Textbook Quantum Measurements.

- Projective
- Instanteous
- Irreversible.

$\longrightarrow$ Need new concepts.

Stern-Gerlach apparatus.

$$H = -\vec{\mu}\cdot\vec{B} \longrightarrow \text{Force}$$

$$F_z = \mu_z \frac{\partial B}{\partial z}$$

Q.M. $\mu_z \to \pm \mu_B\, g/2$

takes 2 values.

$$\{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$$

$$\hat{\mu} = \mu_z \cdot \hat{\sigma}_z$$

Measurements. $\longrightarrow$ projection operator.

$\{\hat{\Pi}_j\}$ j delineates possible # of outcomes.

① $\hat{\Pi}_i \hat{\Pi}_j = \delta_{ij} \hat{\Pi}_j$

Given some state $|\psi\rangle$

"Born rule"

② $\sum_j \hat{\Pi}_j = \underline{\mathbb{1}}$

$P_j = \langle\psi|\hat{\Pi}_j|\psi\rangle$

③ $\hat{\Pi}_j^\dagger = \hat{\Pi}_j$

Eckert $\rightarrow \Pi_j \longrightarrow |x\rangle\langle x|$

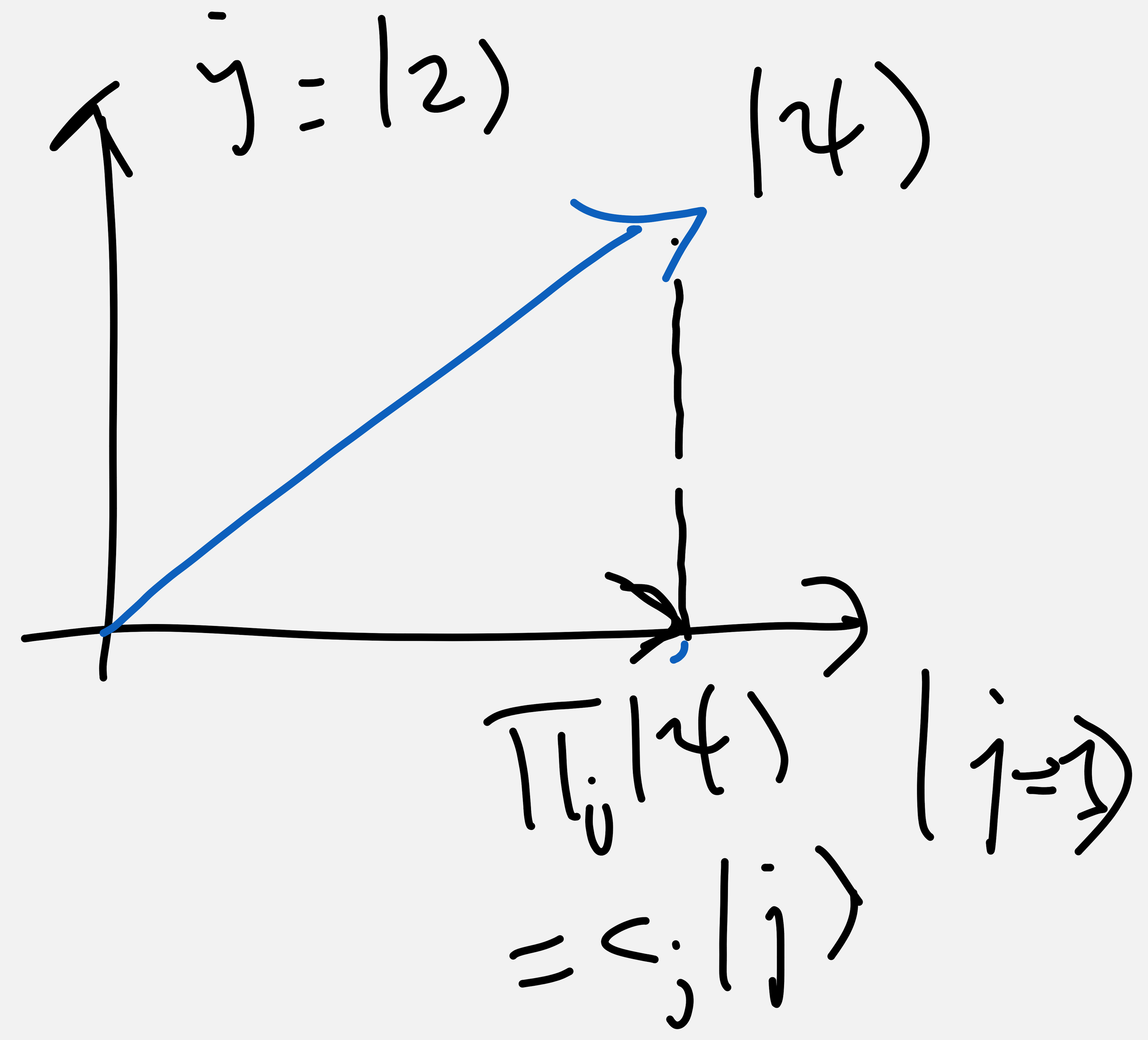
$P(x) = \langle\psi|x\rangle\langle x|\psi\rangle = |\psi(x)|^2$

① + ② + ③ $\hat{\Pi}_j = \underline{|j\rangle\langle j|}$

new basis $|j\rangle$ orthonormal.

$\hat{\Pi}_j = |j\rangle\langle j|$; $|\psi\rangle = \sum_k c_k |k\rangle$; $\sum_k |c_k|^2 = 1$

$\hat{\Pi}_j |\psi\rangle = c_j |j\rangle$



Collapse of wave function.

obtain result "j"

$|\psi\rangle \mapsto e^{i\phi}|j\rangle$ (normalized state)

$\text{prob}(j) = \langle\psi|\Pi_j|\psi\rangle = |\langle j|\psi\rangle|^2$

observable

$\hat{O} = \sum_j \lambda_j \Pi_j$

$\hat{O} = \lambda_A \underline{(\hat{\Pi}_1 + \hat{\Pi}_2)} + \lambda_B \hat{\Pi}_{rest}$

$$(\Pi_1 + \Pi_2)|\psi\rangle = (\Pi_1 + \Pi_2) \sum_j c_j |j\rangle$$

$$= \frac{c_1|1\rangle + c_2|2\rangle}{\sqrt{|c_1|^2 + |c_2|^2}}$$

$$\hat{O} = \sum_j \lambda_j \Pi_j$$

$$\langle\psi|\hat{O}|\psi\rangle = \sum_j \lambda_j \, \mathrm{Prob}(j)$$

$$\langle x \rangle = (-d)\,\frac{\langle \psi_f | \overset{\hat{\sigma}_z}{\overset{\uparrow}{\hat{A}}} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$$

divergent

distance between the beams

$$d = (n_o - n_E)T$$

index of refraction difference.

thickness.

before crystal

$$|\Psi\rangle = |\phi\rangle \otimes |\psi\rangle$$

polarization

transverse profile

$$|\phi\rangle = a|H\rangle + b|V\rangle.$$

$$|\psi\rangle = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{1}{4}} e^{-\frac{x^2}{4\sigma^2}}$$

after crystal

before screen

$$\langle x|\Psi'\rangle = a|H\rangle\,\phi(x) + b|V\rangle\,\phi(x+d)$$

joint state

Entangled

Q2

what is the probability (density) of finding the photon at position x?

$$\text{prob}(x)\,dx = \langle \Psi | \Pi_x | \Psi \rangle$$

$$= |\alpha|^2 \underbrace{|\phi(x)|^2}_{|\phi_u(x)|^2} + |\beta|^2 \underbrace{|\phi(x+d)|^2}_{|\phi_l(x)|^2} \longleftrightarrow \text{Law of total probability.}$$

Abstract away the transverse degree of freedom

$$P(x) = \left(a^* \langle H| + b^* \langle V|\right) \Omega_x^\dagger \Omega_x \left(a|H\rangle + b|V\rangle\right)$$

$$\Omega_x = \begin{pmatrix} \phi_H(x) & 0 \\ 0 & \phi_V(x) \end{pmatrix} = \begin{pmatrix} \phi(x) & 0 \\ 0 & \phi(x+d) \end{pmatrix}$$

in $|H\rangle, |V\rangle$ basis.

polarization rotation:

$$|\psi'\rangle = \frac{\Omega_x |\psi\rangle}{\| \Omega_x |\psi\rangle \|}$$

Simple example of General rule (formalism).

