Doping and Probing

The Original Liquid

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Heavy Ion Collisions: What Next?

By recreating droplets of the matter that filled the microseconds-old universe in ultrarelativistic heavy ion collisions, we have discovered a liquid that, as far as we now know, is:

- The first liquid that ever existed; the "original liquid"...
- The liquid from which the protons and neutrons in today's universe formed, as the liquid fell apart into mist.
- At a few trillion degrees, the hottest liquid that has ever existed.
- The earliest complex form of matter.
- The most liquid liquid that has ever existed, with a specific viscosity $\eta/s \sim 0.1$.
- In a sense the simplest form of complex matter, namely in the sense that it is "close" to the fundamental degrees of freedom of the standard model.

All great discoveries pose new challenges. My lectures on Wednesday will be about some recent advances and What Next?, namely the challenges for the decade to come. But first, today's intro will be vintage 2015...

Quark-Gluon Plasma

- The $T \to \infty$ phase of QCD. Entropy wins over order; symmetries of this phase are those of the QCD Lagrangian.

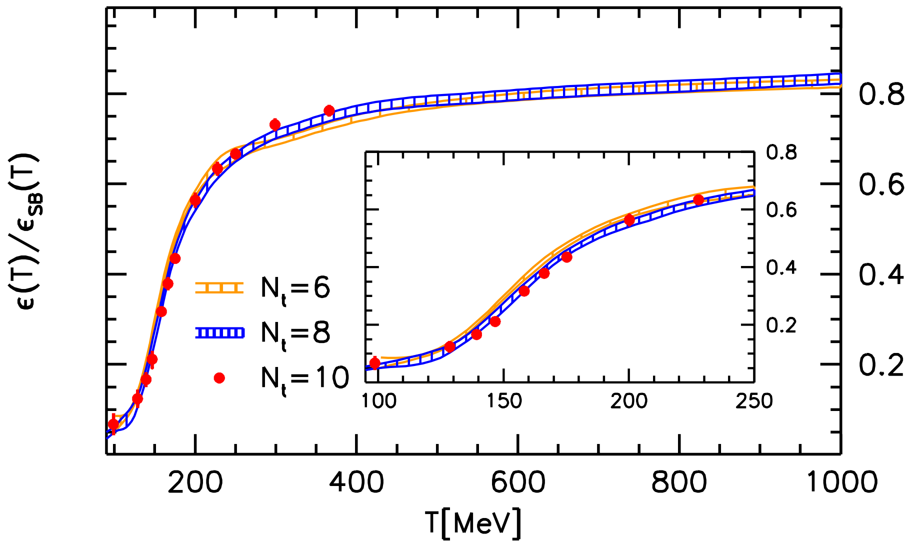
- Asymptotic freedom tells us that, for $T \to \infty$, QGP must be weakly coupled quark and gluon quasiparticles.

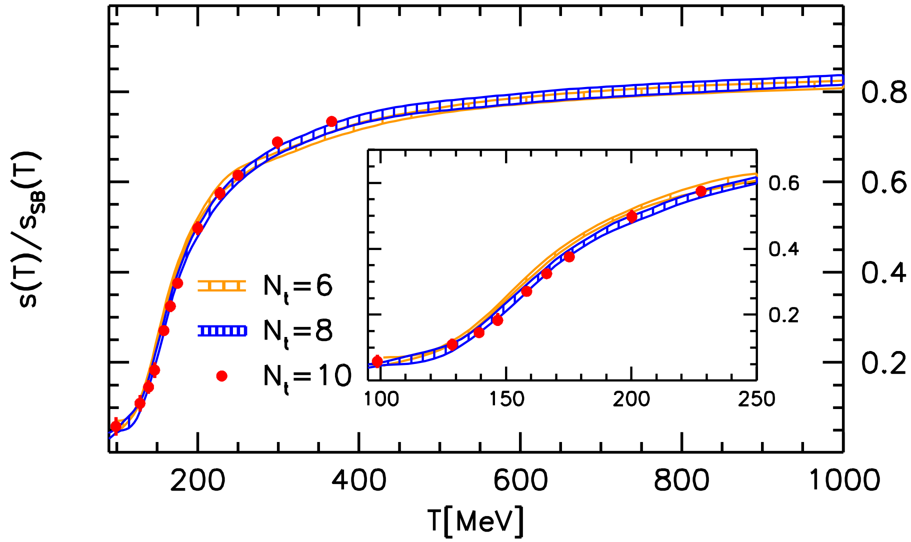
- Lattice calculations of QCD thermodynamics reveal a smooth crossover, like the ionization of a gas, occurring in a narrow range of temperatures centered at a $T_c \simeq 150$ MeV $\simeq 2$ trillion °C ~ 20 μs after big bang. At this temperature, the QGP that filled the universe broke apart into hadrons and the symmetry-breaking order that characterizes the QCD vacuum and gives mass to hadrons developed.

- Heavy ion collisions produce droplets of QGP at temperatures several times T_c, reproducing the stuff that filled the few-microseconds-old universe.

QGP Thermodynamics on the Lattice

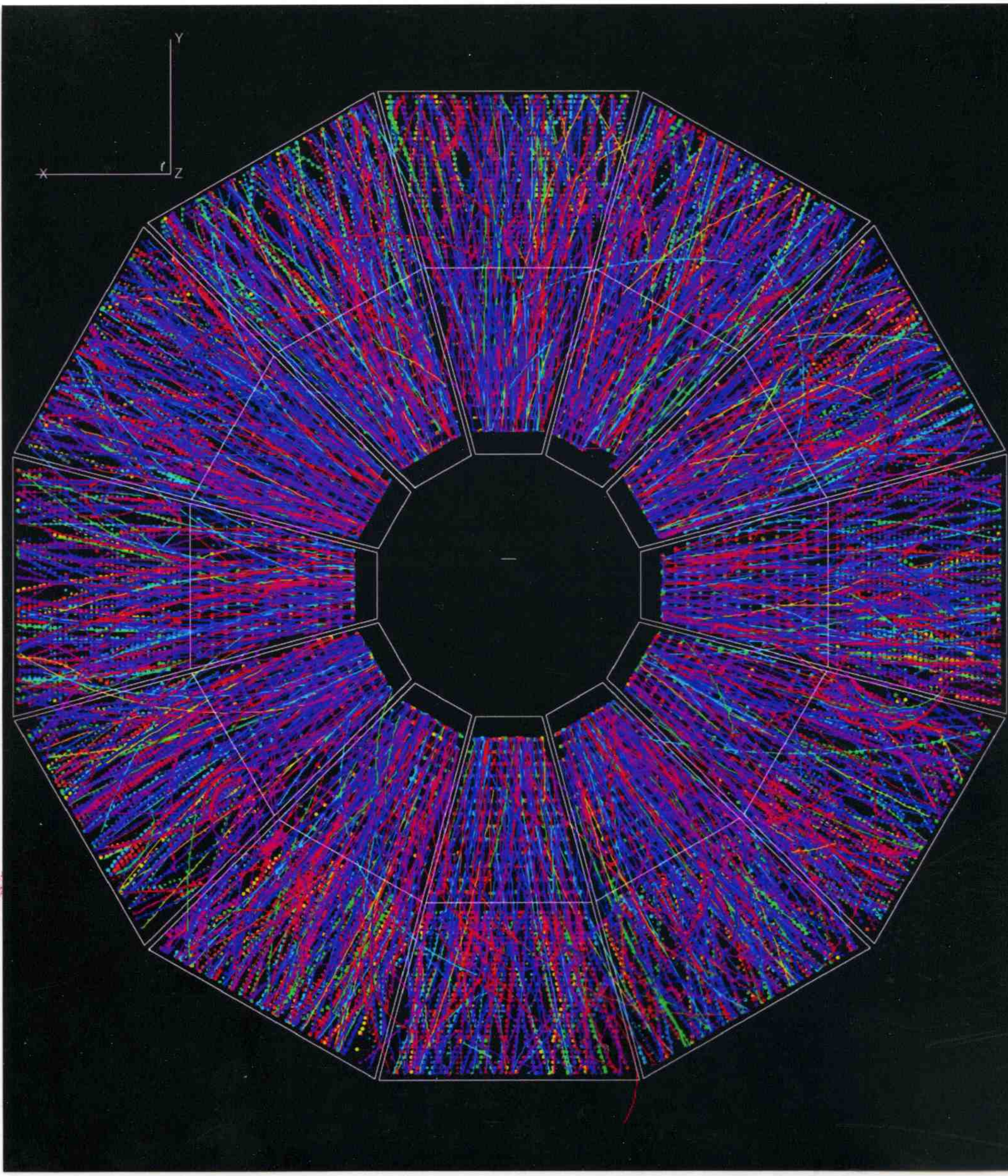
Endrodi et al, 2010





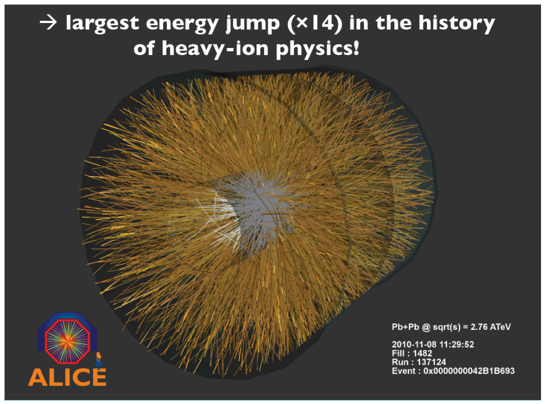
Above $T_{\text{crossover}} \sim$ 150-200 MeV, QCD = QGP. QGP static properties can be studied on the lattice.

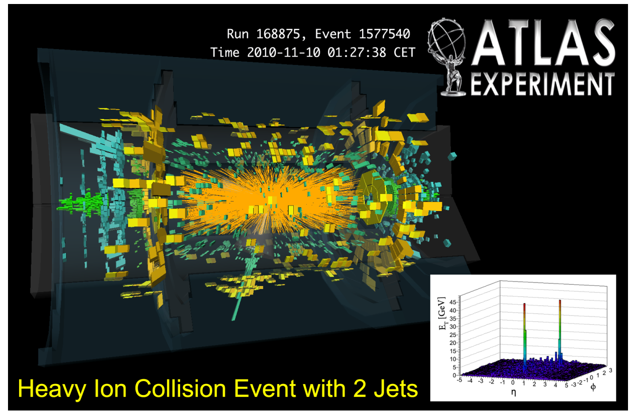
BUT: don't try to infer dynamic properties from static ones! Although its thermodynamics is almost that of ideal, noninteracting gas, QGP, this stuff is very different in its dynamical properties. [Lesson from experiment+hydrodynamics. But, also from the large class of gauge theories with holographic duals whose plasmas have ε and s at infinite coupling 75% that at zero coupling.]



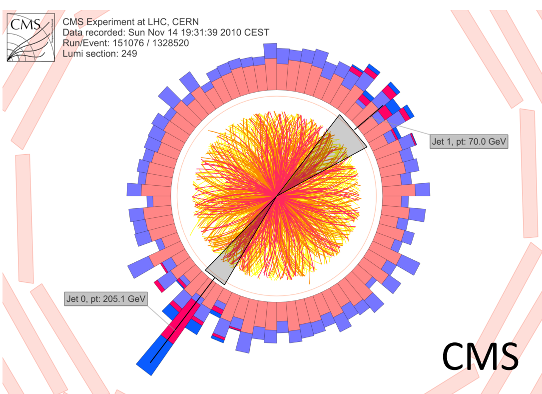
STAR

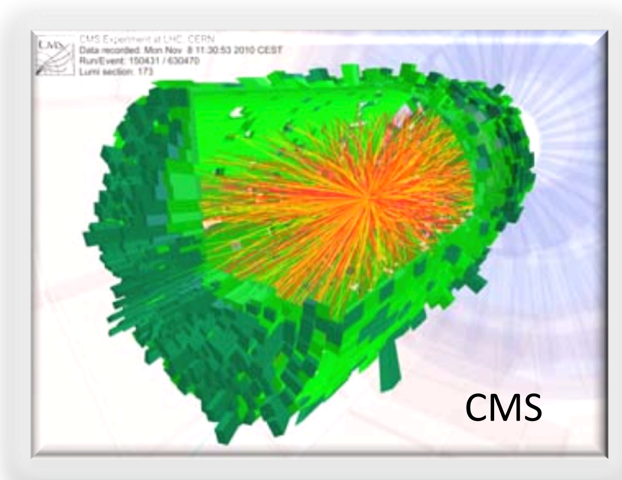
Nov 2010 first LHC Pb+Pb collisions





$\sqrt{s_{NN}}$ = 2760 GeV





Integrated Luminosity = 10 μb^{-1}

Liquid Quark-Gluon Plasma

- Hydrodynamic analyses of RHIC data on how asymmetric blobs of Quark-Gluon Plasma expand (explode) taught us that QGP is a strongly coupled liquid, with (η/s) — the dimensionless characterization of how much dissipation occurs as a liquid flows — much smaller than that of all other known liquids except one.

- Quarks and gluons in QGP diffuse, without being confined in hadrons. QGP flows. Its energy density and coupling are so large that quarks and gluons are always bumping into each other. Far from noninteracting; mean free path hard to define; relaxation times $\sim 1/T$.

- Quarks and gluons in QGP are not confined — but also not free.

- The discovery that it is a strongly coupled liquid is what has made QGP interesting to a broad scientific community.

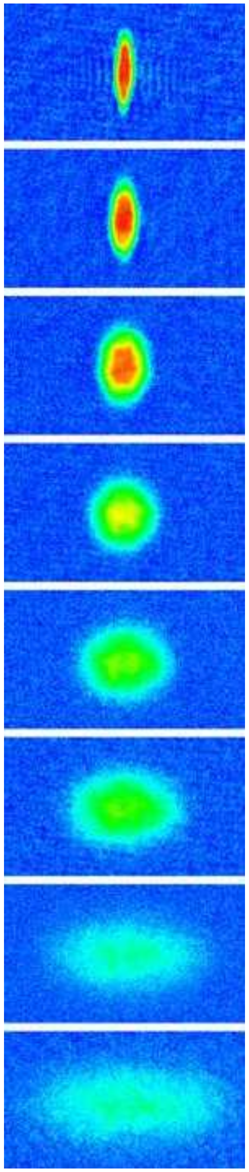
Ultracold Fermionic Atom Fluid

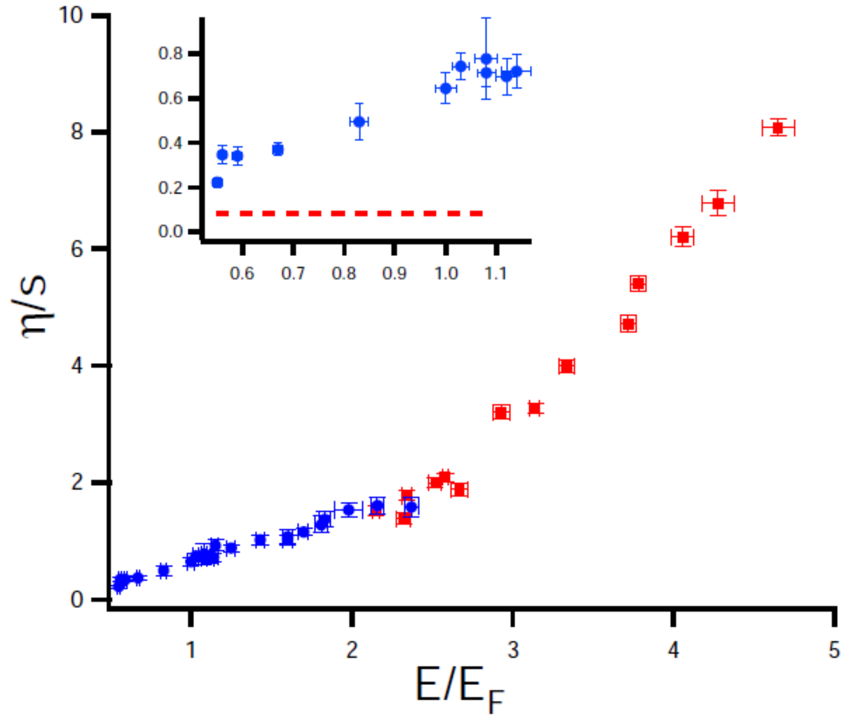
- The one terrestrial fluid with η/s comparably small to that of QGP.
- NanoKelvin temperatures, instead of TeraKelvin.
- Ultracold cloud of trapped fermionic atoms, with their two-body scattering cross-section tuned to be infinite. A strongly coupled liquid indeed. (Even though it's conventionally called the "unitary Fermi gas".)
- Data on elliptic flow (and other hydrodynamic flow patterns that can be excited) used to extract η/s as a function of temperature...

Viscosity to entropy density ratio

consider both collective modes (low T)

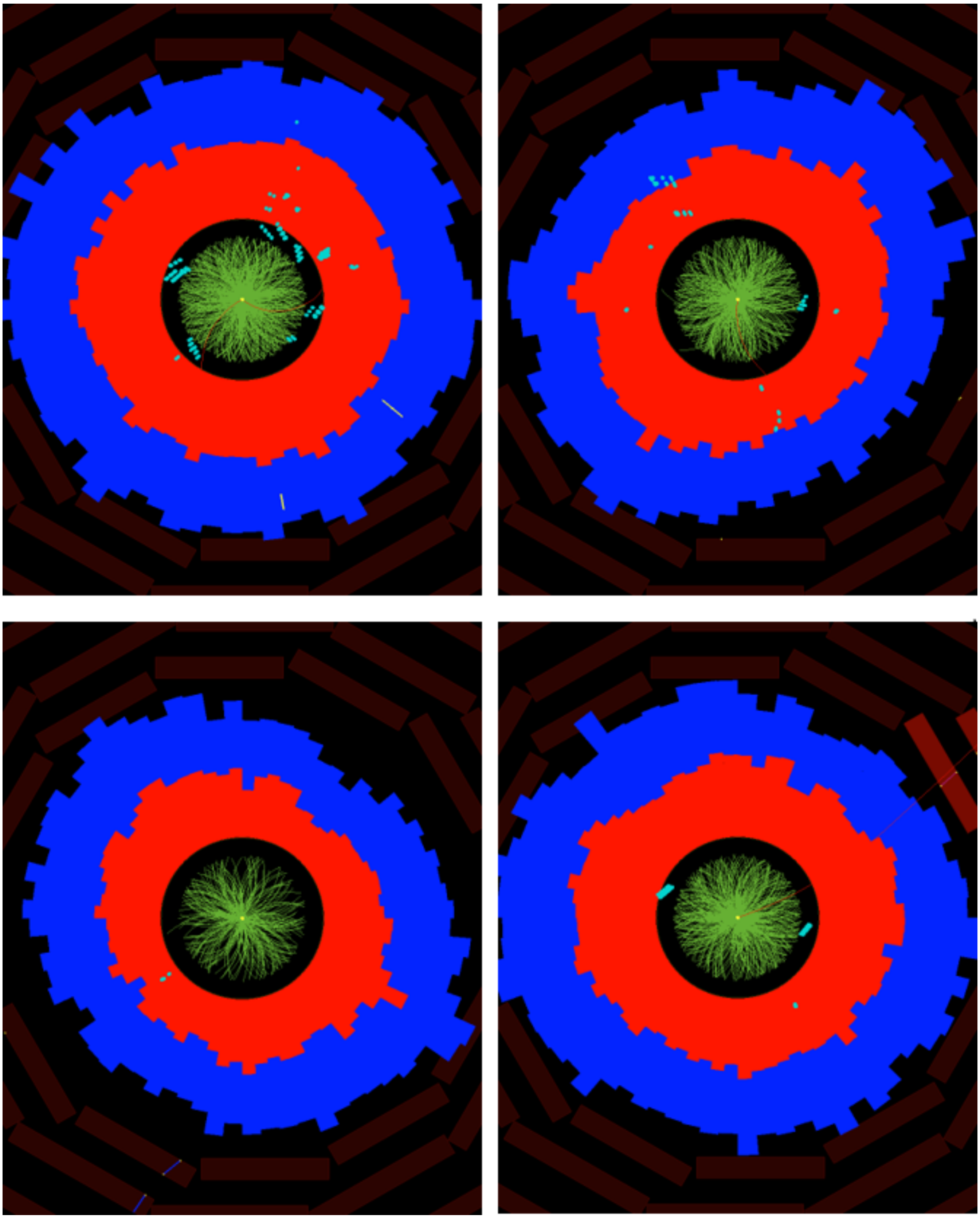
and elliptic flow (high T)





Cao et al., Science (2010)

$$\eta/s \leq 0.4$$



Rapid Equilibration?

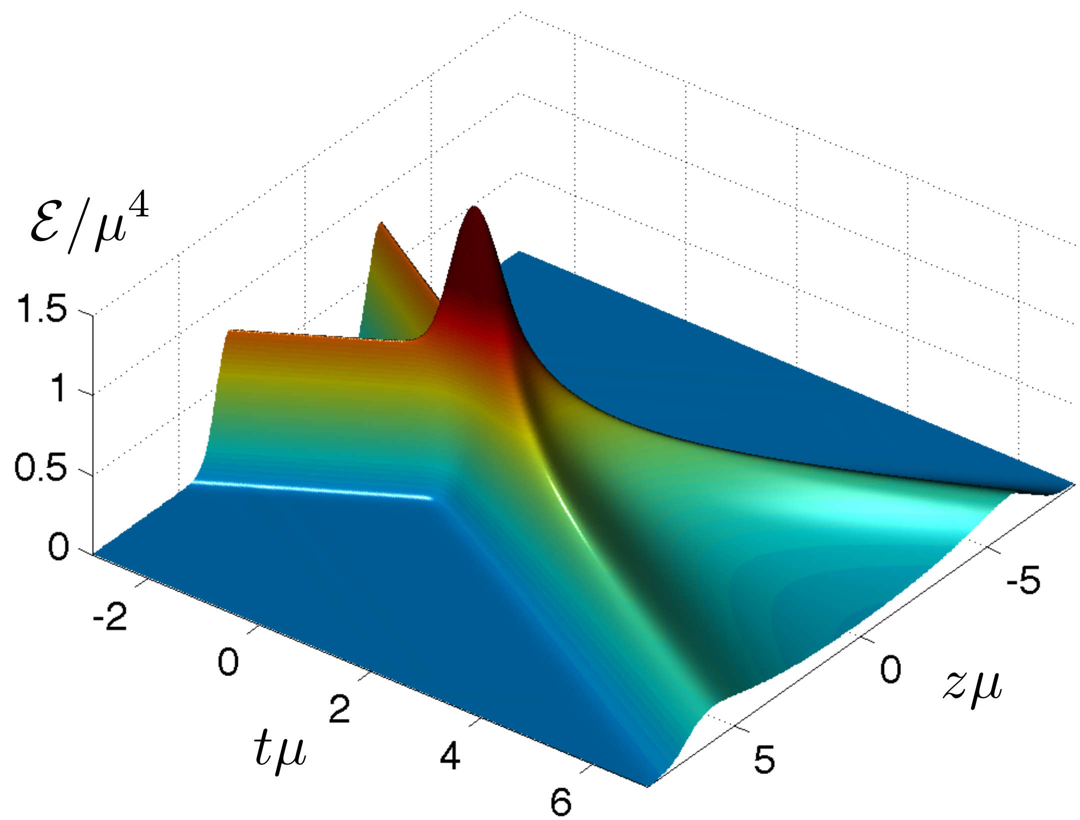
- **Agreement between data and hydrodynamics can be spoiled either if there is too much dissipation (too large η/s) or if it takes too long for the droplet to equilibrate.**

- **Long-standing estimate is that a hydrodynamic description must already be valid only 1 fm/c after the collision.**

- **This is the time it takes light to cross a proton, and was long seen as** *rapid equilibration*.

- **But, is it really? How rapidly does equilibration occur in a strongly coupled theory?**

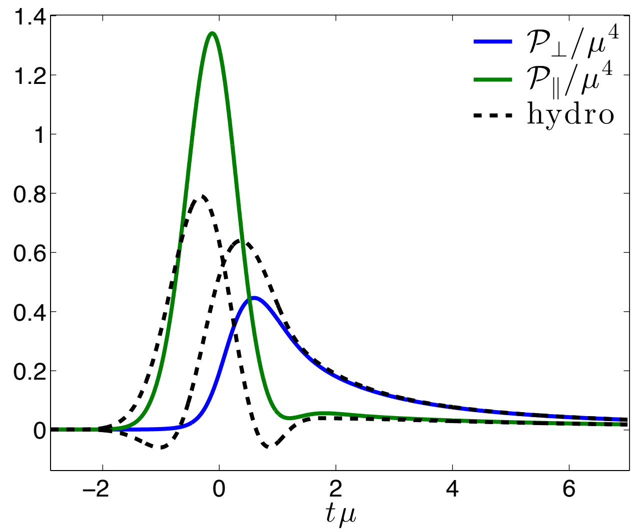
Colliding Strongly Coupled Sheets of Energy



Hydrodynamics valid ~ 3 **sheet thicknesses after the collision, i.e.** ~ 0.35 **fm after a RHIC collision. Equilibration after** ~ 1 **fm need not be thought of as rapid.** Chesler, Yaffe 1011.3562; generalized in C-S,H,M,vdS 1305.4919; CY 1309.1439 **Similarly 'rapid' hydrodynamization times (**$\tau T \lesssim 0.7 - 1$**) found for *many* initial conditions.** 1103.3452, 1202.0981, 1203.0755, 1304.5172.

This was the best answer we had circa 2015.

Anisotropic Viscous Hydrodynamics



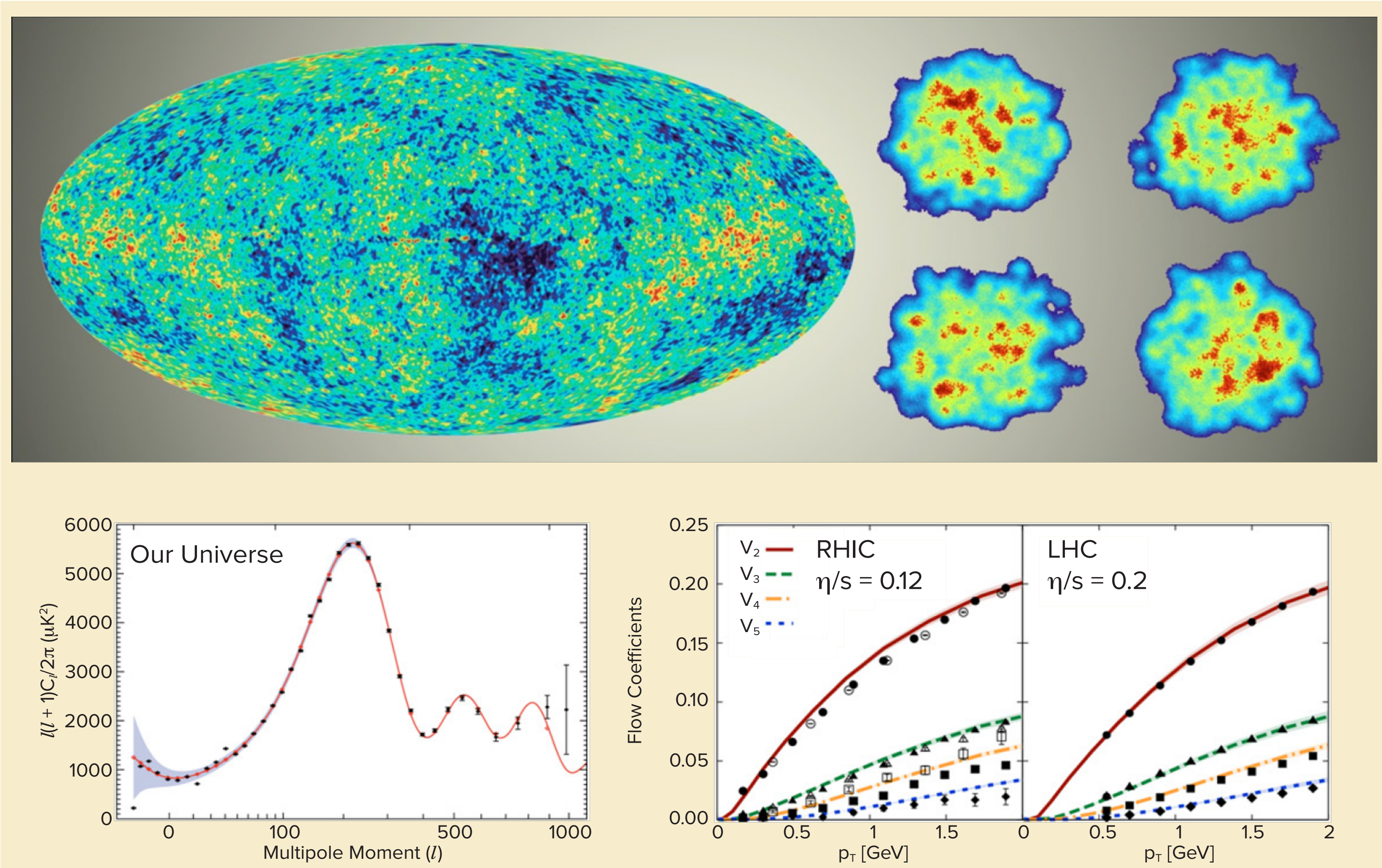
Hydrodynamics valid so early that the hydrodynamic fluid is not yet isotropic. 'Hydrodynamization before isotropization.' An epoch when first order effects (spatial gradients, anisotropy, viscosity, dissipation) important. Hydrodynamics with entropy production.

This has now been seen in very many strongly coupled analyses of hydrodynamization. Janik et al., Chesler et al., Heller et al., ...

Could have been anticipated as a possibility without holography. But, it wasn't — because in a weakly coupled context isotropization happens first.

η/s from RHIC and LHC data

- **I have given you the beginnings of a story that has played out over the past decade. I will now cut to the chase, leaving out many interesting chapters and oversimplifying.**
- **Using relativistic viscous hydrodynamics to describe expanding QGP,** *produced in an initially lumpy heavy ion collision*, **using microscopic transport to describe late-time hadronic rescattering, and using RHIC and LHC data on pion and proton spectra and** v_2 *and* v_3 *and* v_4 *and* v_5 *and* v_6 $\ldots$ **as functions of** p_T **and impact parameter**. . .
- **QGP@RHIC, with** $T_c < T \lesssim 2T_c$**, has** $1 < 4\pi\eta/s < 2$ **and QGP@LHC, with** $T_c < T \lesssim 3T_c$ **has** $1 < 4\pi\eta/s < 3$**.**
Nota bene: this was circa 2015.
- $4\pi\eta/s \sim 10^4$ **for typical terrestrial gases, and** 10 **to** 100 **for all known terrestrial liquids except one. Hydrodynamics works much better for QGP@RHIC than for water.**
- $4\pi\eta/s = 1$ **for any (of the by now very many) known strongly coupled gauge theory plasmas that are the "hologram" of a (4+1)-dimensional gravitational theory "heated by" a (3+1)-dimensional black-hole horizon.**


Our Universe
6000
5000
4000
3000
2000
1000
0
0
100
500
1000
Multipole Moment (l)
l(l + 1)C_l/2π (μK²)
V2
V3
V4
V5
RHIC
η/s = 0.12
LHC
η/s = 0.2
Flow Coefficients
0.25
0.20
0.15
0.10
0.05
0.00
0
0.5
1
1.5
2
pT [GeV]
pT [GeV]

QGP cf CMB

- In cosmology, initial-state quantum fluctuations, processed by hydrodynamics, appear in data as c_ℓ's. From the c_ℓ's, learn about initial fluctuations, and about the "fluid" — eg its baryon content.

- In heavy ion collisions, initial state quantum fluctuations, processed by hydrodynamics, appear in data as v_n's. From v_n's, learn about initial fluctuations, and about the QGP — eg its η/s, ultimately its $\eta/s(T)$ and ζ/s.

- Cosmologists have a huge advantage in resolution: c_ℓ's up to $\ell \sim$ thousands. But, they have only one "event"!

- Heavy ion collisions only up to v_6 at present. But they have billions of events. And, they can do controlled variations of the initial conditions, to understand systematics...

Beyond Quasiparticles

- QGP at RHIC & LHC, unitary Fermi "gas", gauge theory plasmas with holographic descriptions are all strongly coupled fluids with no apparent quasiparticles.

- In QGP, with η/s as small as it is, there can be no 'transport peak', meaning no self-consistent description in terms of quark- and gluon-quasiparticles. [Q.p. description self consistent if $\tau_{\rm qp} \sim (5\eta/s)(1/T) \gg 1/T$.]

- Other "fluids" with no quasiparticle description include: the "strange metals" (including high-T_c superconductors above T_c); quantum spin liquids; matter at quantum critical points;... Among the grand challenges at the frontiers of condensed matter physics today.

- In all these cases, after discovery two of the central strategies toward gaining understanding are *probing* and *doping*. To which we will turn...
  But first, what from 2015 Intro must be updated in 2022? Many improvements, but big picture was solid in 2015! I will highlight two ways in which it has been consolidated.

2023 Updates to 2015 Intro

- **Much more complete understanding now of how hydrodynamization happens in kinetic theory. A weakly coupled picture, applied at intermediate coupling. Hydrodynamization in 1 fm/c is no longer surprising in kinetic theory.** Berges, Heller, Kurkela, Mazeliauskas, Paquet, Schlichting, Spalinski, Strickland, Teaney, Zhu...

- **We had a qualitative, intuitive, understanding of how it can happen on this timescale at strong coupling in 2015. Now we have a qualitative, intuitive, understanding in kinetic theory also:** *adiabatic hydrodynamization.* Brewer, Scheihing-Hitschfeld, Steinhorst, Yan, Yin, KR...

- **Quantification! including uncertainty quantification. Via work of *many* experimentalists and theorists, we now have more, and more precise, experimental data that, together with improved theoretical modeling, are driving Bayesian determinations, by multiple groups, of the "shape" of the fluid at the time of hydrodynamization, and key properties of QGP and their temperature dependence.**

η/s from RHIC and LHC data

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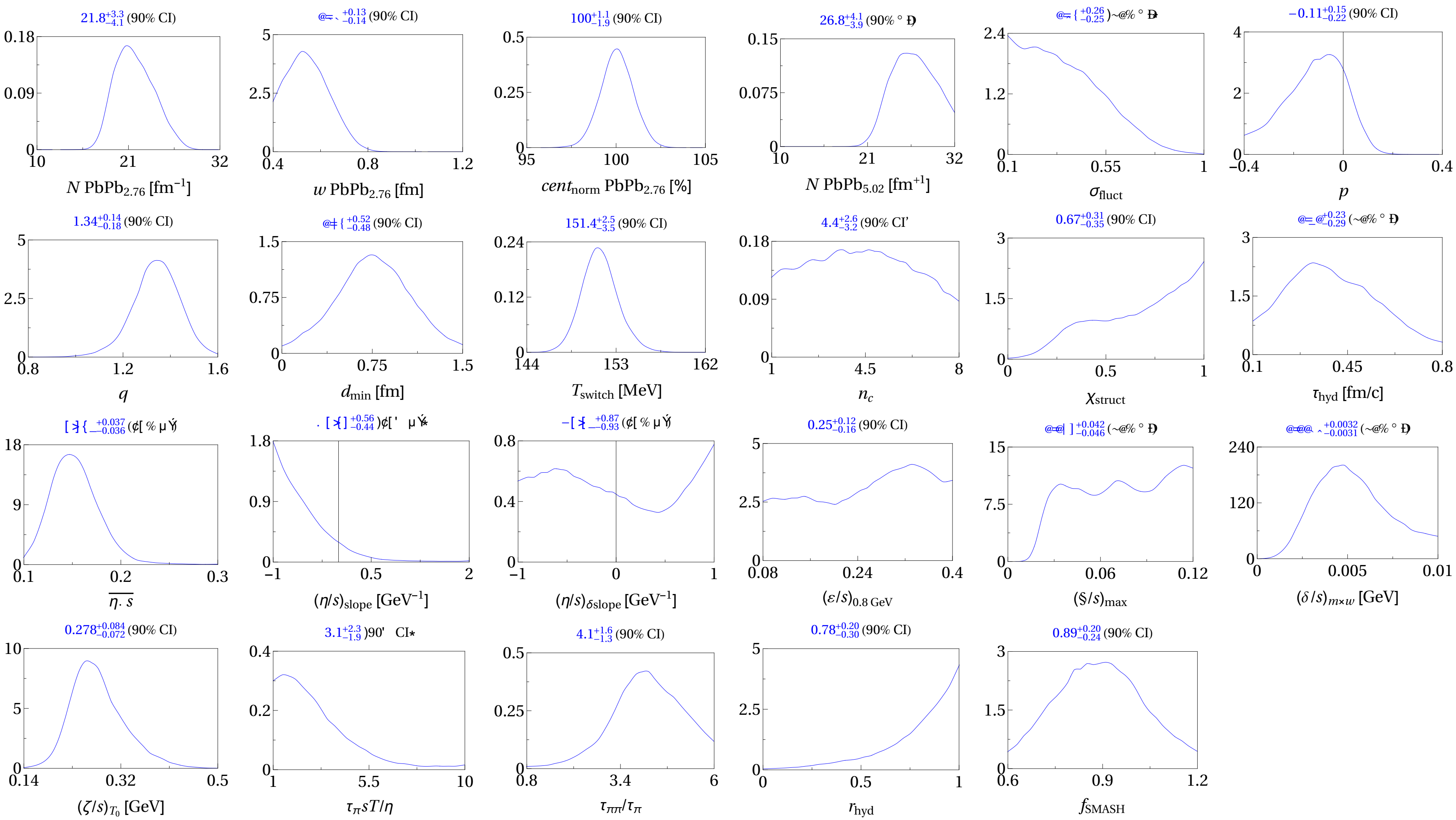
Eg. of Today's State of the Art



Trajectum (Gürsoy, Nijs, Snellings, van der Schee)
this fig: Nijs, van der Schee, arXiv:2304.06191

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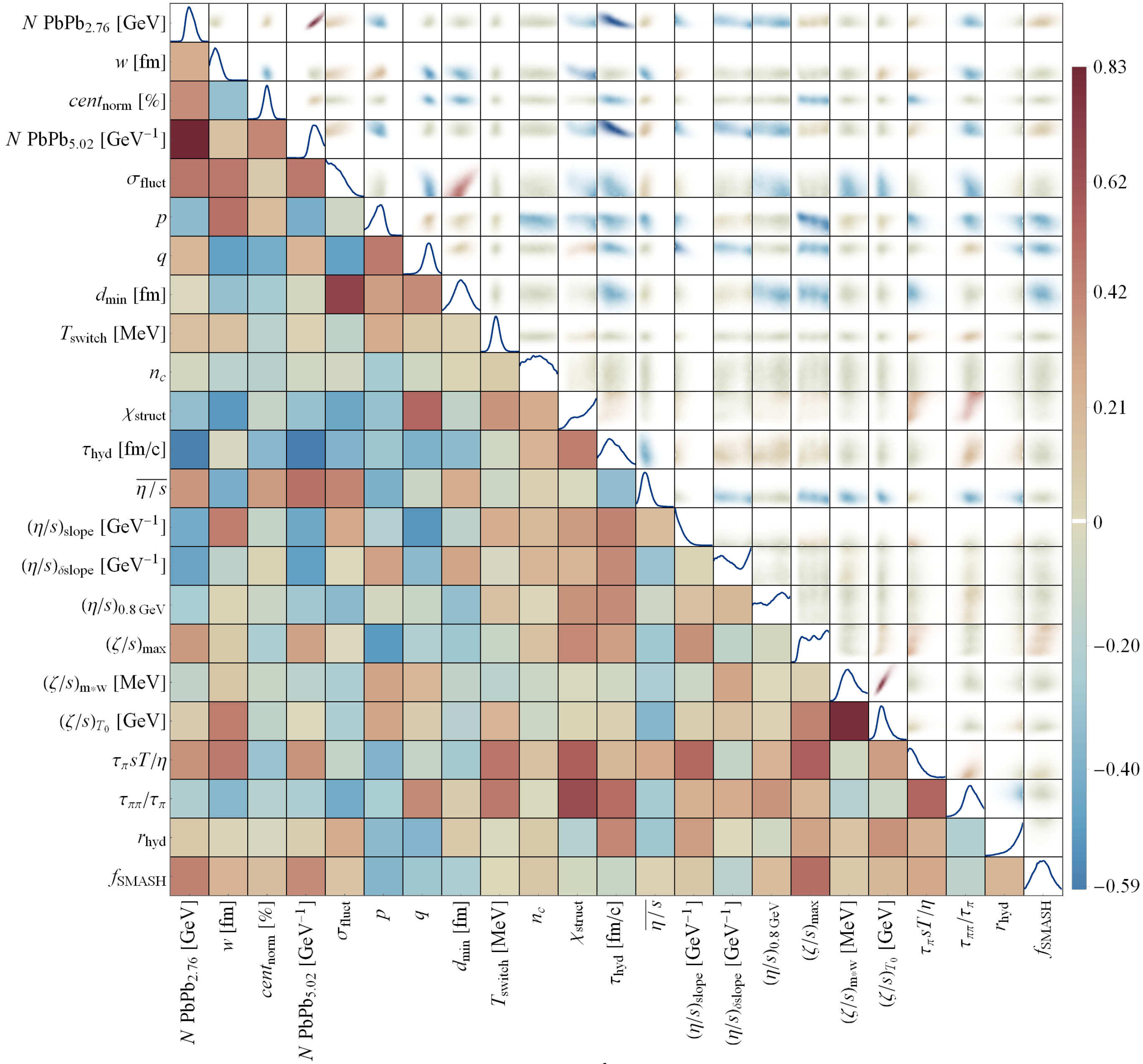

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What the State of the Art Makes Possible...

INT PROGRAM INT-23-1A

Intersection of nuclear structure and high-energy nuclear collisions

January 23, 2023 - February 24, 2023

HIGH-RESOLUTION IMAGES

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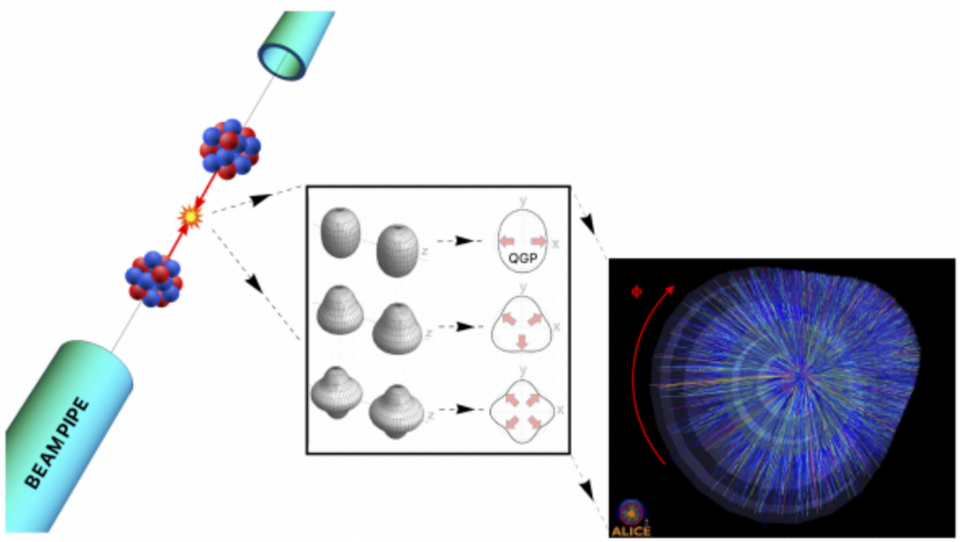

High-energy heavy-ion collisions producing a quark gluon plasma whose energy density profile reflects the collective structure of the colliding ions

APPLICATION FORM - FOR FULL CONSIDERATION, APPLY BY SEPT. 12, 2022

What Next?

Two kinds of What Next? questions for the coming decade...
(and for Parts III and IV of my lectures...)

- A question that one asks after the discovery of any new form of complex matter: What is its phase diagram? For high temperature superconductors, for example, phase diagram as a function of temperature and doping. Same here! For us, doping means excess of quarks over anti-quarks, rather than an excess of holes over electrons.
- A question that we are privileged to have a chance to address, after the discovery of "our" new form of complex matter: How does the strongly coupled liquid emerge from an asymptotically free gauge theory? Maybe answering this question could help to understand how strongly coupled matter emerges in other contexts.

But first, a second introduction....

How to Calculate Properties of Strongly Coupled QGP Liquid?

- **Lattice QCD.** Perfect for THERMODYNAMICS. Calculation of η, heavy quark diffusion coefficient, other transport coefficients, beginning. Hydrodynamization, jet quenching and other dynamical processes not in sight.
- **Perturbative QCD.** The right theory, but the wrong approximation.
- Calculate properties, transport coefficients, hydrodynamization, dynamical processes for hot strongly coupled liquid in other gauge theories that, via **holography,** *are* analyzable at strong coupling. Right approximation, wrong theory.

Are some dynamical properties similar across strongly coupled liquid phases in many theories? How can we use holographic calculations to gain intuition re dynamical questions? Examples have arisen in the first Intro, and will arise again in last lecture. So, a second Intro...

$\mathcal{N}=4$ Supersymmetric Yang Mills

- A gauge theory specified by two parameters: N_c and $g^2 N_c \equiv \lambda$.
- Conformal. (λ does not run.)
- If we choose λ large, at $T \neq 0$ we have a strongly coupled plasma.
- This 3+1 dimensional gauge theory is equivalent to a particular string theory in a particular spacetime: $\underbrace{AdS_5}_{4+1 \text{ "big" dimensions}} \times \underbrace{S_5}_{}$ → 5 "curled up" dim.
- In the $N_c \to \infty$, $\lambda \to \infty$ limit, the string theory reduces to classical gravity. ∴ Calculations easy at strong coupling.

Thermodynamics at Strong Coupling

- In the $N_c \to \infty$ and $\lambda \to \infty$ limit, the thermodynamics of strongly coupled $\mathcal{N} = 4$ SYM plasma are:

$$\frac{\varepsilon_{\lambda=\infty}}{\varepsilon_{\lambda=0}} = \frac{P_{\lambda=\infty}}{P_{\lambda=0}} = \frac{s_{\lambda=\infty}}{s_{\lambda=0}} = \frac{3}{4}$$

- Teaches us that thermodynamics of very weakly coupled plasmas and very strongly coupled plasmas can be very similar.
- Reminds us that (approximate) conformality above T_c need not mean weak coupling.
- But we don't "need" this, in the sense that we have reliable lattice calculations of the thermodynamics of QGP in QCD.

η/s and Holography

- $4\pi\eta/s = 1$ for any (of the very many) known strongly coupled large-N_c gauge theory plasmas that are the "hologram" of a (4+1)-dimensional gravitational theory "heated by" a (3+1)-dimensional black-hole horizon.
- Examples of theories in which this result holds are known which are: conformal or not; confining at $T = 0$ or not; have fundamentals or not; supersymmetric or not.
- cf. $1 < 4\pi\eta/s < 3$ for QGP at RHIC and LHC.
- Suggests a new kind of universality, not yet well understood, applying to dynamical aspects of strongly coupled liquids. To which liquids? Unitary Fermi 'gas'?

η/s and Holography

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- Geometric intuition for dynamical phenomena at strong coupling. Hydrodynamization = horizon formation. Nontrivial hydrodynamic flow pattern = nontrivial undulation of black-hole metric. Dissipation due to shear viscosity = gravitational waves falling into the horizon.
- Conformal examples show that hydrodynamics need not emerge from an underlying kinetic theory of particles. A liquid can just be a liquid.

AdS/CFT

We now know of infinite classes of different gauge theories whose quark-gluon plasmas:

- are all equivalent to string theories in higher dimensional spacetimes that contain a black hole
- all have

$$\frac{\varepsilon}{T^4} = \frac{3}{4}\left(\frac{\varepsilon}{T^4}\right)_0$$

Gubser Klebanov Tseytlin Peet....

$$\eta/s = \frac{1}{4\pi}$$

Son Policastro Starinets Kovtun Buchel Liu....

in the limit of strong coupling and large number of colors.

[Not known whether QCD in this class.]

Why care about the value of η/s?

- **Here is a theorist's answer...**

- **Any gauge theory with a holographic dual has $\eta/s = 1/4\pi$ in the large-N_c, strong coupling, limit. In that limit, the dual is a classical gravitational theory and η/s is related to the absorption cross section for stuff falling into a black hole. If QCD has a dual, since $N_c = 3$ it must be a string theory. Determining $(\eta/s) - (1/4\pi)$ would then be telling us about string corrections to black hole physics, in whatever the dual theory is.**

- **For fun, quantum corrections in dual of $\mathcal{N} = 4$ SYM give:**

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{15\,\zeta(3)}{(g^2 N_c)^{3/2}} + \frac{5}{16}\frac{(g^2 N_c)^{1/2}}{N_c^2} + \ldots\right)$$ Myers, Paulos, Sinha

  with $1/N_c^2$ and N_f/N_c corrections yet unknown. Plug in $N_c = 3$ and $\alpha = 1/3$, i.e. $g^2 N_c = 12.6$, and get $\eta/s \sim 1.73/4\pi$. And, $s/s_{SB} \sim 0.81$, near QCD result at $T \sim 2 - 3T_c$.

- **A more serious answer...**

Beyond Quasiparticles

- QGP at RHIC & LHC, unitary Fermi "gas", gauge theory plasmas with holographic descriptions are all strongly coupled fluids with no apparent quasiparticles.

- In QGP, with η/s as small as it is, there can be no 'transport peak', meaning no self-consistent description in terms of quark- and gluon-quasiparticles. [Q.p. description self consistent if $\tau_{\mathrm{qp}} \sim (5\eta/s)(1/T) \gg 1/T$.]

- Other "fluids" with no quasiparticle description include: the "strange metals" (including high-T_c superconductors above T_c); quantum spin liquids; matter at quantum critical points;...

- Emerging hints of how to look at matter in which quasiparticles have disappeared and quantum entanglement is enhanced: "many-body physics through a gravitational lens." Black hole descriptions of liquid QGP and strange metals are continuously related! But, this lens is at present still somewhat cloudy...

From $\mathcal{N} = 4$ SYM to QCD

- **Two theories differ on various axes. But, their plasmas are *much* more similar than their vacua. Neither is supersymmetric. Neither confines or breaks chiral symmetry.**
- $\mathcal{N} = 4$ **SYM is conformal. QCD thermodynamics is reasonably conformal for** $2T_c \lesssim T <$ **?. In model studies, adding the degree of nonconformality seen in QCD thermodynamics to** $\mathcal{N} = 4$ **SYM has *no* effect on** η/s **and little effect on many other observables.**
- **The fact that the calculations in** $\mathcal{N} = 4$ **SYM are done at strong coupling is a feature, not a bug.**
- **The fact that strongly coupled** $\mathcal{N} = 4$ **SYM is strongly coupled at all scales, including short length scales, is a bug.** $\rightarrow$ **Wednesday.**
- $\mathcal{N} = 4$ **SYM calculations done at** $1/N_c^2 = 0$ **rather than** $1/9$**.**
- **In QCD thermodynamics, fundamentals are as important as adjoints. No fundamentals in** $\mathcal{N} = 4$ **SYM, and so far they have only been added as perturbations.**
- **Our goals are, and must be, limited to qualitative insights.**

A Grand Challenge

- **How can we clarify the understanding of fluids without quasiparticles, whose nature is a central mystery in so many areas of science?**

- **We have two big advantages: (i) direct experimental access to the fluid of interest without extraneous degrees of freedom; (ii) weakly-coupled quark and gluon quasiparticles at short distances.**

- **We can quantify the properties and dynamics of Liquid QGP at its natural length scales.**

- **Can we probe, quantify and understand Liquid QGP at** *short distance scales*, **where it is made of quark and gluon quasiparticles? See** *how* **the strongly coupled fluid emerges from well-understood quasiparticles at short distances.**

- **The LHC and newly upgraded RHIC offer new probes and open new frontiers.**

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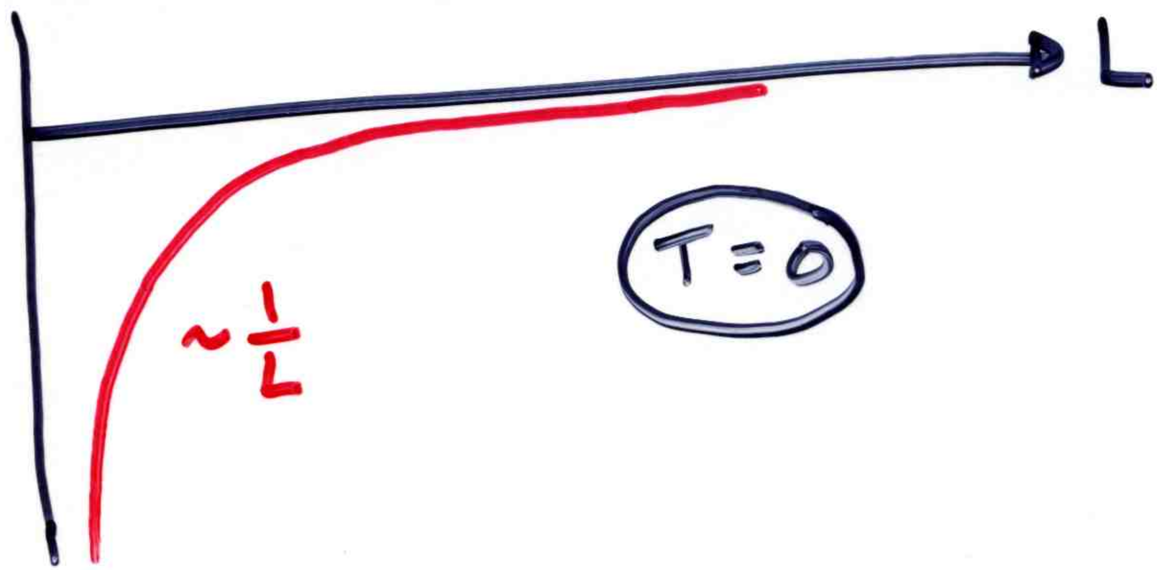
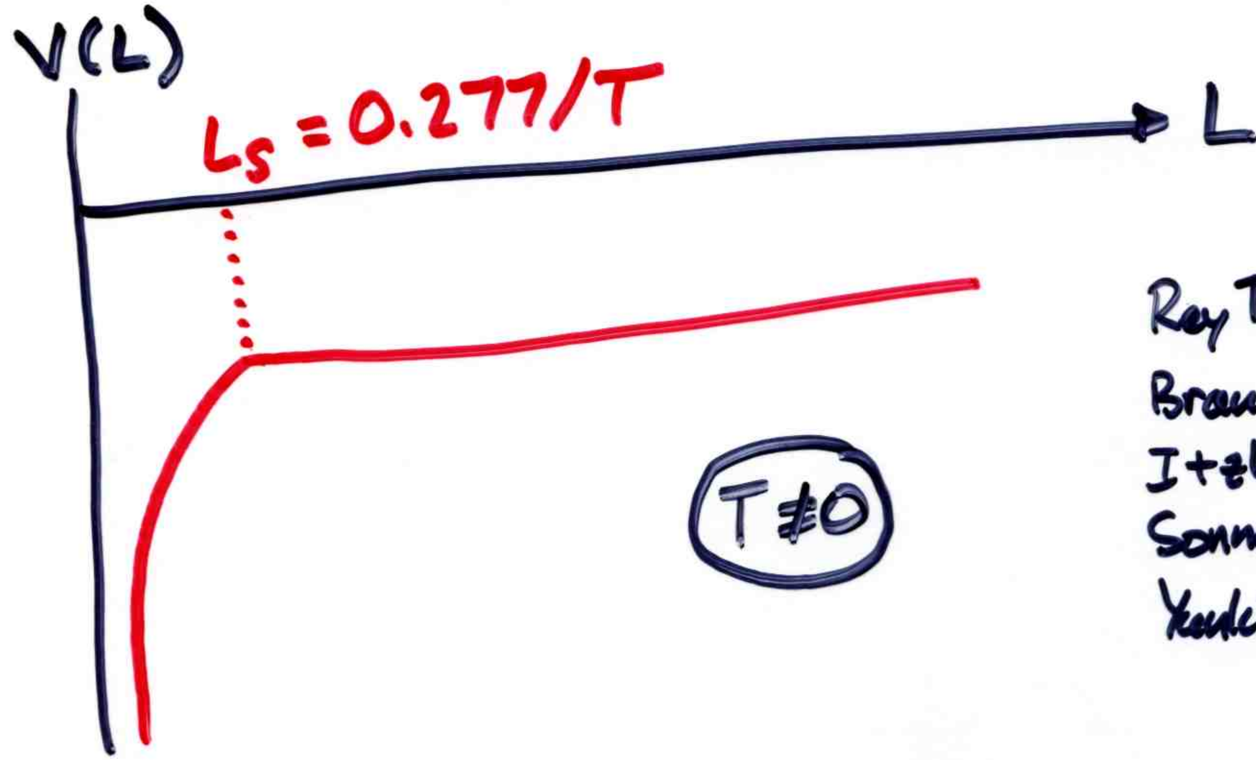
SCREENING IN N=4

$V(L)$ = potential between static Q & $\bar{Q}$

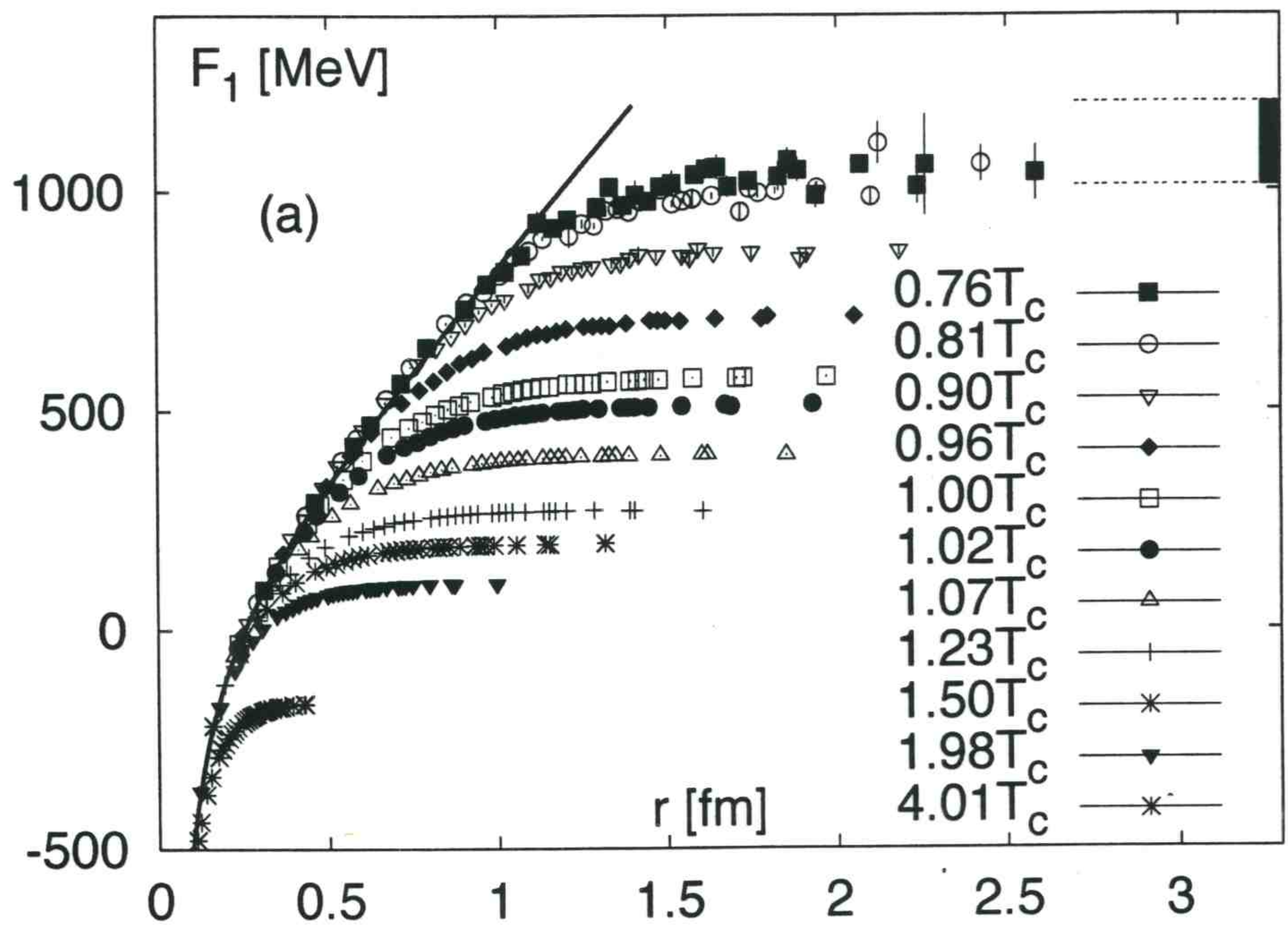




Rey Theisen Yee;
Brandhuber
Itzhaki
Sonnenschein
Yankielowicz

Similar to screening in QCD above QCD's T_c....

SCREENING IN QCD



Kaczmarek, Zantow

lattice QCD calculation

[Unquenched. $N_f = 2$]

Upon defining an L_s, the authors find $L_s \sim 0.5/T$

AdS / CFT

Maldacena; Witten; Gubser Klebanov Polyakov; ....

N=4 SYM is equivalent to Type IIB String theory on $\underbrace{AdS_5}_{\text{4+1 "big" dimensions}} \times \underbrace{S_5}_{\text{5 curled up dimension}}$

Translation Dictionary:

N=4 SYM gauge theory in 3+1 dim — String theory in 4+1(+5) dim

$$\frac{g^2 N_c}{4\pi N_c} = g_{string}$$

[$N_c \to \infty$ at fixed $g^2 N_c$ means $g_{string} \to 0$]

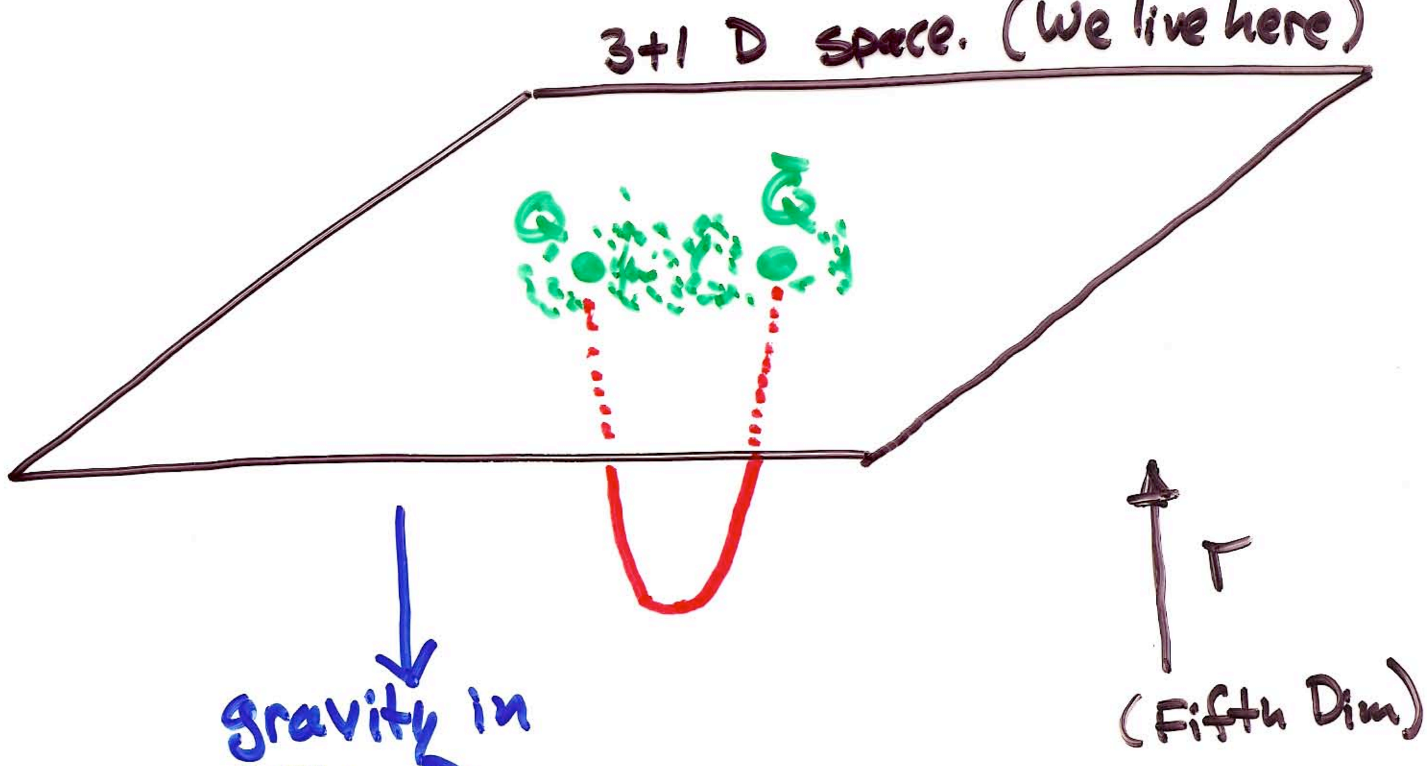
$$\sqrt{g^2 N_c} = R^2/\alpha'$$

[R: AdS curvature
$\frac{1}{2\pi\alpha'}$: string tension]

Heat the gauge theory to a temperature T. = $T_H = r_0/\pi R^2$ — Add a Black hole in the 5^{th} dimension, with

[r_0: location of BH horizon in fifth dim.]

How can strings in $\underline{5D}$ describe, say, force between Q and $\bar{Q}$ in a 4D gauge theory?

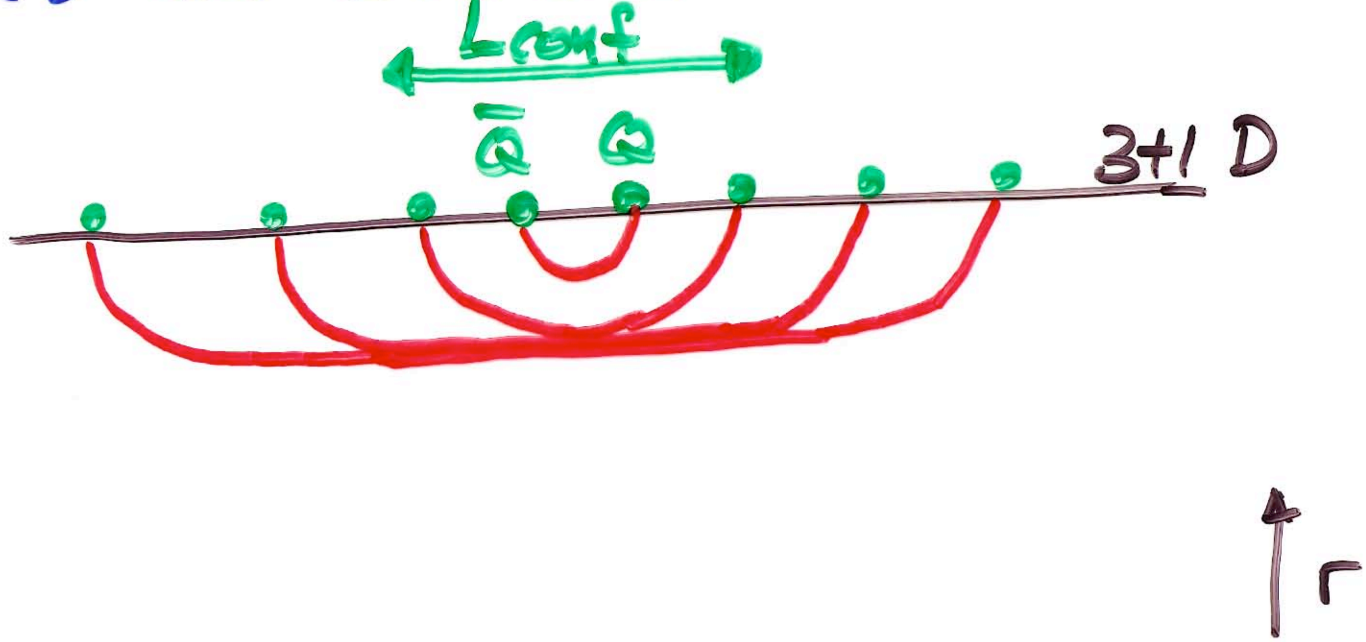


(AdS is a curved space)

- Extremize energy of U string. (Like catenary problem, in unusual gravitational field.)
  - Large $g^2 N_c$ → Large tension → no fluctuation
  - Large N_c → Small g_{string} → no loops break off.
- Force between Q and $\bar{Q}$ $= \dfrac{d}{d\ \text{separation}} \left(\text{Energy of string} \right)$

CONFINEMENT?

