Spinor kinematics $\quad |j\rangle \, , \, |j]$

$$A_3(--+) = \text{const} \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}$$

complex parameter z

$[j, \ell\rangle$

$$|j] \to |j] - z|\ell]$$

$$|\ell\rangle \to |\ell\rangle + z|j\rangle$$

$$k_j^\mu \to k_j^\mu(z) = k_j^\mu - \frac{z}{2}\langle j|\mu|\ell]$$

$$k_\ell^\mu \to k_\ell^\mu(z) = k_\ell^\mu + \frac{z}{2}\langle j|\mu|\ell]$$

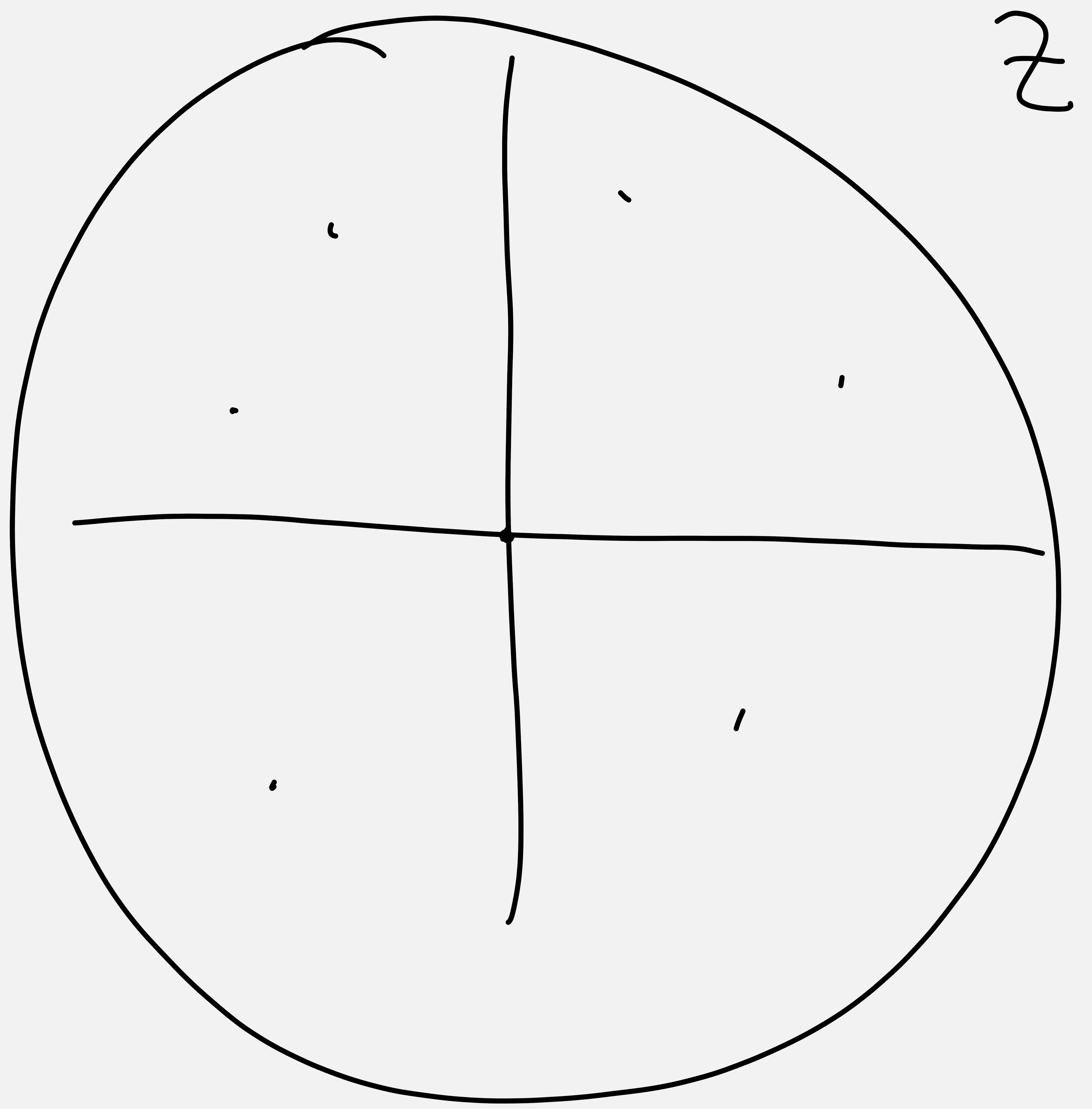
$$k_j^2(z) = 0 = k_\ell^2(z)$$

$$\sum_i k_i(z) = 0$$

$$A_n(z)$$

$A(z) \to 0$ as $z \to \infty$

$(k_i + \cdots + k_r)^2 \neq 0.$



$j \in \{i \dots r\}$ $\quad \ell \notin \{i \dots r\}.$

$$\frac{1}{2\pi i} \int_C dz \, \frac{A(z)}{z} = 0$$

$$K_{i\dots r} = k_i + \cdots + k_r$$

$$K^\mu_{i\dots r}(z) = K^\mu_{i\dots r} - \frac{z}{2} \langle j|\mu|\ell]$$

Residues

$$A(0) = -\sum_{\text{poles } a} \operatorname*{Res}_{z=z_a} \frac{A(z)}{z}$$