→ Kraus operator:

$\{\Omega_j\}$; $j = 1, \ldots, M$.

obeys $\sum_j \Omega_j^\dagger \Omega_j = \mathbb{1}$

j - denotes outcome.

Note $\Omega_j \Omega_k \neq \delta_{jk} \Omega_j$

$\Omega_j^\dagger \Omega_j$ is a positive operator.

$\langle\psi|\Omega_j^\dagger \Omega_j|\psi\rangle$

$$\mathrm{prob}(j) = \langle\psi|\Omega_j^\dagger \Omega_j|\psi\rangle$$

Conditional state

$$|\psi'\rangle_j = \frac{\Omega_j|\psi\rangle}{\||\Omega_j|\psi\rangle\||}$$

Generalizes the Born rule

$$|\psi\rangle \longrightarrow \rho = |\psi\rangle\langle\psi|$$

$$P_j = \text{prob}(j) = \text{Tr}[\Omega_j \rho \Omega_j^\dagger]$$

$$\rho'_{(j)} = \frac{\Omega_j \rho \Omega_j^\dagger}{P_j}$$

Bayes rule

diagonal matrix elements

$$\langle l | \rho'_j | l \rangle = \frac{p(j|l)\rho_{ll}}{P_j}$$

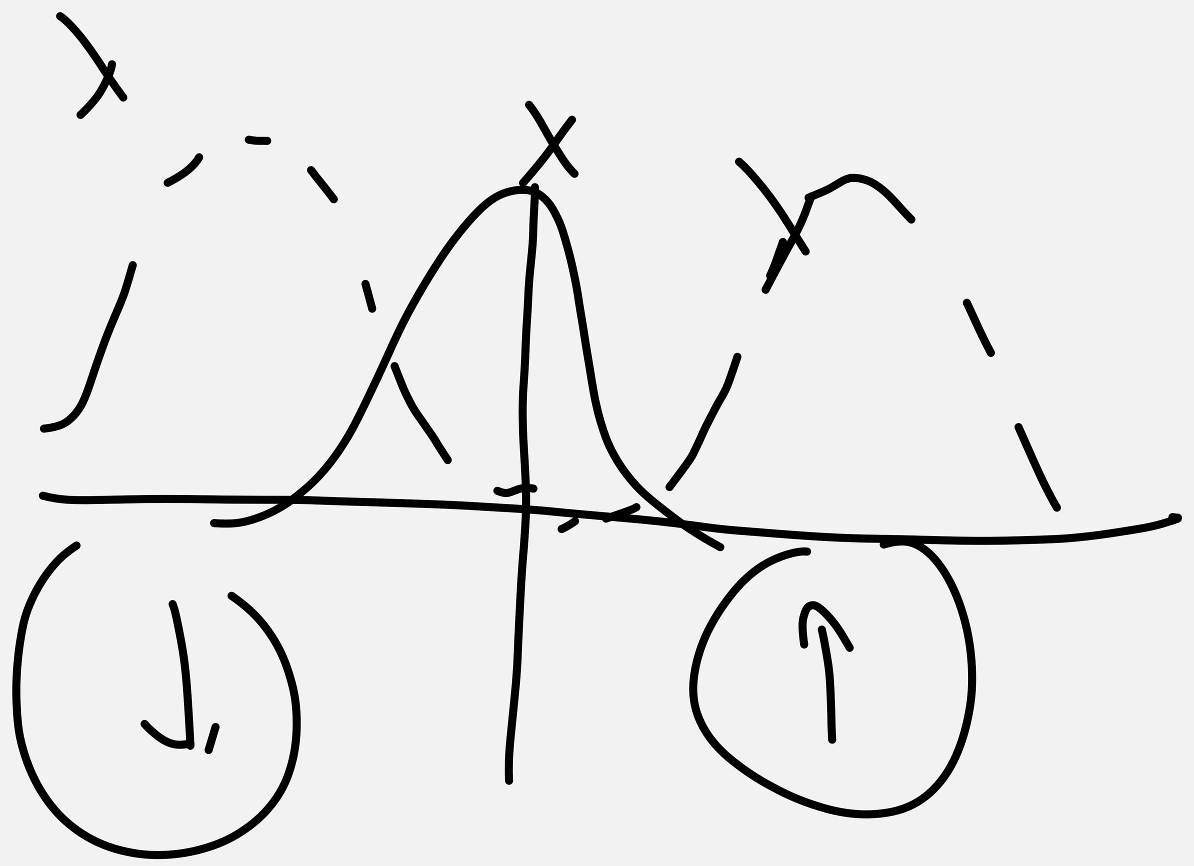
$p(j|l)$ is just the conditional probability of finding result j, given state is in l.

Off diagonal matrix elements:

$$\left(\rho^{(j)}\right)_{\ell,\ell'} = \frac{\sqrt{p(j|\ell)}\sqrt{p(j|\ell')}\ \rho_{\ell,\ell'}}{P_j}$$

Von Neumann

meter: free particle + spin $\frac{1}{2}$ system



$$H_{SM} = g\,\delta(t)\,\hat{\sigma}_z \otimes \hat{p}$$

$$U_{SM} = T e^{-i\int^t dt'\, H_{SM}} = e^{-i\frac{g}{\hbar}\hat{\sigma}_z \otimes \hat{p}}$$

$|\uparrow\rangle$ or $|\downarrow\rangle$

$\sigma_z \to \pm 1$

$$\langle x| e^{\pm \frac{ig}{\hbar}\hat{p}} |\psi\rangle = \psi(x \pm g)$$

poke a hole at position x.