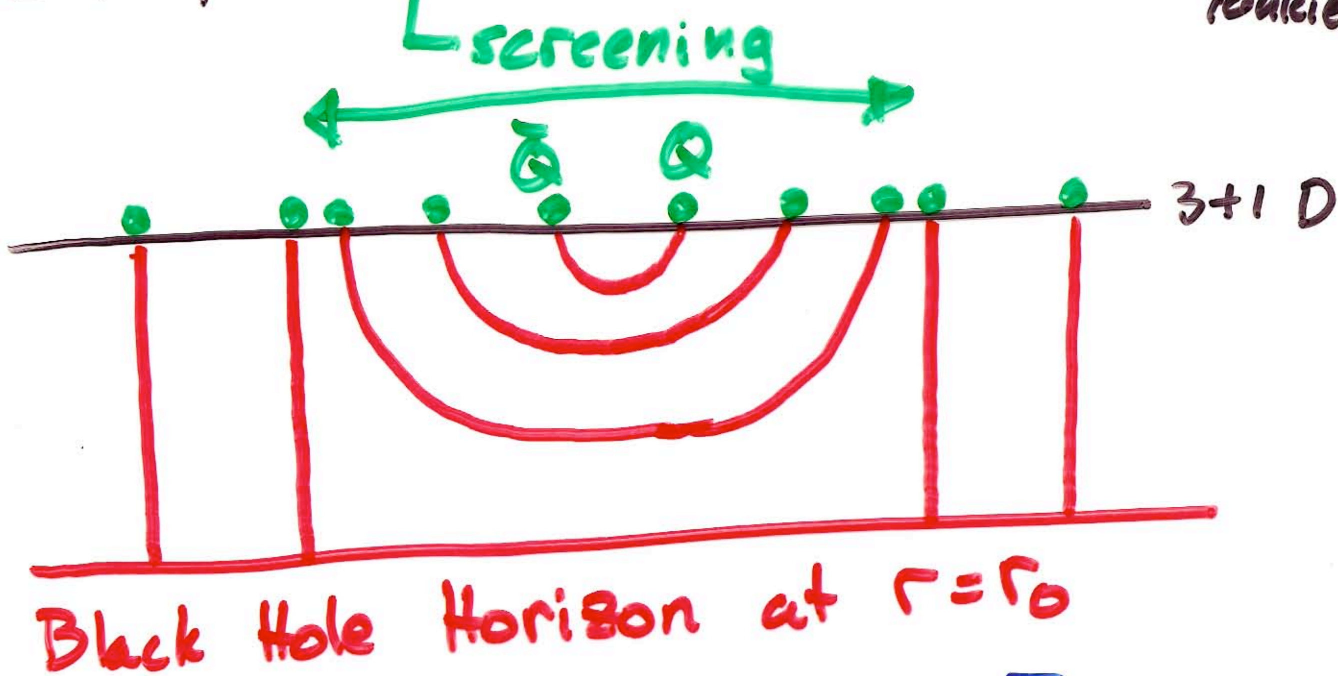
Here's how confinement can arise....



- This does not happen in $N=4$
  - shape of string stays same as L increases. ($N=4$ is conformal)
- Confining gauge theories with dual descriptions like this are known.
- QCD not known to have a description like this.
- Don't use $N=4$ as a guide to QCD at $T=0$.

DECONFINEMENT AT $T \neq 0$

Maldacena; Rey Yee; Rey Theisen Yee; Brandhuber Itzhaki Sonnenschein Yankielowicz



- For $L < L_s$, force between Q & $\bar{Q}$.
- For $L > L_s$, force is screened. Q & $\bar{Q}$ deconfined.
- In $N=4$ SUSY QCD,

$$L_s = \frac{0.277}{T}$$

- In QCD, force between static Q & $\bar{Q}$ in QGP can be calculated. (Lattice QCD) Can define L_s, though it is not a sharp boundary. Find: $L_s \sim \frac{0.5}{T} \rightarrow \frac{0.7}{T}$ Kaczmarek Karsch Zantow Petreczky
- $N=4$ gets this feature of the QCD strongly interacting QGP to within factor of 2!

Dragging a Heavy Quark through Strongly Coupled Plasma

HKKKY, G, 2006

- **One of the first holographic calculations related to *probing* strongly coupled plasma.**

- **To drag a heavy quark, $M \to \infty$, with constant velocity $\vec{\beta}$ through the static, homogeneous, equilibrium strongly coupled plasma with temperature T of $\mathcal{N} = 4$ SYM theory requires exerting a *drag force*:**

$$\vec{f} = \frac{\sqrt{\lambda}}{2\pi} (\pi T)^2 \gamma \vec{\beta} \propto \frac{\vec{p}}{M}$$

with $\lambda \equiv g^2 N_c$ the 't Hooft coupling.

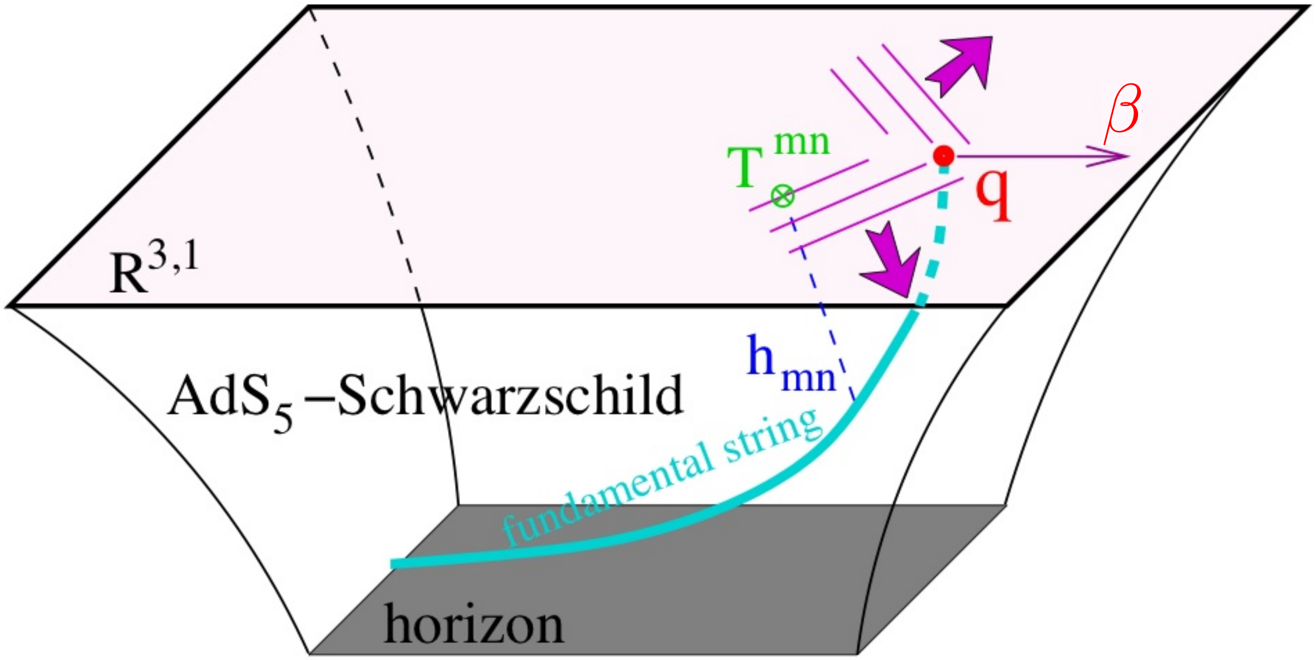
- ***Caveat emptor:* At finite M, this picture only applies for**

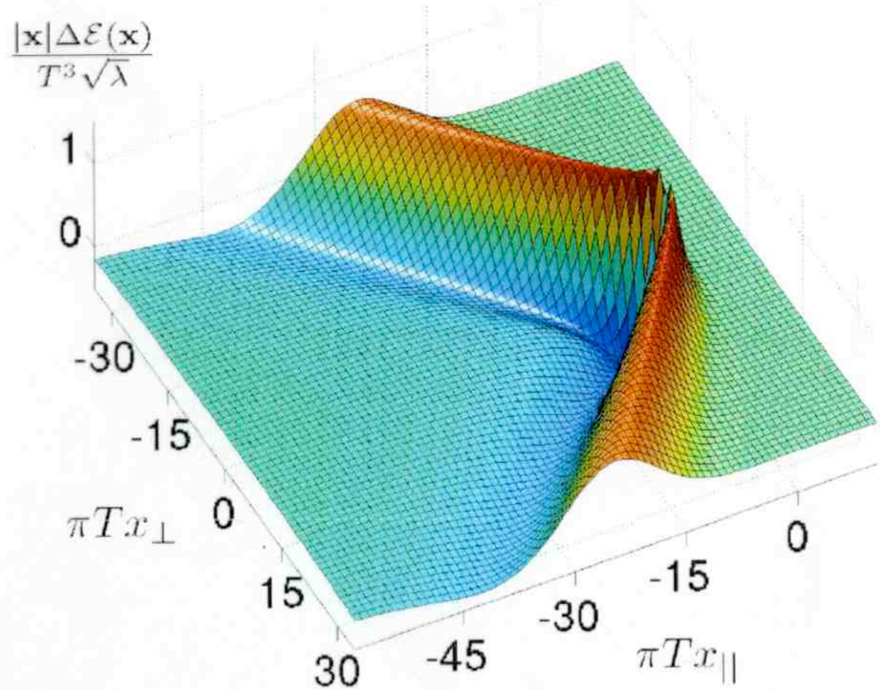
$$\sqrt{\gamma} \ll \frac{M}{T\sqrt{\lambda}} \ .$$

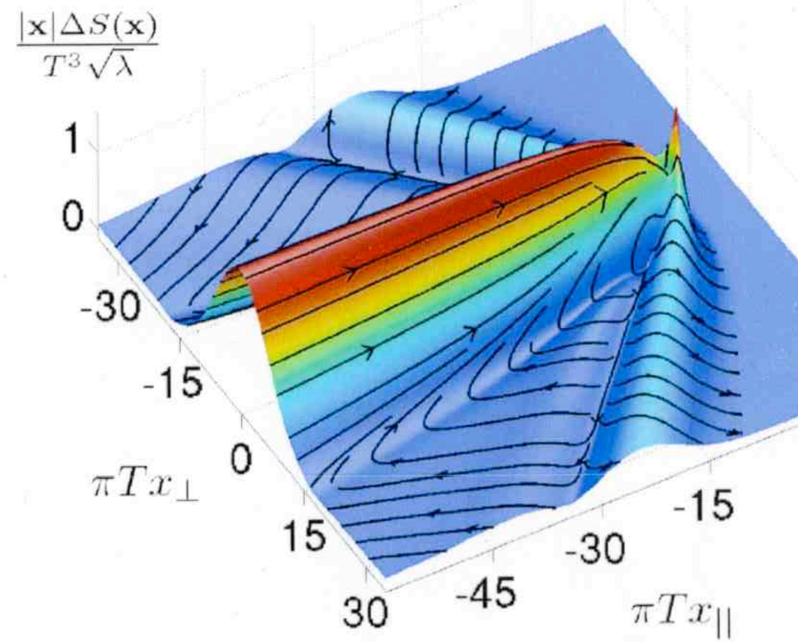
Eg for b quarks at the LHC validity is $p_T \lesssim 20 - 40$ GeV. Higher p_T heavy quarks behave like light quarks.

Dragging a Heavy Quark through Strongly Coupled Plasma

HKKKY, G, 2006







Energy density.

Nb: Specific heat $\propto N_c^2$ amplifies effect of heat over motion in $\mathcal{E}$. So, this plot tells you where there is heating. Ie compression. Ie SOUND.

Momentum flow.

Mach cone and wake.

Chesler + Yaffe

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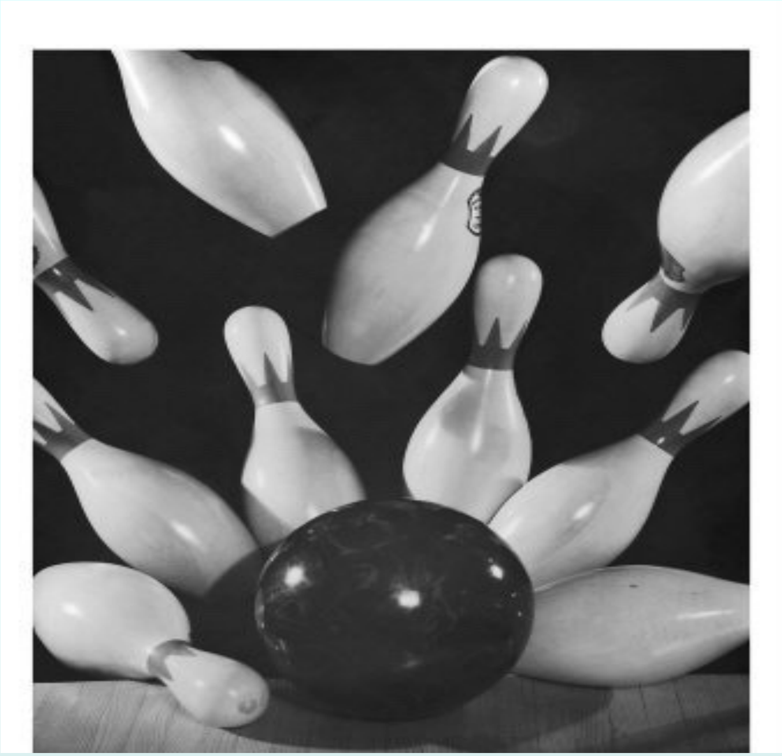
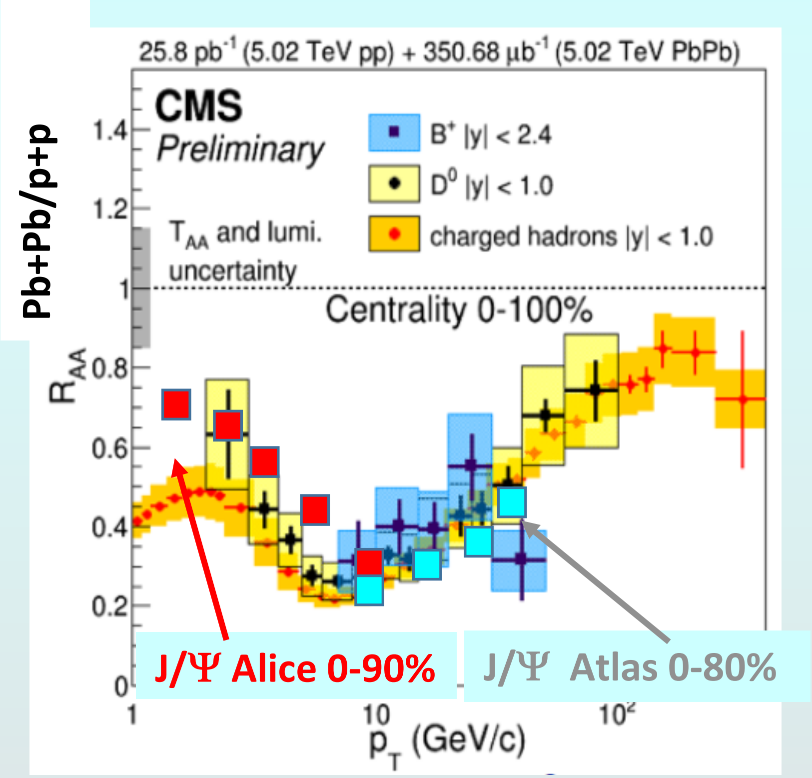
Eg for b quarks at the LHC validity is $p_T \lesssim 20 - 40$ GeV. Higher p_T heavy quarks behave like light quarks.

An astounding result!



Even more surprising than you might think…



Even b quarks lose energy!

Heavy Quark Drag and Diffusion in Strongly Coupled Plasma

HKKKY, G, C-Y&T 2006

- **Under the same conditions as on the previous slide, heavy quark in strongly coupled plasma satisfies:**

$$\frac{dp}{dt} = -\eta_{\text{drag}}\, p + \xi(t) \qquad \langle \xi(t), \xi(t') \rangle = \kappa\, \delta(t - t')$$

where

$$\eta_{\text{drag}} = \frac{\pi\sqrt{\lambda}T^2}{2M} \qquad D \equiv \frac{2T^2}{\kappa} = \frac{4}{\sqrt{\lambda}}\frac{1}{2\pi T} \qquad \kappa = 2MT\eta_{\text{drag}}$$

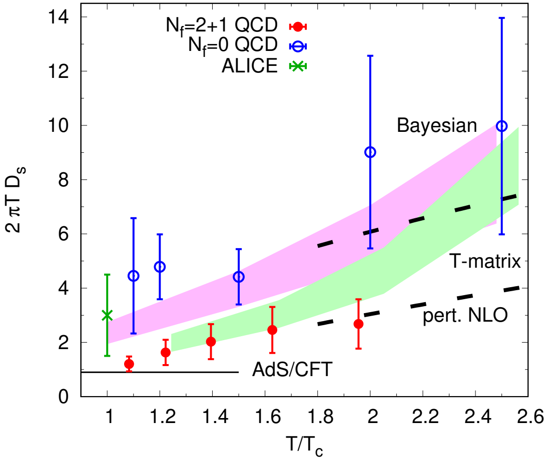
- **So, the calculation of the drag force is at the same time a calculation of the heavy quark diffusion constant D. And, for $\lambda \simeq 12.6$ (the value we used several slides ago) the diffusion constant in strongly coupled plasma is $D \simeq 1.1/(2\pi T)$.**

- **This fifteen year old result agrees** *surprisingly* **well with contemporary lattice calculations of D in QGP. The extraction of D from heavy ion collision data, see Barbara's lectures, is broadly consistent with this also.**

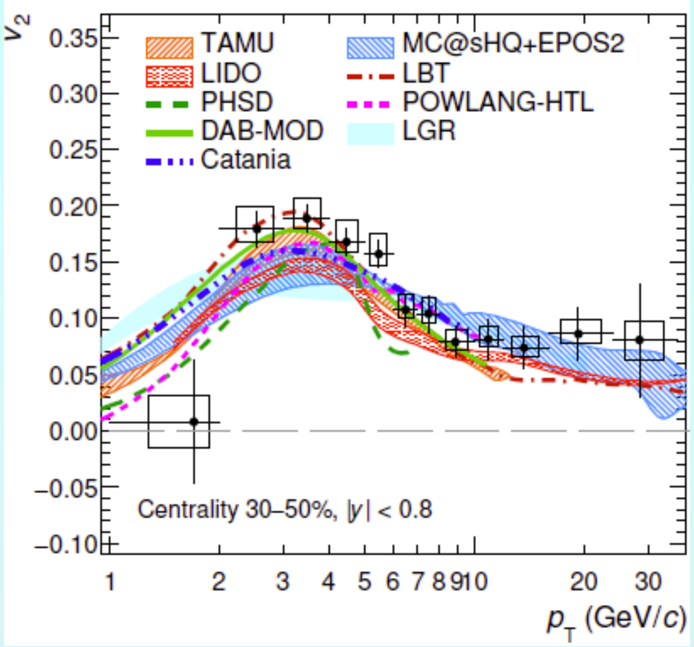
Diffusion coefficient

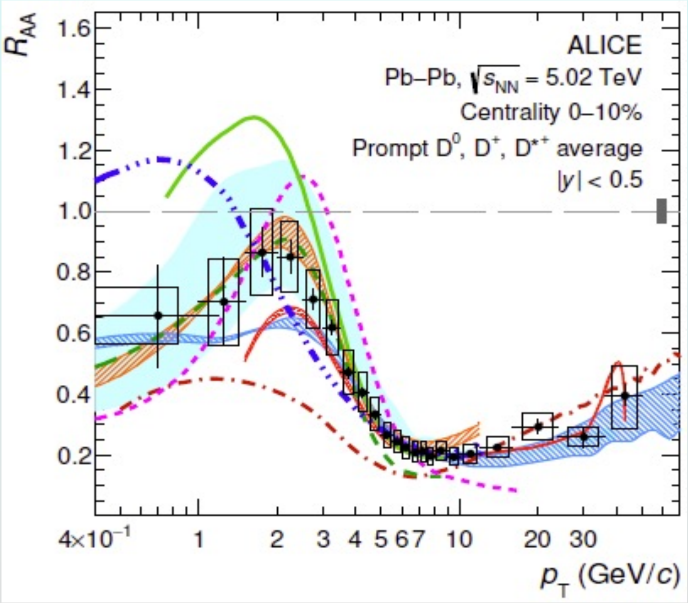
- Results for $D_s = 2T^2/\kappa$ shows lower than quenched behavior

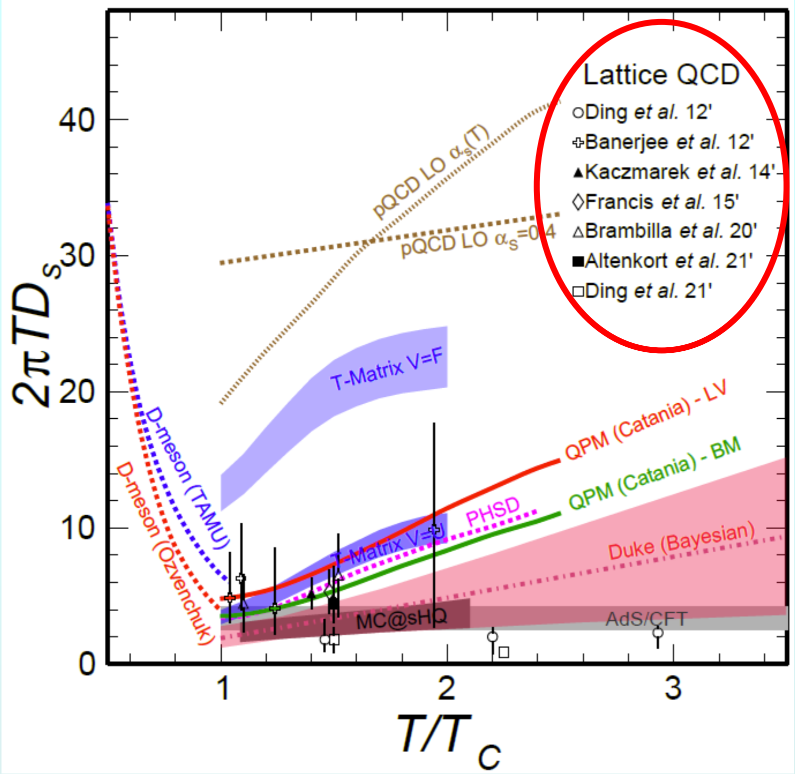
- $6D_s$ is the mean distance squared traveled by unit time
- T-Matrix results updated compared to figure in paper, R. Rapp et al. [arxiv:1612.09318][arxiv:1711.03282]



Heavy quark diffusion from D meson v_2 and R_{AA}







Again use data + models together: radiation, collisions, medium evolution

$D_s(2\pi T)$ = 1.5 - 4.5 near T_c

per models with χ^2/DOF < 5 (2)

for R_{AA} (v_2)

Heavy Quark Drag and Diffusion in Strongly Coupled Plasma

HKKKY, G, C-Y&T 2006

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where

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- **Perhaps best to focus on a striking qualitative feature:**

$$\frac{dp}{dt} \propto \frac{p}{M}$$

which is inevitable at strong coupling, and not the case at weak coupling. Energy loss of a 20 (or 10 or 5) GeV bottom quark same as energy loss of 6 (or 3 or 1.5) GeV charm quark. This qualitative feature has not been tested against data, and should be...

$\hat{q}$ in $\mathcal{N} = 4$ SYM Plasma

Liu, KR, Wiedemann 2006

- **The jet quenching parameter, featured in Barbara's lectures, can also be calculated exactly in holographic theories, in the $N_c^2 \to \infty$, $\lambda \to \infty$ limit. (The calculation involves computing the expectation value of a certain Wilson loop with two light-like sides.) The result is:**

$$\hat{q} = \frac{\pi^{3/2}\Gamma(5/4)}{\Gamma(3/4}\sqrt{\lambda}T^3 = 4.12\sqrt{\lambda}T^3$$

- **If we again take $\lambda \approx 12.6$ this yields $\hat{q} \approx 14.6\, T^3$. This fifteen year old result is about three times larger than that estimated for QGP in QCD – not unreasonable.**

- **$\hat{q}$ is *not* proportional to s or to the number density of scatterers, as at weak coupling. Such quantities are $\propto N_c^2 T^3$, and $\hat{q} \propto \sqrt{\lambda}T^3$ in strongly coupled plasma.**

- **Reminds us that strongly coupled holographic liquids have no well-defined quasiparticles, so $\hat{q}$ cannot count the density of such.**

INSIGHTS I DESCRIBED / SKETCHED

① Thermodynamics within 15–25% of that at zero coupling arises at strong coupling.

② $\eta/s = 1/4\pi$, in $N_c^2 \to \infty$, $\lambda \to \infty$ limit, for plasma of any gauge theory with a gravity dual. η/s in QCD plasma (lattice; RHIC) and for unitary cold atom gas seems comparable.

③ $\hat{q} \propto \sqrt{\frac{S}{N_c^2 T^3}} \sqrt{\lambda}\, T^3$ for an infinite class of strongly coupled plasmas. Jet quenching does not count gluons; all multiple gluon correlations equally important.

$\hat{q} \sim 3\text{–}5\ \mathrm{GeV}^2/\mathrm{fm}$ at $T = 300\ \mathrm{MeV}$. $\frac{\hat{q}_{LHC}}{\hat{q}_{RHIC}} \sim \frac{(dN/d\eta)_{LHC}}{(dN/d\eta)_{RHIC}}$

④ In a strongly coupled plasma, heavy POINT-LIKE quarks drag, diffuse, and excite a Mach cone.

⑤ Heavy quarkonia mesons, bound above T_c, dissociate at lower temperatures when moving. $T_{diss}(v) \simeq T_{diss}(0)\,(1-v^2)^{1/4}$

Also for heavy quark baryons.

WHAT ARE WE LEARNING?

- Qualitative, and semi-quantitative, insights and predictions regarding properties of strongly interacting quark-gluon plasma
- String theory useful as a source of new calculational techniques, opening previously intractable regimes.
- Perhaps QGP in QCD at $T \sim$ few T_c is "close to", or maybe even is, equivalent to a string theory in a curved 4+1 dimensional spacetime containing a black hole. Is there a precise sense of universality here?
- Could successes of this approach to RHIC data hint that QCD is equivalent to a string theory????

What Next?

Two kinds of What Next? questions for the coming decade...
(and for Wednesday)

- A question that one asks after the discovery of any new form of complex matter: What is its phase diagram? For high temperature superconductors, for example, phase diagram as a function of temperature and doping. Same here! For us, doping means excess of quarks over anti-quarks, rather than an excess of holes over electrons.
- A question that we are privileged to have a chance to address, after the discovery of "our" new form of complex matter: How does the strongly coupled liquid emerge from an asymptotically free gauge theory? Maybe answering this question could help to understand how strongly coupled matter emerges in other contexts.

Second introduction concluded....

What Next?

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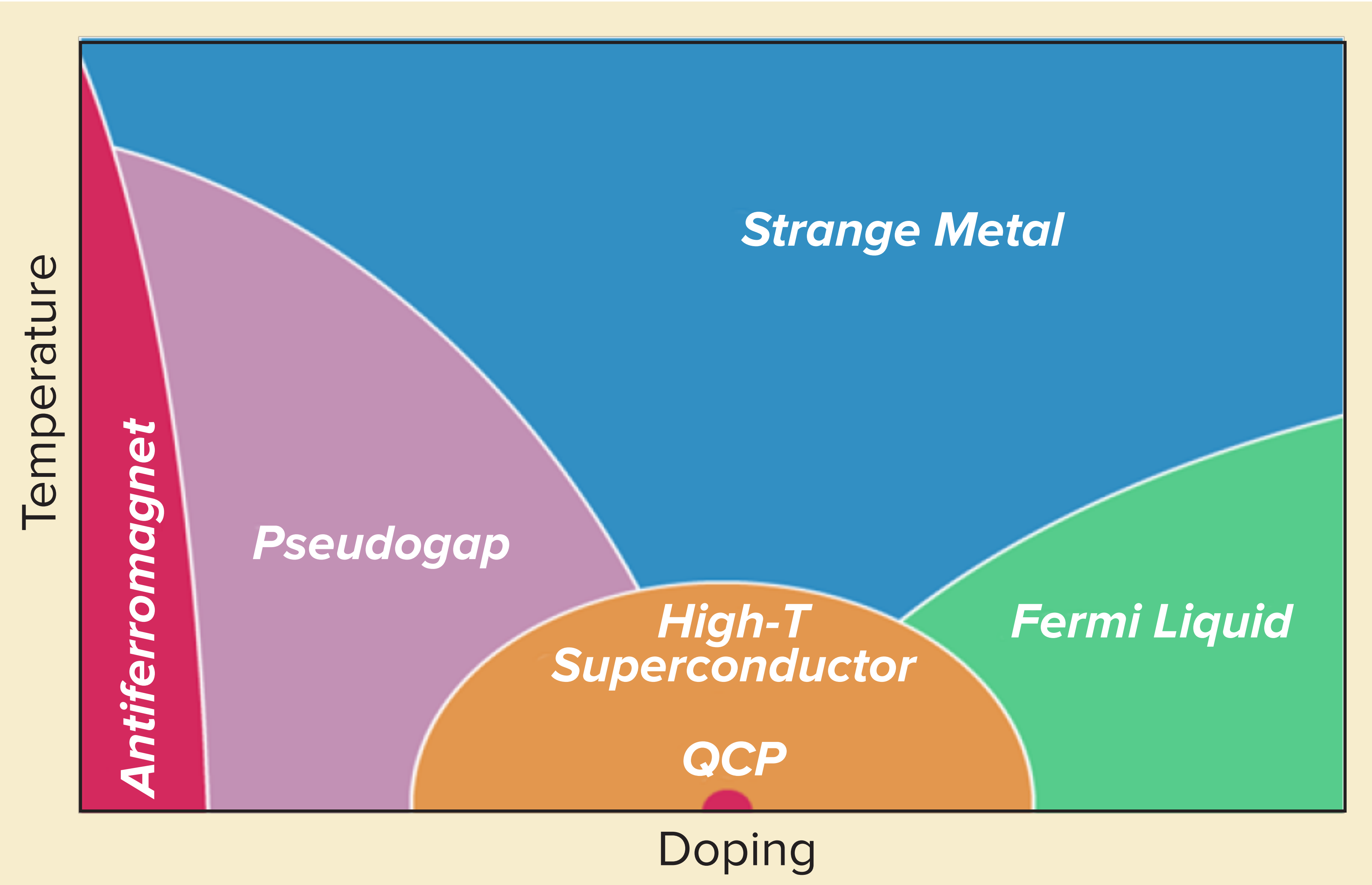
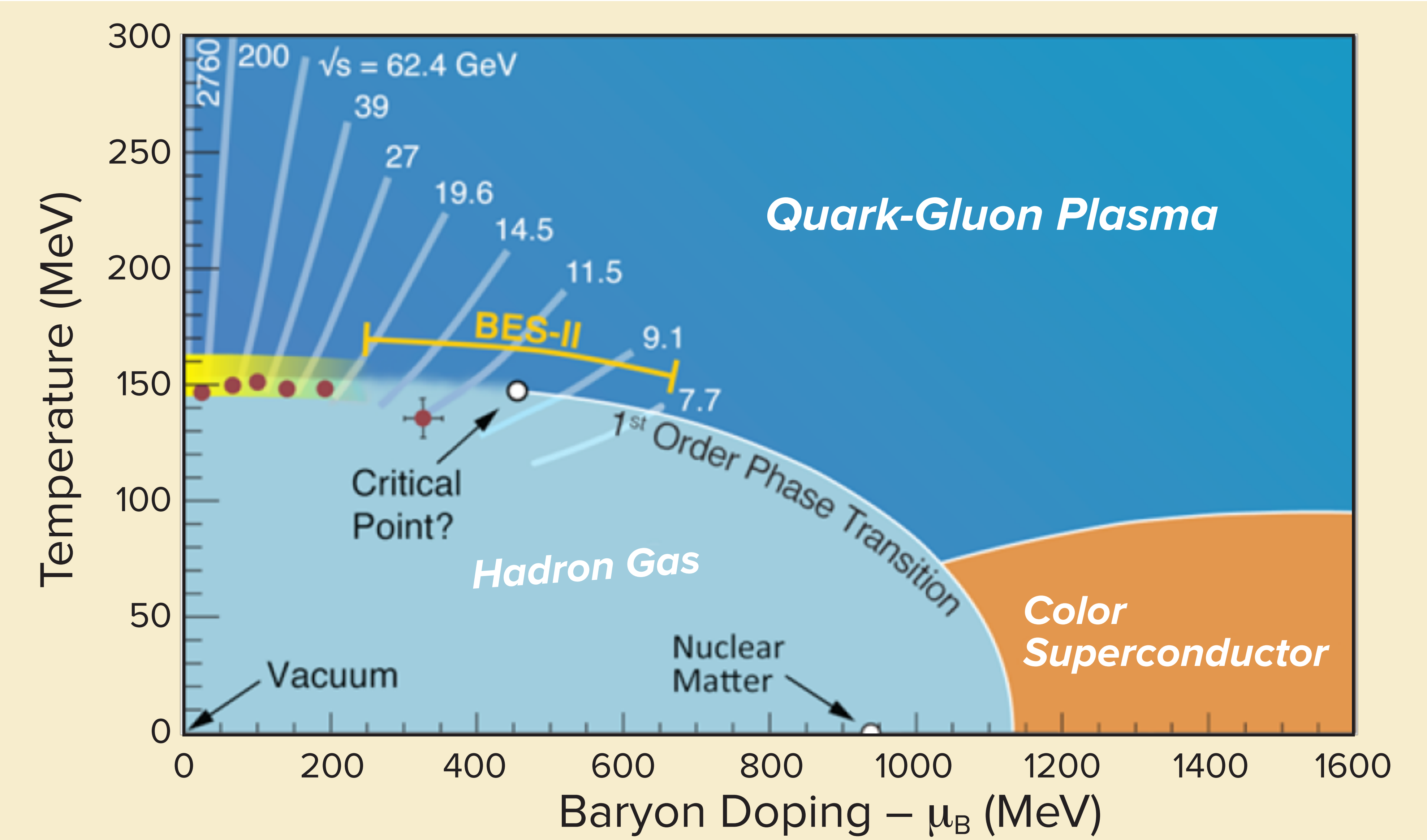
  Three different variants of this question...

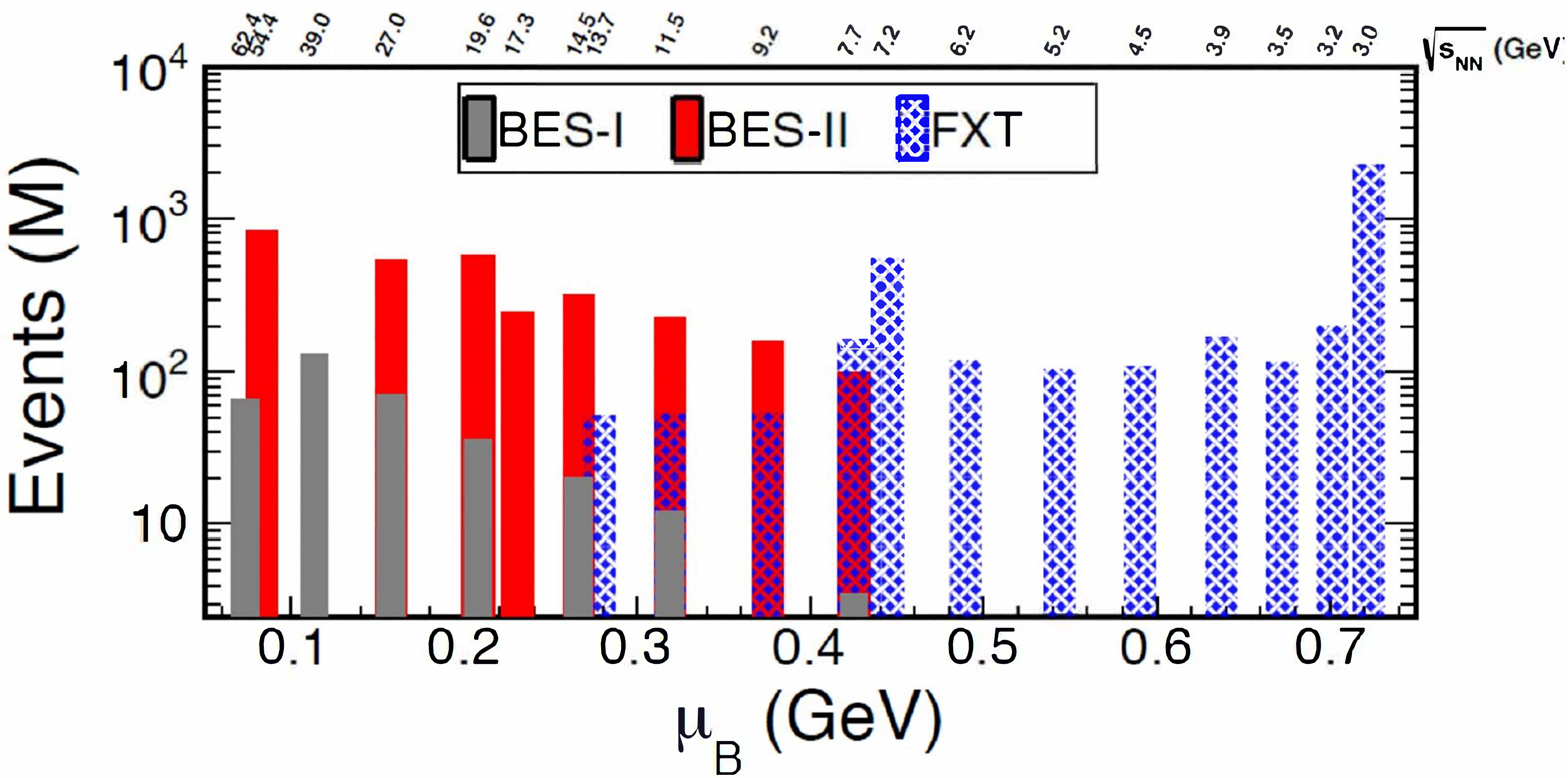

Strange Metal
Pseudogap
Antiferromagnet
High-T
Superconductor
QCP
Fermi Liquid
Temperature
Doping


300
250
200
150
100
50
0
Temperature (MeV)
0
200
400
600
800
1000
1200
1400
1600
Baryon Doping – μB (MeV)
2760
200
√s = 62.4 GeV
39
27
19.6
14.5
11.5
9.1
7.7
BES-II
Quark-Gluon Plasma
Critical Point?
1st Order Phase Transition
Hadron Gas
Color Superconductor
Vacuum
Nuclear Matter

Mapping the QCD Phase Diagram

- How does QGP change as you "dope" it with a larger and larger excess of quarks over antiquarks, i.e. larger and larger μ_B?
- Substantial recent progress... Slides from 2015 almost completely superseded.
- Enormous progress on theory and modeling, by many people. Including by the BEST collaboration – see 2108.13867 for a summary.
- Phase II of the RHIC Beam Energy Scan data taking was completed in 2021. We await results with great interest and anticipation.

RHIC BES II Data Taken...



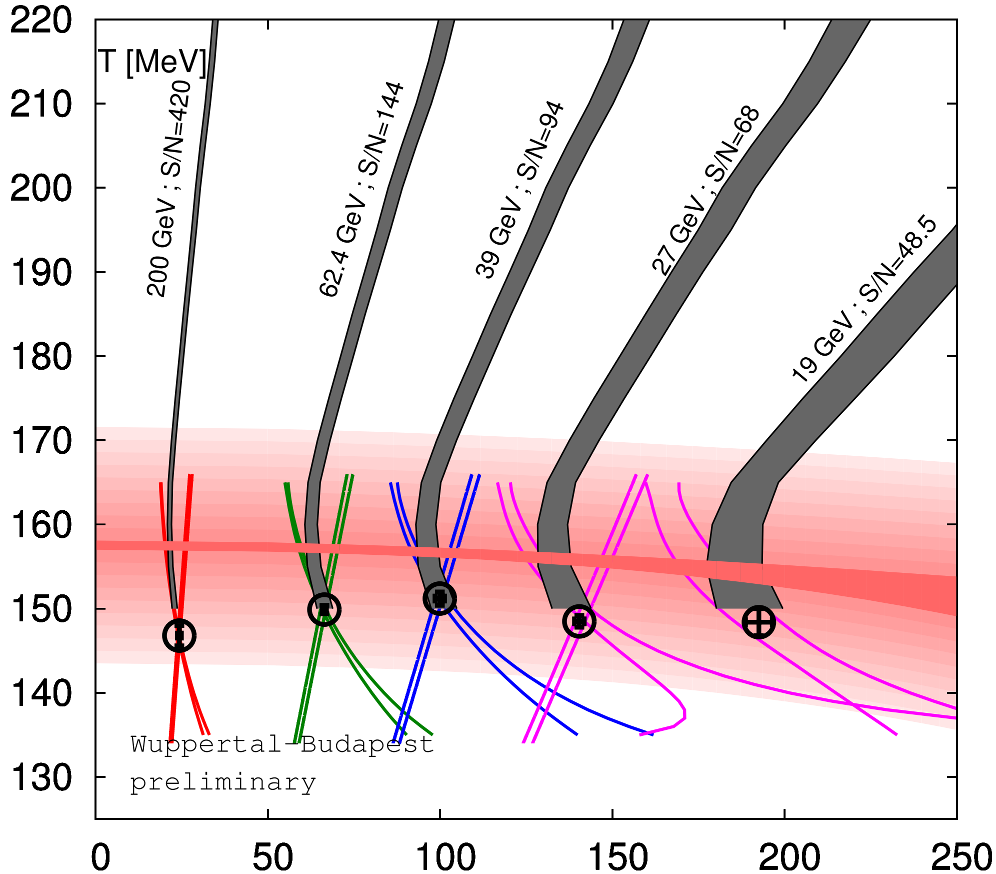
Mapping the QCD Phase Diagram

- How does QGP change as you "dope" it with a larger and larger excess of quarks over antiquarks, i.e. larger and larger μ_B? Substantial recent progress in answering questions like this on the lattice, e.g. doping-dependence of equation of state and susceptibilities, as long as the doping is not too large. Combining lattice and RHIC Beam Energy Scan results to map the crossover region.

- How is the crossover between QGP and hadrons affected by doping? Does it turn into a first order transition above a critical point?

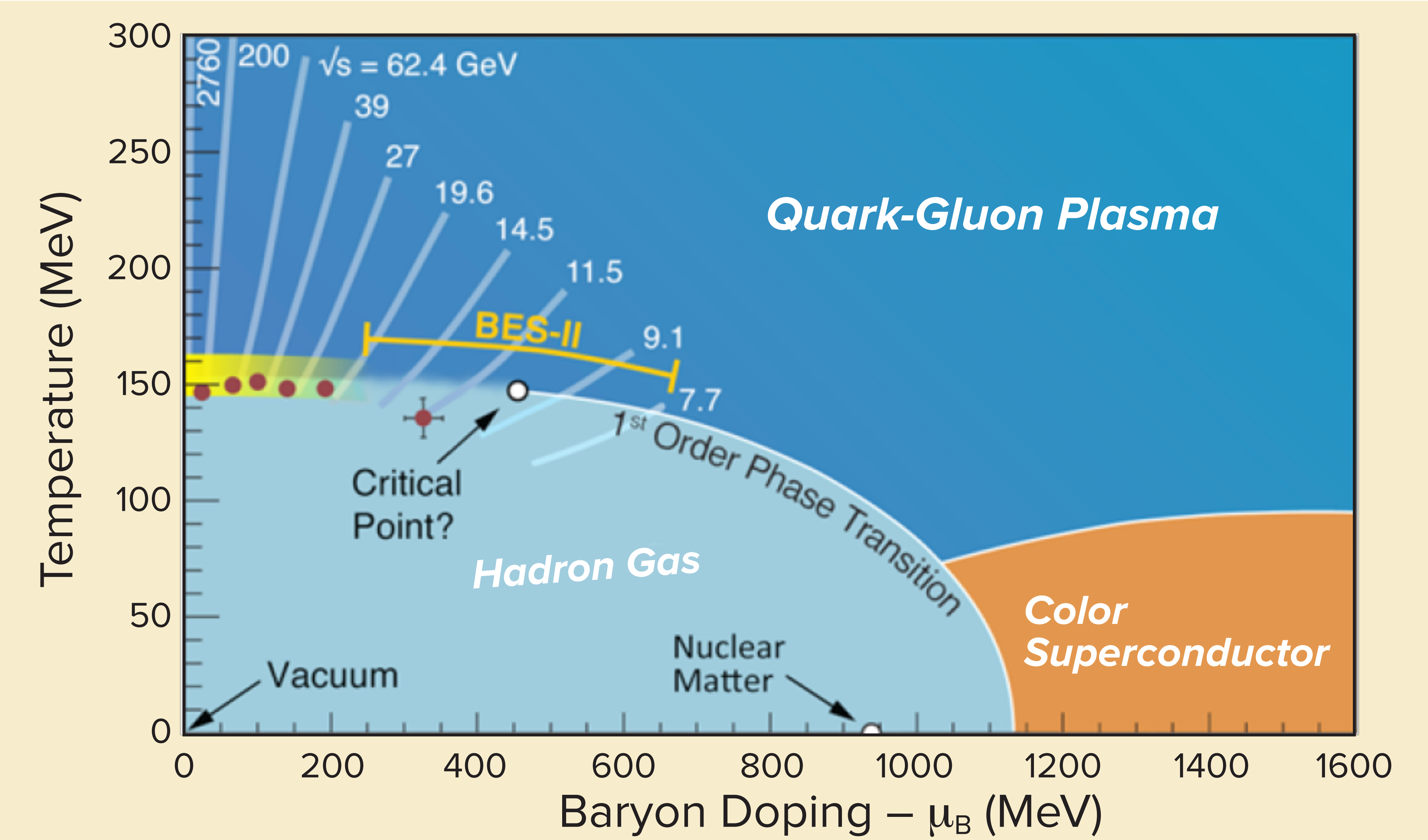
- Answering this question via theory will need further advances in lattice "technology". Impressive recent progress advancing established Taylor-expansion methods. New ideas also being evaluated. Nevertheless, at present theory is good at telling us what happens near a critical point or first order transition, but cannot tell us where they may be located.

Mapping the Crossover Region



Wuppertal-Budapest-Houston, 1601.00466

Lattice determination of crossover region compared with freeze-out points obtained from the intersection of: (i) lattice calculations and BES-I exptl measurements of magnitude of charge fluctuations and proton number fluctuations; (ii) hadron resonance gas calculations of and exptl measurements of S/N.



Quark-Gluon Plasma
Hadron Gas
Color Superconductor
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9.1
7.7
Temperature (MeV)
Baryon Doping – μB (MeV)
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100
150
200
250
300
0
200
400
600
800
1000
1200
1400
1600

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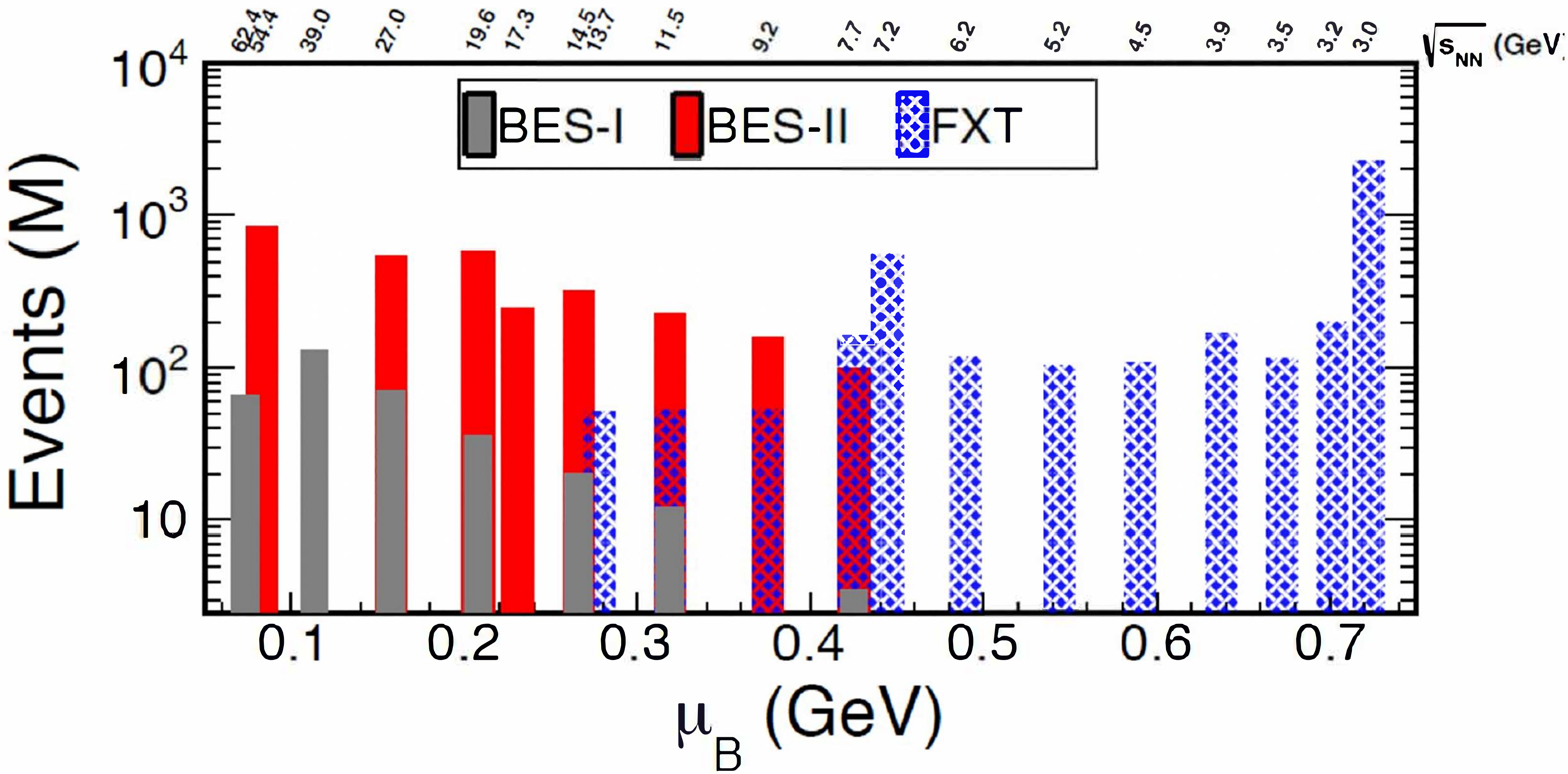
RHIC BES II Data Taken...

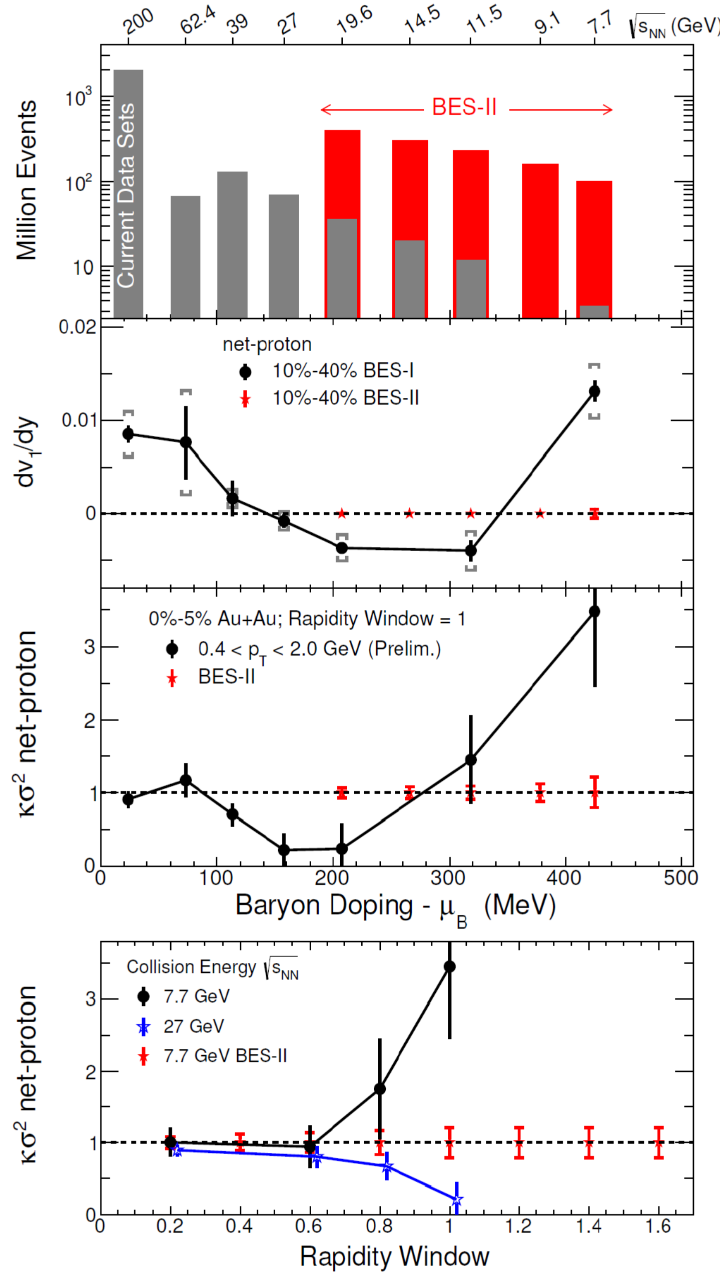


Mapping the QCD Phase Diagram

- Exploring the phase diagram is the goal of the RHIC Beam Energy Scan. Pioneering results from BES-I, 2011-14. Suggestive variations in flow and fluctuation observables as a function of $\sqrt{s}$, and hence μ_B. Strong motivation for higher statistics data below $\sqrt{s} = 20$ GeV $\rightarrow$ BES-II.

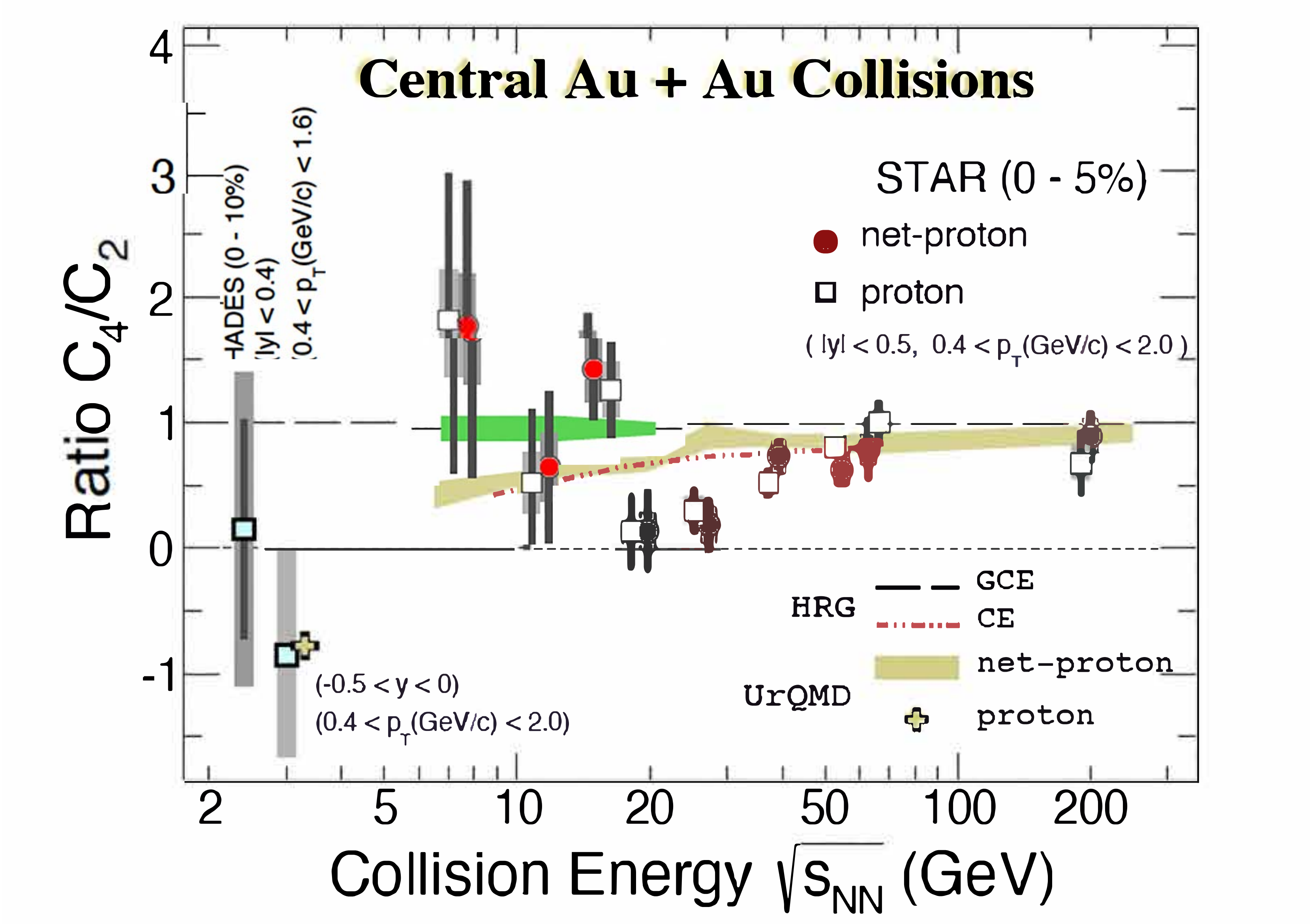
- BES-I results present an opportunity for theory. Interpreting flow (and other) observables requires 3+1-D viscous hydrodynamic calculations at BES energies that evolve j_B^μ in addition to $T^{\mu\nu}$, and must include state-of-the-art treatment of the hadrodynamics: relative importance of hadrodynamic effects on all observables grows. Also need baryon stopping and state-of-the-art initial state. BES-I data demand that the sophistication that has been applied at top energies be deployed at BES energies.

- Theorists, including in the BEST collaboration, have developed these tools; I will focus today on the fluctuation observables used to search for the critical point.



200
62.4
39
27
19.6
14.5
11.5
9.1
7.7
$\sqrt{s_{NN}}$ (GeV)
Million Events
Current Data Sets
BES-II
net-proton
10%-40% BES-I
10%-40% BES-II
dv_1/dy
0%-5% Au+Au; Rapidity Window = 1
$0.4 < p_T < 2.0$ GeV (Prelim.)
BES-II
$\kappa\sigma^2$ net-proton
Baryon Doping - μ_B (MeV)
Collision Energy $\sqrt{s_{NN}}$
7.7 GeV
27 GeV
7.7 GeV BES-II
$\kappa\sigma^2$ net-proton
Rapidity Window

Proton Kurtosis, before BES II

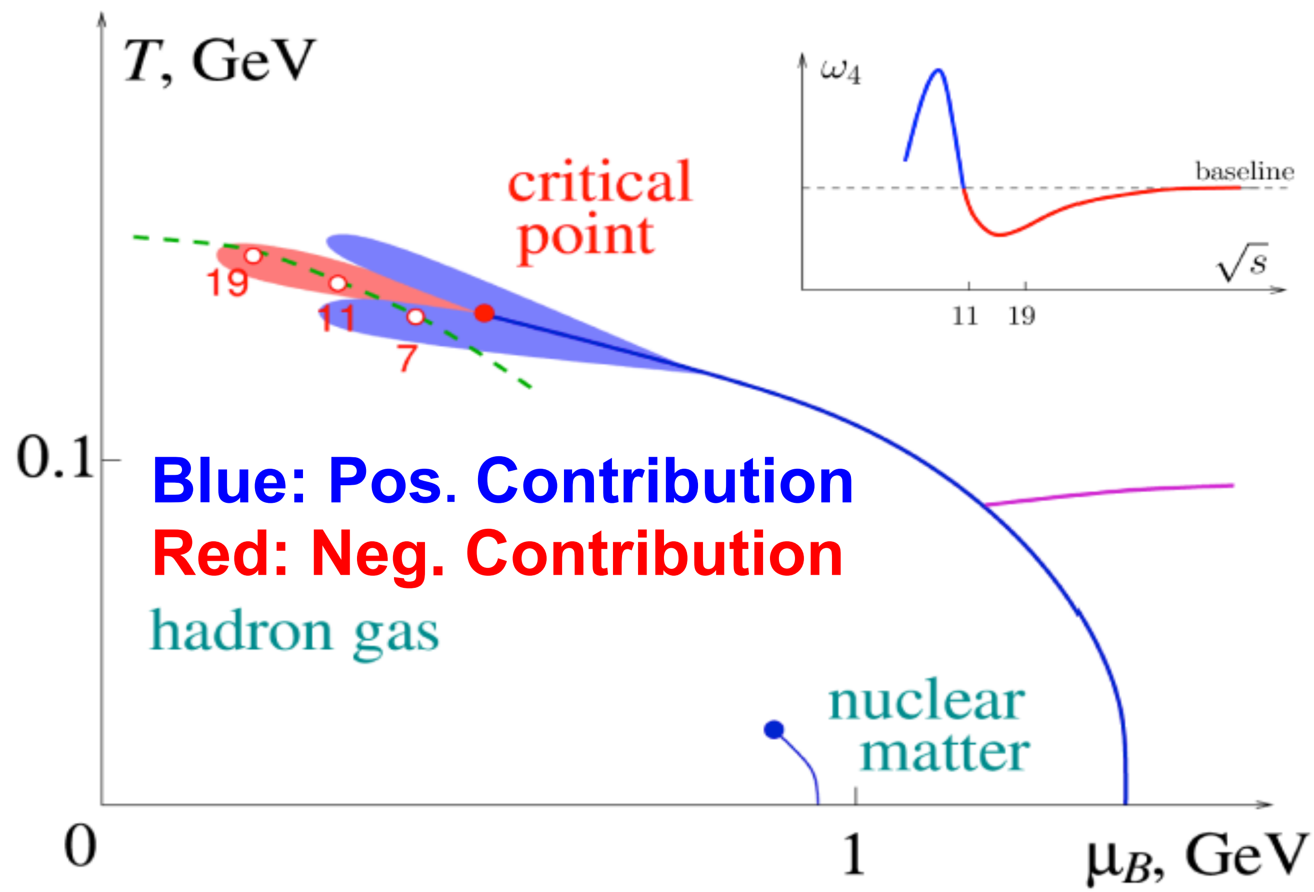


Mapping the QCD Phase Diagram

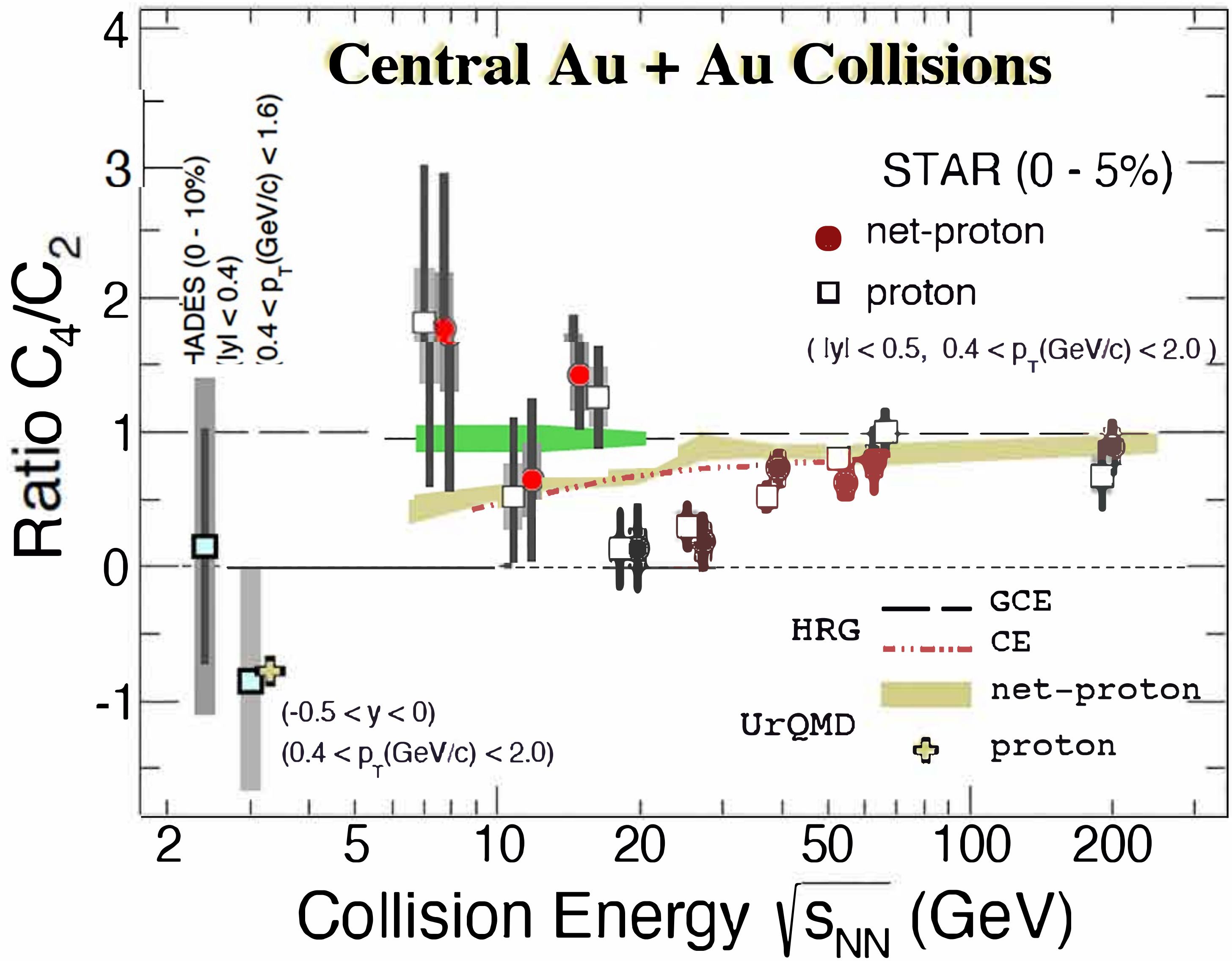
- How can we detect the presence of a critical point on the phase diagram, if there is one, in HIC data?

- A negative contribution to the proton kurtosis at $\mu_B \sim 150 - 200$ MeV is established. Is this a harbinger of the approach toward a critical point at larger μ_B? Signs of an upturn at larger μ_B are inconclusive. Higher statistics data needed. As are substantial advances on the theory side...

- Once you have a validated hydrodynamic model at BES energies, then you can add both hydrodynamic fluctuations and the critical fluctuations of the chiral order parameter. Need to source them, evolve them, and describe their consequences at freezeout. Need self-consistent treatment: fluctuations can't stay in eqbm because of finite-time limitation on growth of the correlation length, how do the fluctuations evolve? Feedback on hydro? Only then can quantify the signatures of, a possible critical point.


T, GeV
0.1
0
1
μB, GeV
critical
point
19
11
7
ω4
baseline
√s
11
19
Blue: Pos. Contribution
Red: Neg. Contribution
hadron gas
nuclear
matter

Proton Kurtosis, before BES II



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- BES-II data-taking completed in 2021; results anticipated soon. Error bars will shrink and today's tantalizing hints, e.g. of non-monotonic behavior in dv_1/dy and in the kurtosis of the proton multiplicity distribution, will become ... ?

Mapping the QCD Phase Diagram

- Finding, or excluding, a critical point requires theory and modeling, with ingredients including:
- Energy and baryon number in initial stages.
- Equation of State (EoS)
  - Known (lattice QCD) at $\mu_B = 0$; universal features known near a critical point. Putting these together into a model EoS with non-universal parameters to be fixed via comparison to data: Parotto, $\ldots$, KR, et al, 1805.05249. Now referred to as the "BEST EoS".
  - Implementing strangeness conservation and neutrality (2110.00622) into BEST EoS
  - Extending BEST EoS to describe first order phase transition (Karthein, Koch, Ratti, in progress)
- Hydrodynamics. Critical fluctuations.
- Freezeout of critical fluctuations.

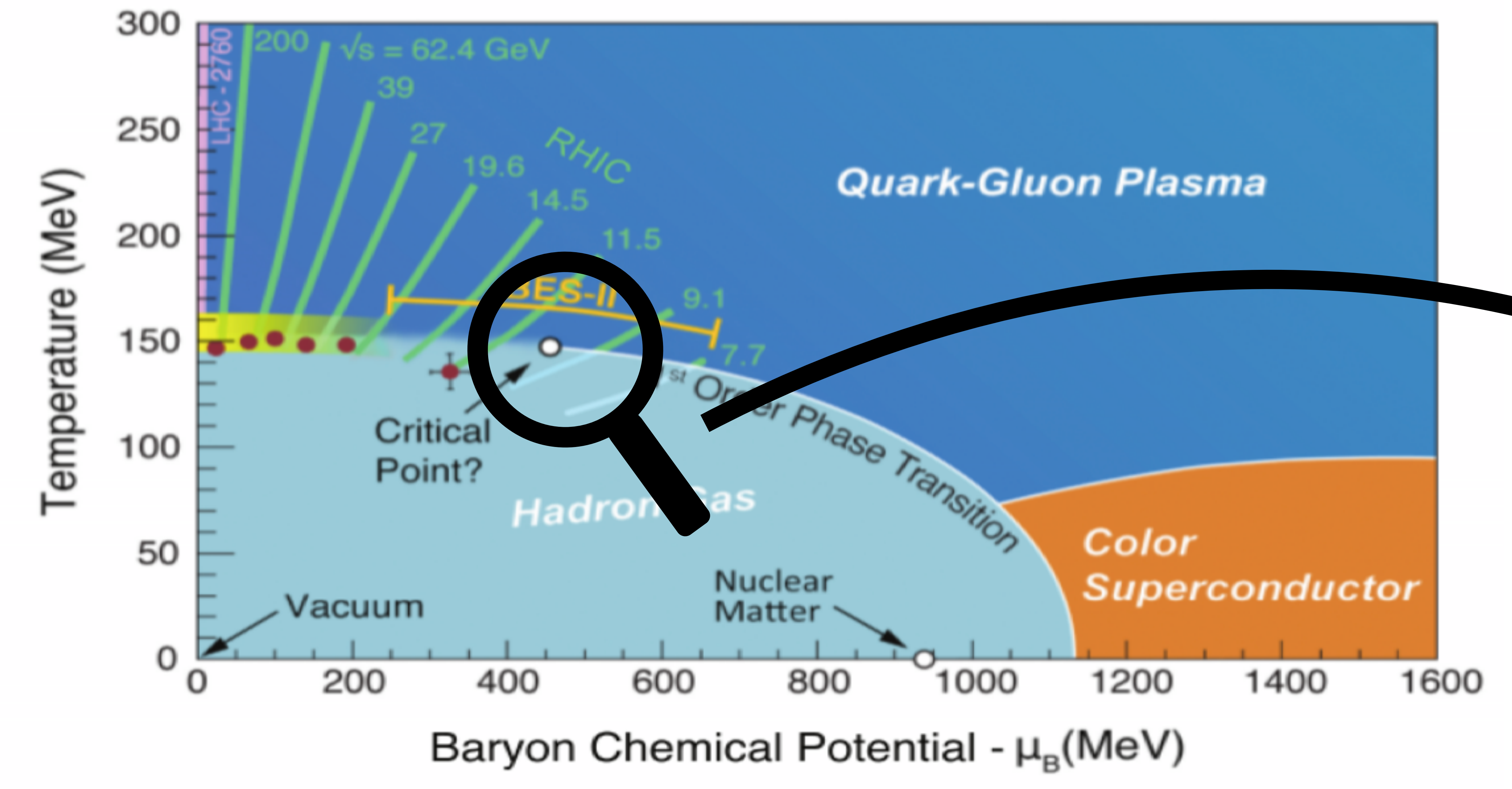
Equilibrium expectations for non-Gaussian fluctuations near a QCD critical point

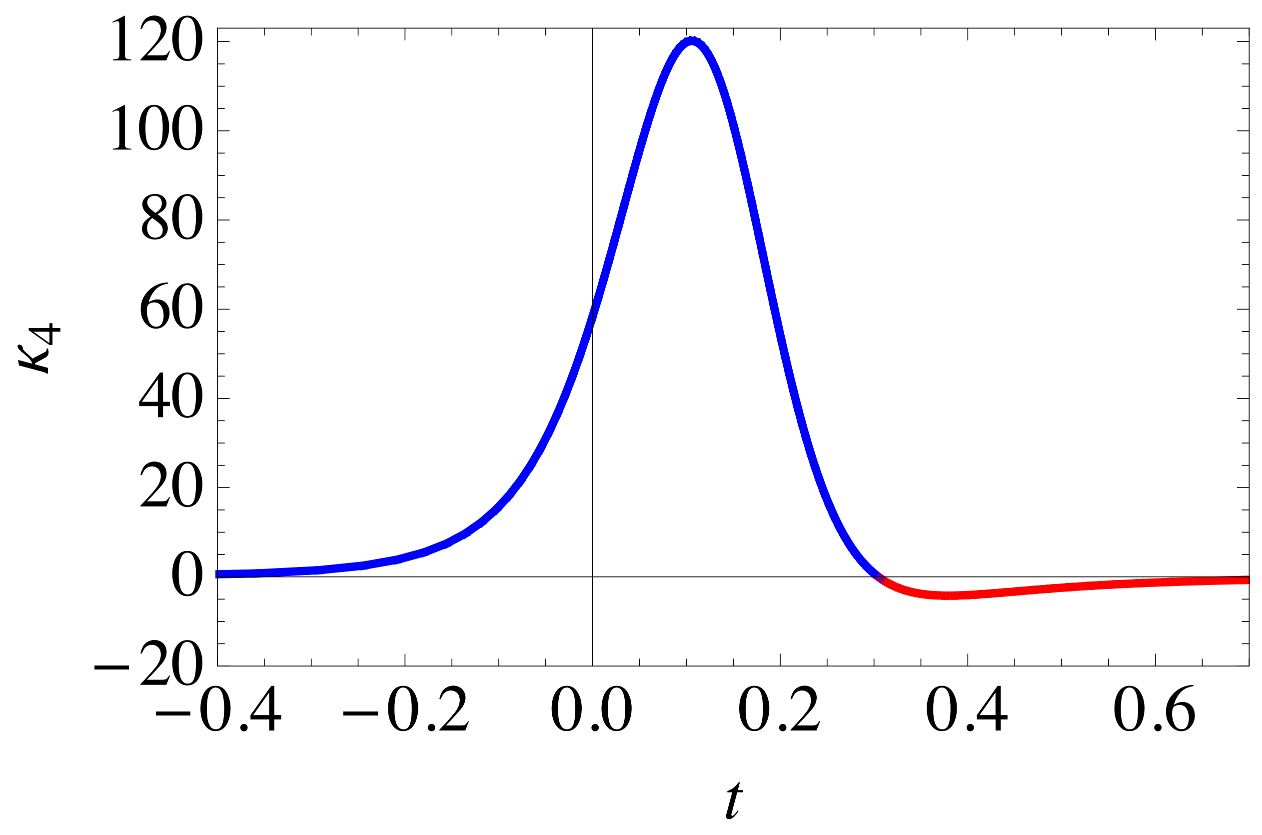
Jamie M. Karthein

Collaborators: Maneesha Pradeep, Misha Stephanov,
Krishna Rajagopal, and Yi Yin

Search for Criticality

- Ongoing search for critical point requires support from theory community to provide candidates for criticality-carrying observables



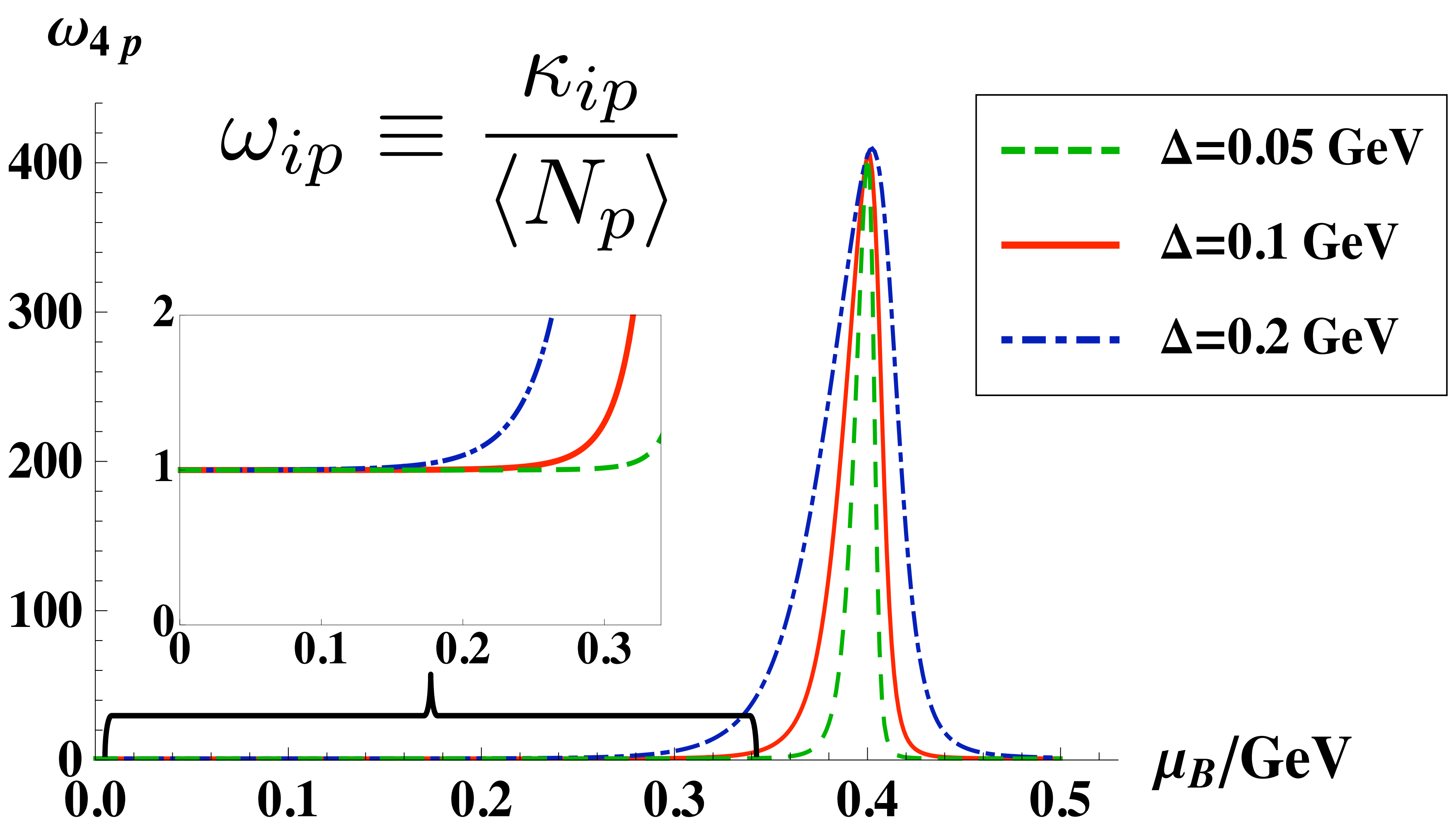


- Higher order susceptibilities diverge with higher power of the correlation length, $\kappa_4 \propto \xi^7$ $\qquad \chi_n^B \equiv \dfrac{\partial^n (p/T^4)}{\partial(\mu_B/T)^n}$
- Related to moments of the net-proton distribution: can be measured experimentally $\qquad \kappa_4\sigma^2 = \chi_4^B/\chi_2^B$

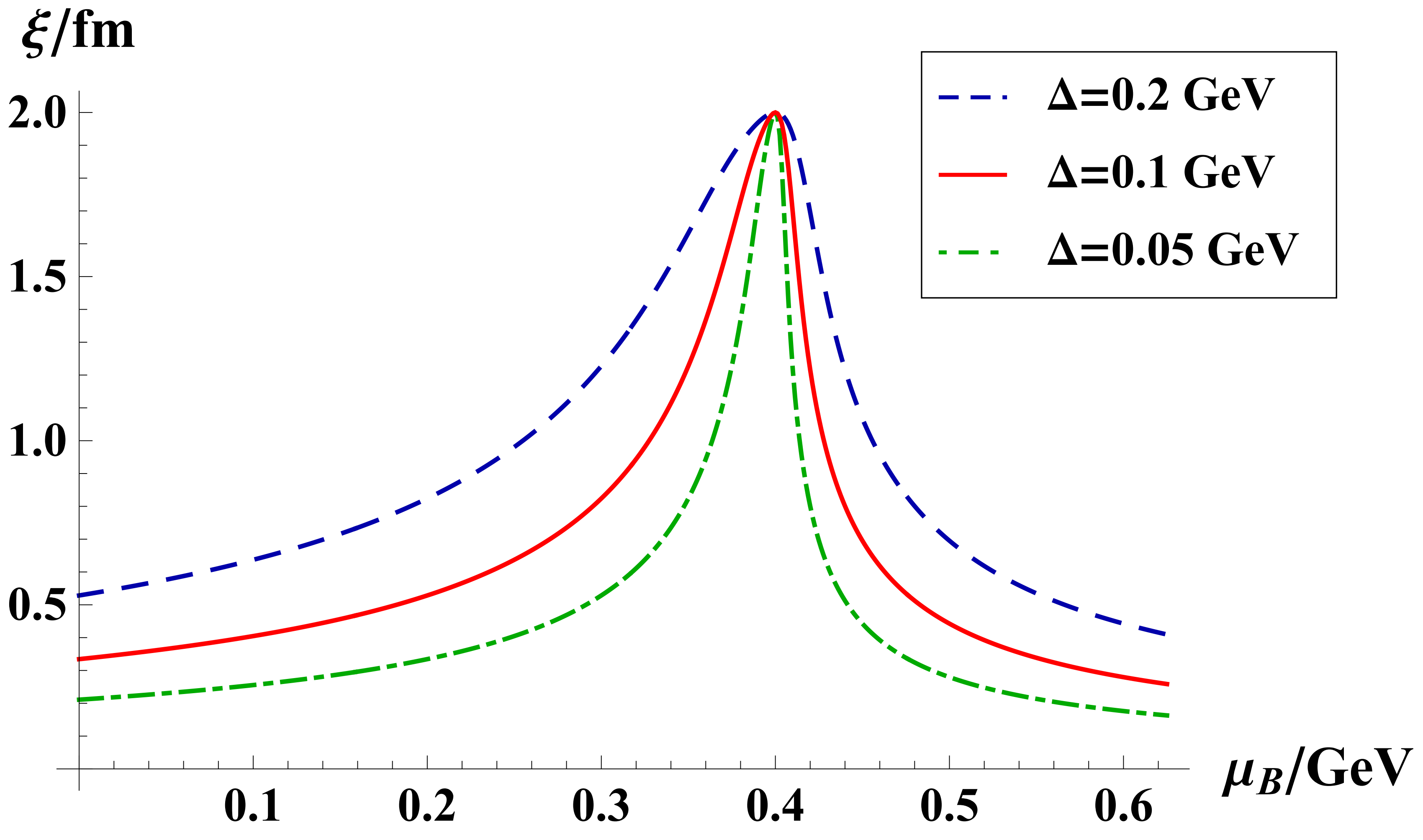
NSAC 2015 Long Range Plan for Nuclear Physics
M. Stephanov, K. Rajagopal and E. Shuryak, PRD (1999)
M. Stephanov, PRL (2011)

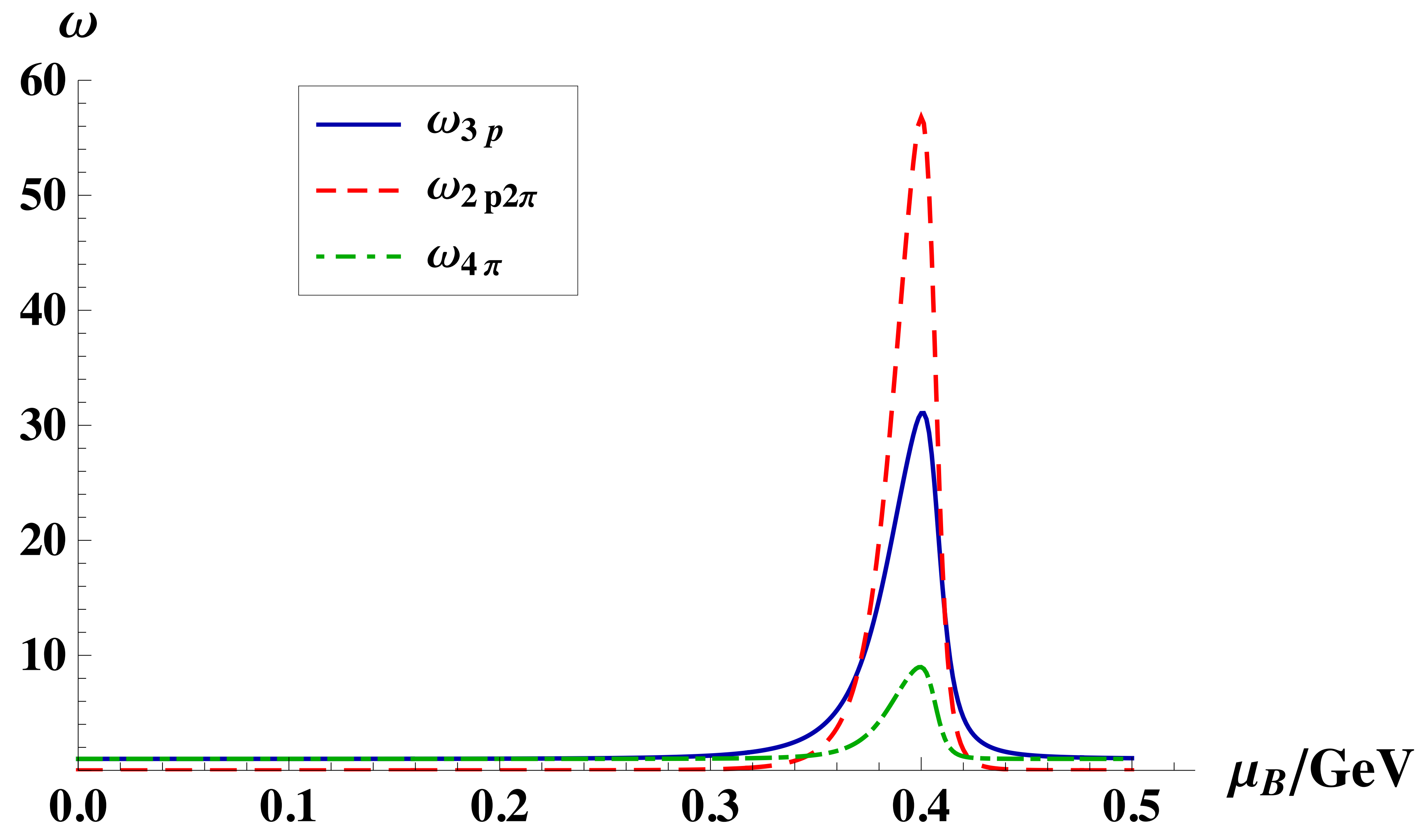
Early Estimates of Equilibrium Fluctuations

- Order-of-magnitude predictions of volume-independent normalized cumulants from 2010 relied on ansätze
  - Original estimates used parametrized correlation length with width Δ



$$\xi(\mu_B) = \frac{\xi_{\max}}{\left[1 + \frac{(\mu_B - \mu_B^c)^2}{W(\mu_B)^2}\right]^{1/3}}$$





C. Athanasiou, K. Rajagopal, M. Stephanov, PRD (2010)

Effective Field Theory for Critical Fluctuations

See talk by M. Pradeep next

- Fluctuations near the critical point are driven by coupling of particles to **σ**-field

$$\Omega = \int d^3\boldsymbol{x} \left[\frac{(\boldsymbol{\nabla}\sigma)^2}{2} + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \dots \right]$$

$$\delta f_{\boldsymbol{p}} = \delta f_{\boldsymbol{p}}^0 + \frac{\partial n_{\boldsymbol{p}}}{\partial m} g\,\delta\sigma$$

$$\delta m = g\delta\sigma$$

- Correlation length diverges as the **σ** mass vanishes: $\xi = m_\sigma^{-1}$
  - Higher order fluctuations depend on larger powers of ξ, introduce higher point couplings

$$\kappa_2 = \langle \sigma_V^2 \rangle = VT\,\xi^2\,; \qquad \kappa_3 = \langle \sigma_V^3 \rangle = 2\lambda_3 VT^2\,\xi^6$$

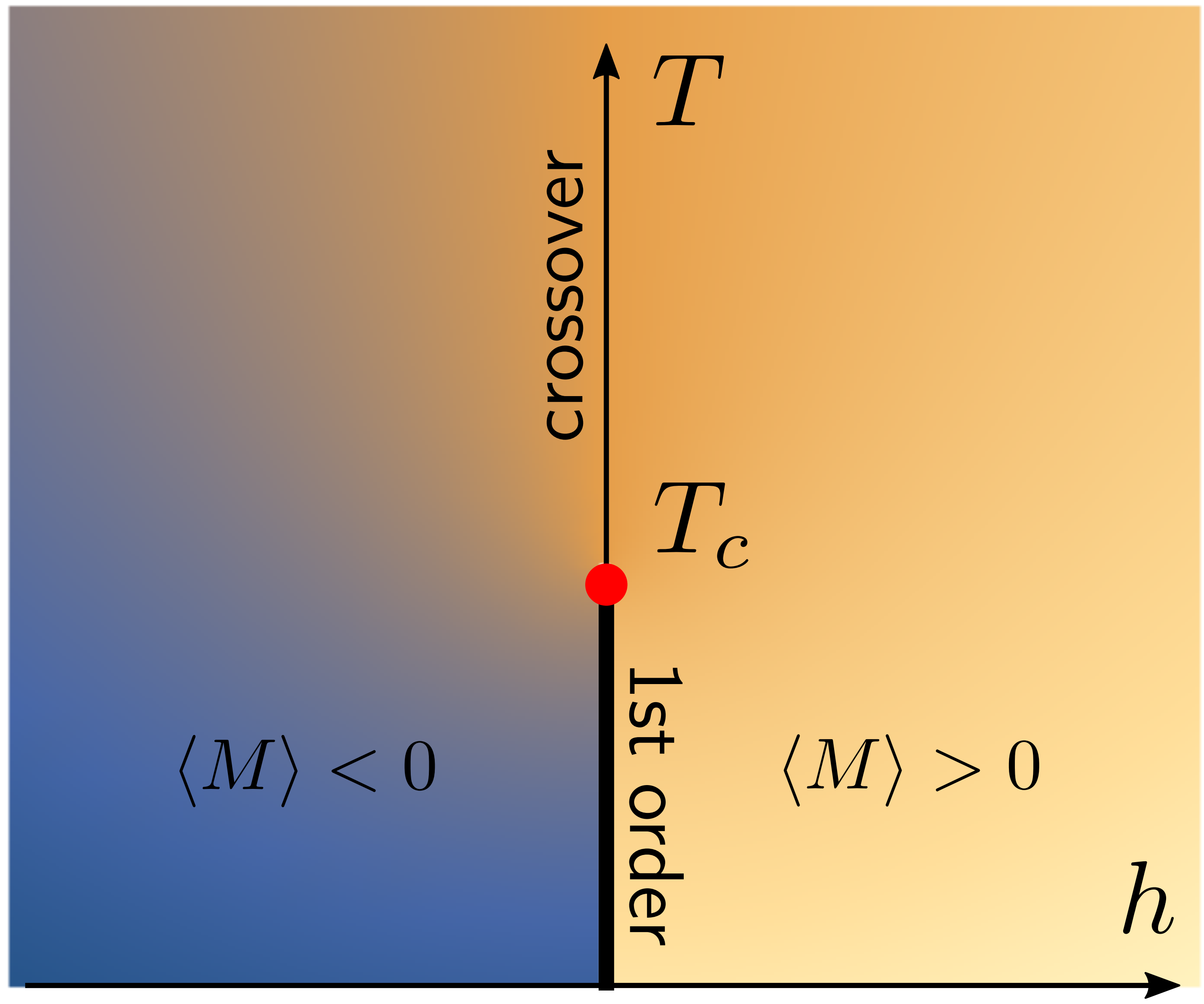
$$\kappa_4 = \langle \sigma_V^4 \rangle_c \equiv \langle \sigma_V^4 \rangle - 3\langle \sigma_V^2 \rangle^2 = 6VT^3\,[\,2(\lambda_3\xi)^2 - \lambda_4\,]\,\xi^8$$

M. Stephanov, K. Rajagopal and E. Shuryak, PRD (1999)
C. Athanasiou, K. Rajagopal, M. Stephanov, PRD (2010)
M. Stephanov, PRL (2011)

Universal Scaling EOS

- Average critical fluctuations of σ give rise to "magnetization": $M = \langle \sigma \rangle$
- Universal critical scaling behavior given by the 3D Ising model equation of state:
  - Magnetic field: $h = h_0 R^{\beta\delta} H(\theta),\ H(\theta) = \theta(3 - 2\theta^2)$
  - Reduced temperature: $t = R(1 - \theta^2)$
  - Magnetization: $M = M_0 R^{\beta} \theta$
- Critical fluctuations calculated in 3D Ising EOS

$$\kappa_{n+1}^{\mathrm{eq}} \propto \left(\frac{\partial^n M^{\mathrm{eq}}(t,h)}{\partial h^n} \right)_t$$



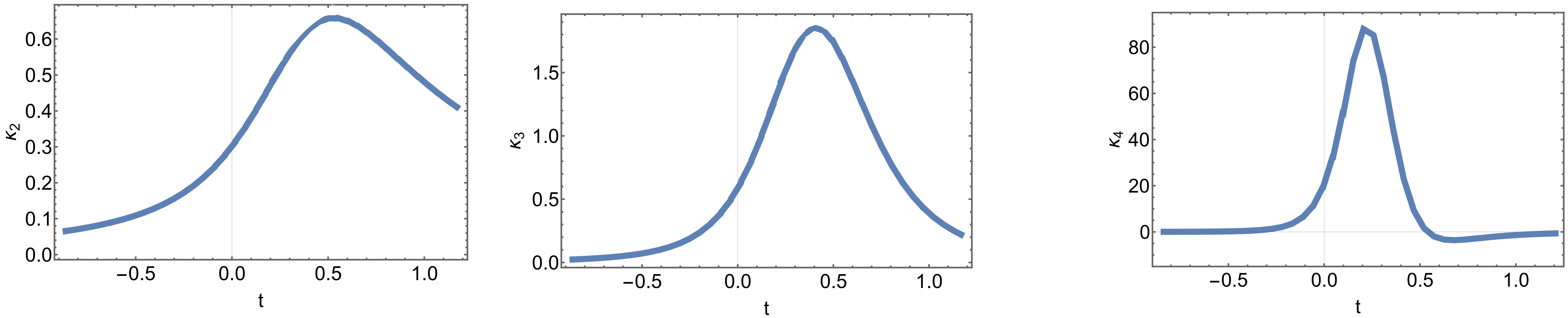
K. Rajagopal and F. Wilczek, Nucl. Phys. B (1993)
J. Zinn-Justin, Quantum Field Theory and Critical Phenomena
S. Mukherjee, R. Venugopalan, Y. Yin, PRC (2015)
A. Bzdak et al, Phys. Rep. (2020)

Equilibrium Fluctuations in 3D Ising Model

- Calculate critical fluctuations as parametric derivatives of universal EOS utilizing approximate critical exponents

$$\kappa_{n+1}^{\mathrm{eq}} \propto \left(\frac{\partial^n M^{\mathrm{eq}}(t,h)}{\partial h^n}\right)_t$$

$$\kappa_2 = \frac{M_0}{h_0}\frac{1}{R^{4/3}\left(3+2\theta^2\right)} \qquad \kappa_3 = \frac{-M_0}{h_0^2}\frac{4\theta\left(9+\theta^2\right)}{R^3\left(3-\theta^2\right)(3+2\theta)^3} \qquad \kappa_4 = \frac{-M_0}{h_0^3}\frac{12\left(2\theta^8-5\theta^6+105\theta^4-783\theta^2+81\right)}{R^{14/3}\left(3-\theta^2\right)^3\left(3+2\theta^2\right)^5}$$



M. Stephanov, PRL (2011)
S. Mukherjee, R. Venugopalan, Y. Yin, PRC (2015)

Equilibrium Correlation Length in 3D Ising Model

- 3D Ising EOS also provides a parametrization of the correlation length in the ϵ-expansion

$$\xi^2(M,t) = R^{-2\nu} g_\xi(\theta)$$

  - New equilibrium calculation to $\mathcal{O}(\epsilon^2)$

$$g_\xi(\theta) = g_\xi(0)\left(1 - \frac{5}{18}\epsilon\theta^2 + \left[\frac{1}{972}(24I - 25)\theta^2 + \frac{1}{324}(4I+41)\theta^4\right]\epsilon^2)\right)$$

$$\text{where:} \quad I \equiv \int_0^1 \frac{\ln[x(1-x)]}{1-x(1-x)}dx \sim -2.3439$$

- Now with the true critical EOS determine the higher order couplings

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J. Zinn-Justin, Quantum Field Theory and Critical Phenomena

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$$\text{where:} \quad I \equiv \int_0^1 \frac{\ln[x(1-x)]}{1 - x(1-x)} dx \sim -2.3439$$

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J. Zinn-Justin, Quantum Field Theory and Critical Phenomena

Mapping to QCD Phase Diagram

- Utilize the BEST EOS mapping between the Ising parametric variables and QCD

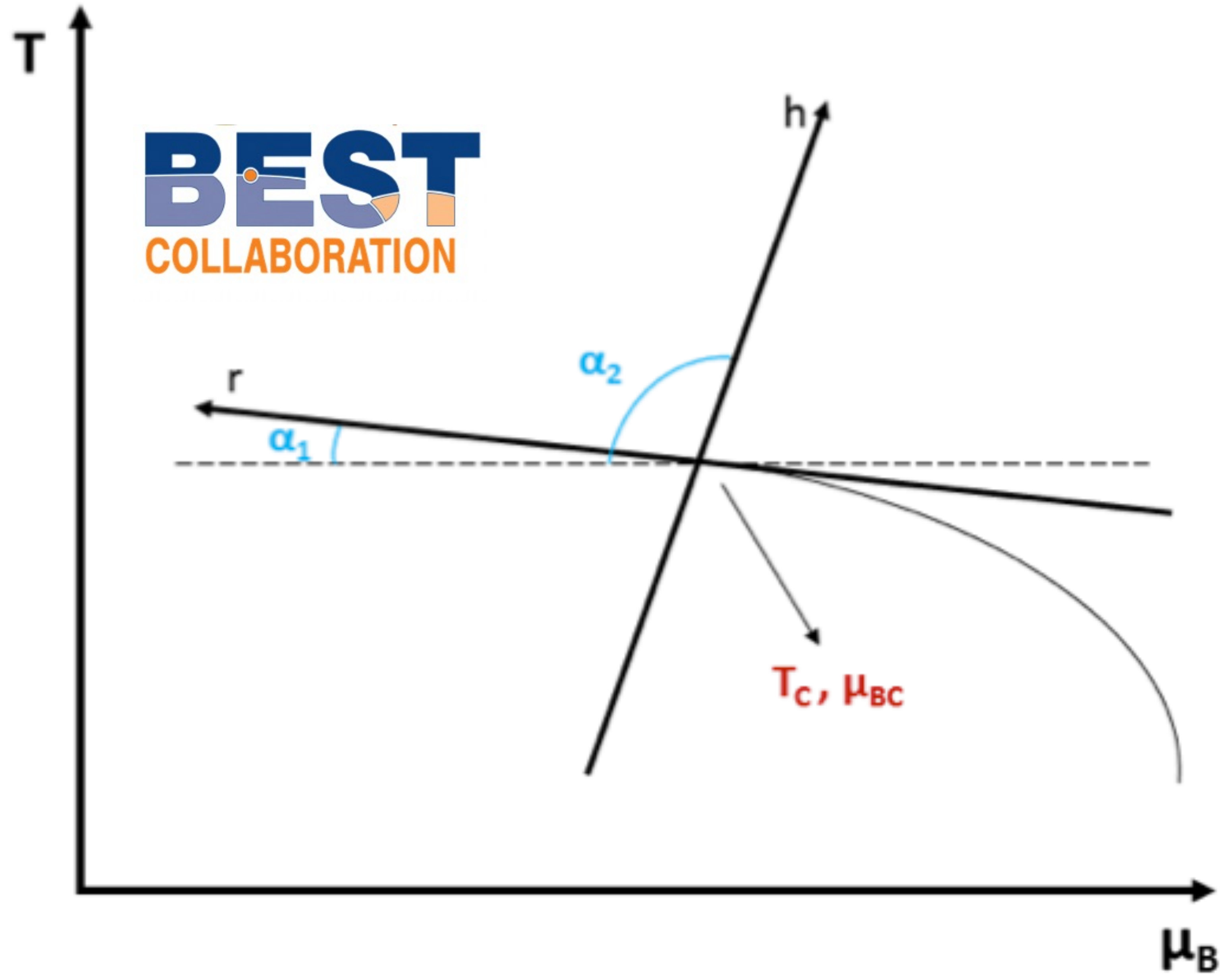
  - Linear map:

$$(\mathbf{r},\mathbf{h}) \longleftrightarrow (\mathbf{T},\mu_\mathbf{B}): \quad \frac{T-\mathbf{T_C}}{\mathbf{T_C}} = \mathbf{w}\,(r\rho\,\sin\alpha_1 + h\,\sin\alpha_2)$$
$$\frac{\mu_B-\mu_\mathbf{BC}}{\mathbf{T_C}} = \mathbf{w}\,(-r\rho\,\cos\alpha_1 - h\,\cos\alpha_2)$$

  - Reduce free parameters by imposing constraints from Lattice QCD

$$T = T_0 + \kappa T_0\left(\frac{\mu_B}{T_0}\right)^2 + O(\mu_B^4), \qquad \alpha_1 = \tan^{-1}\left(2\frac{\kappa}{T_0}\mu_{BC}\right)$$

  - Parameter choice consistent with BEST EOS: $\mu_{B,c} = 350\,\text{MeV}, w=1,\ \rho=2,\ \alpha_2-\alpha_1 = 90^o$



P. Parotto et al, PRC (2020),
***J. M. Karthein** et al, EPJ+ (2021)*

Estimating Equilibrium Fluctuations with BEST EOS

- Re-evaluate equilibrium estimates for normalized cumulants $\omega_{ip} \equiv \frac{\kappa_{ip}}{\langle N_p \rangle}$ with realistic critical EOS
  - Updates: $\xi, \lambda_3, \lambda_4$ (dimensionless, ξ-independent: $\tilde{\lambda}_3 = \lambda_3 T^{1/2} \xi^{3/2}$, $\tilde{\lambda}_4 = \lambda_4 T \xi$)
  - Remaining dependence on coupling: g_p

$$\omega_{4p,\sigma} = \frac{6(2\tilde{\lambda}_3^2 - \tilde{\lambda}_4)}{T^2 n_p} \xi^7 \left(d_p \, g_p \int_k \frac{v_k^2}{\gamma_k} \right)^4 \xrightarrow{\textit{generalize}} \omega_{ip} = 1 + \omega_{ip}^{\text{prefactor}} \left(\frac{n_p}{n_0} \right)^{i-1} \left(\frac{\xi}{\xi_{\max}} \right)^{\frac{5}{2}i-3}$$

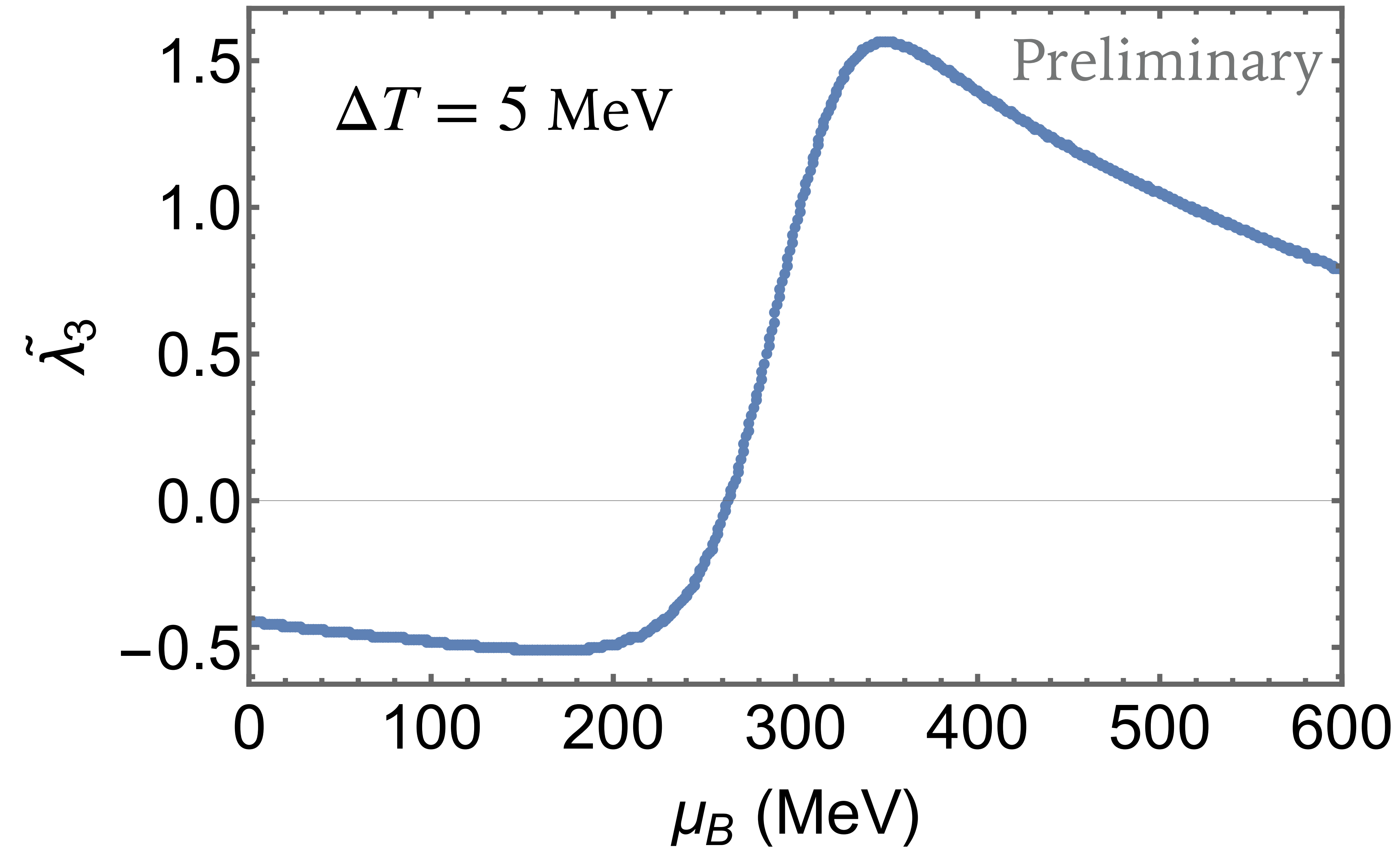
$$\omega_{ip}^{\text{prefactor}} = \frac{\tilde{\lambda}_i'(i-1)!\,\xi_{\max}^{\frac{5}{2}i-3}}{T^{i/2} n_p} \left(\int_k d_p \, g_p \frac{v_k^2}{\gamma_k} \right)^i \left(\frac{n_0}{n_p} \right)^{i-1}$$

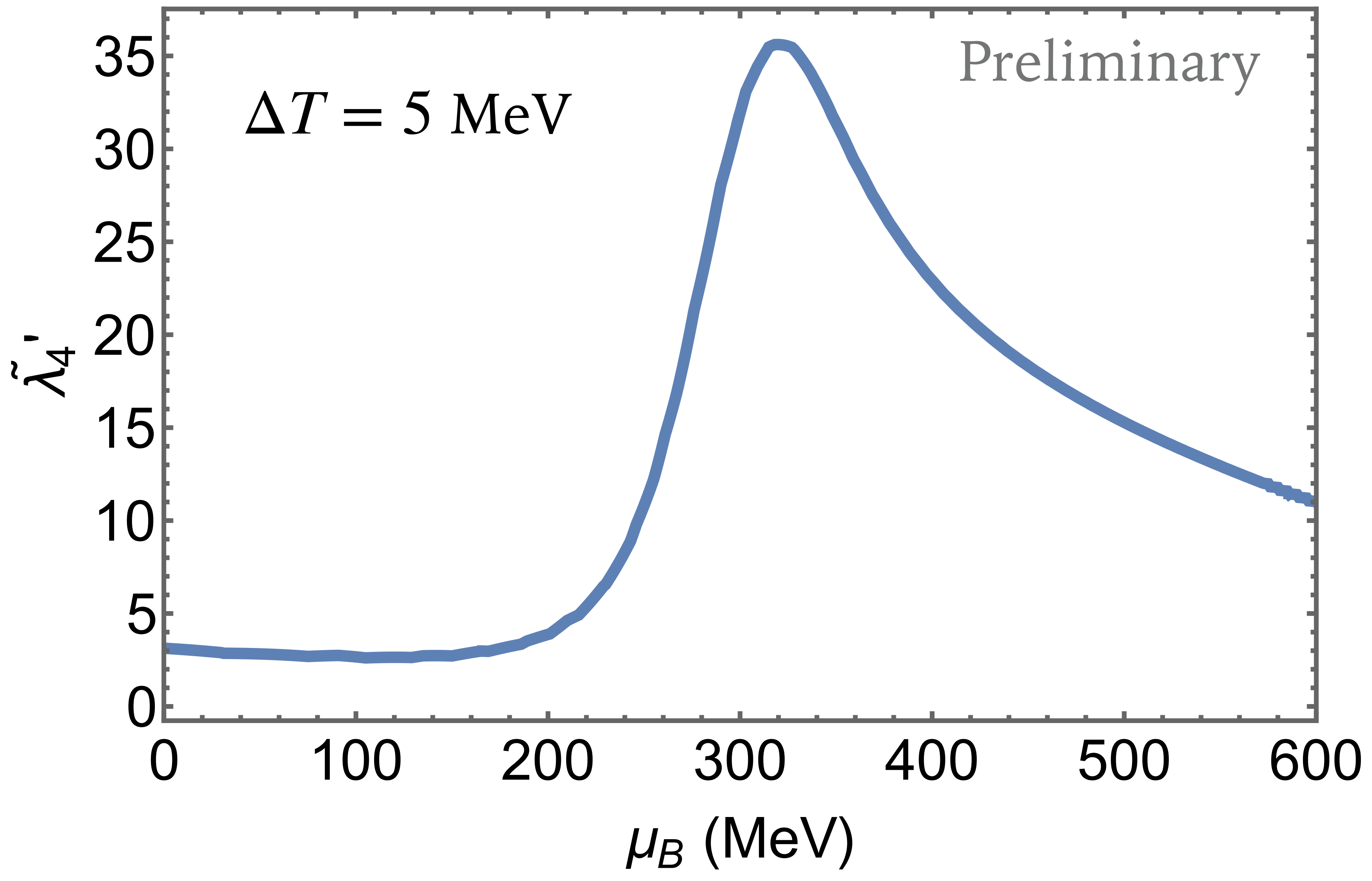
$$\tilde{\lambda}_3' \equiv \tilde{\lambda}_3 \quad \text{and} \quad \tilde{\lambda}_4' \equiv 2\tilde{\lambda}_3^2 - \tilde{\lambda}_4$$

C. Athanasiou, K. Rajagopal, M. Stephanov, PRD (2010)

Extracting Higher-point Couplings

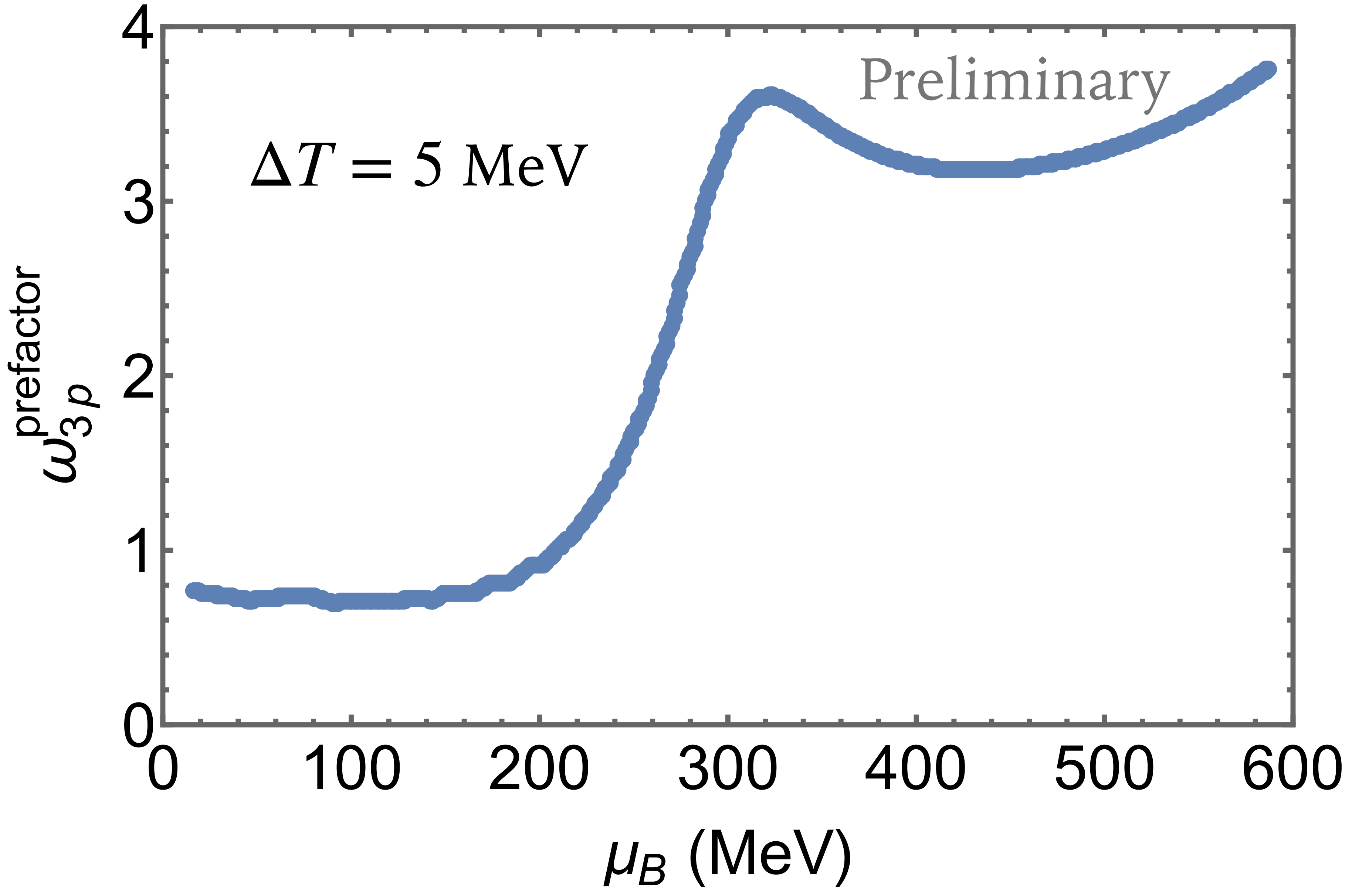
- Determine dimensionless couplings and their μ_B-dependence along chemical freeze-out lines parallel to the transition line from Lattice QCD $\Delta T = 5$ MeV below critical point

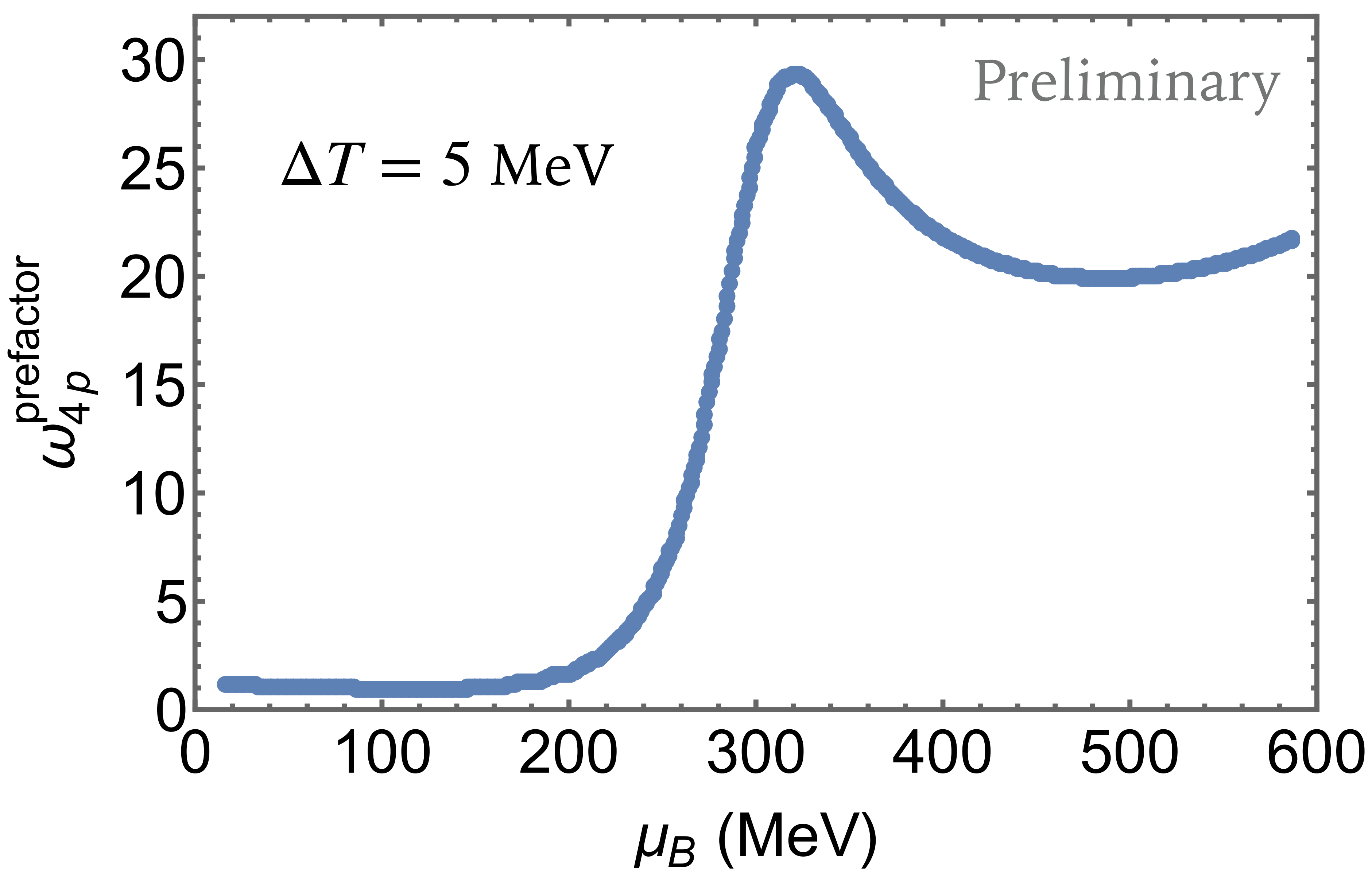




Pre-factors for Equilibrium Normalized Cumulants

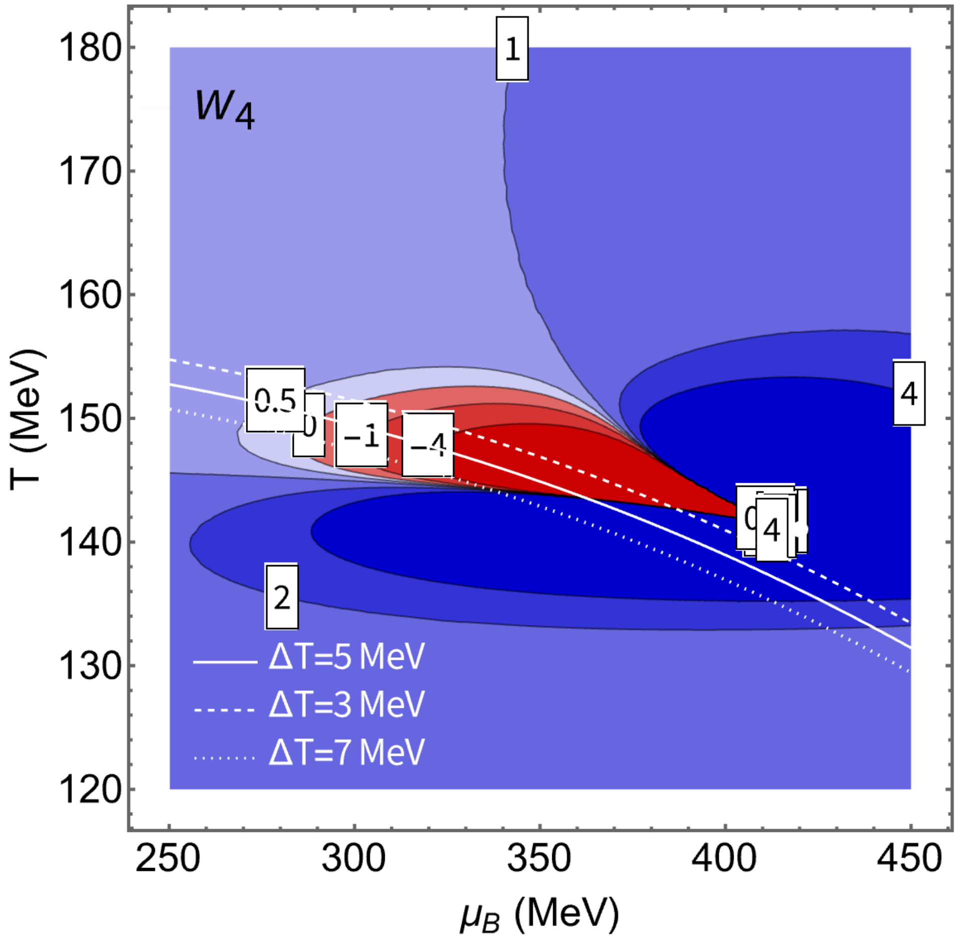
- Update non-critical pre-factors along the same freeze-out line
  - Carry stronger μ_B-dependence than early estimates due to λ's





$$\omega_{ip}^{\text{prefactor}} = \frac{\tilde{\lambda}_i'(i-1)!\,\xi_{\max}^{\frac{5}{2}i-3}}{T^{i/2}\,n_p}\left(\int_k d_p\, g_p \frac{v_k^2}{\gamma_k}\right)^i \left(\frac{n_0}{n_p}\right)^{i-1}$$

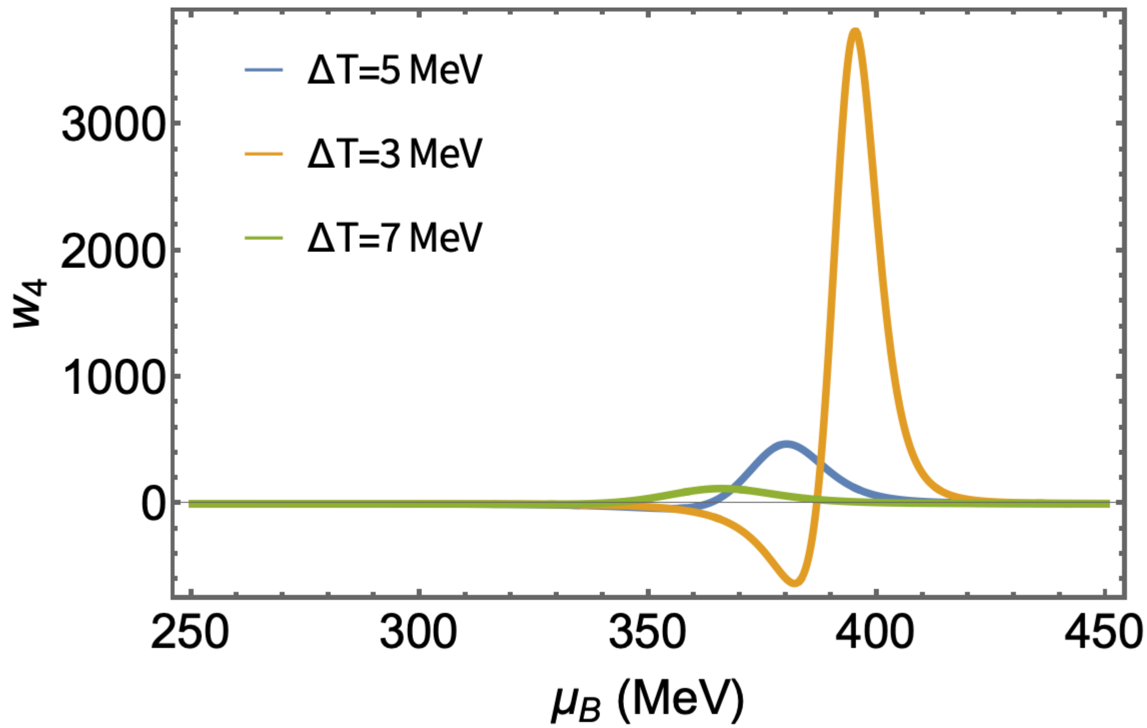
Equilibrium Normalized Cumulants with Realistic EoS



Karthein, Pradeep, KR, Stephanov, Yi, in progress

$$\omega_{4p} = 1 + \omega_{4p}^{\text{prefactor}} \left(\frac{n_p}{n_0}\right)^3 \left(\frac{\xi}{\xi_0}\right)^7$$

Equilibrium Normalized Cumulants with Realistic EoS



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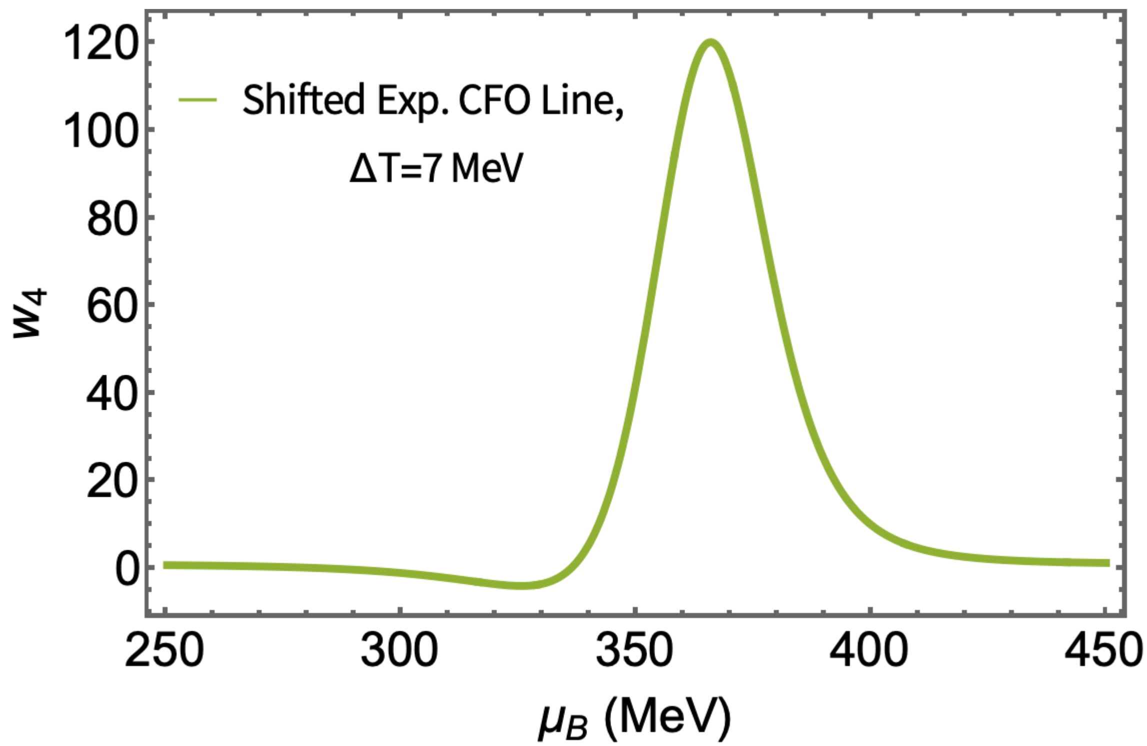

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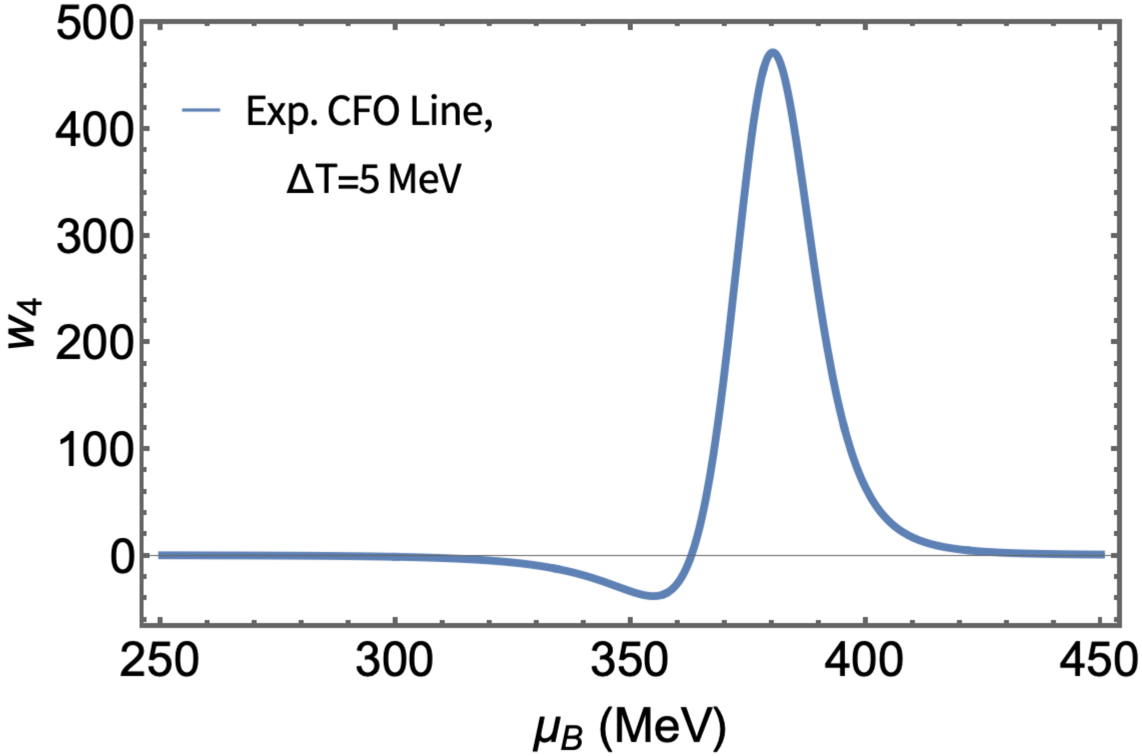

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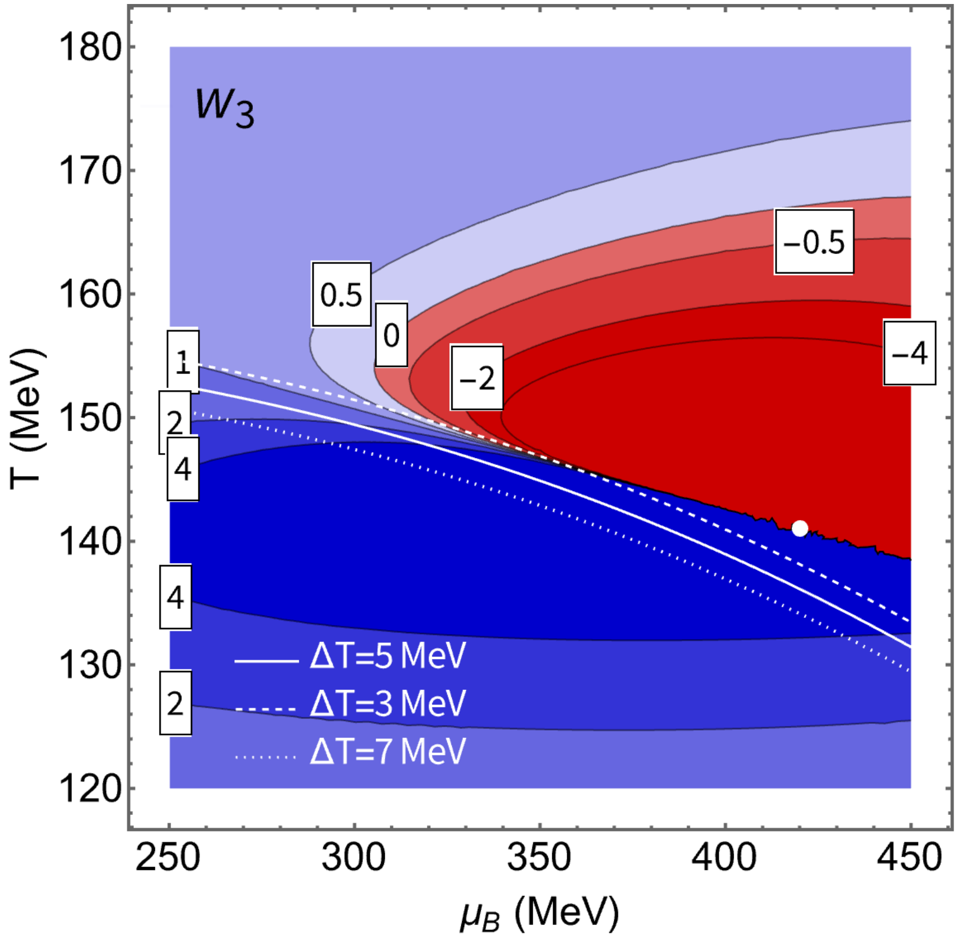
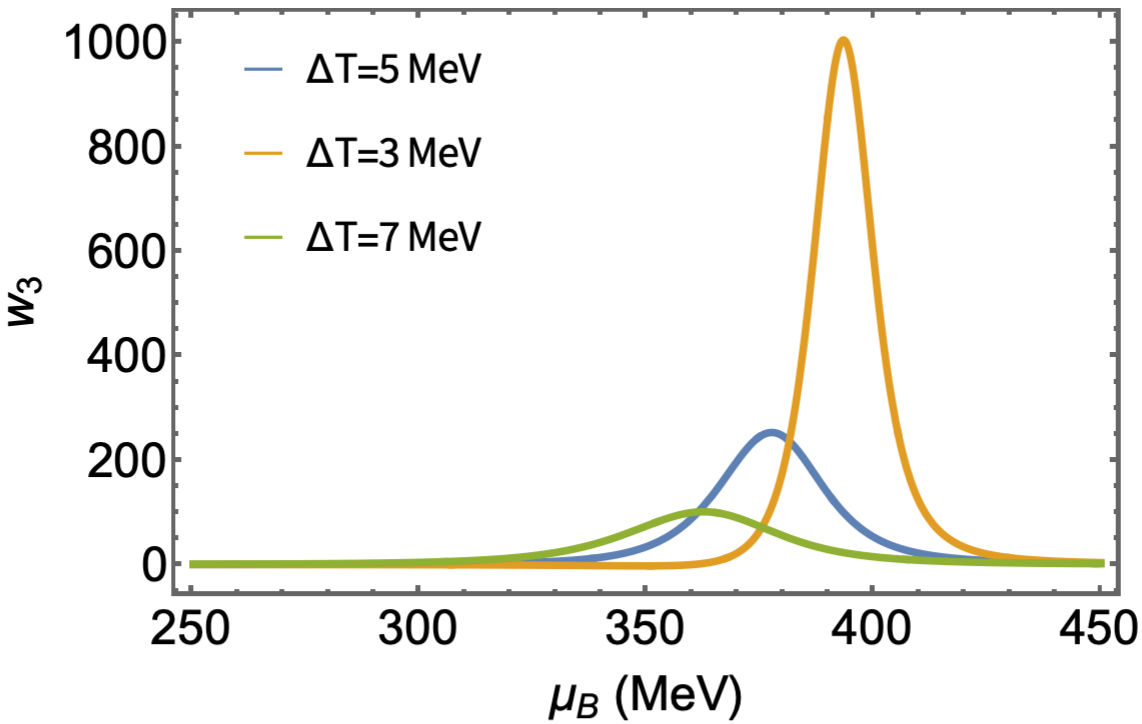


Karthein, Pradeep, KR, Stephanov, Yi, in progress

$$\omega_{3p} = 1 + \omega_{3p}^{\text{prefactor}} \left(\frac{n_p}{n_0}\right)^2 \left(\frac{\xi}{\xi_0}\right)^{9/2}$$

Equilibrium Normalized Cumulants with Realistic EoS



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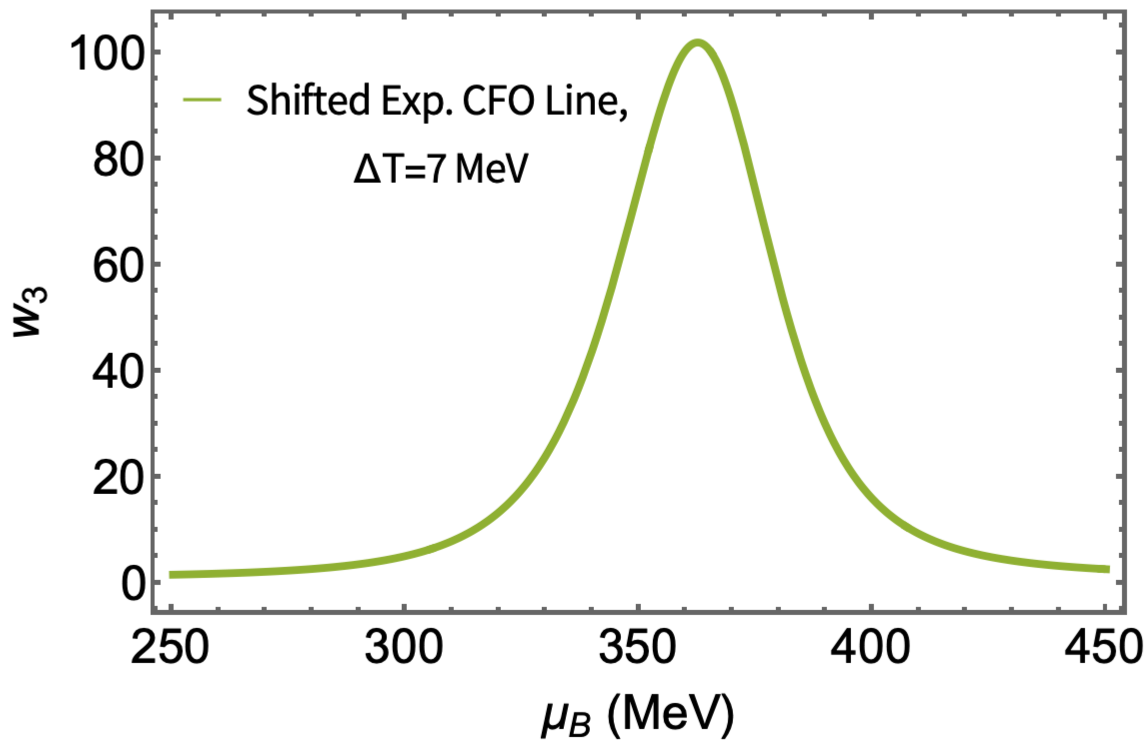

Karthein, Pradeep, KR, Stephanov, Yi, in progress

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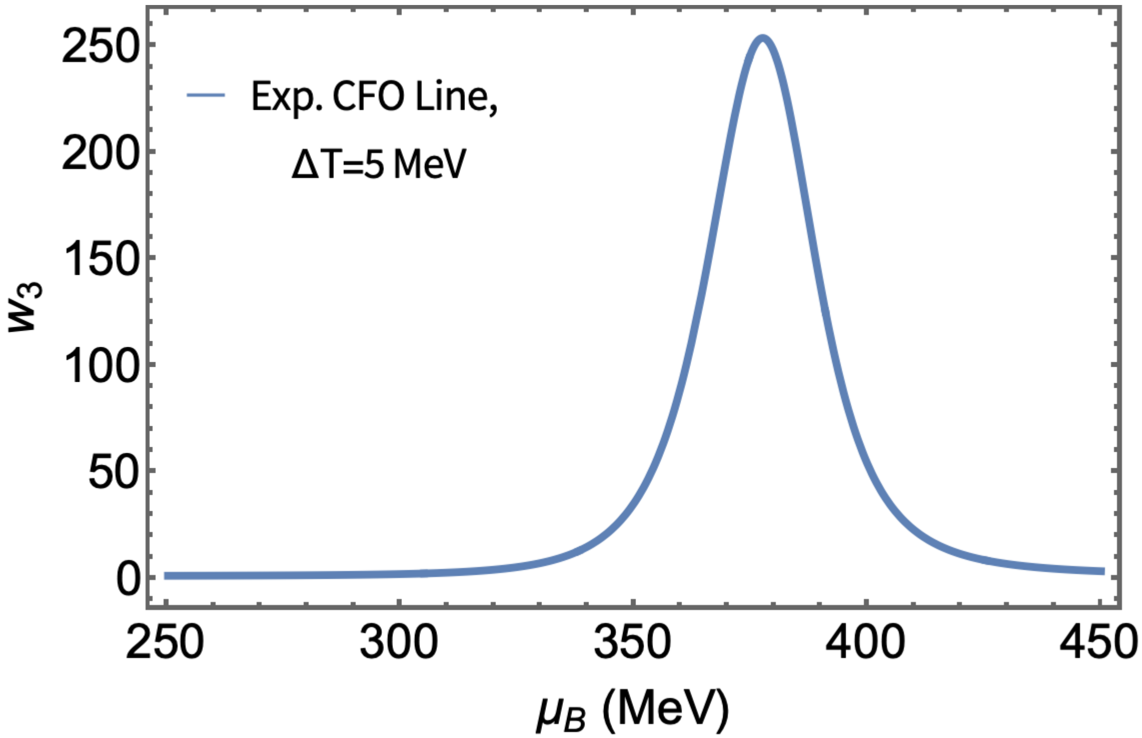

Karthein, Pradeep, KR, Stephanov, Yi, in progress

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Equilibrium Normalized Cumulants with Realistic EoS

- Normalized non-Gaussian cumulants (skewness and kurtosis) at freeze-out $\Delta T \simeq 5 - 7$ MeV below critical point, assumed to be in equilibrium, are comparable in magnitude to what was estimated much more crudely, with more ad hoc assumptions, in 2010.
- New calculations use BEST EoS plus the new calculations of the universal behavior of ξ, λ_3, λ_4.
- In reality, the critical fluctuations, and the consequent non-Gaussian cumulants of the proton multiplicity distribution, will *NOT* be in equilibrium.
- Critical slowing down will prevent them from growing anywhere near this big, and will also slow their subsequent relaxation.
- Magnitude will be *much* less. Sensitivity to ΔT will be less...
- Great recent progress toward full dynamical calculation...

