Quantum Connections in Sweden - 11, Summer School,
Högberga Gård 11.-24 June 2023

Programmable Quantum Simulators
→ *Programmable* Quantum Sensors

Quantum Many-Body Physics → Quantum Metrology

Basic Quantum Science → Applied Quantum Technology

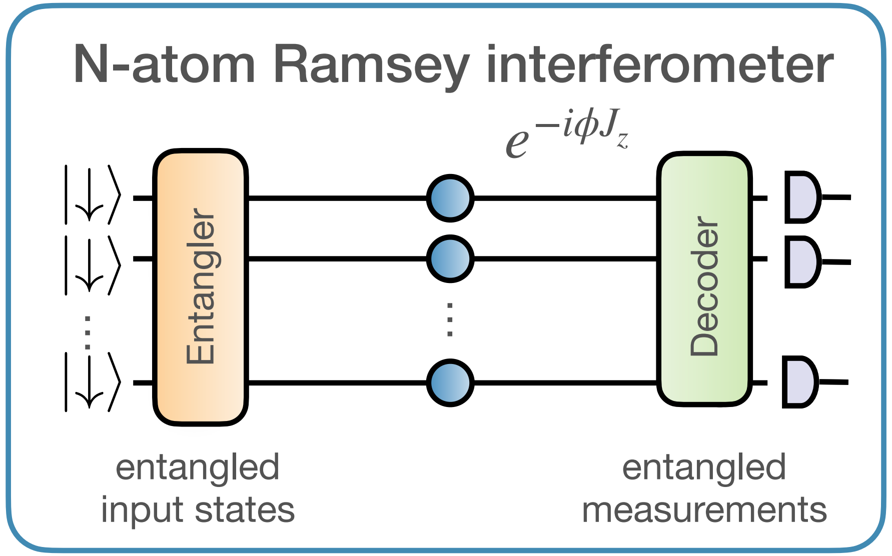
Peter Zoller

350 universität innsbruck
IQI
PASQuanS
FWF Der Wissenschaftsfonds.
AFOSR MURI (JILA)
Q-SEnSE
Simons Collaboration on Ultra-Quantum Matter
AQT

Lecture 3:

Optimal and *Variational* Quantum Metrology

- Entanglement enhanced quantum sensing
- At the interface of quantum information, precision measurement & quantum simulation



Topics of interest:

- identify *optimal* sensors* allowed by quantum physics
- implement via *variational* algorithm/quantum circuits
- *Bayesian* approach [vs. Fisher] (single shot measurement)

*optimality is task-specific: metrological cost-function

… with *Programmable* Quantum Sensors

Lecture 3:


R Kaubrügger D Vasilyev R van Bijnen C Marciniak T Feldker

PHYSICAL REVIEW X **11**, 041045 (2021)

theory

Featured in Physics

Quantum Variational Optimization of Ramsey Interferometry and Atomic Clocks

Raphael Kaubruegger[1,2,*] Denis V. Vasilyev[1,2,*] Marius Schulte[3] Klemens Hammerer[3] and Peter Zoller[1,2]

Optimal metrology with programmable quantum sensors

604 | Nature | Vol 603 | 24 March 2022

Christian D. Marciniak[1,5], Thomas Feldker[1,5], Ivan Pogorelov[1], Raphael Kaubruegger[2,3], Denis V. Vasilyev[2,3], Rick van Bijnen[2,3], Philipp Schindler[1], Peter Zoller[2,3], Rainer Blatt[1,2] & Thomas Monz[1,4]✉

theory + experiment

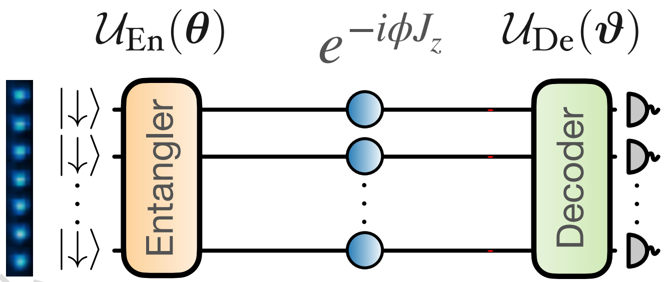
PRX QUANTUM **4**, 020333 (2023)

theory

Optimal and Variational Multiparameter Quantum Metrology and Vector-Field Sensing

Raphael Kaubruegger[1,2,*] Athreya Shankar[1,2,3] Denis V. Vasilyev[1,2] and Peter Zoller[1,2]

trapped ion experiment




C Marciniak et al
PRX Quantum 2021

Variational Principle for Optimal Quantum Controls in Quantum Metrology
J Yang, SP, Zekai Chen, AN Jordan, and A del Campo, PRL (2022)

Preparation of metrological states in dipolar-interacting spin systems
TX Zheng, A Li, J Rosen, S Zhou, M Koppenhöfer, Z Ma, FT Chong, AA Clerk, L Jiang & PC Maurer
npj Quantum Information (2022)

Lecture 3:

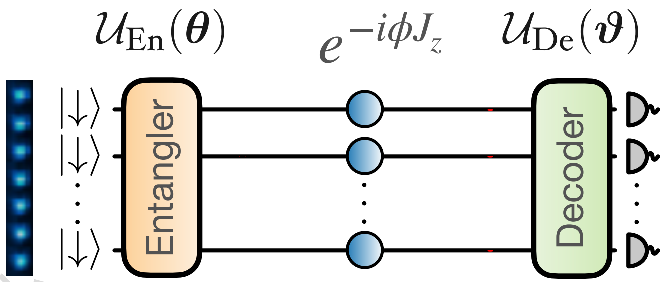
PHYSICAL REVIEW X **11,** 041045 (2021) — theory

Featured in Physics

Quantum Variational Optimization of Ramsey Interferometry and Atomic Clocks

Raphael Kaubruegger,[1,2,*] Denis V. Vasilyev,[1,2,*] Marius Schulte,[3] Klemens Hammerer,[3] and Peter Zoller[1,2]

trapped ion experiment



Optimal metrology with programmable quantum sensors

604 | Nature | Vol 603 | 24 March 2022

theory + experiment

Christian D. Marciniak[1,5], Thomas Feldker[1,5], Ivan Pogorelov[1], Raphael Kaubruegger[2,3], Denis V. Vasilyev[2,3], Rick van Bijnen[2,3], Philipp Schindler[1], Peter Zoller[2,3], Rainer Blatt[1,2] & Thomas Monz[1,4]✉

PRX QUANTUM **4,** 020333 (2023) — theory

Optimal and Variational Multiparameter Quantum Metrology and Vector-Field Sensing

Raphael Kaubruegger,[1,2,*] Athreya Shankar,[1,2,3] Denis V. Vasilyev,[1,2] and Peter Zoller[1,2]

Background reading/reviews on entanglement enhanced quantum metrology

L. Pezzè, A. Smerzi, M. K. Oberthaler, R. Schmied, and P. Treutlein, *Quantum Metrology with Nonclassical States*, RMP (2018).
A. D. Ludlow, M. M. Boyd, J. Ye, E. Peik, and P. O. Schmidt, *Optical Atomic Clocks*, RMP (2015).
R. Demkowicz-Dobrzański, M. Jarzyna, and J. Kołodyński, *Quantum Limits in Optical Interferometry*, in *Progress in Optics (2015)*.
R. Demkowicz-Dobrzański, W. Górecki, and M. Guţă, Multi-Parameter Estimation beyond Quantum Fisher Information, JPA (2020).

AQTION AQT

from Lecture 1:

Quantum Many-Body Physics and Quantum Simulation

- analog quantum simulation
- digital quantum simulation
- → variational quantum simulation

… on Atomic NISQ Devices

Self-Verifying Variational Quantum Simulations …
C Kokail, et al., Nature 569, 355 (2019)

Variational Approach to …
Quantum Many-Body Physics

target Hamiltonian (e.g. lattice model)

$$\hat{H}_T = \sum_{n\alpha} h_n^\alpha \hat{\sigma}_n^\alpha + \sum_{n\ell\alpha\beta} h_{n\ell}^{\alpha\beta} \hat{\sigma}_n^\alpha \hat{\sigma}_\ell^\beta + \dots$$

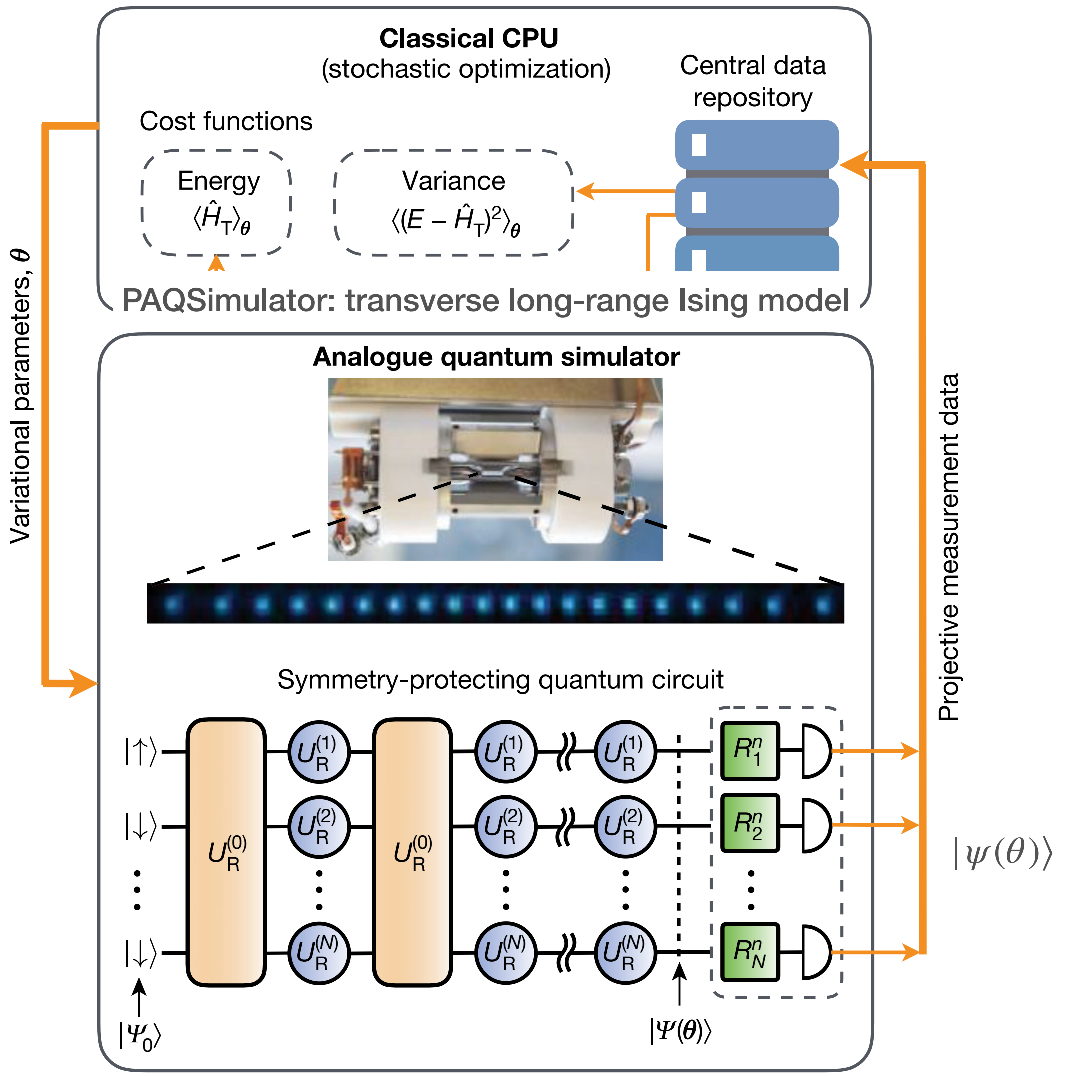
Cost function: Variational Quantum Eigensolver

$$C(\theta) = \langle \psi(\theta) | \hat{H}_T | \psi(\theta) \rangle \rightarrow \text{min}$$

lowest energy
~ ground state

optimize on classical machine
variational parameters

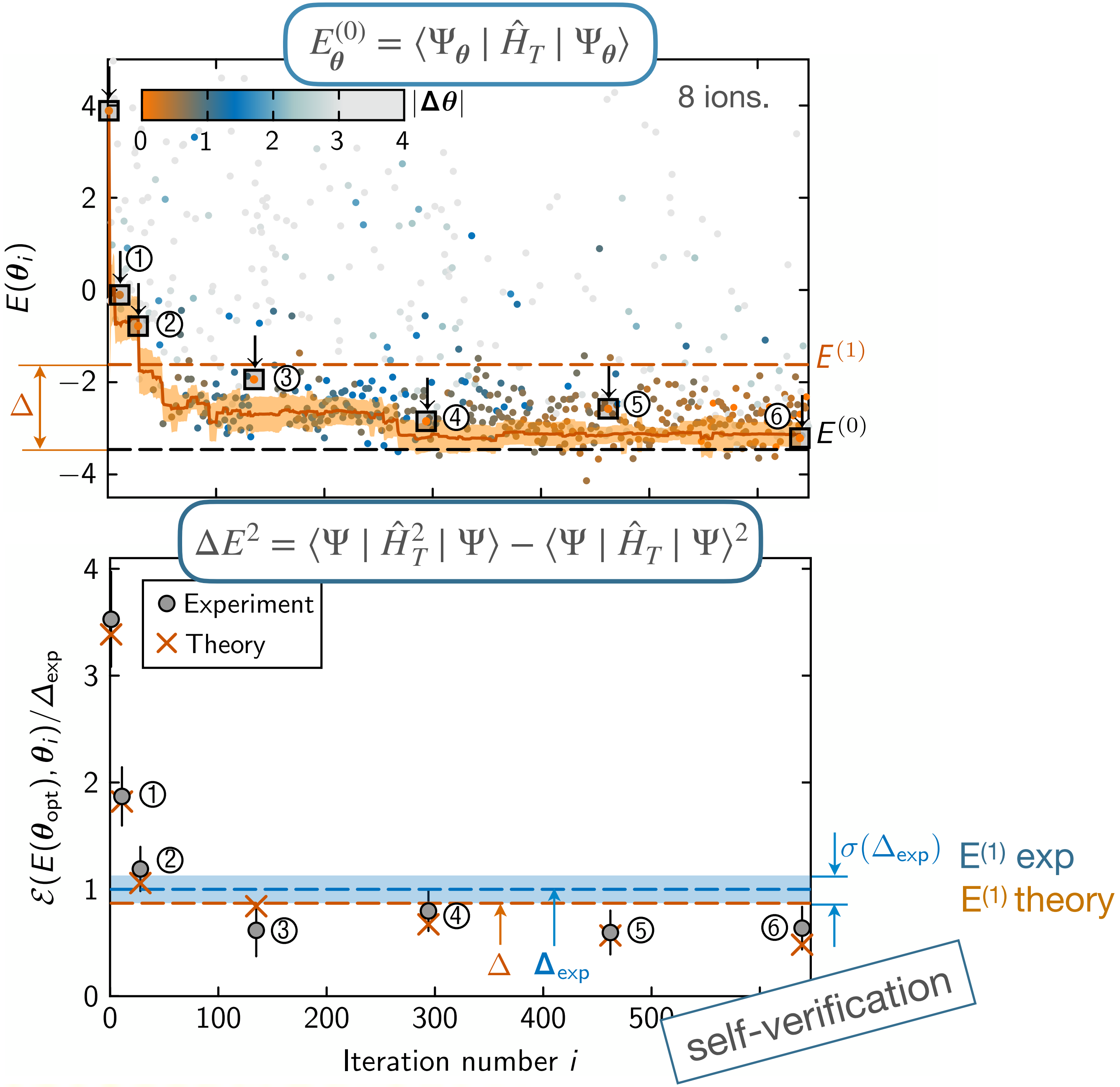
evaluate on quantum machine …
efficiently



… on Atomic NISQ Devices

Energy Optimization Trajectory for Ground State (VQE)

C Kokail   C Maier   R. van Bijnen



N=20 ions

Lattice Schwinger Model (1D QED)

$$H_S = J \sum c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

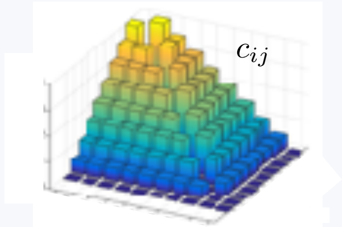
long - range interaction

$$+w \sum \left(\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^- \right)$$

particle - antiparticle creation/annihilation

$$+m \sum c_i \hat{\sigma}_i^z + J \sum \tilde{c}_i \hat{\sigma}_i^z$$

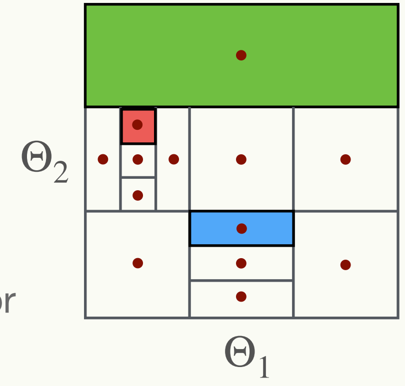
effective particle masses



DIviding RECTangles (DIRECT)

global optimization
in noisy landscape

- 15 parameters
- circuit depth = 6
- budget: 10^5 calls to simulator



C Kokail, C Maier, R van Bijnen, T Brydges, MK Joshi, P Jurcevic, CA Muschik, P Silvi, R Blatt, CF Roos & P.Z., Nature (2019)

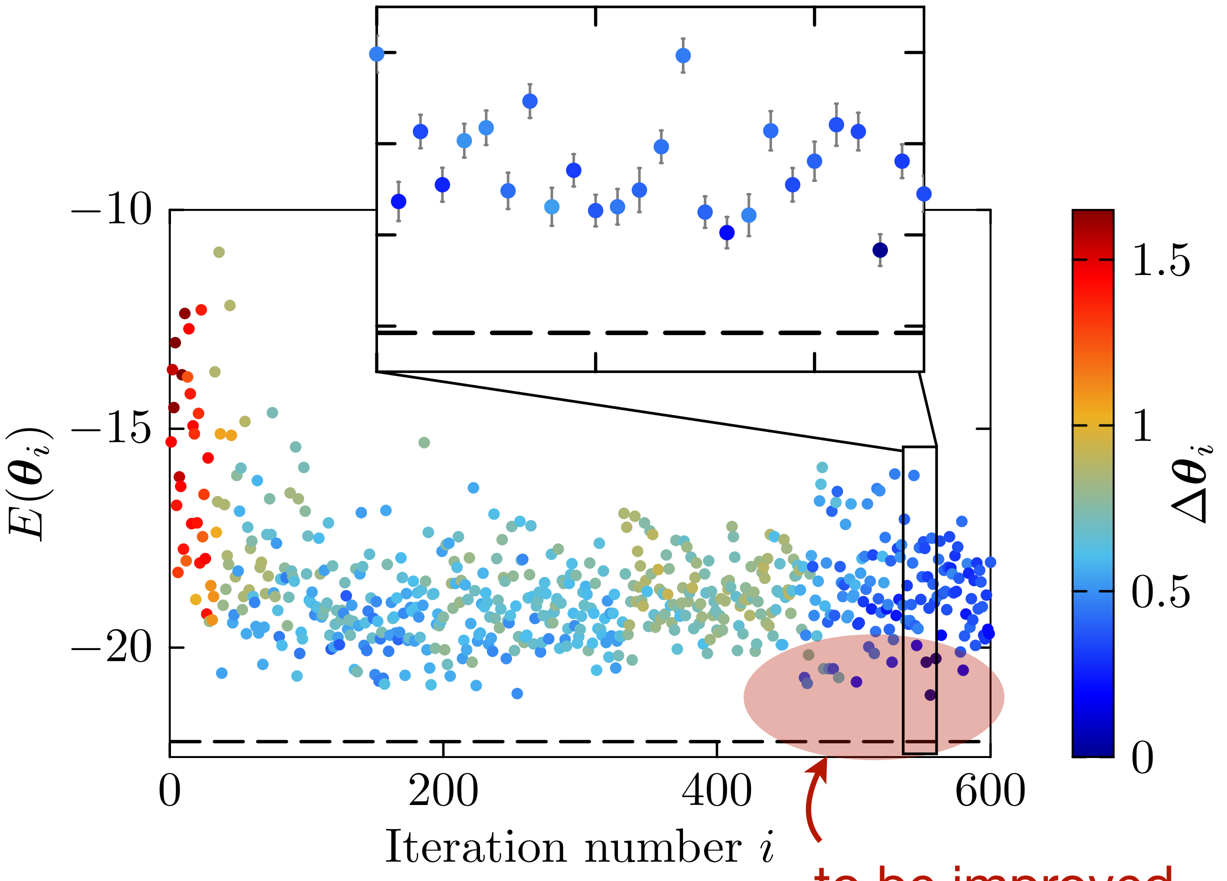
q-technology 2018

Experimental Energy Optimization Trajectory for Ground State (VQE)

Theory: C Kokail, R van Bijnen et al.
Experiment: M Joshi et al.

arXiv:2306.00057

N = 51 ions



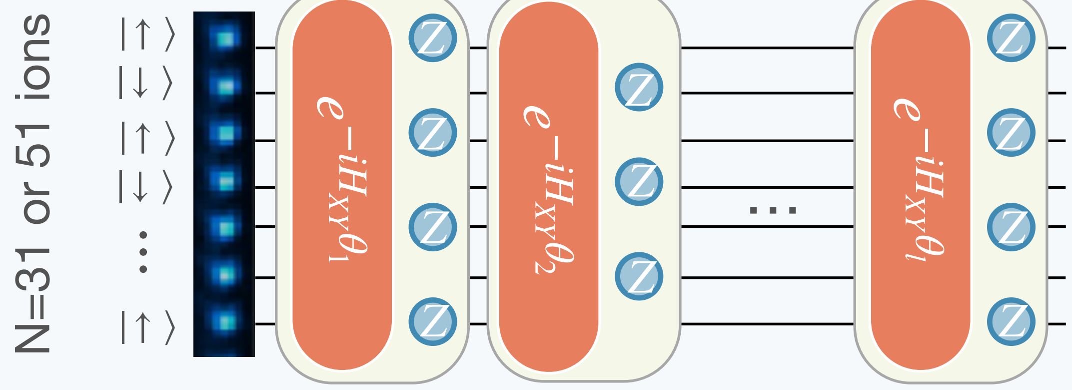
to be improved

~ low temperature state $T \sim$ few J

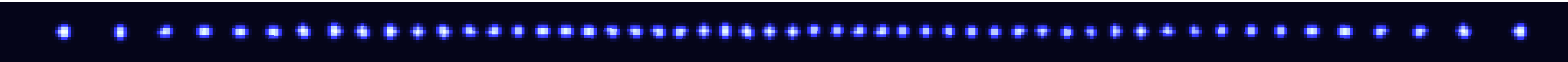
Heisenberg Model (spin-½)

$$\hat{H} = J \sum_{i=1}^{N-1} \left(\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y \right) + \Delta \sum_{i=1}^{N-1} \hat{S}_i^z \hat{S}_{i+1}^z$$

VQE Circuit with Trapped Ion Resources



5 layers and 10 parameters



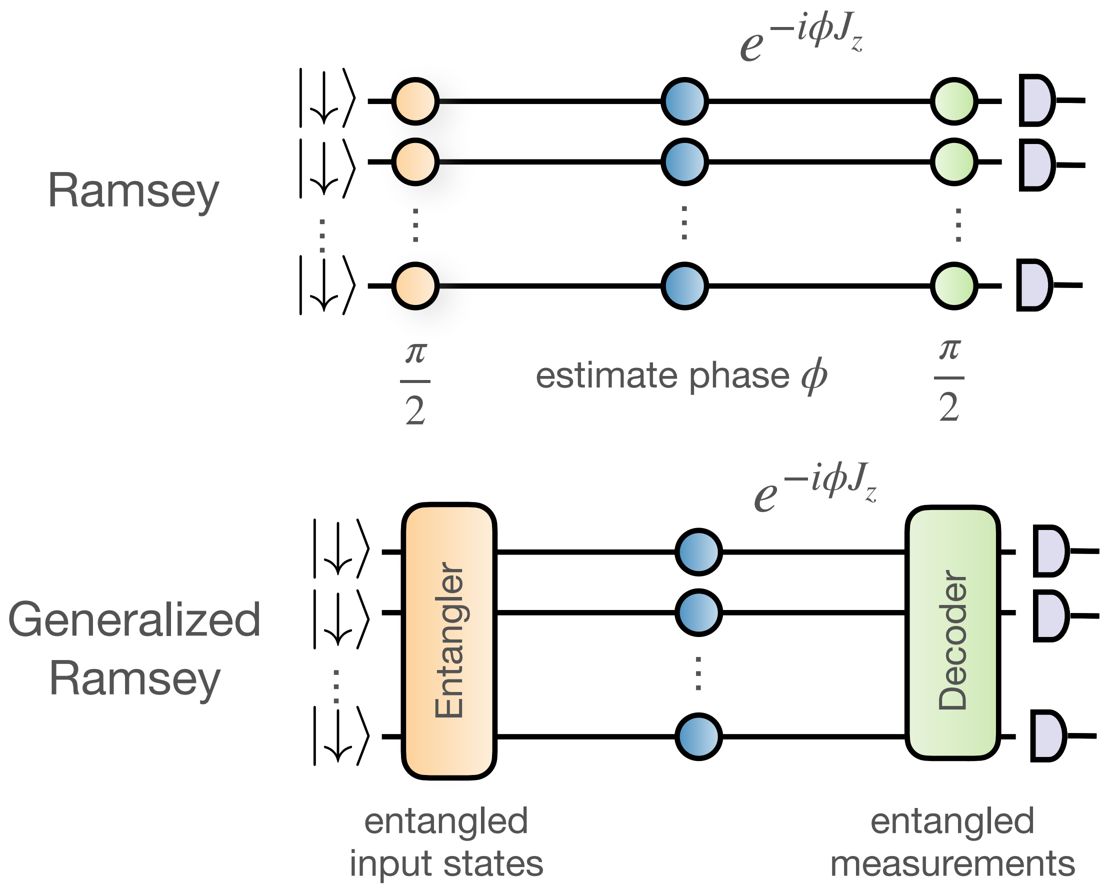
Variational Approach to …

Optimal Quantum Sensing

… on Atomic NISQ Devices

Variational Approach to …

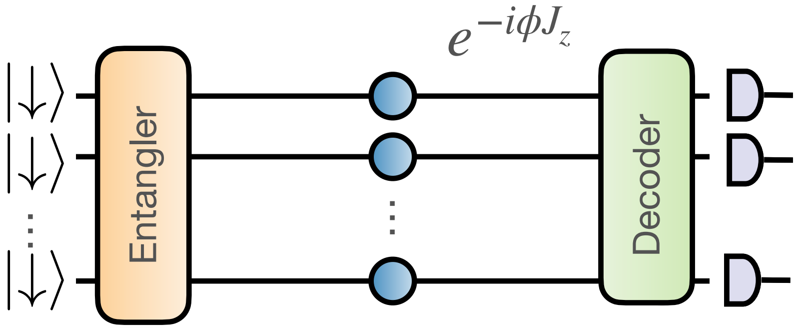
Optimal Ramsey Interferometry



Variational approximations for entangling/decoding quantum circuits, implemented on `programmable quantum sensor' (many-body dynamics)

What is the cost function?

… on Atomic NISQ Devices



Optimal Ramsey Interferometry

- **What is *optimal* … ?**

  Optimality is defined via a metrological cost function

$$\mathcal{C}_{\text{metrological}} \rightarrow \text{max/min}$$

wishlist:

✓best signal to noise ratio for N atoms

✓*finite* dynamic range $\delta\phi$ GHZ ~~states~~!

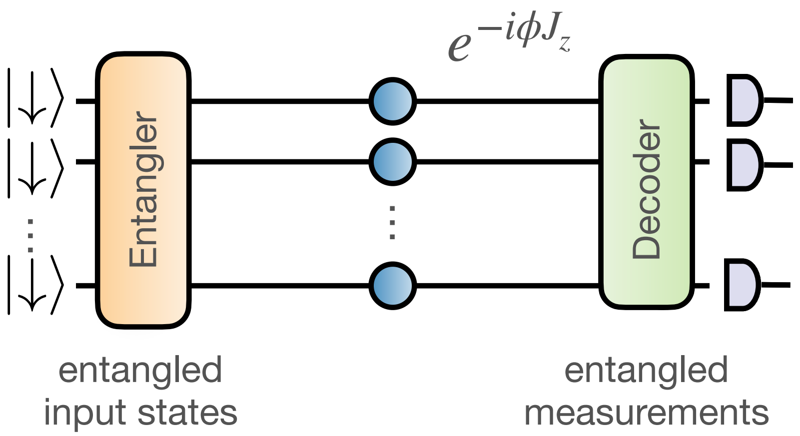
✓…

atomic clocks

L. Pezzè et al.,, Rev. Mod. Phys. **90**, 035005 (2018). R. Demkowicz-Dobrzański et al., in *Progress in Optics Vol 60* 2015) pp. 345; K. Macieszczak et al., New J. Phys. **16**, 113002 (2014)

Variational Approach to …

Optimal Ramsey Interferometry



- **What is *optimal* … ?**

  Optimality is defined via a metrological cost function

$$\mathcal{C}_{\text{metrological}} \rightarrow \max/\min$$ (to be optimized in a variational algorithm)

- **How to *implement* the optimal Ramsey interferometer?**

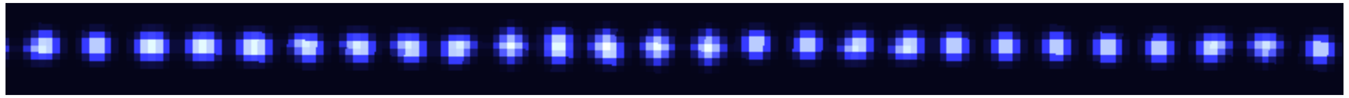
  Below: variational quantum circuits for entangler/decoder built from available q-resources

  … with shallow-depth circuits (!?)

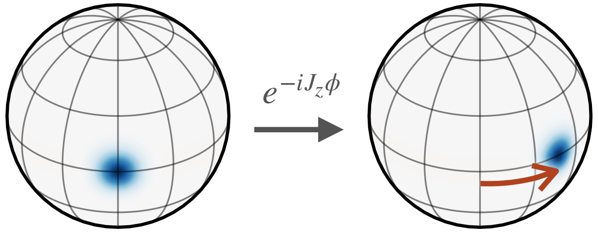
  … solve a complex quantum many-body problem

… on Atomic NISQ Devices

Ramsey Interferometry

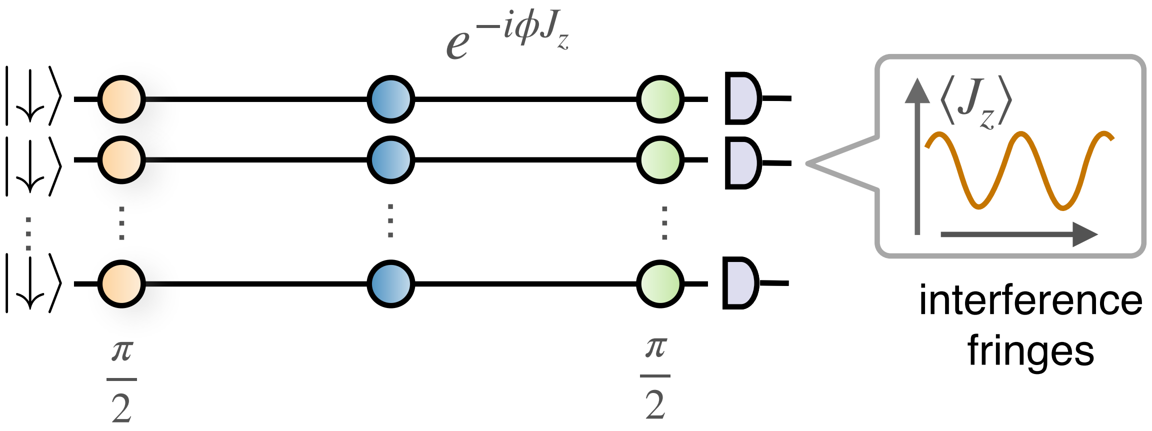


quantum sensor



coherent spin state
on Bloch sphere

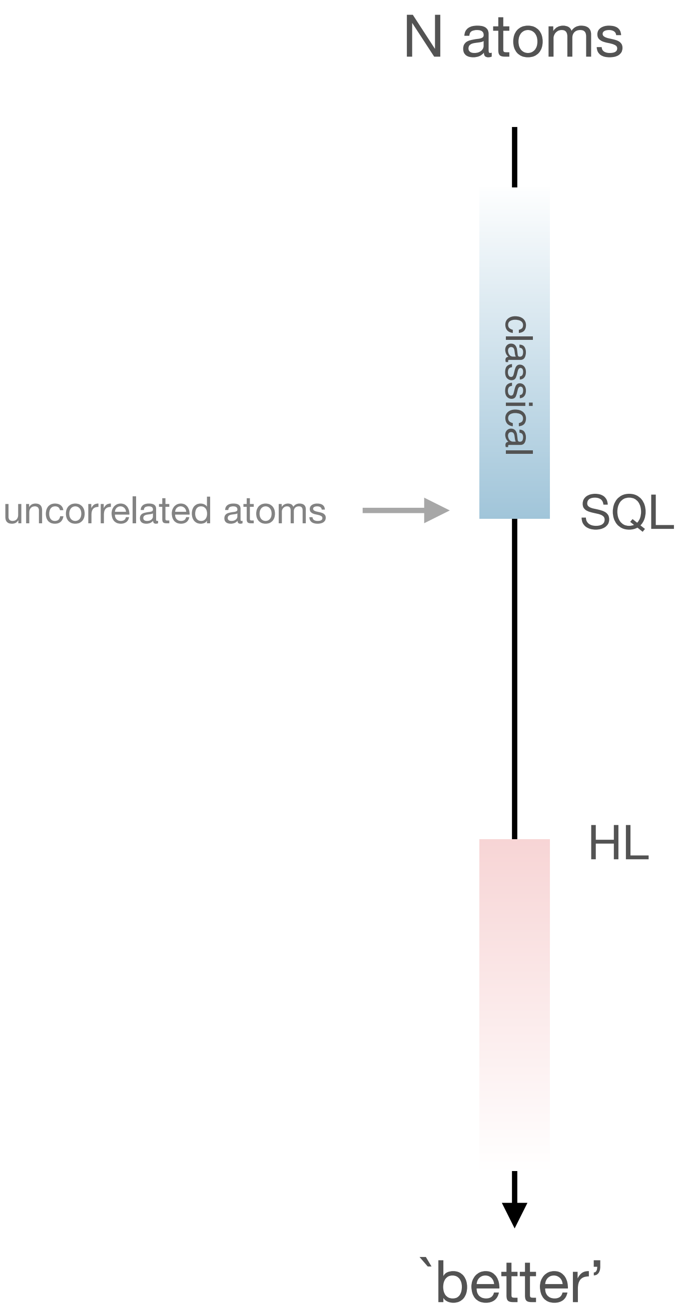
Ramsey interferometry



interference
fringes

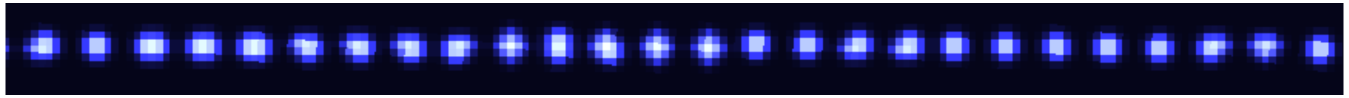
application: atomic clocks

here: uncorrelated atom, Standard Quantum Limit (SQL)

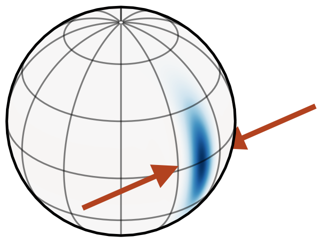


Ramsey Interferometry

S/N ratio for N atoms

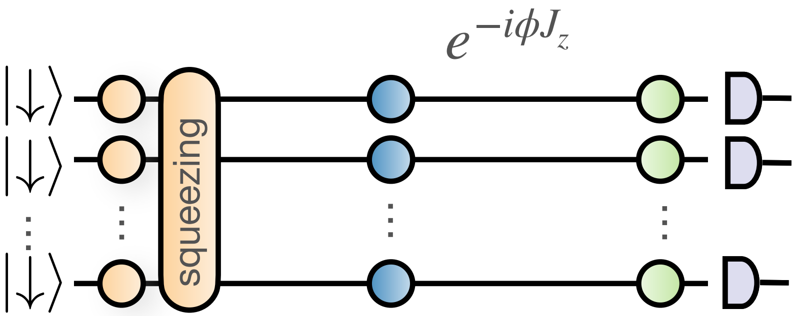


quantum sensor



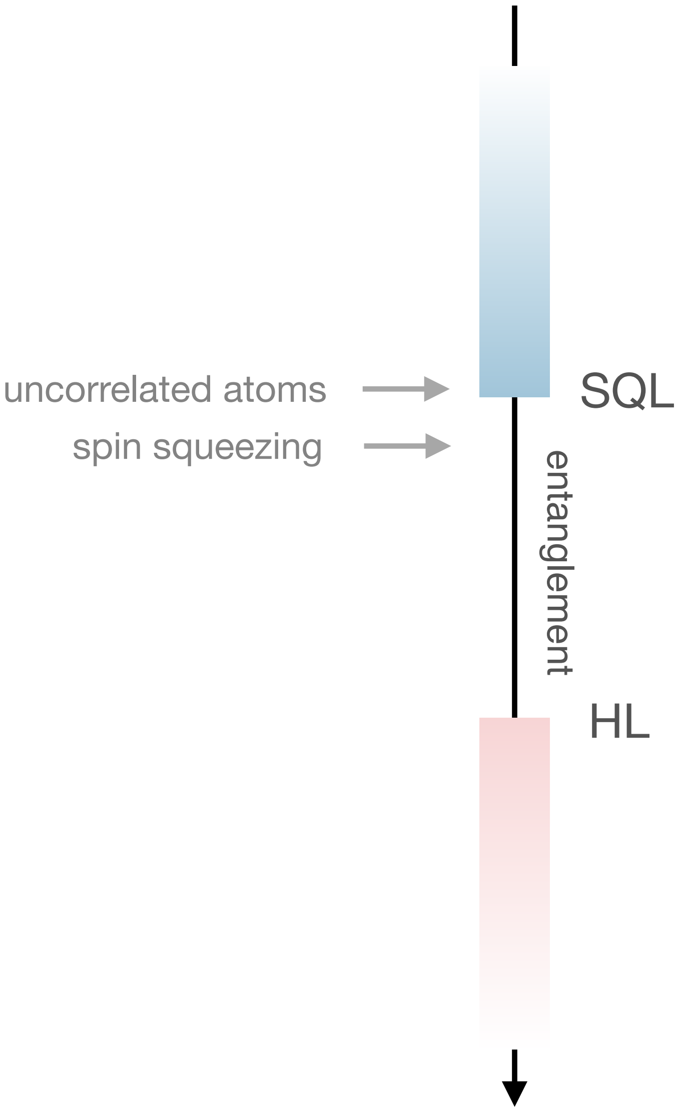
spin squeezed state
on Bloch sphere

Ramsey interferometry



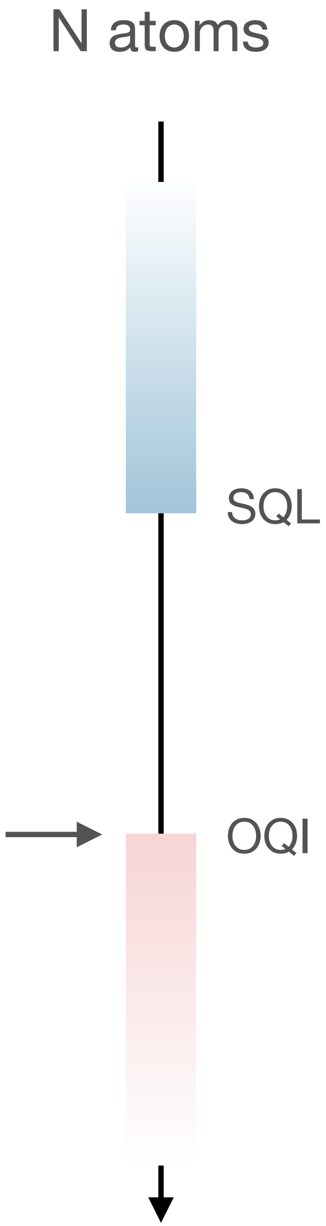
$T_z = e^{-i\chi J_z^2}$ One-Axis Twisting as native q-gate for ions

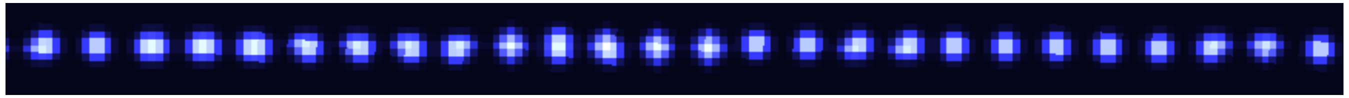
infinite range interaction



\`better’

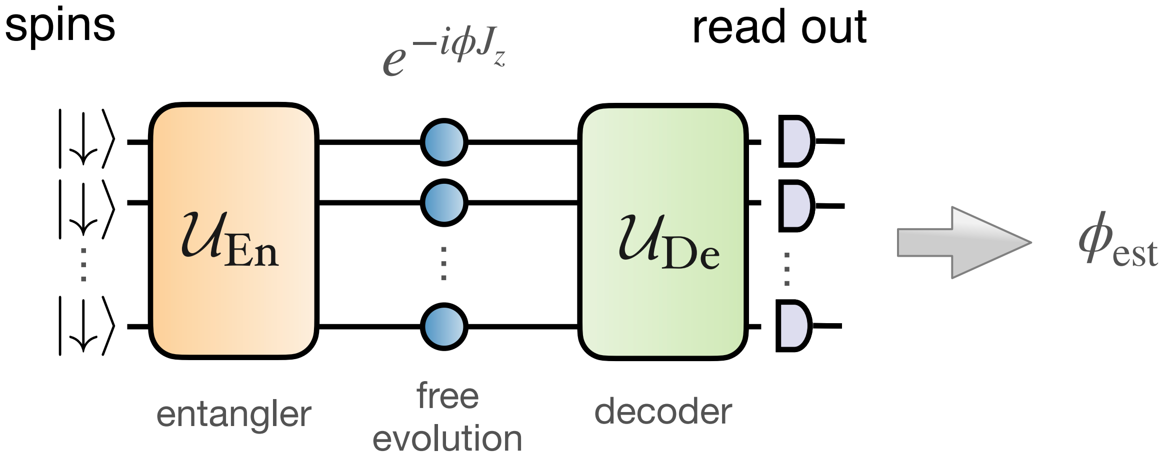
Optimal Ramsey Interferometry





programmable **quantum sensor**

Generalized Ramsey interferometer



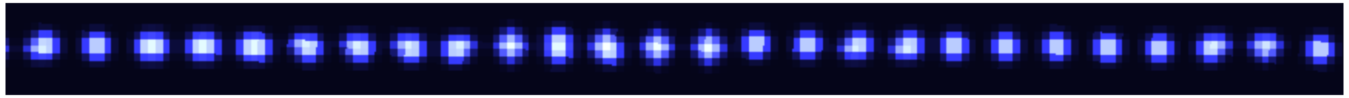
Optimal quantum interferometer (OQI)?

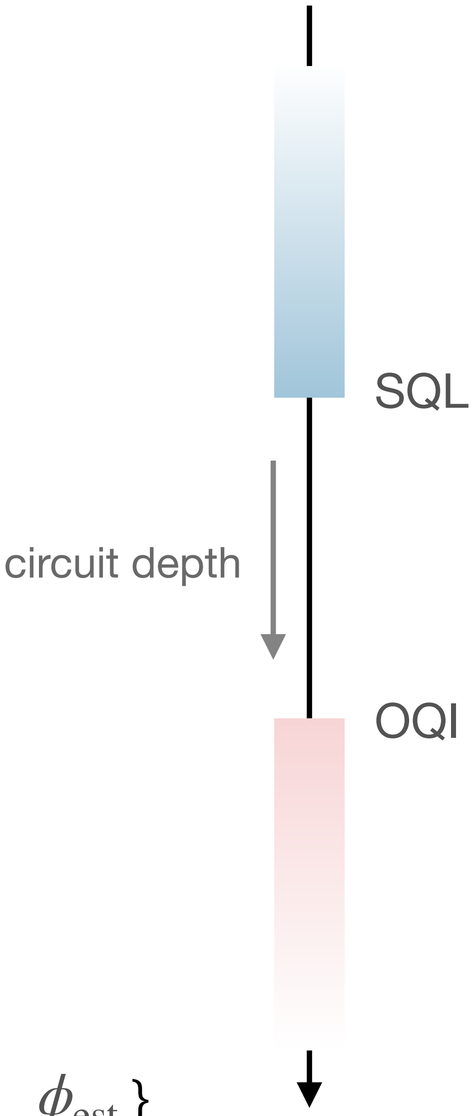
cost function: $\mathscr{C}_{\mathrm{metrological}} \rightarrow \mathrm{opt}$ over all possible $\{\mathscr{U}_{\mathrm{En}}, \mathscr{U}_{\mathrm{De}}, \phi_{\mathrm{est}}\}$

input states, measurements, estimators

L. Pezzè et al.,, Rev. Mod. Phys. **90**, 035005 (2018). R. Demkowicz-Dobrzański et al., in *Progress in Optics Vol 60* 2015) pp. 345; K. Macieszczak et al., New J. Phys. **16**, 113002 (2014)

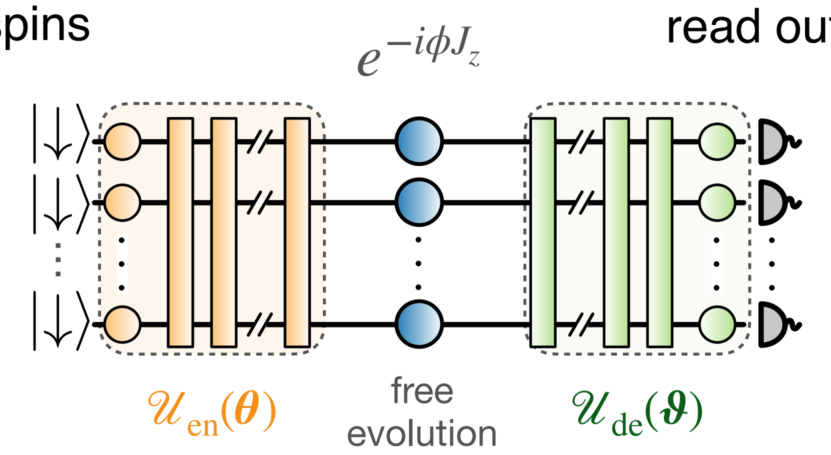
Optimal Ramsey Interferometry - Variational Approach





programmable **quantum sensor**

Generalized Ramsey interferometer

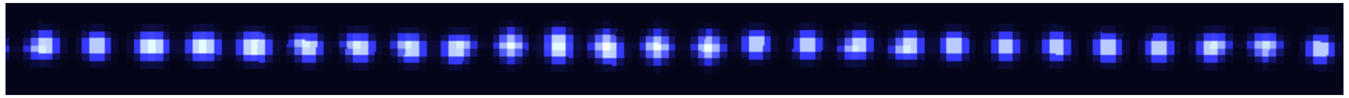


Variational ansatz for Entangler and Decoder

cost function: $\mathscr{C}_{\mathrm{metrological}}(\boldsymbol{\theta}, \boldsymbol{\vartheta}) \rightarrow \max/\min$ over all possible $\{\, \mathscr{U}_{\mathrm{en}}(\boldsymbol{\theta}) \,,\ \mathscr{U}_{\mathrm{de}}(\boldsymbol{\vartheta}) \,,\ \phi_{\mathrm{est}} \,\}$

variational parameters to be optimized in theory, or `on-device' in quantum feedback loop (i.e., in presence of imperfections and decoherence)

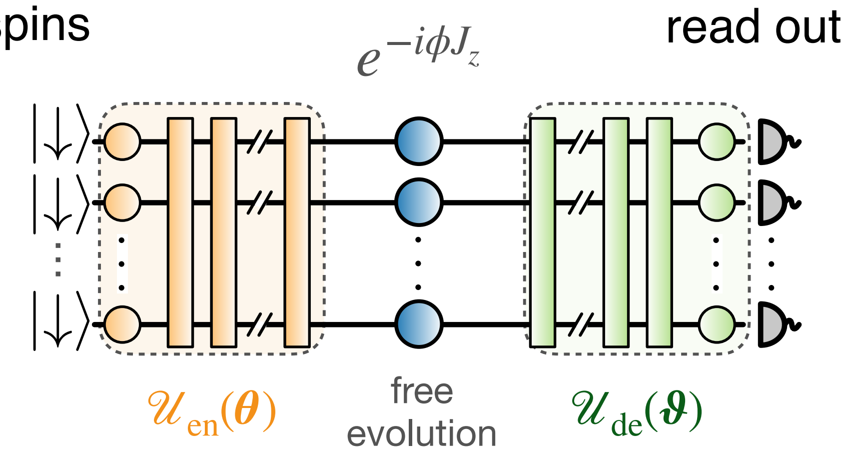
Quantum Hardware 1: Trapped Ion Quantum Computer



Innsbruck N=26 ion quantum computer

programmable quantum sensor

Generalized Ramsey interferometer



native resources on quantum computer

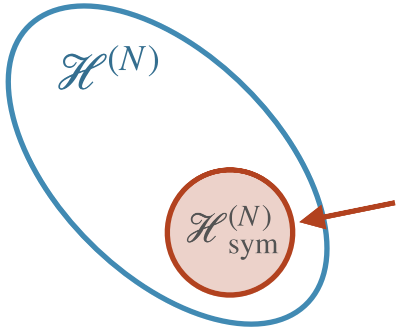
- global rotations
- one-axis-twisting

MS gate: $e^{-i\chi J_z^2}$

infinite range interaction

Choosing, and implementing variational circuits

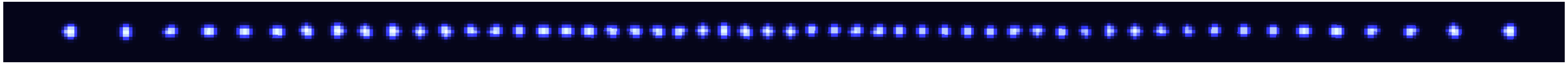
✓ variational circuits from quantum resources on given hardware platform



dynamics restricted to symmetric subspace $\dim = N + 1$

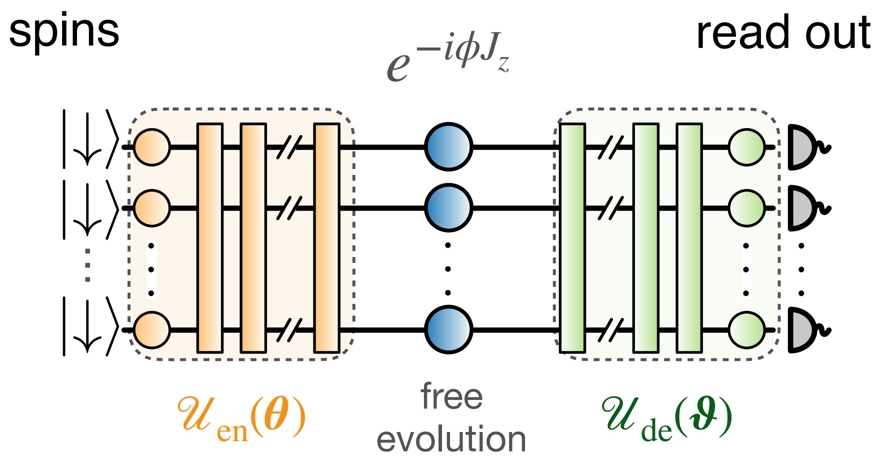
Quantum Hardware 2: Programmable Analog Q-Simulator

Innsbruck N=51 ion PAQS



programmable quantum sensor

Generalized Ramsey interferometer



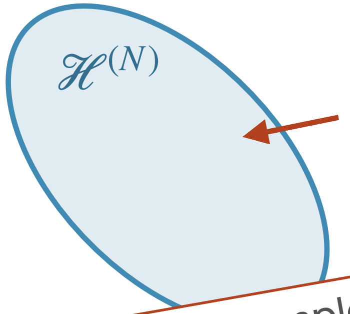
native resources on quantum simulator

- global rotations
- transverse Ising

$$e^{-iH_{\rm Ising}t}$$

finite range interaction

scalable



dynamics explores potentially all Hilbert space $\dim = 2^N$

many-body complexity
('relevant' quantum advantage)

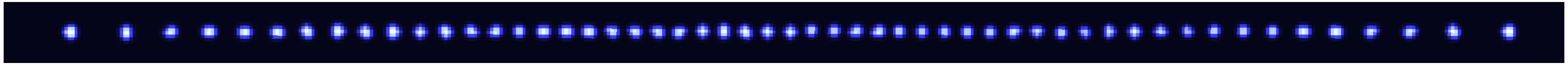
Choosing, and implementing variational circuits

✓ variational circuits from quantum resources on given hardware platform

Other Atomic Platforms: Quantum Simulator Resources

Trapped Ions

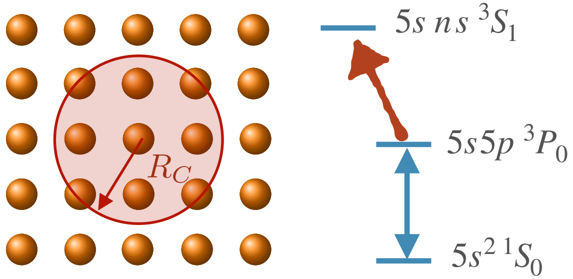
51 ion simulator @ IQOQI



entangling resource: $\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + B \sum_i \hat{\sigma}_i^z$ $\quad J_{ij} \sim \frac{1}{|i-j|^\alpha}$ $\quad \alpha = 0 \ldots 3$

finite range interaction

Rydberg Tweezer Arrays

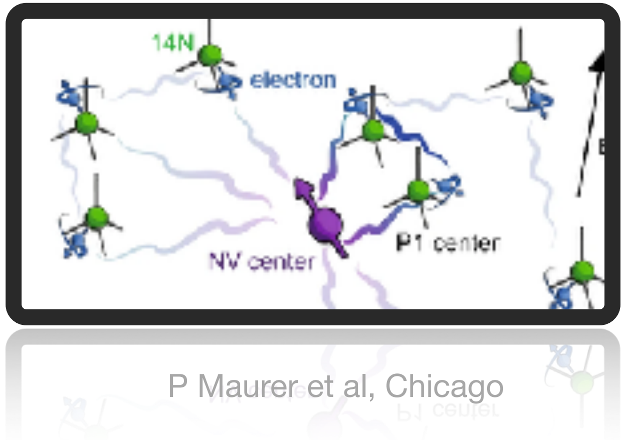


$$\hat{H} = \sum_i \frac{1}{2} \Omega_i \hat{\sigma}_x^i - \sum_i \Delta_i \hat{n}_i + \sum_{i<j} V_{ij} \hat{n}_i \hat{n}_j$$

finite range

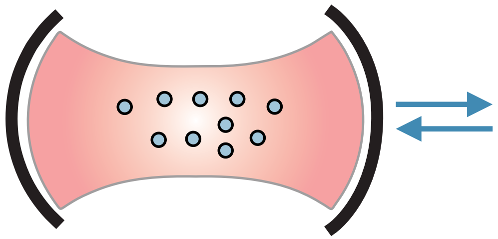
Tweezer clock: JILA, Caltech, …

NV Centers & Dipolar Interactions



P Maurer et al, Chicago

Cavity QED



$\hat{H} = \chi \hat{J}_z^2$ one-axis twisting

infinite range

MIT, JILA, Copenhagen, Stanford, …

Quantum Metrology & Quantum Parameter Estimation

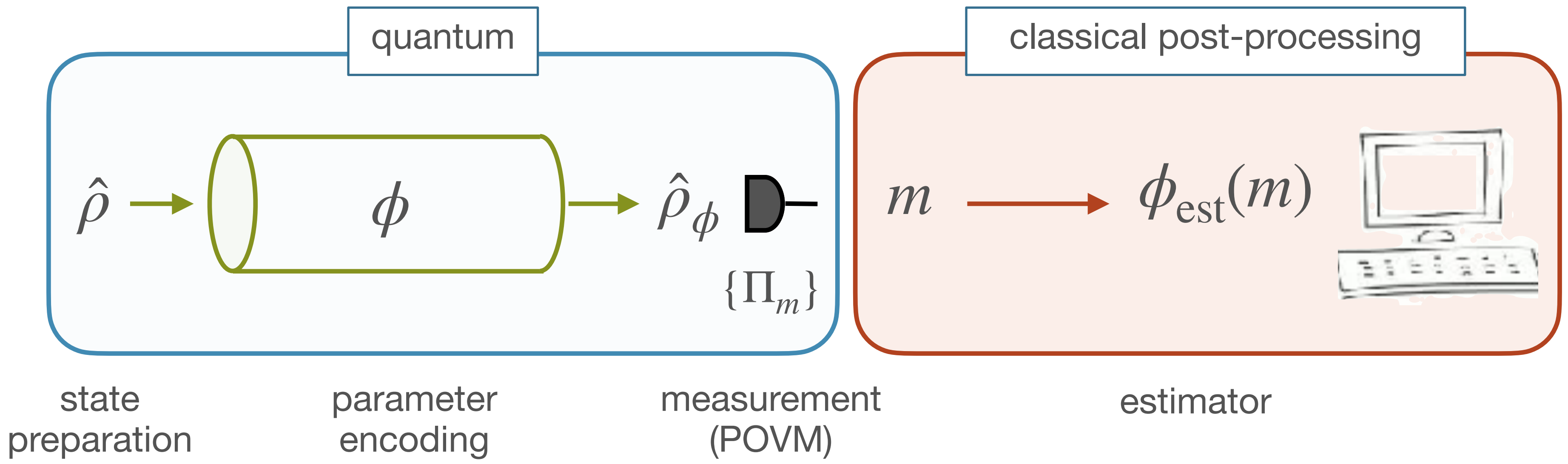
Frequentist Approach - Quantum Fisher Information

⇨ Bayesian Approach (single shot measurement)

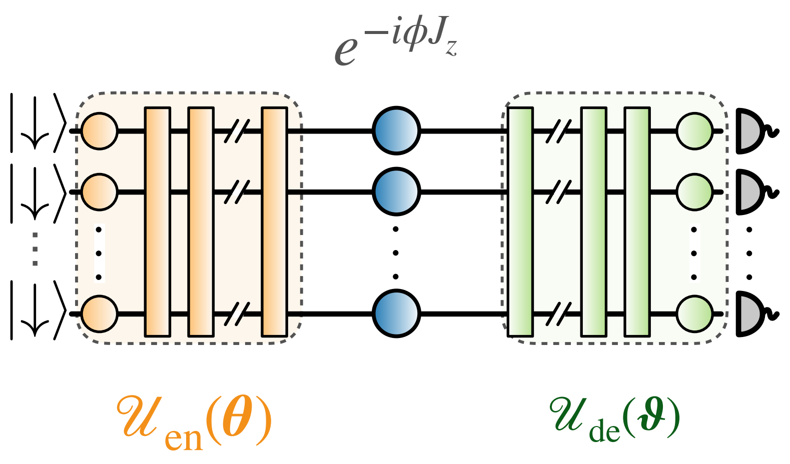
Quantum Parameter Estimation

$\{\hat{\rho}_\theta, \Pi_m, \phi_{est}(m)\}$

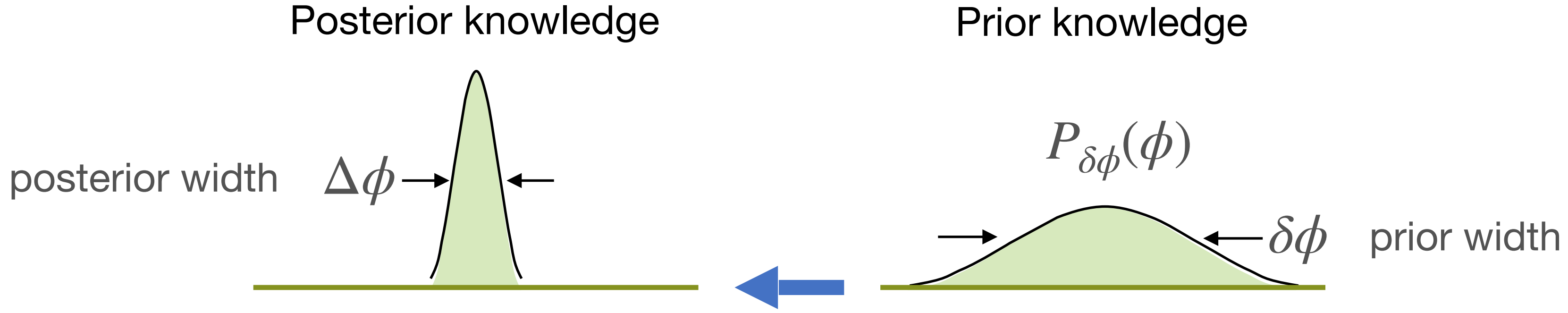
optimal
- ✓ input
- ✓ measurement
- ✓ estimator



Optimal & Variational Ramsey Interferometry with finite dynamic range



Bayesian interpretation / approach:



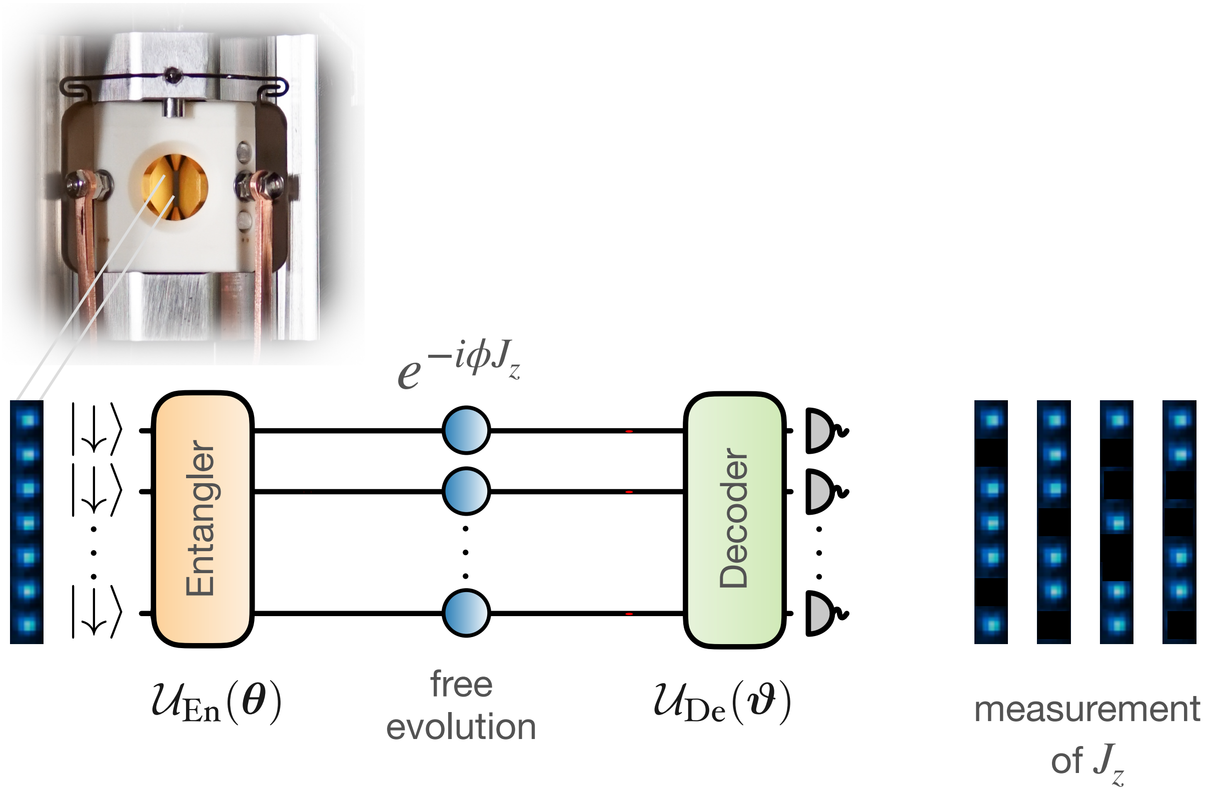
Bayesian update (single shot measurement)

finite dynamic range

Cost function: posterior width $\mathscr{C}(\boldsymbol{\theta}, \boldsymbol{\vartheta}) \equiv (\Delta\phi)^2 \rightarrow \min$ for given prior $\delta\phi$

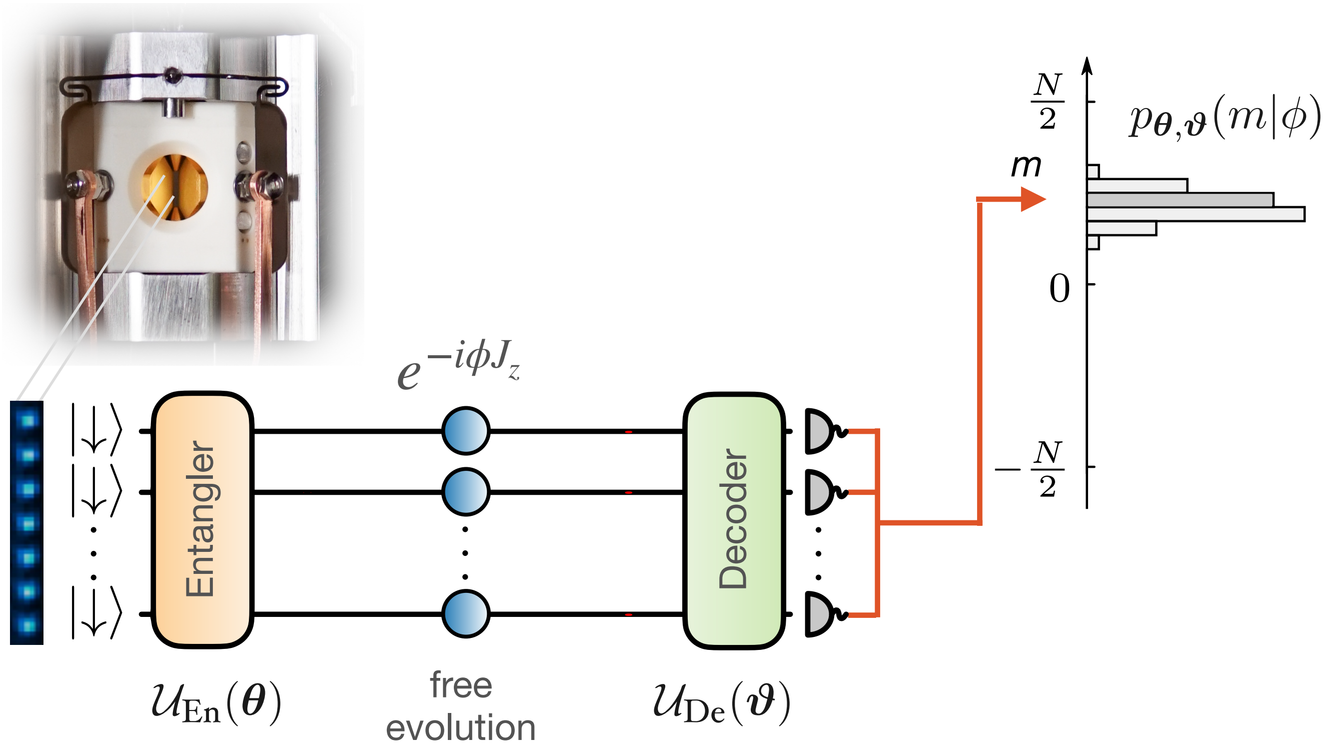
… learn as much as possible about parameter ϕ from single measurement

Ramsey Interferometer



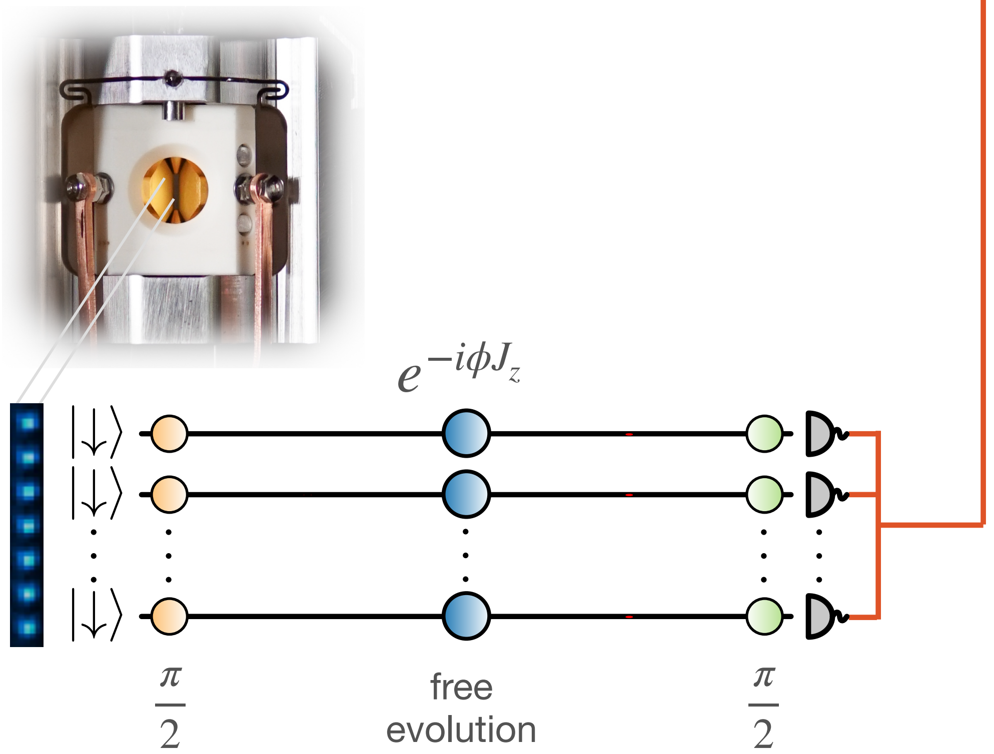
$$m = \frac{1}{2}(\#\text{up} - \#\text{down})$$

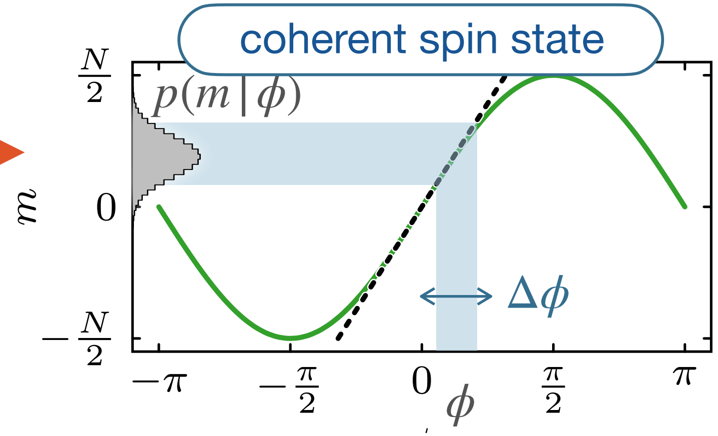
Ramsey Interferometer

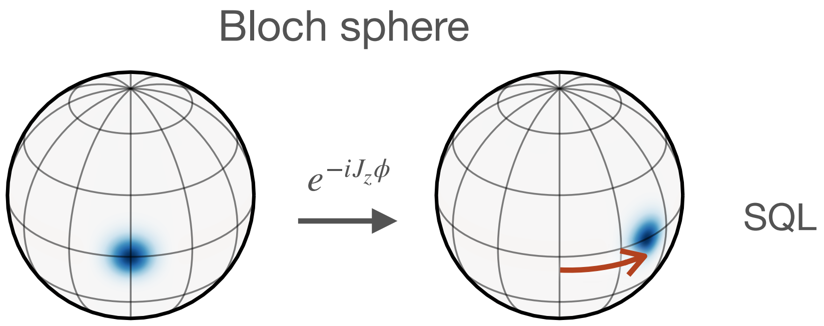


$$p_{\boldsymbol{\theta},\boldsymbol{\vartheta}}(m|\phi) = \left| \langle m | U_{\mathrm{de}}(\boldsymbol{\vartheta}) e^{-i\phi J_z} U_{\mathrm{en}}(\boldsymbol{\theta}) \, |\downarrow\rangle^{\otimes N} \right|^2$$

Ramsey Interferometer



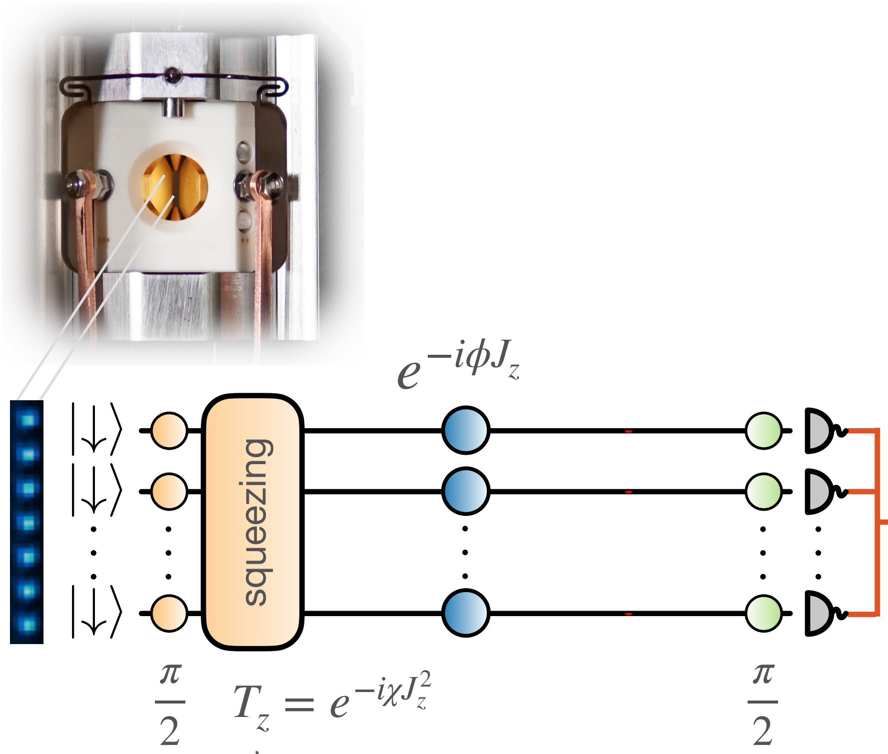




product state: $|CSS\rangle \sim \left(|\uparrow\rangle + |\downarrow\rangle\right)^{\otimes N}$

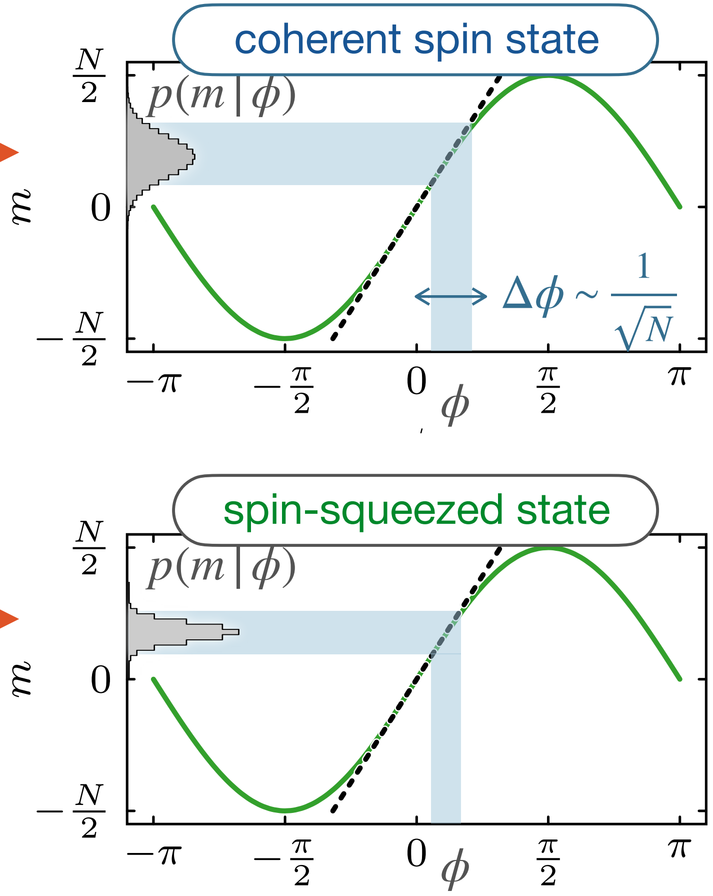
Standard Quantum Limit (SQL): $\Delta\phi \sim \frac{1}{\sqrt{N}}$

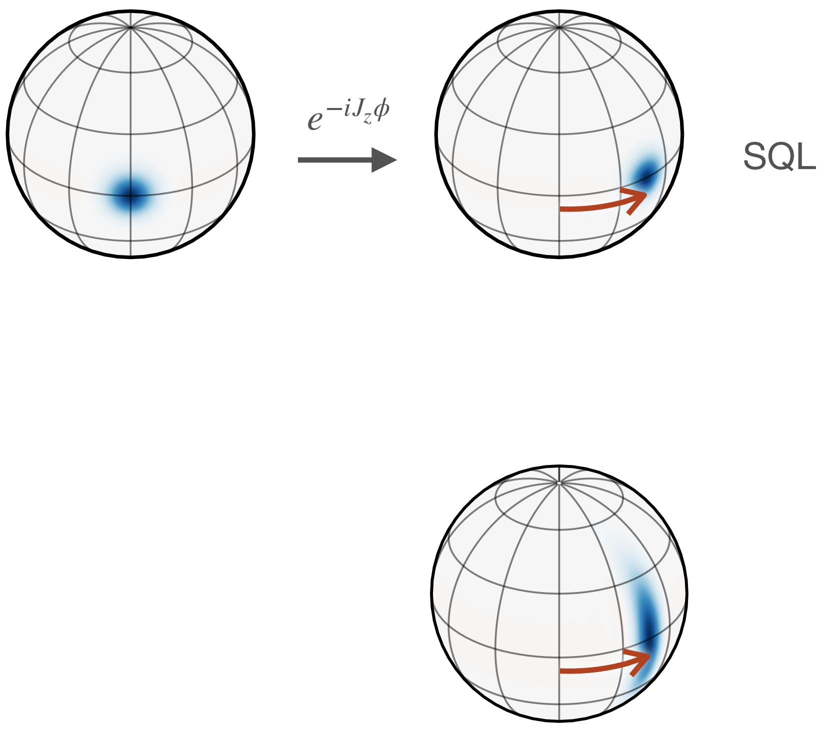
Ramsey Interferometer



One-Axis Twisting $J_z^2 = \sum_{i,j=1}^{N} \sigma_z^i \sigma_z^j$

infinite range interaction

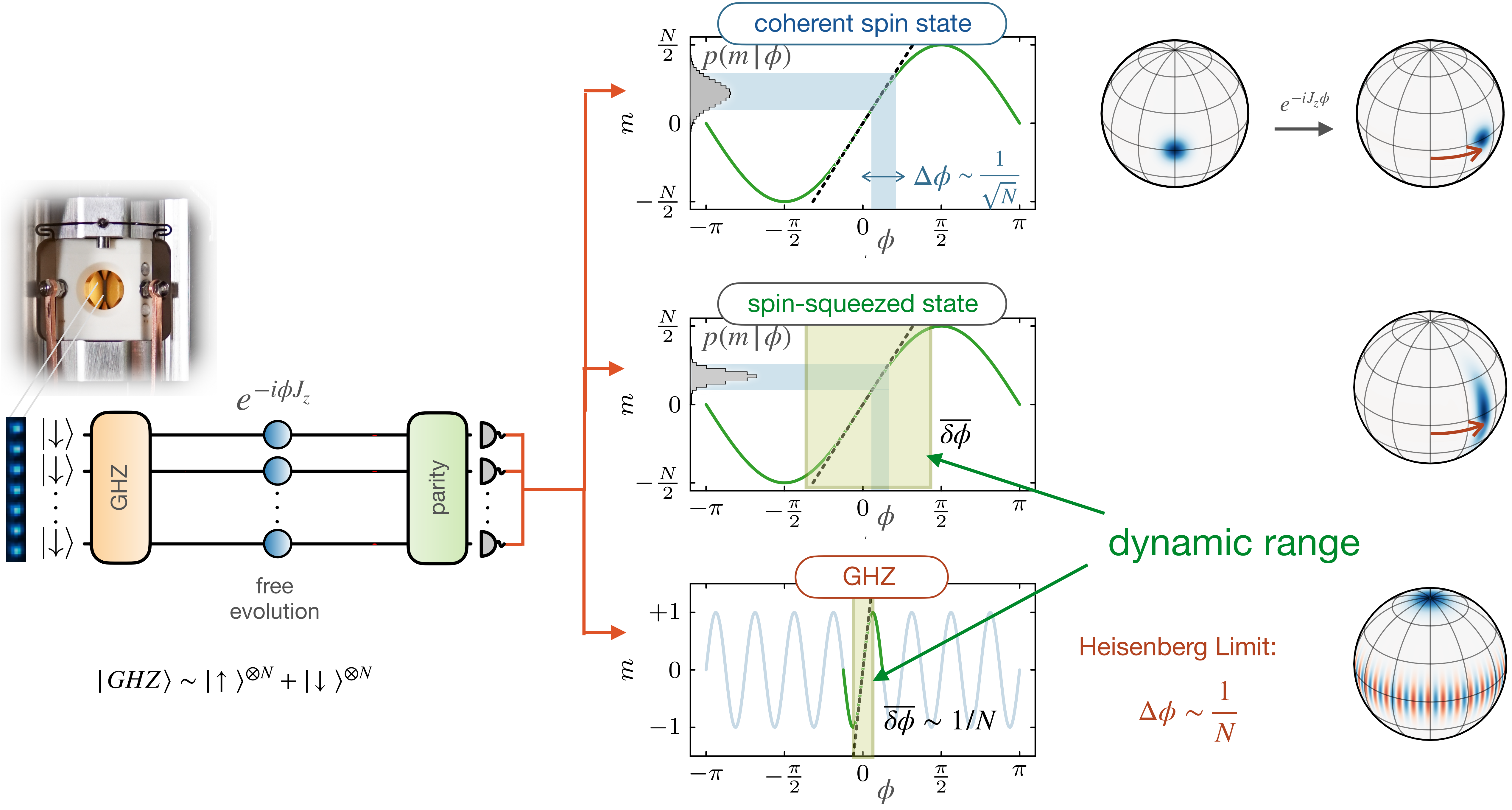




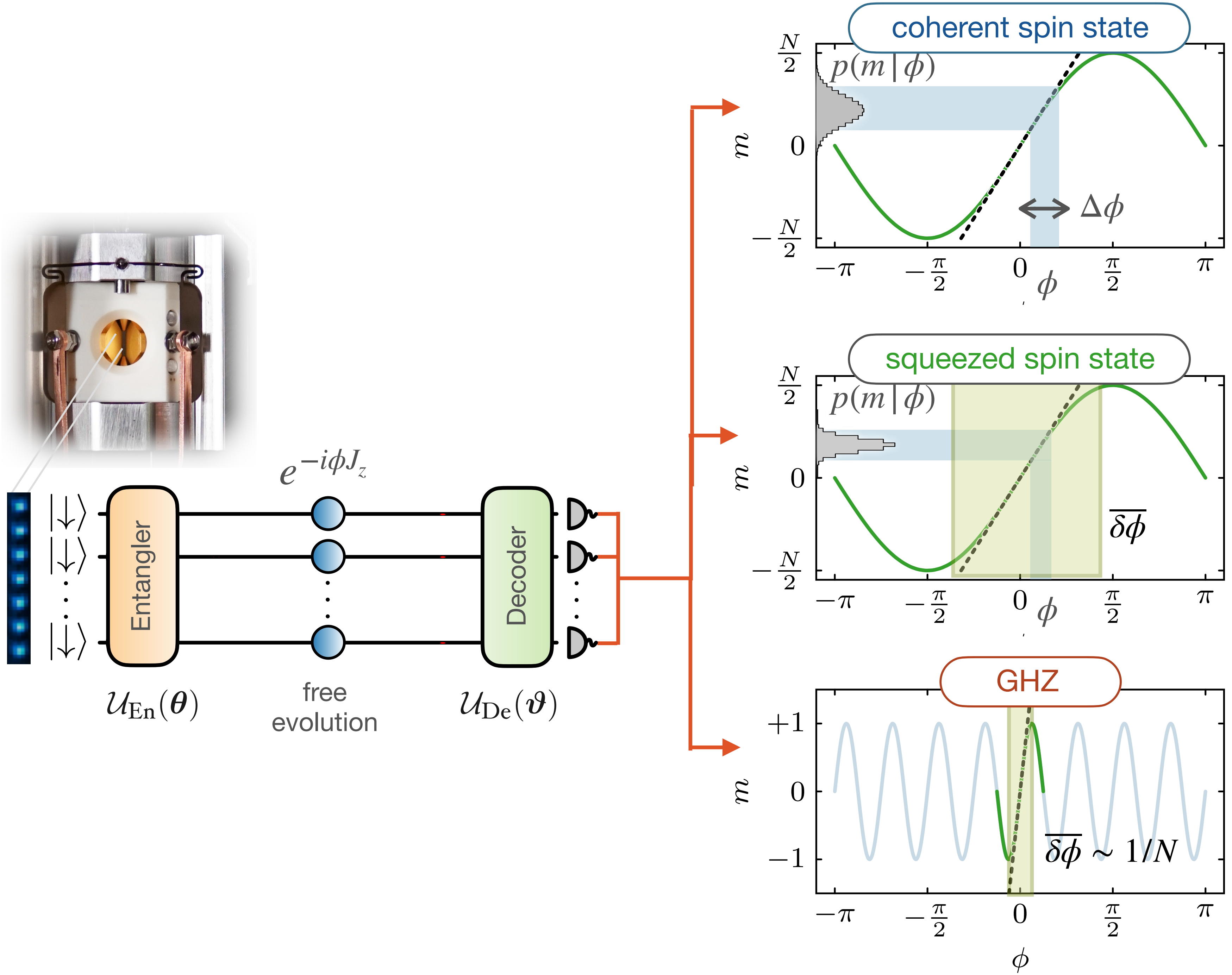
spin-squeezing ~ entangled state

below SQL: $\Delta\phi < \frac{1}{\sqrt{N}}$

Ramsey Interferometer

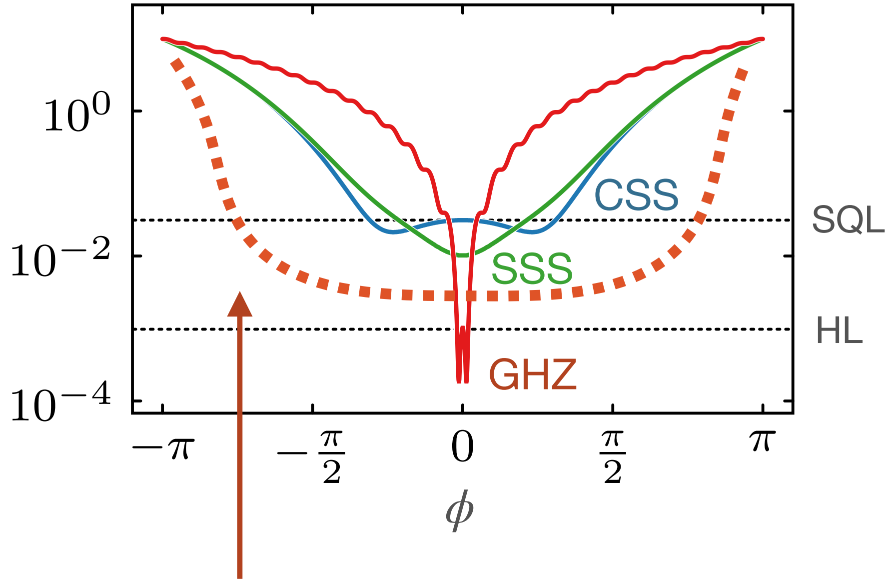


Ramsey Interferometer



- mean square error with respect to phase ϕ

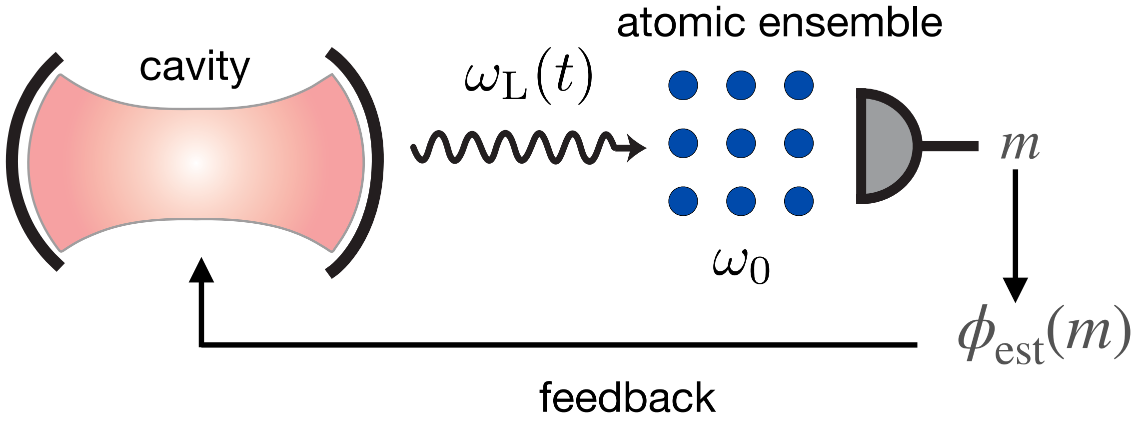
$$\mathrm{MSE}(\phi) = \sum_m \left[\phi - \phi_{\mathrm{est}}(m)\right]^2 p_{\boldsymbol{\theta},\boldsymbol{\vartheta}}(m|\phi)$$



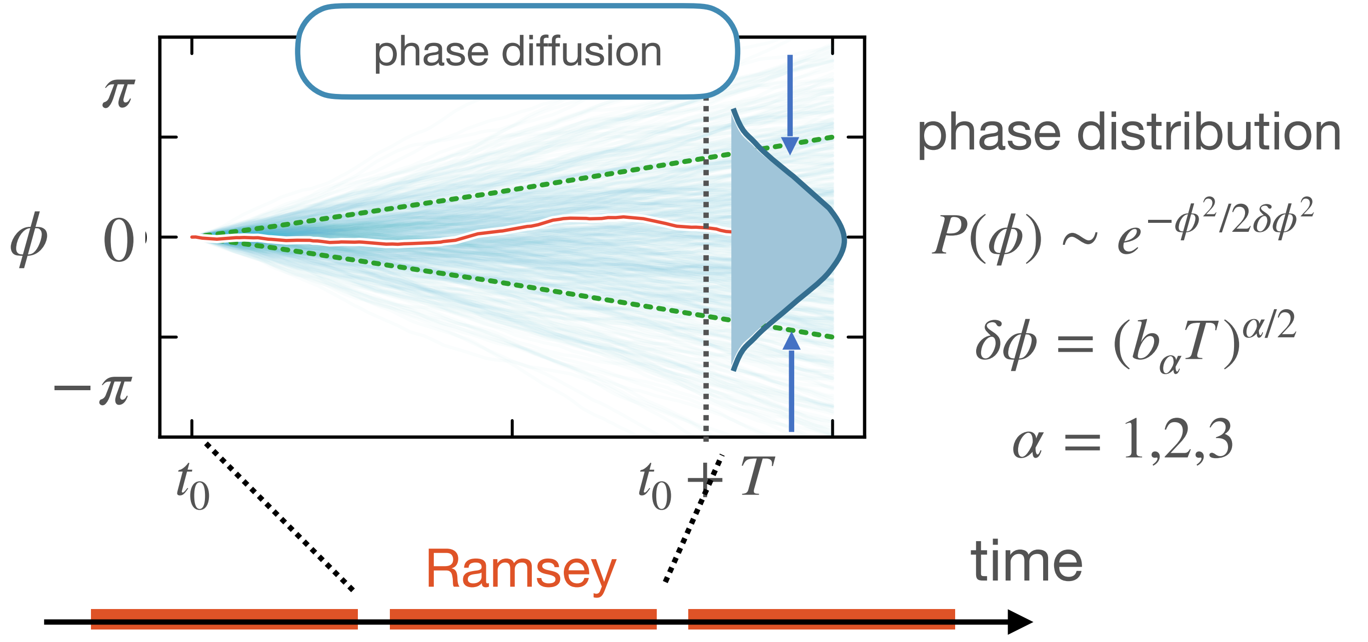
interferometer we wish to have

- best Signal /Noise ratio
- for broad dynamic range $\delta\phi$

Atomic Clock



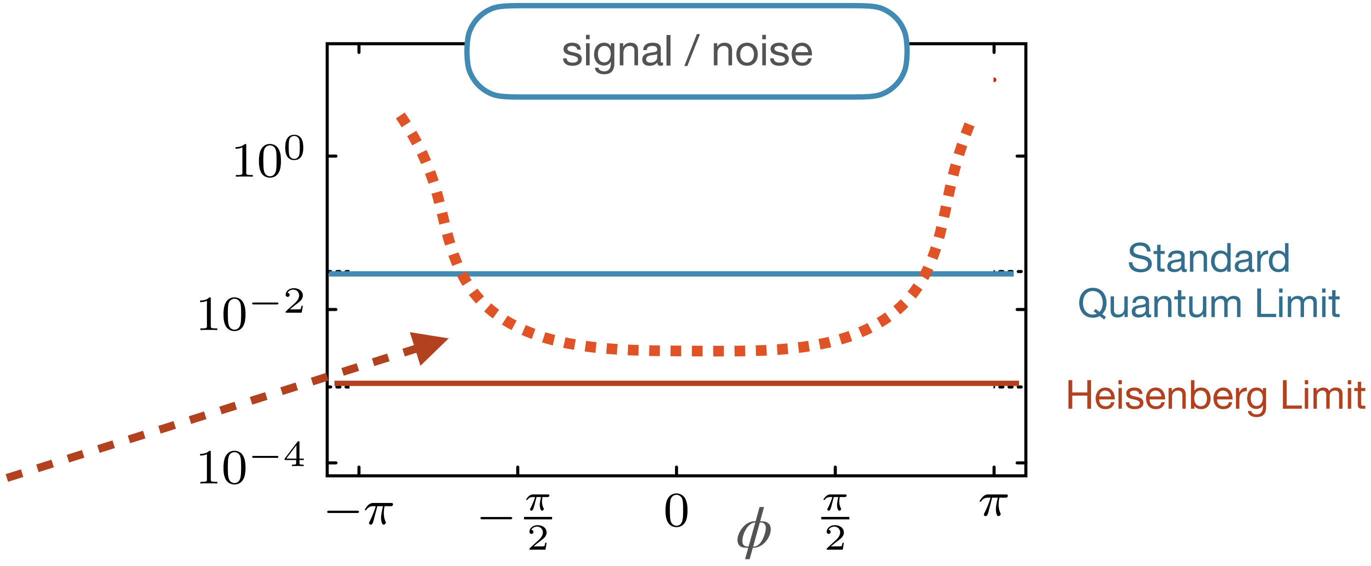
$$\phi = \int_{t_0}^{t_0+T} \left(\omega_L(t) - \omega_0\right) dt$$



Variational Classical-Quantum Algorithms

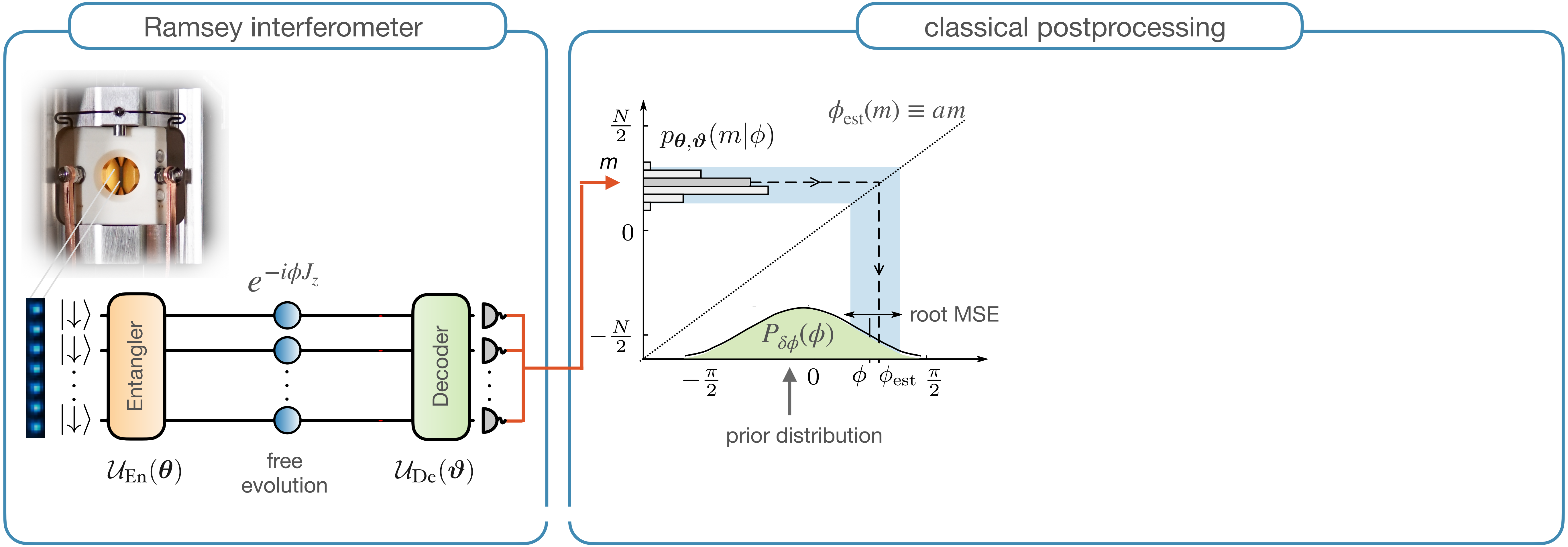
$\mathscr{C}_{\text{metrological}} \rightarrow \text{opt}$

wishlist:
- ✓ signal / noise ratio
- ✓ finite dynamic range $\delta\phi$



A. D. Ludlow, M. M. Boyd, J. Ye, E. Peik, and P. O. Schmidt, *Optical Atomic Clocks*, Rev. Mod. Phys. **87**, 637 (2015).

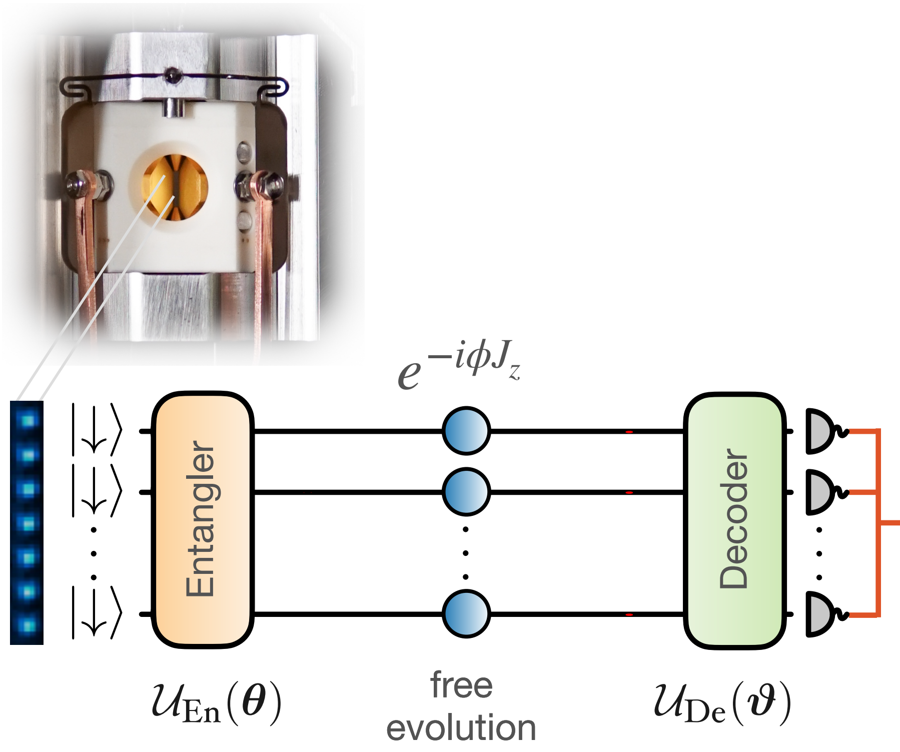
Variational Quantum Algorithm for Optimal Ramsey Interferometry



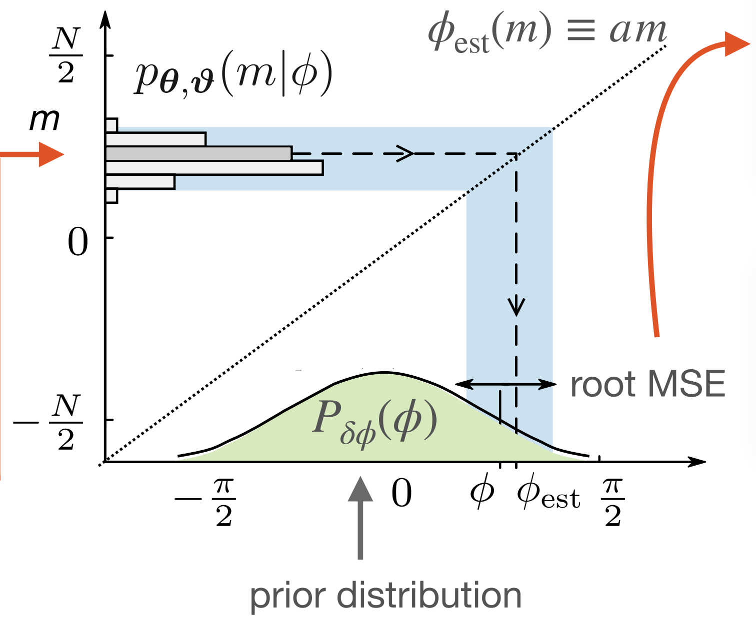
Find optimal entangled state and optimal measurement for given prior (dynamic range) $\delta\phi$

Variational Quantum Algorithm for Optimal Ramsey Interferometry

Ramsey interferometer



classical postprocessing



- mean square error with respect to phase ϕ

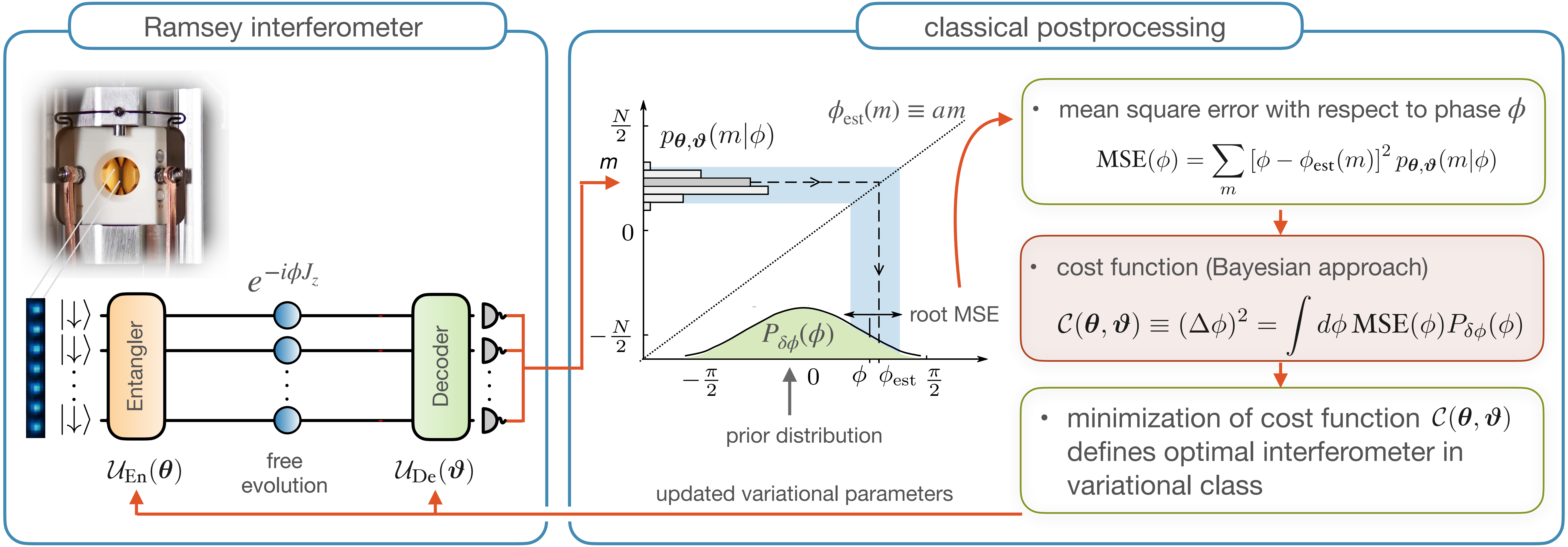
$$\mathrm{MSE}(\phi) = \sum_m [\phi - \phi_{\mathrm{est}}(m)]^2 \, p_{\boldsymbol{\theta},\boldsymbol{\vartheta}}(m|\phi)$$

- cost function (Bayesian approach)

$$\mathcal{C}(\boldsymbol{\theta},\boldsymbol{\vartheta}) \equiv (\Delta\phi)^2 = \int d\phi \, \mathrm{MSE}(\phi) P_{\delta\phi}(\phi)$$

Find optimal entangled state and optimal measurement for given prior (dynamic range) $\delta\phi$

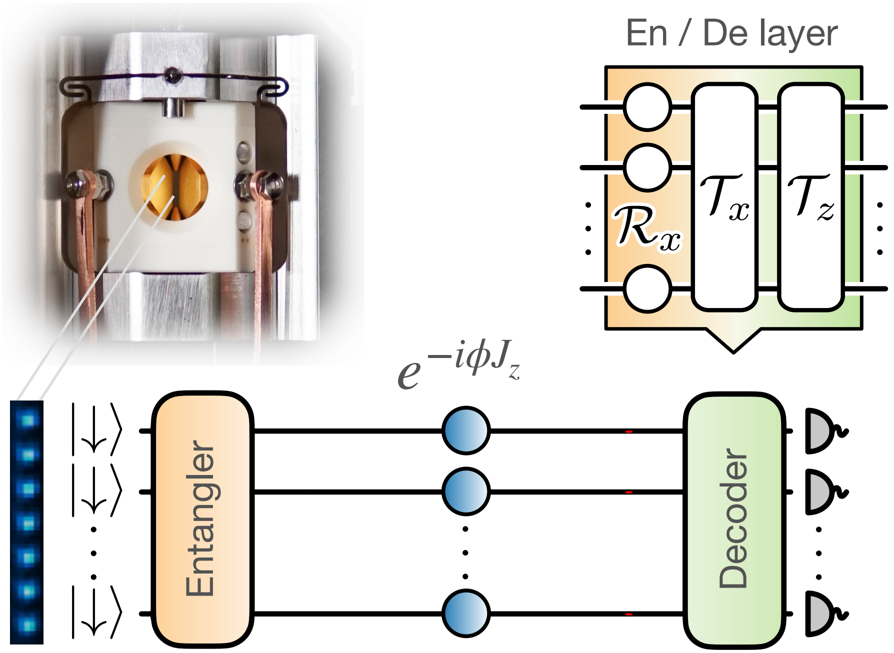
Variational Quantum Algorithm for Optimal Ramsey Interferometry



Find optimal entangled state and optimal measurement for given prior (dynamic range) $\delta\phi$

Variationally Optimized Ramsey Interferometer

Ramsey interferometer



Ion q-computer— native building blocks

$$\mathcal{U}_{\mathrm{En}} = \prod_{k=1}^{n_{\mathrm{En}}} \mathcal{R}_x(\theta_k^3)\mathcal{T}_x(\theta_k^2)\mathcal{T}_z(\theta_k^1)$$

$$\mathcal{U}_{\mathrm{De}} = \prod_{k=1}^{n_{\mathrm{De}}} \mathcal{T}_z(\vartheta_k^1)\mathcal{T}_x(\vartheta_k^2)\mathcal{R}_x(\vartheta_k^3)$$

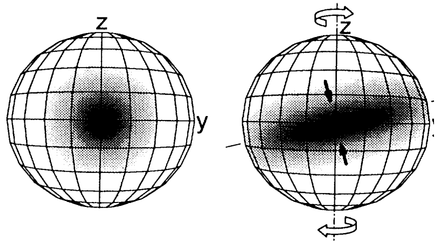
#parameters = $3(n_{\mathrm{en}} + n_{\mathrm{de}})$

Our resource operations:

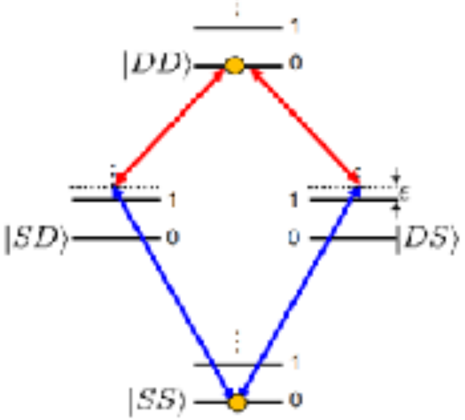
$$\mathcal{R}_{x,y,z}(\beta) = e^{-i\beta \hat{J}_{x,y,z}}$$

$$\mathcal{T}_{x,y,z}(\chi) = e^{-i\chi \hat{J}^2_{x,y,z}}$$

one-axis-twisting



M Kitagawa & M Ueda, *PRA* 1993

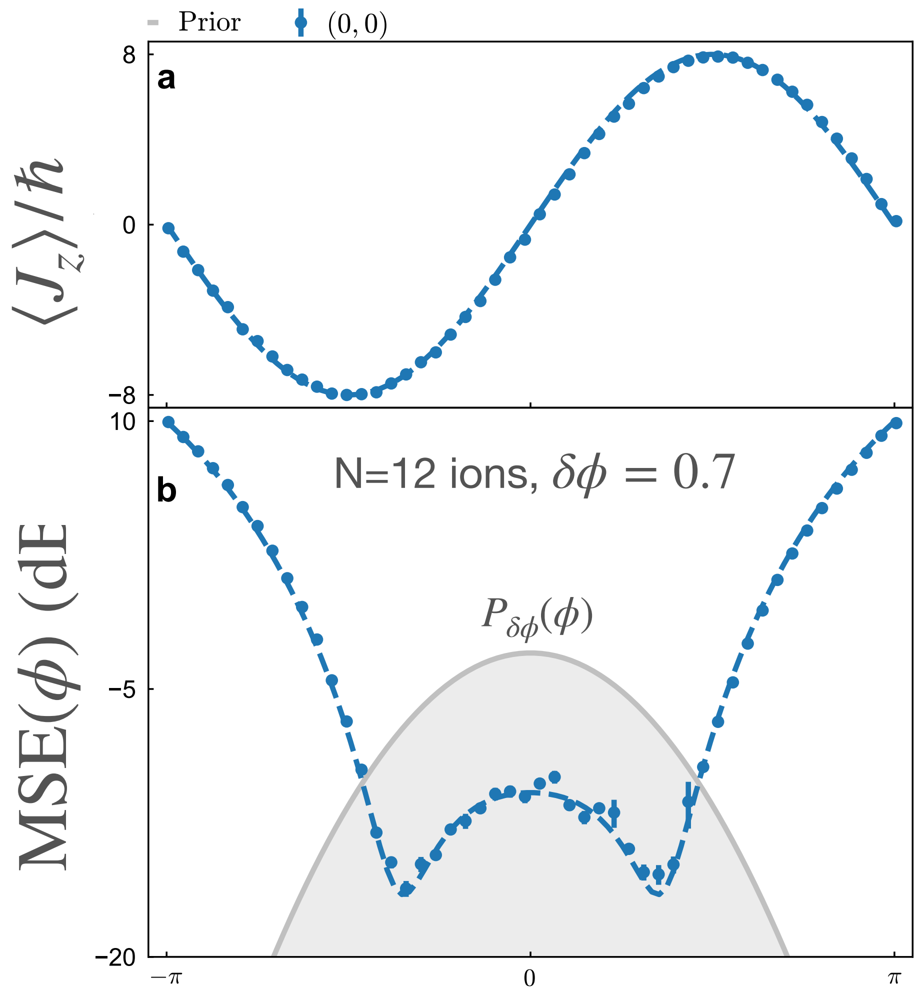


M Sorensen, K Molmer, PRL 1999

here: symmetric subspace

1. Theory prediction for $\boldsymbol{\theta}_{\text{opt}}$, $\boldsymbol{\vartheta}_{\text{opt}}$ → Trapped Ion QC Experiment

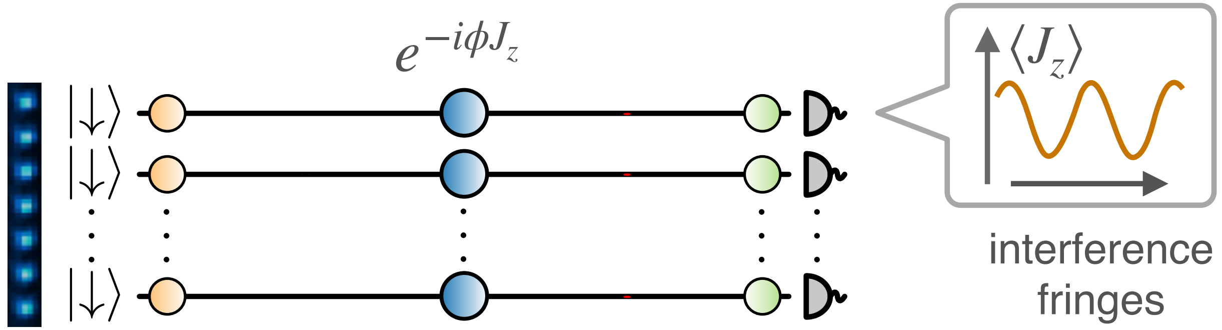
CSS



phase ϕ
evaluation pulse

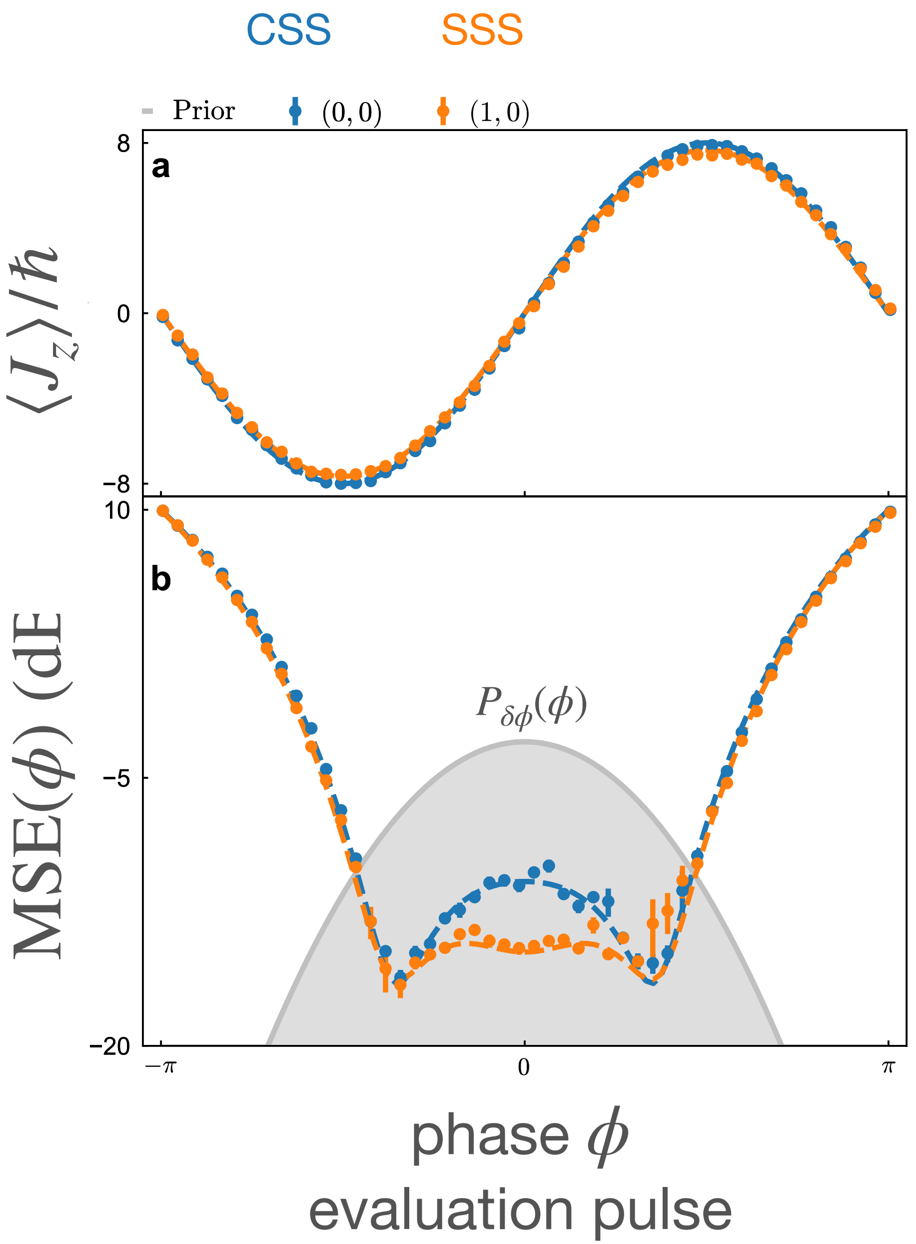
$(n_{\text{en}}, n_{\text{de}})$

(0, 0): Coherent spin state (classical interferometry), CSS



product state

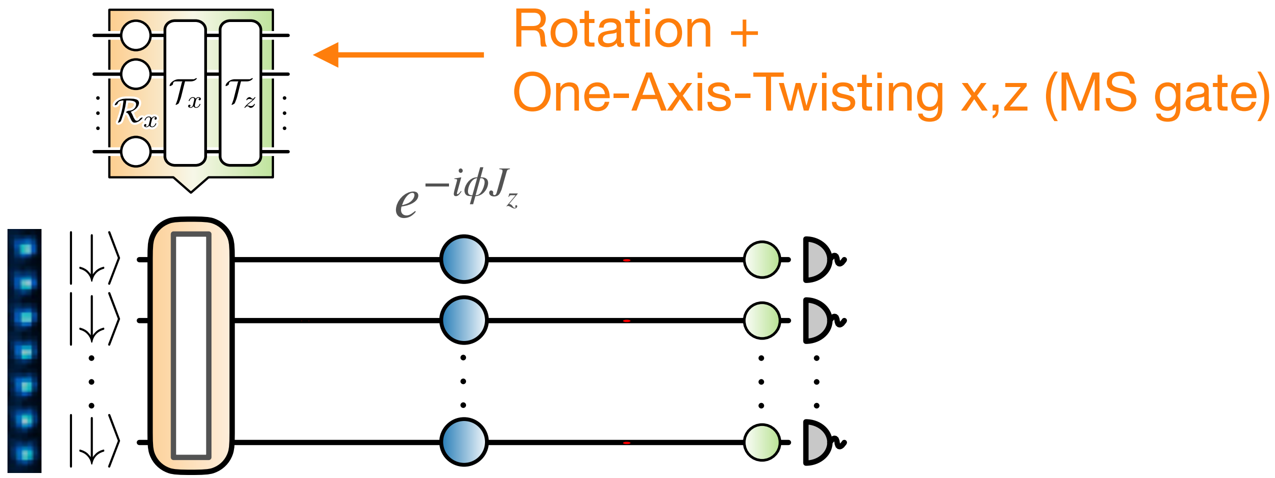
1. Theory prediction for θ_{opt}, ϑ_{opt} → Trapped Ion QC Experiment



(n_{en}, n_{de})

(0, 0): Coherent spin state (classical interferometry), CSS

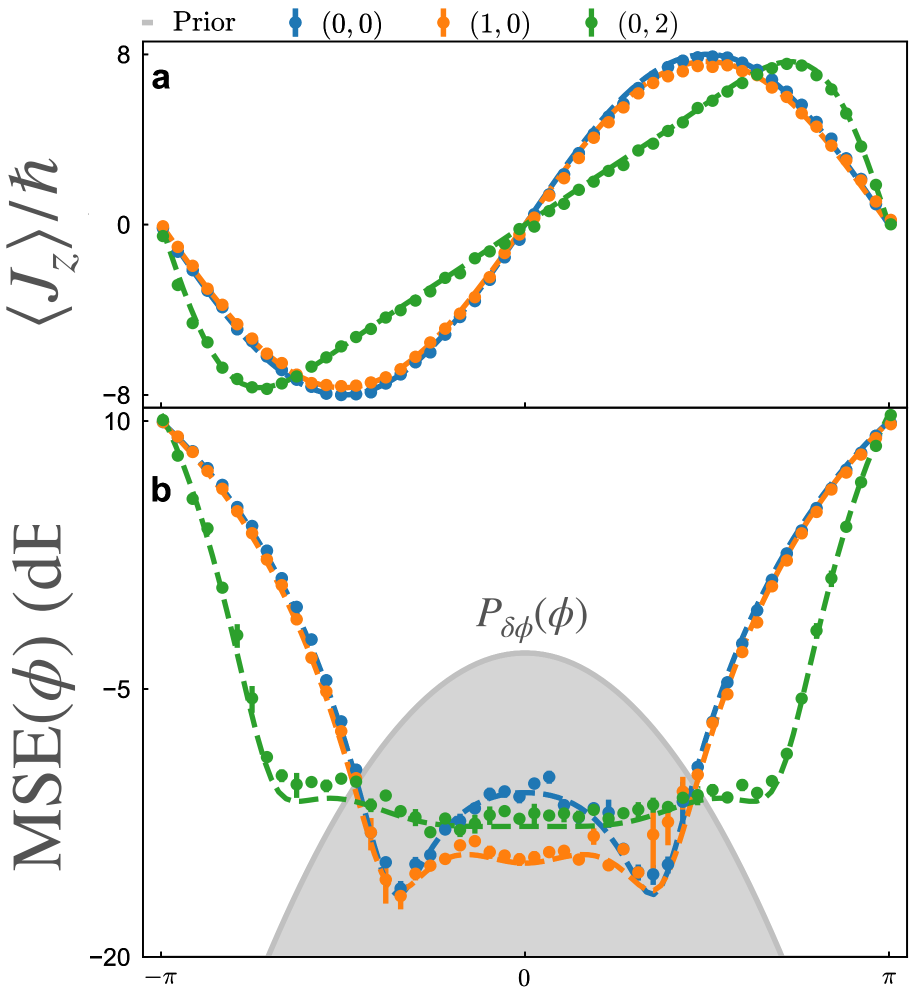
(1, 0): Squeezed spin state, SSS



Encoding increases sensitivity around 0

1. Theory prediction for $\boldsymbol{\theta}_{\text{opt}}, \boldsymbol{\vartheta}_{\text{opt}}$ → Trapped Ion QC Experiment

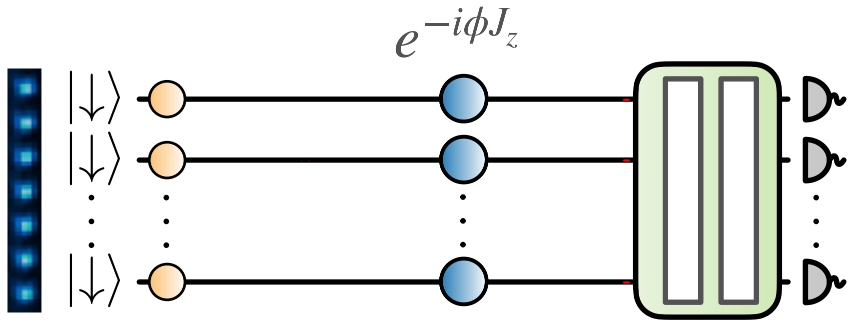
CSS SSS



phase ϕ
evaluation pulse

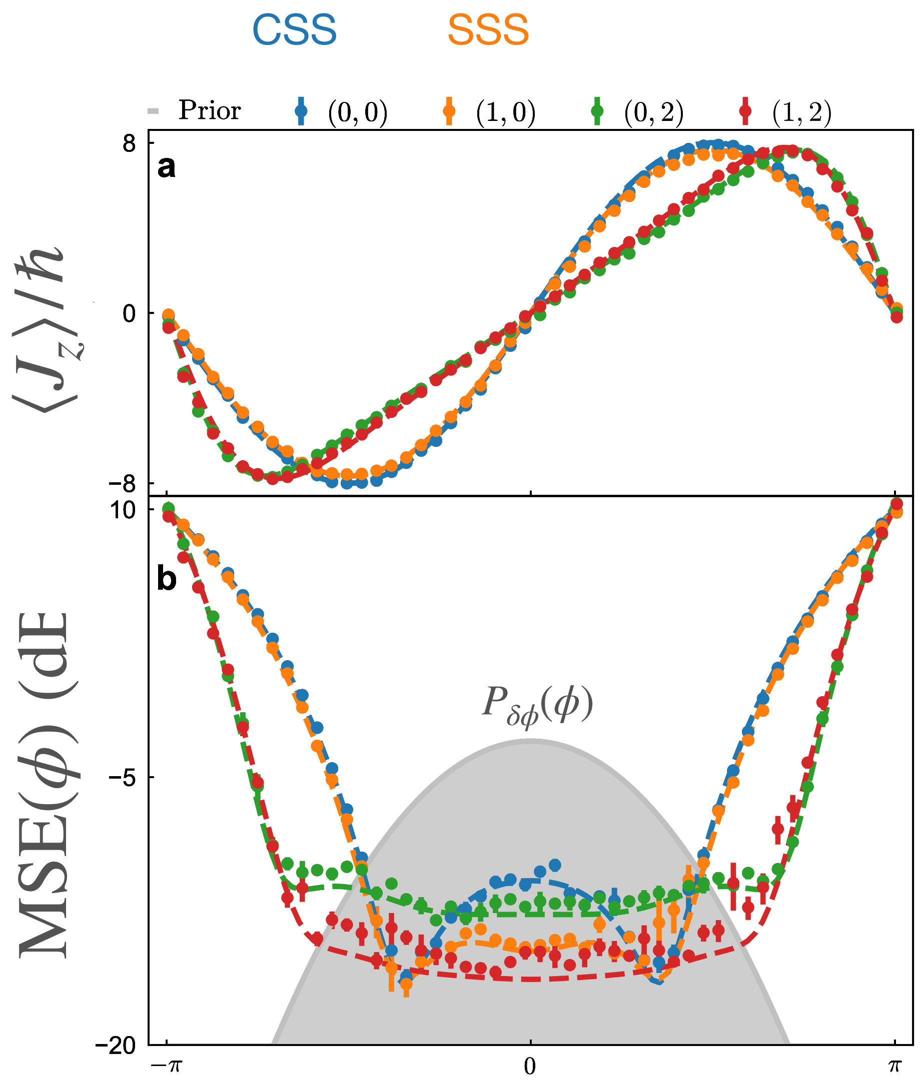
$(n_{\text{en}}, n_{\text{de}})$

(0, 0): Coherent spin state (classical interferometry), CSS
(1, 0): Squeezed spin state, SSS
(0, 2): CSS with Decoding



Decoding increases dynamic range

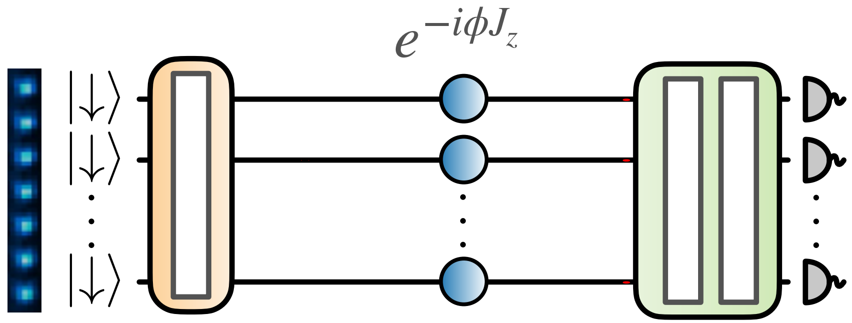
1. Theory prediction for $\boldsymbol{\theta}_{\mathrm{opt}}$, $\boldsymbol{\vartheta}_{\mathrm{opt}}$ → Trapped Ion QC Experiment



phase ϕ
evaluation pulse

$(n_{\mathrm{en}}, n_{\mathrm{de}})$

(0, 0): Coherent spin state (classical interferometry), CSS
(1, 0): Squeezed spin state, SSS
(0, 2): CSS with Decoding
(1, 2): Encoding + Decoding



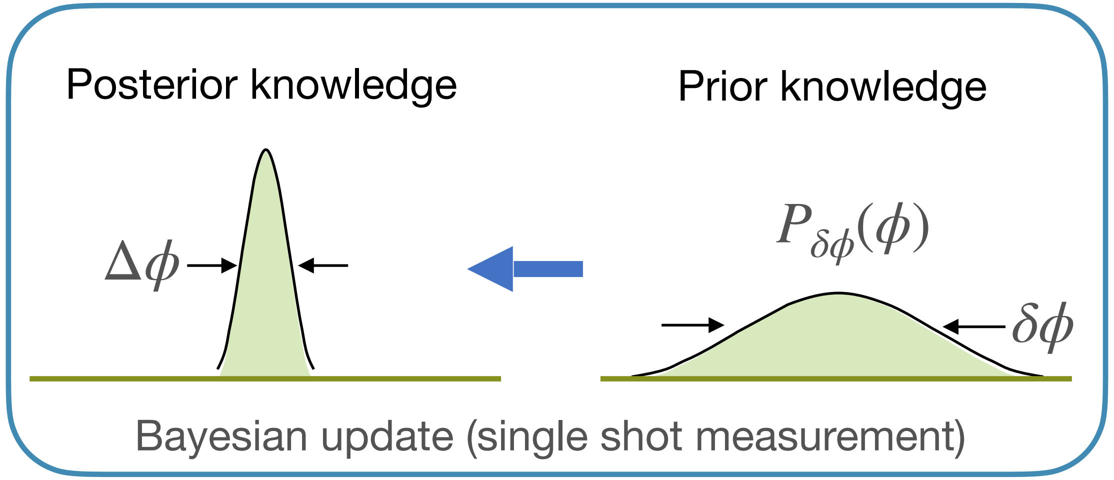
Combining increases sensitivity and range

… on an *optical* clock transition!

1. Experiment vs. Theory: ‘Reducing Ignorance’ in Bayesian Update

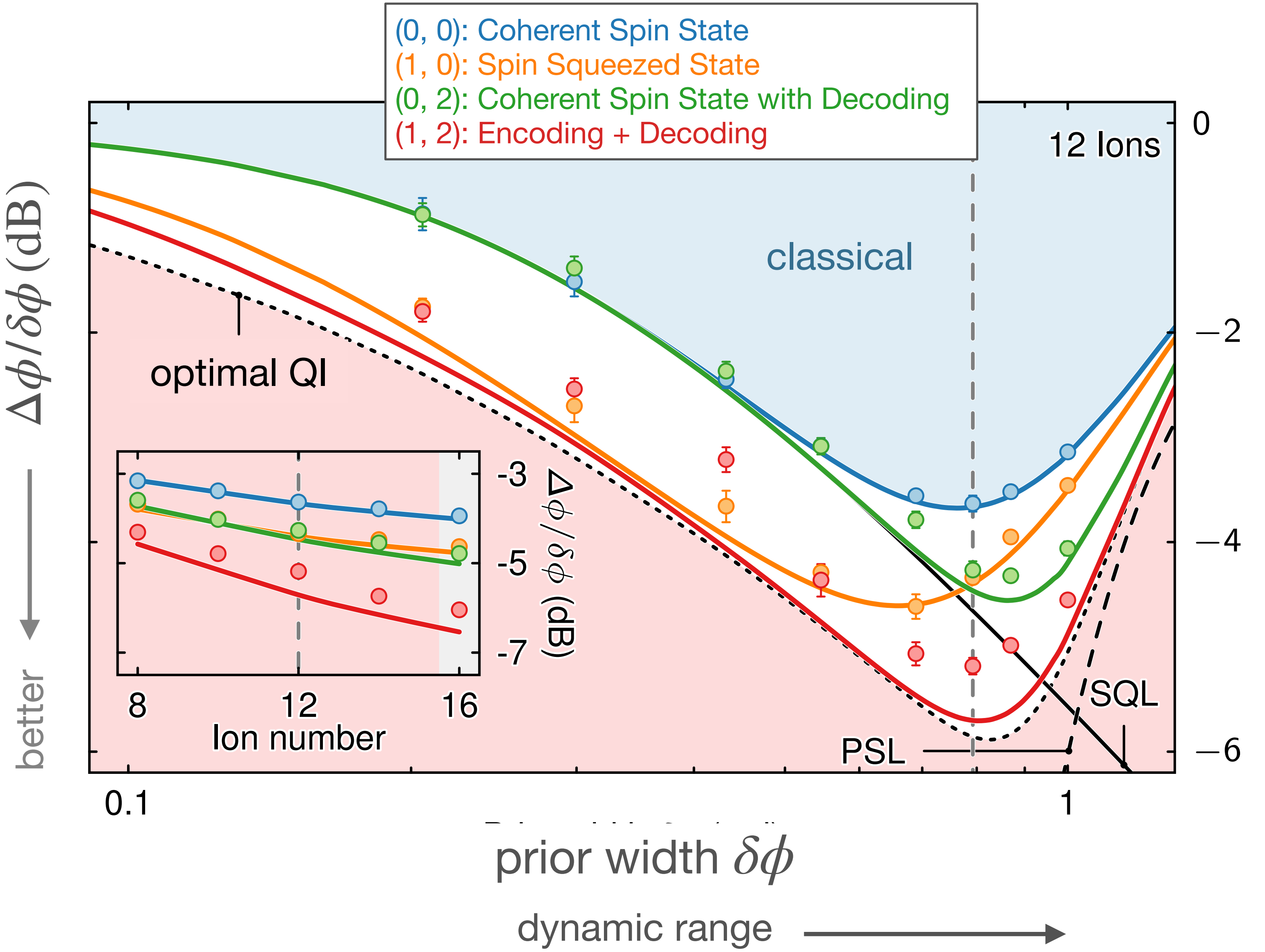
Prior knowledge $\delta\phi$

Posterior knowledge $\Delta\phi$

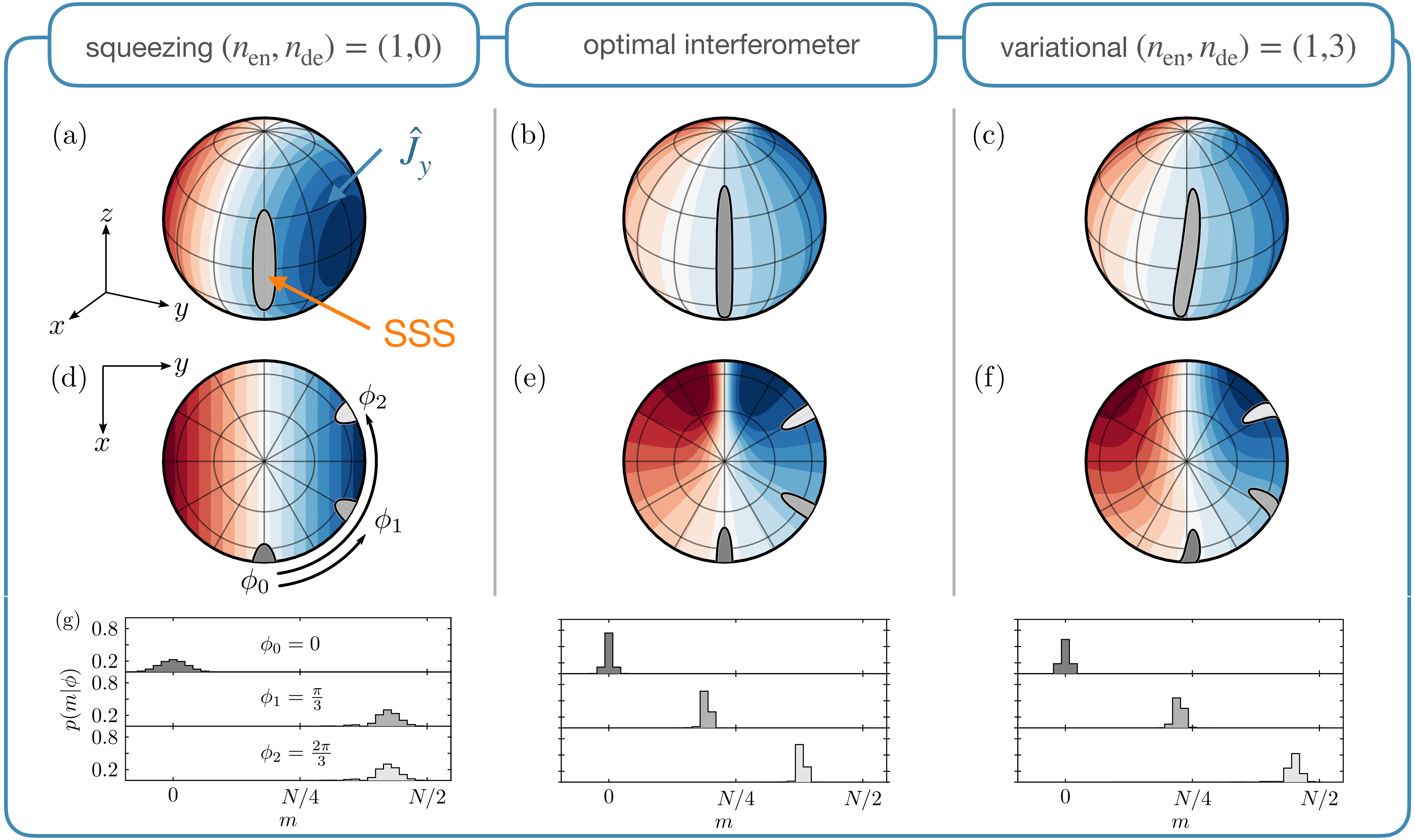


Uncertainty reduction
in single measurement

$\Delta\phi/\delta\phi$



Theory: Interpretation of Results



- Wigner plots of input states

$$|\psi_{in}\rangle = \mathcal{U}_{en} |\downarrow\rangle^{\otimes N}$$

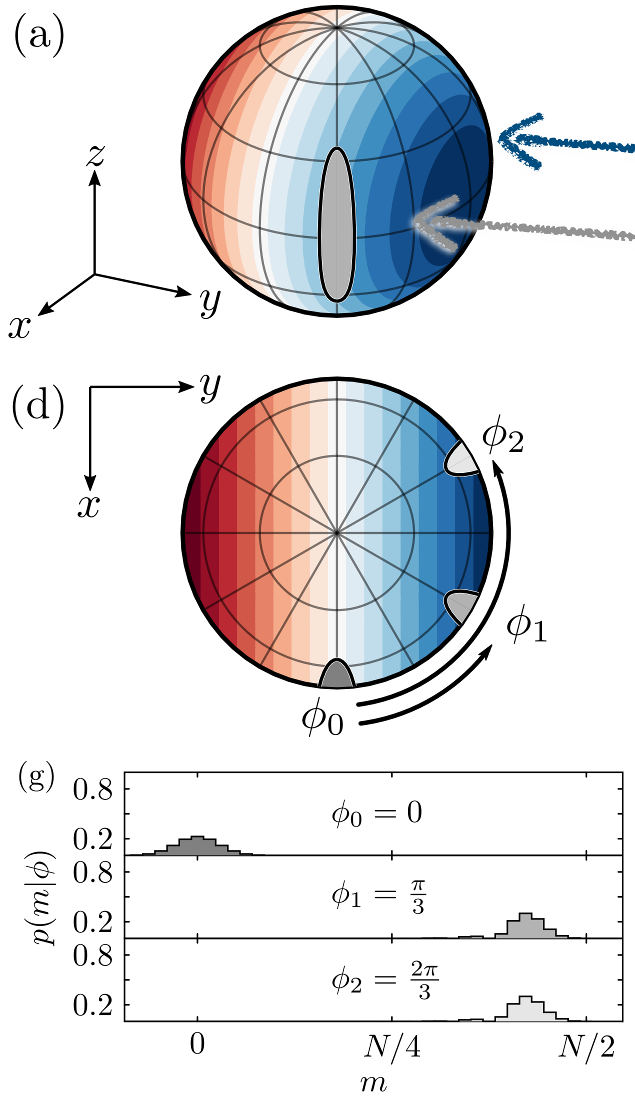
and measurement operators

$$\mathcal{U}_{de} \hat{J}_y \mathcal{U}_{de}^{\dagger}$$

- contour lines of input states and measurement operators match for broad range $\delta\phi$

Theory: Interpretation of Results

squeezing $(n_{\text{en}}, n_{\text{de}}) = (1{,}0)$



(1,0)-interferometer: SSS with J_y measurement

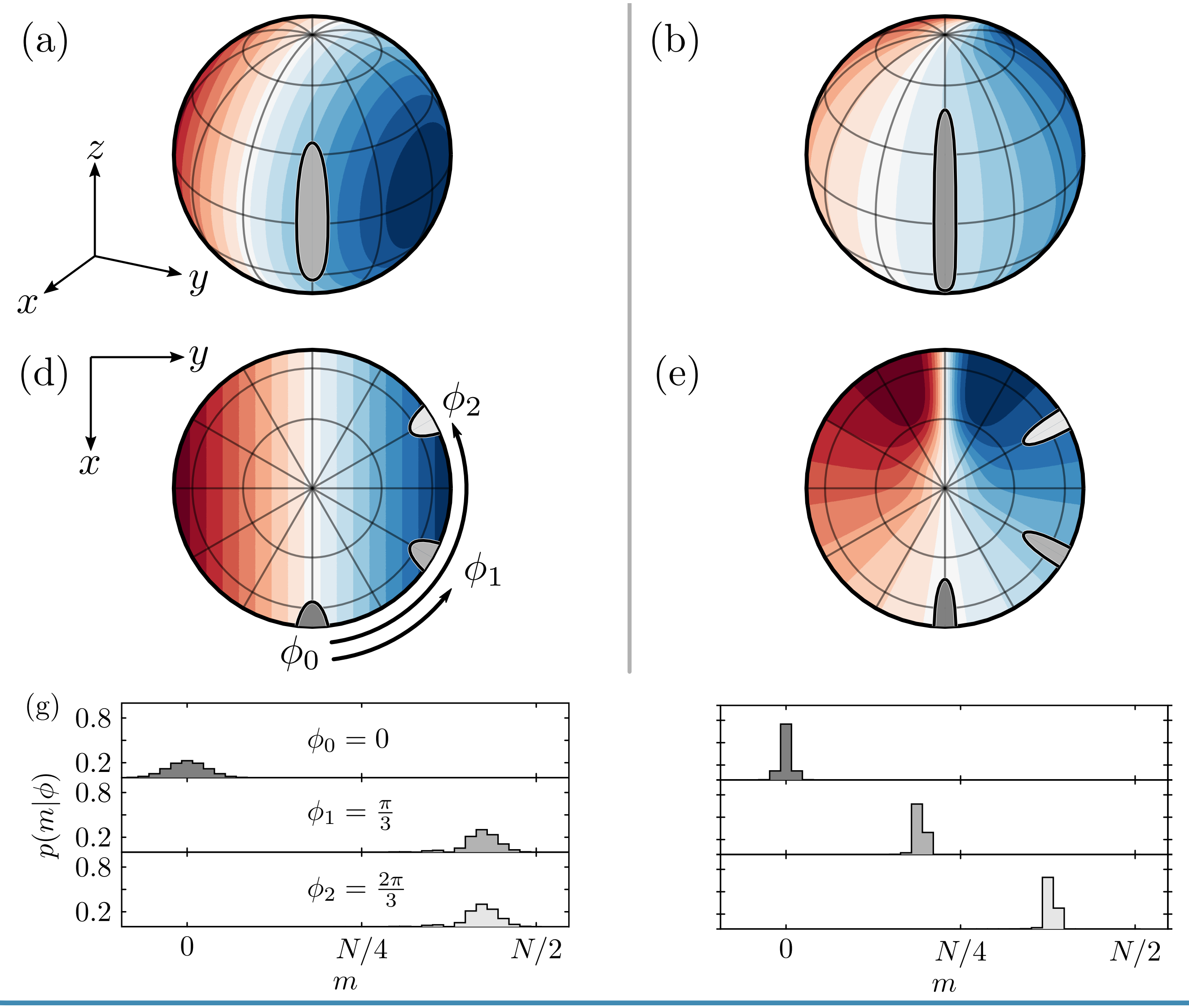
Wigner function of the measurement operator

Wigner function of the state (SSS)

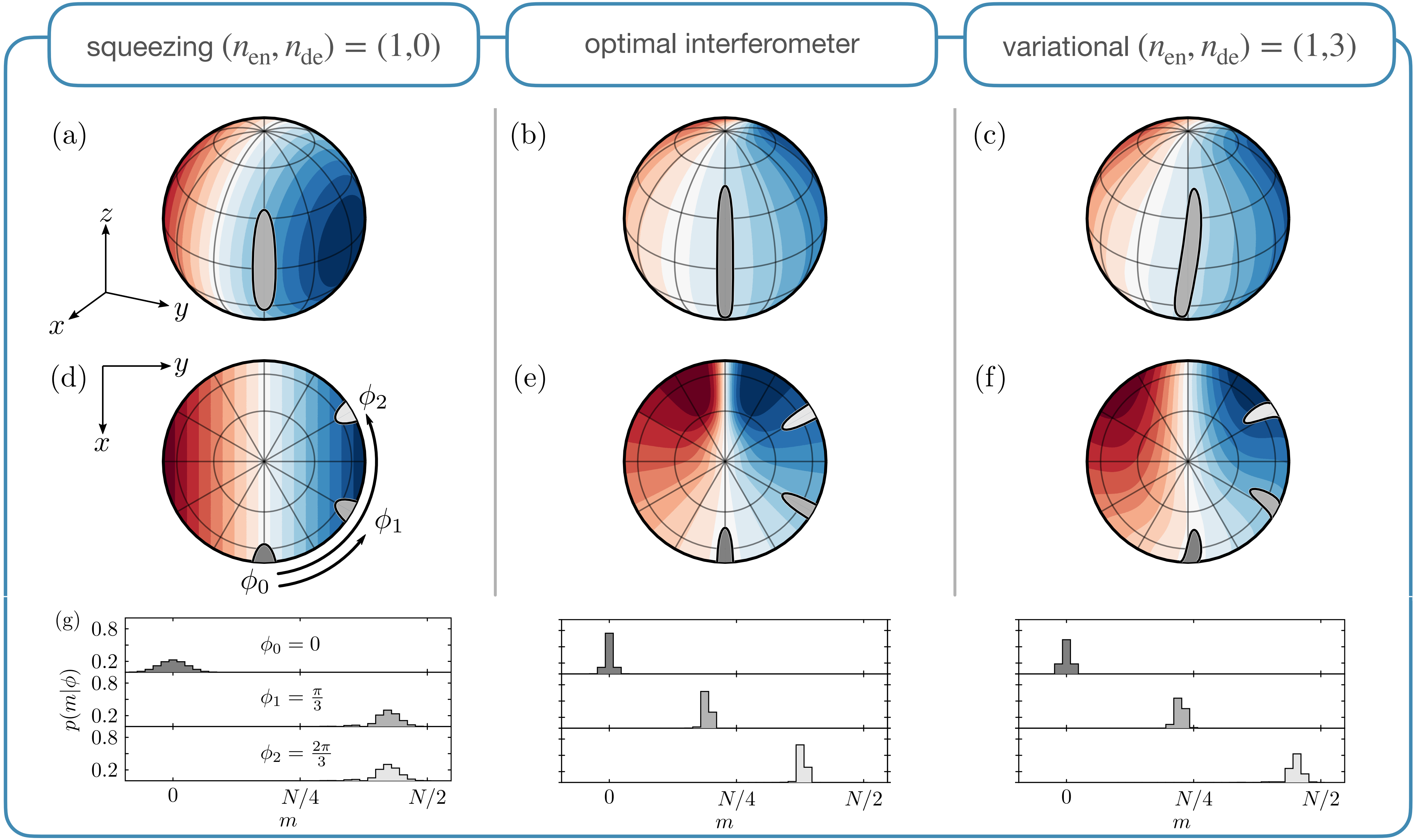
Theory: Interpretation of Results

squeezing $(n_{\text{en}}, n_{\text{de}}) = (1,0)$

optimal interferometer



Theory: Interpretation of Results



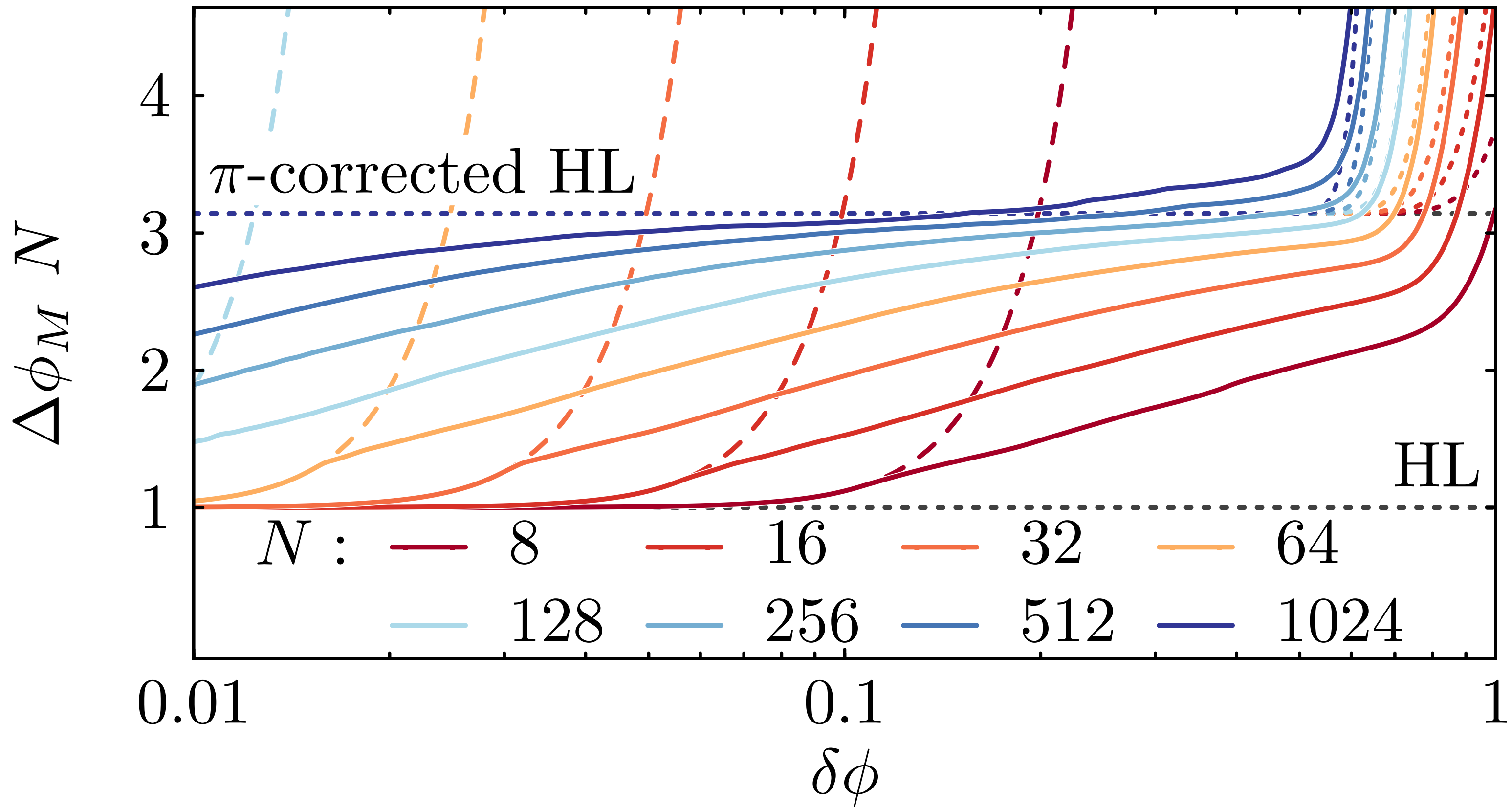
Toward the Heisenberg Limit [Bayesian]

effective measurement variance

$$(\Delta\phi_M)^2 \geq \frac{1}{\bar{F}_\phi} \geq \frac{1}{N^2}$$

based on Van-Tres inequality

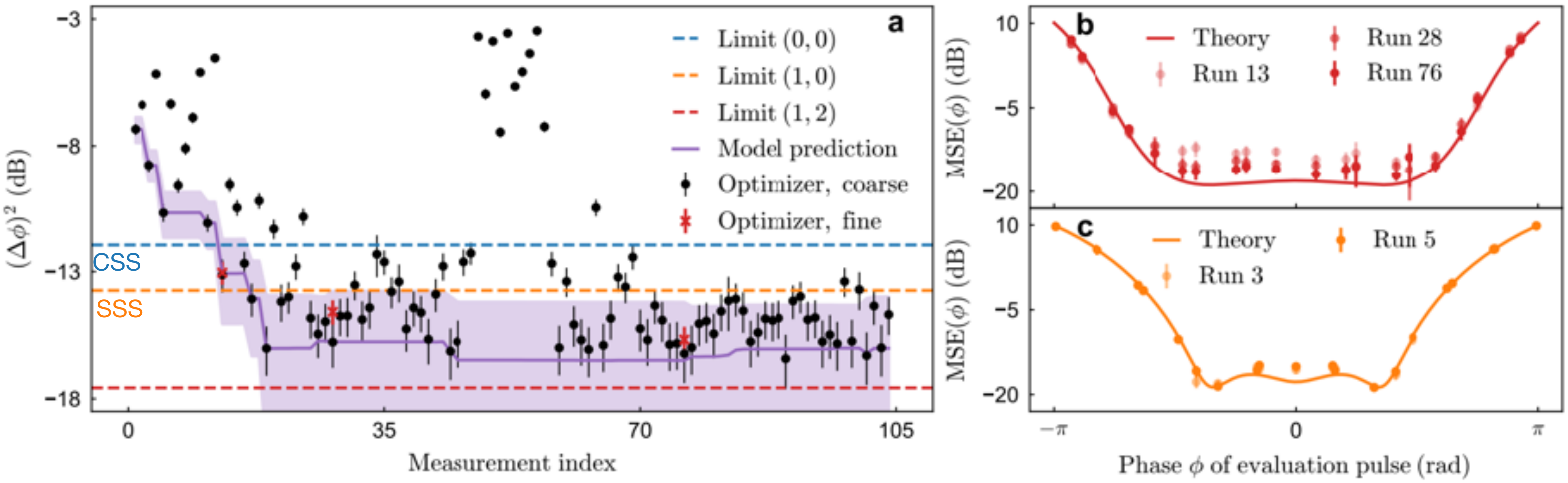
~ Cramer-Rao in Bayesian



prior width

2. `On-device' optimization for $\boldsymbol{\theta}_{\text{opt}}, \boldsymbol{\vartheta}_{\text{opt}}$ in experiment

26 ion optimizer* run of (1, 2) sequence, 7 free parameters, twisting angles <u>not calibrated</u>



*Modified DIRECT global optimizer with trigonometric covariance kernel and GP meta-model [R. van Bijnen; and C Kokail et al., Nature 2019]

`On-device' optimization in regime of quantum advantage

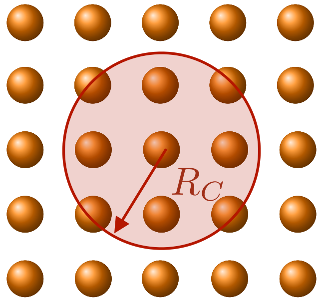
- Classical optimization of variational entangler and decoder is challenging in regime $N > 50$ spins, and in 2D etc.

Science **366**, 93–97 (2019) 4 October 2019

ATOMIC PHYSICS

Seconds-scale coherence on an optical clock transition in a tweezer array

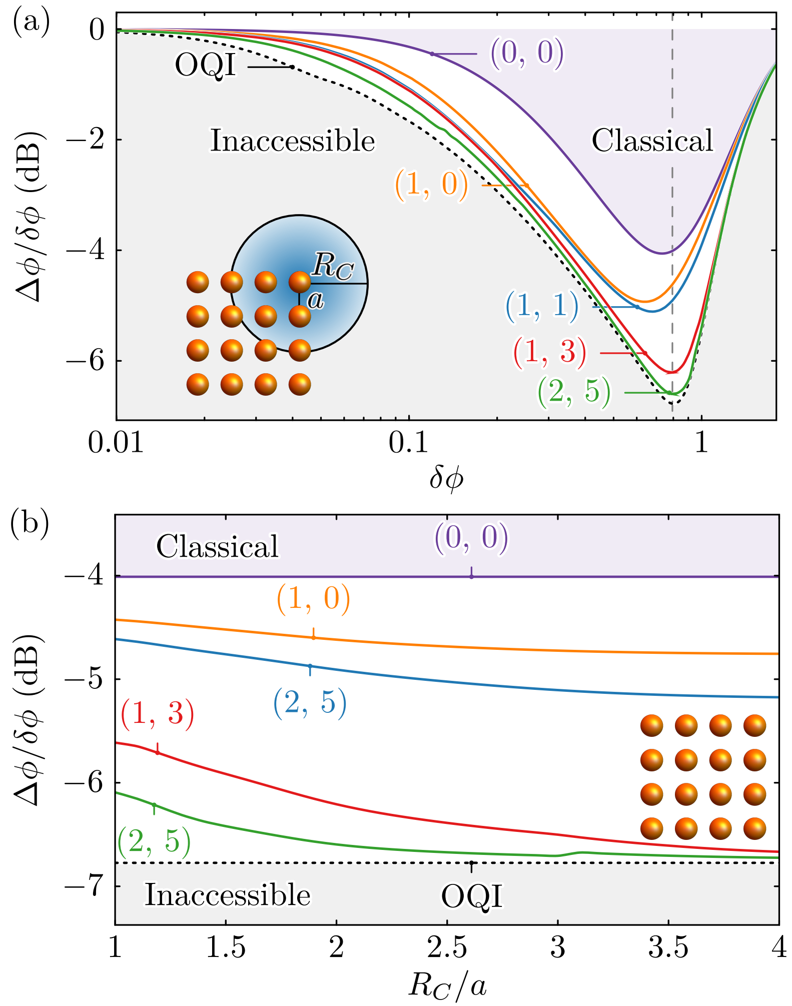
Matthew A. Norcia, Aaron W. Young, William J. Eckner, Eric Oelker, Jun Ye, Adam M. Kaufman*



$$V_{ij} = \frac{R_C^6}{R_C^6 + |\boldsymbol{r}_i - \boldsymbol{r}_j|^6}$$

finite range Rydberg interactions

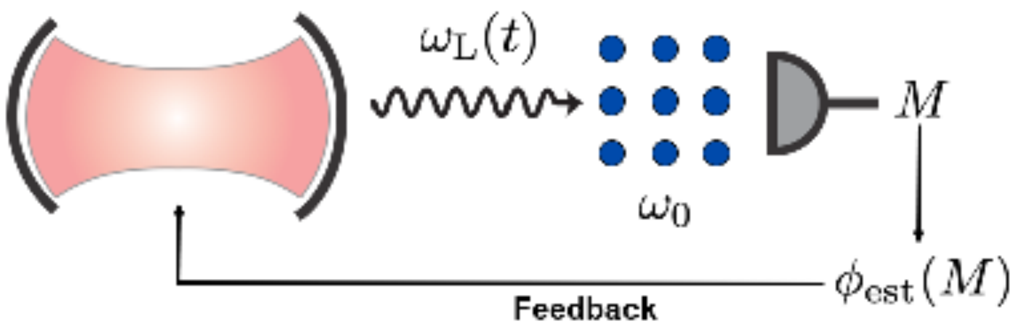
see also Endres (Caltech), …



- `On-device' optimization in presence of decoherence & imperfections

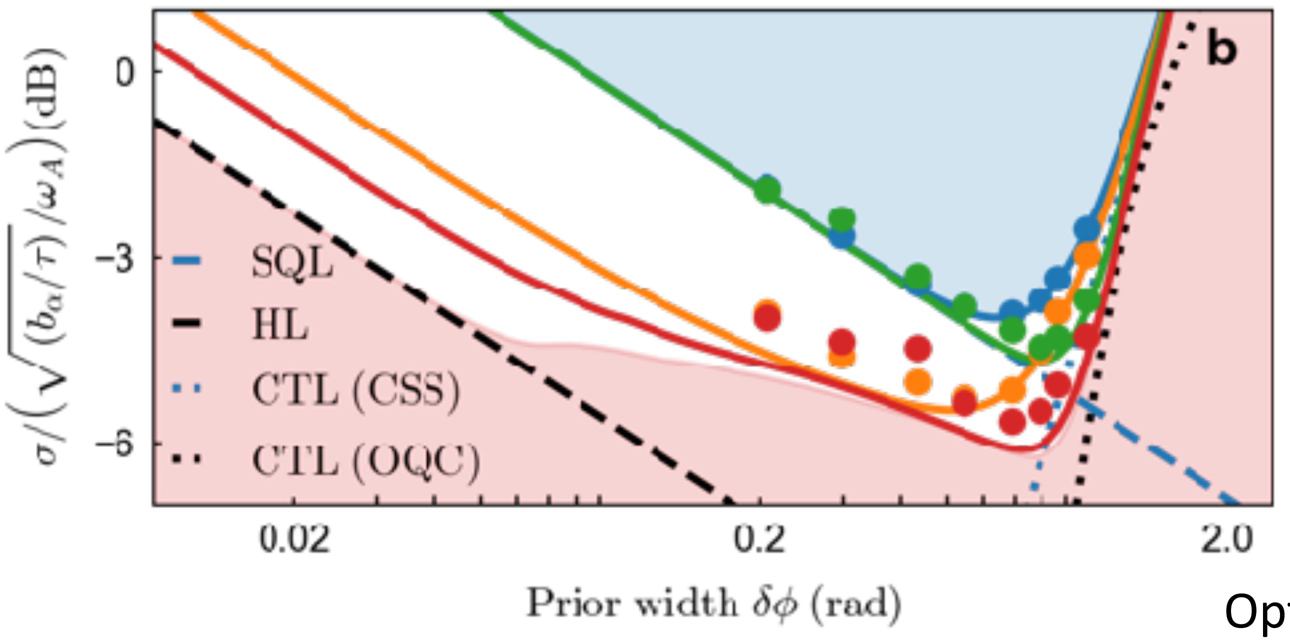
T.-X. Zheng, … P. C. Maurer, *Preparation of Metrological States in Dipolar-Interacting Spin Systems*, Npj Quantum Information (2022).

Improving clocks



$$\sigma(\tau) = \frac{1}{\omega_A}\frac{\Delta\phi_M}{T_R}\sqrt{\frac{T_R}{\tau}} = \frac{1}{\omega_A}\frac{\Delta\phi_M}{T_R\sqrt{n}}$$

$$\Delta\phi_M = \Delta\phi / \sqrt{1 - \left(\frac{\Delta\phi}{\delta\phi}\right)^2} = \sqrt{\frac{\xi_W^2}{N}}$$

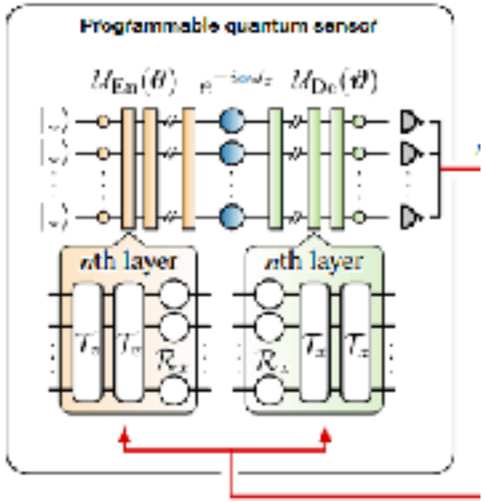


N	Approach	(1,0)	(1,2)
12	Theory	1.49(0) dB	2.13(0) dB
	Direct	1.38(1) dB	1.75(2) dB
26	Theory	2.12(0) dB	2.70(0) dB
	Direct	1.47(8) dB	2.02(8) dB
	Optimizer	1.54(9) dB	1.77(8) dB
362	Theory	4.53(0) dB	7.50(0) dB

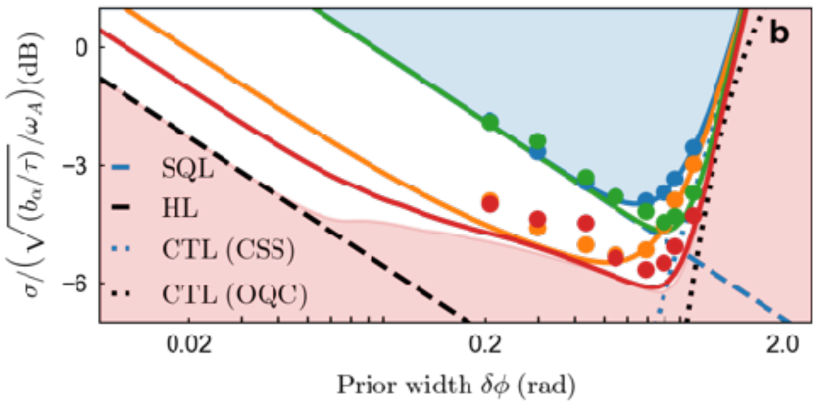
Optimized sequences' longer Ramsey times reduce Dick effect

Conclusion & Outlook

- Optimal quantum metrology & parameter estimation with variational quantum circuits
  - cost function optimized with low-depth variational circuits native to device
  - on-device optimization



Trapped ion experiment



- Optimization in classically inaccessible regime

Science **366**, 93–97 (2019) 4 October 2019

ATOMIC PHYSICS

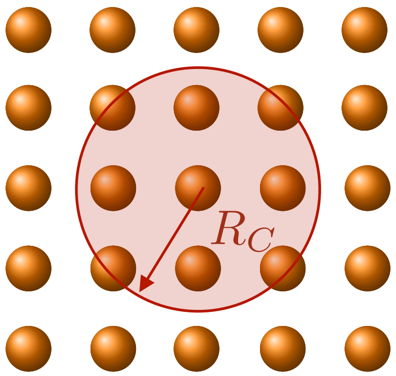
Seconds-scale coherence on an optical clock transition in a tweezer array

Matthew A. Norcia, Aaron W. Young, William J. Eckner, Eric Oelker, Jun Ye, Adam M. Kaufman*

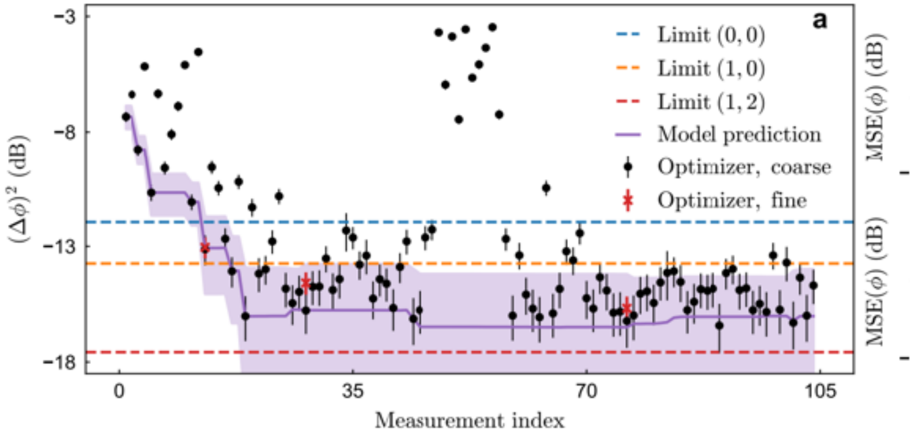
$$V_{ij} = \frac{R_C^6}{R_C^6 + |\boldsymbol{r}_i - \boldsymbol{r}_j|^6}$$

finite range Rydberg interactions

see also Endres (Caltech), …



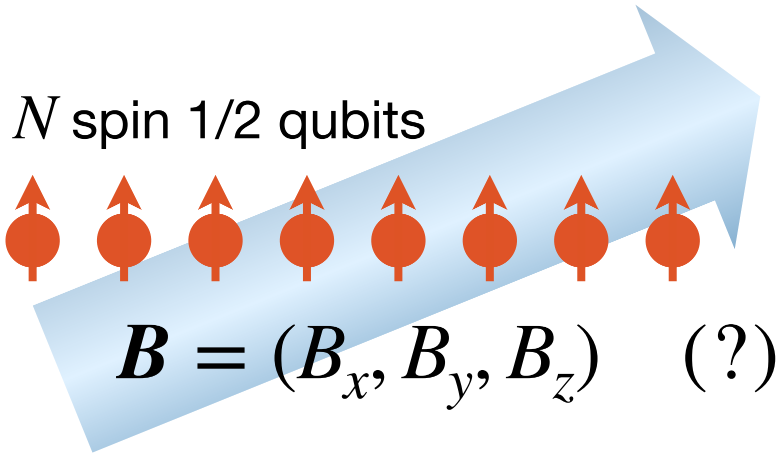
complexity of quantum many-body problem !



Outlook

- Single → **Multi-parameter** q-metrology and field sensing

R Kaubruegger, A Sankar, D Vasilyev & PZ, PRX Quantum (2023)



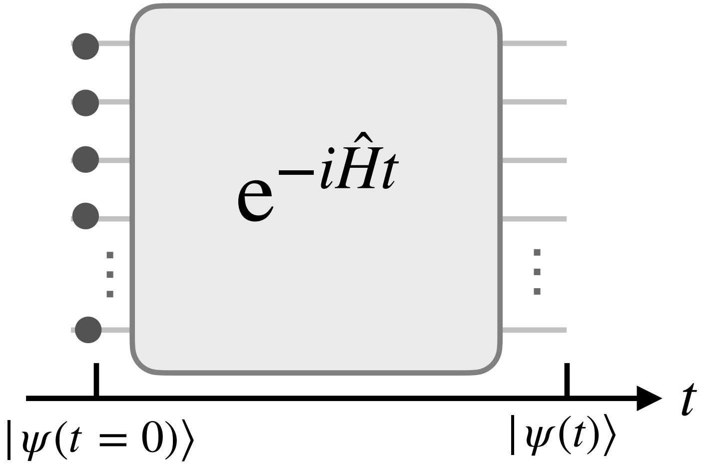
- Parameter Estimation in Quantum Metrology vs. **Hamiltonian Learning** in Quantum Simulation

L Pastori et al., PRX Quantum (2022)

$$U(\boldsymbol{\phi}) = \exp\left[-i(\phi_x J_x + \phi_y J_y + \phi_z J_z)\right]$$

non-commuting

many-body quench



$$\hat{H} = \sum_{j=1}^{N-1} \sum_{\mu=x,y,z} J_j^{\mu} \hat{\sigma}_j^{\mu} \hat{\sigma}_{j+1}^{\mu} + \sum_{j=1}^{N} B_j^x \hat{\sigma}_j^x + \ldots \quad (?)$$

learn the structure and couplings of the many-body Hamiltonian from *many* preparations and measurements