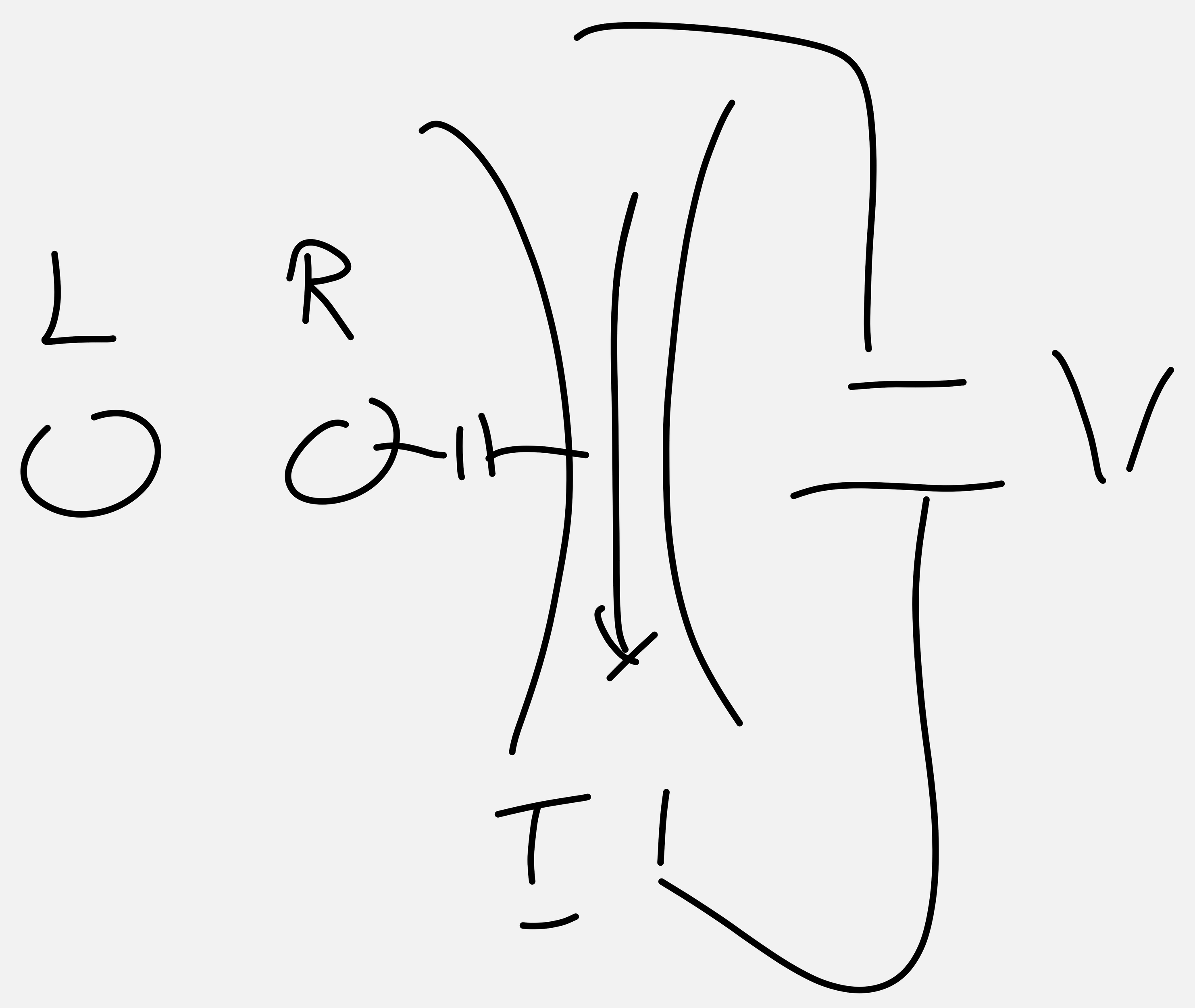
detection $\mapsto$ $\Pi_x |\Psi\rangle$

<u>Question</u>: Suppose the photon passes. what is polarization?

$$|\psi''\rangle = \Big(c_V(x)|V\rangle + c_H(x)|H\rangle\Big)|x\rangle$$

$$|\psi''\rangle = \frac{a\,\phi(x)\,|H\rangle + b\,\phi(x+d)\,|V\rangle}{\sqrt{|a|^2|\phi|^2 + |b|^2|\phi(x+d)|^2}}$$

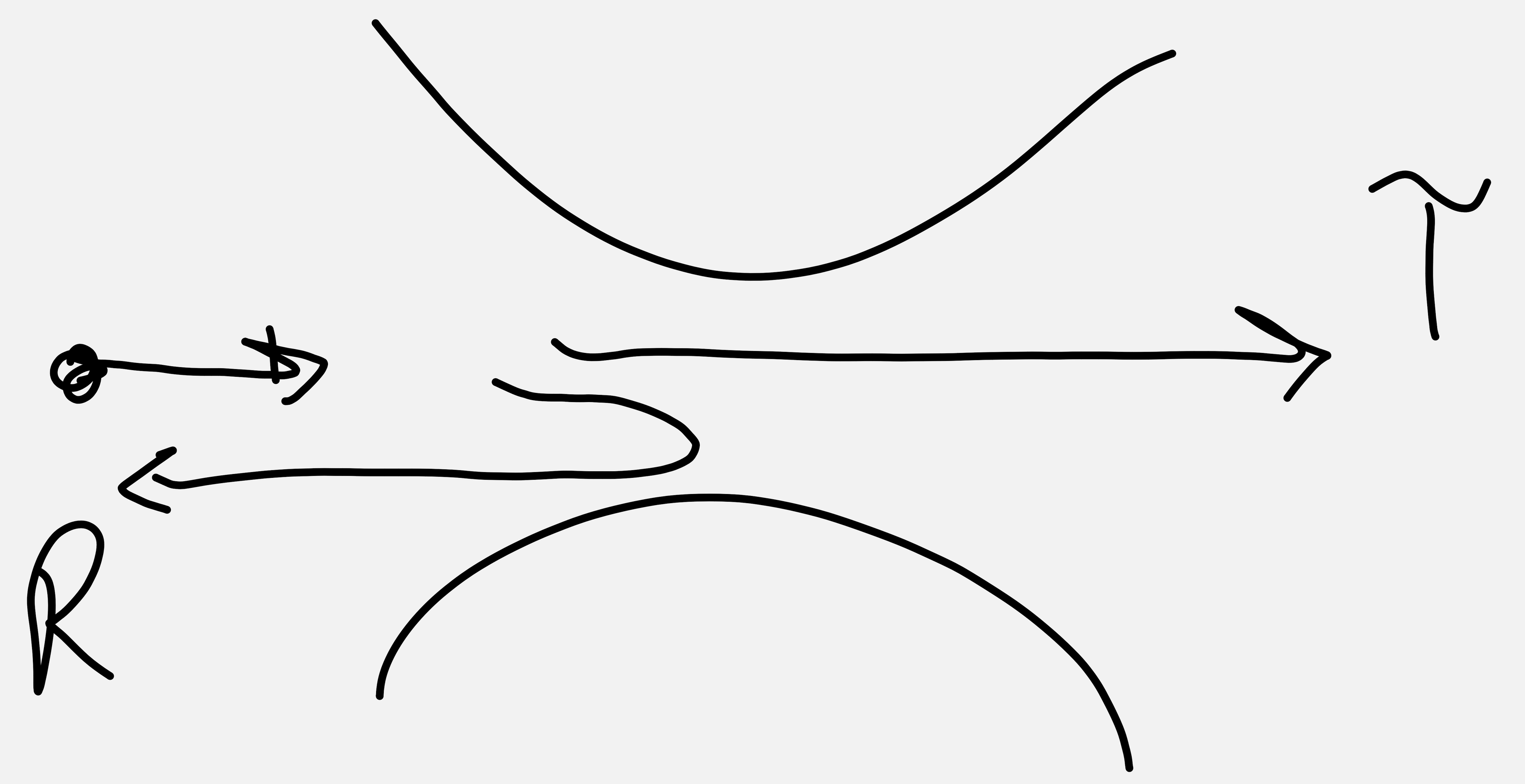
Quantum Dot // Quantum point contact.



Depending on dot position

$$\bar{I} = I_L , I_R$$

$$I_L - I_R << \frac{I_L + I_R}{2}$$



Assume weakly responding limit

Landauer - Büttiker

$$G = \frac{e^2}{h} T$$

conductance

$I(t) = I_0 + \xi$

Apply voltage V: $\tau_0 \sim \frac{h}{eV} \ll$ system time scales.

$\int dt \langle \delta I(t)\, \delta I(0) \rangle = S_I$; zero frequency noise power.