Mapping the QCD Phase Diagram

- Energy and baryon number in initial stages.
- Equation of State (EoS)
- Hydrodynamics. Critical fluctuations.
  - Critical fluctuations develop in those collisions that pass near a critical point as they cool
  - Critical slowing down $\rightarrow$ fluctuations cannot stay in equilibrium (Berdnikov+KR, 1999). Must describe out-of-equilibrium critical fluctuations and hydrodynamics self-consistently. Two formalisms developed; we use Hydro+ (Stephanov, Yin, 2017)
  - First use of Hydro+ to model fluctuation dynamics near a QCD critical point (KR, Ridgway, Weller, Yin, 2019; Du, Heinz, 2020; Pradeep, KR, Stephanov, Yin, 2022)
  - Cooling+critical slowing down $\rightarrow$ growth of critical fluctuations "lags" what it would be in equilibrium, fluctuations also persist longer than they would; expansion, radial flow $\rightarrow$ critical fluctuations advected outward; back-reaction on hydrodynamics turns out to be small.
- Freezeout of critical fluctuations.

Dynamics and freeze-out of fluctuations near the QCD critical point

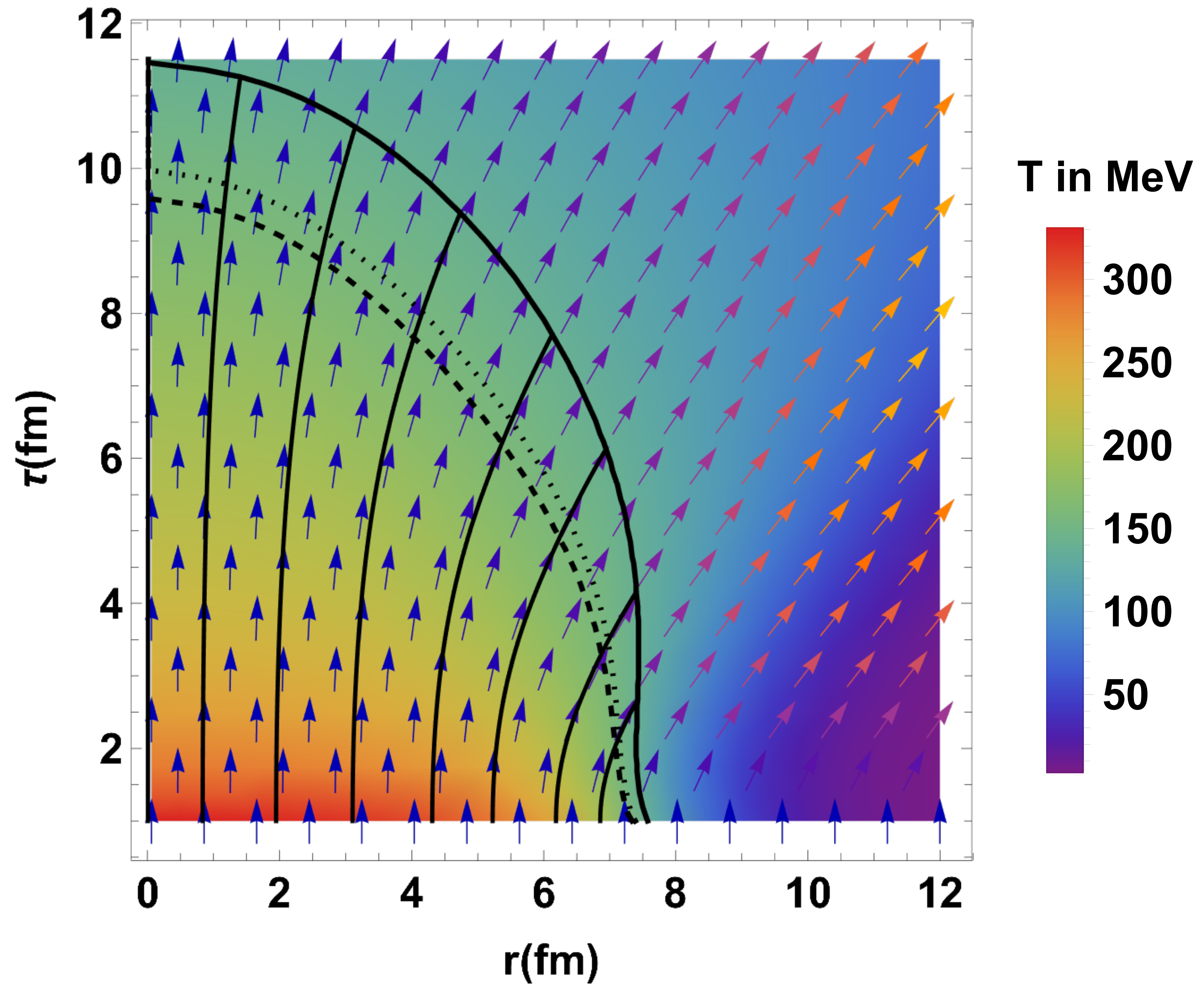
(arXiv: 2204.00639)

Maneesha Pradeep[1*], Krishna Rajagopal[2], Misha Stephanov[1], Yi Yin[3]
1 University of Illinois at Chicago, 2 Massachusetts Institute of Technology, 3 Institute of Modern Physics, Lanzhou

Hydro+ simulation

* Hydrodynamics + relaxation equation for the slowest non-hydrodynamic mode

Stephanov & Yin, 2017



Back reaction of out-of-equilibrium fluctuations on the EoS neglected as they have been found to be less than sub-percent level in Rajagopal et al, 19, Du et al, 20

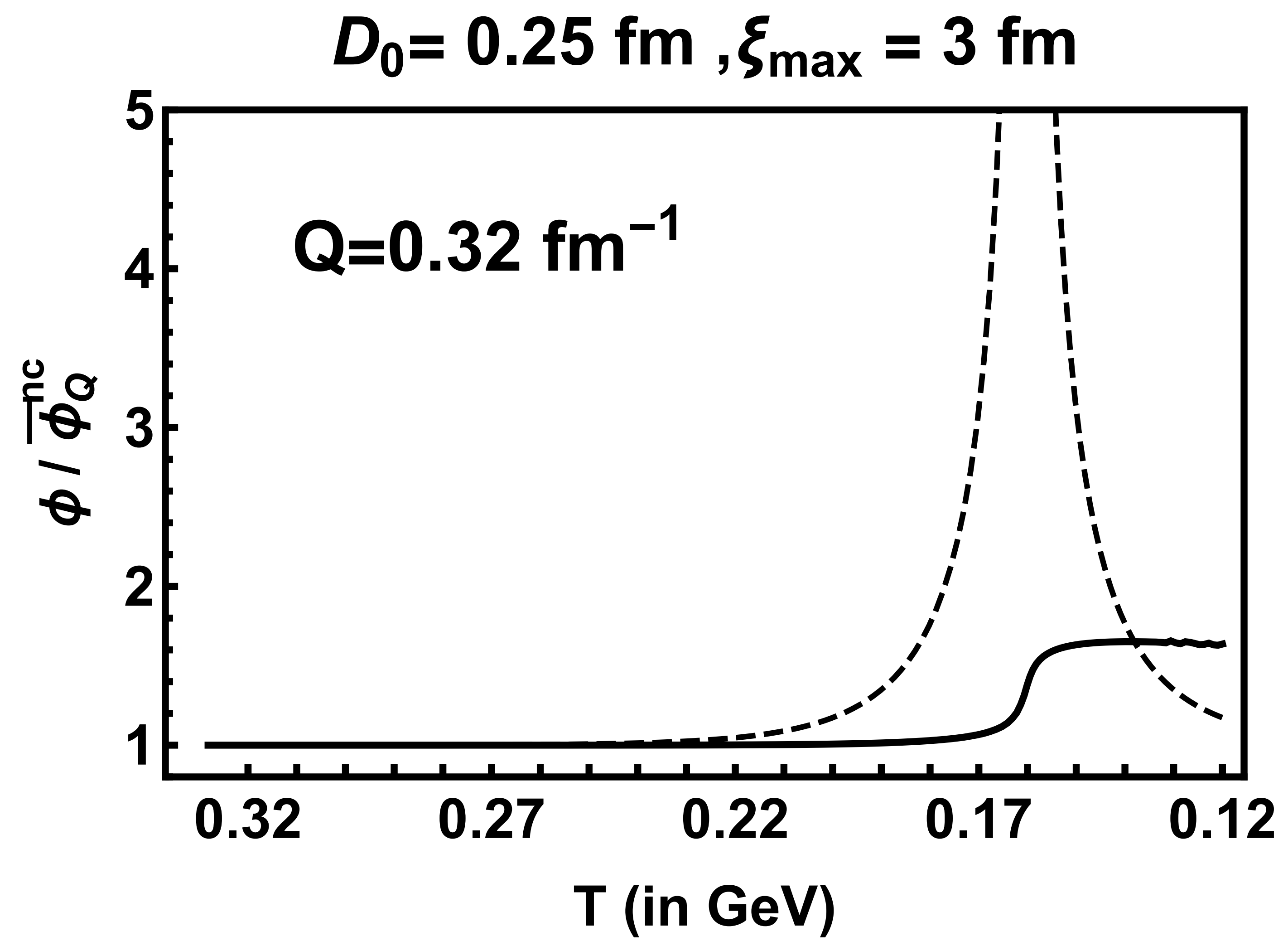
Baier and Romatschke, 2007

This talk :

Azimuthally symmetric, boost invariant hydrodynamic background with radial expansion with fluctuations discussed in Rajagopal, Ridgway, Weller, Yin, 19

Evolution of fluctuations

Stephanov & Yin, 2017



(2204.00639)

- The slowest and the most singular mode near the critical point corresponds to fluctuations of $\hat{s} \equiv \frac{s}{n}$
- The relaxation rate $\Gamma \sim \xi^{-3}$
- Equilibrium fluctuations $\propto C_p \sim \xi^2$

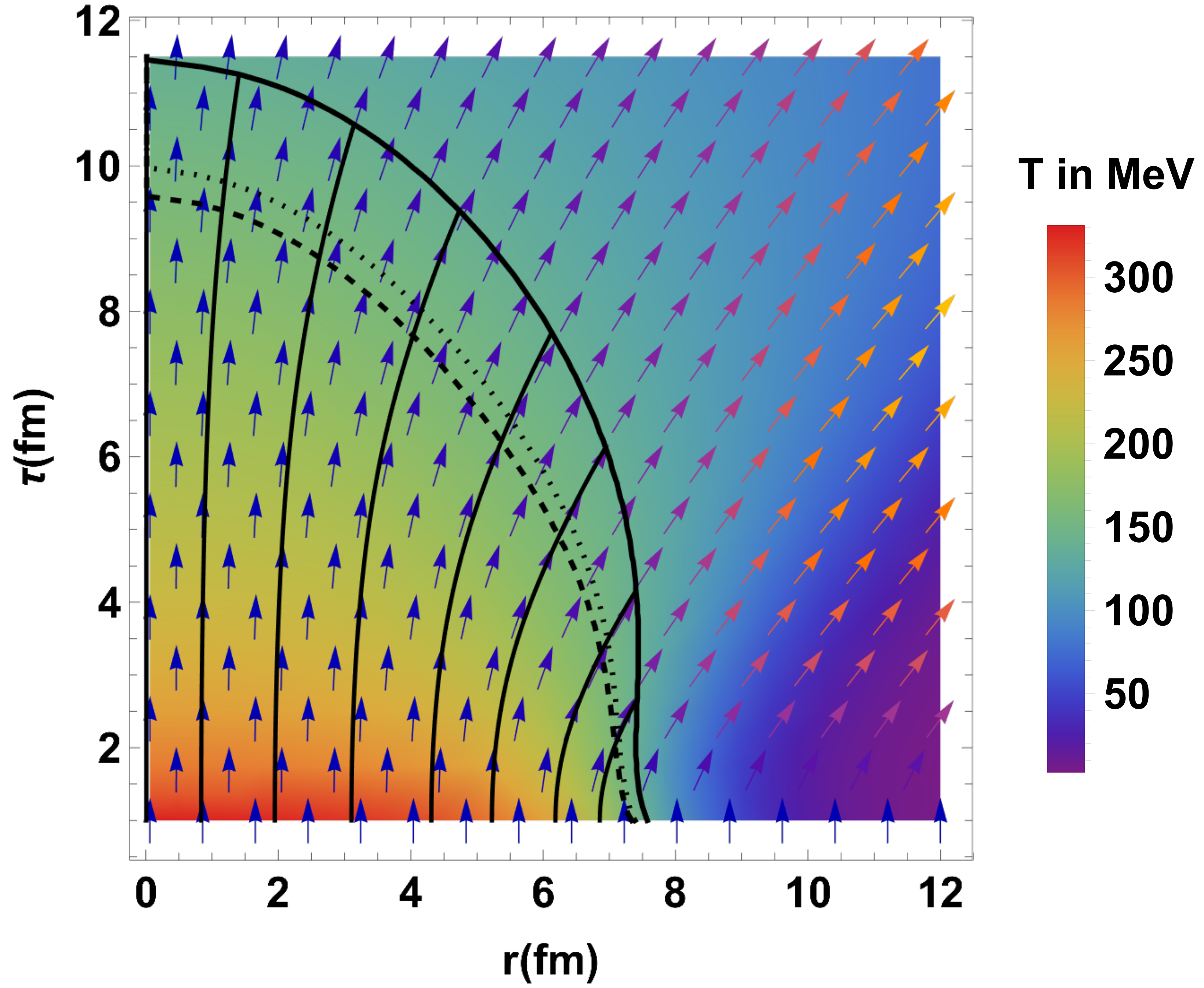
$$\phi_{\mathbf{Q}} = \int_{\Delta \mathbf{x}} e^{-i\mathbf{Q}\cdot\Delta\mathbf{x}} \left\langle \delta\hat{s}(x_+)\,\delta\hat{s}(x_-) \right\rangle$$

$$u \cdot \partial \phi_{\mathbf{Q}} = -\Gamma(\mathbf{Q}) \left(\phi_{\mathbf{Q}} - \bar{\phi}_{\mathbf{Q}}\right)$$

$\Gamma(\mathbf{Q}) = \frac{2D_0\xi_0}{\xi^3} K(|\mathbf{Q}\xi|),\, K(x) \sim x^2$ for $x << 1$

Zero mode doesn't evolve

Dynamics of fluctuations near a critical point



Stephanov, Yin, 17

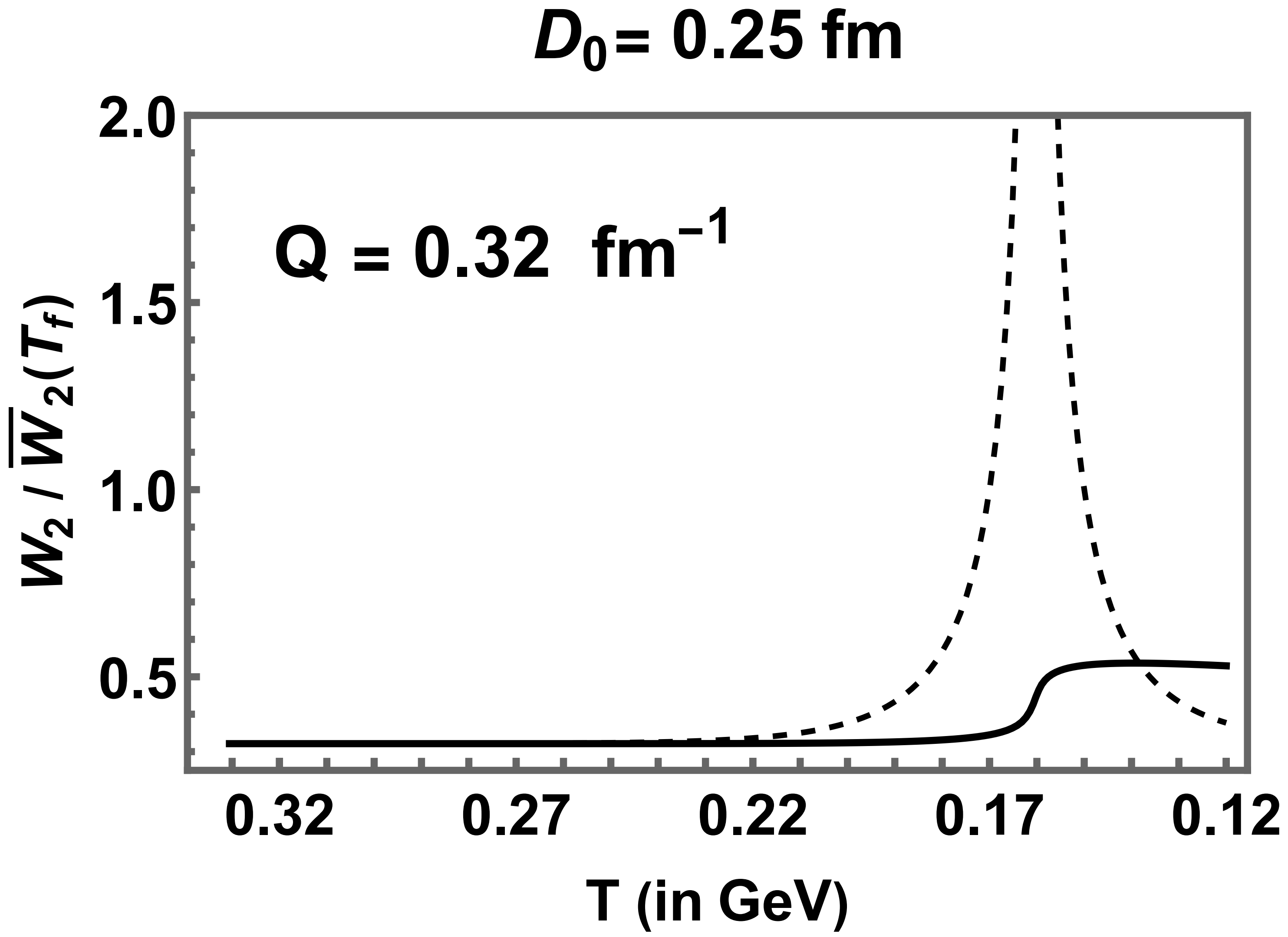
Rajagopal, Ridgway,Weller, Yin, 19

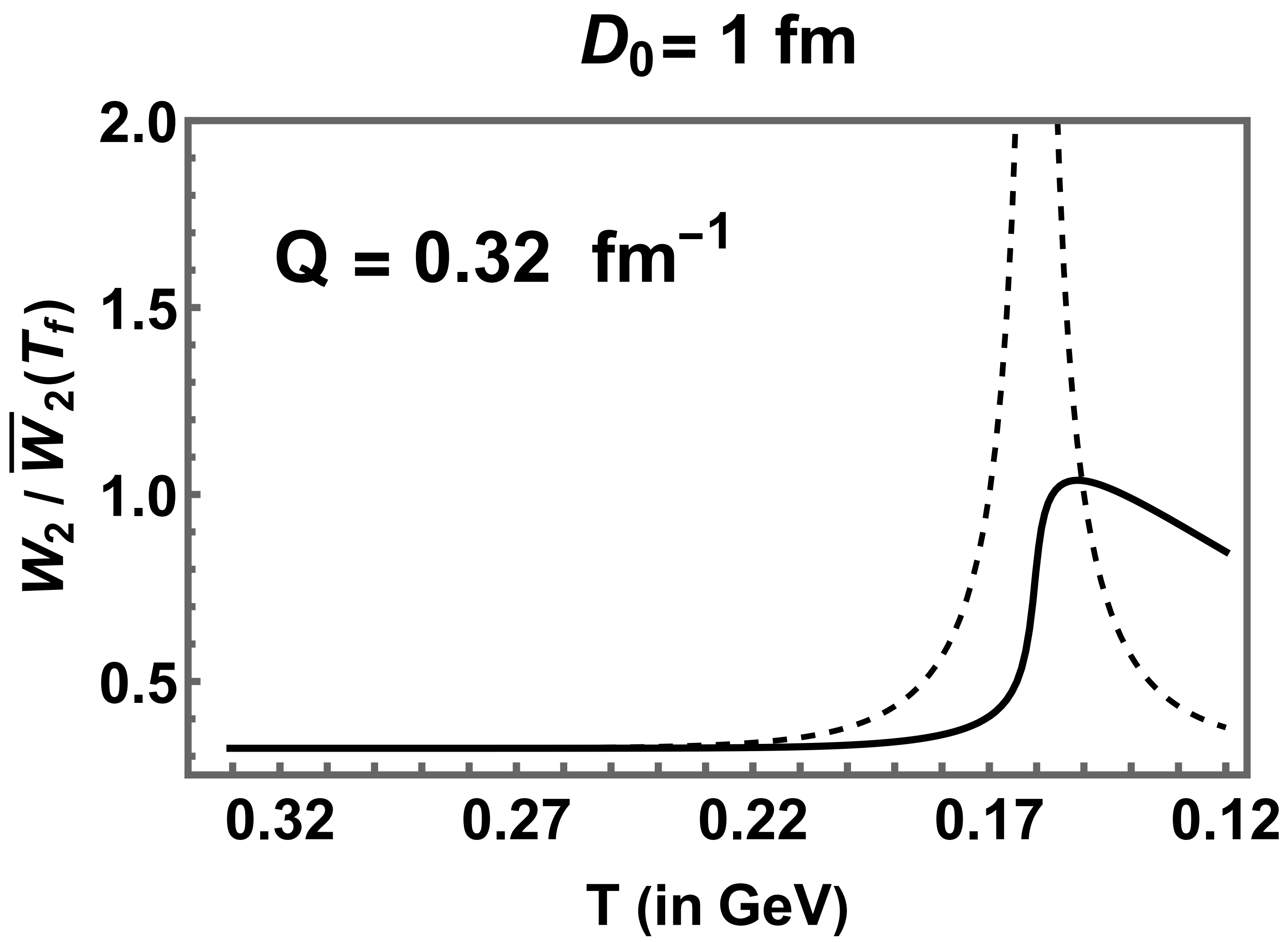
MP, Rajagopal, Stephanov, Yin, 22

$$\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-)\rangle = \int_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\mathbf{\Delta x}} W_2(x,\mathbf{Q})$$

The contribution of low Q modes dominate the particle correlations $Q \leq \tau_f^{-1}\sqrt{m/T}$

$$T_f \;=\; 150\,\mathrm{MeV}$$





$$u\cdot\partial W_2(x,\mathbf{Q}) \;=\; -\Gamma(|\mathbf{Q}|\xi)\left(W_2(x,\mathbf{Q})-\bar{W}_2(x,\mathbf{Q})\right)$$

$$\Gamma(x) = \frac{D_0\xi_0}{\xi^3}K(x),\; K(x)\sim x^2 \text{ for } x\ll 1$$

Model H

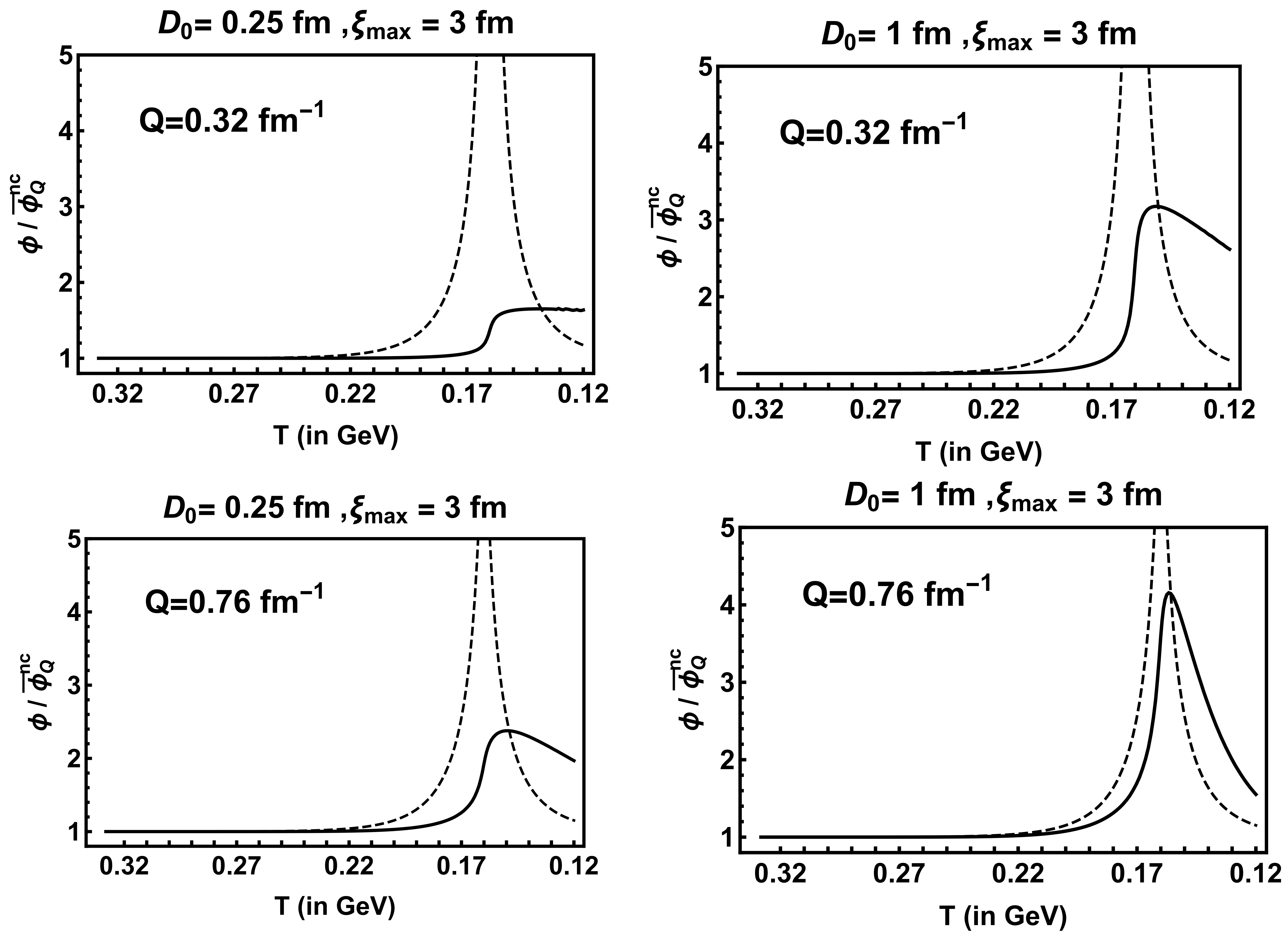
Demonstrating critical slowing down

(2204.00639)

Lower Q modes are suppressed strongly due to conservation and relax more slowly

$$\bar{\phi}^{nc}_{\mathbf{Q}} \sim \frac{\xi_0^2}{1 + (\mathbf{Q}\xi_0)^2}$$

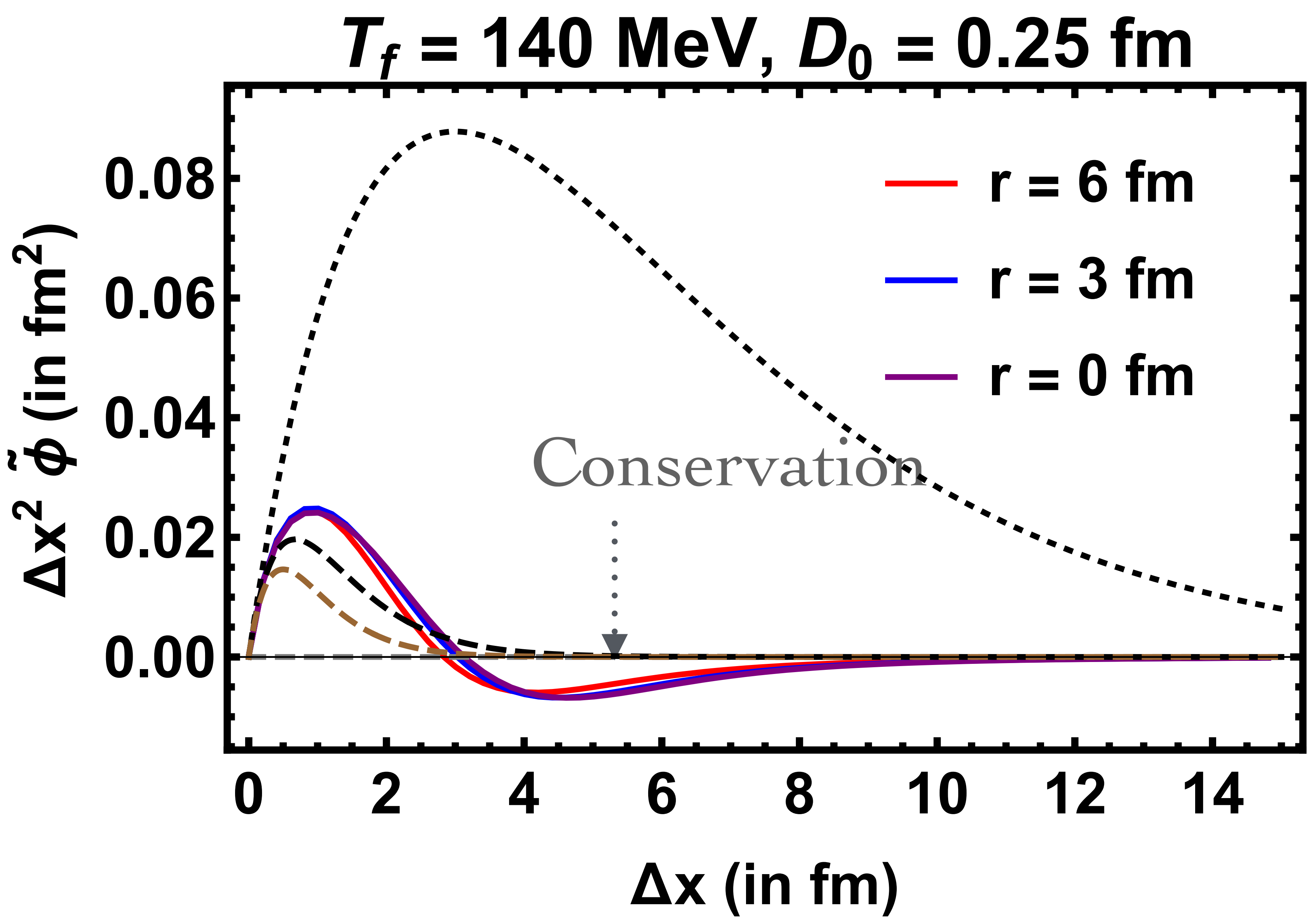
Normalized
out-of-equilibrium
fluctuations
for two Q modes
and two relaxation
rates

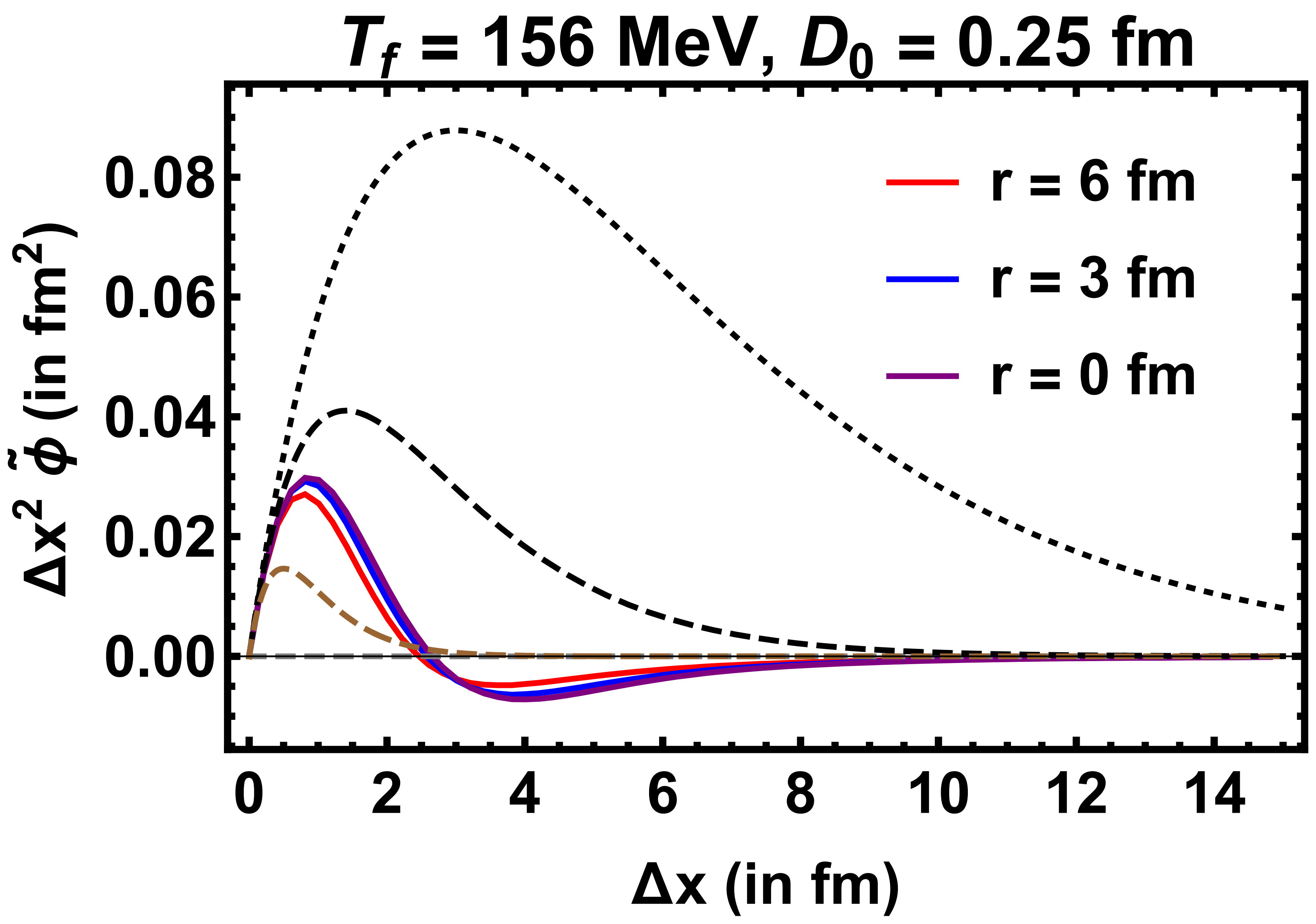


Critical correlations in space

(2204.00639)

We consider two isothermal freeze-out scenarios: T=140 MeV and T=156 MeV





Memory

Out-of equilibrium fluctuations "remember" their past, so the difference between the two freeze-out scenarios is not too large

Conservation $\int d\Delta x\, \Delta x^2\, \tilde{\phi}(\Delta x) = \phi_0$

Zero mode doesn't evolve

Mapping the QCD Phase Diagram

- **Finding, or excluding, a critical point requires theory and modeling, with ingredients including:**
- **Energy and baryon number in initial stages.**
- **Equation of State (EoS)**
- **Hydrodynamics. Critical fluctuations.**
- **Freezeout of critical fluctuations**
  - **Freezing out Hydro+ so as to faithfully turn the critical fluctuations described via Hydro+ into fluctuations of observed proton multiplicities: 2204.00639 Pradeep, KR, Stephanov, Yin**
  - . . . **faithfully turn the** *higher moments of* **the critical fluctuations into the** *skewness and kurtosis of* **observed proton multiplicities (in progress) Karthein, Pradeep, KR, Stephanov, Yin**
- **Phase diagram mapping theory+modeling tools vastly better than in 2015; being completed; data coming soon!**

Dynamics and freeze-out of fluctuations in heavy-ion collisions

- Work in progress with **Jamie Karthein, Bruno Sebastian Scheihing Hitschfeld**, **Krishna Rajagopal, Misha Stephanov,** and **Yi Yin**
- *Phys.Rev.D* 106 (2022) 3, 036017 with Krishna Rajagopal, Misha Stephanov, and Yi Yin
- arXiv 2211.09142 with Misha Stephanov

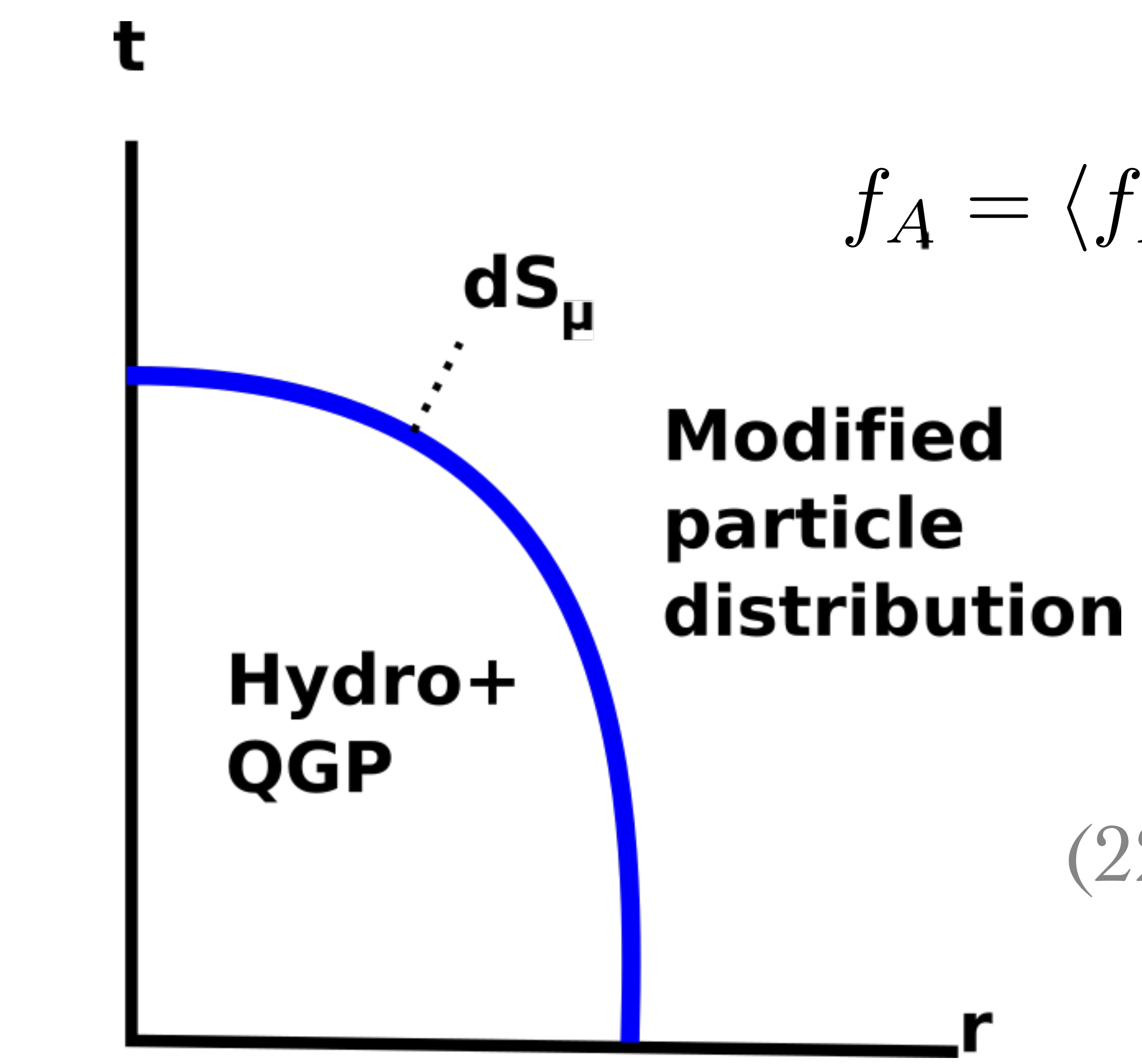
CPOD 2022

Maneesha Pradeep, University of Illinois at Chicago

Critical fluctuations in hadron resonance gas

* We incorporate the effects of critical fluctuations via the modification of particle masses due to their interaction with a critical sigma field

$$\delta m_A \approx g_A \sigma$$



$$f_A = \langle f_A \rangle + g_A \frac{\partial \langle f_A \rangle}{\partial m_A} \sigma$$

(2204.00639)

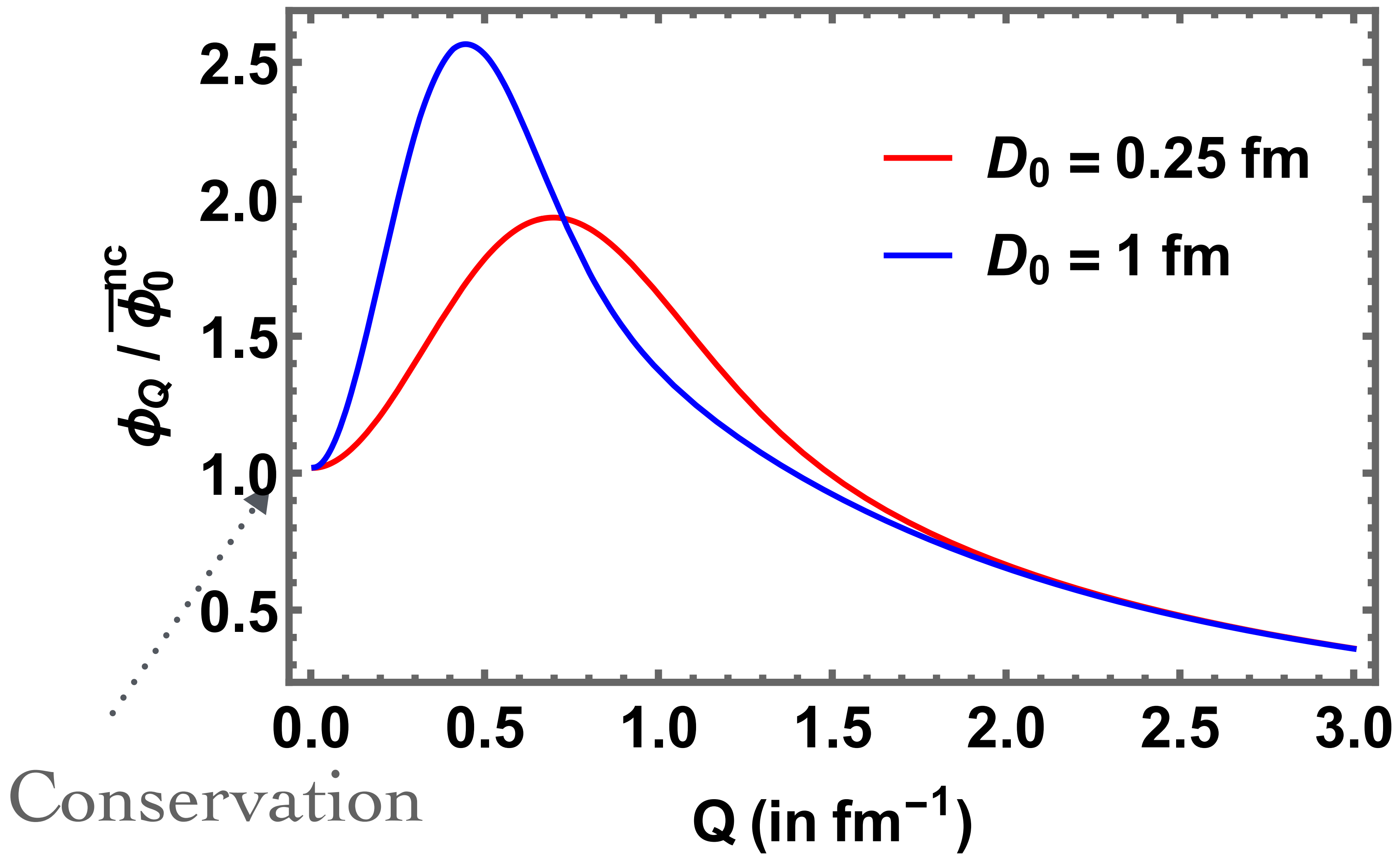
$$\langle \delta N_A^2 \rangle = \langle N_A \rangle + \langle \delta N_A^2 \rangle_\sigma$$

We match the two point function of σ to the two point function of the Hydro+ mode, $\hat{s} \equiv s/n$

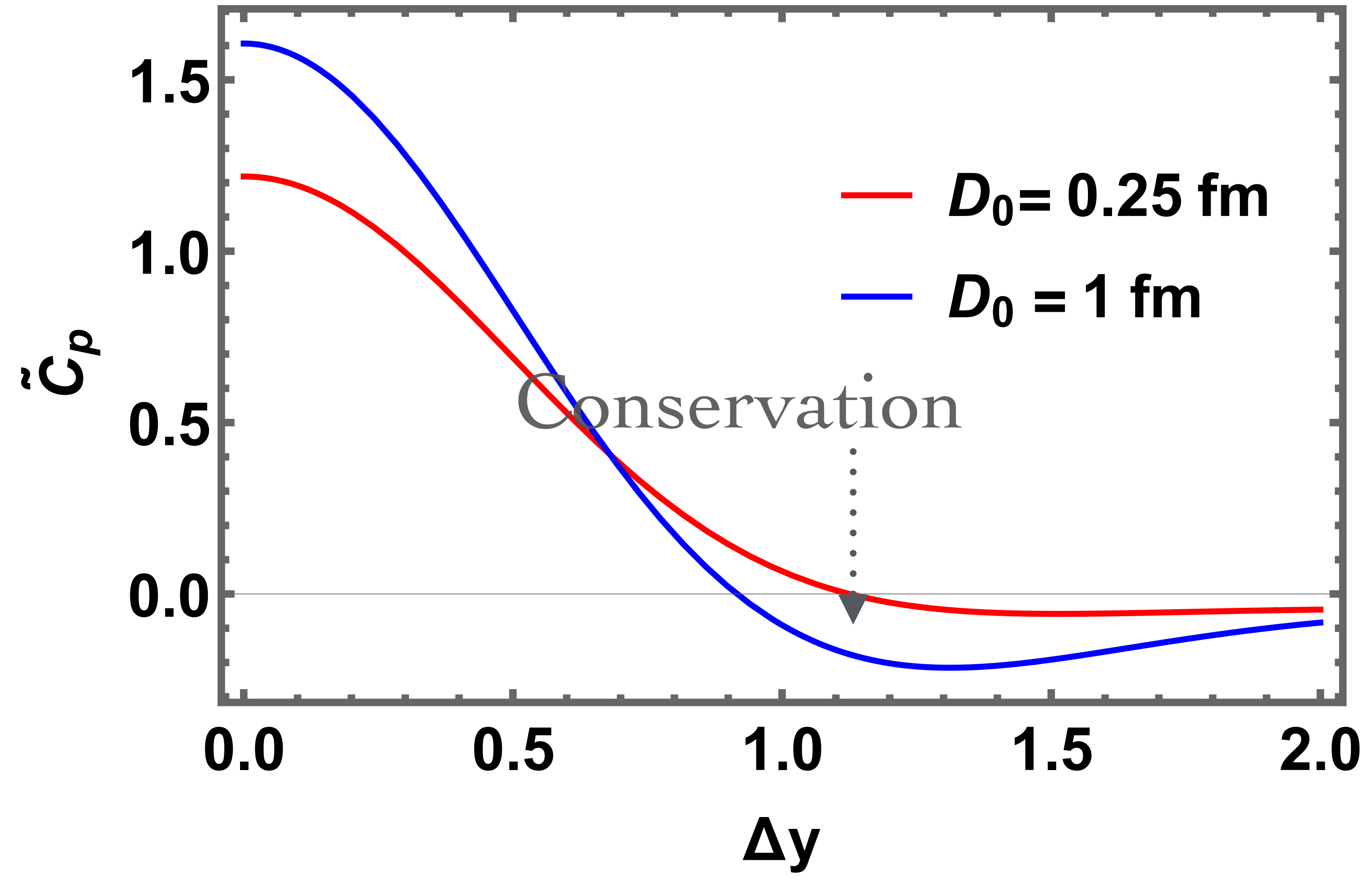
$$\left\langle \sigma(x_+)\sigma(x_-) \right\rangle \approx Z^{-1} \left\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-) \right\rangle$$

$$\left\langle \delta N_A^2 \right\rangle_\sigma = g_A^2 Z^{-1} \int dS_\mu J_A^\mu(x_+) \int dS_\nu J_A^\nu(x_-) \left\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-) \right\rangle$$

Effect of conservation laws on particle (anti)correlations at freeze-out



$$C_A(y_+, y_-) = \left\langle \delta \frac{dN_A}{dy_+} \delta \frac{dN_A}{dy_-} \right\rangle$$



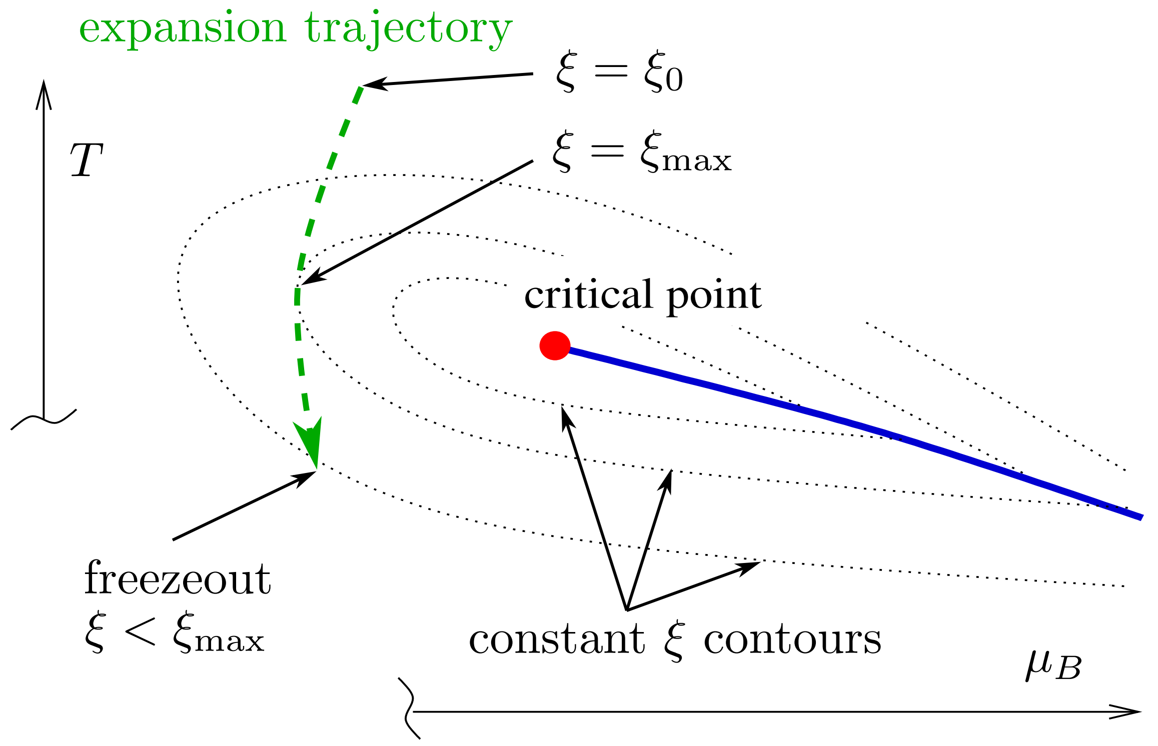
Enhancement at low Δy, anti-correlations at large Δy

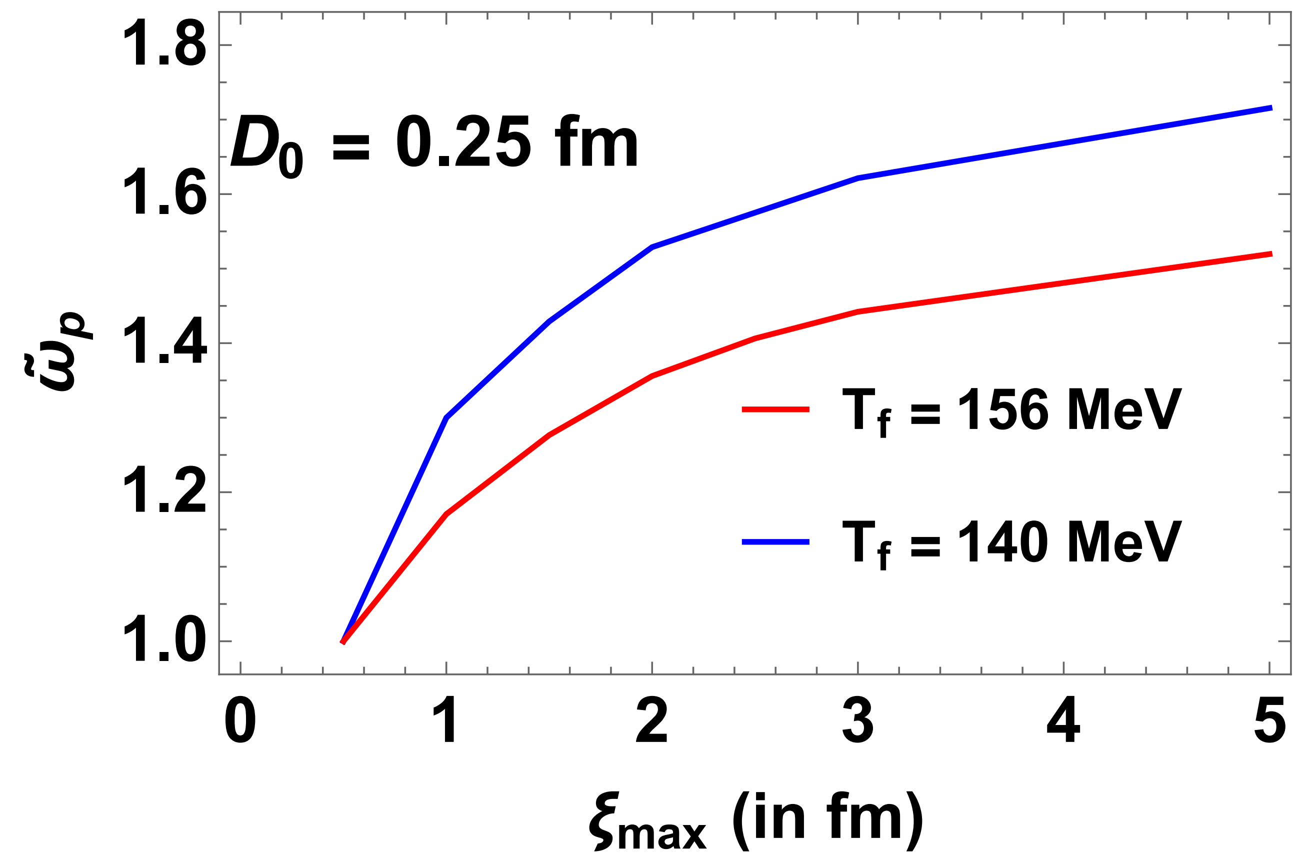
The low Q modes contribute the most to rapidity correlations

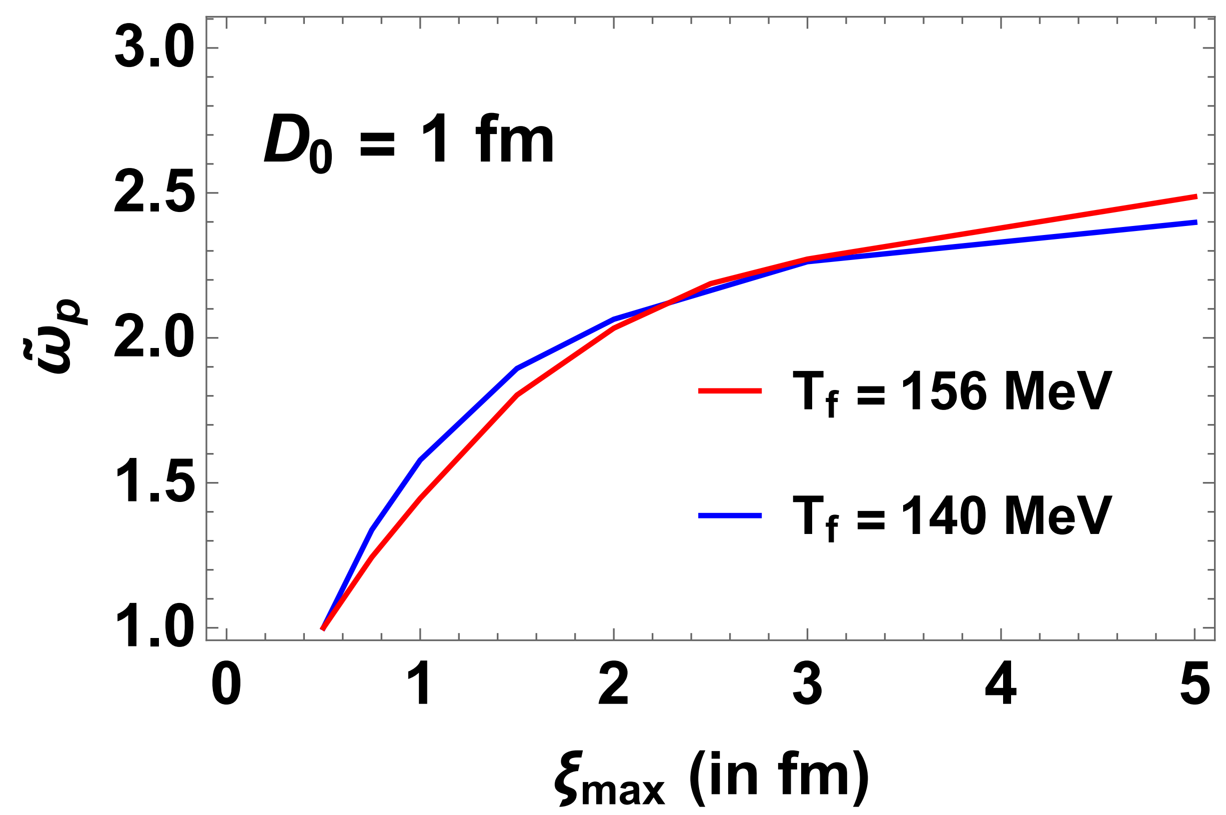
(2204.00639)

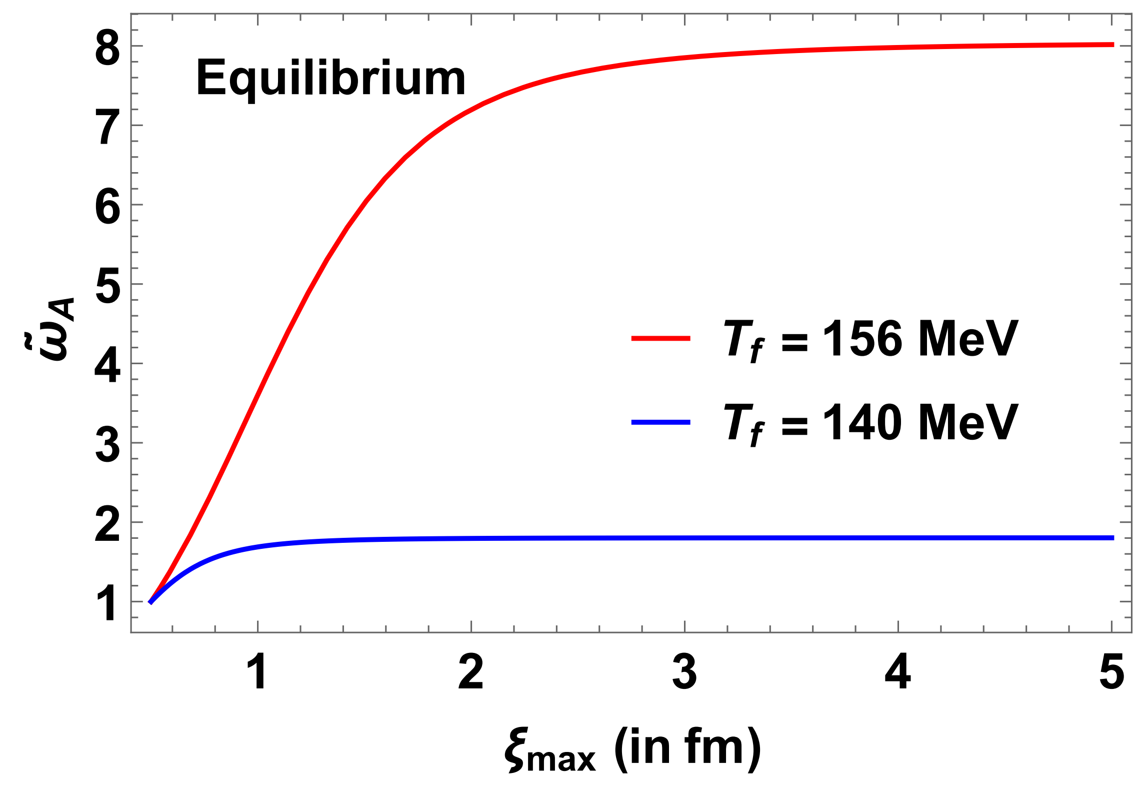
Critical contribution to variance of proton multiplicities

$$\omega_p \equiv \frac{\left\langle \delta N_p^2 \right\rangle_\sigma}{\left\langle N_p \right\rangle}$$









$$\tilde{\omega}_p \equiv \frac{\omega_p}{\omega_p^{nc}}$$

- The fluctuations are reduced relative to equilibrium value (due to conservation laws)
- The fluctuations are found to increase with D_0 (faster diffusion)
- Compared to the equilibrium scenario, the fluctuations are less sensitive to freeze-out temperature

(2204.00639)

Summary

* We have generalized the Cooper-Frye freeze-out procedure so that not only the averages, but also the critical fluctuations of the conserved densities are matched on the freeze-out hypersurface
* We have demonstrated the freeze-out in a semi-realistic scenario and estimated the dynamical effects for the critical contribution to the Gaussian cumulants of proton multiplicity
* The fluctuations are less sensitive to the freeze-out temperature in an out-of-equilibrium scenario unlike in an equilibrium case

Mapping the QCD Phase Diagram

- **Finding, or excluding, a critical point requires theory and modeling, with ingredients including:**
- **Energy and baryon number in initial stages.**
- **Equation of State (EoS)**
- **Hydrodynamics. Critical fluctuations.**
- **Freezeout of critical fluctuations**
  - **Freezing out Hydro+ so as to faithfully turn the critical fluctuations described via Hydro+ into fluctuations of observed proton multiplicities: 2204.00639 Pradeep, KR, Stephanov, Yin**
  - ... **faithfully turn the** *higher moments of* **the critical fluctuations into the** *skewness and kurtosis of* **observed proton multiplicities (in progress) Karthein, Pradeep, KR, Stephanov, Yin**
- **Phase diagram mapping theory+modeling tools vastly better than in 2015; being completed; data coming soon!**

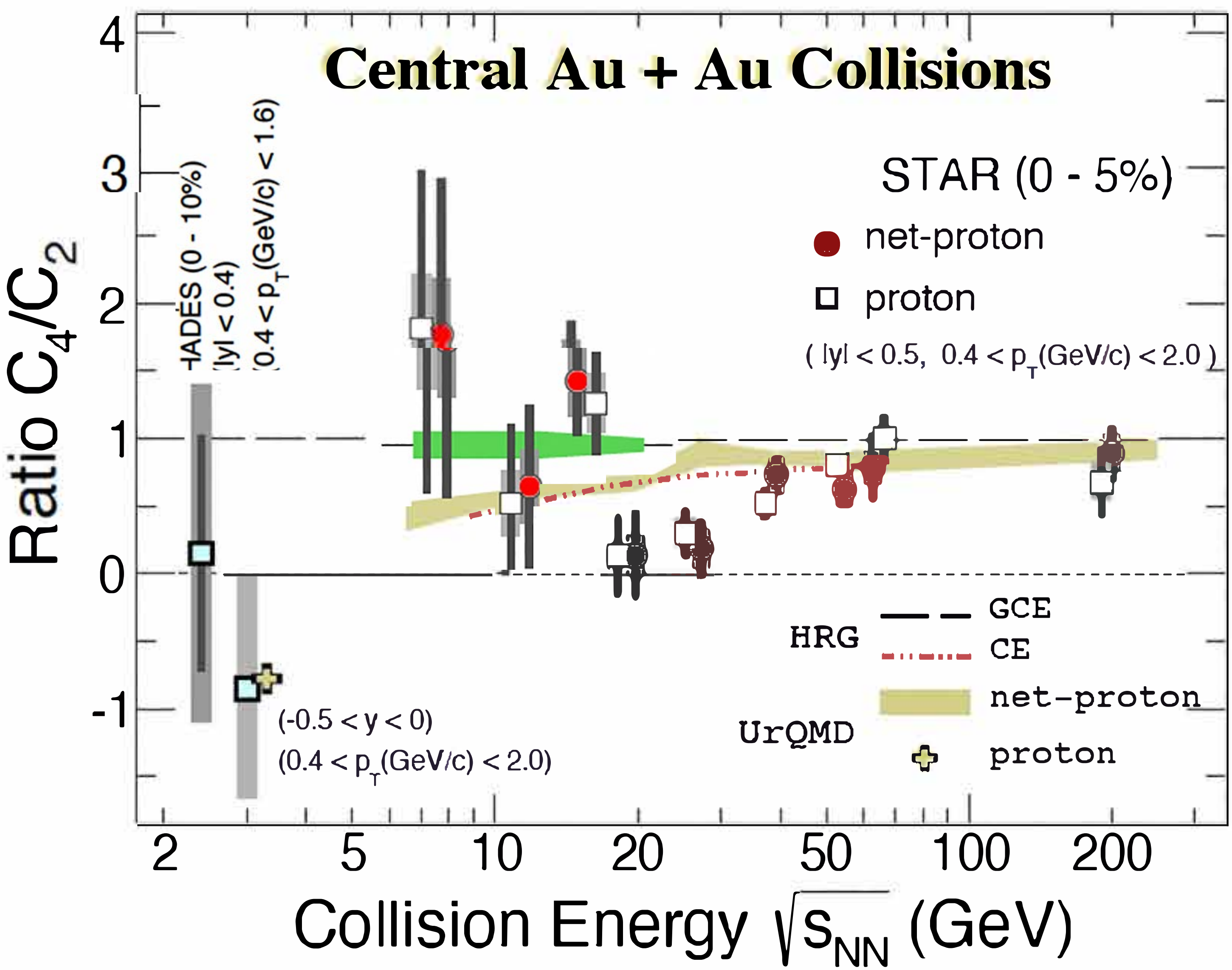
What Next?

