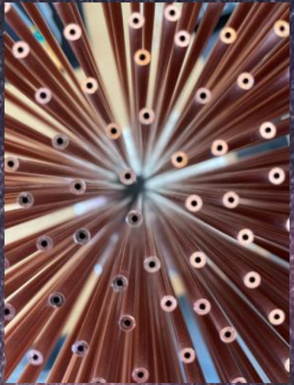
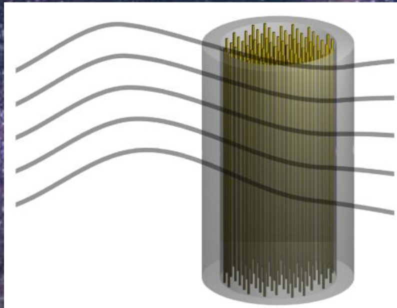
Emergent axion response in metamaterials

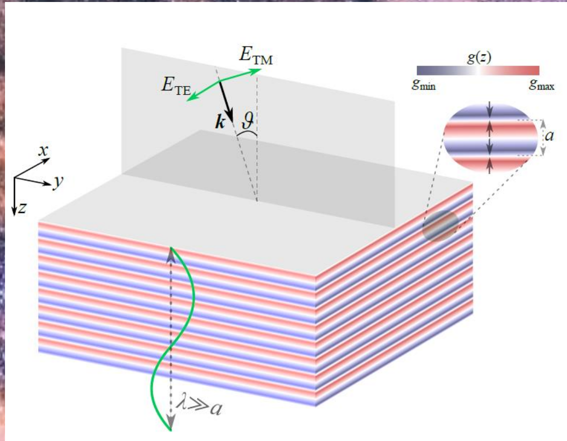




Maxim Gorlach,

ITMO University

L. Shaposhnikov, M. Mazanov, D.A. Bobylev, F. Wilczek, M.A. Gorlach. "Emergent axion response in multilayered metamaterials", arXiv: 2302.05111 (2023)



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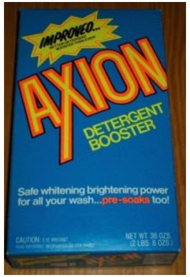
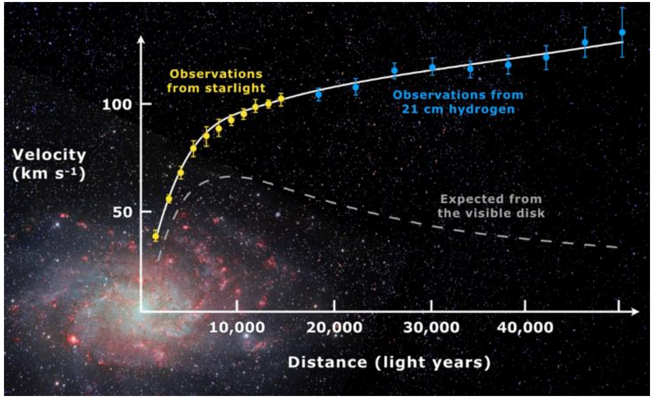
The concept of axions



Axions were originally introduced to resolve strong CP problem in quantum chromodynamics [1,2]

↳ name of the hypothetic particle suggested by Frank Wilczek comes from the mark of laundry detergent

Axion is one of the promising dark matter candidates



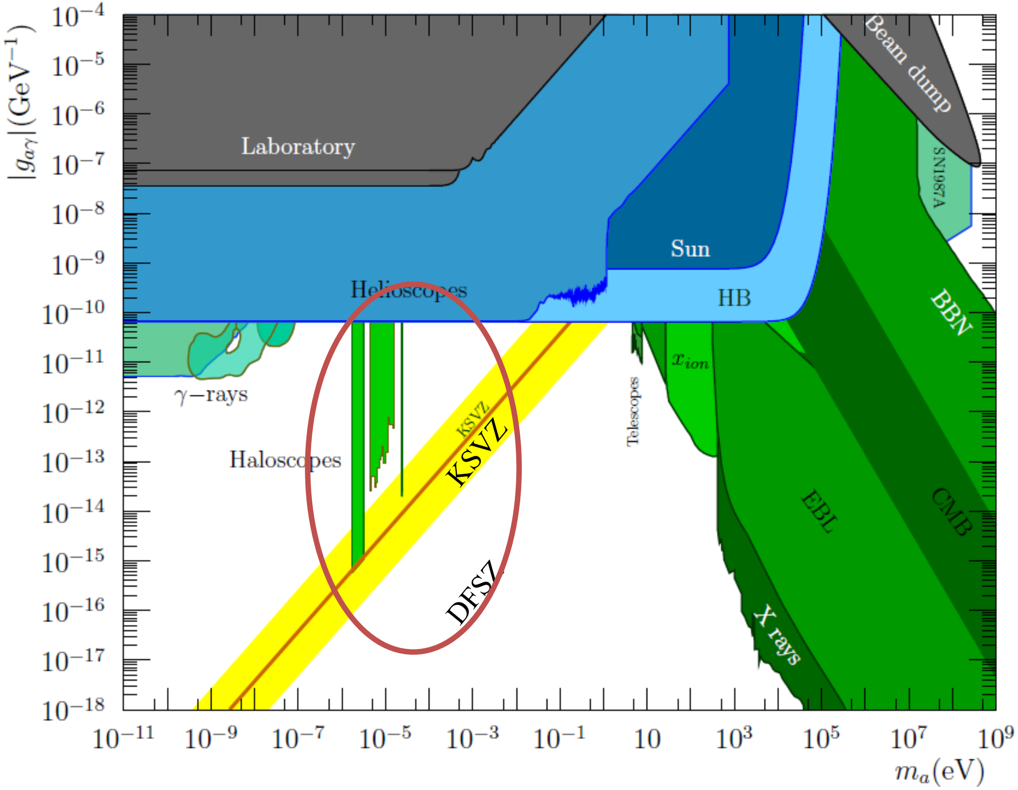
Astronomers postulated some invisible or **dark mass**

currently its origin **remains unclear,** multiple candidates have been suggested

None of the candidate particles is detected so far

[1] F. Wilczek. "Problem of Strong P and T Invariance in the Presence of Instantons", Physical Review Letters **40**, 279 (1978).
[2] S. Weinberg. "A New Light Boson?", Physical Review Letters **40**, 223 (1978).