Fermions $S_I = (eV)\, 2\frac{e^2}{h} \mathcal{T}(1-\mathcal{T})$

$I = G \cdot V$; $G_{L,R} = \frac{e^2}{h} \mathcal{T}_{L,R}$

Consider $Q = \int_0^T dt\, I(t)$ = random variable average

$$\langle Q_L - Q_R \rangle = \underline{\underline{T}} (I_L - I_R)$$

$$\langle \delta Q^2 \rangle = \int_0^T dt_1\, dt_2 \langle \delta I(t_1)\, \delta I(t_2) \rangle$$

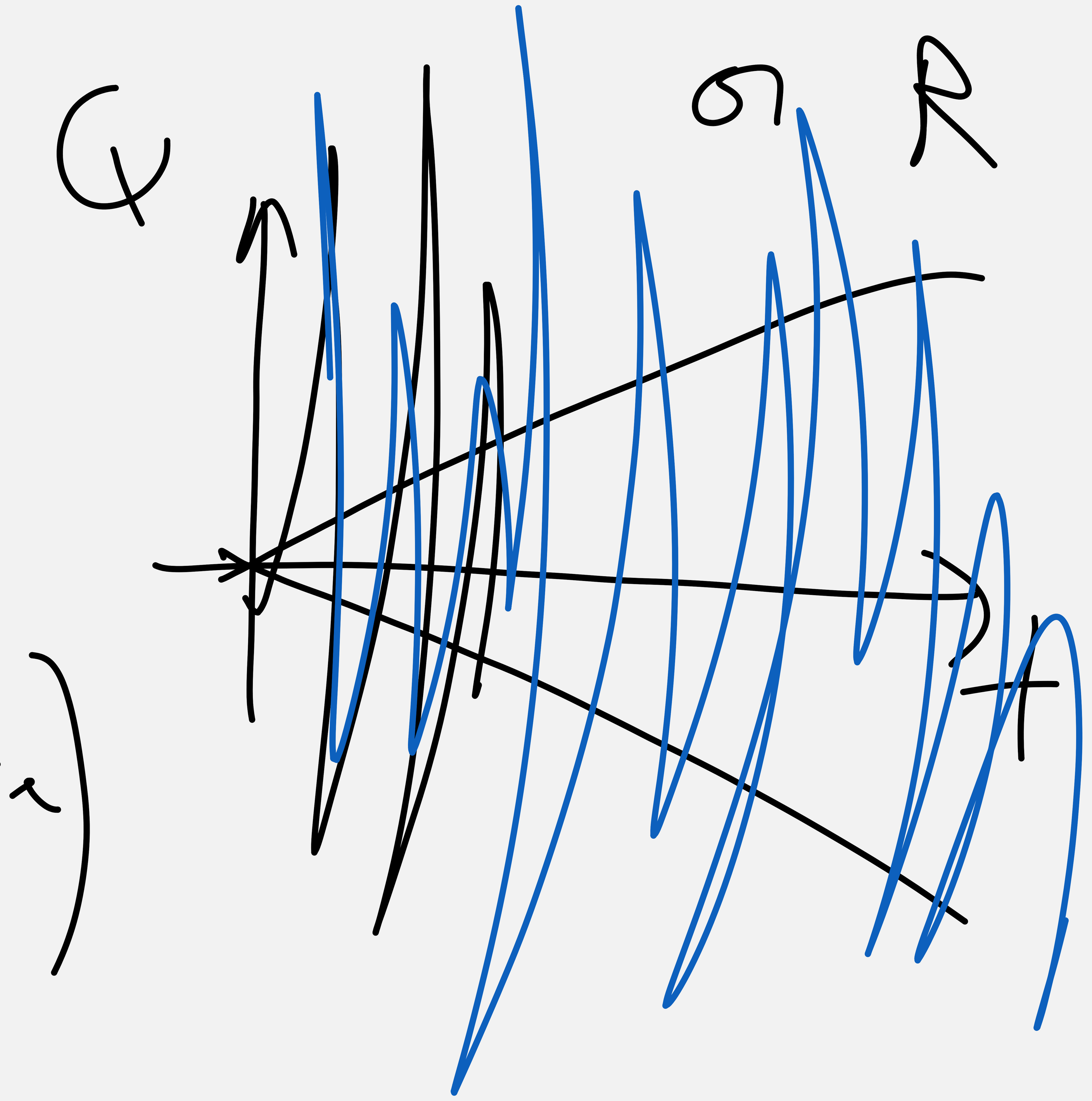
$$\doteq T S_I \qquad \text{(white noise approx.)}$$

want to let T be big enough, so that $\langle Q_L - Q_R \rangle \geq \sqrt{\langle \delta Q^2 \rangle}$