$$K^2_{i\dots r}(z) = K^2_{i\dots r} - z \langle j|K_{i\dots r}|\ell]$$

$$\langle j|i+\dots+r|\ell]$$

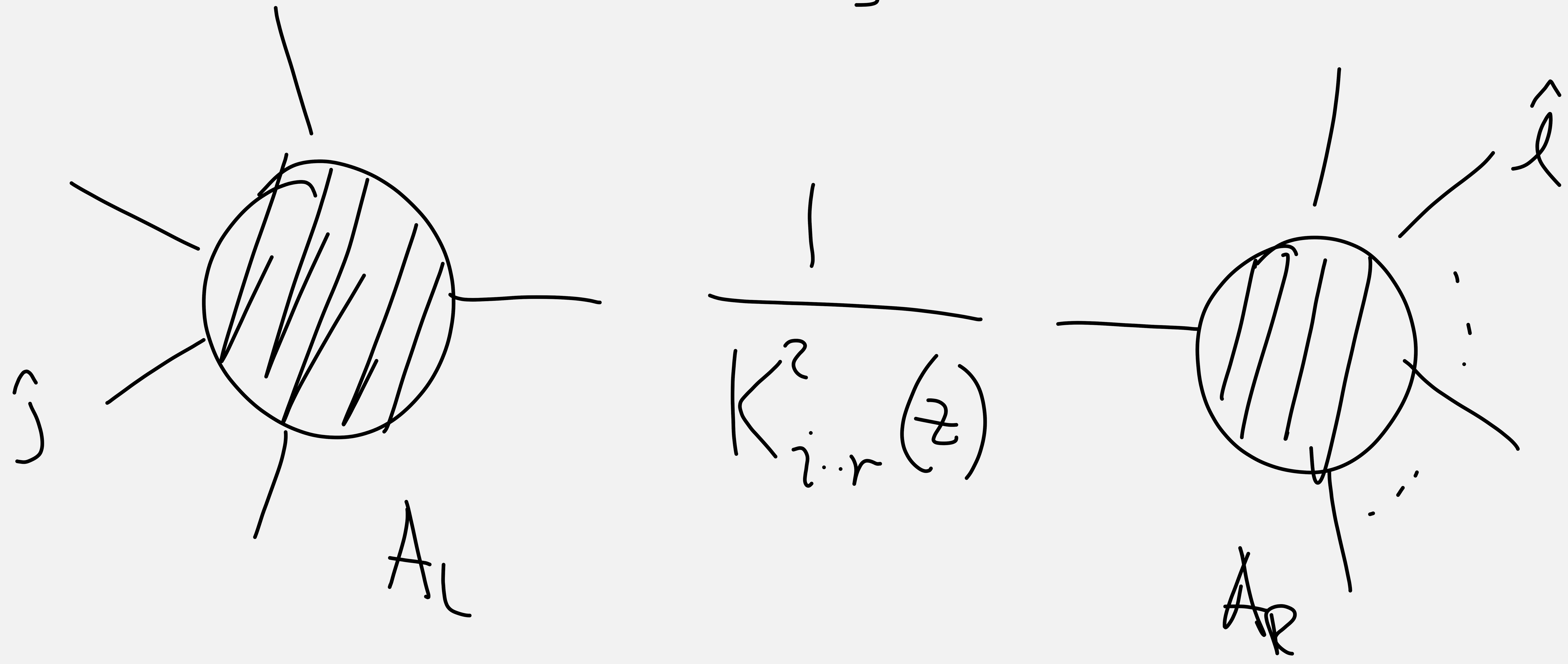
$$z = z_{ir} \equiv \frac{K^2_{i\cdots r}}{\langle j|K_{i\cdots r}|\ell]}$$

$$\mathop{\mathrm{Res}}_{z=z_{ir}} \frac{f(z)}{z\,K^2_{i\cdots r}(z)}$$

$$= \mathrm{Res}\left[-\frac{A_L(z)\, i\, A_R(z)}{z(z-z_{ir})\langle j|K_{i\cdots r}|\ell]}\right]$$

$$= -i\,\frac{A_L(z_{ir})\,A_R(z_{ir})}{z_{ir}\langle j|K_{i\cdots r}|\ell]}$$

$$= -\frac{i}{K^2_{i\cdots r}}\,A_L(z_{ir})\,A_R(z_{ir})$$



$$\sum_{\text{poles } a} = \sum_{\substack{\text{partitions}\\ P}}$$

$$A_n(1,\dots,n) = A(0) =$$

$$\sum_{\substack{\text{partitions } P \\ h=\pm}} A_{\#P+1}(\dots \hat{j} \dots - \hat{P}^{h})\Big|_{z=z_{ir}}$$

$$\times \frac{i}{P^2} \times A_{\#\bar{P}+1}(\dots \hat{l} \dots \hat{P}^{-h})\Big|_{z=z_{ir}}$$