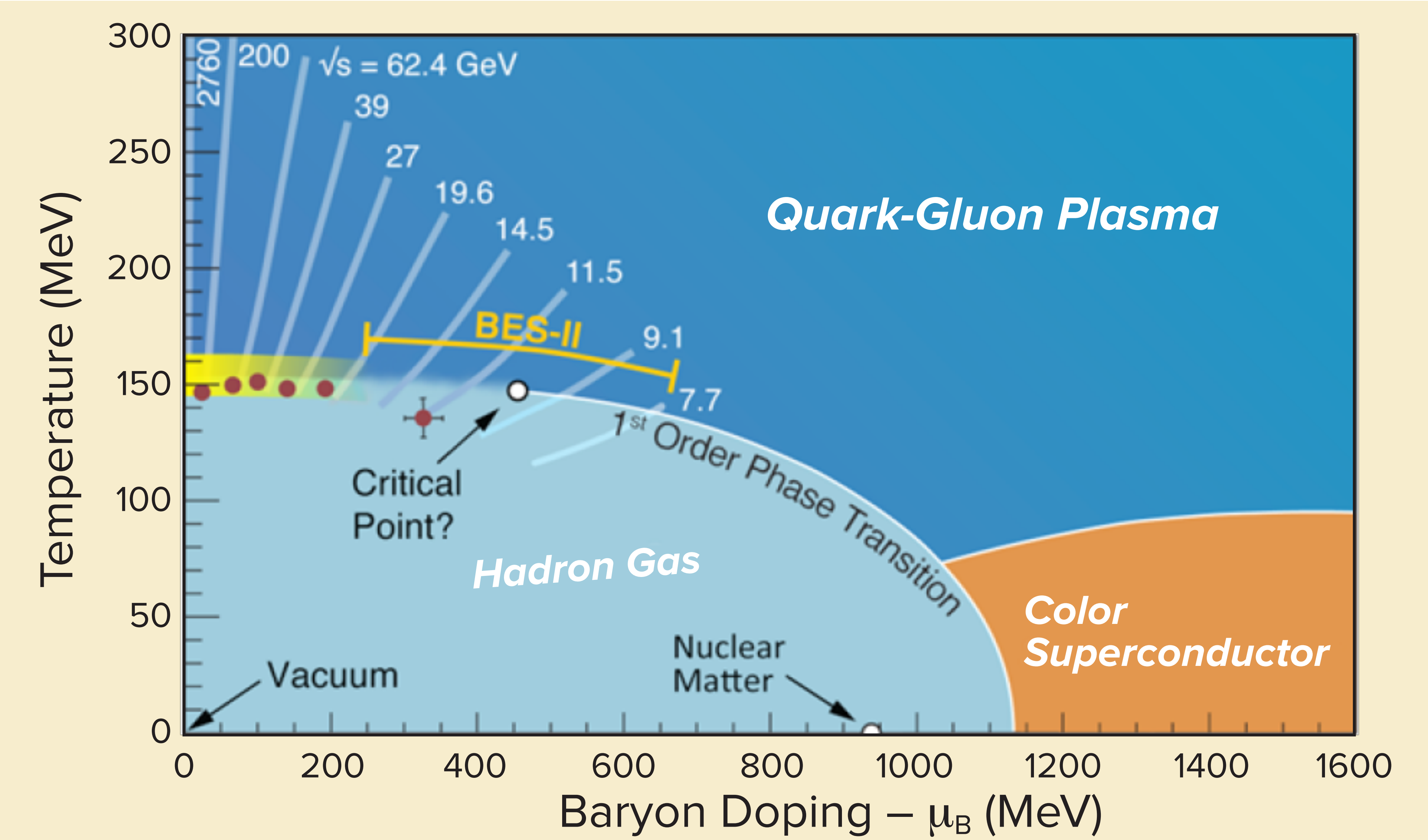
Two kinds of What Next? questions for the coming decade...

- A question that one asks after the discovery of any new form of complex matter: What is its phase diagram? For high temperature superconductors, for example, phase diagram as a function of temperature and doping. Same here! For us, doping means excess of quarks over antiquarks, rather than an excess of holes over electrons.
- A question that we are privileged to have a chance to address, after the discovery of "our" new form of complex matter: How does the strongly coupled liquid emerge from an asymptotically free gauge theory? Maybe answering this question could help to understand how strongly coupled matter emerges in other contexts.

  Three different variants of this question...

Proton Kurtosis, before BES II





Temperature (MeV)
Baryon Doping – μB (MeV)
Quark-Gluon Plasma
Hadron Gas
Color Superconductor
1st Order Phase Transition
Critical Point?
Vacuum
Nuclear Matter
BES-II
√s = 62.4 GeV
2760
200
39
27
19.6
14.5
11.5
9.1
7.7
300
250
200
150
100
50
0
0
200
400
600
800
1000
1200
1400
1600

What Next?

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  Three different variants of this question. . .

Probing the Original Liquid

The question How does the strongly coupled liquid emerge from an asymptotically free gauge theory? can be thought of in three different ways, corresponding to three meanings of the word "emerge": as a function of resolution, time, or size.

- How does the liquid emerge as a function of resolution scale? What is the microscopic structure of the liquid? Since QCD is asymptotically free, we know that when looked at with sufficient resolution QGP must be weakly coupled quarks and gluons. How does a liquid emerge when you coarsen your resolution length scale to $\sim 1/T$?
- Physics at $t = 0$ in an ultrarelativistic heavy ion collision is weakly coupled. How does strongly coupled liquid form? How does it hydrodynamize?
- How does the liquid emerge as a function of increasing system size? What is the smallest possible droplet of the liquid?

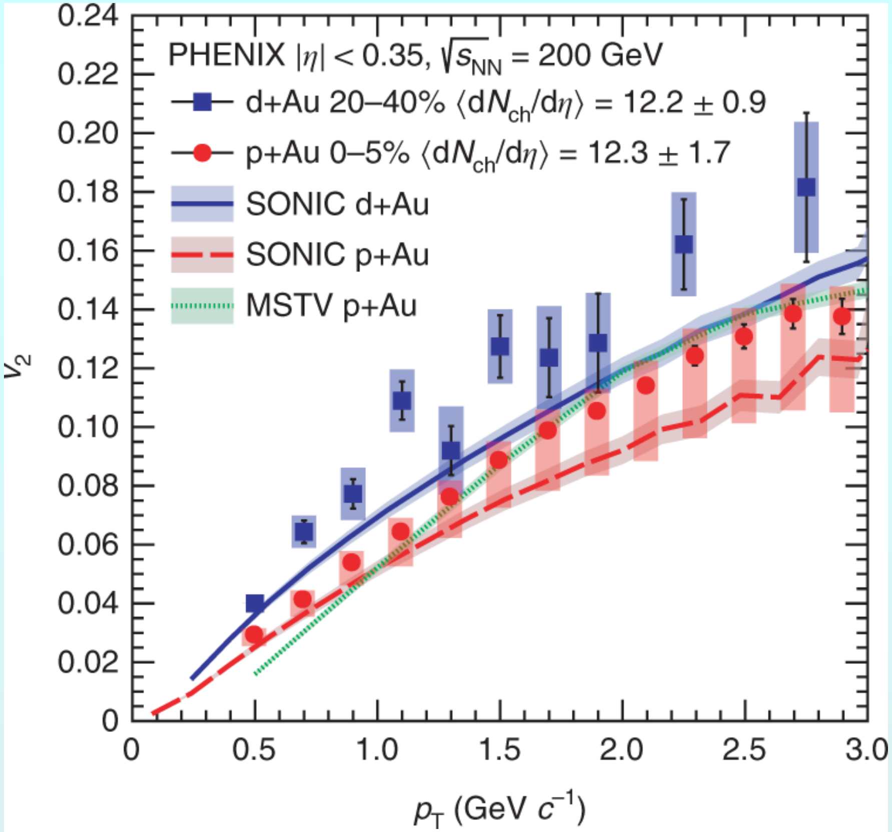
Each, in a different way, requires stressing or probing the QGP. Each can tell us about its inner workings.

Smallest possible droplet of liquid?

- What is the smallest possible droplet of QGP that behaves hydrodynamically? **Anyone doing holographic calculations at strong coupling, or anyone seeing effects of small lumps in the initial state visible in the final state, could have asked this question, but didn't. Question was asked by data: pPb collisions @LHC; pAu, dAu and ^{3}HeAu data @RHIC.**
- **Subsequently, holographic calculations of a "proton" of radius R colliding with a sheet show hydrodynamic flow in the final state as long as the collision has enough energy such that $RT_{\text{hydrodynamization}} \gtrsim 0.5$ to 1.**
- **Many recent theoretical advances. Hydrodynamic behavior in small-big collisions at top RHIC energy and LHC energy less surprising, *a posteriori*. But still remarkable.**
- **Not our focus today. For today, tells us that to see "inside" the liquid we will need probes which resolve short length scales...**

Eeek! Hydrodynamics in small systems!

PHENIX Collaboration
Nature Physics (2018)

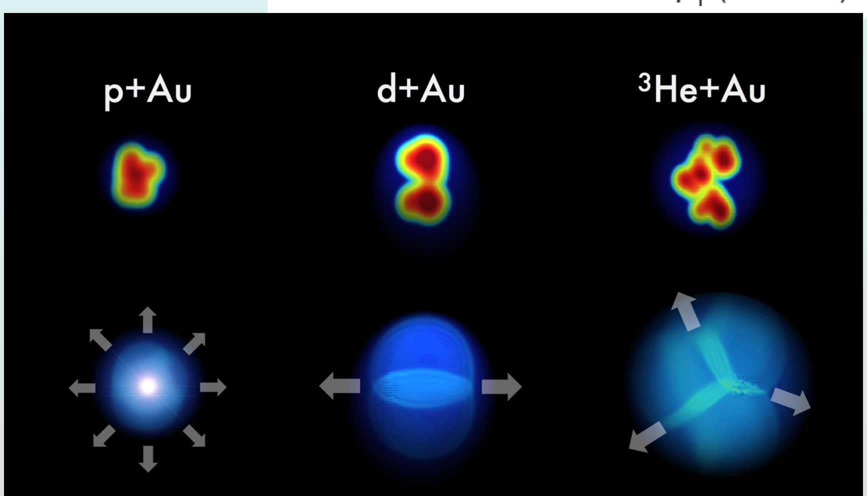


Not big & dense

But, we see collective flow!

Seeded by the initial geometry

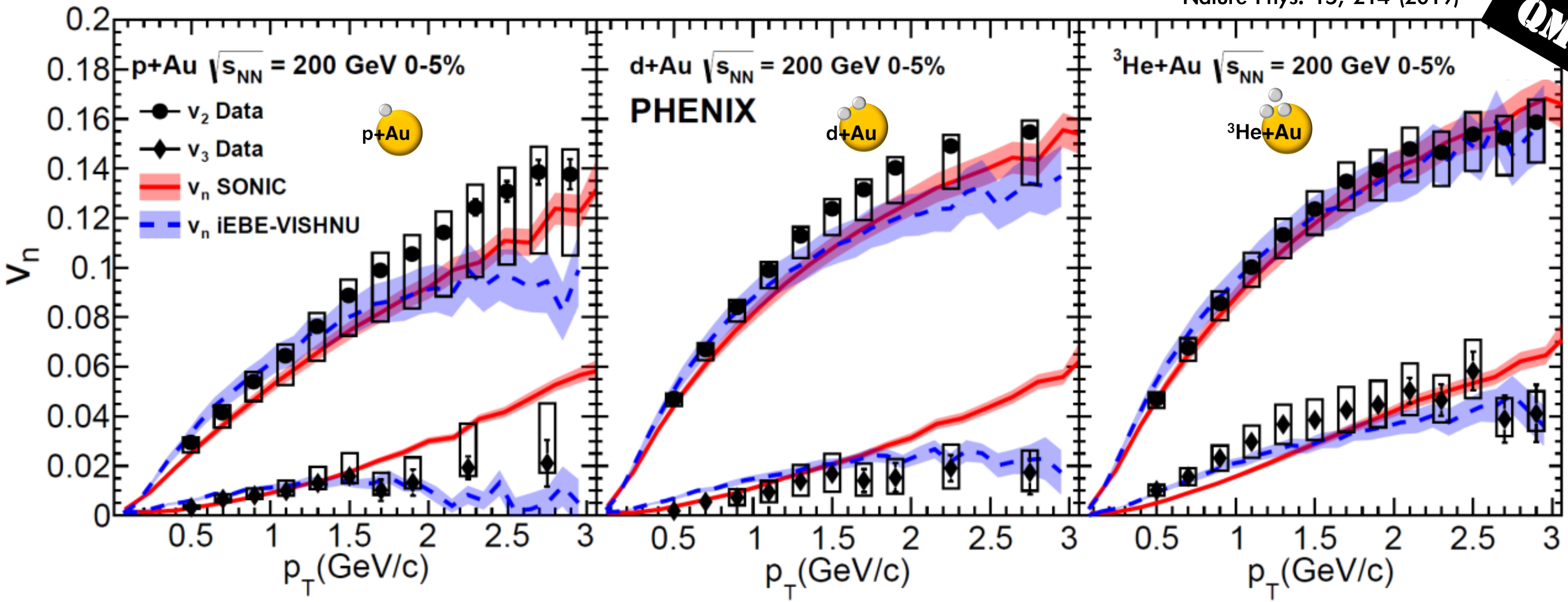
A small droplet of QGP?!



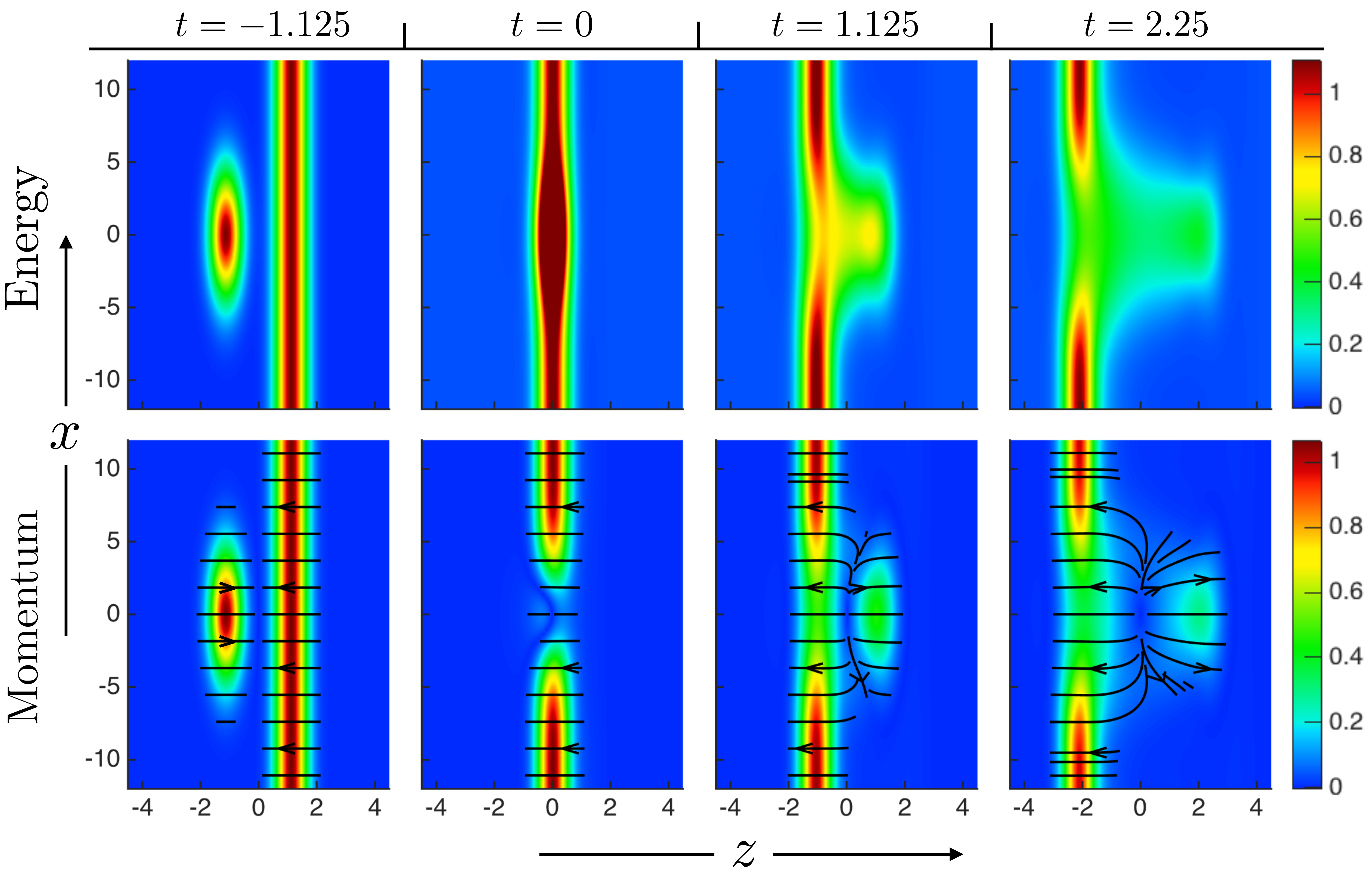
Collectivity in small systems

Nature Phys. 15, 214 (2019)

QM19



- Evidence of QGP droplets in small collision systems
- Smaller v_2 in p+Au and larger v_3 in ^{3}He+Au



t = −1.125
t = 0
t = 1.125
t = 2.25
Energy
Momentum
x
z
10
5
0
-5
-10
-4
-2
0
2
4
1
0.8
0.6
0.4
0.2
0

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Why Jets?

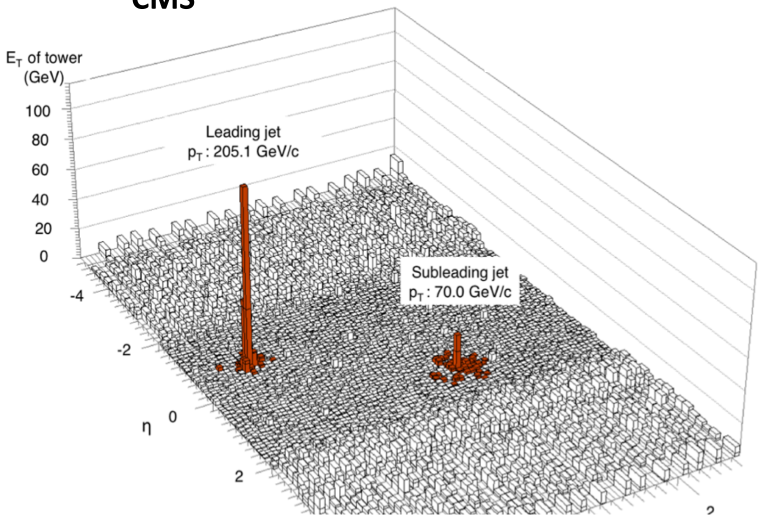
- The remarkable utility of hydrodynamics, for example in describing the dynamics of small lumps in the initial state in AA collisions, tells us that to see the inner workings of QGP, namely to see how the liquid is put together from quarks and gluons, we will need probes with much finer resolution.
- Need resolution scale that is $\ll$ size of a proton, $\ll$ size of lumps coming from the initial state that behave hydrodynamically, $\ll 1/T_{\rm hydrodynamization}$.
- Jets are multiscale probes. (Scales associated with: hard production, splittings in the shower, momentum transfers as jet partons interact with the medium, response of medium. So, from very hard to very soft.)
- They provide best+only chance for a scattering experiment off a droplet of QGP and seeing its inner workings.
- Our best shot at getting experimental evidence for point-like scatterers in QGP when QGP is probed with large momentum transfer.

Why Jets?

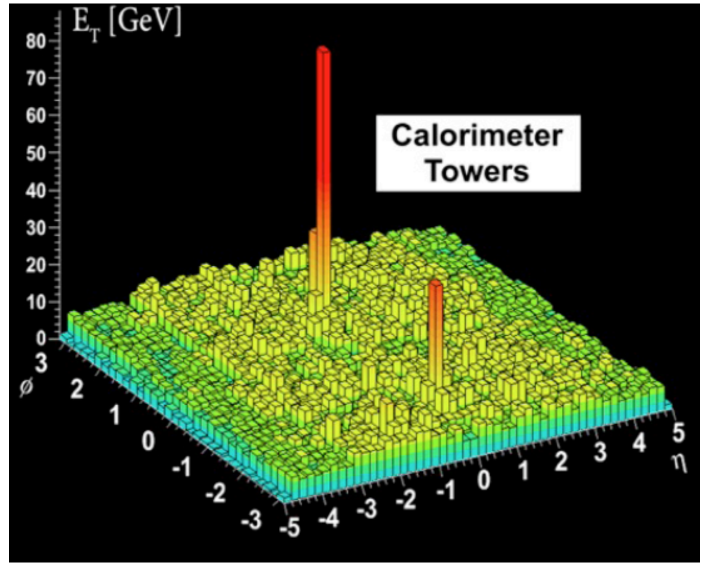
- Nature gives us two multi-resolution-scale probes: Upsilons and jets.
- Upsilons tell us whether the QGP can screen color forces over length scales of order the size of the Υ(1S), Υ(2S), Υ(3S). LHC data indicate that the dissociation pattern of these quarkonia states depends on their binding energy, which is to say on their size, as long expected. More to come, for example as p_T-dependence is studied.
- Upsilons can tell us about the screening length of the QGP, not about how it is put together. And, since the screening length is $\sim 1/T$ at strong coupling, and even longer at less strong coupling, the QGP is liquid-like at this resolution. And, if an Upsilon state is smaller than the screening length, it doesn't tell us anything beyond that fact. Bottom line: Upsilons are a three-scale probe that will tell us about screening but they do not see the inner workings.

Jet Quenching, in brief

CMS



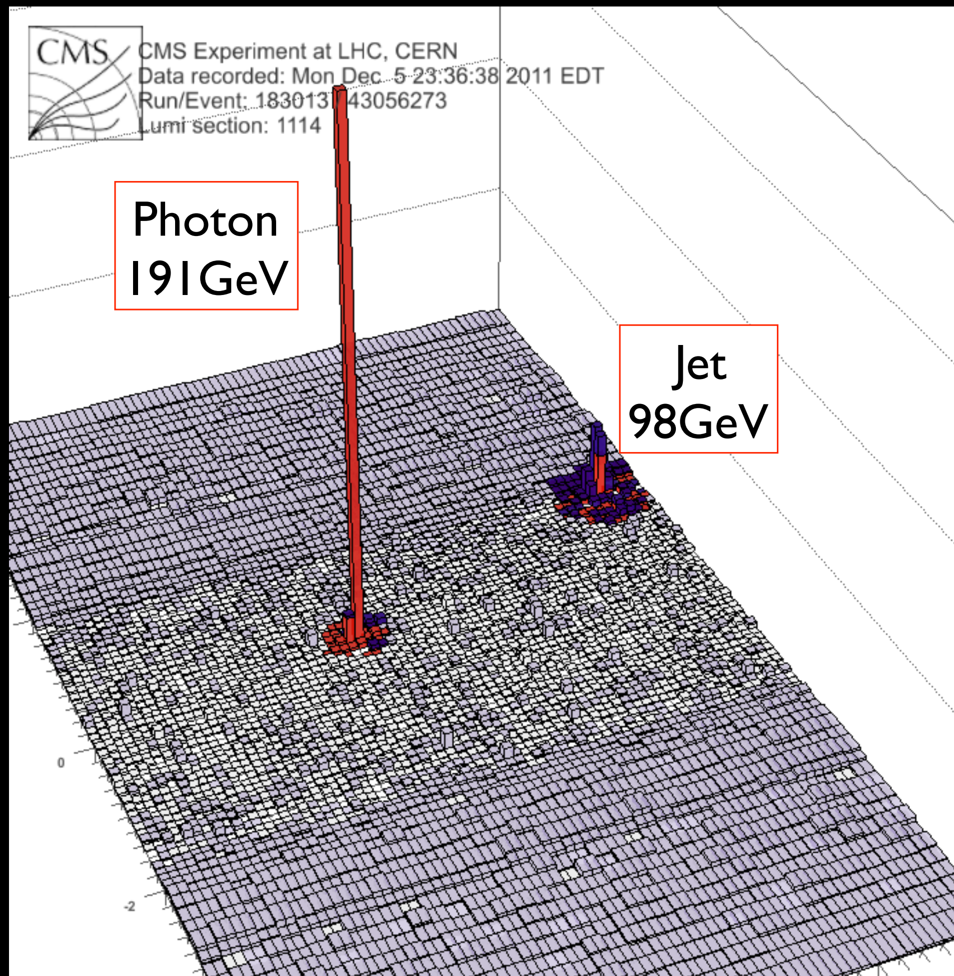
ATLAS



Jet quenching discovered @ RHIC; @ LHC, seen instantly!

- 200+ GeV jets lose many tens of GeV passing through the liquid QGP.
- Lost energy turns into many soft particles, around jet and at all angles.
- There might have been a third jet in these events? If so it has been turned entirely into soft particles.
- Lower energy jets, seen by ALICE and at RHIC, can emerge, surrounded by their debris.

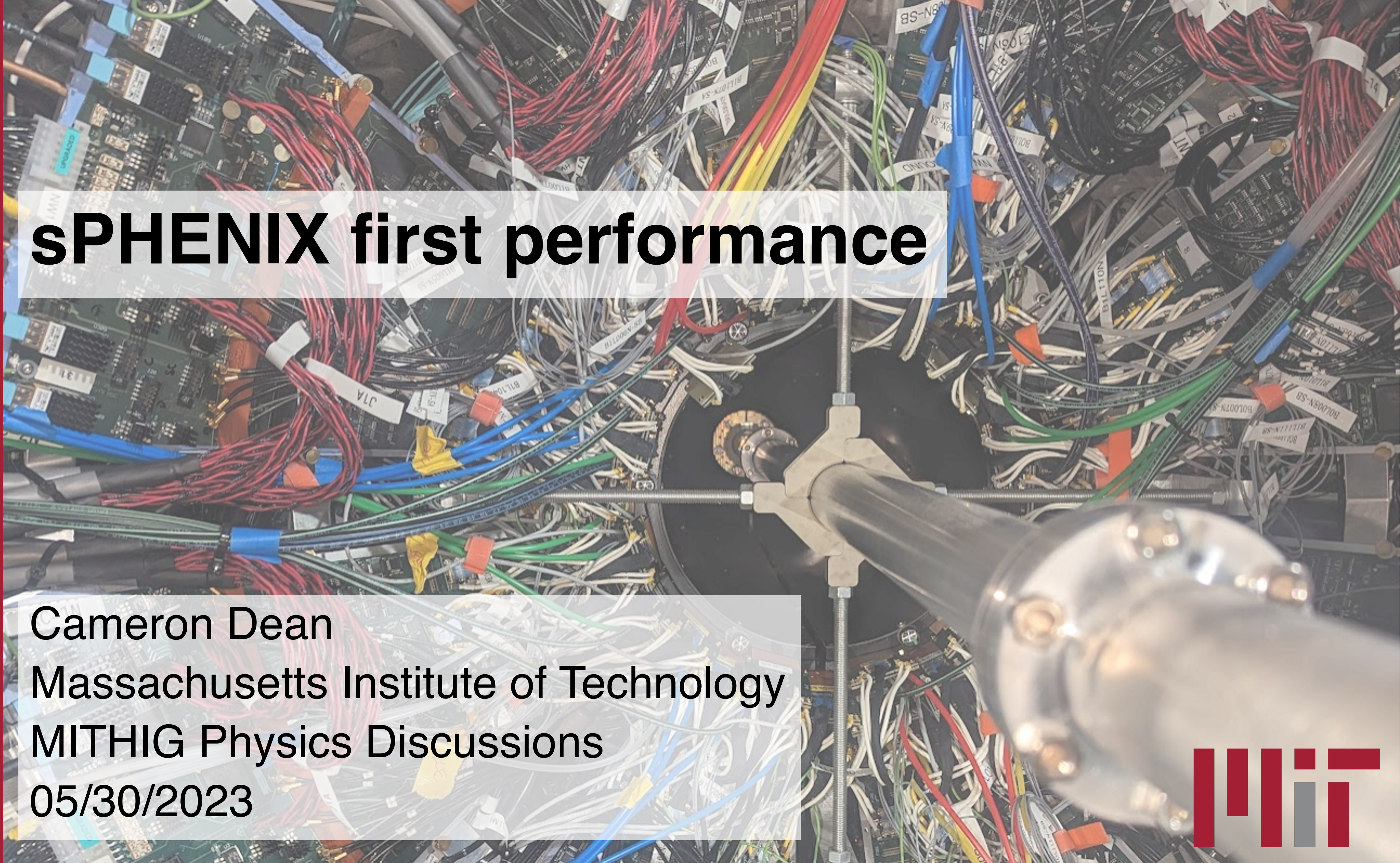
2011: Detected 3000 photon-jet pairs in 10^9 PbPb collisions



Unbalanced photon-jet event in PbPb

Jets as Probes of QGP

- Closest we will ever come to doing a scattering experiment off a droplet of Big Bang matter.
- Jets in heavy ion collisions *also* offer the best chance of watching how QGP hydrodynamizes. Jets leave a wake in the medium. Can we see how it hydrodynamizes, and then flows? Best shot at experimental access to this physics.
- But, jets sure don't make it easy to decode the info about the nature of QGP at various length scales encoded in the modification of their energies, shapes, and structure.
- We need high statistics sPHENIX and LHC data on rare events in which jet partons scatter off QGP partons by a sufficient angle to yield observable consequences.
- And, theorists are using the data of today to build the baseline of understanding with and against which to look for and interpret such effects.
- For example, how do we separate observable effects due to wake from those due to scattering off quasiparticles?

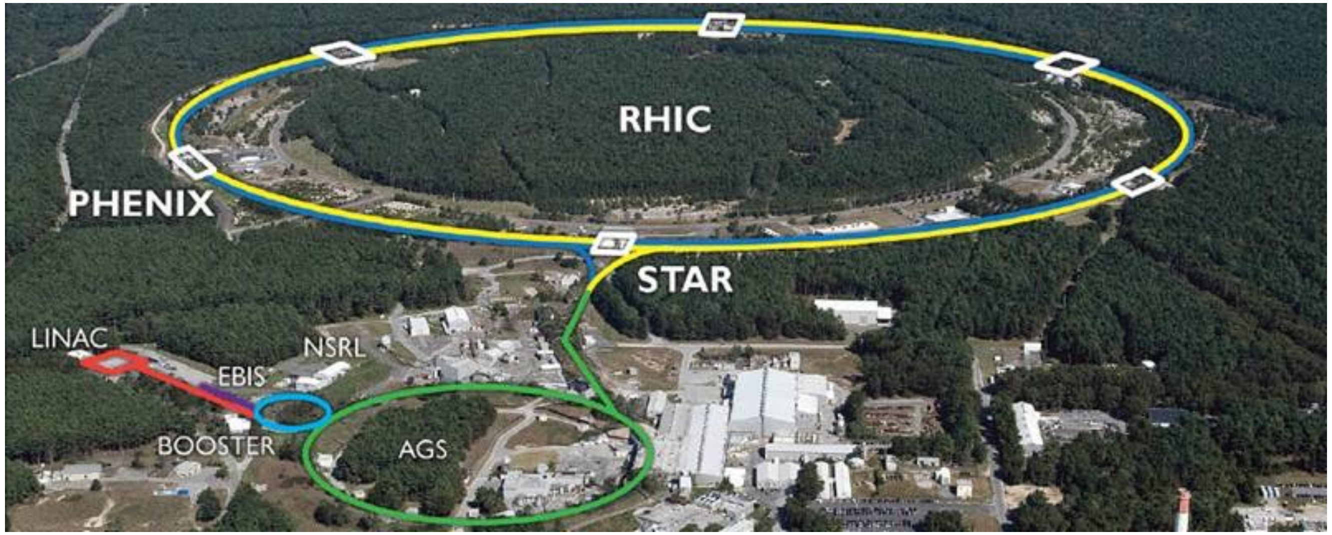


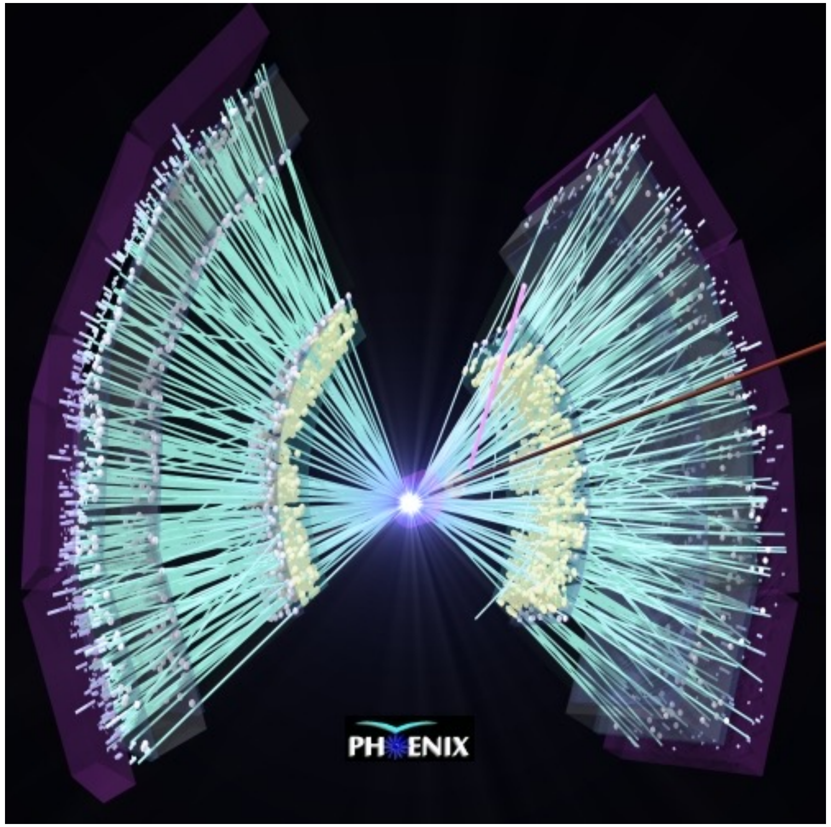
sPHENIX first performance

Cameron Dean
Massachusetts Institute of Technology
MITHIG Physics Discussions
05/30/2023

What is sPHENIX?

- Super PHENIX is the successor to the Pioneering Hadron Electron Nuclear Interaction eXperiment (PHENIX)
- A barrel detector designed to study heavy flavor and jet physics in a heavy ion environment
- Uses both new technology and technology shared with other experiments

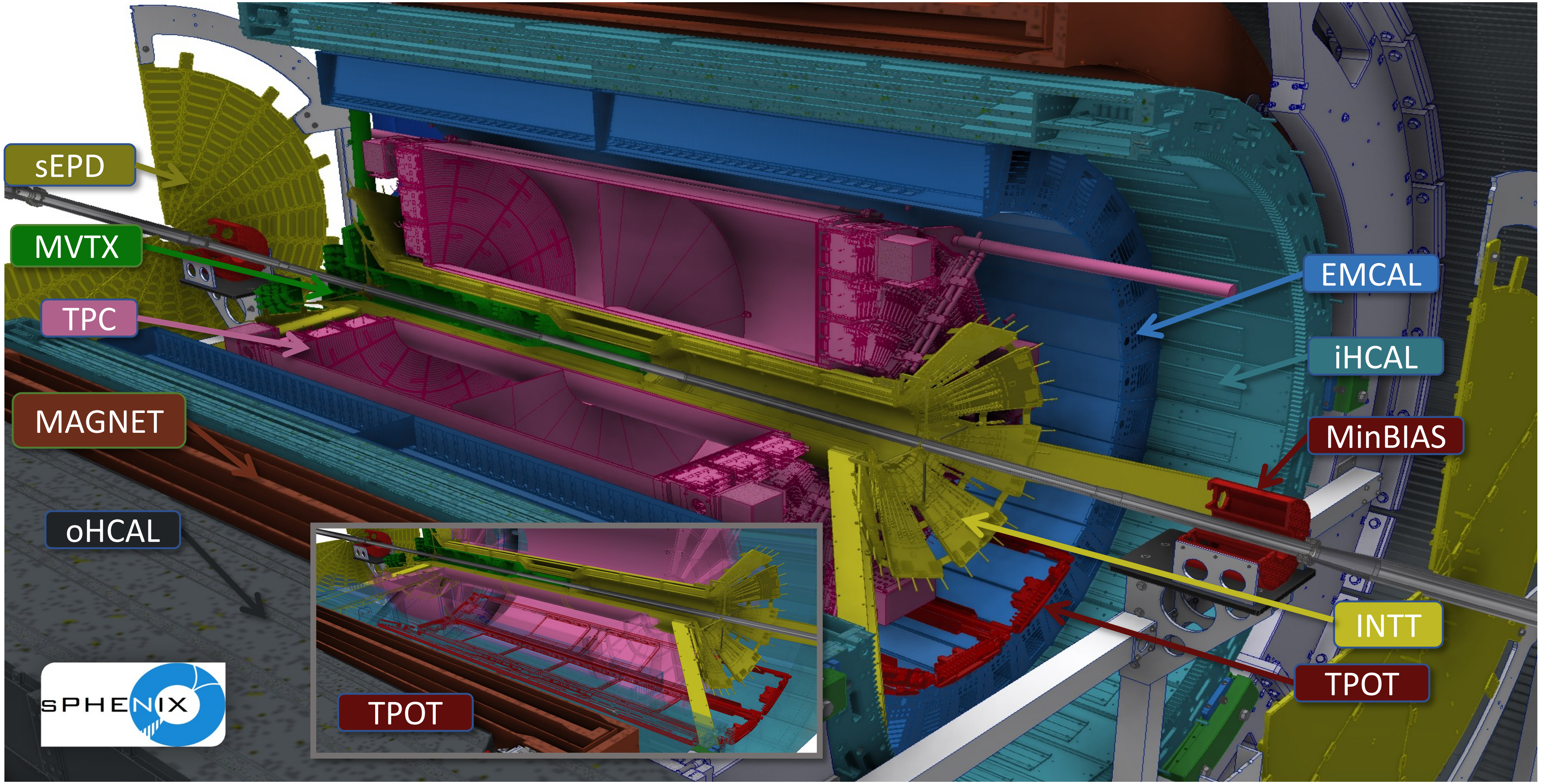




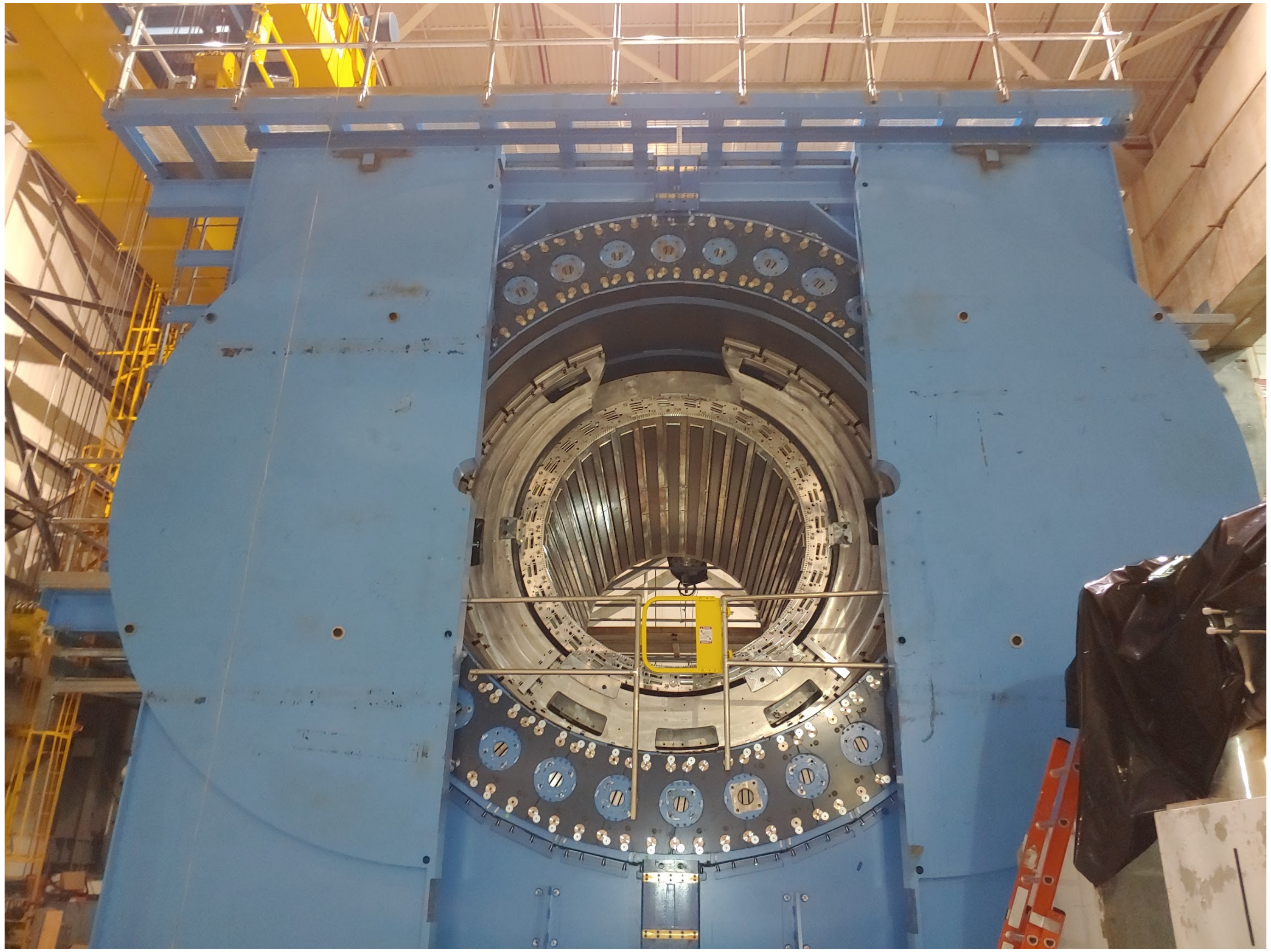
- Located in the PHENIX experimental hall, IP-8
- Last PHENIX data taking was 2016
- Data taking began on May 18th 2023

Top – The location of (s)PHENIX at RHIC
Left – A PHENIX event display

sPHENIX layout



sPHENIX scale

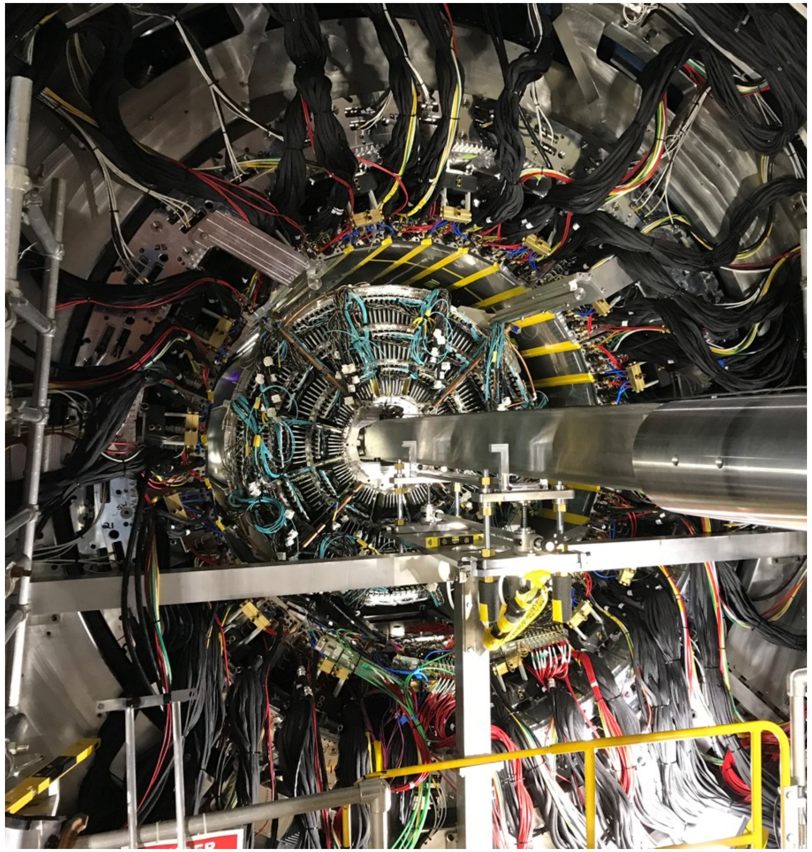


First run year	2023
$\sqrt{s_{NN}}$ [GeV]	200
Trigger Rate [kHz]	15
Magnetic Field [T]	1.4
First active point [cm]	2.5
Outer radius [cm]	270
$\lvert\eta\rvert$	$\leqslant$1.1
$\lvert z_{vtx}\rvert$ [cm]	10
N(AuAu) collisions*	1.43x10^{11}

* In 3 years of running

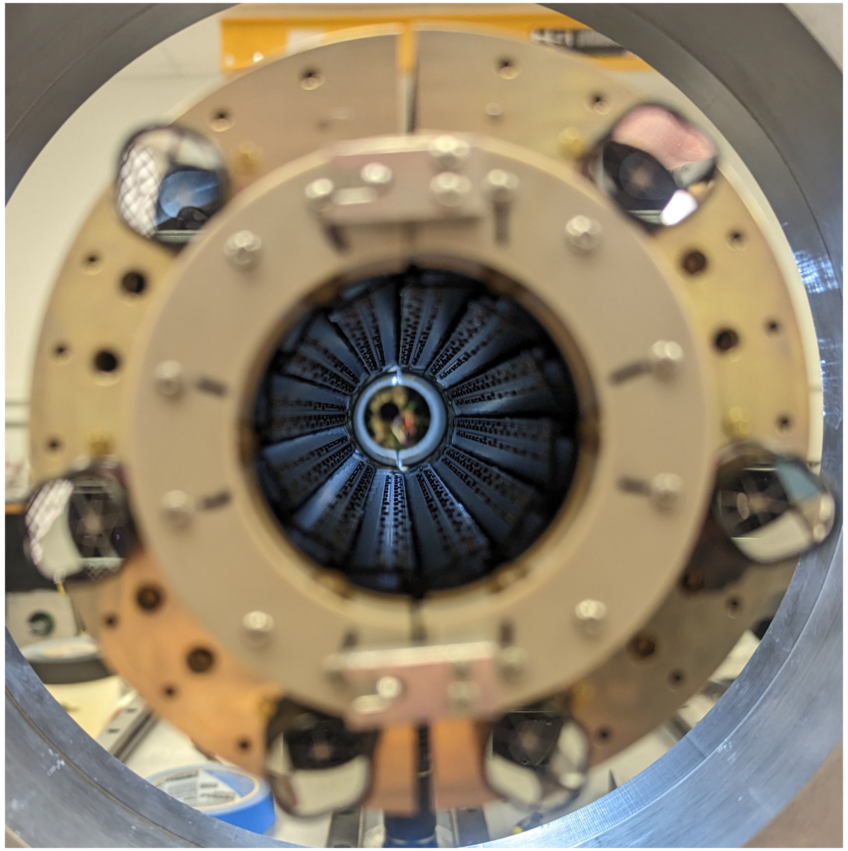
Tracking

TPC after insertion,
January 2023



INTT ladder placement
at BNL, June 2022

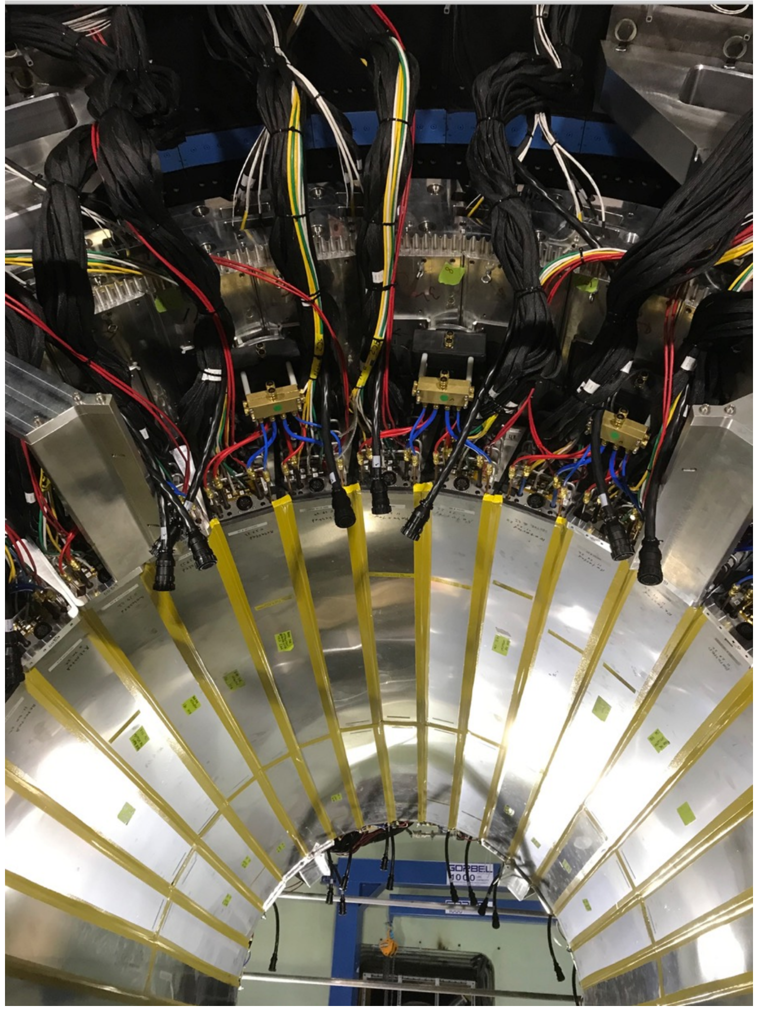
MVTX closure tests,
February 2023



Calorimetry

Left: Inner hadronic calorimeter installation, June 2022

Right: Electromagnetic calorimeter cabling, December 2022

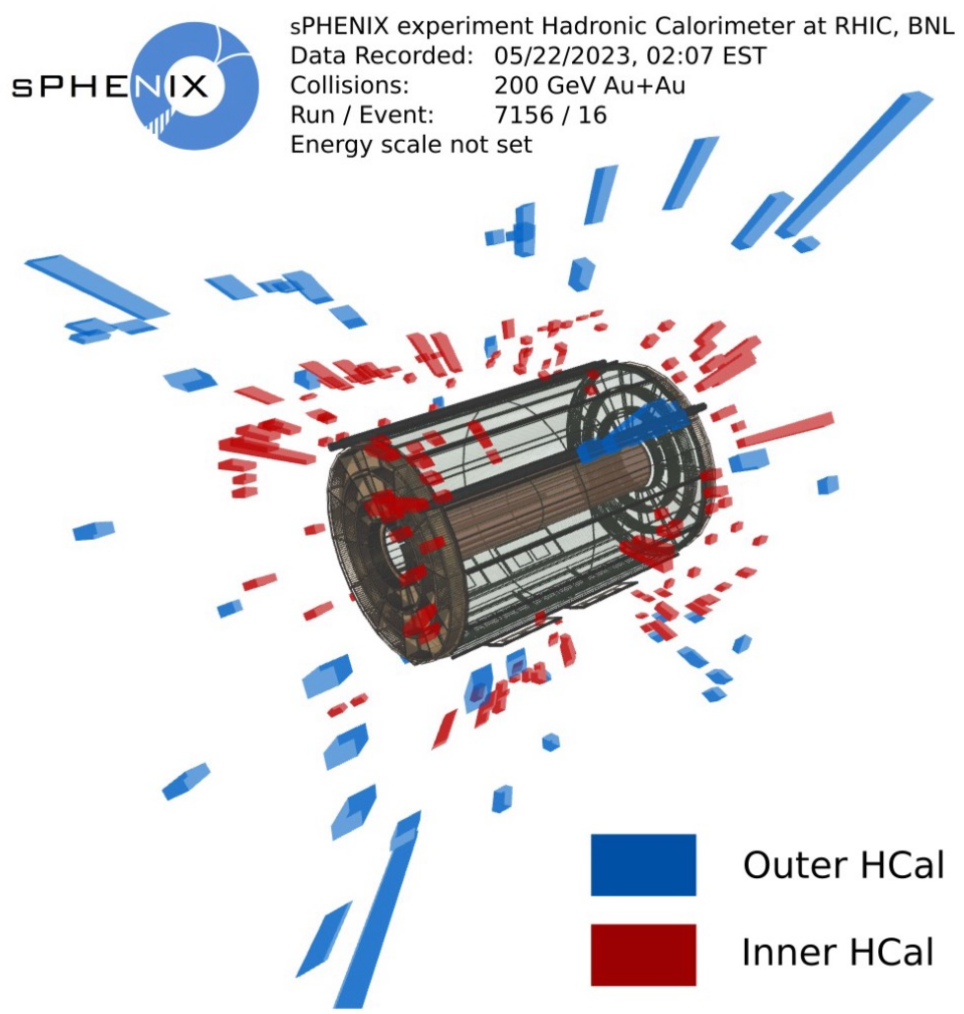


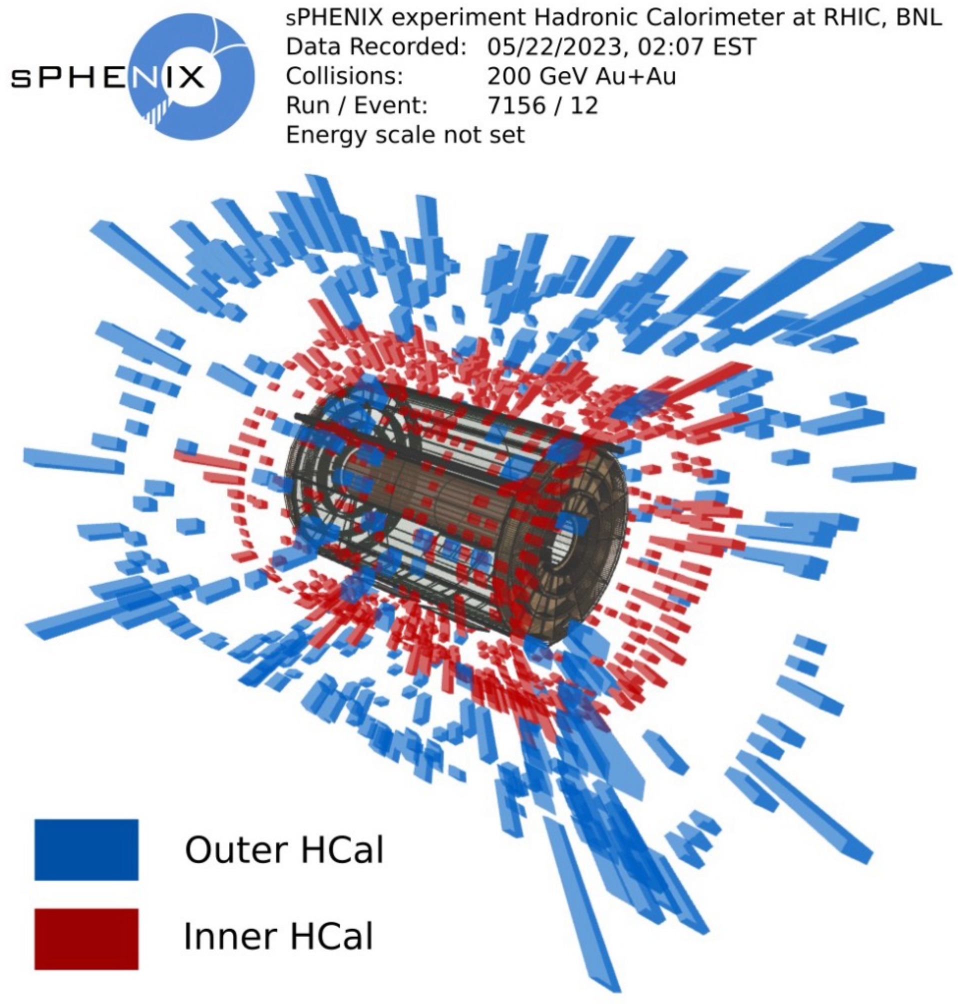
Commissioning plan

- Commissioning started on May 18th when we got approval to cool down the magnet
  - First detectors on were ZDC and MBD
  - HCals were timed in next
  - Followed by EMCal
  - Started timing in INTT and TPOT last week
  - Started timing in TPC this week
  - Magnet switch on was yesterday, 5/30!
  - Will also start bringing in MVTX tomorrow
  - sEPD installation to start mid-July during maintenance period
- **Currently trying collisions with 56-56 bunches!**

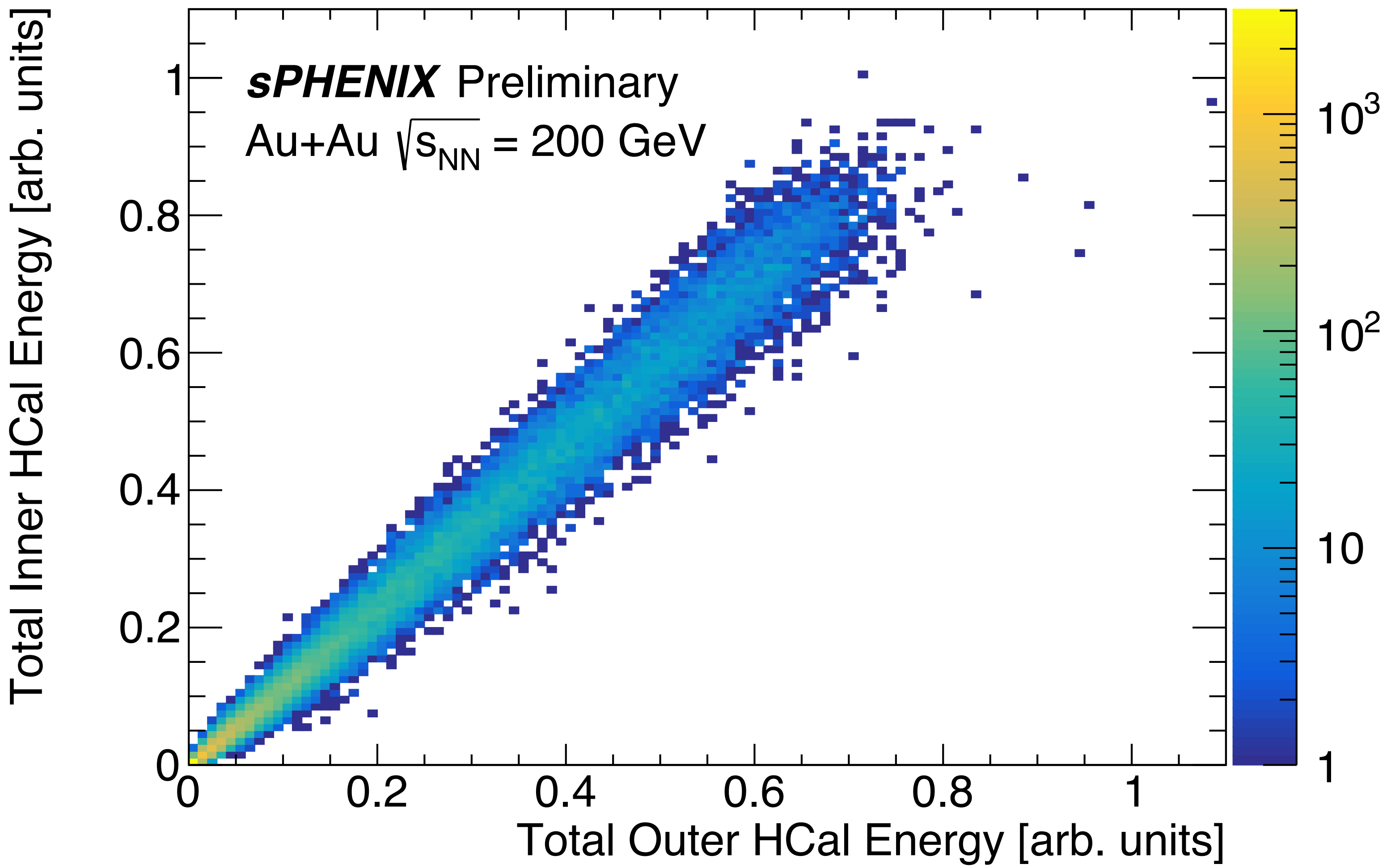
Weeks	Details
2.0	low rate, 6-28 bunches
2.0	low rate, 111 bunches, MBD L1 timing
1.0	low rate, crossing angle checks
1.0	low rate, calorimeter timing
4.0	medium rate, TPC timing, optimization
2.0	full rate, system test, DAQ throughput
12.0	**total**

First-look event displays!





Hadronic calo correlations



Conclusions

- sPHENIX is the first new hadron detector in > 10 years
- Data taking started less than 2 weeks ago!
- Magnet turned on yesterday!
- First approval process occurred on Friday!
- Phenomenal effort underway to time in all detectors
  - ZDC, MBD, inner and outer HCal, and EMCal are timed in
  - INTT, TPC and TPOT are getting added in now
  - MVTX expected to start timing in tomorrow (locked to global trigger ~5pm yesterday)
- First physics ideas have been circulating for months now, let's discuss!

How you can learn from a model

- There are things you can do with a model (here, the Hybrid Model) that you cannot do with experimental data. (Eg, turn physical effects off and on) ...
- ... but that nevertheless teach us important lessons for how to look at, and learn from, experimental data.
- TODAY's EXAMPLE: identifying which jet observables are more sensitive to the presence of quasiparticles — scatterers — in the QGP-soup. And, which are more sensitive to the wakes that jets make in the soup.
- Disentangling effects of jet modification from effects of jet selection. In simulations; in Z+jet or γ+jet data. 2110.13159 Brewer, Brodsky, KR
- Using jet substructure modification to probe QGP resolution length. Can QGP "see" partons within a jet shower (rather than losing energy coherently)? 1707.05245 ZH, DP, KR; 1907.11248 Casalderrey-Solana, Milhano, DP, KR. (Apparent answer: yes. Eg., 2303.13347 ALICE)
- But first, an intro to the Hybrid Model...

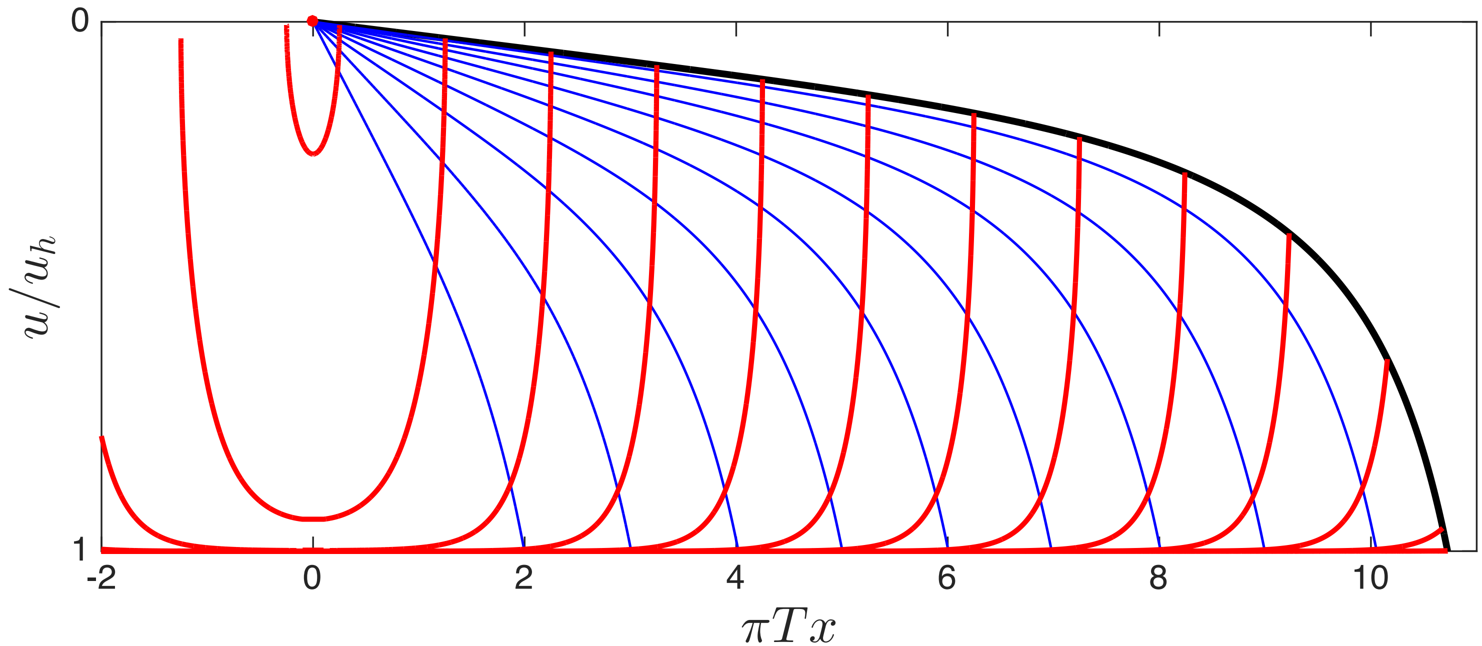
A Hybrid Approach

Casalderrey-Solana, Gulhan, Milhano, Pablos, KR, 2014,15,16; Hulcher, DP,KR, '17; JCS,ZH,GM,DP,KR, '18; JCS,GM,DP,KR, '19; JCS,GM,DP,KR, Yao, '20

- **Hard scattering and the fragmentation of a hard parton produced in a hard scattering are weakly coupled phenomena, well described by pQCD.**
- **The medium itself is a strongly coupled liquid, with no apparent weakly coupled description. And, the energy the jet loses seems to quickly become one with the medium.**
- **Try a hybrid approach. Think of each parton in a parton shower à la PYTHIA losing energy à la dE/dx for light quarks in strongly coupled liquid.**
- **Look at R_{AA} for jets and for hadrons, dijet asymmetry, jet fragmentation function, photon-jet and Z-jet observables. Upon fitting one parameter, *lots* of data described well. Value of the fitted parameter is reasonable: x_{therm} (energetic parton thermalization distance) 3-4 times longer in QGP than in $\mathcal{N} = 4$ SYM plasma at same T.**
- **Then: add the wake in the plasma; add resolution effects; look at jet shapes, jet masses jet substructure observables; add Molière scattering...**

Quenching a Light Quark "Jet"

Chesler, Rajagopal, 1402.6756, 1511.07567



- Take a highly boosted light quark and shoot it through strongly coupled plasma...
- A fully geometric characterization of energy loss. Which is to say a new form of intuition. Energy propagates along the blue curves, which are null geodesics in the bulk. When one of them falls into the horizon, that's energy loss! Precisely equivalent to the light quark losing energy to a hydrodynamic wake in the plasma.

Implementation of Hybrid Model

Casalderrey-Solana, Gulhan, Milhano, Pablos, KR, 1405.3864,1508.00815

- **Jet production and showering from PYTHIA.**
- **Embed the PYTHIA parton showers in hydro background. (2+1D hydro from Heinz and Shen.)**
- **Between one splitting and the next, each parton in the branching shower loses energy according to**

$$\frac{1}{E_{\text{in}}}\frac{dE}{dx} = -\frac{4x^2}{\pi x_{\text{therm}}^2}\frac{1}{\sqrt{x_{\text{therm}}^2 - x^2}}$$

where $x_{\text{therm}} \equiv E_{\text{in}}^{1/3}/(2\kappa_{\text{sc}}T^{4/3})$ with κ_{sc} one free parameter that to be fixed by fitting to one experimental data point. ($\kappa_{\text{sc}} \sim 1 - 1.5$ in $\mathcal{N} = 4$ SYM; smaller κ_{sc} means x_{therm} is longer in QGP than in $\mathcal{N} = 4$ SYM plasma with same T.)

- **Turn energy loss off when hydrodynamic plasma cools below a temperature that we vary between 145 and 170 MeV. (This, plus the experimental error bar on the one data point, becomes the uncertainty in our predictions.)**
- **Reconstruct jets using anti-k_T.**

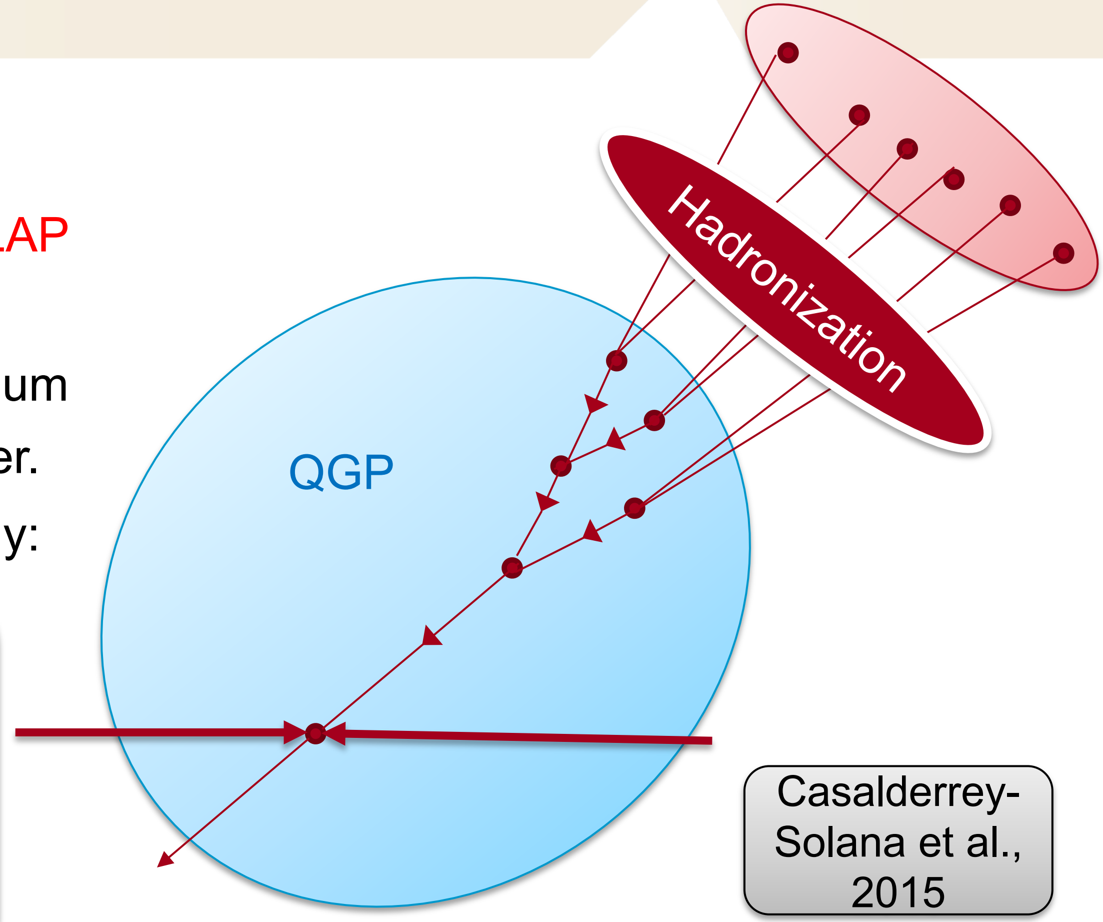
Perturbative Shower … Living in Strongly Coupled QGP

- High Q^2 parton shower up until hadronization described by DGLAP evolution (PYTHIA).
- For QGP with $T\sim\Lambda_{QCD}$, the medium interacts strongly with the shower.
  - Energy loss from holography:

$$\frac{1}{E_{in}}\frac{dE}{dx} = -\frac{4}{\pi}\frac{x^2}{x_{stop}^2}\frac{1}{\sqrt{x_{stop}^2 - x^2}}$$

$$x_{stop} = \frac{1}{2\kappa_{sc}}\frac{E_{in}^{\frac{1}{3}}}{T^{\frac{4}{3}}} \qquad \tau = \frac{2E}{Q^2}$$

O(1) fit const.



Casalderrey-Solana et al., 2015

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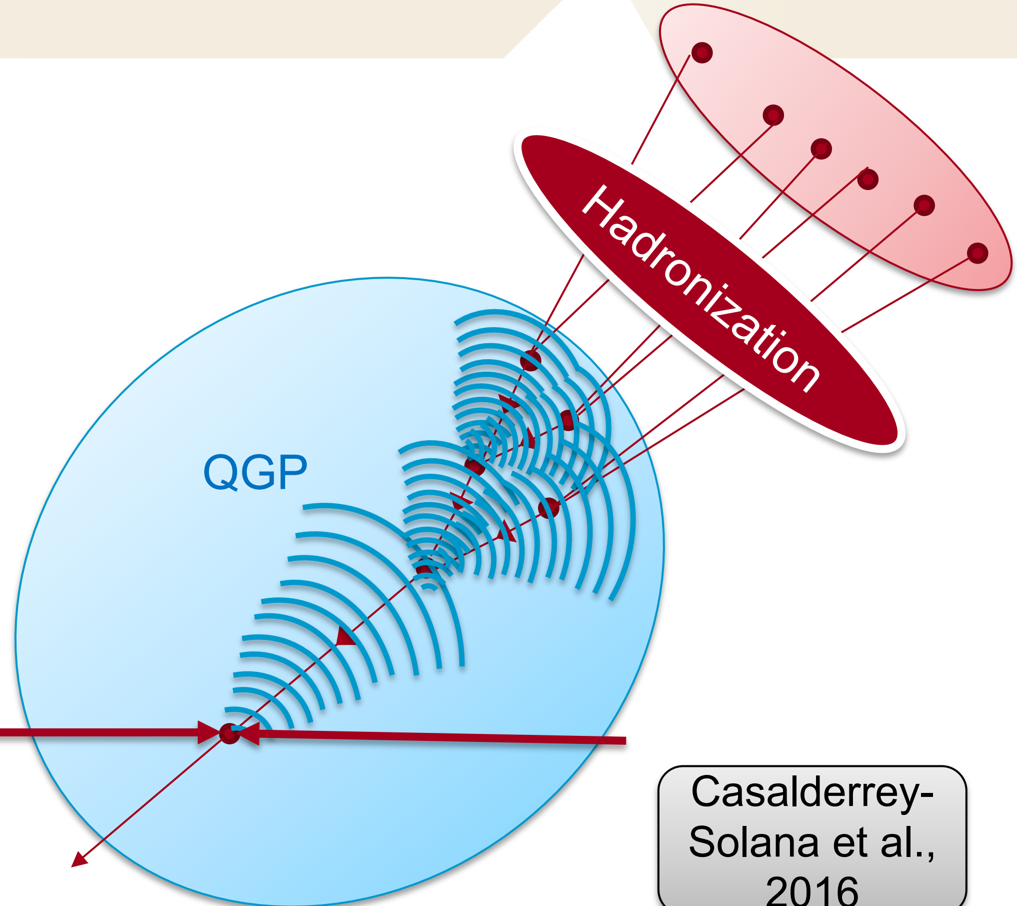


Casalderrey-Solana et al., 2016

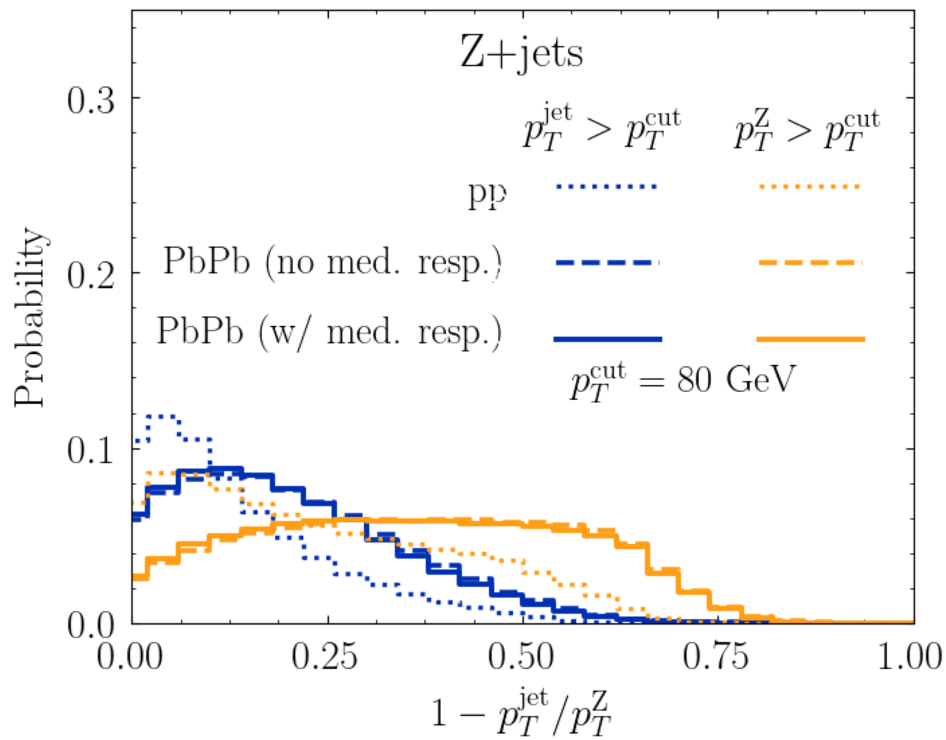
Energy and momentum conservation → deposit hydrodynamic wake in QGP liquid

$$\frac{d\Delta N}{p_T dp_T d\phi dy} = \frac{1}{(2\pi)^3}\int \tau dx dy d\eta_s m_T \cosh(y-\eta_s)\left[f\left(\frac{u^\mu p_\mu}{T_f+\delta T}\right) - f\left(\frac{u_0^\mu p_\mu}{T_f}\right)\right]$$

Jets as Probes of QGP

- Theorists are taking key steps toward realizing the vision of using jets as probes. Four examples here, all relying upon the Hybrid Model.
- Disentangling effects of jet modification from effects of jet selection. In simulations; in Z+jet data (LHC); in γ+jet data (sPHENIX). 2110.13159 Brewer, Brodsky, KR
- Using jet substructure modification to probe the QGP resolution length. Can the QGP "see" partons within a jet shower, or does it lose energy coherently? 1707.05245 Hulcher, Pablos, KR; 1907.11248 Casalderrey-Solana, Milhano, Pablos, KR
- Jet wakes in droplet of QGP
- Selecting substructure observables sensitive to scattering of jet partons off QGP partons

Disentangling Jet Modification from Selection



Orange: $p_T^{Z} > 80$ GeV; $p_T^{\rm jet} > 30$ GeV

Blue: $p_T^{\rm jet} > 80$ GeV; $p_T^{Z} > 30$ GeV — jet selection biases toward those jets that lose less energy

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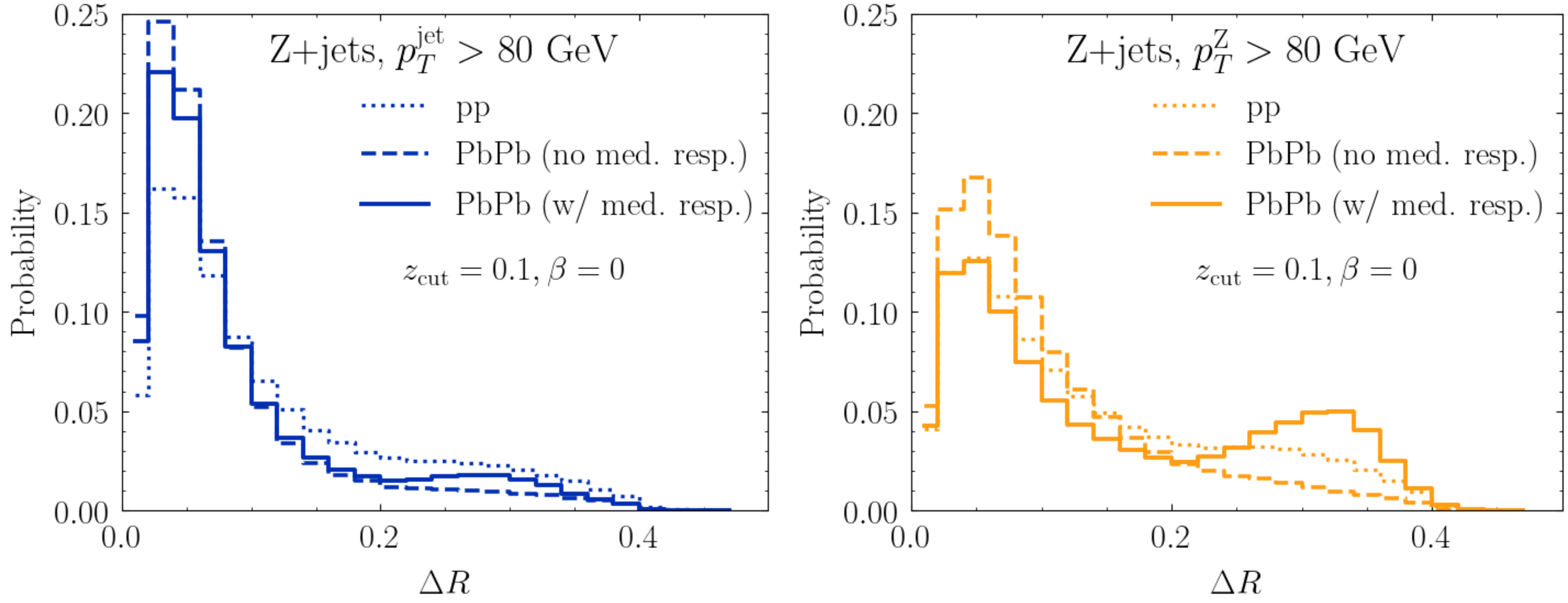

Orange: $p_T^Z > 80$ GeV; $p_T^{\rm jet} > 30$ GeV. See jet modification.

Blue: $p_T^{\rm jet} > 80$ GeV; $p_T^Z > 30$ GeV — jet selection biases toward those jets that lose less energy. These jets are skinnier. And the bias is toward less jet modification.

Jets as Probes of QGP

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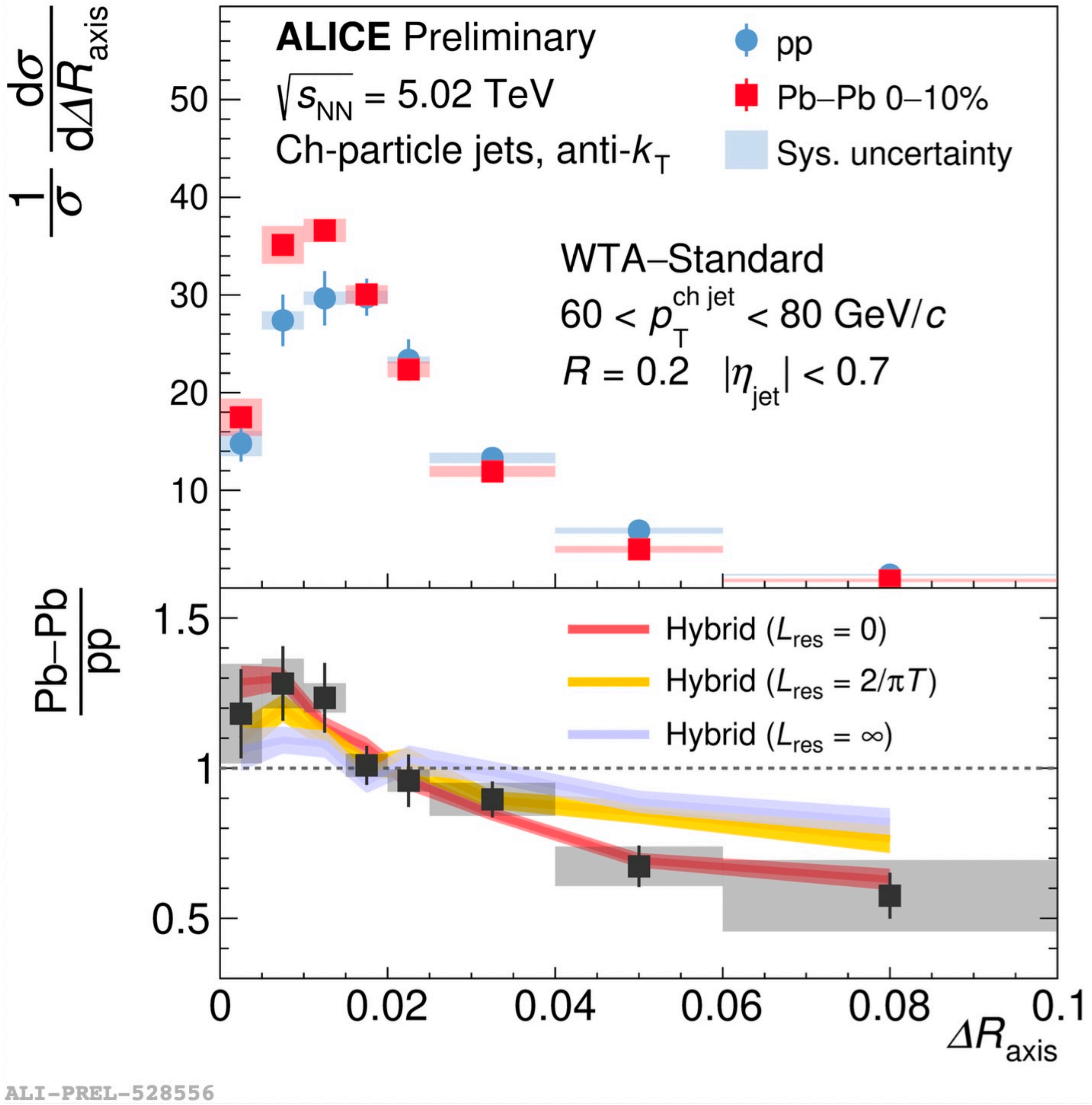
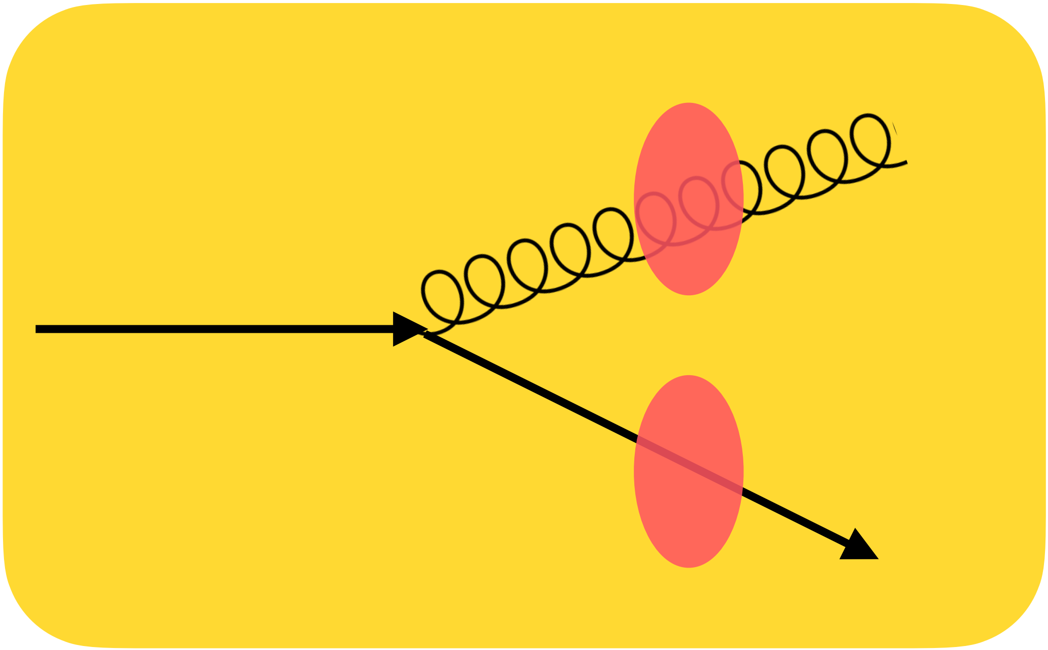
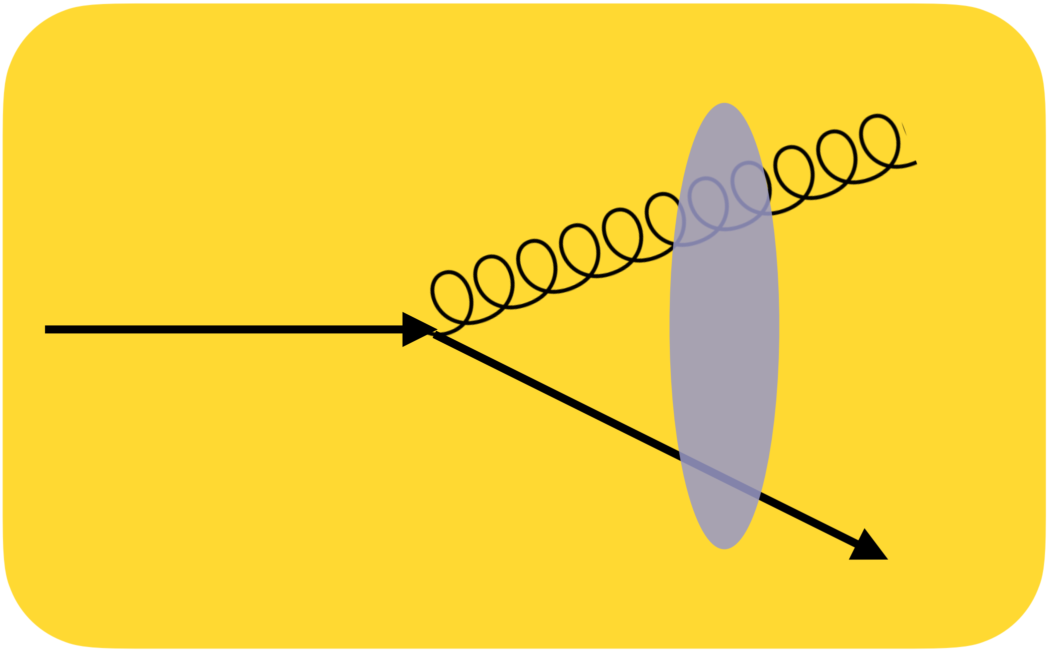
Medium resolution length, L_{res}





L_{res} = 0: medium resolves splitting immediately after parton fragments.
Fully-incoherent energy loss



L_{res} = ∞: medium does not resolve splitting.
Fully-coherent energy loss

Data favors mechanisms of incoherent energy loss in the QGP

Jets as Probes of QGP

- Jet wakes in droplets of QGP.
  - Momentum/energy "lost" by parton shower $\rightarrow$ wake in the fluid $\rightarrow$ spray of soft hadrons, many in the jet. Jets in HIC are not just the parton shower hadronized.
  - To use jets as probes, must calculate, or understand+avoid, wake. Wake also interesting: study equilibration.
  - Crude calculation of particles in jet originating from wake has been a part of the Hybrid Model since 2016, it's weaknesses and strengths known...
  - Full hydrodynamic calculation of wake due to every parton in every jet in a sample of 100,000 jets is unfeasible. Jet wake from *linearized* hydrodynamics will suffice, and will modify Hybrid Model predictions for soft particles in jets in the direction indicated by data: 2010.01140 Casalderrey-Solana, Milhano, Pablos, KR, Yao
  - Use the linearity of linearized hydro to speed up calculation of wake by $\sim 10,000$ and of its hadronization by ~ 100 (in progress).

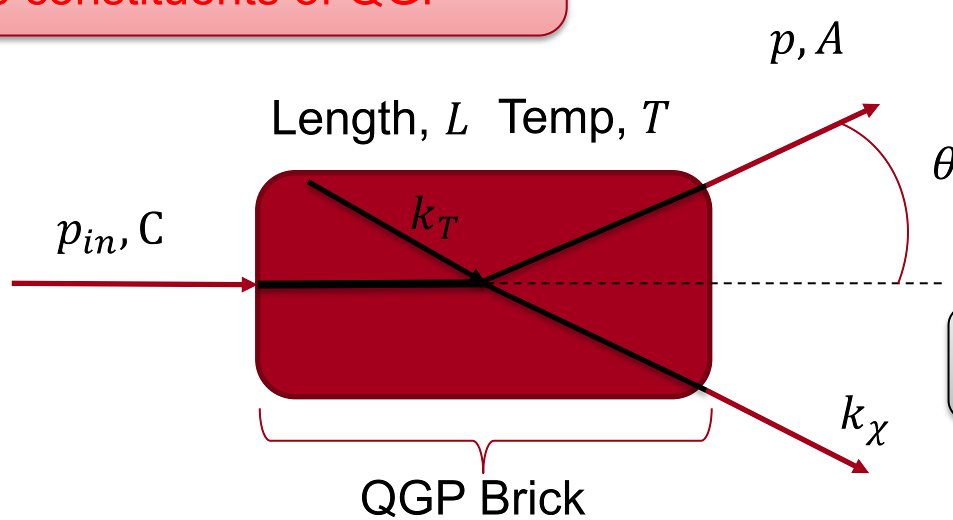
Why Molière scattering?
Why add to Hybrid Model?

- QGP, at length scales $\mathcal{O}(1/T)$, is a strongly coupled liquid. Flow, and jet observables sensitive to parton energy loss, are well-described (eg in hybrid model) in such a fluid, without quasiparticles.
- At shorter length scales, probed via large momentum-exchange, asymptotic freedom $\rightarrow$ quasiparticles matter.
- High energy partons in jet showers *can* probe particulate nature of QGP. Eg via power-law-rare, high-momentum-transfer, large-angle, Molière scattering
- "Seeing" such scattering is first step to probing microscopic structure of QGP.
- What jet observables are sensitive to effects of high-momentum-transfer scattering? To answer, need to turn it off/on.
- Start from Hybrid Model – in which any particulate effects are definitively off! Add Molière, and look at effects...

Moliere Scattering in a brick of QGP (D'Eramo, KR, Yin, 2019)

Power-law-rare medium kicks which can probe particle constituents of QGP

In JEWEL, LBT, MARTINI, harder to turn off



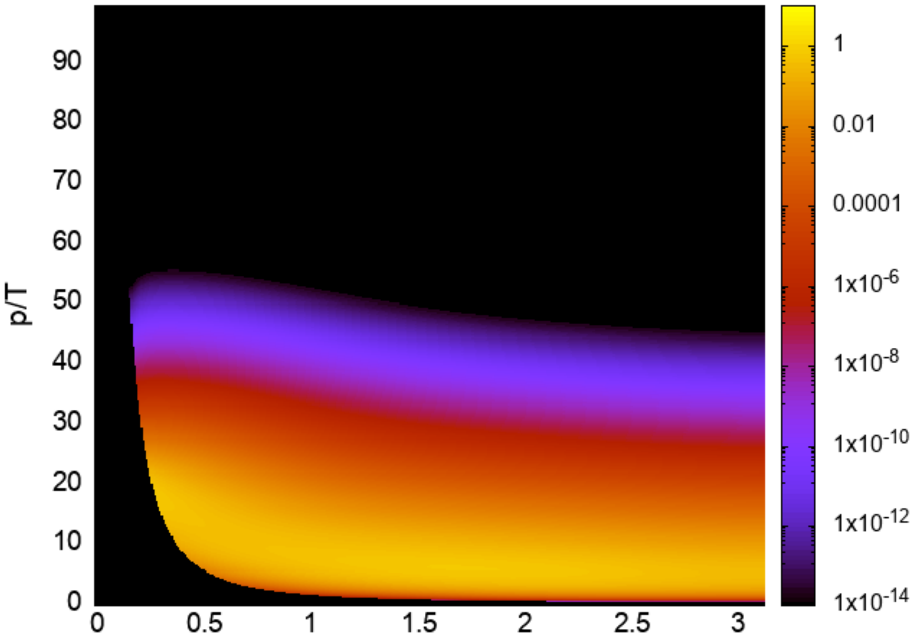
D'Eramo et al., 2019

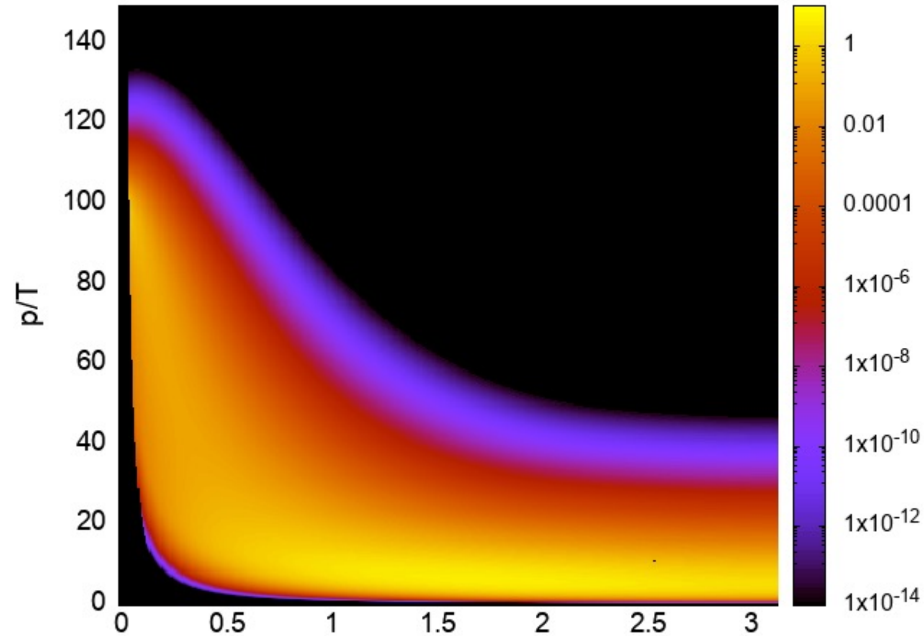
- Sufficiently hard scattering should be perturbative.
- High p_T particle can be deflected, changing its energy and direction.
- Recoiling particle, k_χ, a new particle to be quenched
- Thermal particle, k_T, from BE/FD distribution, removed from medium.

Tree-Level 2-2 massless scattering amplitudes

$$F^{C\to A}(p,\theta;p_{in}) = \sum_{nDB} \frac{c_{DBn}^{C\to A}}{2(4\pi)^3}\left(\frac{p\sin(\theta)}{p_{in}|\boldsymbol{p}-\boldsymbol{p_{in}}|T}\right)\int_{k_{min}}^{\infty} dk_T\, n_D(k_T)[1\pm n_B(k_\chi)]\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{\left|M^{(n)}\right|^2}{g_s^4}$$

Results (for a QGP brick)



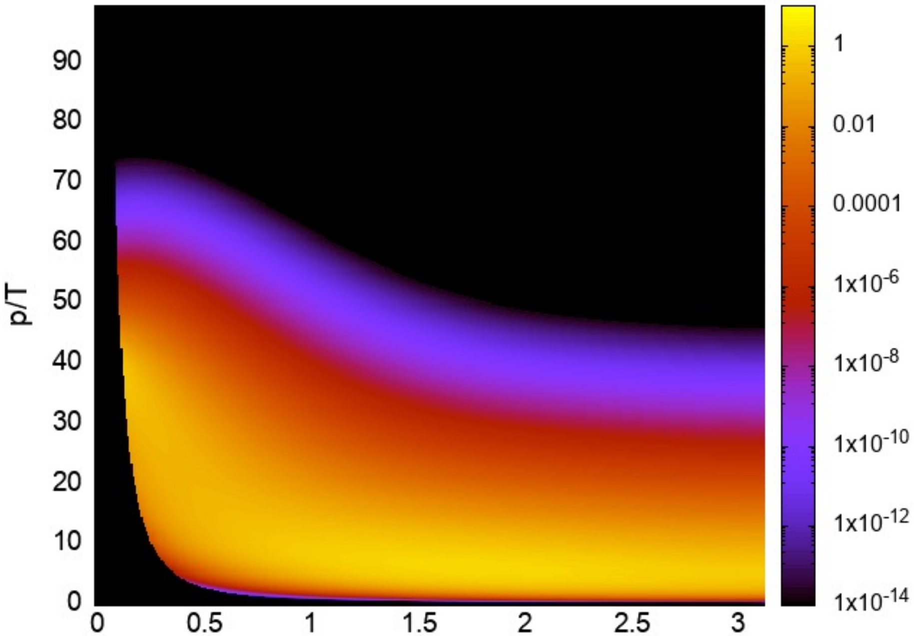


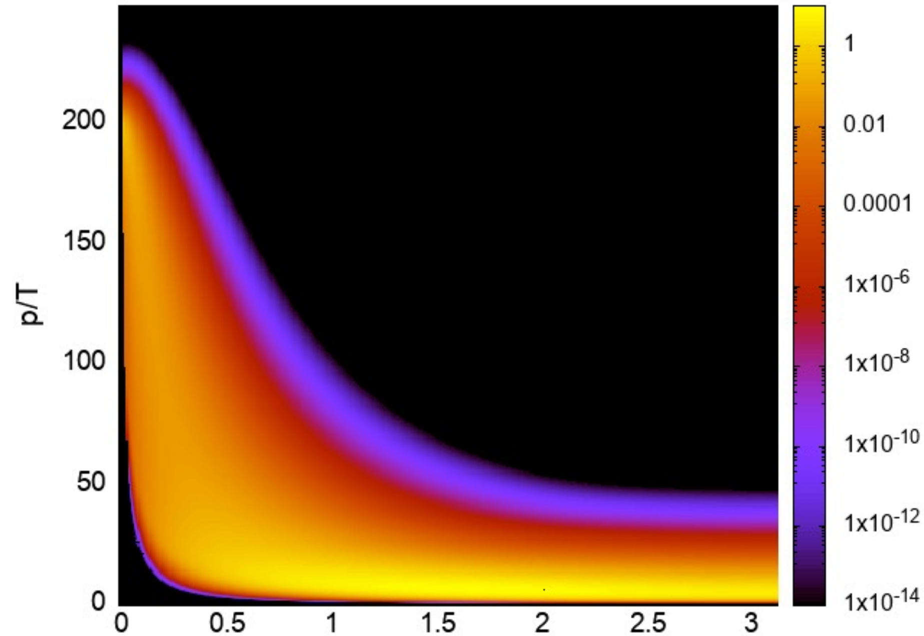
Incoming gluon, $p_{in} = 20T, \mathrm{L} = 15/T$

Incoming gluon, $p_{in} = 100T, \;\; \mathrm{L} = 15/T$

- Excluding $\tilde{u} > 4\, m_D^2$ not a simple curve on this plot, but effects visible
- Restricting to $\tilde{u}, \tilde{t} > 4\, m_D^2$ excludes soft scatterings; justifies assumptions made in amplitudes; avoids double counting
- Analytical results → fast to sample
- Apply at every time step, to every rung, in every shower, in Hybrid Model Monte Carlo…. And, if a scattering happens, two subsequent partons then lose energy a la Hybrid

Results (for a QGP brick)





Incoming gluon, $p_{in} = 40T, \mathrm{L} = 15/T$

Incoming gluon, $p_{in} = 200T, \ \mathrm{L} = 15/T$

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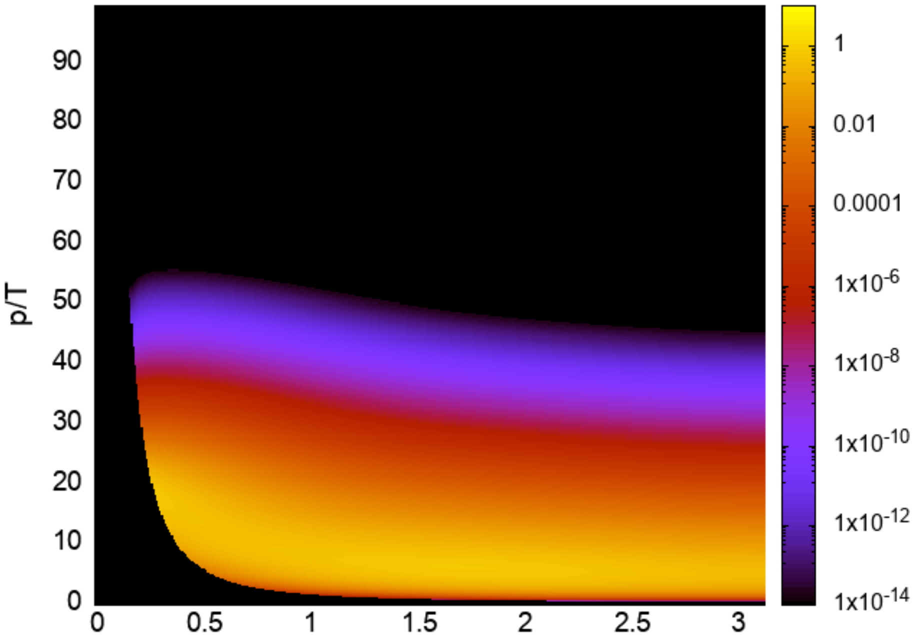

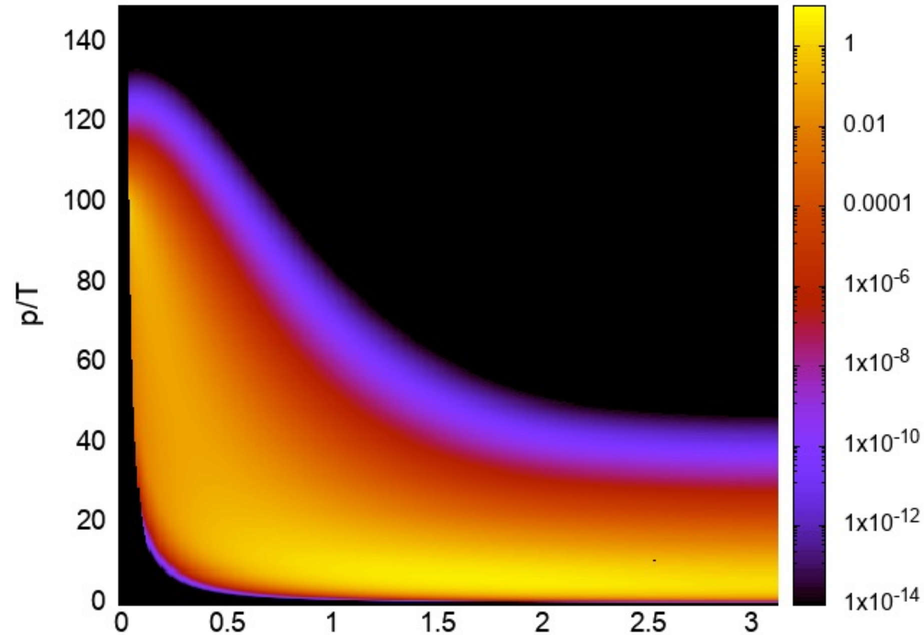

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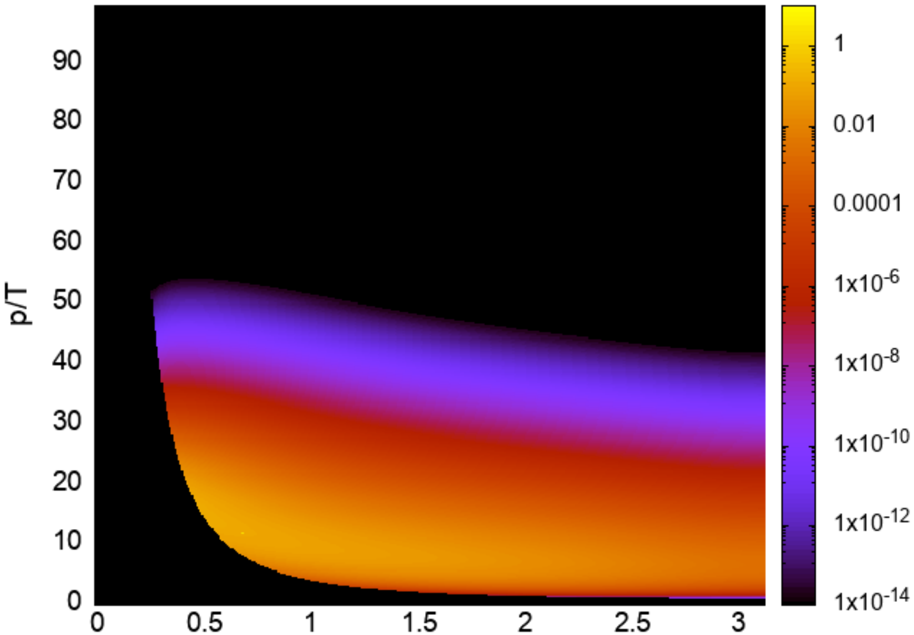

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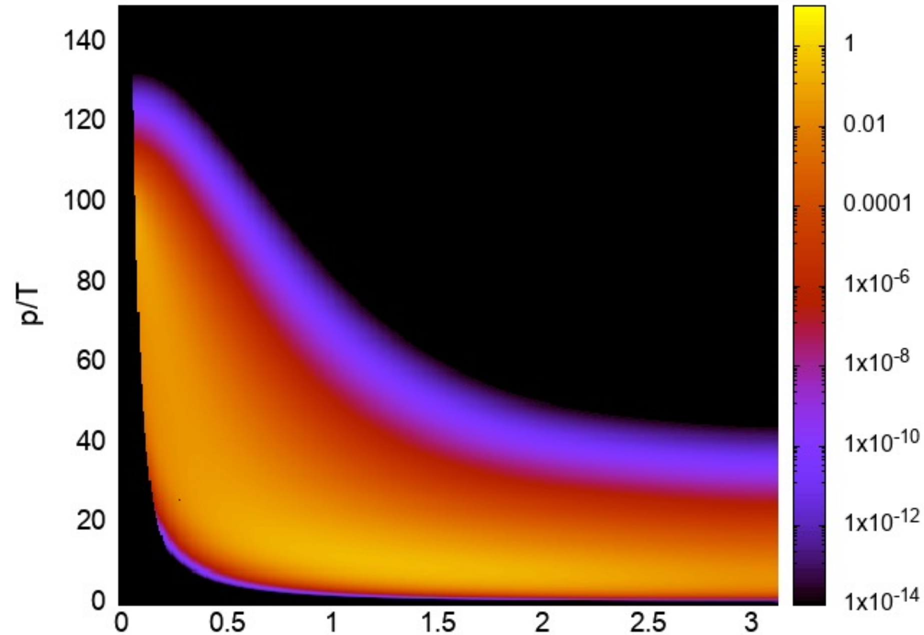

Incoming gluon, $p_{in} = 100T, \;\; \mathrm{L} = 15/T$

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- Restricting to $\tilde{u}, \tilde{t} > 10\, m_D^2$ excludes soft scatterings; justifies assumptions made in amplitudes; avoids double counting. Can vary where to set this cut…
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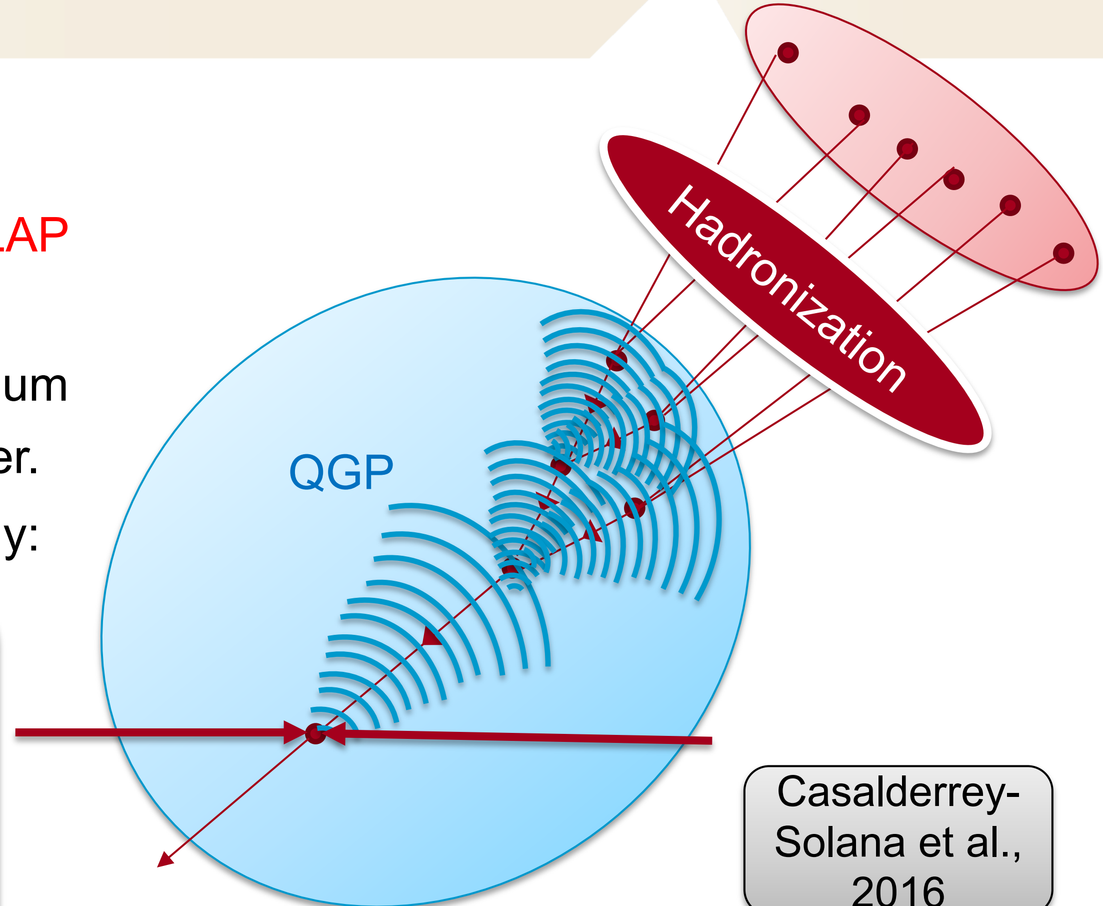

Casalderrey-Solana et al., 2016

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Adding Moliere Scattering to Hybrid Model

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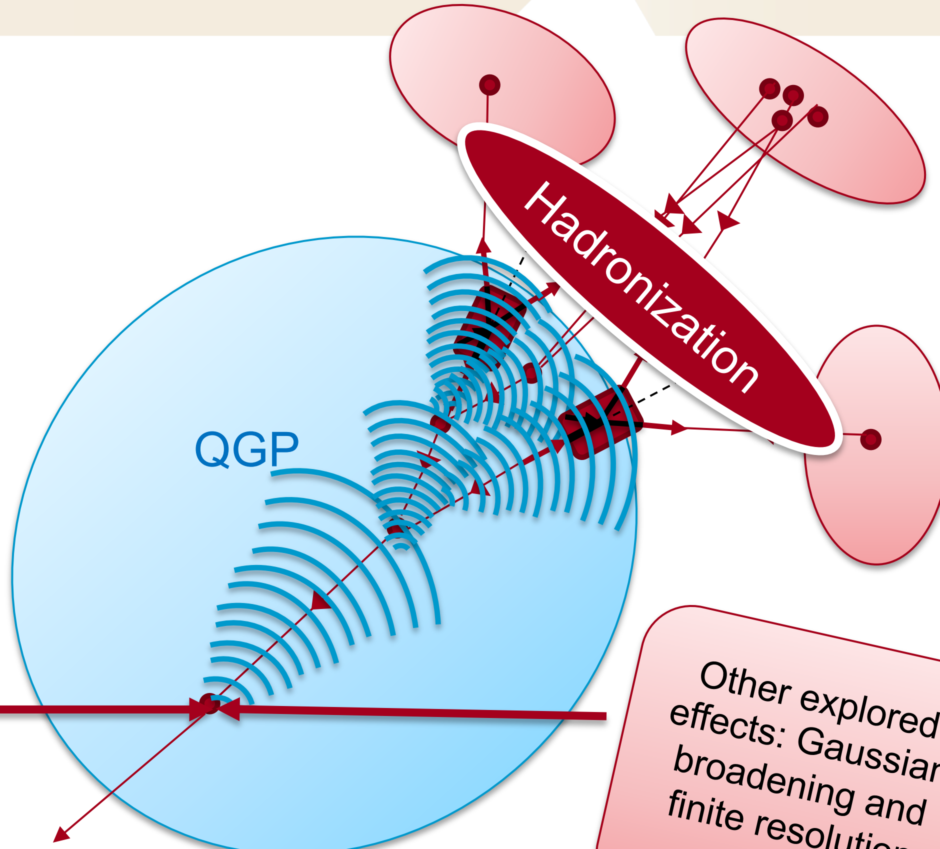


Other explored effects: Gaussian broadening and finite resolution effects

Energy and momentum conservation → activate hydrodynamic modes of plasma

$$\frac{d\Delta N}{p_T dp_T d\phi dy} = \frac{1}{(2\pi)^3}\int \tau dx dy d\eta_s m_T \cosh(y - \eta_s)\left[f\left(\frac{u^\mu p_\mu}{T_f + \delta T}\right) - f\left(\frac{\mu_0^\mu p_\mu}{T_f}\right)\right]$$

Gaussian Broadening vs Large Angle Scattering

- Elastic scatterings of exchanged momentum $\sim m_D$

  → Gaussian broadening due to multiple soft scattering

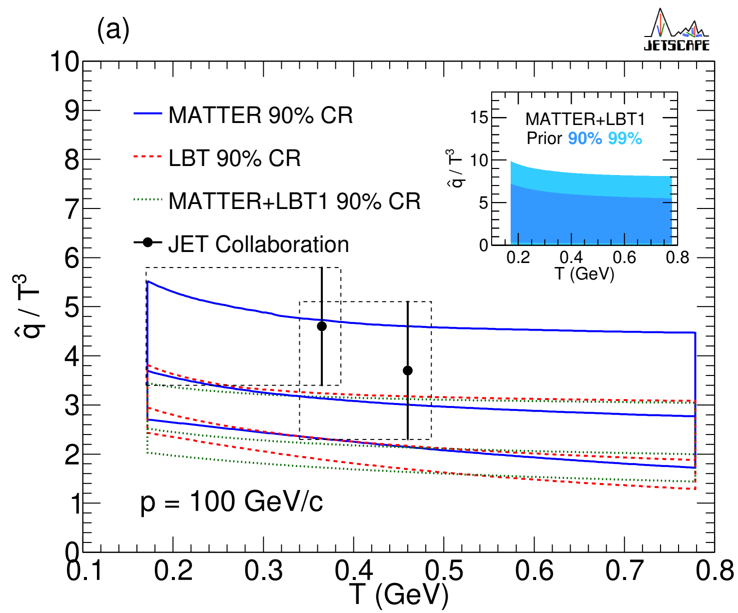
- At strong coupling, holography predicts Gaussian broadening without quasi-particles (eg: N=4 SYM)

$$P(k_\perp) \sim \exp\left(-\frac{\sqrt{2}k_\perp^2}{\hat{q}L^-}\right) \qquad \hat{q} = \frac{\pi^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\sqrt{\lambda}T^3$$

Adding this in hybrid model (C-S et al 2016) yields very little effect on jet observables

- Restrict to momentum exchanges $> m_D$

  → focus on perturbative regime with a power-law distribution

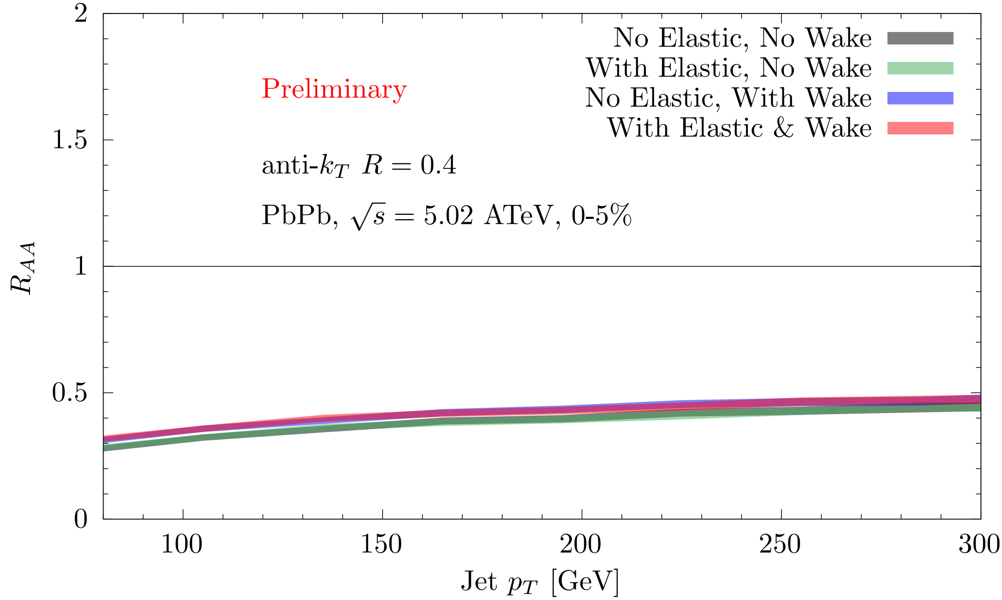


D'Eramo et al., 2011, 2018
+
Mehtar-Tani et al., PRD 2021

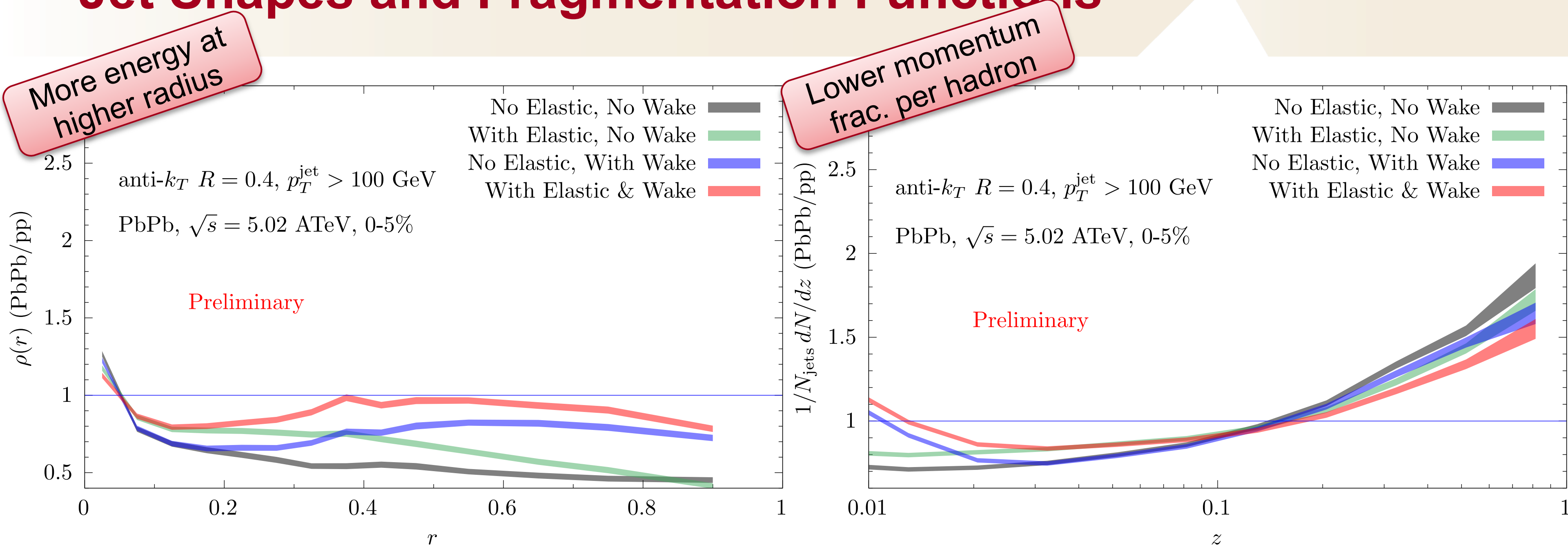
Jet R_{AA}

Casalderrey-Solana et al. 2019

- κ_{sc} previously fit with jet and hadron suppression data from ATLAS+CMS at 2.76+5.02 TeV
- Elastic scatterings lead to slight additional suppression; refit κ_{sc} . That means red is on top of blue in this plot by construction. (Addition of the elastic scatterings yields only small change to value of κ_{sc}.)
- Adding the hadrons from the wake allows the recovery of part of the energy within the jet cone; blue and green slightly below red and blue.
- All results, here on, are Preliminary.



Jet Shapes and Fragmentation Functions



→ Elastic scattering effects look very similar to wake effects, but smaller.

- Moliere scattering transfers jet energy to high angle and lower momentum fraction particles. So does energy loss to wake in fluid.
- In **these observables**, effect of Moliere looks like just a bit more wake.
- In principle sensitive to Moliere, but in practice not: more sensitive to wake.
- Moliere effects are even slightly smaller if $\tilde{u}, \tilde{t} > a\, m_D^2$ with a=10.
- What if we look at groomed observables? Less sensitive to wake…

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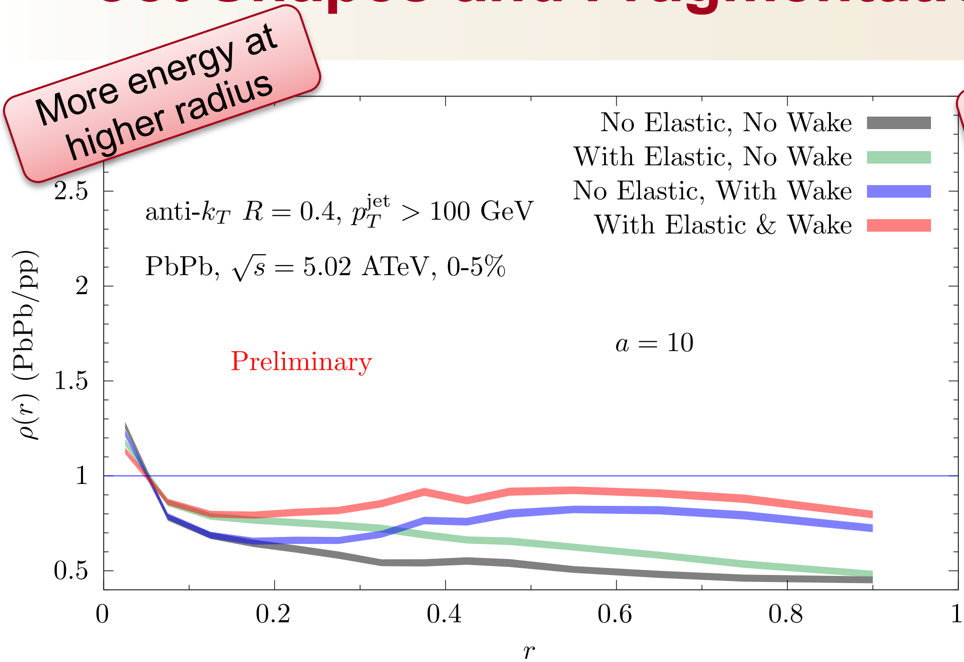

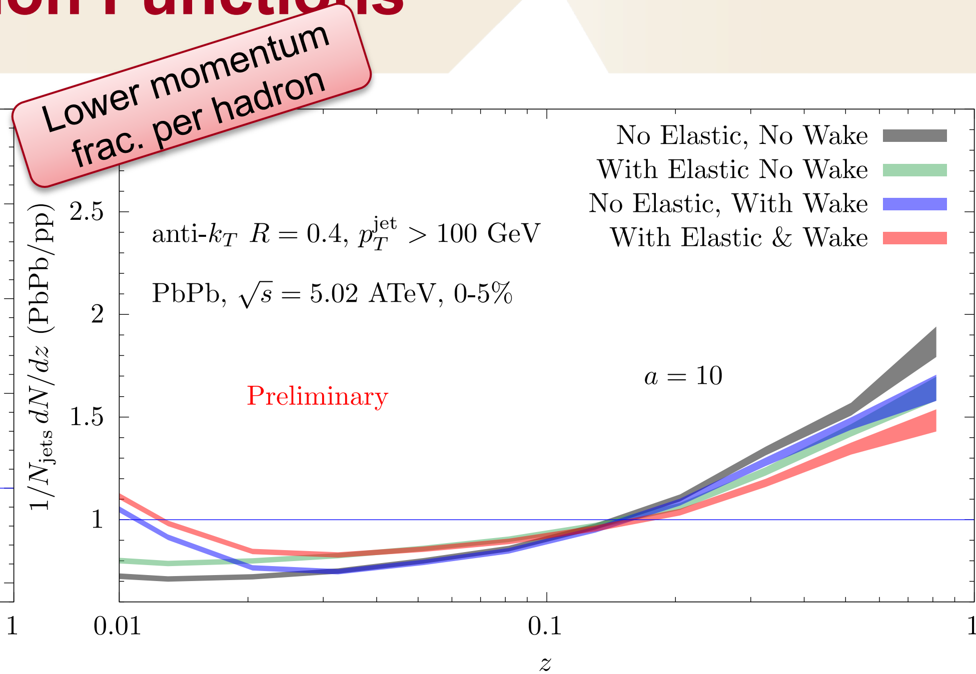

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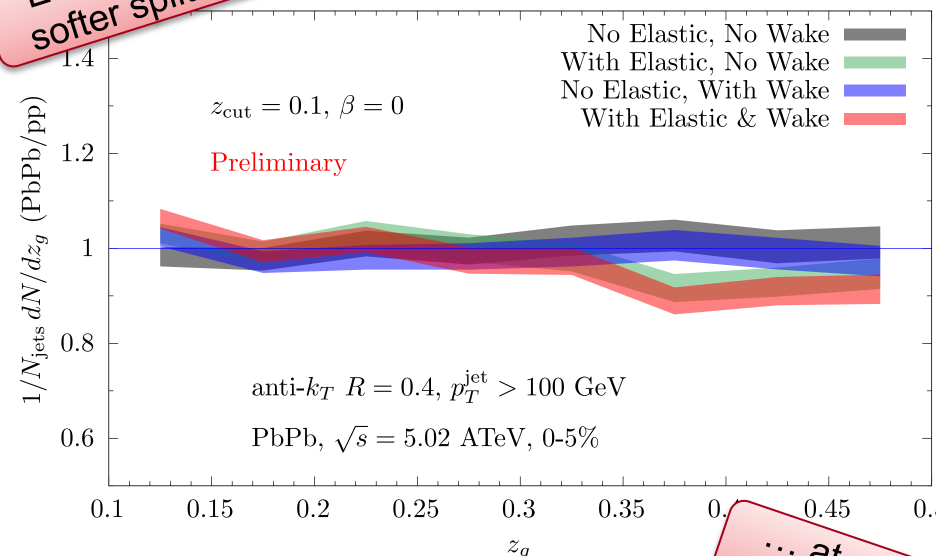
Groomed z_g and R_g

<u>Soft Drop ($\beta = 0$)</u>

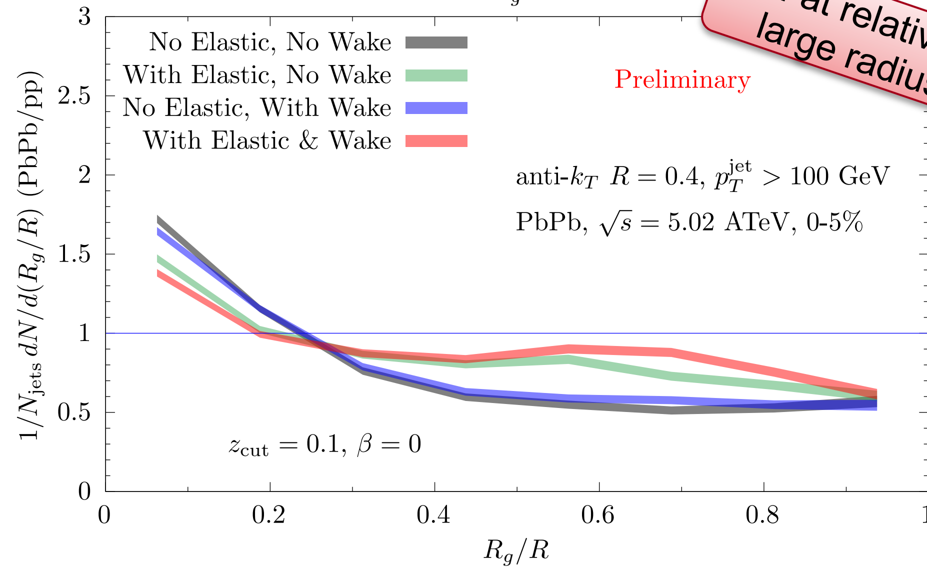
1. Reconstruct jet with anti-k_T
2. Recluster with Cambridge-Aachen
3. Undo last step of 2, resulting in subjets 1 and 2, separated by angle R_g
4. If $\frac{\min(p_{T1}, p_{T2})}{p_{T1}+p_{T2}} \equiv z_g > z_{cut}$, then original jet is the final jet. Otherwise pick the harder of subjets 1 and 2 and repeat

Much less sensitivity to wake; Moliere scattering shows up; effects of Moliere and wake are again similar in shape, but here effects of Moliere are dominant, with a=4 or 10.

Enhancement of softer splittings…



… at relatively large radius.



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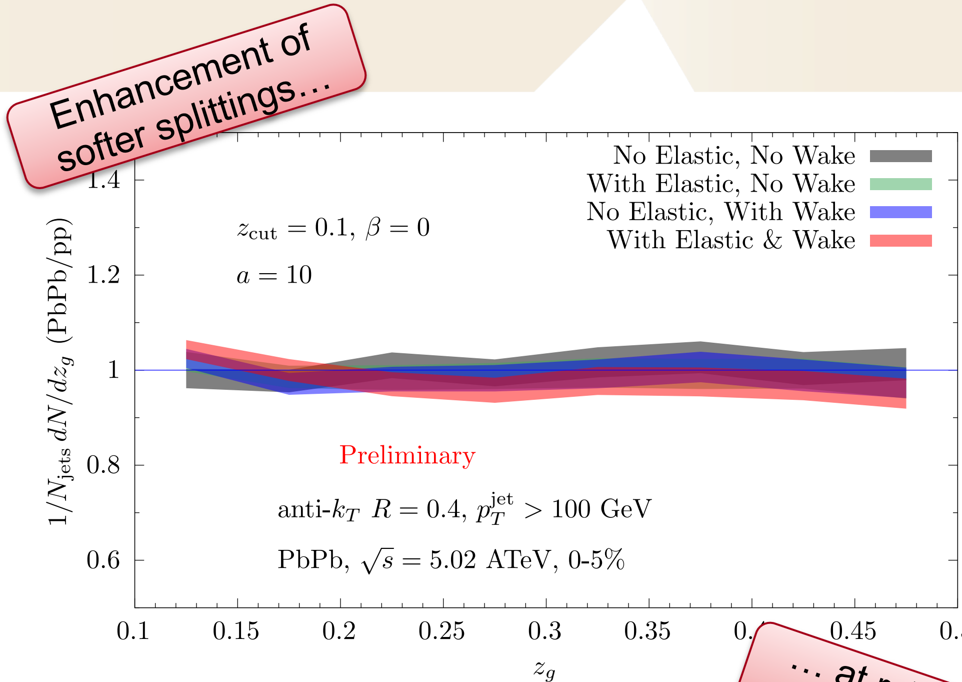
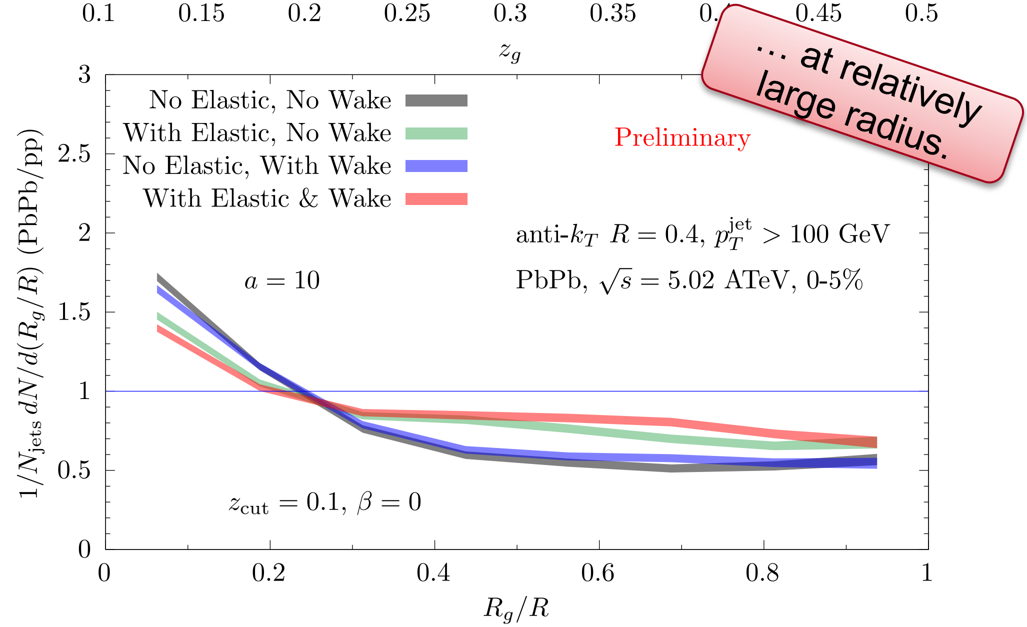




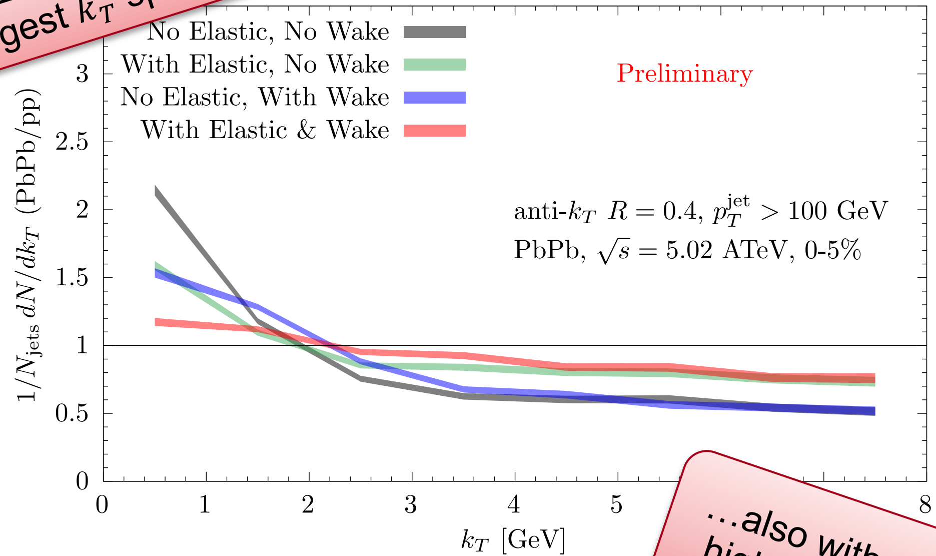
Leading k_T

1. Reconstruct jet with anti-k_T
2. Recluster with Cambridge-Aachen
3. Undo last step of 2, resulting in subjets 1 and 2
4. Note k_T of splitting
5. Follow primary branch until the end.
6. Record largest k_T

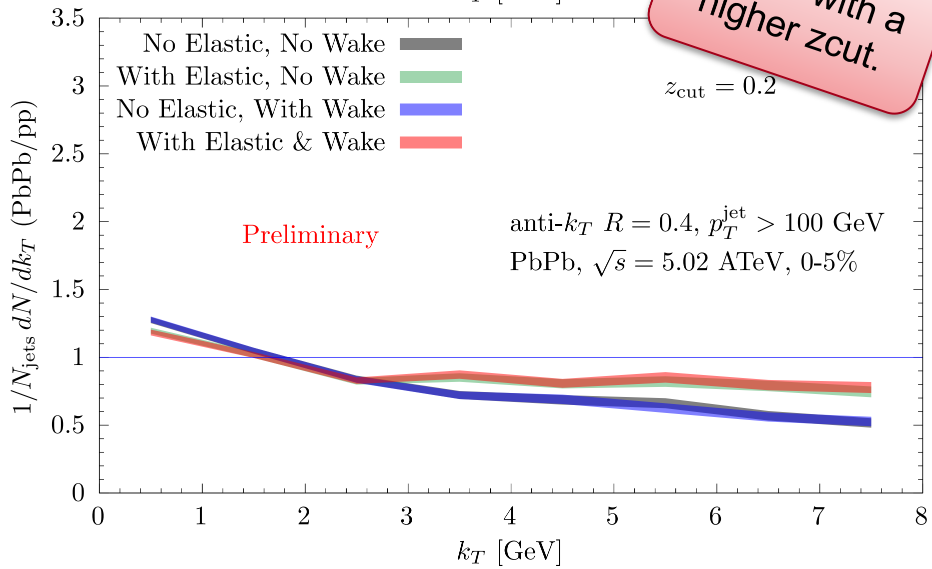
$$k_T = \min(p_{T1}, p_{T2})\sin(R_g)$$

Similar message also for this groomed observable: Moliere scattering effects show up; much larger than wake effects.

Enhancement of largest k_T splittings…



…also with a higher zcut.



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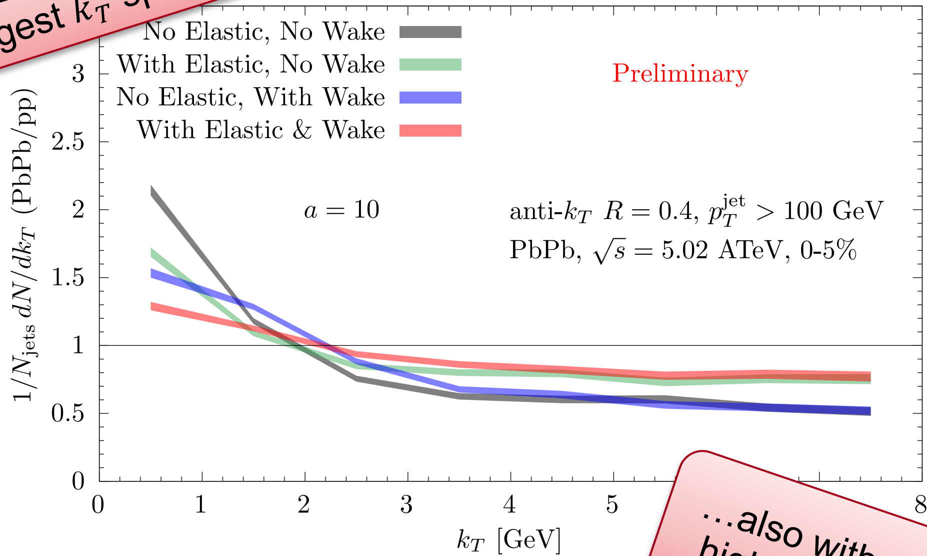

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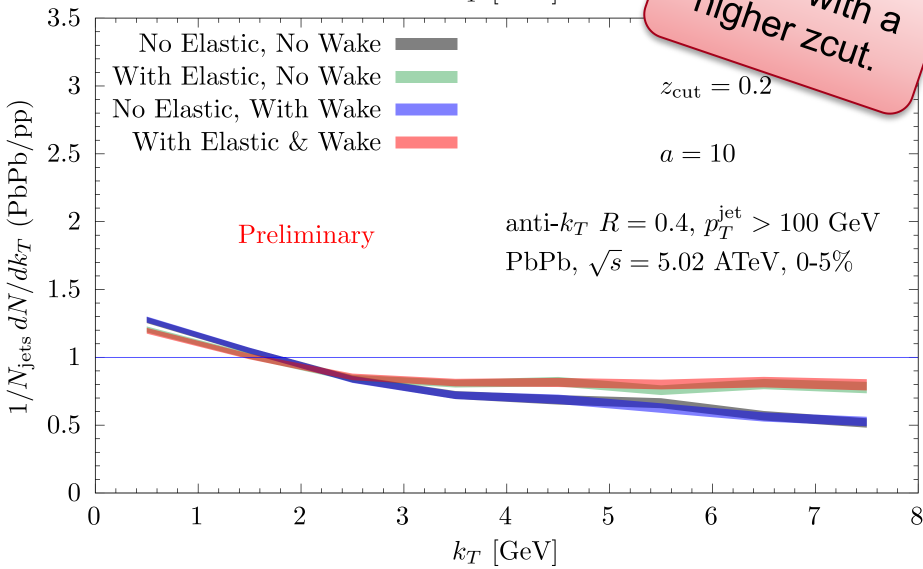
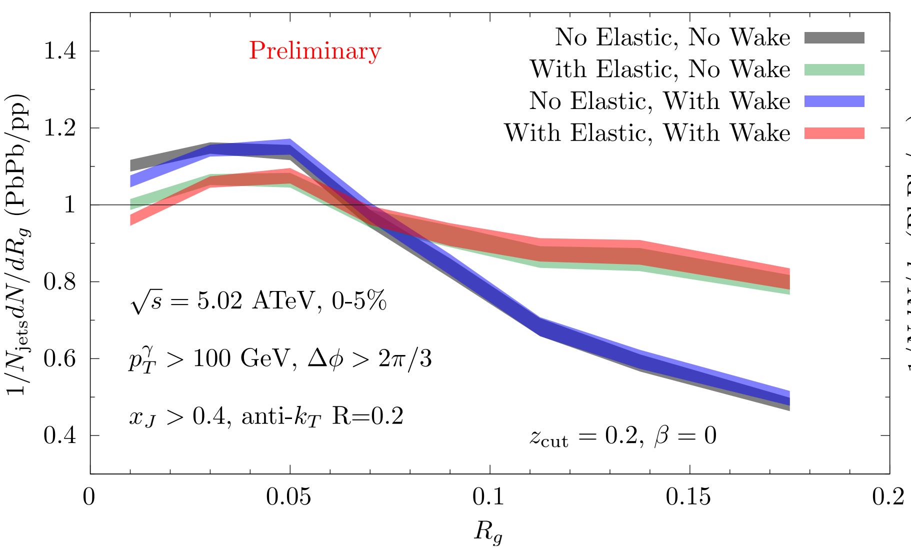
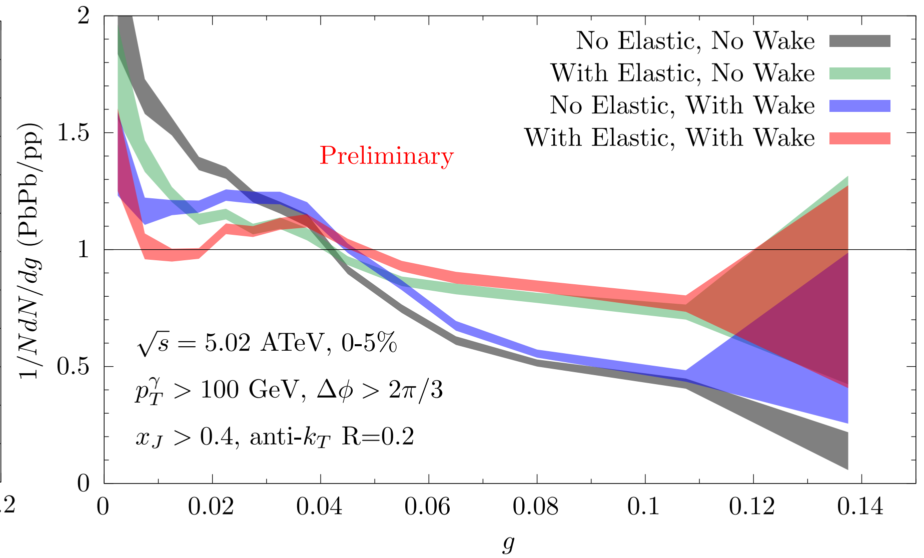
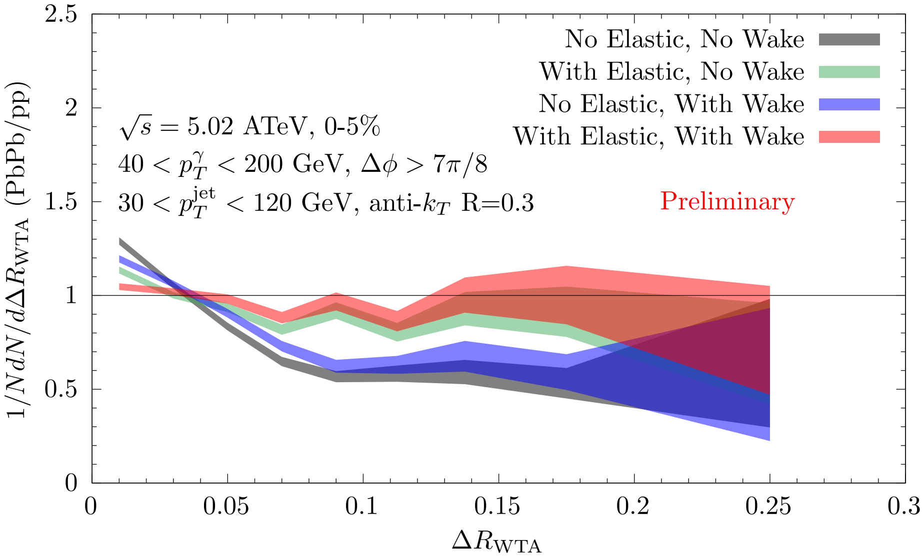


Three "groomed" gamma-Jet Observables: R_g, Girth, and angle between standard and WTA axes

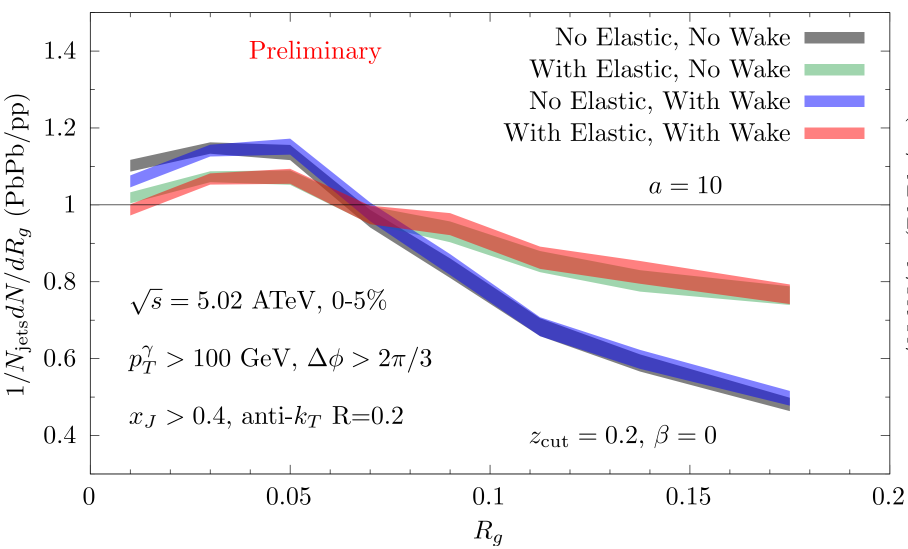


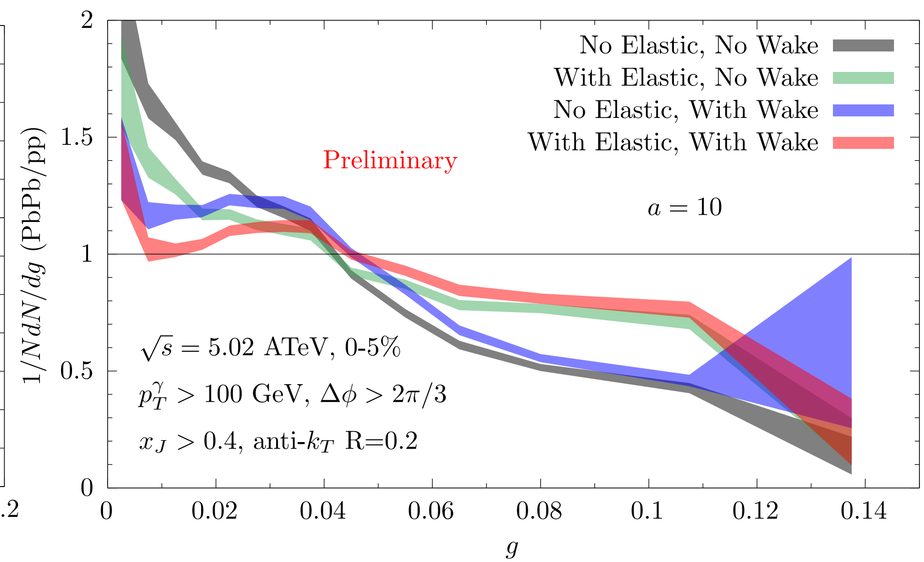


All show much less sensitivity to wake: R=0.2; Moliere scattering shows up; effects of Moliere and wake are again similar in shape, but here effects of Moliere are very much dominant.



Three "groomed" gamma-Jet Observables: R_g , Girth, and angle between standard and WTA axes





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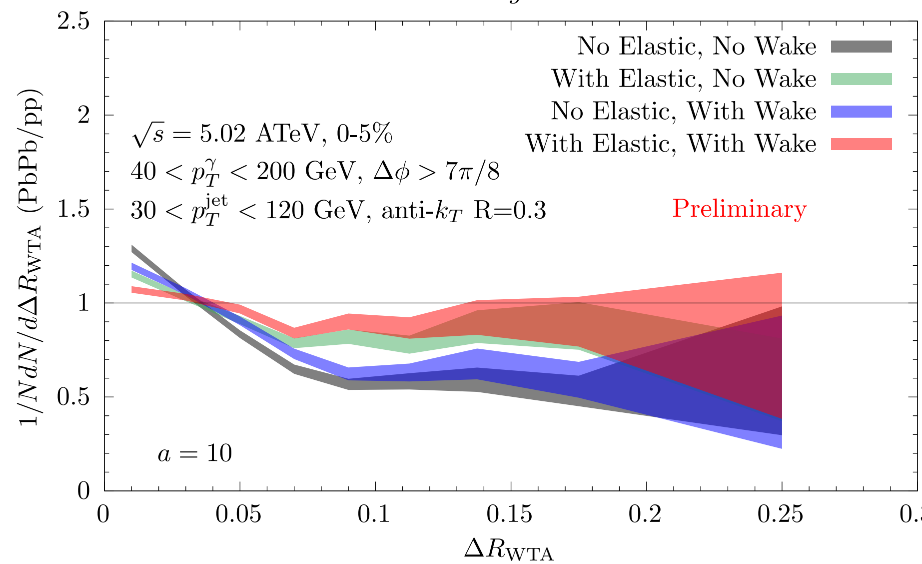

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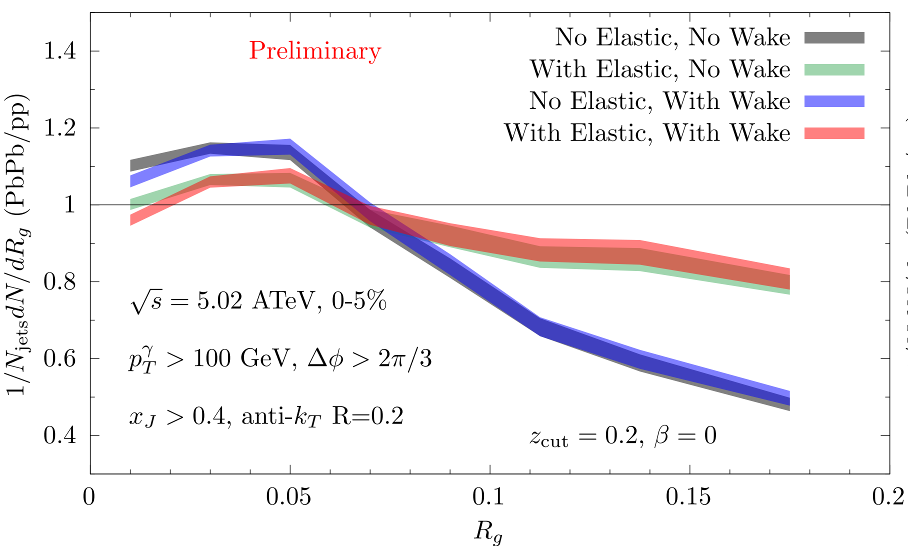

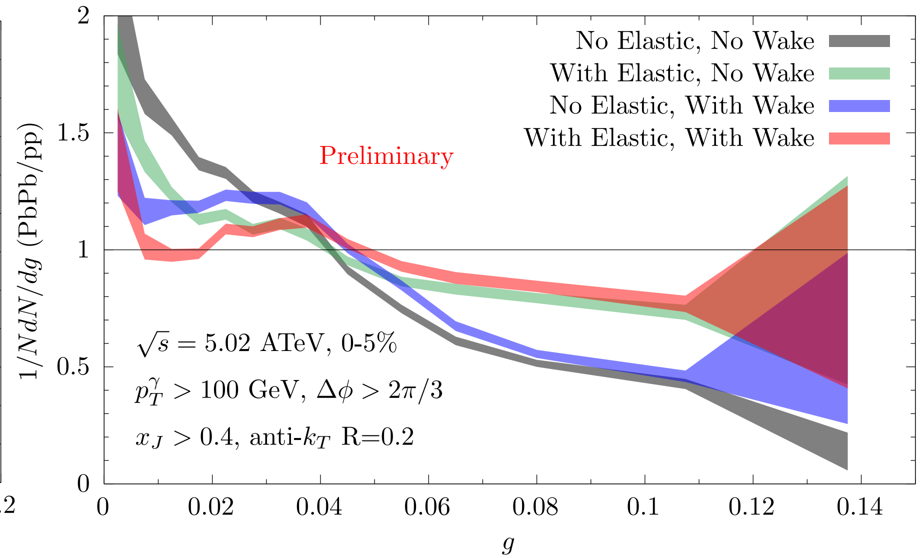

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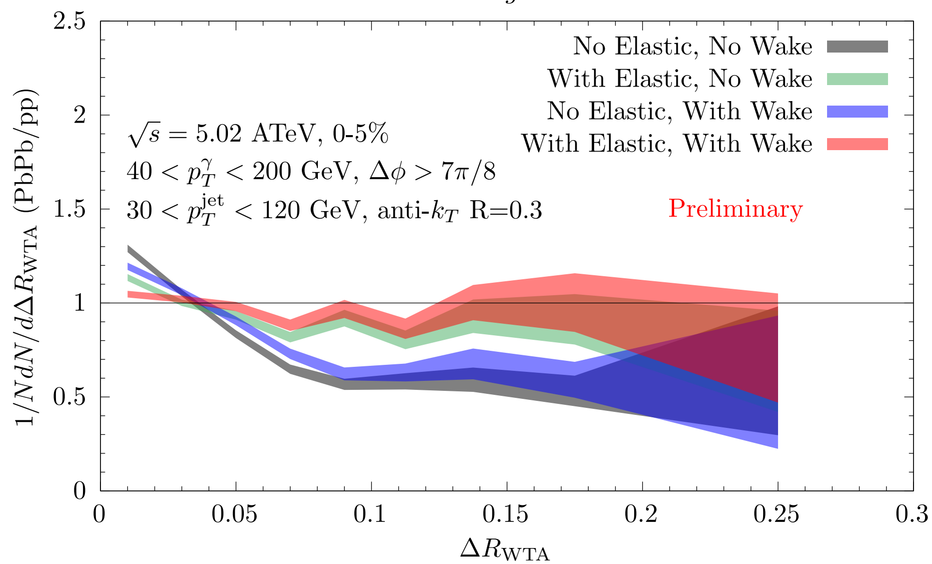
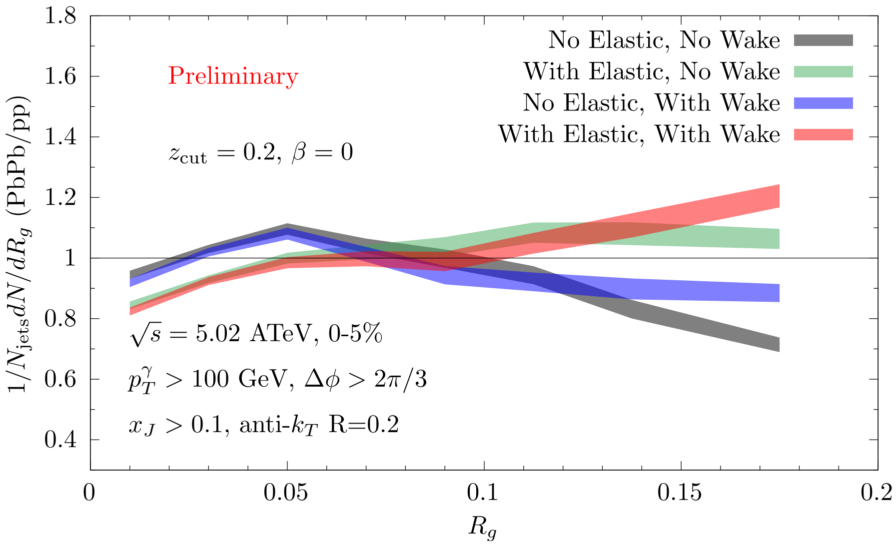
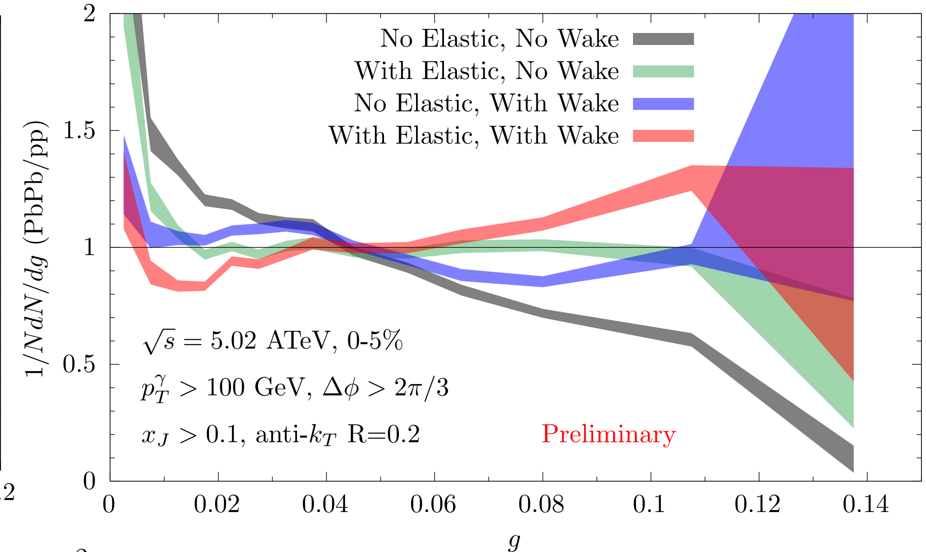


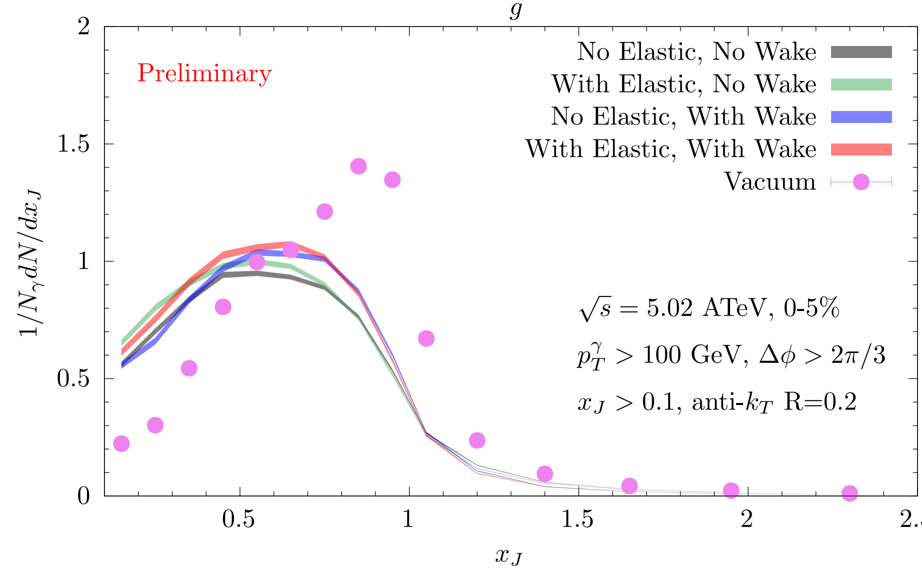
Gamma-Jet Observables: R_g and Girth, with xJ>0.1





On previous slides, Rg and Girth with xJ>0.4: missing the most modified jets. Here, xJ>0.1. Moliere scattering important. Some effects of wake.

Selection bias reduced (cf Brewer+Brodsky+KR); some effects of wake visible.



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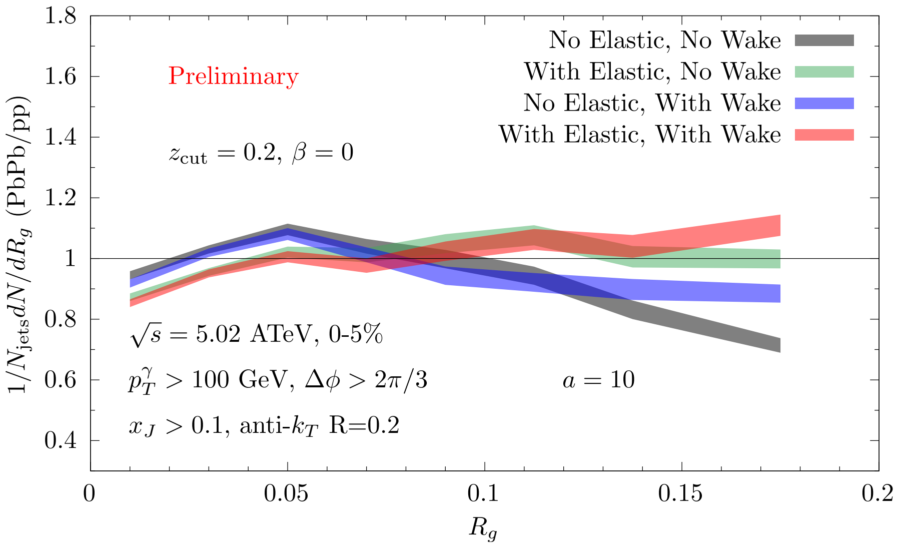

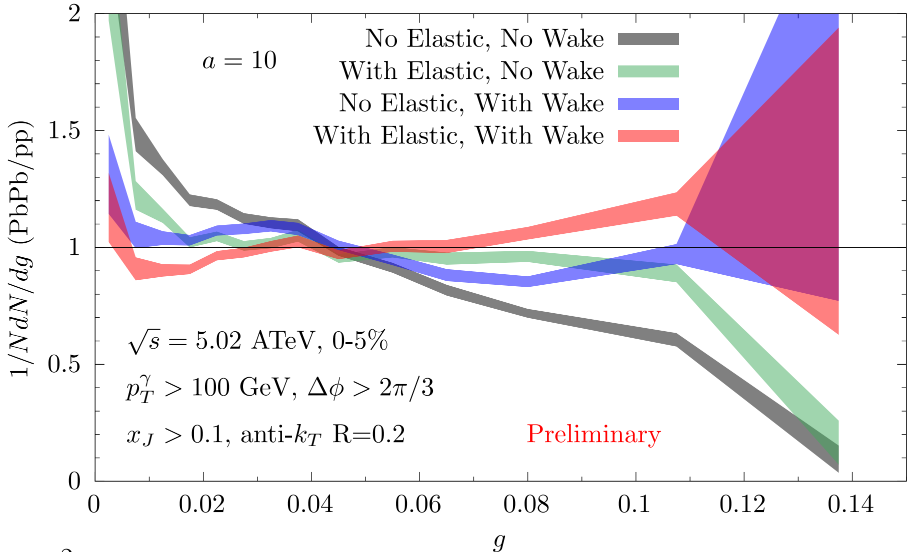

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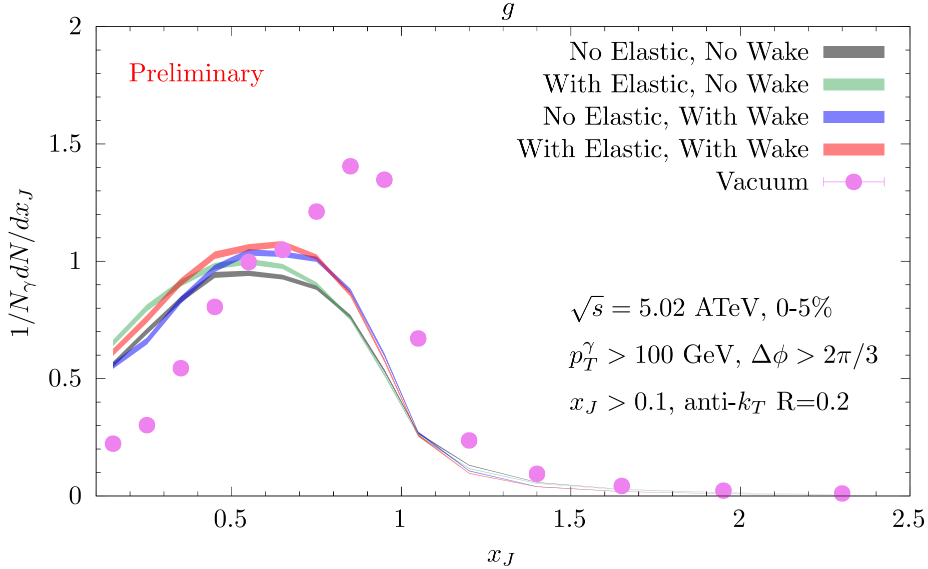
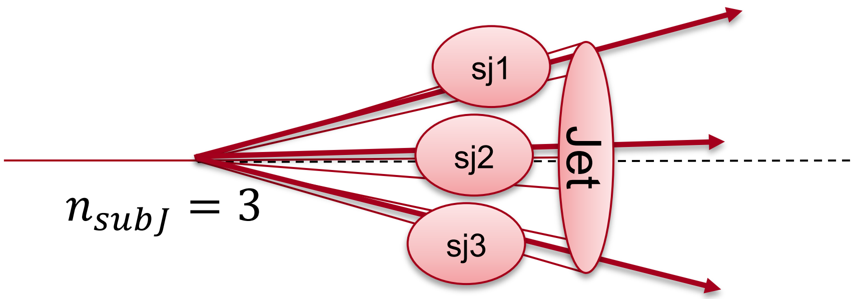
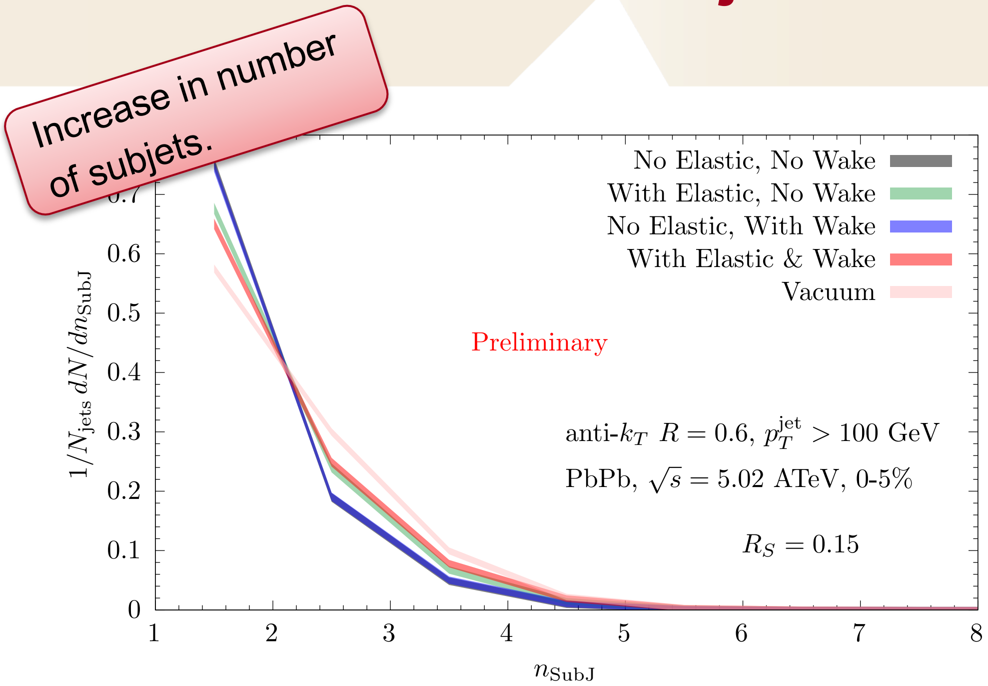


Inclusive Jets within Inclusive Jets: Inclusive Subjets

1. Reconstruct jet with R=0.6
2. Recluster each jet's particle content into subjets with R=0.15





Moliere scattering visible as increase in number of subjets; no such effect coming from wake at all.

Moliere scattering also yields more separated subjets…

These observables are directly sensitive to "sprouting a new subjet" the intrinsic feature of Moliere scattering which makes it NOT just a bit more wake.

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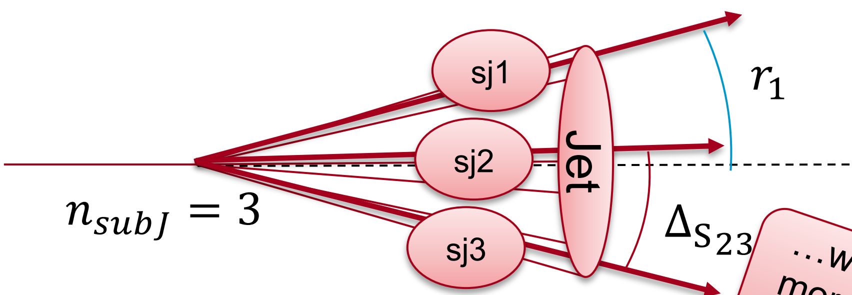
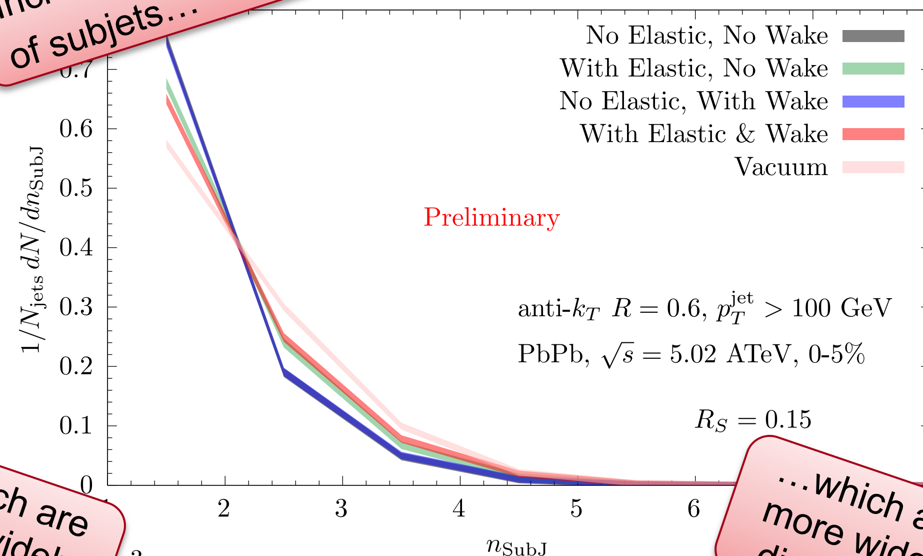
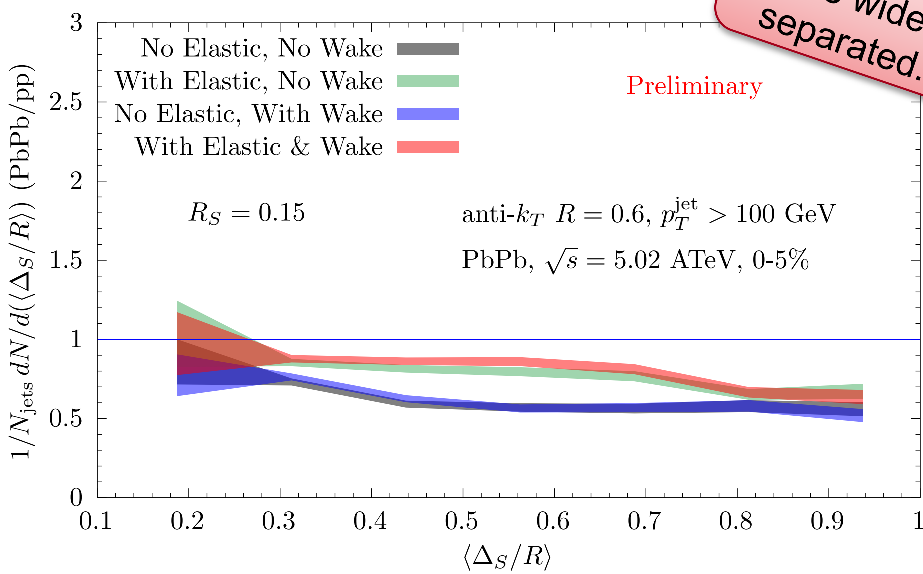


Increase in number of subjets…



…which are more widely separated.



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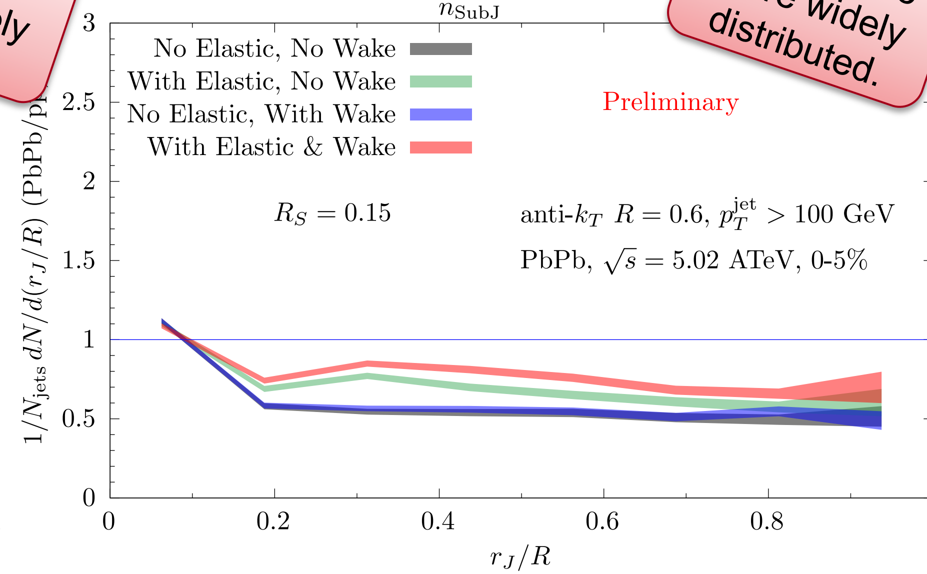
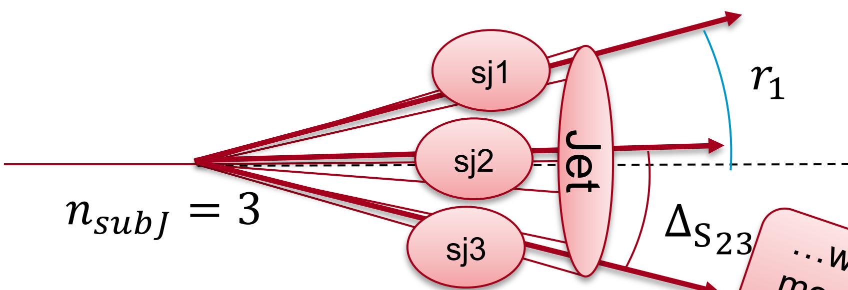
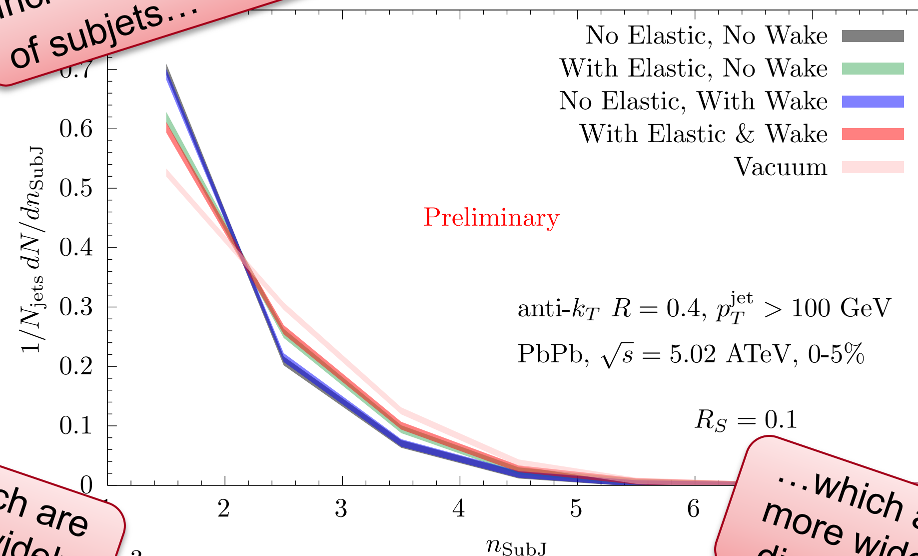


Inclusive Subjets

1. Reconstruct jet with R=0.4
2. Recluster each jet's particle content into subjets with R=0.1



Increase in number of subjets…



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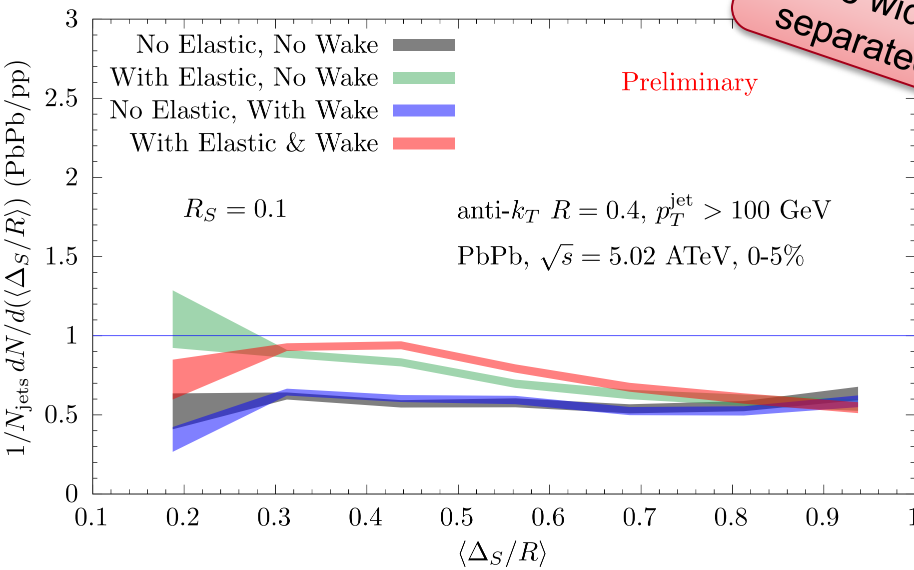

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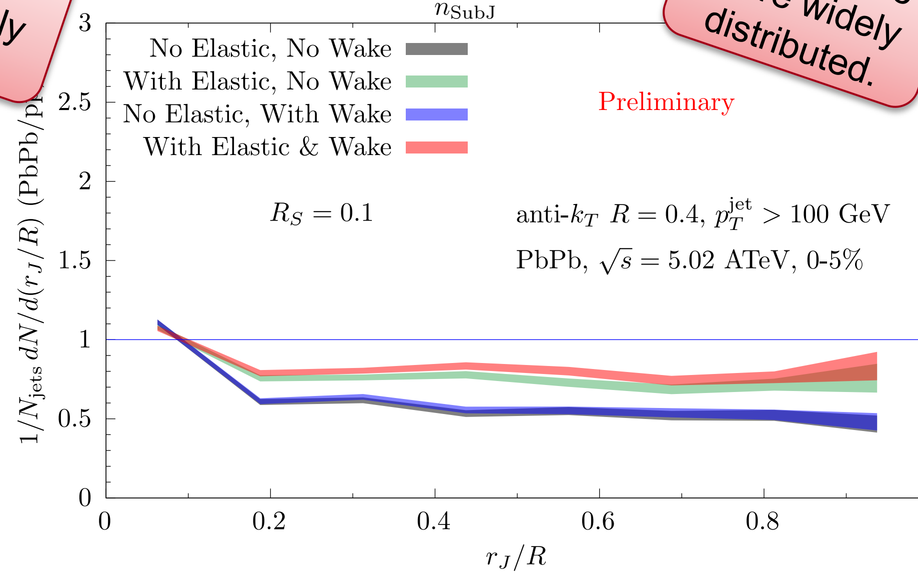

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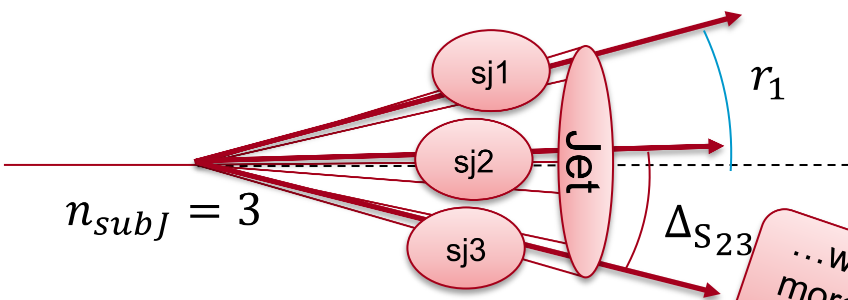
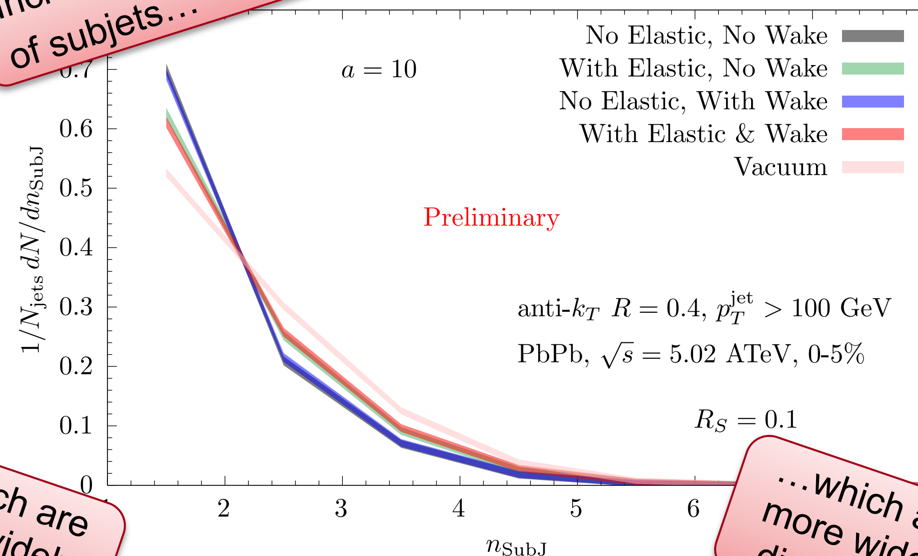


Increase in number of subjets…



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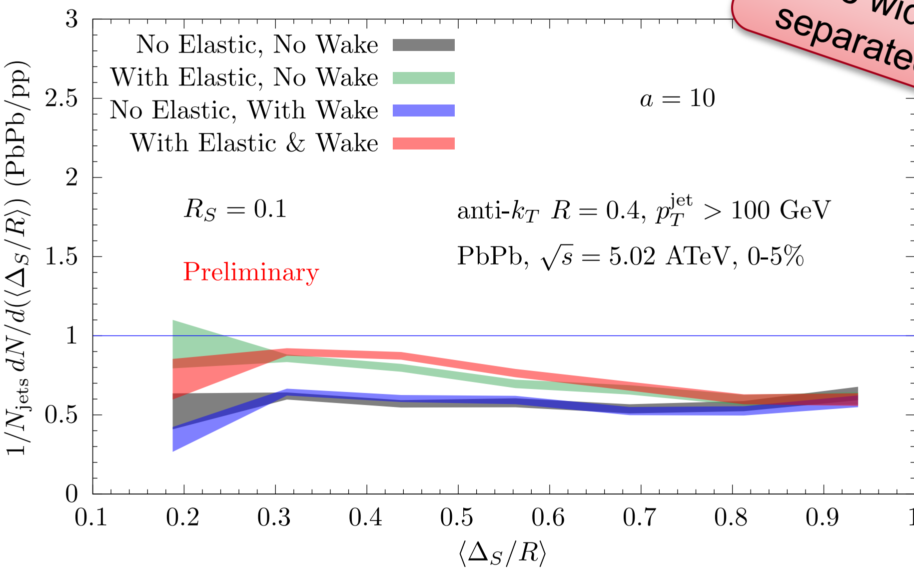

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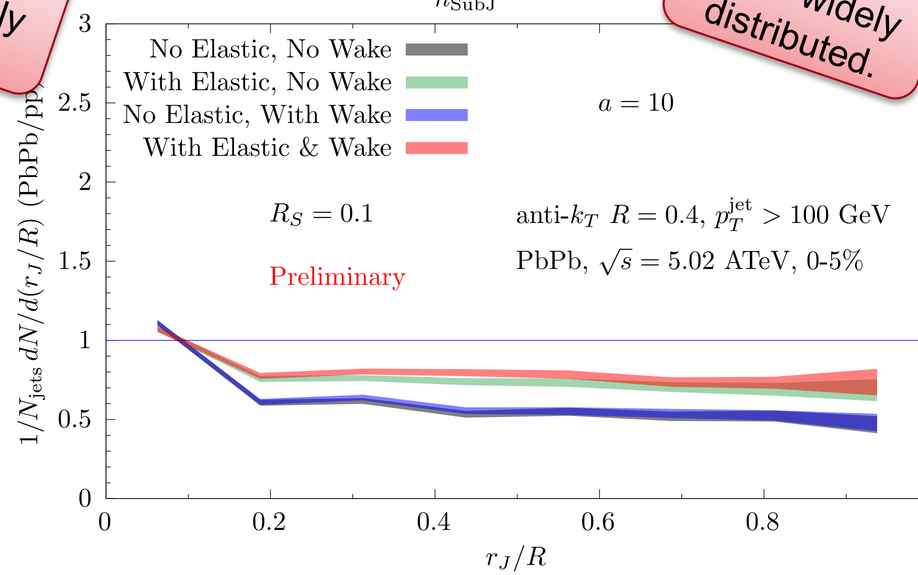
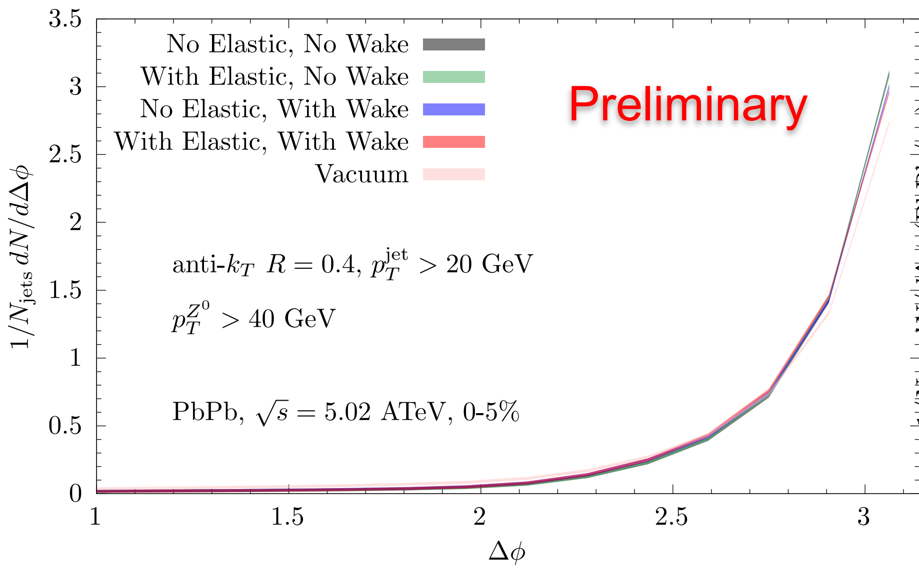
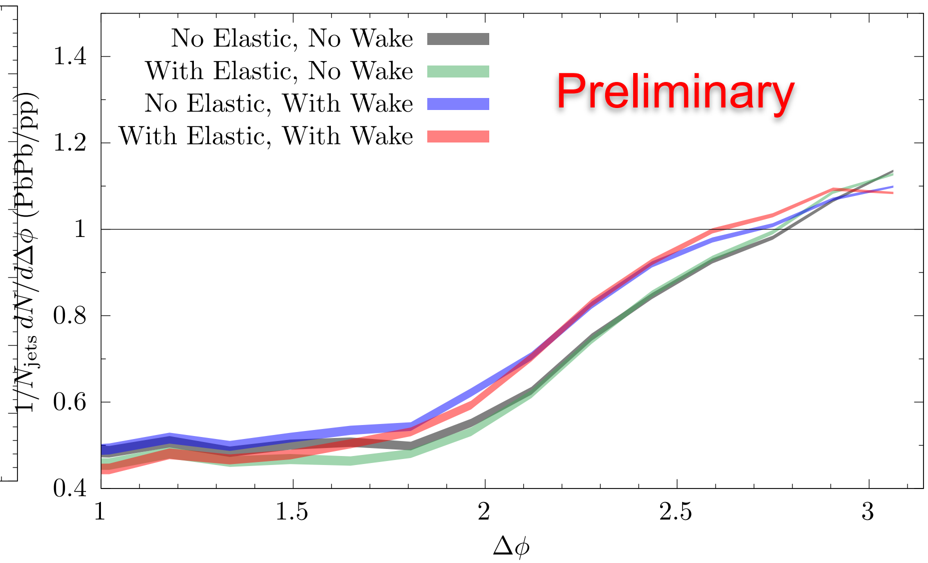


Z-Jet Acoplanarity





- Study acoplanarity in boson-jet system: Z-jet.
- Very little effect from Moliere scattering; increase in acoplanarity that we see is almost entirely due to the wake.
- Similar conclusions for acoplanarities at even lower p_T , via hadron—charged-jet correlations. Should look also Gamma-D, $D\overline{D}$ correlations….
- Groomed z_g and R_g , leading kT, and in particular inclusive subjet observables all more sensitive to Moliere scattering.
- Moliere scattering: jet sprouts added prongs, not much overall deflection

Conclusions

- Studied the effect of power-law-rare, large-angle, scattering on jet observables in the perturbative regime.
- Moliere scattering affects many “shape observables”. But, for “overall shape observables” (jet shapes; FF) effects are similar to, and smaller than, effects of wake.
- Grooming helps, by grooming away the soft particles from the wake. Effects of Moliere scattering dominate the modification of several groomed observables (Rg, Leading kT, Girth, WTA axis angle. Inclusive jets, and gamma-jets; for the latter, selection biases can be reduced.)
- Not all groomed observables are sensitive to Moliere scattering; cf groomed jet mass.
- Modification of inclusive subjet observables (number, and angular spread, of subjets) are especially sensitive to the presence of Moliere scatterings. These observables are unaffected by the wake. They reflect what it is that makes the effects of scattering different from those of the wake.
- Subjet observables may also be influenced by other ways in which jet shower partons “see” particulate aspects of the QGP. That’s great!
- Acoplanarity observables that we have investigated to date show little sensitivity to Moliere scattering; significant sensitivity to the wake in many cases.
- Future: studying charm observables (gamma-D, $D\overline{D}$, D within jets …)

Jets as Probes of QGP

- Theorists taking key steps. . .
- Disentangling jet modification from jet selection.
- Showing that QGP *can* resolve structure within jet shower.
- Jet wakes in droplets of QGP.
- Selecting those jet substructure observables that *are* sensitive to scattering of jet partons off QGP partons, and are *not* sensitive to particles coming from the wake: 2208.13593 and in progress, Hulcher, Pablos, KR.
  - Builds upon theoretical framework for computing Molière scattering in QGP, and finding point-like scatterers in a liquid developed in: 1808.03250 D'Eramo, KR, Yin
- Next several years will be the golden age of HIC jet physics: sPHENIX, LHC runs 3 and 4, new substructure observables. *Many* theory advances, here and elsewhere, whet our appetite for the feast to come. We shall learn about the microscopic structure of QGP, and the dynamics of rippling QGP.

Probing the Original Liquid

The question How does the strongly coupled liquid emerge from an asymptotically free gauge theory? can be thought of in three different ways, corresponding to three meanings of the word "emerge": as a function of resolution, time, or size.

- How does the liquid emerge as a function of resolution scale? What is the microscopic structure of the liquid? Since QCD is asymptotically free, we know that when looked at with sufficient resolution QGP must be weakly coupled quarks and gluons. How does a liquid emerge when you coarsen your resolution length scale to $\sim 1/T$?
- Physics at $t = 0$ in an ultrarelativistic heavy ion collision is weakly coupled. How does strongly coupled liquid form? How does it hydrodynamize?
- How does the liquid emerge as a function of increasing system size? What is the smallest possible droplet of the liquid?

Each, in a different way, requires stressing or probing the QGP. Each can tell us about its inner workings.

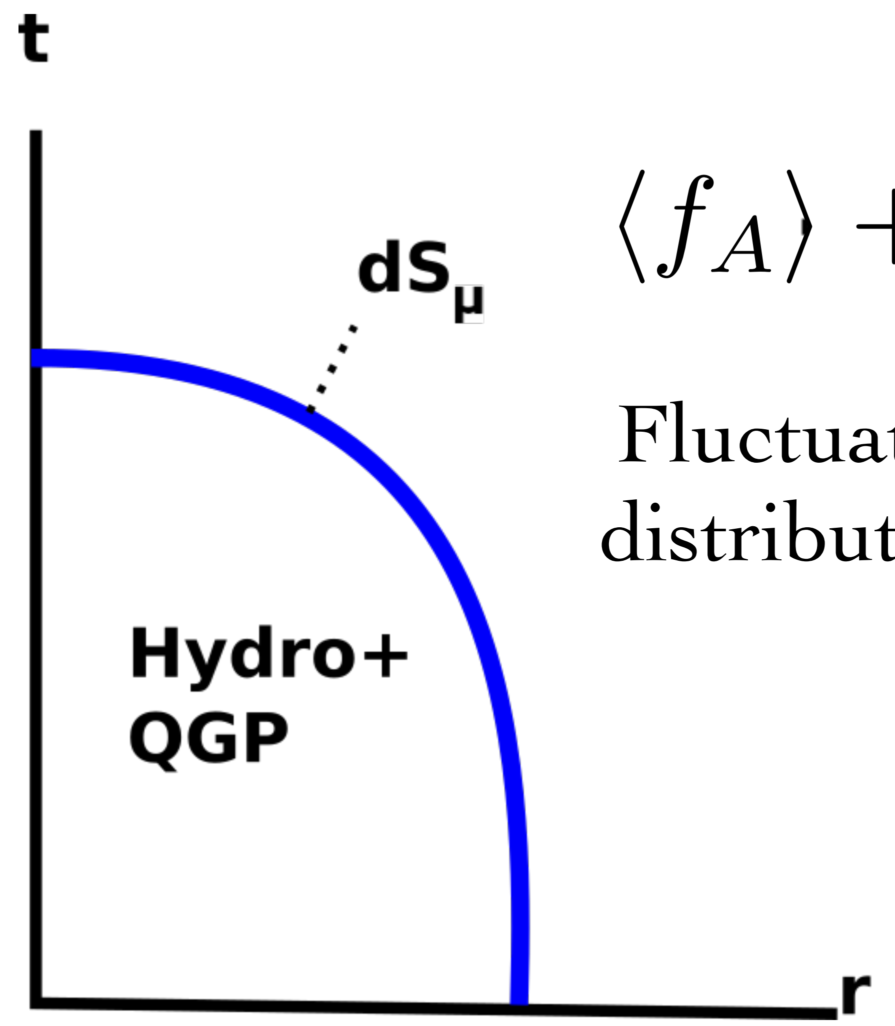
What Next?

Two kinds of What Next? questions for the coming decade...