$$T = T_M \qquad T_M^2 (I_L - I_R)^2 \geq S_I T_M$$

Hypothesis

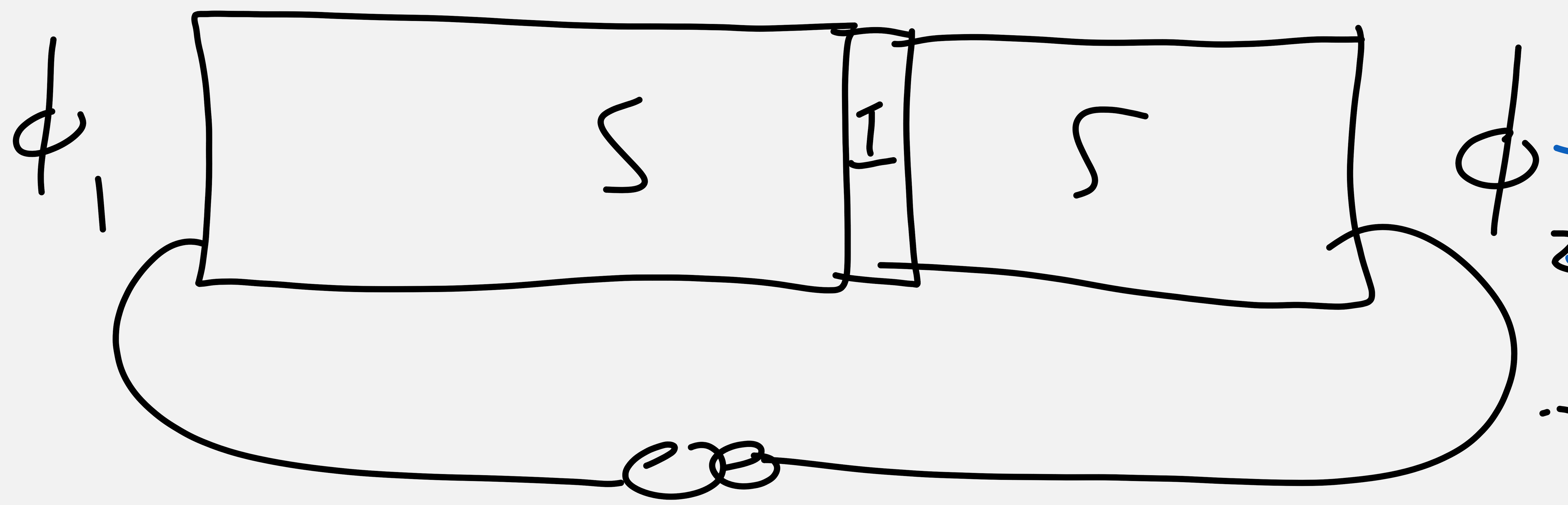
electron is L or R

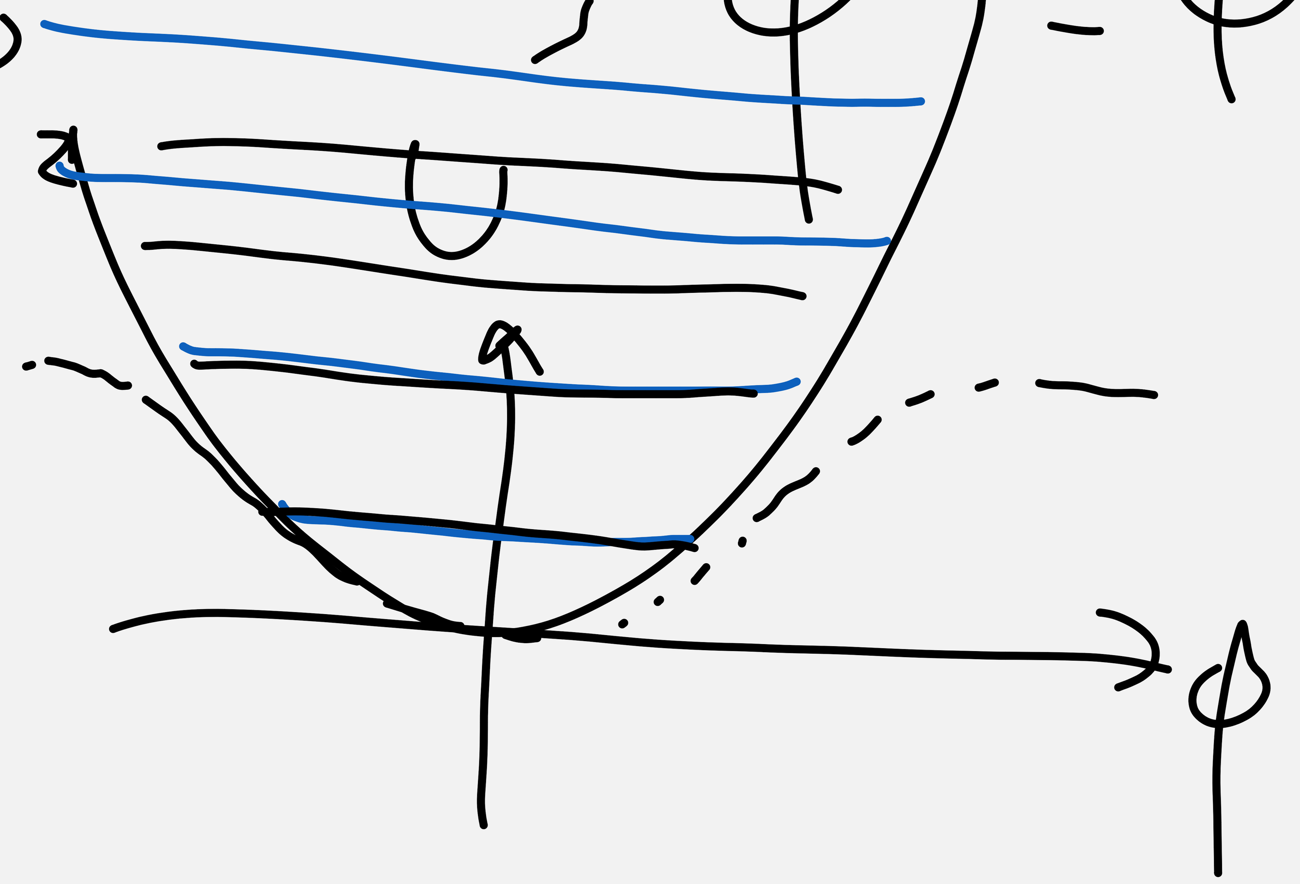


$\Rightarrow$ New time scale!

$$\tau_\mu \sim \frac{S_I}{(I_L - I_R)^2} \longrightarrow \text{new ingredient in formalism.}$$

Josephson junction circuit qubits.





; $\phi = \phi_2 - \phi_1$

Josephson Eqns.
$$\begin{cases} V = \dfrac{\hbar}{2e}\dfrac{\partial \phi}{\partial t} \\ I = I_c \sin\phi \end{cases}$$

$$\text{Energy stored} = \int I V\,dt = -I_c \frac{\hbar}{2e} \cos\phi$$

Hamiltonian

2-level approx.

$$H = \hbar\omega_c a^\dagger a + \hbar\omega_q \frac{\hat{\sigma}_z}{2} + \hbar g (a^\dagger \sigma_- + a \sigma_+)$$

Jaynes-Cummings.

Dispersive limit $\Delta = \omega_q - \omega_c$; detuning.

$$\Delta \gg g$$

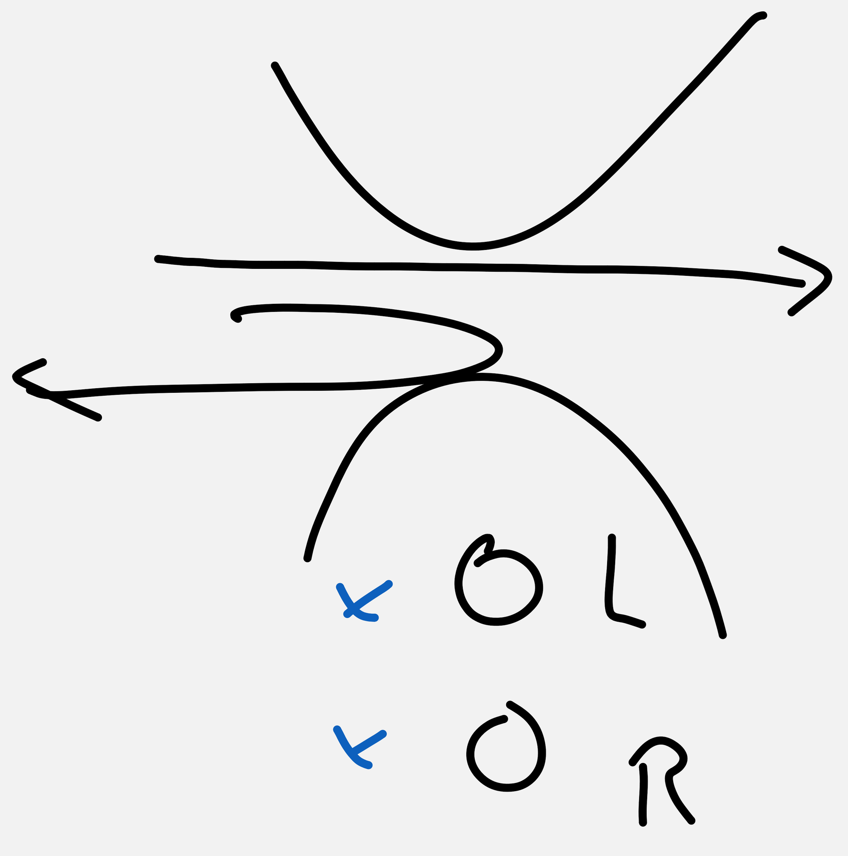
$$H \simeq \hbar\left(\omega_c + \frac{g^2}{\Delta}\sigma_z\right)\left(a^\dagger a + \frac{1}{2}\right) + \frac{1}{2}\hbar\omega_q \sigma_z$$

$$\omega_c \longrightarrow \omega_c + \frac{g^2}{\Delta}\sigma_z \qquad \chi$$

Very similar to QPC readout.

Build up description from elementary events.