$\sim |l-j| \times n-3$

l, j neighbors

P : j, l are on opposite sides

$\#P \geq 2$

$\#\bar{P} \geq 2$

MHV – Parke-Taylor

$$A(1^+ \cdots m_1^- \cdots m_2^- \cdots n^+) = i \frac{\langle m_1 m_2 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle (n-1) n \rangle \langle n 1 \rangle}$$

$m_1 = 1, \; m_2 = 4$

$[1^-, 2^+\rangle$: $|2] \to |2] + z|1]$

$\langle 12 \rangle \to \langle 12 \rangle$

$\langle 23 \rangle \to \langle 23 \rangle + z \langle 13 \rangle$

$A \sim \frac{1}{z}$ as $z \to \infty$

$[1^-, 3^+\rangle$: $|3] \to |3] + z|1]$

$\langle 23 \rangle \to \langle 23 \rangle - z \langle 12 \rangle$

$\langle 34 \rangle \to \langle 34 \rangle + z \langle 14 \rangle$

$A \sim \frac{1}{z^2}$ $z \to \infty$

$[2^+ \, 3^+\rangle$

$$|3\rangle \to |3\rangle + z|2\rangle$$
$$\langle 23\rangle \to \langle 23\rangle$$
$$\langle 34\rangle \to \langle 34\rangle + z\langle 24\rangle$$
$$A(z) \sim \frac{1}{z}$$

$[-,+\rangle, \quad [+\,+\rangle \quad [-\,-\rangle$

legitimate $\frac{1}{z}, \frac{1}{z^2}$

$[+,-\rangle$ bad shift

$[2^+ \, 1^-\rangle$:

$$|1\rangle \to |1\rangle + z|2\rangle$$
$$\langle 14\rangle \to \langle 14\rangle + z\langle 24\rangle$$
$$\langle 12\rangle \to \langle 12\rangle$$
$$\langle n1\rangle \to \langle n1\rangle + z\langle n2\rangle$$
$$A(z) \sim z^3$$