Different approaches to cosmic axion searches

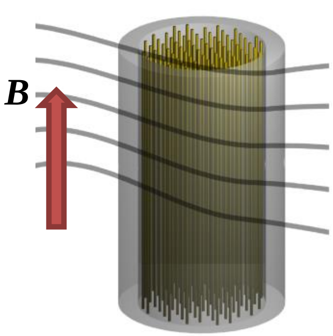


Helioscopes or stellar physics

Cosmological bounds

Haloscopes

Plasma haloscopes as a novel approach[2]
masses: $(20 \div 190)\mu$eV
frequencies: $(5 - 45)$ GHz



[1]I. Irastorza, J. Redondo. Progress in Particle and Nuclear Physics **102**, 89-159 (2018).
[2]A.J. Millar, *et al*. "ALPHA: Searching For Dark Matter with Plasma Haloscopes", Phys. Rev. D **107**, 055013 (2023).

alpha

Description of the axion field

Landau, Lifshitz. The Classical Theory of Fields.

Equations of motion can be recovered from the least action principle

$$S = \int \mathcal{L}\, d^3\boldsymbol{r}\, cdt$$ action

Lagrangian of axion electrodynamics. Should be scalar and Lorentz invariant

$$\mathcal{L} = \frac{1}{8\pi c}\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right) - \frac{1}{c}\rho\varphi + \frac{1}{c^2}\boldsymbol{A}\cdot\boldsymbol{j} + \mathcal{L}_m + \mathcal{L}_a + \frac{\kappa}{4\pi c}\, a\,(\boldsymbol{E}\cdot\boldsymbol{B})$$

Lagrangian of classical electrodynamics — free axion field — axion coupling to the electromagnetic field

$$\mathcal{L}_a = \frac{1}{8\pi c}\left(\frac{1}{c^2}\left(\frac{\partial a}{\partial t}\right)^2 - (\nabla a)^2 - m_a^2 a^2\right)$$

m_a axion mass

Time reversal $t \to -t$:

$$\boldsymbol{E} \to \boldsymbol{E}, \qquad \boldsymbol{B} \to -\boldsymbol{B}$$

hence $a \to -a$

Spatial inversion $\boldsymbol{r} \to -\boldsymbol{r}$:

$$\boldsymbol{E} \to -\boldsymbol{E}, \qquad \boldsymbol{B} \to \boldsymbol{B}$$

hence $a \to -a$

Axion field a is odd under **T** and **P** (pseudoscalar). But it is even under **PT** operation

Equations of axion electrodynamics

$$\begin{cases} \operatorname{rot}\left(\mu^{-1}\mathbf{B}\right) = \frac{1}{c}\frac{\partial(\varepsilon\mathbf{E})}{\partial t} + \frac{4\pi}{c}\mathbf{j} + \varkappa\left[\nabla\mathfrak{a}\times\mathbf{E}\right] + \frac{\varkappa}{c}\frac{\partial\mathfrak{a}}{\partial t}\mathbf{B} \\ \operatorname{div}\left(\varepsilon\,\mathbf{E}\right) = 4\pi\rho - \varkappa\left(\nabla\mathfrak{a}\cdot\mathbf{B}\right) \\ \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \\ \operatorname{div}\mathbf{B} = 0 \end{cases}$$

a is a pseudoscalar **axion field** (P-odd, T-odd)

Homogeneous axion field is not manifested

Gradients in the axion field or its temporal variation are detectable

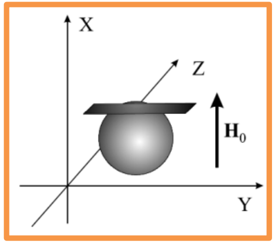
Can we realize this physics in some material platform?

F. Wilczek. Phys. Rev. Lett. **58**, 1799-1802 (1987)

Maxwell's equations in the medium

We bring the equations to the form

$$\begin{cases} \mathrm{rot}\,\mathbf{H} = \dfrac{1}{c}\dfrac{\partial \mathbf{D}}{\partial t} + \dfrac{4\pi}{c}\mathbf{j} \\ \mathrm{div}\,\mathbf{D} = 4\pi\rho \\ \mathrm{rot}\,\mathbf{E} = -\dfrac{1}{c}\dfrac{\partial \mathbf{B}}{\partial t} \\ \mathrm{div}\,\mathbf{B} = 0 \end{cases}$$



Where the constitutive relations

$$\begin{cases} \mathbf{D} = \varepsilon\,\mathbf{E} + \chi\,\mathbf{B} \\ \mathbf{H} = -\chi\,\mathbf{E} + \mu^{-1}\,\mathbf{B} \end{cases}$$

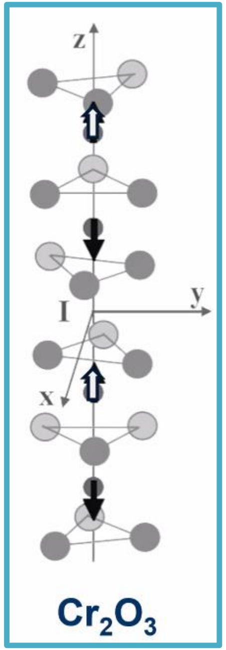
$\chi = \varkappa\,\mathfrak{a}$ plays the role of the effective axion field

Photonics: bianisotropic media, Tellegen-type bianisotropy

Condensed matter: magneto-electrics, multiferroics

Cr_2O_3 is a canonical example, multiple other materials have been suggested

Want to tailor the effective axion response on demand



A. P. Pyatakov and A. K. Zvezdin. Uspekhi Fizicheskih Nauk **182**, 593 (2012).

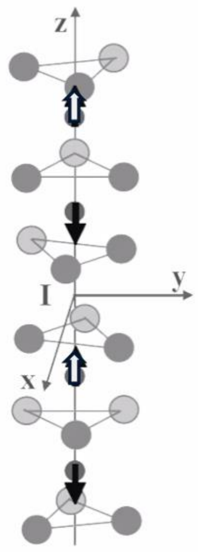
Deriving predictions of axion electrodynamics

If χ is homogeneous and time-independent, electrodynamics of such media is identical to isotropic media with ε and μ

The difference arises in two cases:

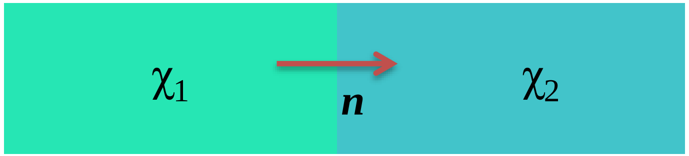
1. $\chi(t)$ - dynamic axion field

2. $\chi(z)$ – boundaries or gradients



Cr_2O_3

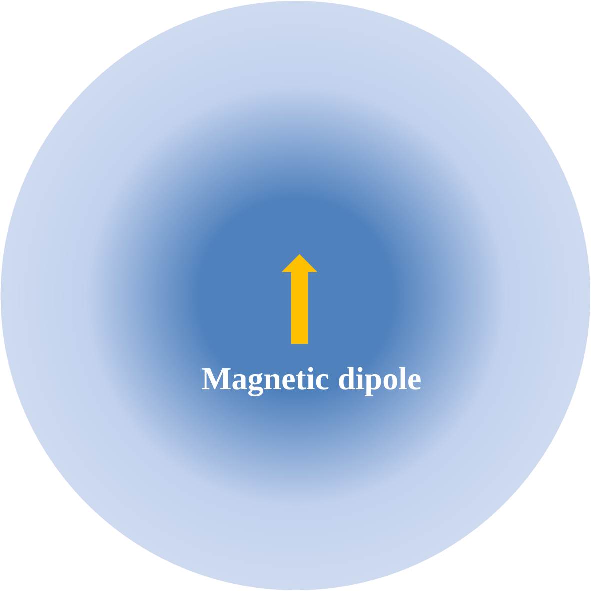
Stepwise time-independent axion field



$$\begin{cases} \mu_1^{-1}\boldsymbol{B}_{1t} - \chi_1\,\boldsymbol{E}_{1t} = \mu_2^{-1}\boldsymbol{B}_{2t} - \chi_2\,\boldsymbol{E}_{2t} \\ \varepsilon_1 E_{1n} + \chi_1 B_{1n} = \varepsilon_2 E_{2n} + \chi_2 B_{2n} \\ \boldsymbol{E}_{1t} = \boldsymbol{E}_{2t} \\ B_{1n} = B_{2n} \end{cases}$$

Effects of axion electrodynamics

Axion shell (homogeneous)



Outside observer

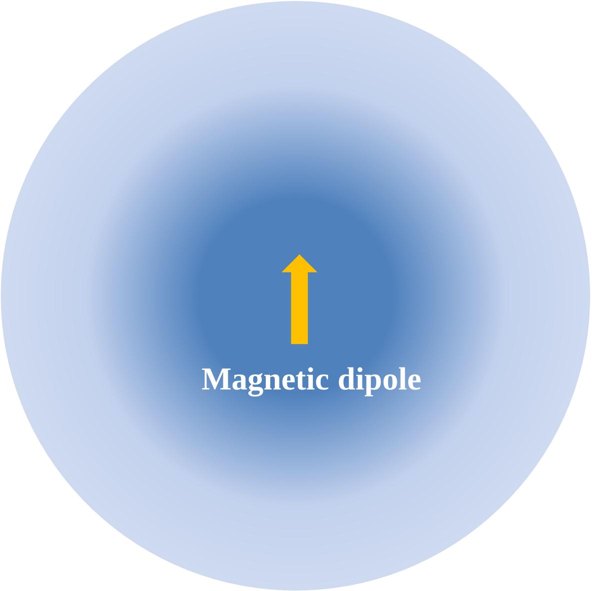
Boundary conditions

$$\begin{cases} \boldsymbol{n} \times \boldsymbol{E}^{in} = \boldsymbol{n} \times \boldsymbol{E}^{out} \\ \boldsymbol{n} \cdot \boldsymbol{B}^{in} = \boldsymbol{n} \cdot \boldsymbol{B}^{out} \\ \boldsymbol{n} \times \boldsymbol{B}^{in} - \chi^{in}\, \boldsymbol{n} \times \boldsymbol{E}^{in} = \\ \qquad \boldsymbol{n} \times \boldsymbol{B}^{out} - \chi^{out}\, \boldsymbol{n} \times \boldsymbol{E}^{out} \\ \boldsymbol{n} \cdot \boldsymbol{E}^{in} + \chi^{in}\, \boldsymbol{n} \cdot \boldsymbol{B}^{in} = \\ \qquad \boldsymbol{n} \cdot \boldsymbol{E}^{out} + \chi^{out}\, \boldsymbol{n} \cdot \boldsymbol{B}^{out} \end{cases}$$

What kind of field is perceived by the observer?
(consider the static case for simplicity)

Effects of axion electrodynamics

Axion shell (homogeneous)



Outside observer

$$\boldsymbol{B}_0 = -\frac{4\chi^2}{9a^3}\frac{\boldsymbol{m}_0}{1+2\chi^2/9}$$

$$\boldsymbol{m} = \frac{\boldsymbol{m}_0}{1+2\chi^2/9}$$

$$\boldsymbol{d} = \frac{2\chi}{3}\frac{\boldsymbol{m}_0}{1+2\chi^2/9}$$

Solution inside

dipole magnetic field

$$\boldsymbol{B}^{in} = \frac{1}{r^3}\left[3(\boldsymbol{n}\cdot\boldsymbol{m}_0)\boldsymbol{n} - \boldsymbol{m}_0\right] + \boldsymbol{B}_0$$

homogeneous electric field

$$\boldsymbol{E}^{in} = -\frac{2\chi}{3}\frac{\boldsymbol{m}_0}{1+2\chi^2/9}$$

Solution outside

dipole magnetic field

$$\boldsymbol{B}^{out} = \frac{1}{r^3}\left[3(\boldsymbol{n}\cdot\boldsymbol{m})\boldsymbol{n} - \boldsymbol{m}\right]$$

dipole electric field

$$\boldsymbol{E}^{out} = \frac{1}{r^3}\left[3(\boldsymbol{n}\cdot\boldsymbol{d})\boldsymbol{n} - \boldsymbol{d}\right]$$

effective electric dipole from the axion shell!

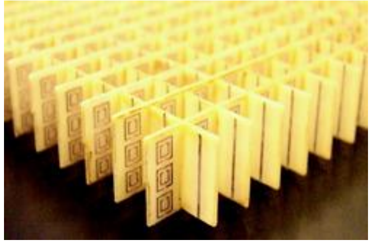
Witten effect

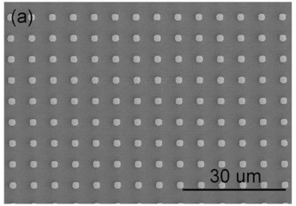
F. Wilczek. Phys. Rev. Lett. **58**, 1799-1802 (1987)

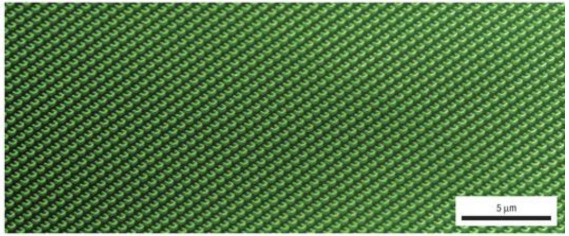
The concept of axion metamaterials

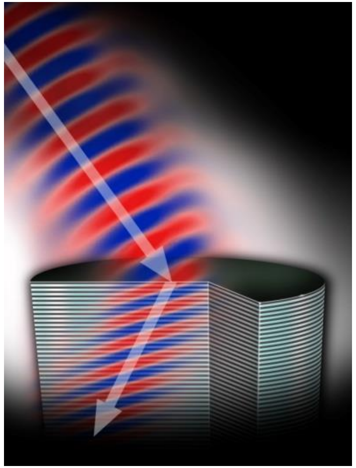
Metamaterials are artificial media with unconventional electromagnetic properties

Typically $a \ll \lambda$ (subwavelength period) → effective material parameters could be applied









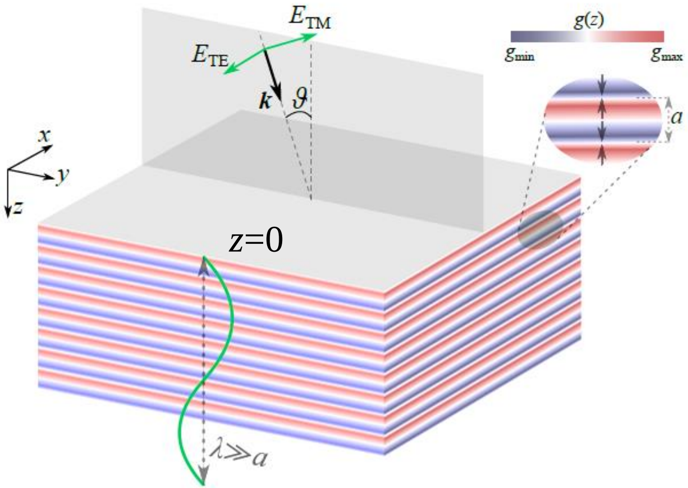
What if we tailor the metamaterial such that it is described by the constitutive relations

$$\mathbf{D} = \varepsilon\,\mathbf{E} + \chi\,\mathbf{B}$$
$$\mathbf{H} = -\chi\,\mathbf{E} + \mu^{-1}\,\mathbf{B}$$

That would provide a tabletop platform to test the effects of axion electrodynamics controlling the strength of the axion response

Axion metamaterial

Designing axion metamaterial



Each layer is made of the conventional gyrotropic material

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon & i\,g(z) & 0 \\ -i\,g(z) & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$$

off-diagonal terms due to static magnetization

Magnetization of the layers is periodically modulated with period *a*

$$g(z) = \sum_{n \neq 0} g_n\, e^{inbz} \qquad b = \frac{2\pi}{a}$$

Floquet expansion

$$\boldsymbol{E}(\boldsymbol{r}) = \sum_{n=-\infty}^{\infty} \boldsymbol{E}_n \exp[ik_x x + i(k_z + nb)z]$$

$\boldsymbol{E}_0$ is averaged (macroscopic) field

other Floquet harmonics are rapidly oscillating; their amplitude is $\sim\xi^2$

Small parameter

$$\xi = \frac{a}{\lambda} = \frac{q}{b} \ll 1$$

$$q = \omega/c$$

Key idea of derivation

Microscopic fields at the boundary of metamaterial are continuous

$$\begin{cases} \sum_{n=-\infty}^{\infty} \boldsymbol{e}_z \times \boldsymbol{E}_n = \boldsymbol{e}_z \times \boldsymbol{E}^{out} \\ \sum_{n=-\infty}^{\infty} \boldsymbol{e}_z \cdot \boldsymbol{B}_n = \boldsymbol{e}_z \cdot \boldsymbol{B}^{out} \\ \sum_{n=-\infty}^{\infty} \boldsymbol{e}_z \times \boldsymbol{B}_n = \boldsymbol{e}_z \times \boldsymbol{B}^{out} \\ \varepsilon \sum_{n=-\infty}^{\infty} \boldsymbol{e}_z \cdot \boldsymbol{E}_n = \boldsymbol{e}_z \cdot \boldsymbol{E}^{out} \end{cases}$$

calculate higher-order Floquet harmonics

keep the terms up to ~ξ

$$\begin{cases} \boldsymbol{e}_z \times \boldsymbol{E}_0 = \boldsymbol{e}_z \times \boldsymbol{E}^{out} \\ \boldsymbol{e}_z \cdot \boldsymbol{B}_0 = \boldsymbol{e}_z \cdot \boldsymbol{B}^{out} \\ \mathbf{B}_{0t} - \mathbf{B}_t^{\mathrm{out}} = \chi\, \mathbf{E}_{0t} \\ \varepsilon\, E_{0z} - E_z^{\mathrm{out}} = -\chi\, B_{0z} \end{cases}$$

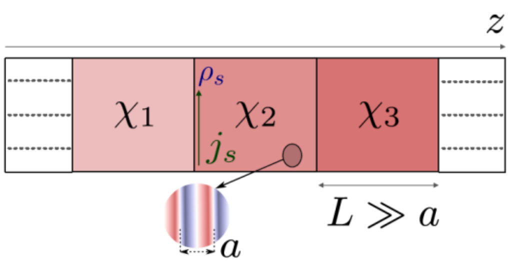
there is a discontinuity of the averaged fields!

Boundary conditions for axion electrodynamics!

quantifies the strength of the effective axion response

$$\chi = -i\,\frac{a}{\lambda} \sum_{n \neq 0} \frac{g_n}{n} \quad \Longleftrightarrow \quad \chi = \frac{1}{\lambda} \int_0^a g(z)\,(\pi - b\,z)\,dz$$

Gradients of axion response



Examine slowly varying effective axion response

At the boundary between the blocks we have:
$$\begin{cases} \mathbf{B}_{2t} - \mathbf{B}_{1t} = (\chi_2 - \chi_1)\, \mathbf{E}_t \,, \\ \varepsilon\, E_{2z} - \varepsilon\, E_{1z} = -(\chi_2 - \chi_1)\, B_z \end{cases}$$

Hence, there are some surface charges and currents
$$\begin{cases} \frac{4\pi}{c}\mathbf{j}_s = \mathbf{e}_z \times [\mathbf{B}_2 - \mathbf{B}_1] = (\chi_2 - \chi_1)\, [\mathbf{e}_z \times \mathbf{E}] \\ 4\pi\, \rho_s = \varepsilon\, E_{2z} - \varepsilon\, E_{1z} = -(\chi_2 - \chi_1)\, B_z \,. \end{cases}$$

After averaging $\mathbf{j}_s/L \to \mathbf{j},\ \rho_s/L \to \rho,\ (\chi_2 - \chi_1)\,\mathbf{e}_z/L \to \nabla\chi$ ⟹
$$\frac{4\pi}{c}\mathbf{j} = [\nabla\chi \times \mathbf{E}]$$
$$4\pi\rho = -\nabla\chi \cdot \mathbf{B}$$

This yields the equations of axion electrodynamics

$$\mathrm{rot}\,\mathbf{B} = \frac{1}{c}\frac{\partial}{\partial t}(\varepsilon\,\mathbf{E}) + [\nabla\chi \times \mathbf{E}] \,,$$
$$\mathrm{div}\,(\varepsilon\,\mathbf{E}) = -\nabla\chi \cdot \mathbf{B} \,,$$
$$\mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \,, \quad \mathrm{div}\,\mathbf{B} = 0 \,.$$

Thus, our structure is indeed an axion metamaterial

Manipulating effective axion response

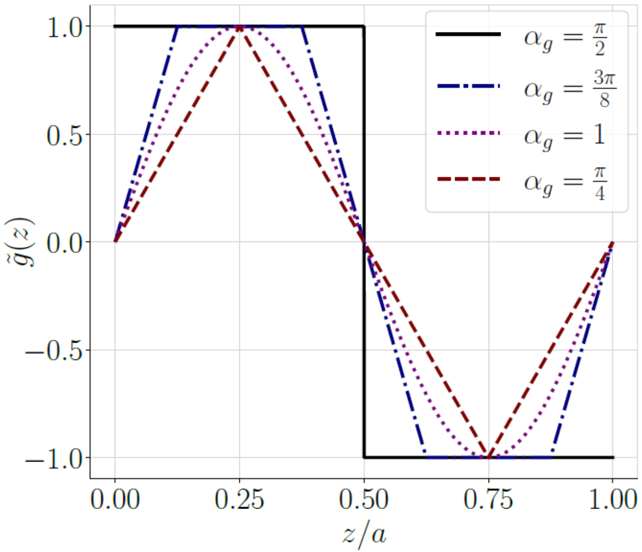
$$\chi = -i\frac{a}{\lambda}\sum_{n\neq 0}\frac{g_n}{n} = \frac{2\pi}{\lambda}\int_0^a\left(\frac{1}{2}-\frac{z}{a}\right)g(z)dz$$

χ vanishes in the static case

1. By tailoring magnetization distribution, we can tailor effective χ

$$\chi = \alpha_g\, g_{\max}\frac{a}{\lambda}$$

2. Effective axion response depends on the termination of the structure (unlike other bulk material parameters)



For instance, if $g(z) = g_{\max}\sin(bz+\gamma)$

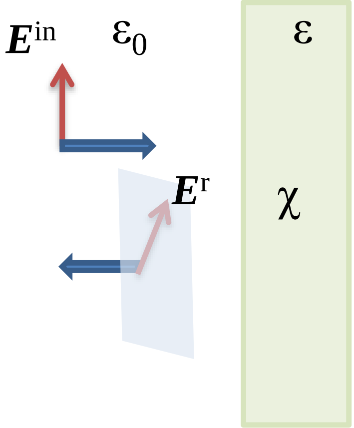
$$\chi = -g_{\max}\frac{a}{\lambda}\cos\gamma$$

So the effective axion response can be continuously varied in a wide range by changing the termination of the structure

↓

new and powerful degree of freedom to shape axion response

Validating effective description



We examine the reflection of light from the free-standing slab of axion metamaterial

Prediction of effective medium theory:

$$r_{xx} = r_{yy} = -\frac{(\chi^2 + \varepsilon - \varepsilon_0)\sin\tilde{L}}{(\chi^2 + \varepsilon + \varepsilon_0)\sin\tilde{L} + 2i\sqrt{\varepsilon\varepsilon_0}\cos\tilde{L}},$$

co-polarized reflectance

$$r_{xy} = -r_{yx} = \frac{2\chi\sqrt{\varepsilon_0}\sin\tilde{L}}{(\chi^2 + \varepsilon + \varepsilon_0)\sin\tilde{L} + 2i\sqrt{\varepsilon\varepsilon_0}\cos\tilde{L}},$$

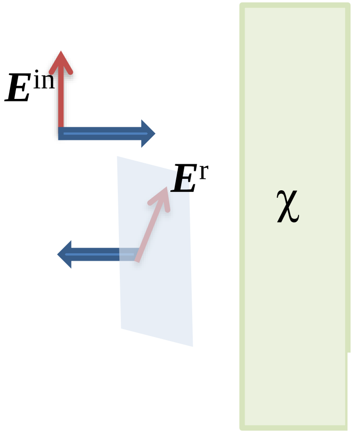
cross-polarized reflectance

$$\tilde{L} = 2\pi\sqrt{\varepsilon}L/\lambda_0 = 2\sqrt{\varepsilon}\pi N a/\lambda_0$$

We compare this result with the rigurous numerical calculation

How well does the effective medium approach work?

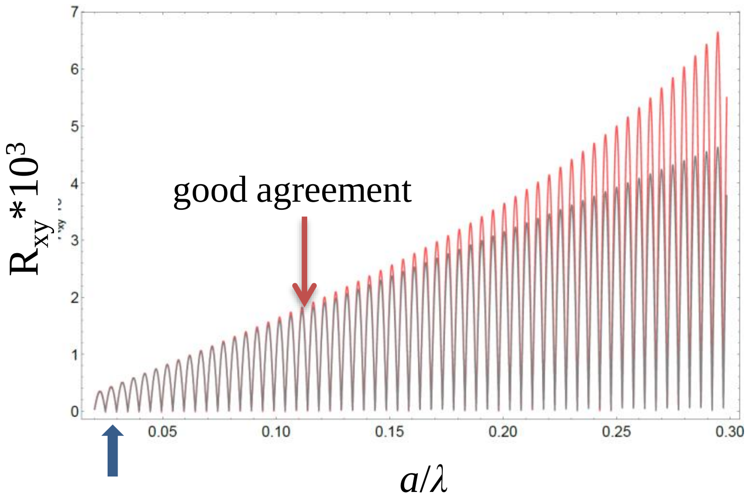
Validating effective description

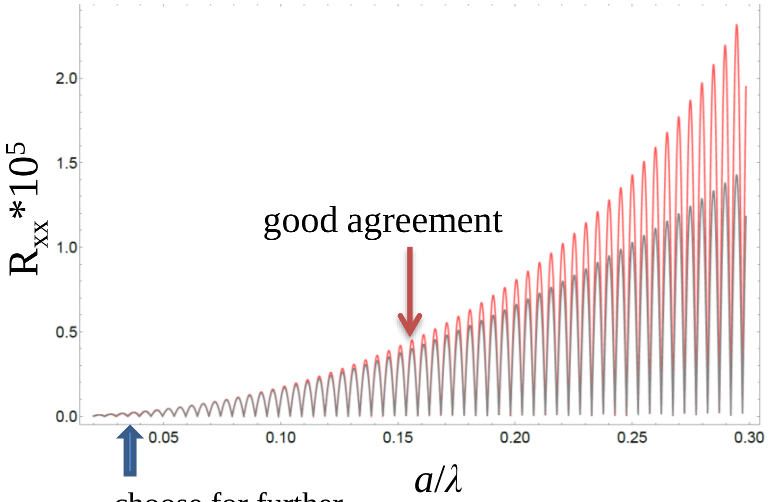


We examine the reflection of light from the free-standing slab of axion metamaterial

cross-polarized reflectance

co-polarized reflectance





effective medium approach works reasonably well if $\lambda > 7a$

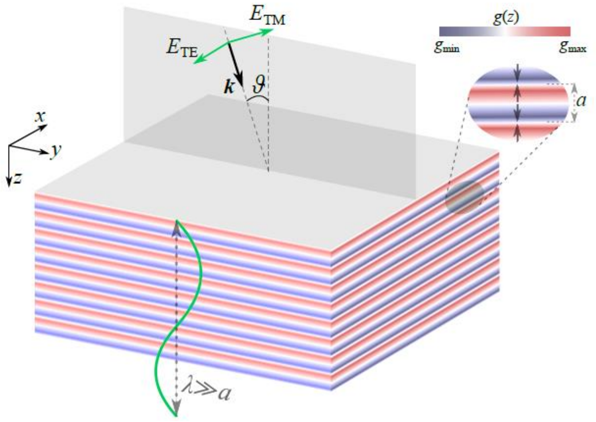
choose for further simulations

$\varepsilon_0 = \varepsilon = 1$ $g = 0.01$ 100 layers

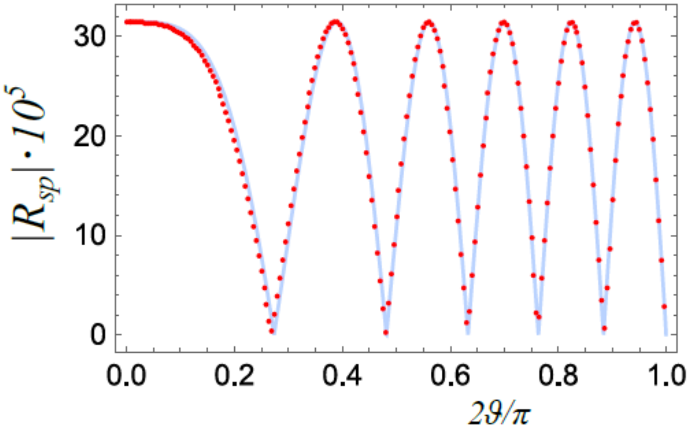
stepwise magnetization distribution

effective medium

transfer matrix method



Cross-polarized reflectance $|R_{sp}| = |R_{ps}|$



effective medium

transfer matrix method

$L = 2.75\,\lambda$

$\varepsilon_0 = \varepsilon = 1$

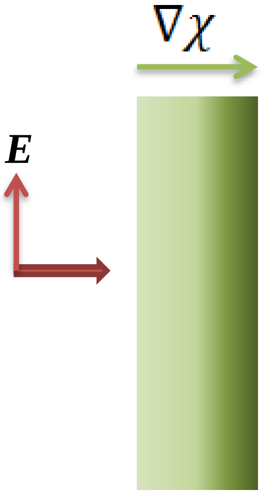
$g = 0.01$

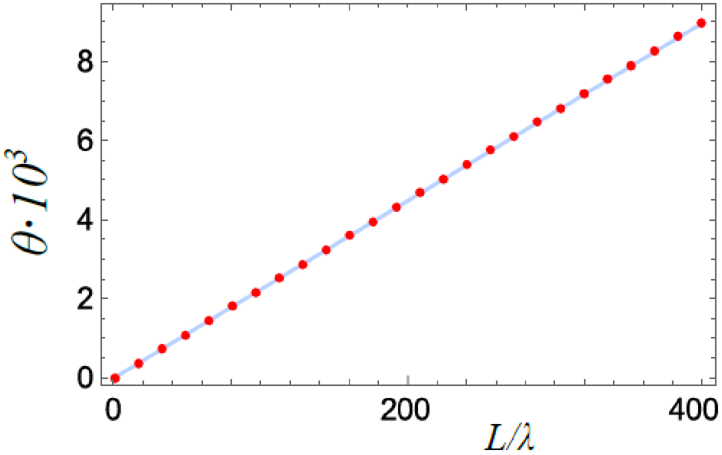
$a = \dfrac{\lambda}{50}$

Oblique incidence: effective medium picture works well for all incidence angles

Gradient of the effective axion response

Rotation of polarization plane





effective medium

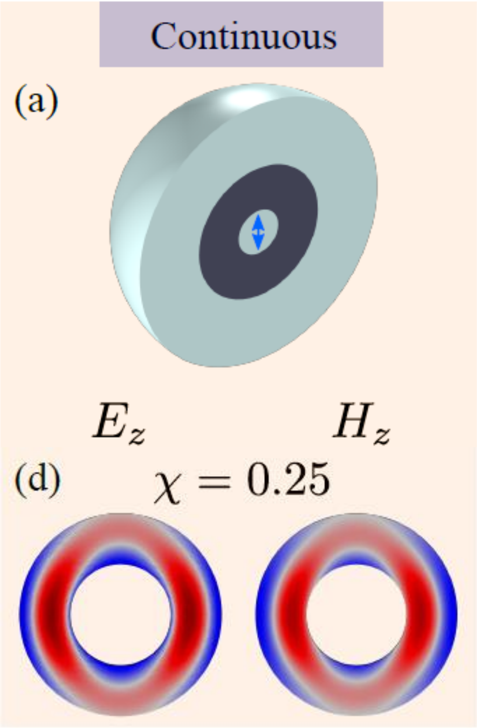
$$\theta = \frac{\Delta\chi}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

transfer matrix method

polarization of the ***averaged field*** is considered

Amplitude of the layers magnetization varies linearly from 0 to $g_{\max} = 0.01\,\frac{L}{400\lambda}$

Effective electric and magnetic dipoles



We verify this prediction for our multilayered metamaterial

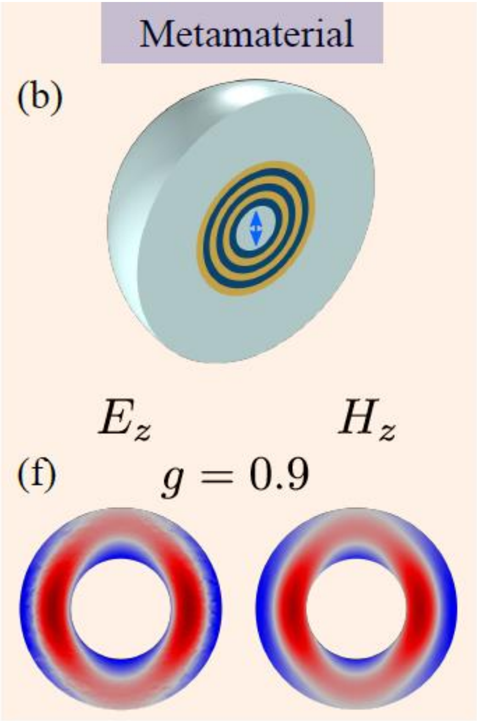
f=1.0 GHz

$$r^{\text{in}} = \lambda/4$$

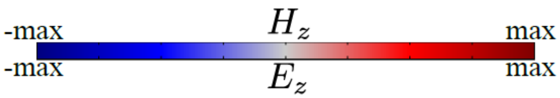
$$r^{\text{out}} = 3\lambda/4$$

$$a = \lambda/6$$

$$\varepsilon = \mu = 1$$



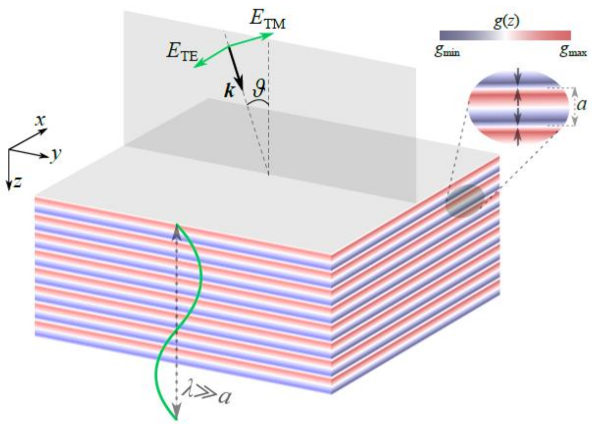
both magnetic and electric dipole fields are observed



Our prediction holds for the designed metamaterial!

Effective axion response in metamaterial

Discussion

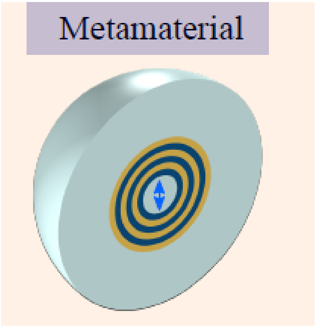


Implications for metamaterials physics

Consistent theory of effective axion response, classical derivation, role of the structure termination

Implications for condensed matter:

Pathways to achieve tunable effective axion response

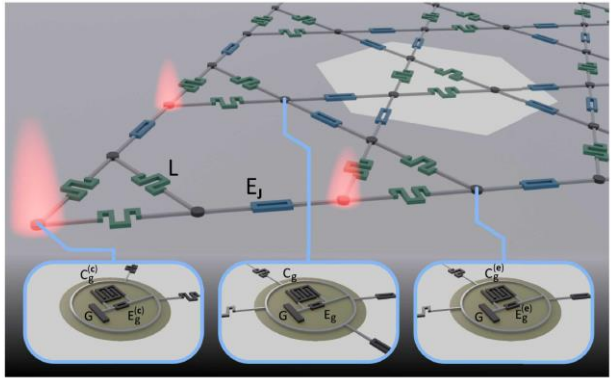


Implications for axion physics:

Possible detection scheme: conversion of dark matter axions into the emergent ones (?)
Need to realize dynamic axion fields for that

Other recent highlights

Topological multiphoton states & quantum simulations



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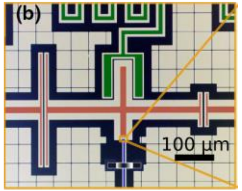

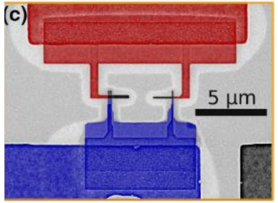

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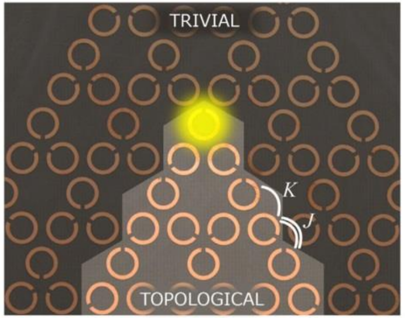
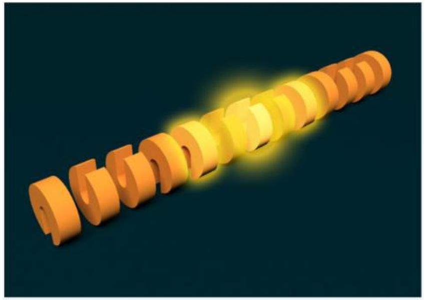
Novel strategy to tailor and tune photonic topological states





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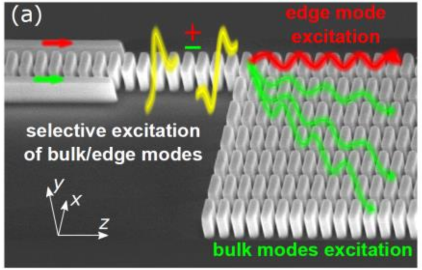

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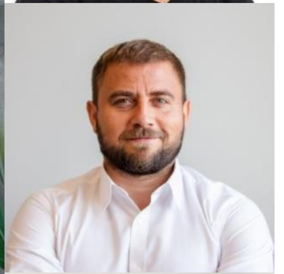
Our team at ITMO



Thank you for attention

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L. Shaposhnikov, M. Mazanov, D.A. Bobylev, F. Wilczek, M.A. Gorlach. arXiv: 2302.05111 (2023)