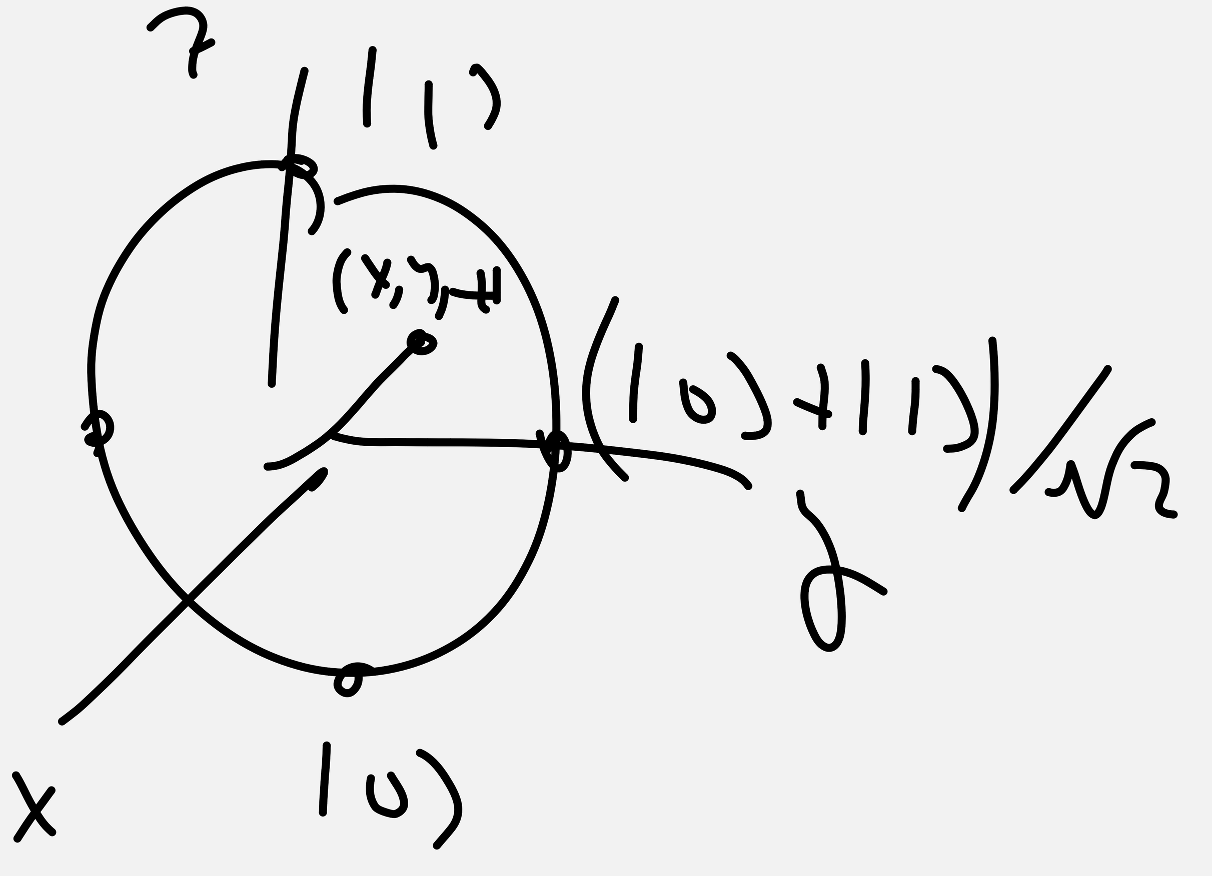
Source



drain

(x,y,z) Bloch coordinates

$$H = \frac{\varepsilon}{2}\sigma_z + \frac{\Delta}{2}\sigma_x$$



Kraus operators.

$$\Omega_S = \begin{pmatrix} r_L & 0 \\ 0 & r_R \end{pmatrix}; \quad \Omega_D = \begin{pmatrix} t_L & 0 \\ 0 & t_R \end{pmatrix}$$

Equations of motion.

$$\dot{x} = -\frac{x}{2\tau_m} - \frac{xz\xi}{\sqrt{\tau_m}} - \varepsilon y.$$

$$\dot{y} = -\frac{y}{2\tau_m} - \frac{yz\xi}{\sqrt{\tau_m}} + \varepsilon x - \Delta z$$

$$\dot{z} = \frac{1-z^2}{\sqrt{\tau_m}}\xi + \Delta y$$

Itô interpretation.

$$r = \frac{I - I_0}{I_1 - I_0} = z(t) + \sqrt{\tau_m}\,\xi.$$

$$\langle \xi(t)\xi(0)\rangle = \delta(t)$$

Stochastic Path integral.

Paths in Hilbert space.

$(x, y, z) \longrightarrow \vec{q}$

$(\vec{q}_0, \vec{q}_1, \vec{q}_2 \cdots\cdots \vec{q}_N)$

$(r_0, r_1, r_2 \cdots\cdots r_N)$

$$\underline{P}(\{\vec{q}\}, \{r\} \mid t, \vec{q}_i, \vec{q}_f) = \int \mathcal{D}p \, e^{S}$$

$$S = \int_0^t dt' \left[-i\vec{p} \cdot \left(\dot{\vec{q}} - \mathcal{L}(q, r) \right) + \underline{F(q, r)} \right]$$