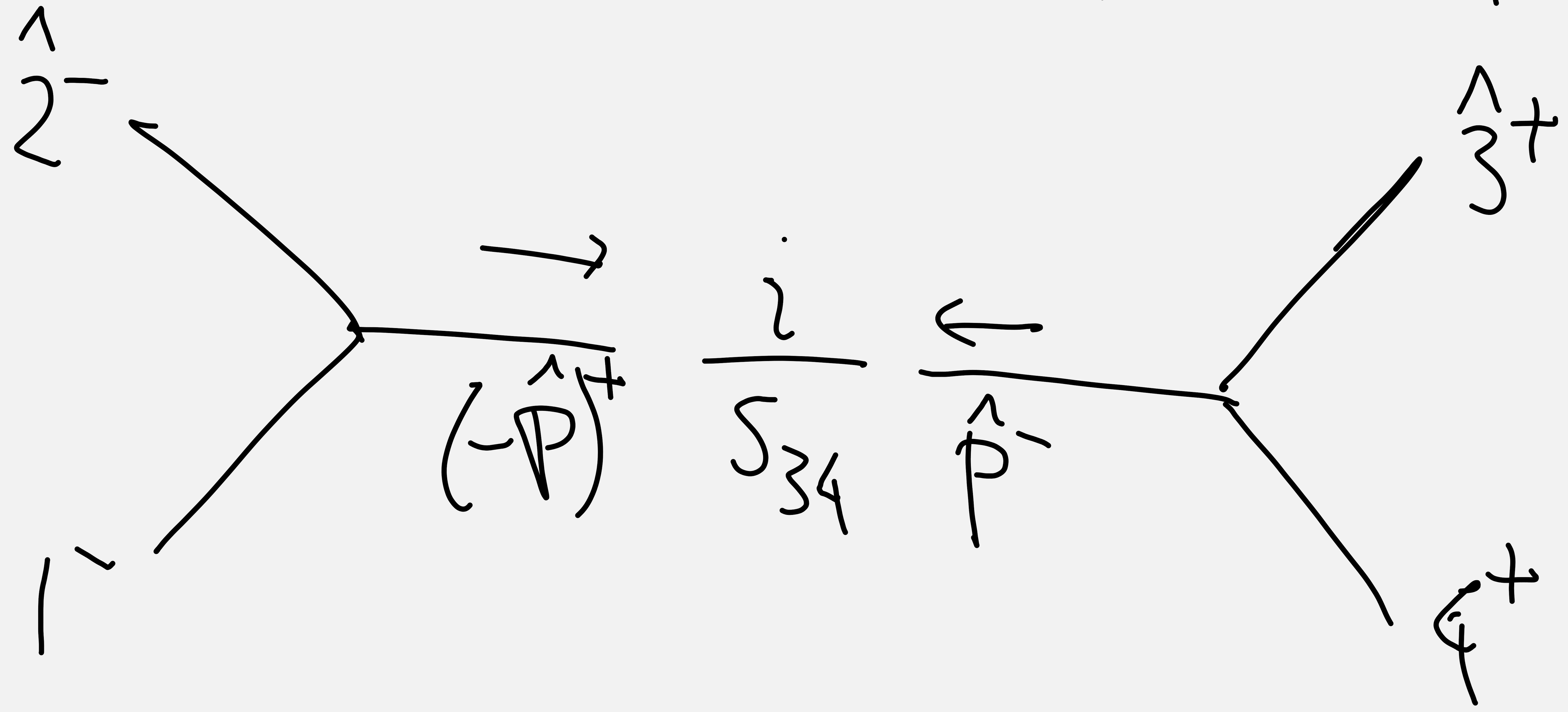
$$A_4(1^- 2^- 3^+ 4^+)$$

$$[2^- 3^+\rangle$$

$$|2] \to |2] - z|3]$$

$$|3\rangle \to |3\rangle + z|2\rangle$$

$$|2\rangle \to |2\rangle, \quad |3] \to |3]$$



$$i\,\frac{\langle 1\hat{2}\rangle^3}{\langle \hat{2}(-\hat{P})\rangle\langle(-\hat{P})1\rangle}\;\frac{i}{s_{34}}\;(-i)\,\frac{[\hat{3}4]^3}{[\hat{P}3][4\hat{P}]}$$

$$|-\hat{P}\rangle = i\,|\hat{P}\rangle$$

$$A_4 = -i\,\frac{\langle 12\rangle^3[34]^2}{\langle 43\rangle\langle 1|\hat{P}|3]\,\langle 2|\hat{P}|4]}$$

$$\hat{P}^\mu = P^\mu - \frac{z_{12}}{2}\langle 2|\mu|3]$$

$$\langle 1|\hat{P}|3] = \langle 1|P|3] - \frac{z_{12}}{2}\underbrace{\langle 1|\mu|3]\langle 2|\mu|3]}_{0}$$

