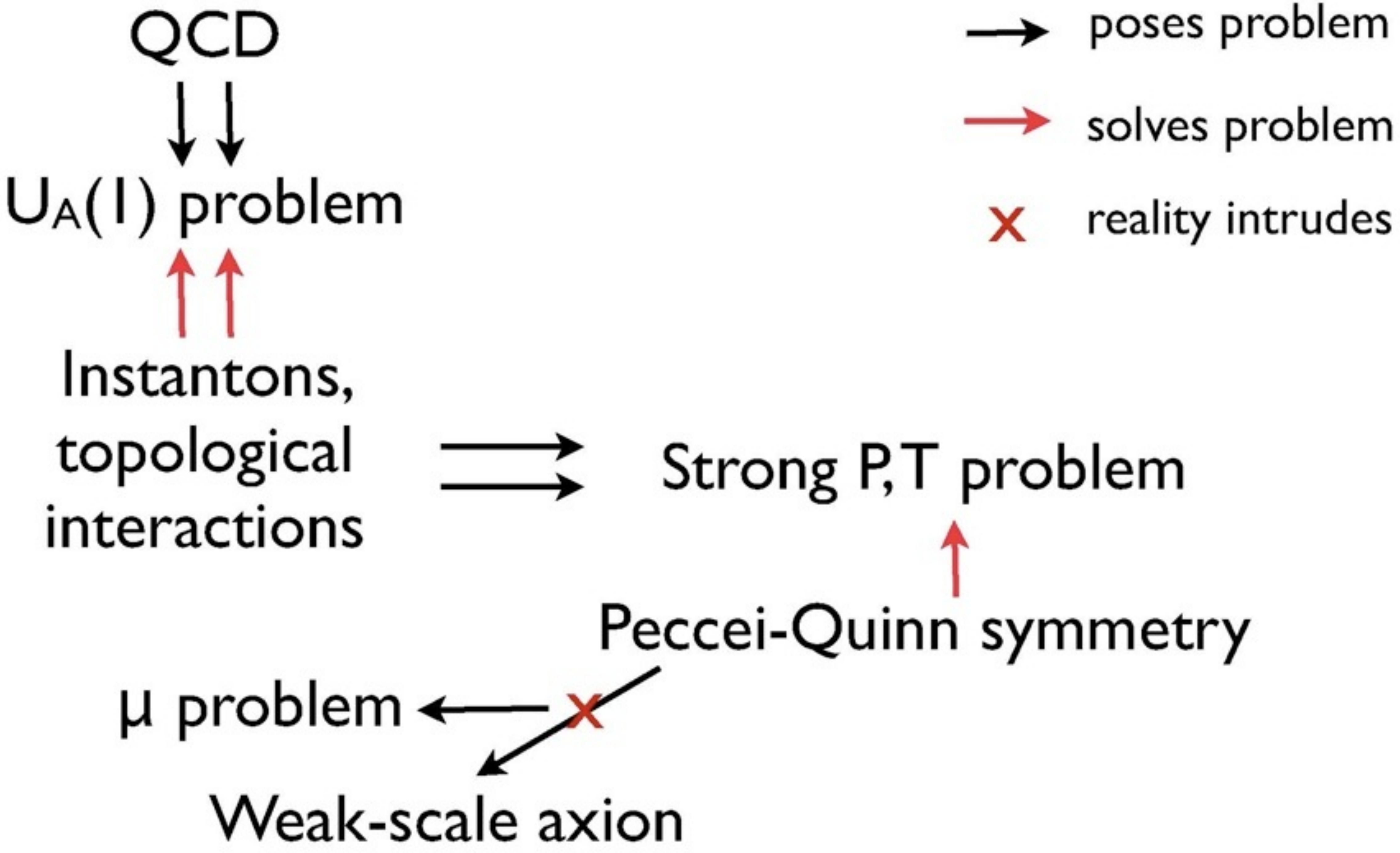
QCD

$U_A(1)$ problem

Instantons, topological interactions

Strong P,T problem

Peccei-Quinn symmetry



→ poses problem

→ solves problem

x reality intrudes

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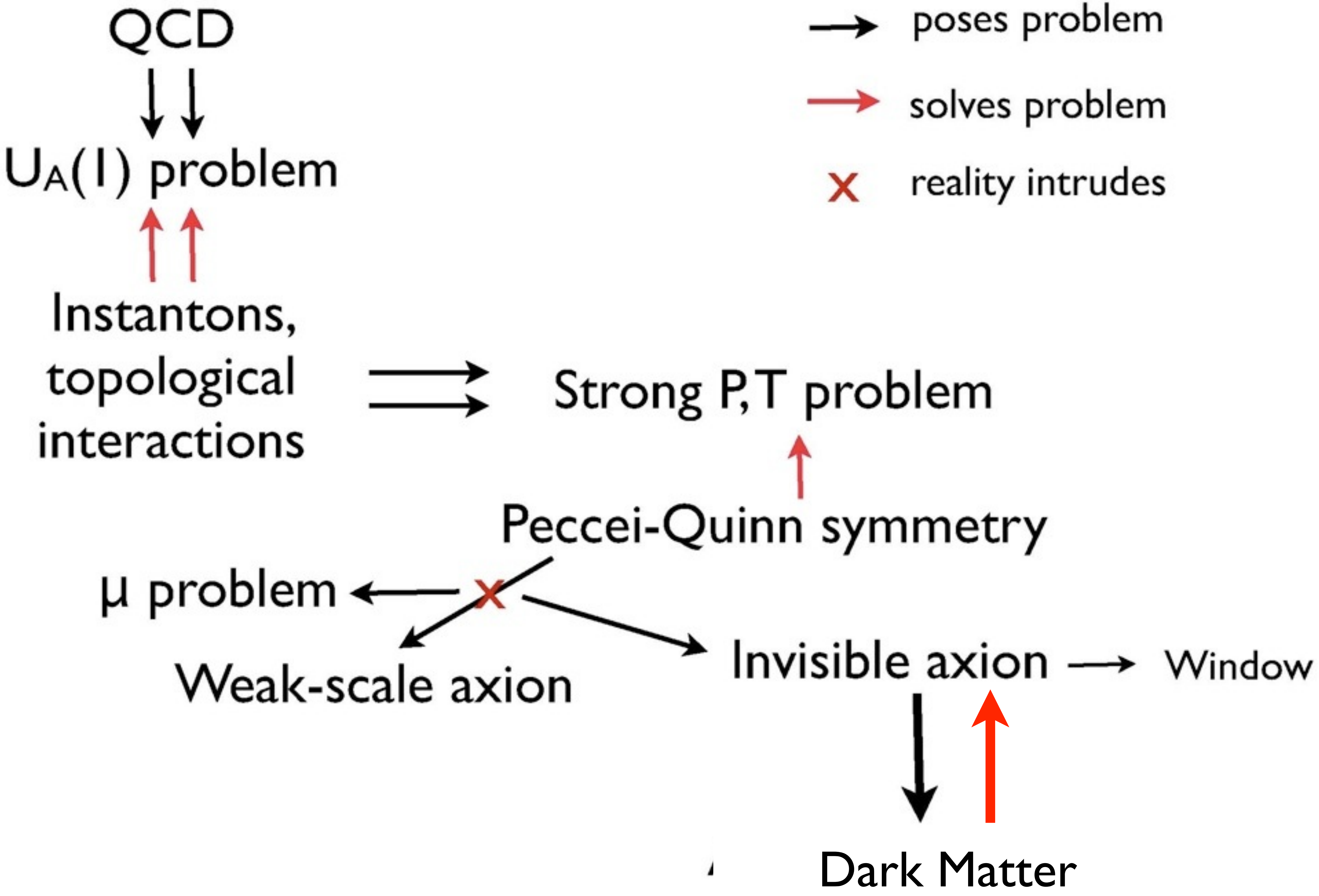
μ problem

Weak-scale axion



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QCD

U_A(1) problem

Instantons, topological interactions

Strong P,T problem

Peccei-Quinn symmetry

μ problem

Weak-scale axion

Invisible axion

Window

Dark Matter

Two deep (and deeply related) aspects of quantum field theory come into play: topology and anomalies.

(4a) Physical Relevance of the θ Term

Mechanism and Result

We have analyzed transformations of the standard model Lagrangian.

In constructing the quantum theory, we use that Lagrangian to weight different field configurations by $\exp\, i \int d^4x\, L$

There are significant issues around the measure of the integral, and the kind of fields it allows.

Quantum fluctuations bring in fields that support some specific kinds of “topological” singularities.

These can make it invalid to throw away surface terms.

In a semiclassical approximation, they appear with amplitudes $\sim \, e^{-c/g^2} e^{in\theta}$

In QCD, there are fully non-perturbative formal and numerical calculations that demonstrate the theory does have this non-trivial, periodic θ dependence.

That’s a relief, because we need it, phenomenologically.

(4b) Time Reversal, Revisited

The Struggle Continues!

T transforms $\theta \to -\theta$.

T symmetry holds only in two physically distinct cases: $\theta \equiv 0, \pi \pmod{2\pi}$.

Thus, there is a second consistent T-violating parameter, different from the one contemplated by Kobayashi and Maskawa.

Empirically, one finds $|\theta| \lesssim 10^{-10}$, based on electric dipole moment limits.

!

?

The bottom line:

We’ve come a long way toward tracing the observed pattern of T symmetry and its breaking to deeper principles - but our work is not quite done.

[supplement on renormalization, unused]

If we are interested in energy and momenta below some ambient mass scale M, contributions of the latter kind would come in with factors p/M.

In an effective low-energy theory we can ignore them, or bring them in perturbatively.

Alternatively, (perturbative) renormalizability:

If we bring in a regulator mass Λ, positive powers of Λ, or $\log \Lambda$, will (only) occur in coefficients of terms with mass dimension ≤ 4.

Such terms are associated with *divergent* radiative corrections, so we must allow for them. Higher dimension terms will have finite coefficients, so they can (and should) be excluded.

Scholium

Quantum mechanics resolved the ultraviolet catastrophe of thermal blackbody radiation, by discretizing the thermal fluctuations.

But quantum mechanics introduces new kinds of fluctuations, and teeters on the brink of ultraviolet catastrophe even at

$$T \; (\equiv \text{Temperature}) = 0.$$

This creative tension leads us to the yoga of restraint. We want the honey, but we must be mindful of the bees.

The underlying buzz is manifested in concrete physical phenomena, notably including the topological anomalies discussed below and the “scaling anomaly” of asymptotic freedom.

We have symmetries of the classical field theory that do not survive its quantization.

[other unused slides follow]

(6) Anomalies in Lower Dimensions

A Serious Playground

(6a) Chiral Vacuum Polarization

A Two-Sided Triangle

In 1+1 dimensions we can apply similar logic to the AV vacuum polarization graph as we had for the triangle graph in 3+1 dimensions. The calculation is much simpler:

[evaluation]

$$\partial_\mu \bar{f}\gamma^\mu\gamma^5 f \propto \epsilon^{\alpha\beta}F_{\alpha\beta}$$

(6b) Physical Interpretation of the Anomaly Equation

Chirality as Motion, and Pair Creation

We can take $\gamma^0 = \sigma_1, \gamma^1 = -i\sigma_2$, and define $\gamma^5 = \gamma^0\gamma^1 = \sigma_3$ to make chiral projectors $P_\pm = \dfrac{1 \pm \sigma_3}{2}$.

Then the projected spinors are 1-component objects, and the Dirac equation just says that they propagate in definite (opposite) directions!

Left-handed → Left-moving

$\partial_\mu \bar{f}\gamma^\mu\gamma^5 f \propto \epsilon^{\alpha\beta}F_{\alpha\beta}$ can be read as

$$\partial_\mu (j_R^\mu - j_L^\mu) \propto E$$

This has a lovely physical interpretation.

A background electric field will produce pairs. The positive particle and the negative antiparticle will move in opposite directions.

In this process, we produce two units of chiral charge.

Note that this process connects left- and right-movers, which does not happen in the classical Lagrangian.

(6c) Physical Interpretation of the θ Term

A Quantum Capacitor

The associated θ term has a simple physical interpretation.

$$\Delta L = \frac{e\theta}{4\pi}\epsilon^{\alpha\beta}F_{\alpha\beta} = \frac{e\theta}{2\pi}E$$

(Here I assume the normalized Maxwell kinetic term $L = \frac{1}{2}E^2$. Note that A has mass dimension 0 and that e has mass dimension 1, so that θ is a pure number.)

Combining the Maxwell and θ terms, we have

$$L + \Delta L \;=\; \frac{1}{2}\,(\,E + \frac{e\theta}{2\pi}\,)^2 \;+\; \text{constant}$$

Thus, the effect of the θ term is to encourage a background electric field $-\dfrac{e\theta}{2\pi}$.

This is what we get from a 1D capacitor with charges $\mp e\dfrac{\theta}{2\pi}$ at $\pm\infty$

(6d) Edge Currents from Anomaly Cancellation

QHE Made “Simple”

A quantum Hall fluid, in bulk, responds as

$$j^{\alpha} \propto \epsilon^{\alpha\beta\gamma} F_{\beta\gamma}$$

We can describe this using an effective Lagrangian density $\Delta L \;\propto\; \epsilon^{\alpha\beta\gamma} A_{\alpha} F_{\beta\gamma}$

If the sample defines a shape Γ, we have the action $S \;\propto\; \int dt \int_{\Gamma} d^2x \; \epsilon^{\alpha\beta\gamma} A_{\alpha} F_{\beta\gamma}$

Under a gauge transformation
$A_\alpha \rightarrow A_\alpha + \partial_\alpha \Lambda$, we have
$\epsilon^{\alpha\beta\gamma} A_\alpha F_{\beta\gamma} \rightarrow \partial_\alpha (\Lambda \epsilon^{\alpha\beta\gamma} F_{\beta\gamma})$.

The action is not quite gauge invariant, because there's a surface term.

We can cancel off the surface term if we have an anomalous (chiral) 1+1 dimensional theory on the boundary!

(The gauge transformation is a chiral rotation, which translates the θ term.)

This is a “simple” way to understand the emergence of unidirectional edge currents in the QHE.