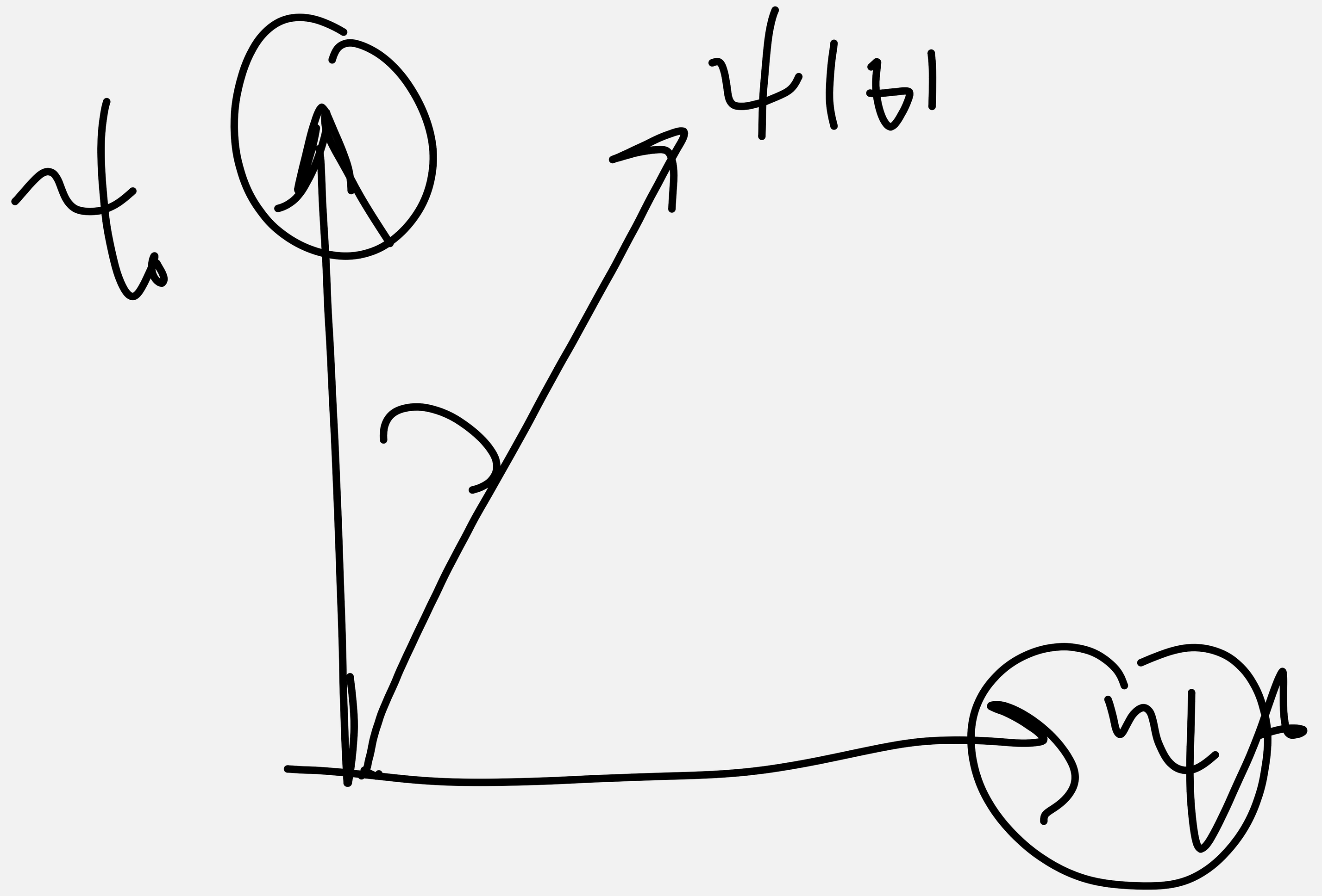
$\mathcal{L}, F$

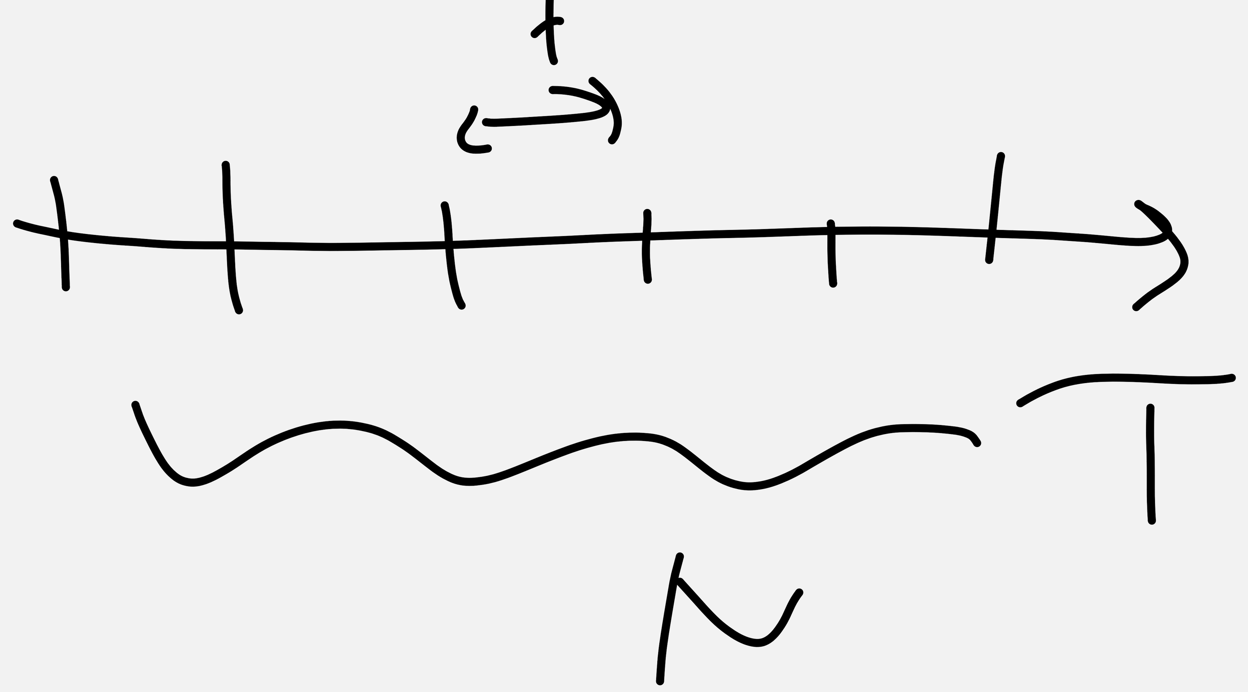
encode backaction + probability density.

Quantum Zeno effect → Many fast measurements freeze state.

$$|\psi_t\rangle = e^{-iHt}|\psi_0\rangle$$

$$\{|\psi_0\rangle, |\psi_0^\perp\rangle\}$$





$$P_t = \left|\langle\psi_0| e^{-iHt} |\psi_0\rangle\right|^2$$

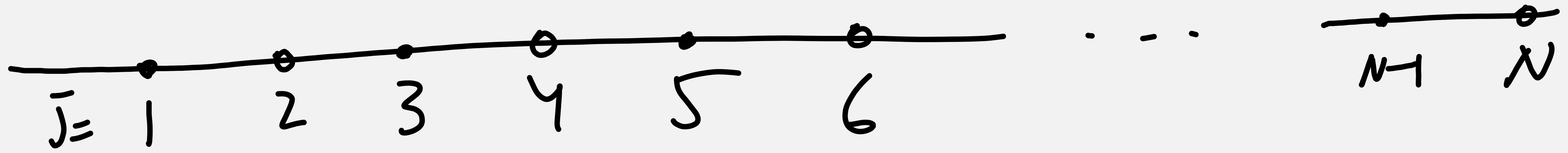
$$\approx 1 - t^2 (\Delta H)^2/\hbar^2$$

$$\Delta H^2 = \mathrm{Var}[H]_{\psi_0}$$

$$P_{T,N} = \left(1 - \frac{T^2}{N^2}\frac{(\Delta H)^2}{\hbar^2}\right)^N \simeq \exp\left[-\frac{T^2(\Delta H)^2}{\hbar^2 N}\right] \to 1$$

Adventures in Lattice Fermions

For N sites, have 2^N states



N is even so the lattice is bipartite

$\{c_j^\dagger, c_{j'}\} = \delta_{jj'}$; $\{c_j, c_{j'}\} = 0$,

One site $j=1$; $c_1|0\rangle = 0$, $|1\rangle = c_1^\dagger|0\rangle$, 2-state system

$$c_1 = \sigma_1^- ; \quad c_1^\dagger = \sigma_1^+ ; \quad \sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}\left(\sigma^x + i\sigma^y\right)$$

$$q_1 = \frac{1}{2}\left[c_1^\dagger, c_1\right] = \frac{1}{2}\sigma_1^z ,$$

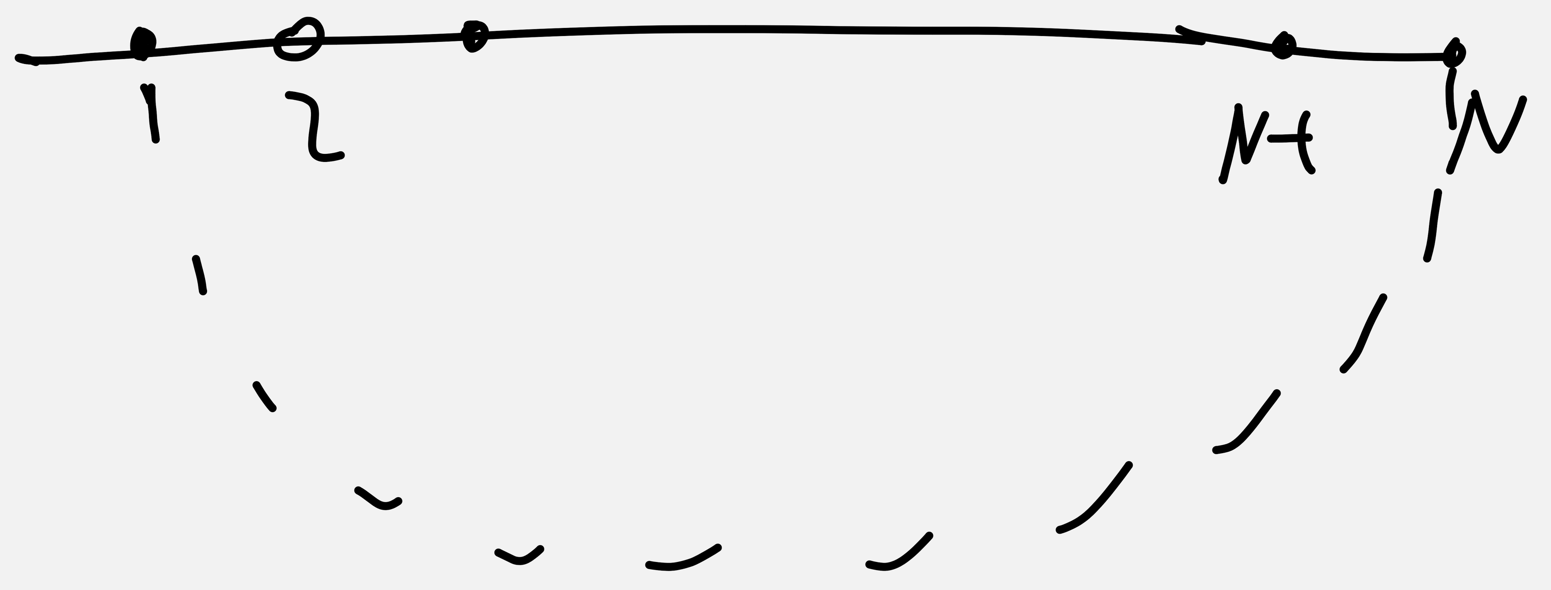
Hopping Hamiltonian: PBC $c_{N+1} = c_1$;

$$H_t = -t \sum_{j=1}^{N-1} \left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j \right) - t \left(c_N^\dagger c_1 + c_1^\dagger c_N \right)$$