- A question that one asks after the discovery of any new form of complex matter: What is its phase diagram? For high temperature superconductors, for example, phase diagram as a function of temperature and doping. Same here! For us, doping means excess of quarks over antiquarks, rather than an excess of holes over electrons.
- A question that we are privileged to have a chance to address, after the discovery of "our" new form of complex matter: How does the strongly coupled liquid emerge from an asymptotically free gauge theory? Maybe answering this question could help to understand how strongly coupled matter emerges in other contexts.

Ansatz for a fluctuating particle distribution function near the critical point

We incorporate the effects of critical fluctuations via the modification of particle masses due to their interaction with the critical sigma field



$\langle f_A \rangle + \delta f_A$

Fluctuating particle distribution function

$\delta m_A \approx g_A \sigma$ **Stephanov, Rajagopal, Shuryak, 1999**

Fluctuating particle distribution function

$$f_A = \langle f_A \rangle + g_A \frac{\partial \langle f_A \rangle}{\partial m_A} \sigma$$

$$\langle \sigma \rangle = 0,\ \langle \sigma(x_+) \sigma(x_-) \rangle = Z^{-1} \langle \delta \hat{s}(x_+) \delta \hat{s}(x_-) \rangle$$

MP, Rajagopal, Stephanov, Yin, 22

For more details about the EFT, refer to previous talk by J.M.Karthein

Freeze-out of Gaussian fluctuations near the critical point

$$\Delta G_{AB} \equiv \langle \delta f_A \delta f_B \rangle = \frac{g_A g_B}{Z T^2} \frac{m_A}{E_A} \frac{m_B}{E_B} f_A f_B \langle \delta \hat{s} \delta \hat{s} \rangle$$

$$\Delta \langle \delta N_A \delta N_B \rangle_\sigma = d_A \, d_B \int D p_A \int D p_B \int (dS \cdot p_A) \int (dS \cdot p_B) \Delta G_{AB}$$

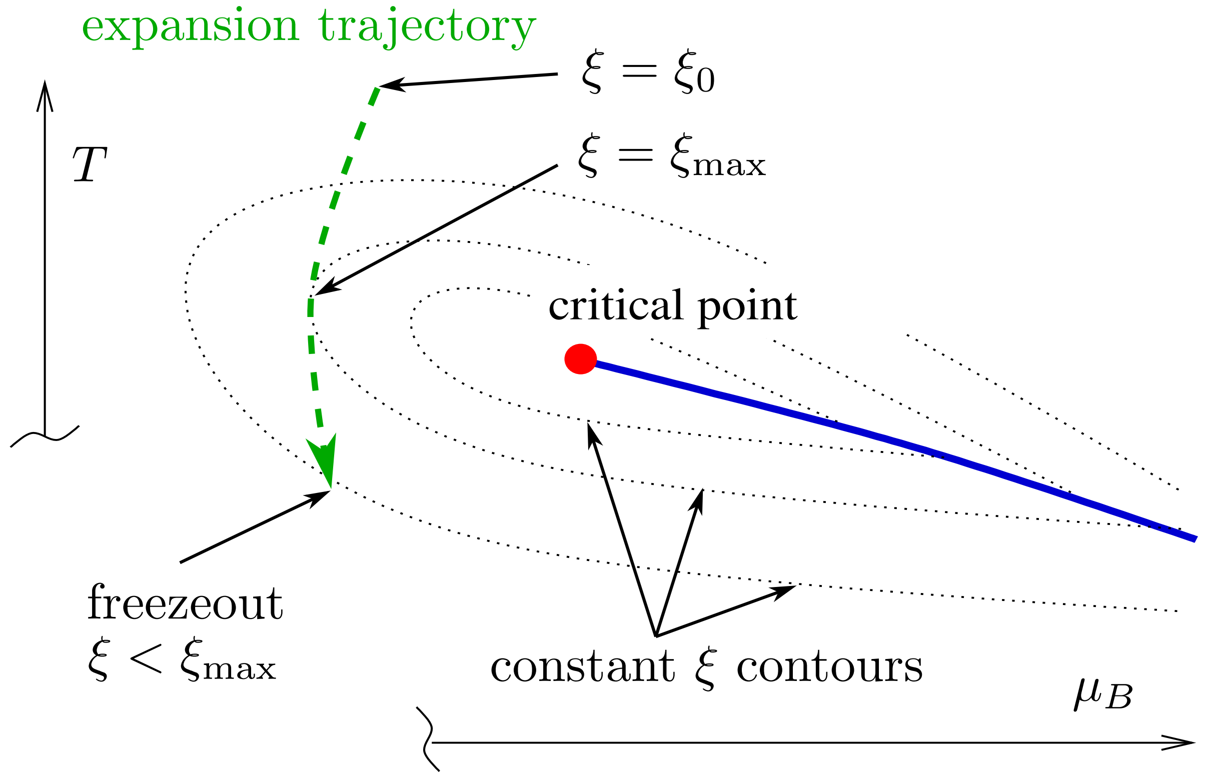
Critical contribution

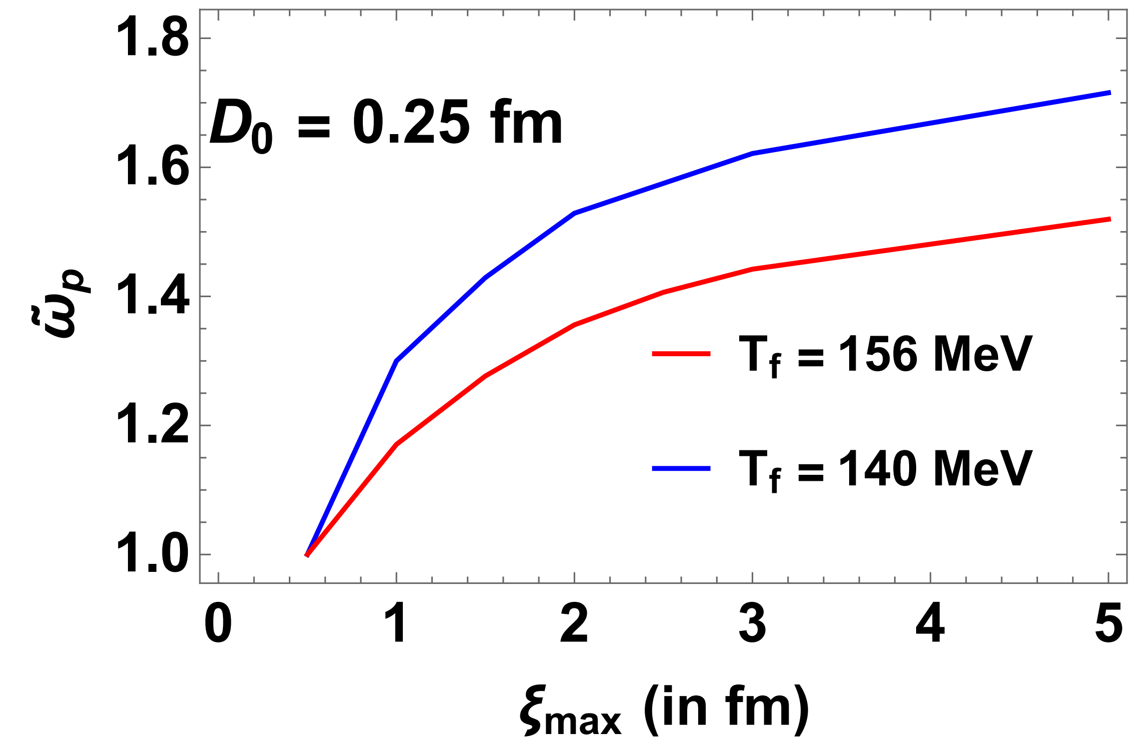
$$\langle \delta N_A \delta N_B \rangle = \langle N_A \rangle \, \delta_{AB} + \Delta \langle \delta N_A \delta N_B \rangle_\sigma$$

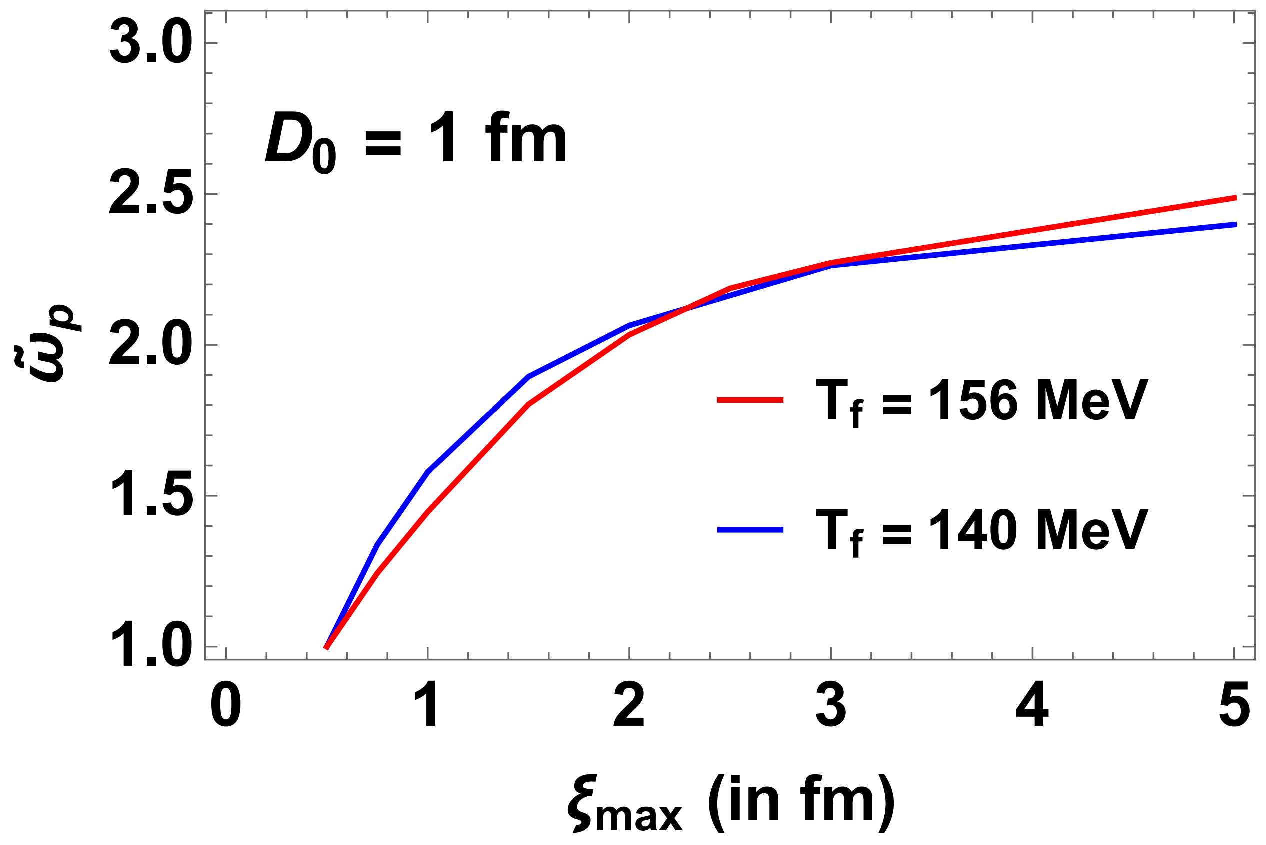
Poisson contribution

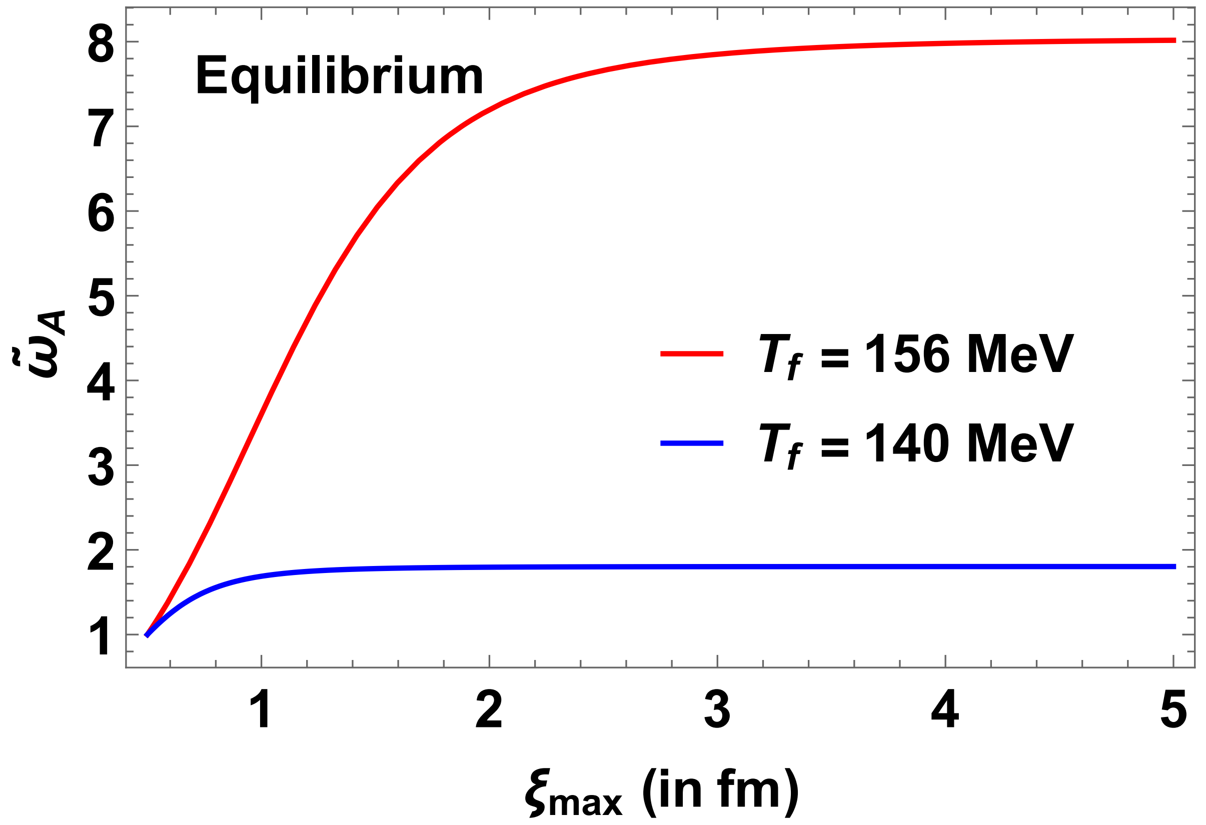
MP, Rajagopal, Stephanov, Yin, 22

Critical contribution to variance of proton multiplicity









$\xi_{\max}$ Proximity of the trajectory to critical point

T_f Proximity of freeze-out point to critical region

MP, Rajagopal, Stephanov, Yin, 22

$T_c = 160$ MeV (T when $\xi = \xi_{max}$)

$$\omega_A(y_{max}) \equiv \frac{\langle \delta N_A^2(y_{max}) \rangle_\sigma}{\langle N_A(y_{max}) \rangle}$$

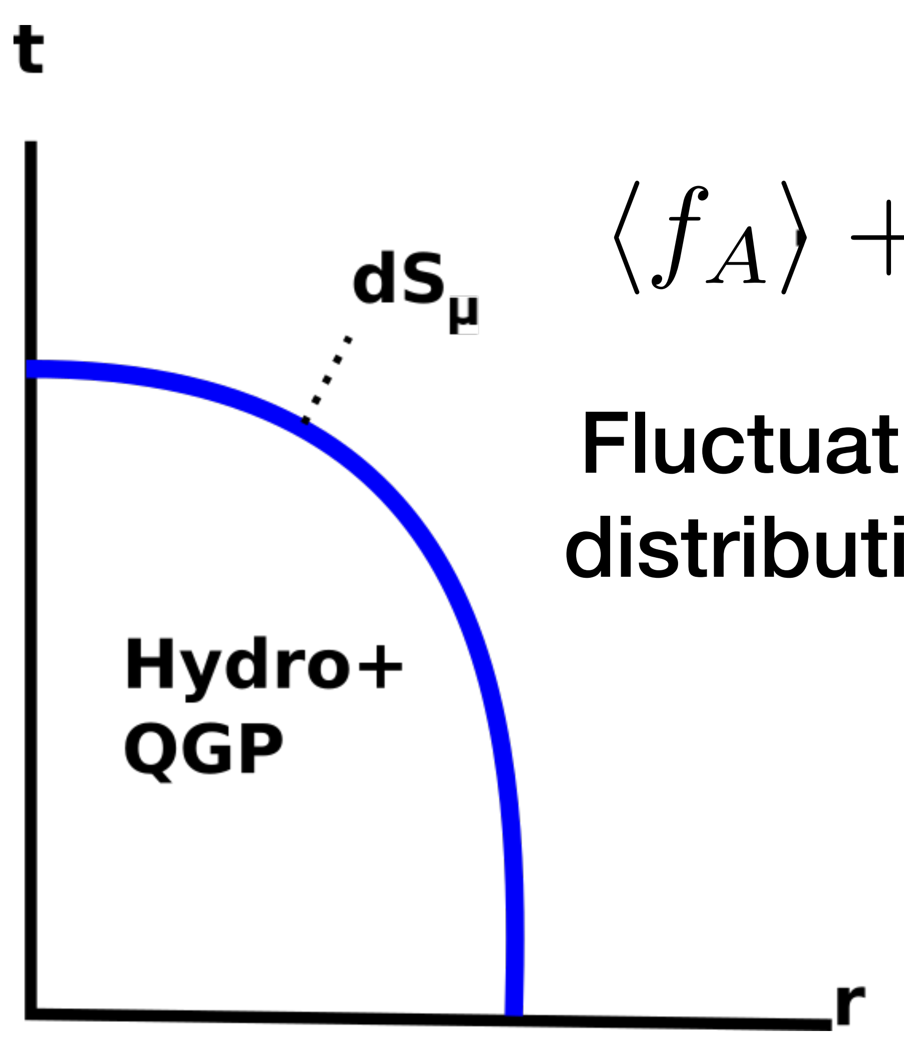
$$\tilde{\omega}_A \equiv \frac{\omega_A}{\omega_A^{nc}}$$

- The fluctuations are **reduced relative to equilibrium** value (conservation laws)
- Fluctuations increase with D_0 (faster diffusion)
- Compared to the equilibrium scenario, the fluctuations are **less sensitive to freeze-out temperature**

We'll now discuss a recently developed approach to freeze-out , called the maximum entropy freeze-out.

- Admits a generalization to non-critical fluctuations and higher-order fluctuations
- Provides crucial information about g_As

Freeze-out in heavy-ion collisions consistent with conservation laws



$\langle f_A \rangle + \delta f_A$

Fluctuating particle distribution function

Mean and higher-point correlation functions of conserved densities must be equated between both the descriptions

More degrees of freedom on the kinetic side

Infinitely many solutions to this set of conservation laws

Maximum entropy approach to freeze-out

MP, Stephanov, 22

- A general method for freeze-out based on the principle of maximum entropy, not relying on universality near critical point

What is the most likely ensemble of free streaming particles after freeze-out consistent with conservation laws?

- We obtain this by *maximizing the entropy* associated with the fluctuations of the particle distribution function f, subject to the *constraints of conservation equations*

Probability distribution functional of f

$$S[\bar{f}, G_2, G_3, G_4, \dots] = \int_f P[f] \log P[f]$$

Similar to n-PI entropy in QFT

Berges, 04, Stephanov, Yin, 17…

How to obtain g_As

Applying *maximum-entropy freeze-out* to a *Hydro+* simulation

where there is only one mode which is singular and out of equilibrium:

$$\Delta G_{AB} = \left(\frac{n_c}{\bar{c}_p T_c}\right)^2 \left[E_A - \frac{w_c}{n_c} q_A\right] \left[E_B - \frac{w_c}{n_c} q_B\right] f_A f_B \Delta \left\langle \delta\hat{s}\delta\hat{s}\right\rangle$$

Agrees with the prescription obtained using the *EFT with sigma field*:

$$\Delta G_{AB} = \frac{g_A g_B}{Z T^2} \frac{m_A}{E_A} \frac{m_B}{E_B} f_A f_B \Delta \left\langle \delta\hat{s}\delta\hat{s}\right\rangle$$

if g_As have a specific energy dependence

Phenomenological implications

Depends on non-critical information from the QCD EoS

Measure of the size of the critical region

$$g_A \equiv \hat{g}_A \frac{\sin\alpha_1}{w\sin(\alpha_1-\alpha_2)}$$

BEST EoS parameters defined in previous talk by J. M. Karthein

$$\hat{g}_A(E_A) \propto \frac{E_A}{m_A}\left(\frac{E_A}{w_c}-\frac{q_A}{n_c}\right)$$

n_c & w_c Baryon density and enthalpy at the critical point

Estimates using BEST EoS

$\mu_c = 350\,\mathrm{MeV}$

$$\hat{g}_{p,\mathbf{0}} \approx -3.1,\ \hat{g}_{\pi,\mathbf{0}} \approx 0.18,\ \hat{g}_{\bar{p},\mathbf{0}} \approx 5.5$$

$$q_p = 1,\ q_\pi = 0$$

Mixed correlations of protons and pions can have negative sign

Freeze-out of higher point fluctuations

Irreducible relative cumulants (IRCs)

Self correlations systematically subtracted for higher point correlations

$$\widehat{\Delta}G_{AB} \equiv \Delta G_{AB}\,;$$

$$\widehat{\Delta}G_{ABC} \equiv \Big[\Delta G_{ABC} - 3\widehat{\Delta}G_{AD}(\bar{G}^{-1}\bar{G}_3)_{DBC}\Big]_{\overline{ABC}}$$

$$\widehat{\Delta}G_{ABCD} \equiv \Big[\Delta G_{ABCD} - 6\widehat{\Delta}G_{ABF}(\bar{G}^{-1}\bar{G}_3)_{FCD} - 4\widehat{\Delta}G_{AF}(\bar{G}^{-1}\bar{G}_4)_{FBCD} - 3\widehat{\Delta}G_{EF}(\bar{G}^{-1}\bar{G}_3)_{EAB}(\bar{G}^{-1}\bar{G}_3)_{FCD}\Big]_{\overline{ABCD}}$$

General freeze-out prescription (linearized)

H denotes hydrodynamic fluctuations

$$P_A = \begin{bmatrix} p^\mu_A \\ q_A \end{bmatrix}$$

$$\widehat{\Delta}G_{AB\ldots} = \widehat{\Delta}H_{ab\ldots}(\bar{H}^{-1}P\bar{G})^a_A(\bar{H}^{-1}P\bar{G})^b_B\ldots,$$

For full non-linear freeze-out prescription, refer MP, Stephanov, 22

For classical gas, IRCs reduce to factorial cumulants.

Summary - Freeze-out

- We have demonstrated the freeze-out of Gaussian fluctuations near a critical point in a semi-realistic scenario.
- A general prescription for freeze-out has been recently developed - Maximum entropy approach
- Previously, unknown parameters crucial for the freeze-out of fluctuations near the QCD critical point in terms of the QCD equation of state
- Numerical implementation of freeze-out of higher-point fluctuations needs to be performed..

Thank you!

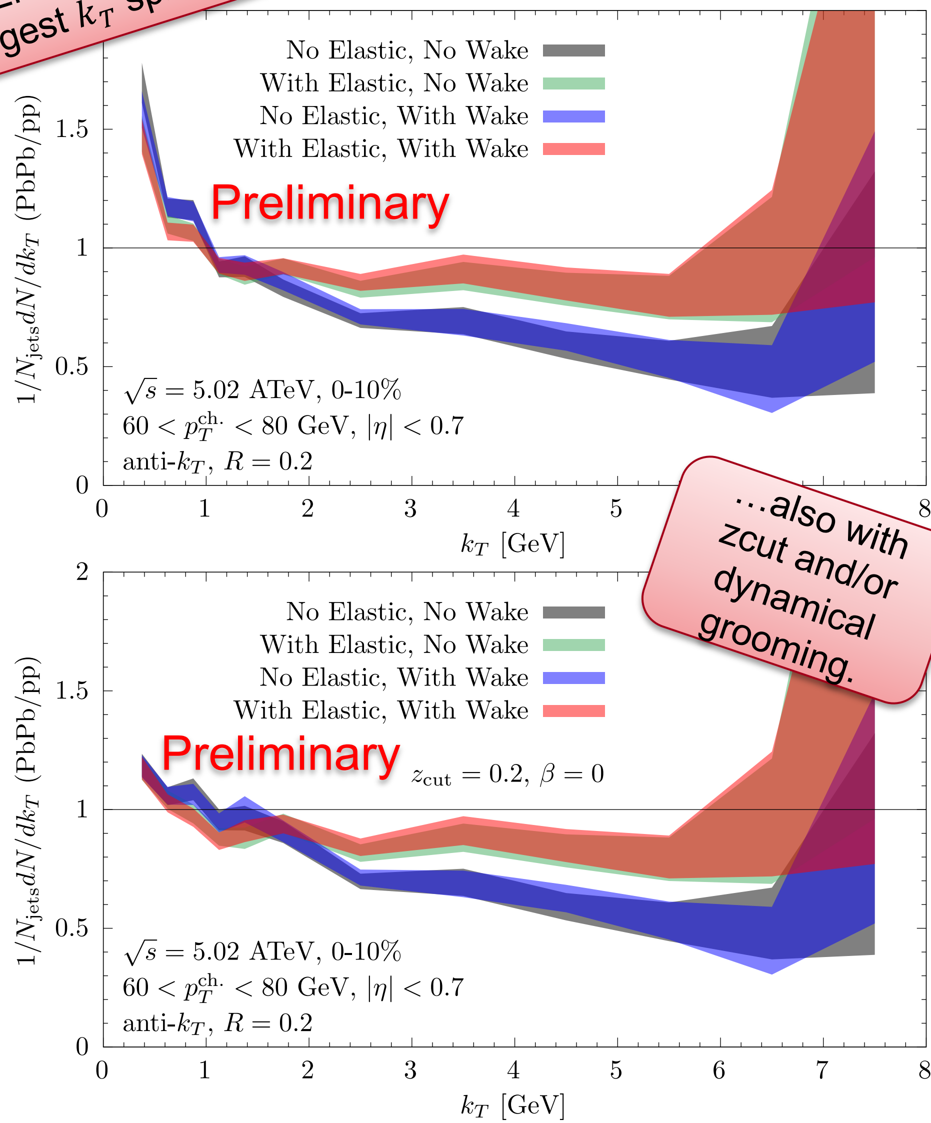
Leading k_T

1. Reconstruct jet with anti-k_T
2. Recluster with Cambridge-Aachen
3. Undo last step of 2, resulting in subjets 1 and 2
4. Note k_T of splitting
5. Follow primary branch until the end.
6. Record largest k_T

$$k_T = \min(p_{T1}, p_{T2})\sin(R_g)$$

Similar message also for this groomed observable: Moliere scattering effects show up; much larger than wake effects.

Enhancement of largest k_T splittings…



…also with zcut and/or dynamical grooming.

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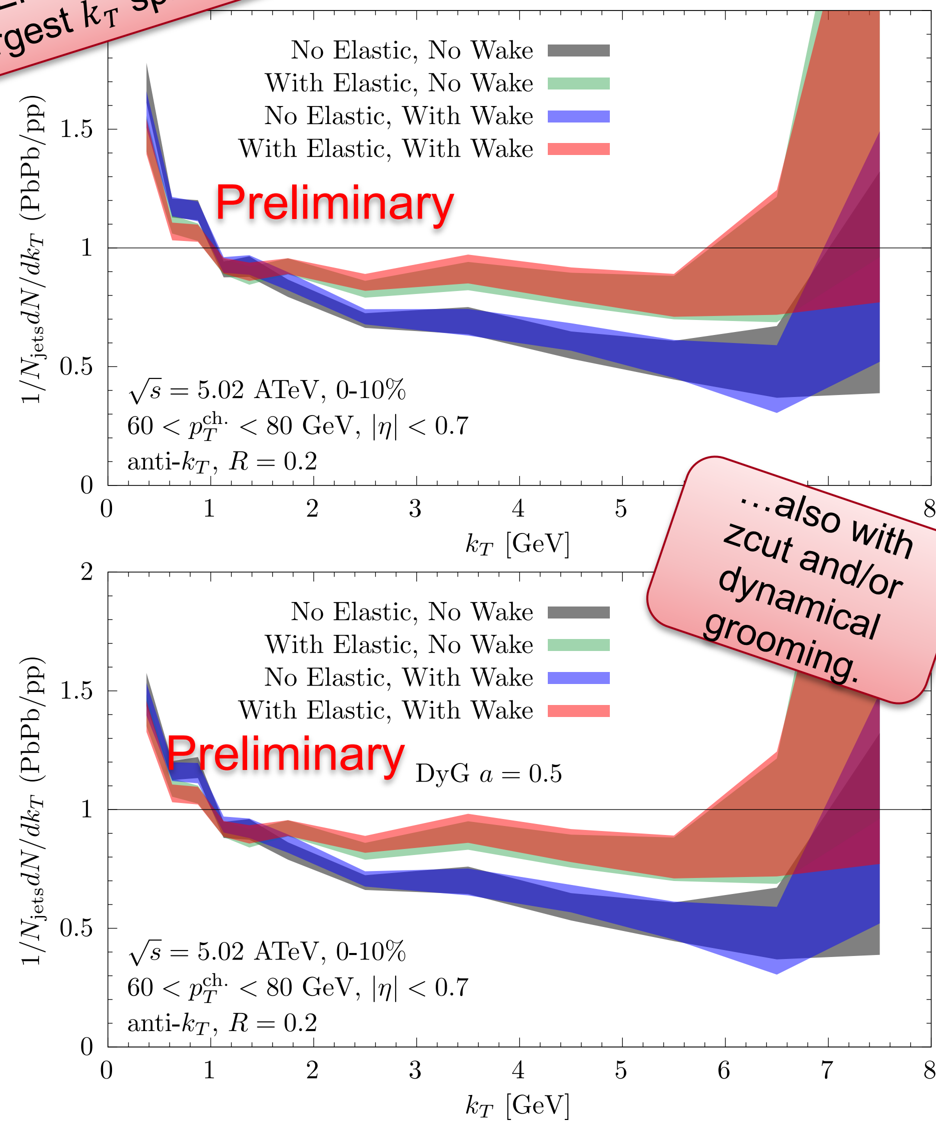

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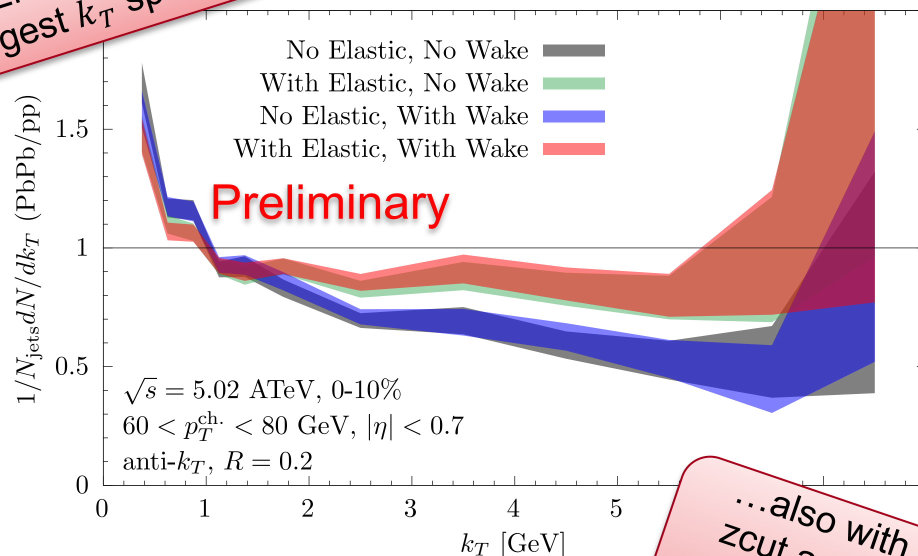

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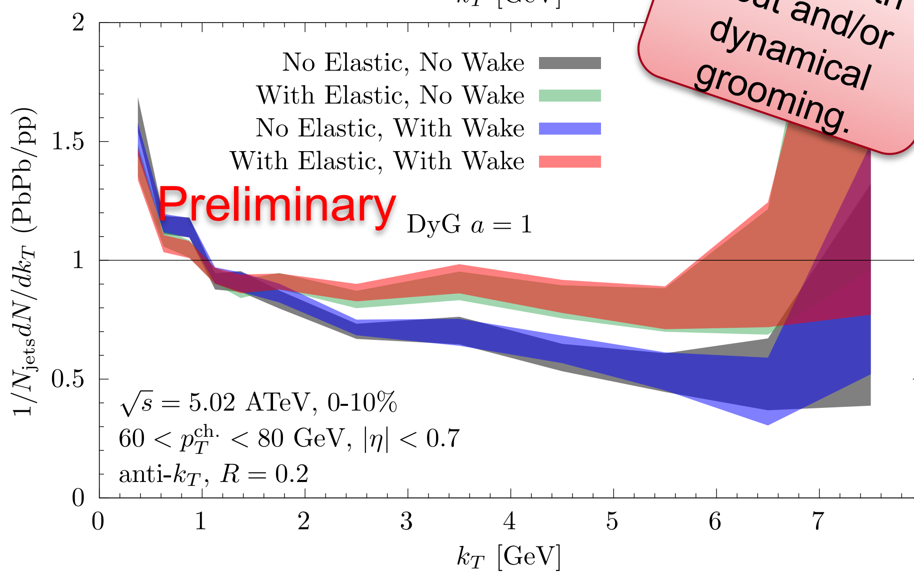

Similar message also for this groomed observable: Moliere scattering effects show up; much larger than wake effects.

Leading k_T

1. Reconstruct jet with anti-k_T
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3. Undo last step of 2, resulting in subjets 1 and 2
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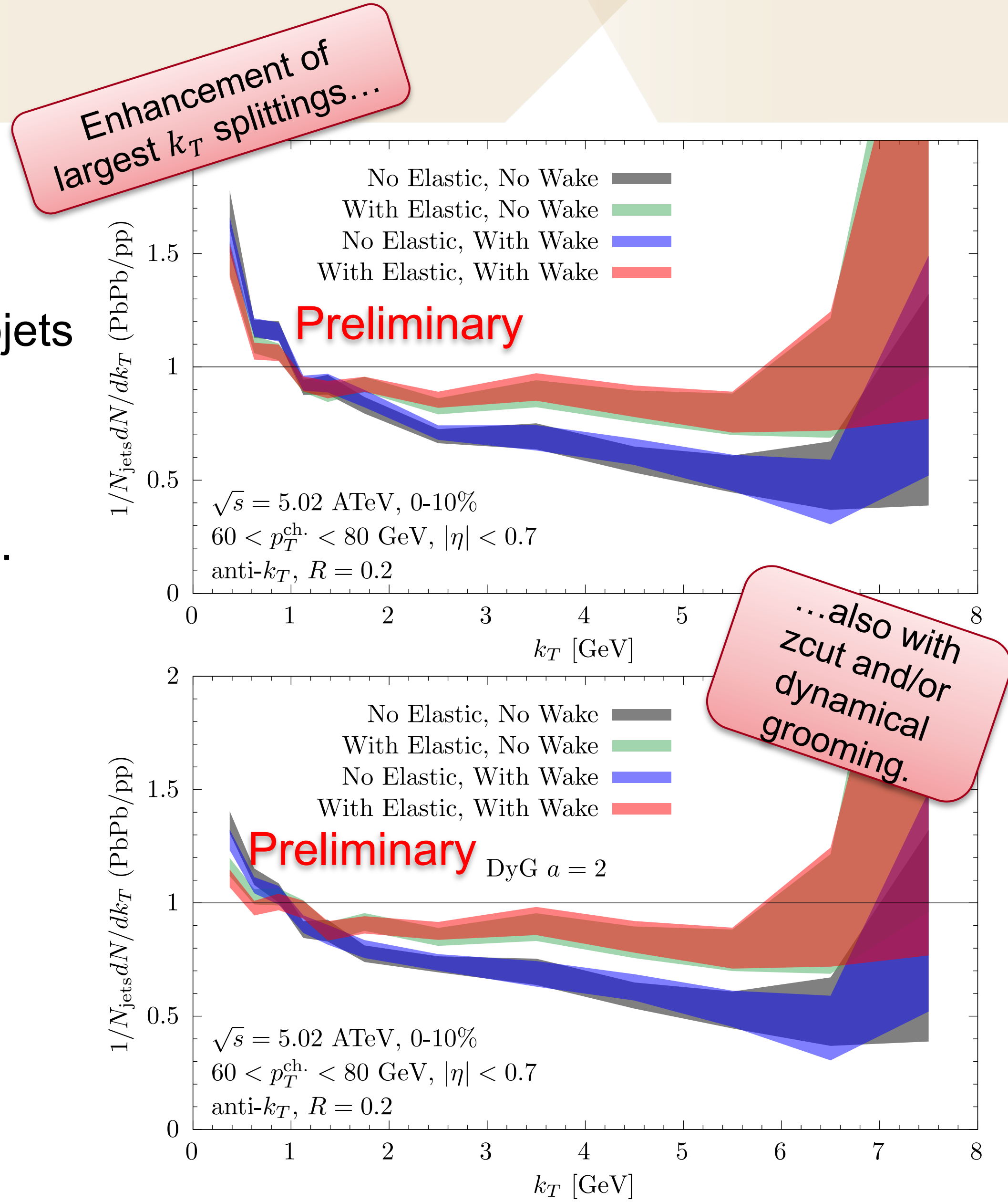

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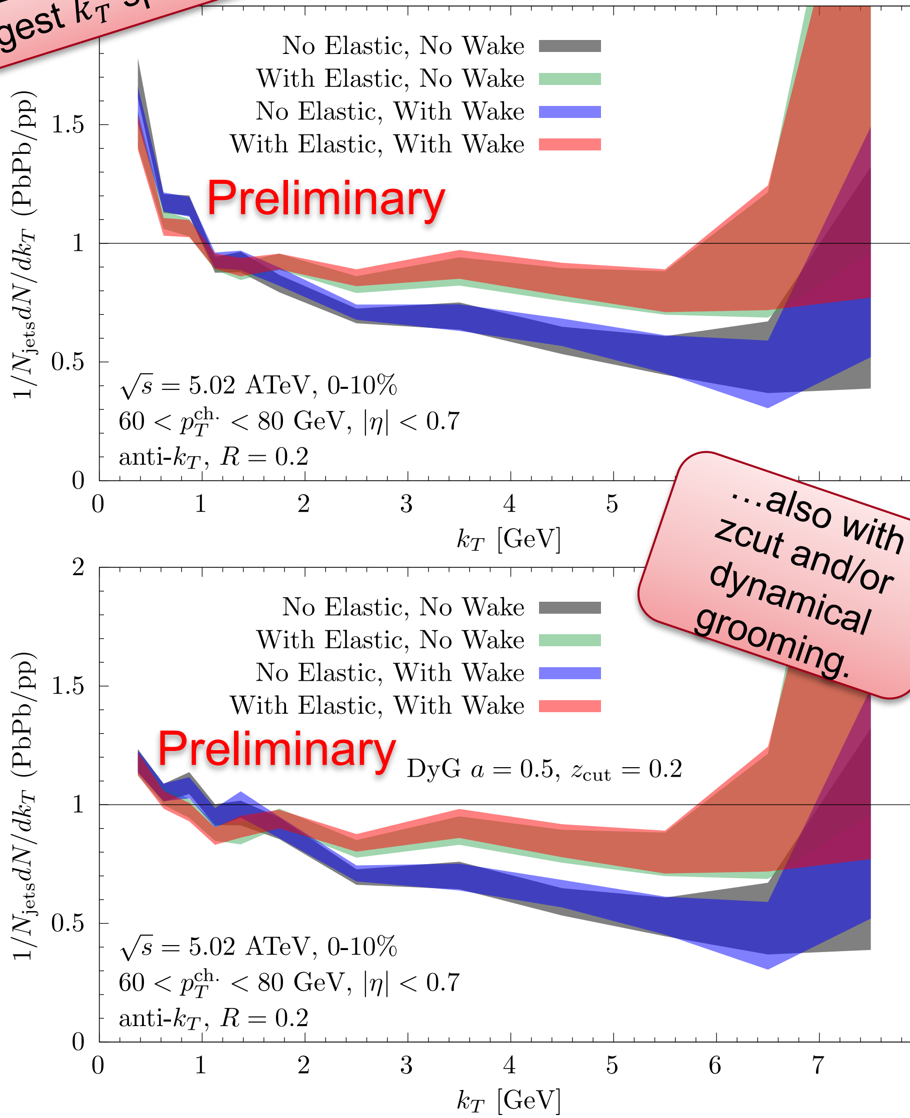

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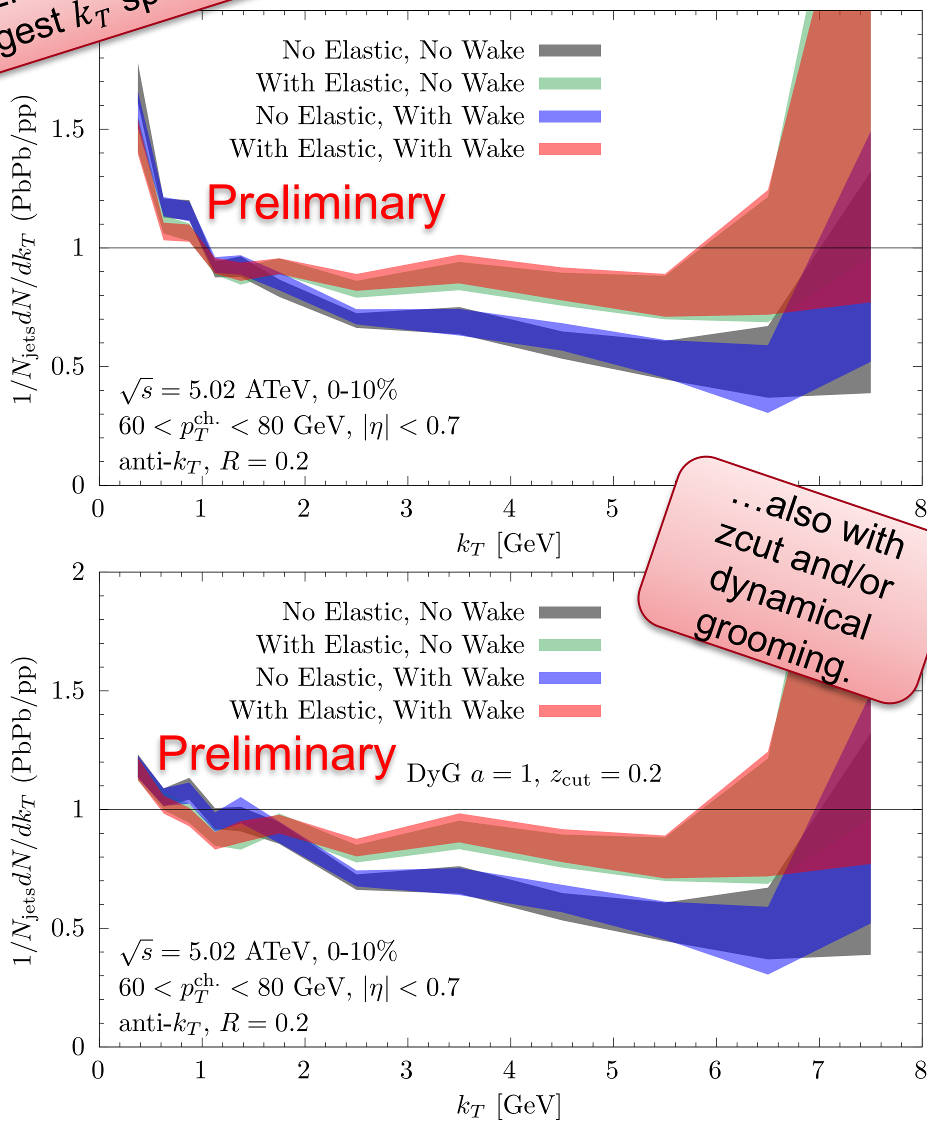

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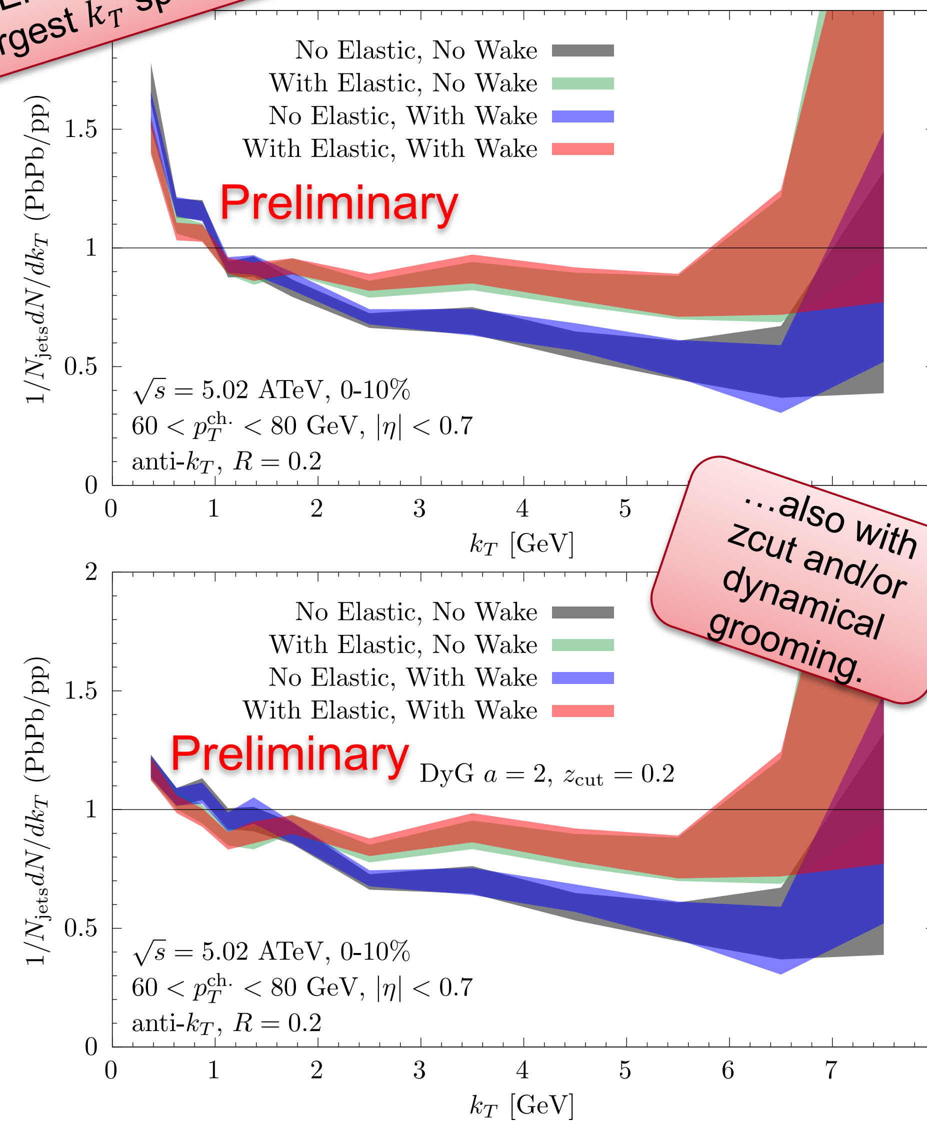
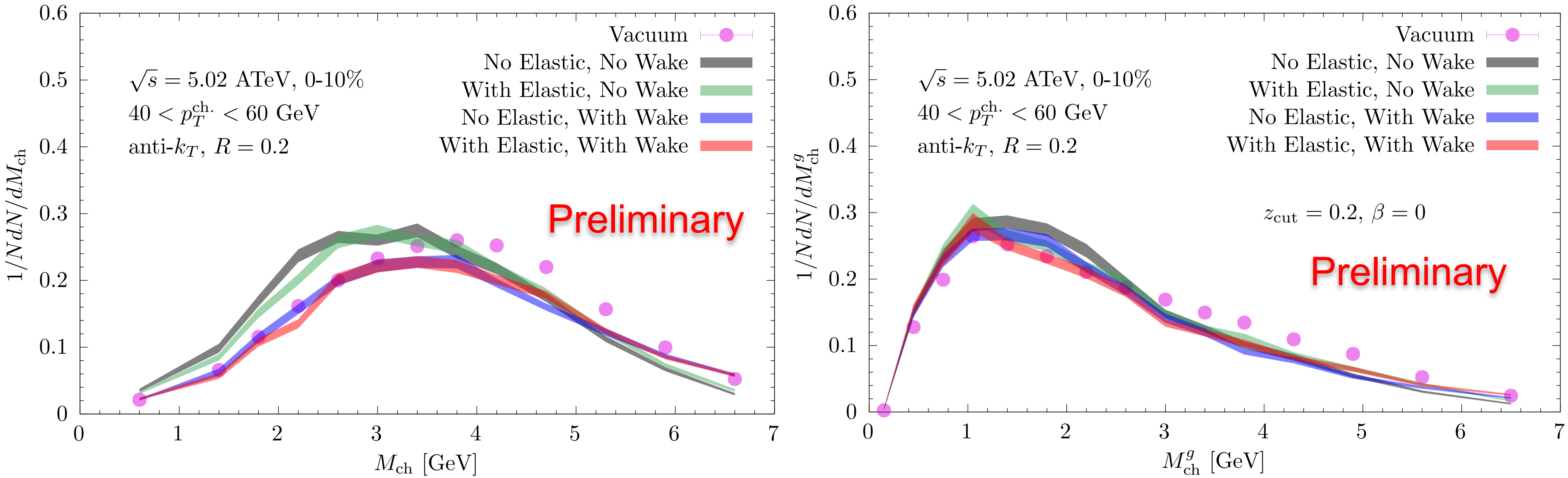


Jet Mass, and Groomed Jet Mass



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Grooming removes wake, but still little sensitivity to Moliere scattering.

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- Yes, but not every groomed observable is sensitive to hard scattering…

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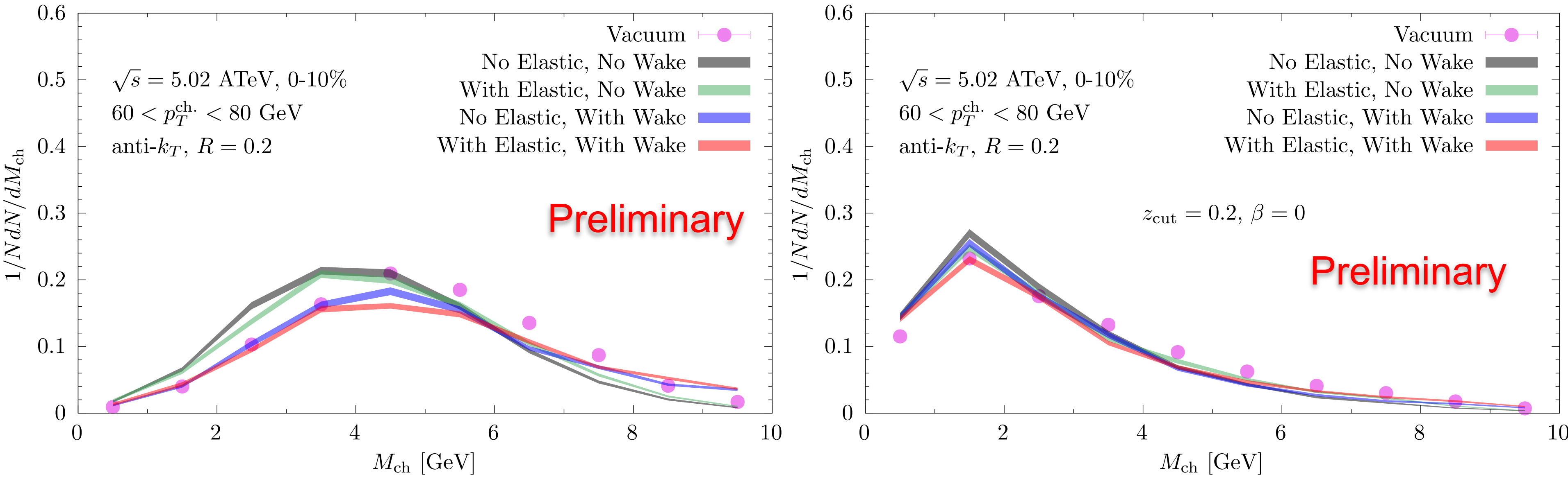

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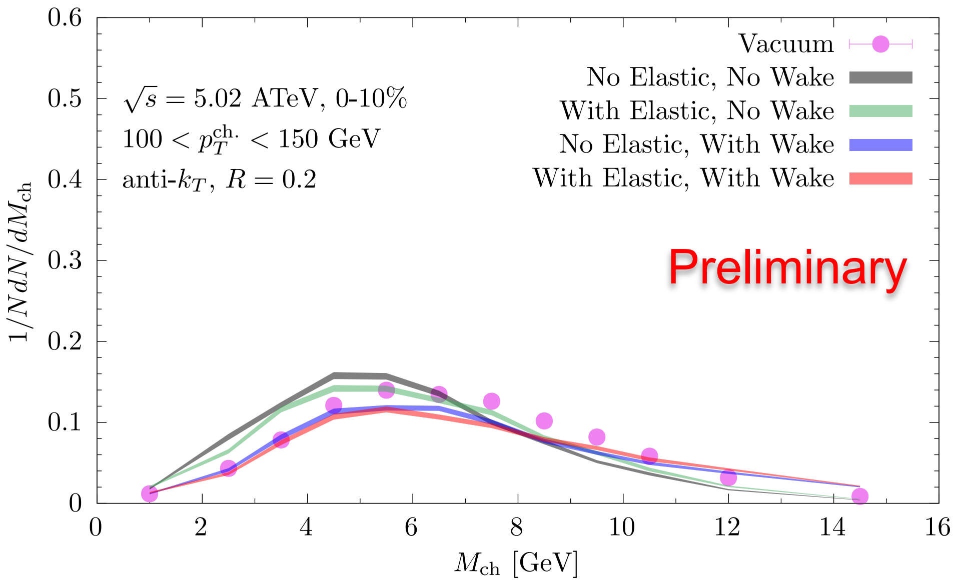

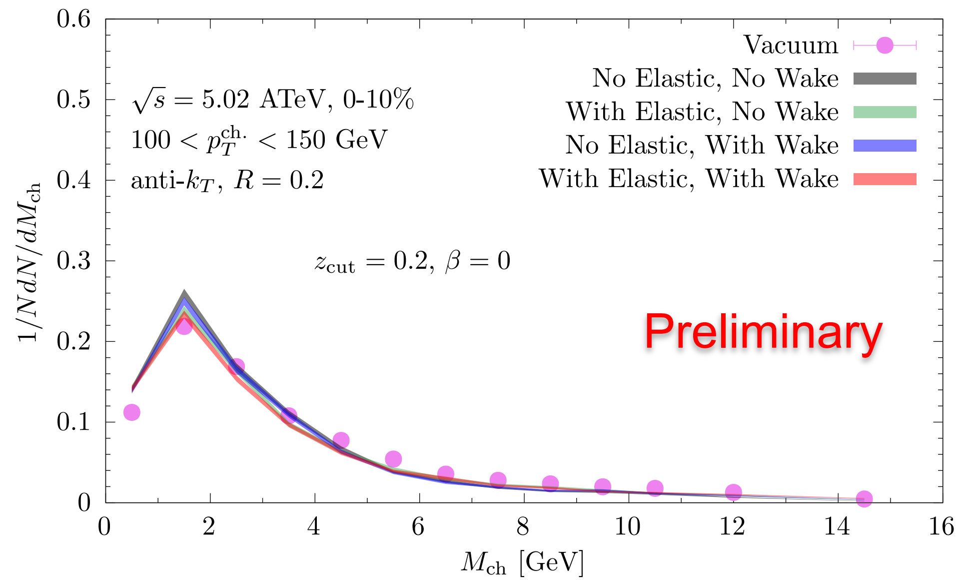

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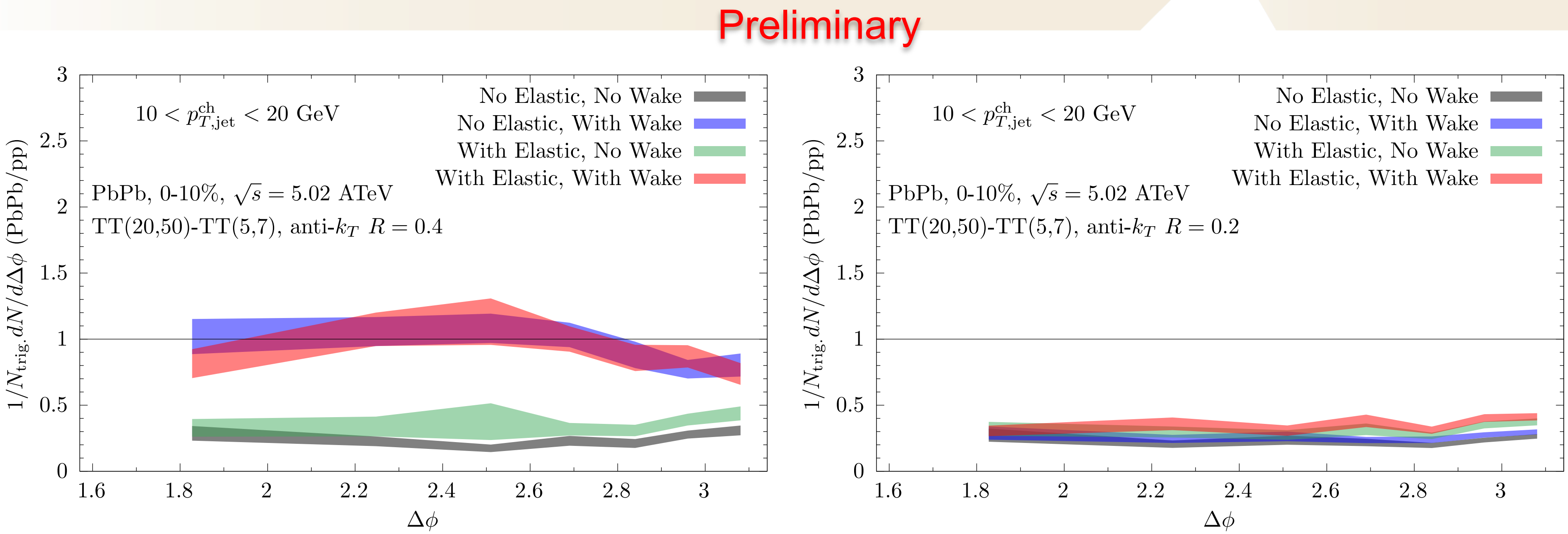
Hadron--Charge-Jet Acoplanarity, LHC energy



- Study acoplanarity in hadron - charged jet system.
- Parameters similar to ALICE
- Very little effect from Moliere scattering; increase in acoplanarity that we see is almost entirely due to the wake.
- Significant effect caused by the wake seen for R=0.4 jets, not for R=0.2
- I_{AA} indicates effect of wake enhances number of jets at these p_T
- And indeed effect of wake seen only in the lower charged jet p_T bin
- Moliere scattering: jet sprouts added prongs, not much overall deflection

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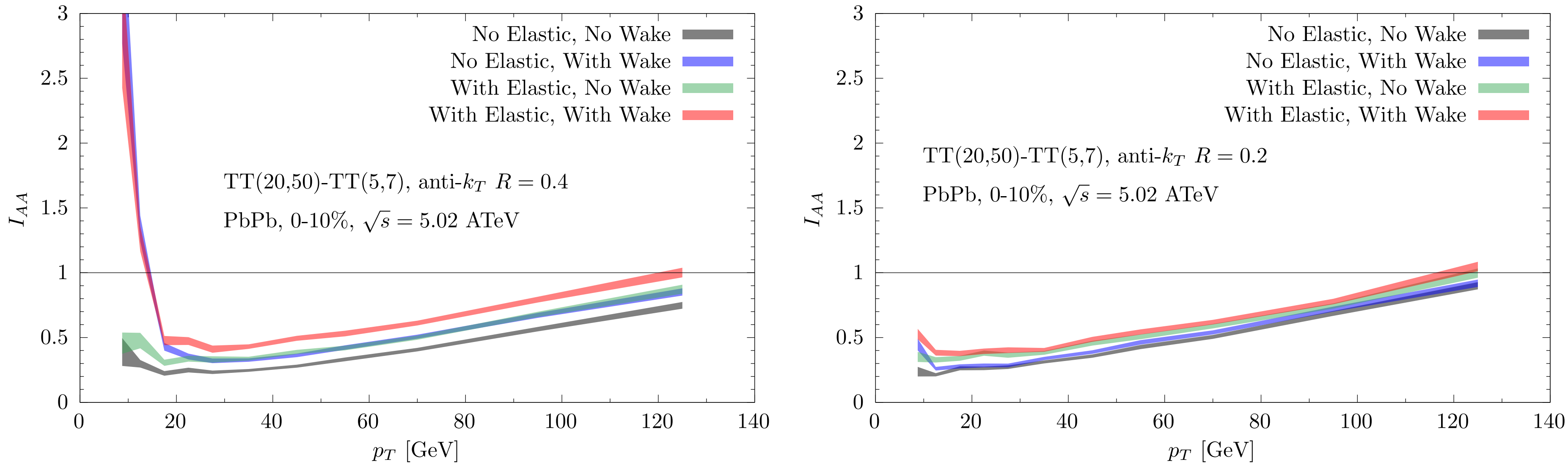
Preliminary



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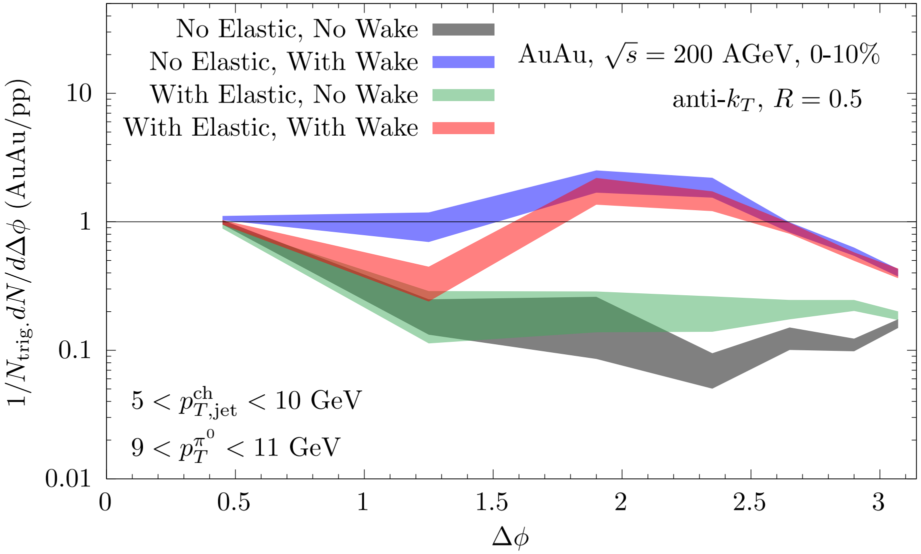

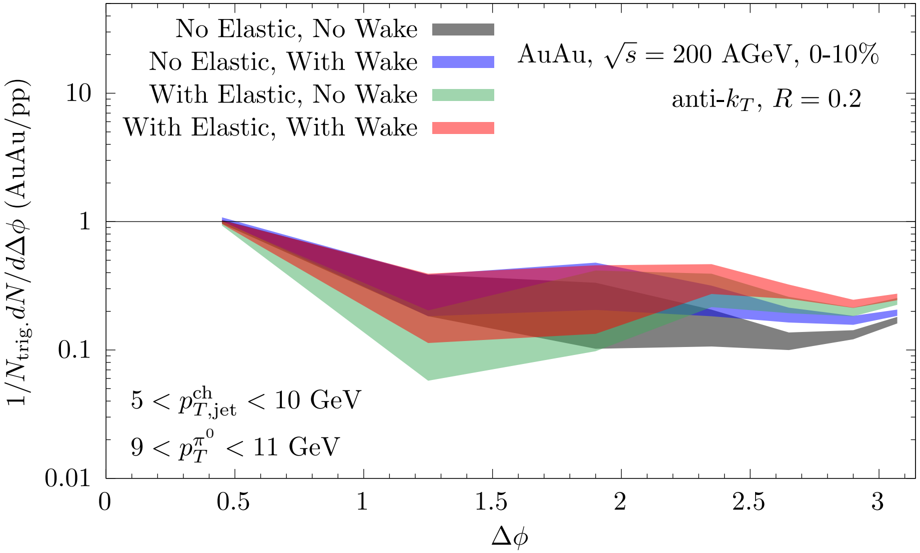

- Study acoplanarity in pi0 - charged jet system.
- Parameters similar to but not same as STAR
- Very little effect from Moliere scattering; increase in acoplanarity that we see is almost entirely due to the wake.
- Significant effect caused by the wake seen for R=0.5 jets, not for R=0.2
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Hadron--Charge-Jet Acoplanarity, RHIC energy

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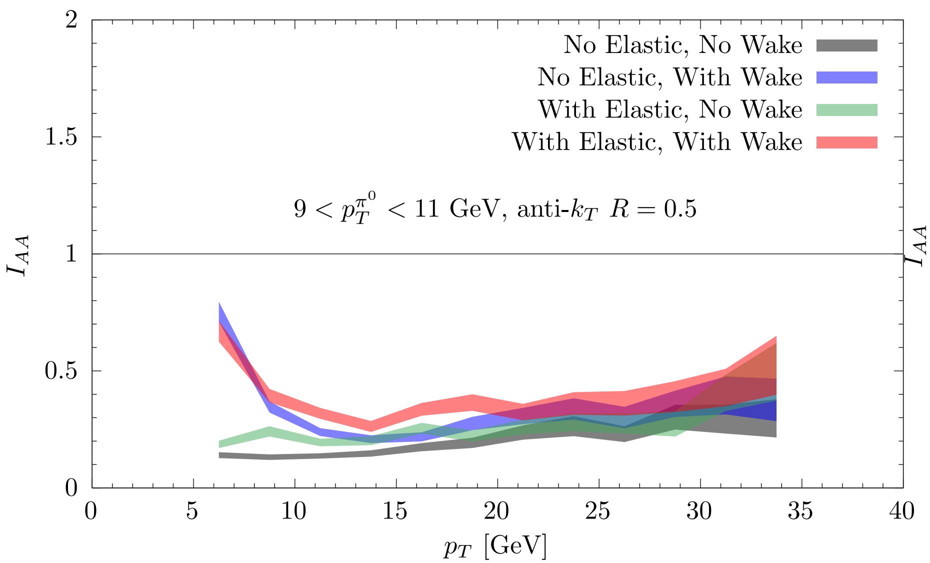

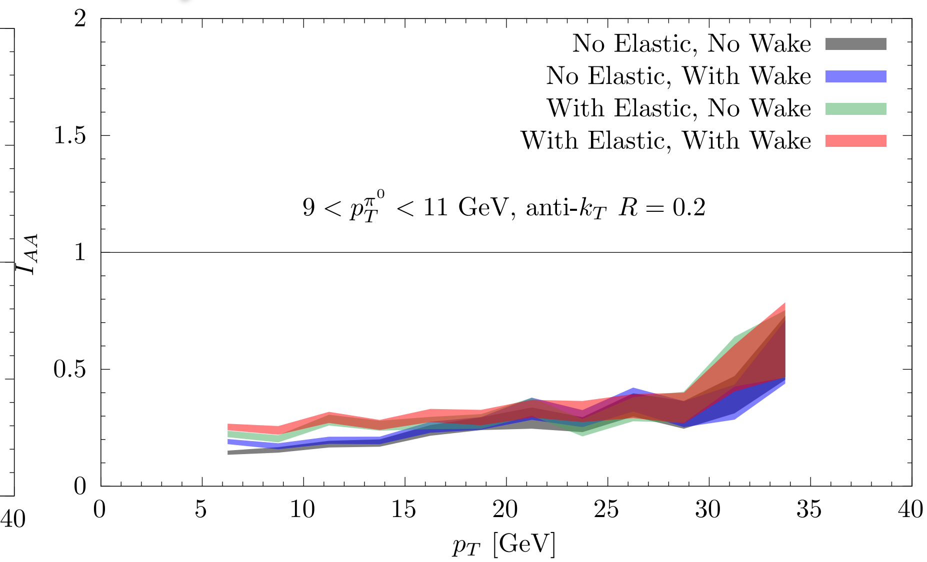

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