$$= \langle 14\rangle[43]$$

$$A_4 = +i \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}$$

<u>Exercise</u> Prove the Parke-Taylor formula by induction

$$A_n(+ \cdots +) = 0$$

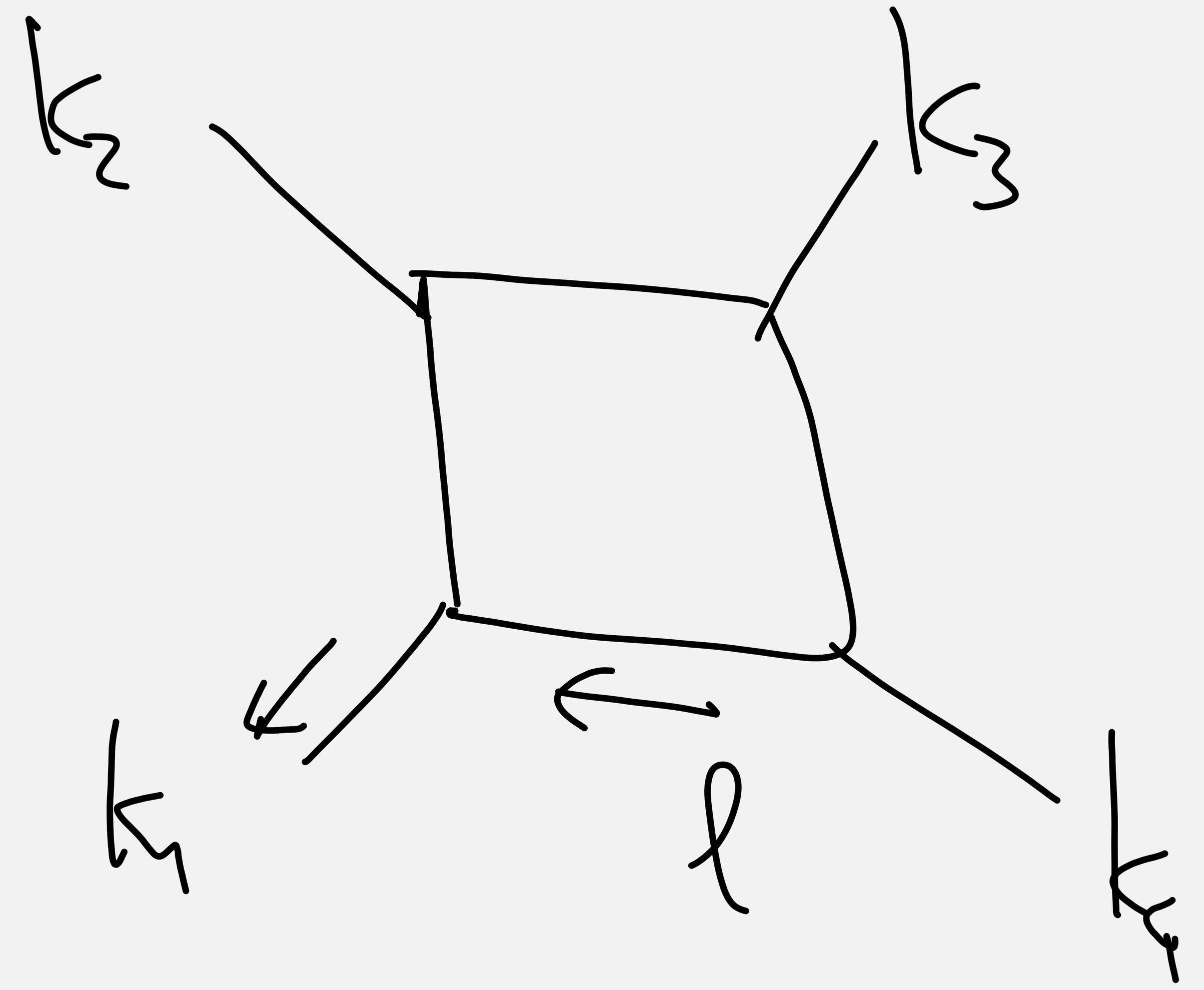
$$A_n(- + \cdots +) = 0$$

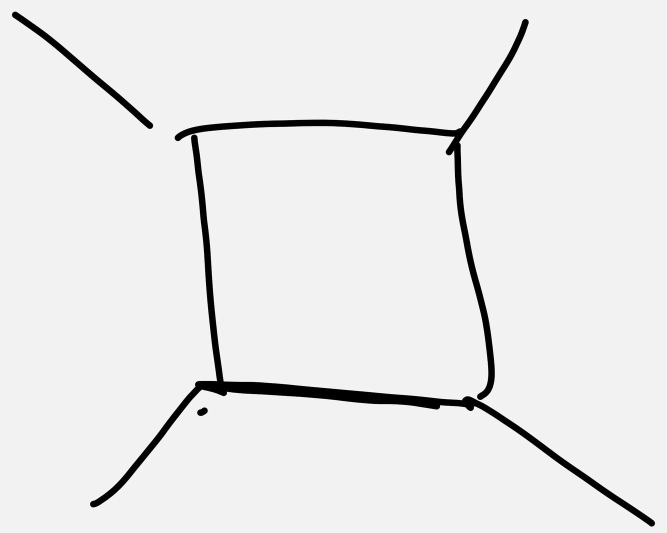
$$A_n^{\text{tree}}\left(\{k_i, h_i, a_i\}\right) = \sum_{\rho \in S_n / Z_n} \mathrm{Tr}\left(T^{a_{\rho(1)}} \cdots T^{a_{\rho(n)}}\right) A_n\left(\rho(1)^{h_{\rho(1)}} \cdots \rho(n)^{h_{\rho(n)}}\right)$$

$$A_n^{\text{1-loop}}\left(\quad \cdots \quad \right) = \sum_J n_J \sum_{c=1}^{\lfloor n/2 \rfloor + 1} \sum_{\rho \in S_n / S_{n;c}} \mathrm{Tr}\left(T^{a_{\rho(1)}} \cdots T^{a_{\rho(c-1)}}\right) \mathrm{Tr}\left(T^{a_{\rho(c)}} \cdots T^{a_{\rho(n)}}\right) A_{n;c}^{[J]}\left(\rho(1) \cdots \rho(n)\right)$$

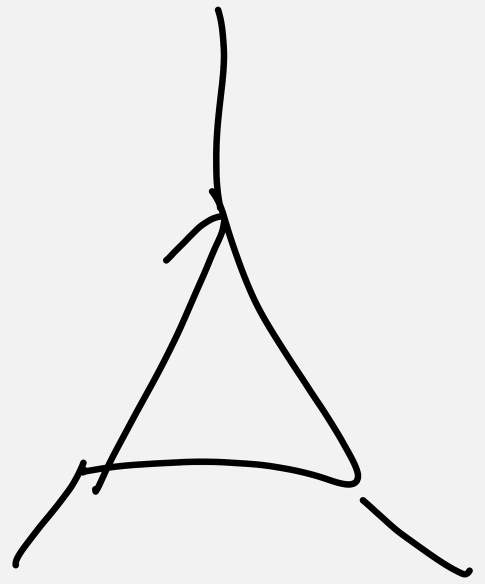
$$\int \frac{d^4\ell}{(2\pi)^4} \frac{1}{\left[\ell^2+i\varepsilon\right]\left[(\ell-k_1)^2+i\varepsilon\right]\left[(\ell-k_{12})^2+i\varepsilon\right]\left[(\ell+k_4)^2+i\varepsilon\right]}$$

$$4 \longrightarrow D = 4-2\epsilon$$

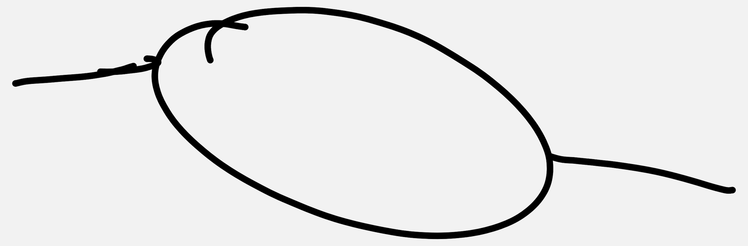




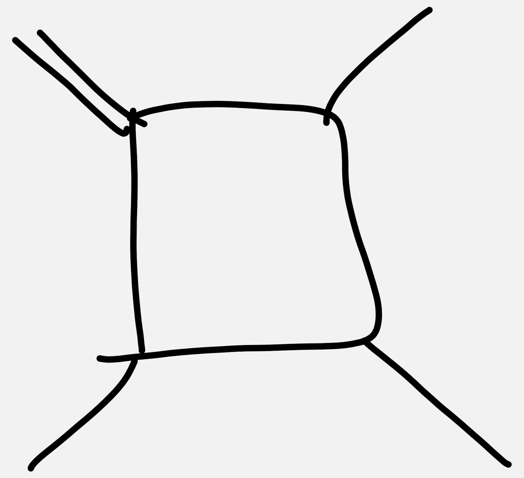
boxes



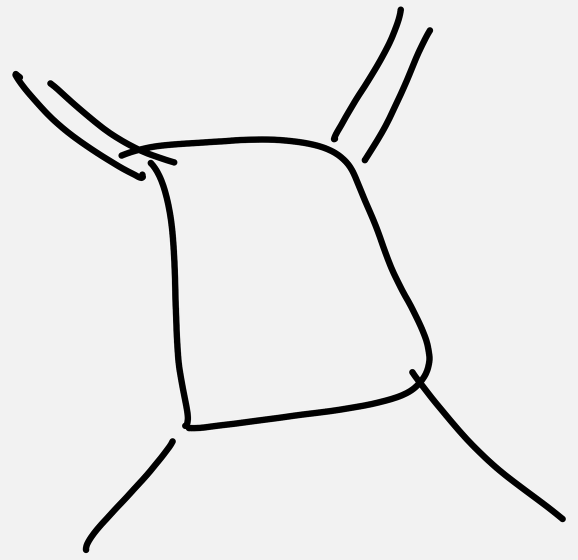
triangles



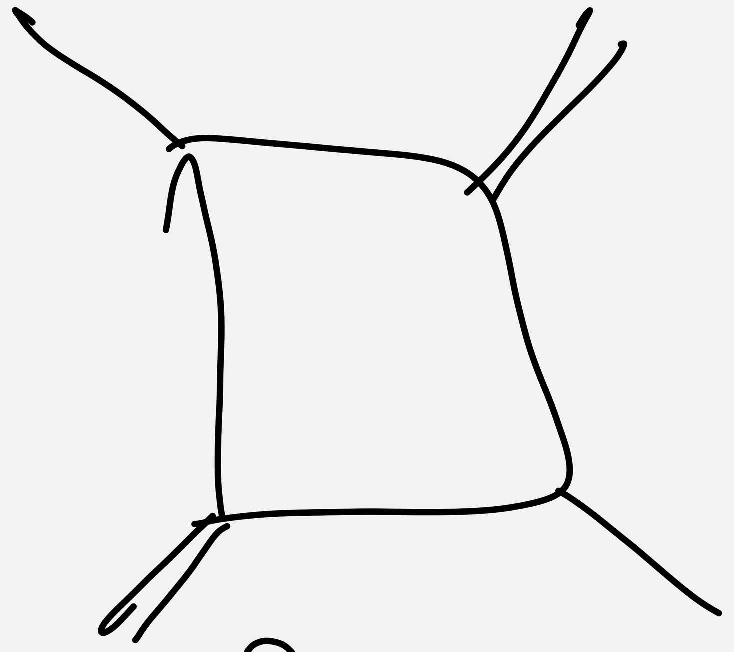
bubble



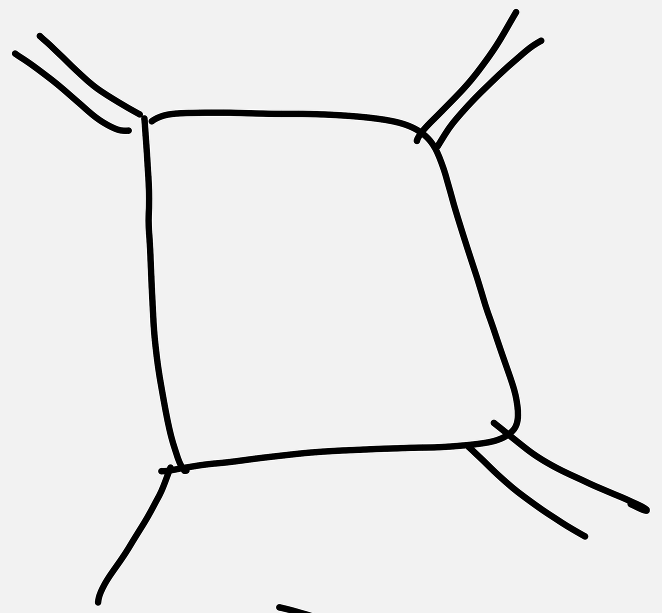
1m



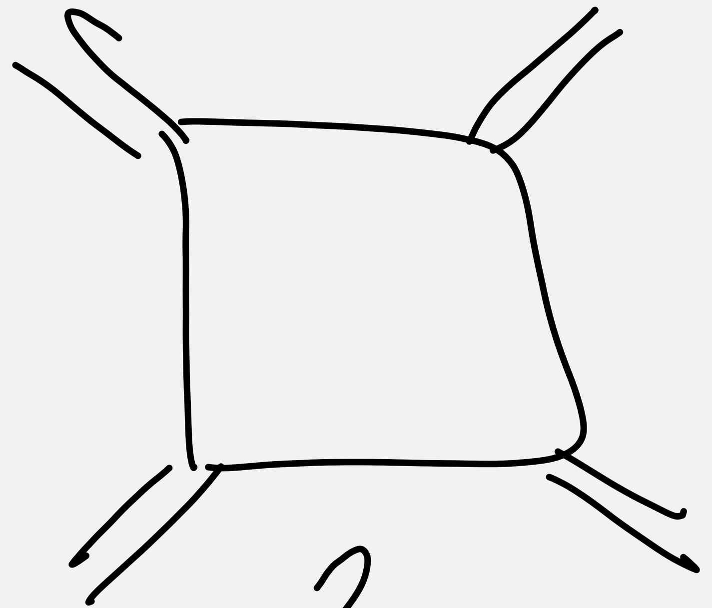
2mh



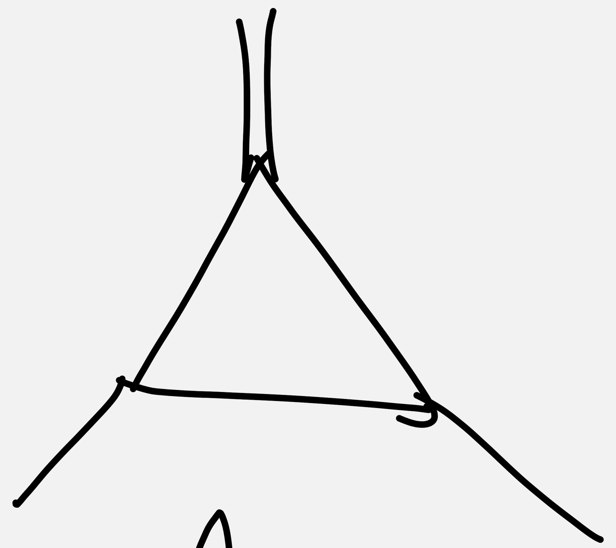
2me



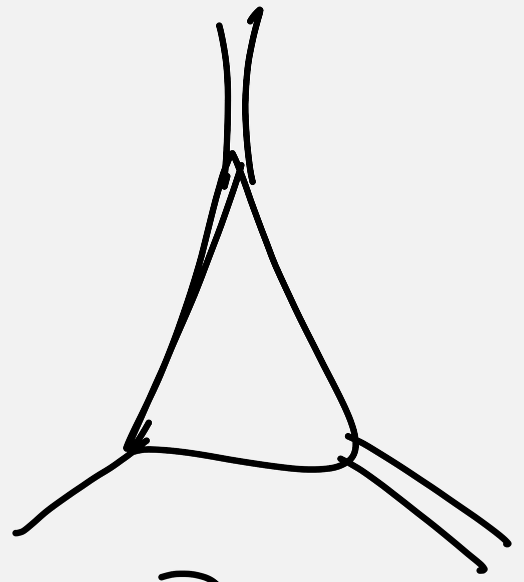
3m



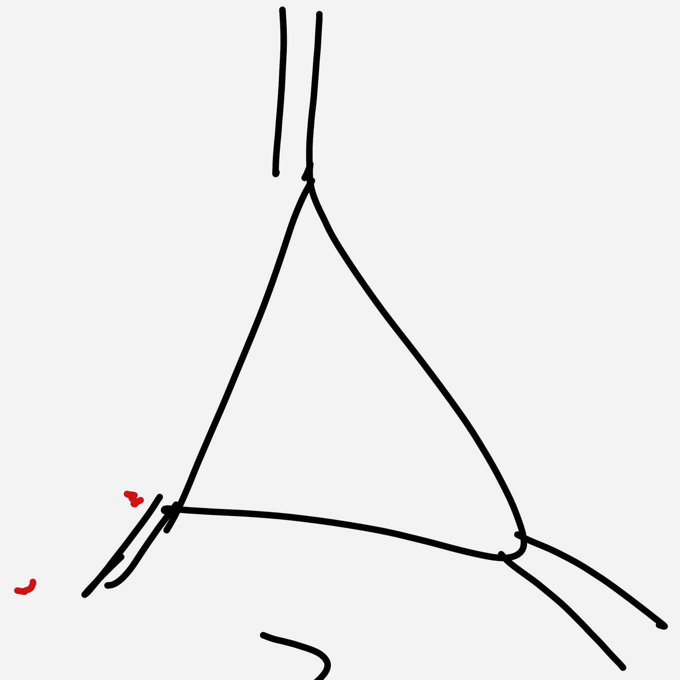
4m



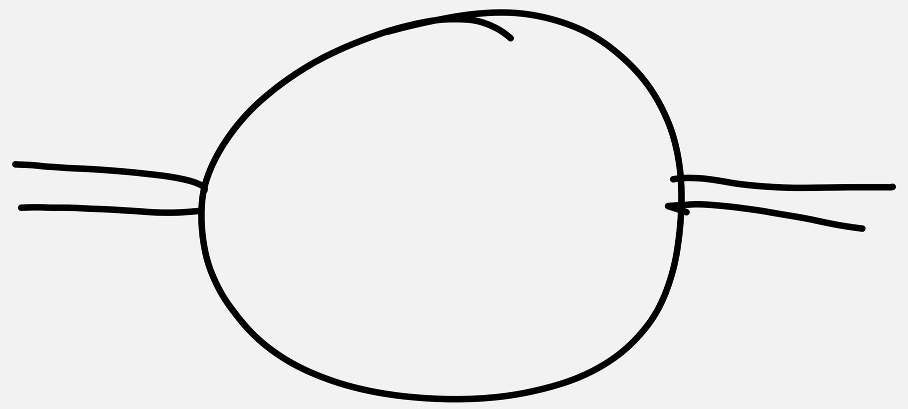
1m



2m

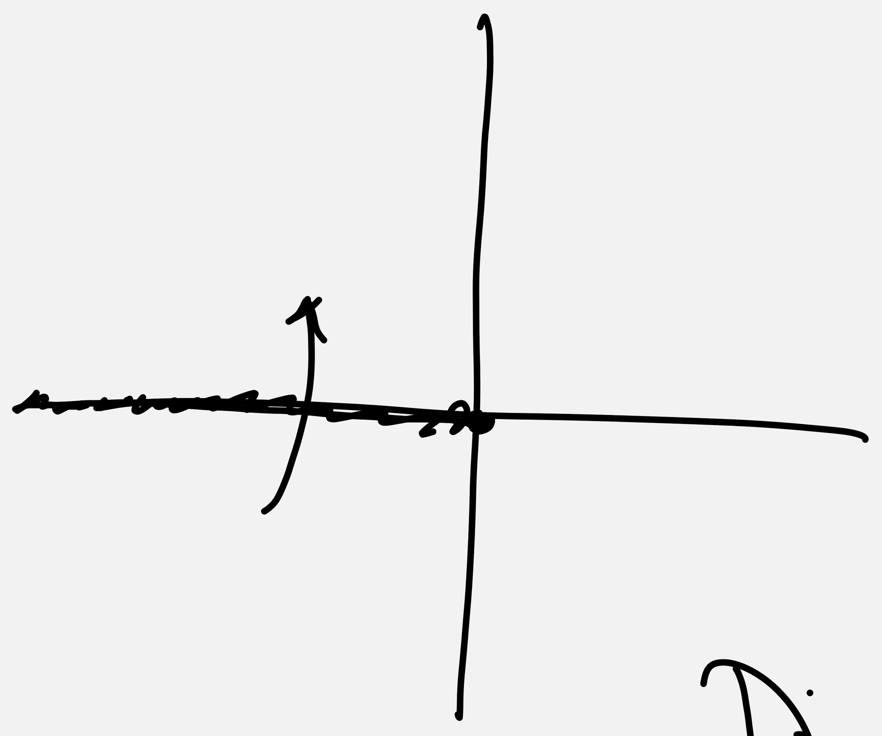


3m



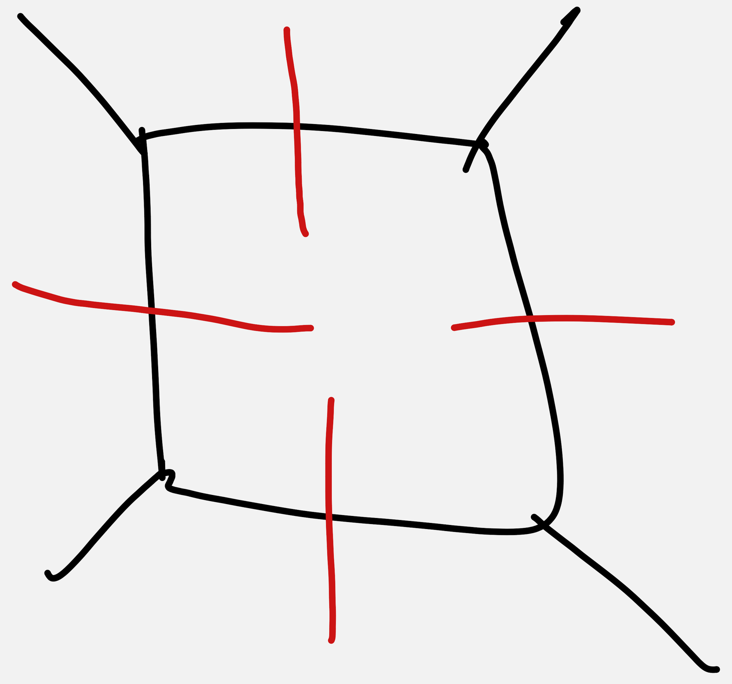
$$A_n = \sum_{j \in \text{basis}} c_j \, \text{Int}_j + \text{Rational} : c_0$$

rational fns of the spinor products



$\text{Disc}_{-x} \ln x = 2\pi i$

$$\longrightarrow \int d^4\ell \; \delta^{(+)}(\ell^2)\, \delta^{(+)}((\ell-k_1)^2)\, \delta^{(+)}((\ell-k_{12})^2)\, \delta^{(+)}((\ell+k_4)^2)$$



$c_j \sim A_A A_B A_C A_D$