$t > 0$; Can flip the sign of t by

$c_j \to (-1)^j c_j$;

Either open or closed



Fourier basis

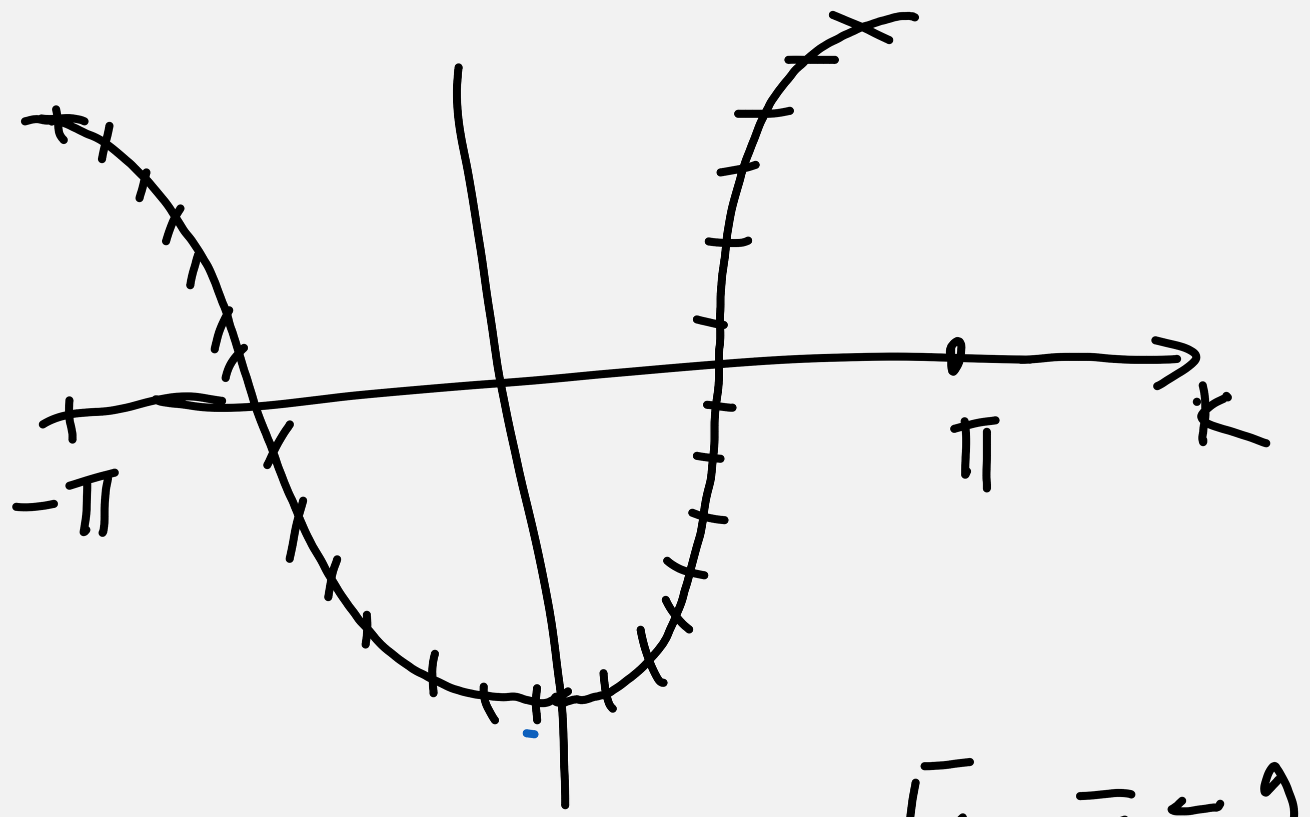
$$c_j = \frac{1}{\sqrt{N}} \sum_k e^{ikj} c_k ,$$

$k = \frac{2\pi n}{N}$, $k \sim k + 2\pi$; $n = -\frac{N}{2}+1, -\frac{N}{2}+2, \dots, 0, \dots, \frac{N}{2}$, Jordan-Wigner trafo.

$$H = -2t \sum_k (\cos k)\, c_k^\dagger c_k ;$$

$$c_j = \left(\prod_{n=1}^{j-1} \sigma_n^z \right) \sigma_j^- ; \quad \{c_j, c_k\} = 0 ;$$

$$c_j^\dagger = \left(\prod_{n=1}^{j-1} \sigma_n^z \right) \sigma_j^+ ,$$



Ground state

$$\prod_{k \in [-\frac{\pi}{2}, \frac{\pi}{2})} c_k^\dagger |0\rangle ; \quad \text{As } N \to \infty$$

$$\Delta k = \frac{2\pi}{N} \Delta n$$

Lieb, Schultz, Mattis

$$E_0 = -2t \frac{N}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos k \, dk = -\frac{2t}{\pi} N ,$$

XY model ;

$$H = -\frac{t}{2} \sum_j \left(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y \right) =$$

$$= -t \sum_j \left(\sigma_j^+ \sigma_{j+1}^- + \sigma_j^- \sigma_{j+1}^+ \right)$$

Acts on 2^N states of N qubits; $[\sigma_j^+, \sigma_{j+2}^-] = 0$;

Hubbard chain

$(c_{j\uparrow}; c_{j\downarrow}) \sim c_{j,\sigma}$

$\sigma = (\uparrow, \downarrow)$

$c_{j,\sigma} \to i^{j} c_{j,\sigma}$

change real to Im hopping parameter. $SU(2)_{spin}$ is manifest, also has hidden $SU(2)_{pseudospin}$

$$H = it \sum_j \left(c^\dagger_{j+1,\sigma} c_{j,\sigma} - c^\dagger_{j,\sigma} c_{j+1,\sigma} \right) = t \sum T_{j,j+1} + U \sum_j n_{j\uparrow} n_{j\downarrow}$$

Free "staggered" lattice Dirac fermion

$$R \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ \\ c_N \end{pmatrix} ; \quad R \in SO(N); \; R^T R = I_{N\times N},$$

"Scars are $SO(N)$ singlets"

Pseudospin generators :

$$SU(2)_{\text{pseudo}} \begin{cases} \eta = \sum_j c_{j\uparrow} c_{j\downarrow} = \frac{1}{2} \sum_j c_{j\alpha} c_{j\beta} \epsilon_{\alpha\beta} \\ \eta^\dagger \\ Q = \frac{1}{2} \sum [c_{j\sigma}^\dagger, c_{j\sigma}] \end{cases}$$

$$[\eta^\dagger, \eta] = Q$$

In 1d : Yang

$(-1)^{j_x + j_y}$ removed by

$c_{j_x, j_y} \to i^{j_x + j_y} c_{j_x, j_y}$,

Yang's ~~eta~~-pairing states

$$(\eta^\dagger)^k |0\rangle \; ; \quad k = 0, \ldots, N$$

Pseudospin $\frac{N}{2}$ multiplet

: