Hunting Axions with Metamaterials
the ALPHA haloscope

A. Gallo Rosso

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OVERVIEW

1. Axions
2. ALPHA Haloscope
3. Statistical inference

1. Axions

What, how and why

arXiv:1801.08127, 2003.01100, 2012.05029
2104.07634, 2105.01406, 2109.07376

STRONG CP PROBLEM

Violating CP term

$$\mathcal{L}_{\not{CP}} = \theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \widetilde{G}^{\mu\nu}_a$$

Neutron electric dipole moment

$$d_{\mathrm{n}} = (2.4 \pm 1.0) \cdot 10^{-3}\,\mathrm{e\,fm} \times \theta$$

The vacuum of the theory is determined by angle $\theta \in [0, 2\pi]$

STRONG CP PROBLEM

$$d_{\mathrm{n}} < 1.8 \cdot 10^{-13}\,\mathrm{e\,fm}$$
$$|\theta| < 8 \cdot 10^{-11}$$

[1] C. Abel *et al.*, Phys. Rev. Lett. 124, 081803.

THE AXION

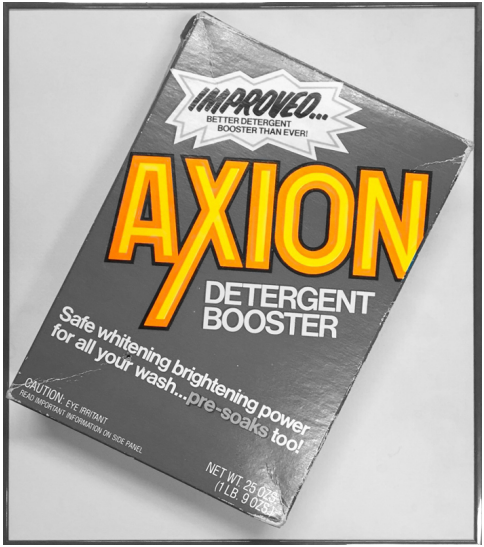
A clean solution

Dynamic variable $\theta(\vec{x}, t)$

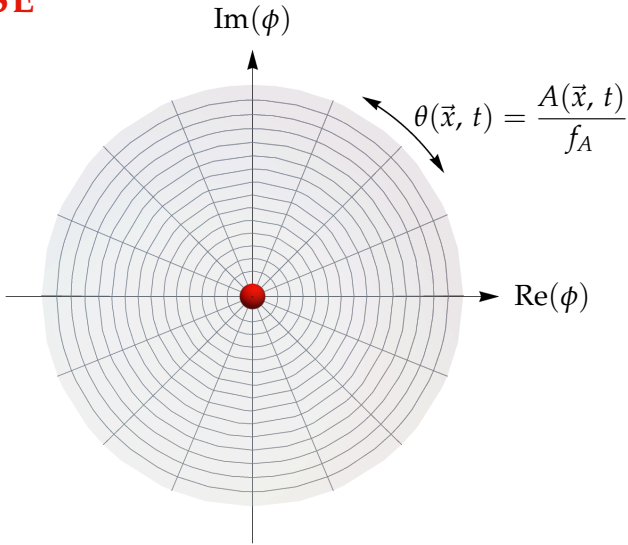
Pseudo Nambu-Goldstone boson

Peccei Quinn $U(1)$ symmetry

Spontaneously broken at f_A



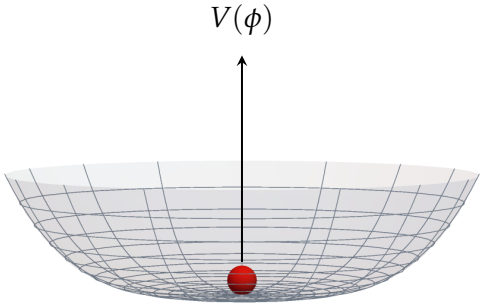
EARLY UNIVERSE



[2]See e.g. arXiv: 1801.08127, 2003.01100, 2012.05029, 2104.07634, 2105.01406, 2109.07376.

EARLY UNIVERSE

1. $E \gg f_A$
2. PQ transition
3. QCD transition



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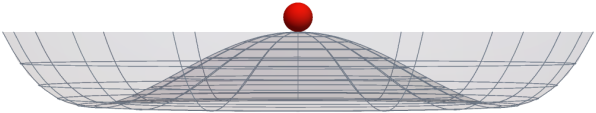

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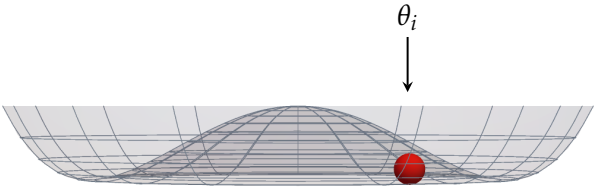

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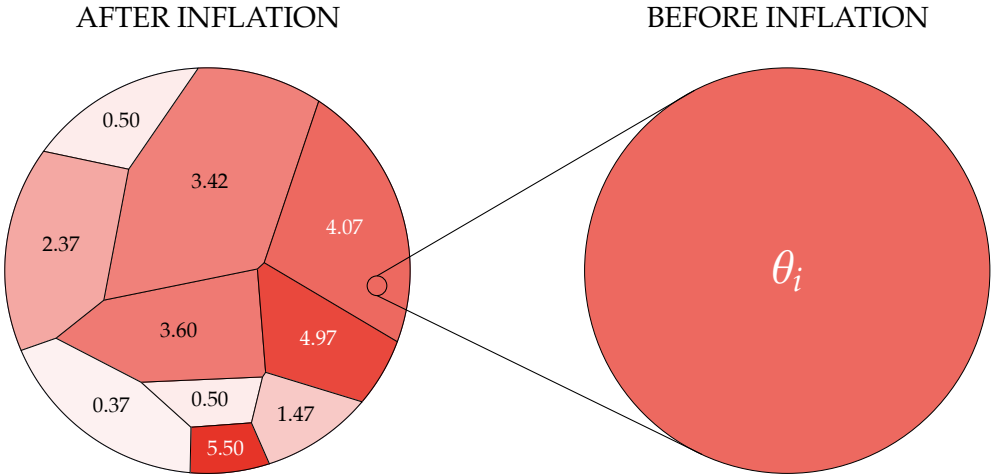

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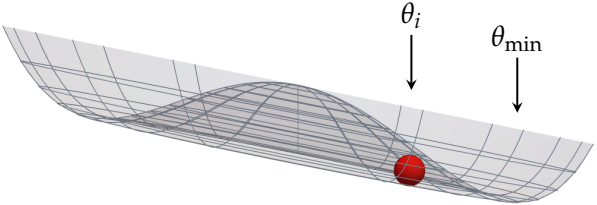

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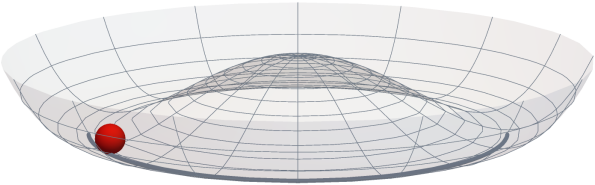

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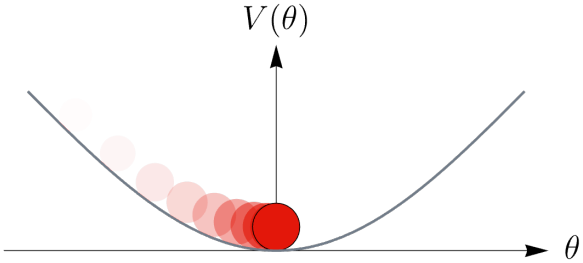

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A FREE MEAL

Strong CP problem

Evolution from θ_i to CP-conserving value θ_{min}

Dark matter

Non-thermal production from mismatch $\theta_i \neq \theta_{\text{min}}$

Inflation

(Possible) explanation of dynamics of inflation field

[2] See e.g. arXiv: 1801.08127, 2003.01100, 2012.05029, 2104.07634, 2105.01406, 2109.07376.

PQ-AXION©

Models

- PQWW: $f_A \sim$ EW-scale
- KSVZ: hadronic axions
- DFSZ: GUT axions

Massive boson

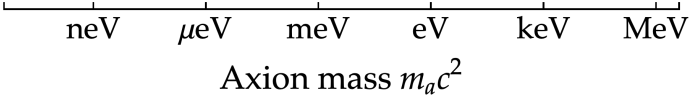
- $m_A \propto f_A$
- Light ($<$ MeV)
- Non-relativistic

Coupling

- $g \propto f_A^{-1}$
- Hadrons & gluons
- Photons
- Electrons (DFSZ only)

[2] See e.g. arXiv: 1801.08127, 2003.01100, 2012.05029, 2104.07634, 2105.01406, 2109.07376.

A LAND TO EXPLORE



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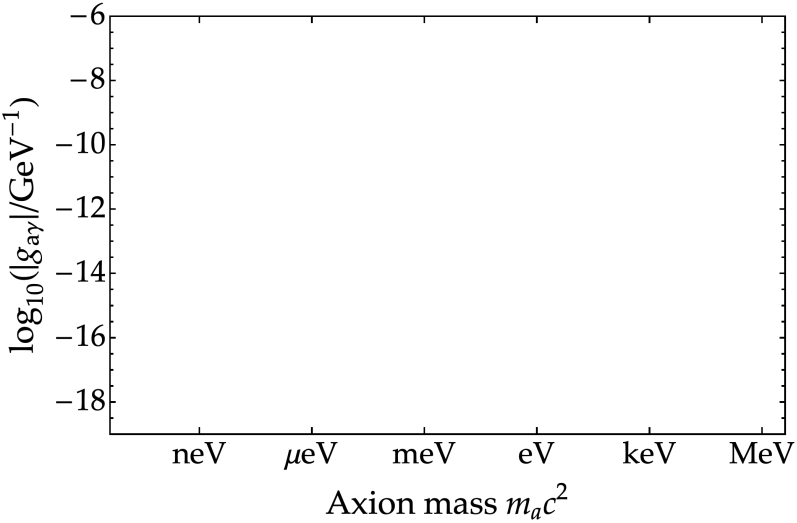

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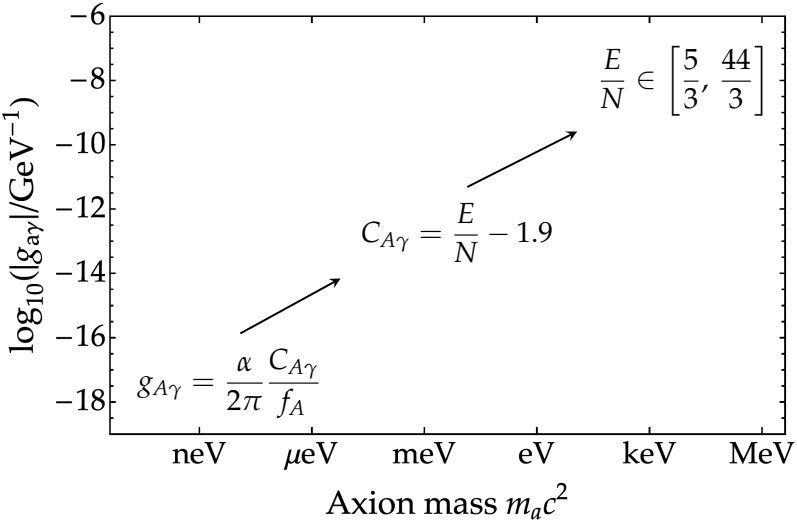

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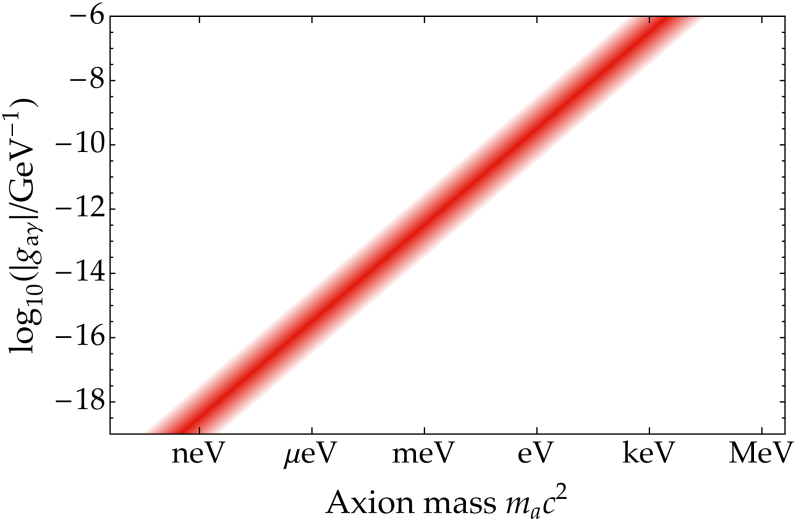

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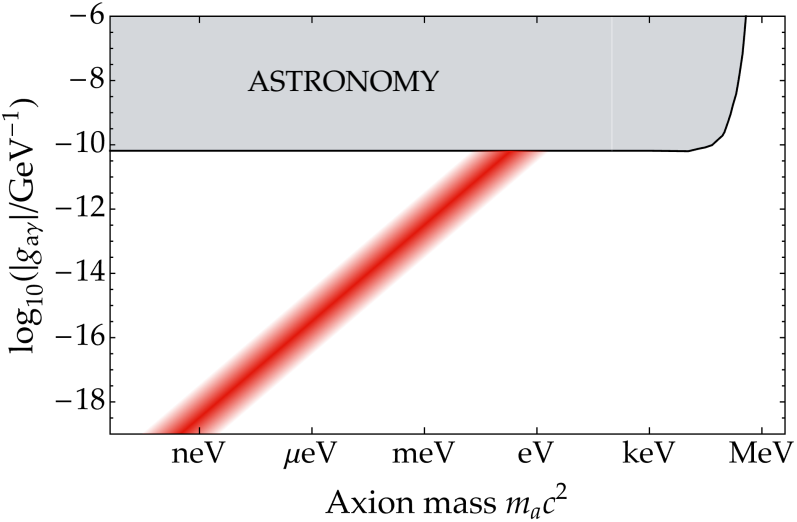

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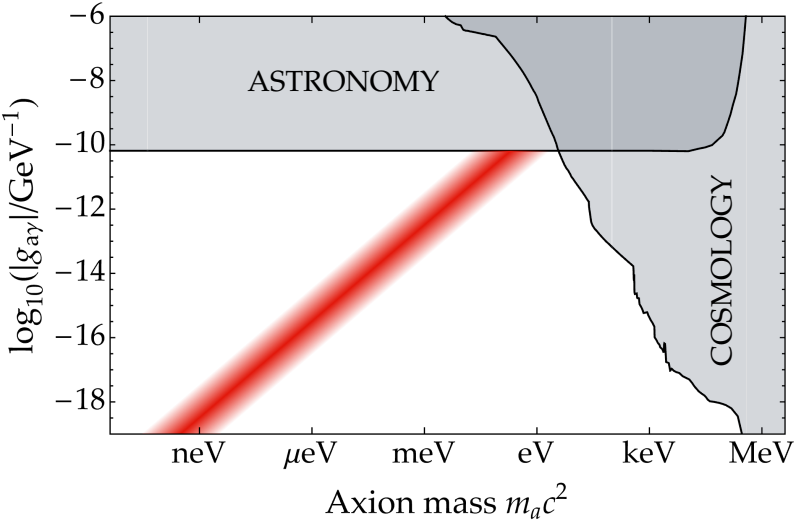

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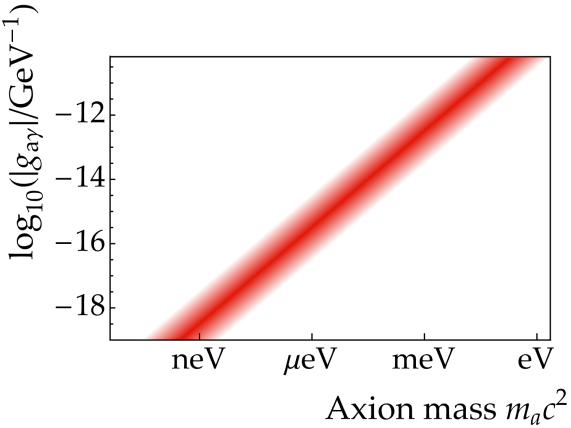

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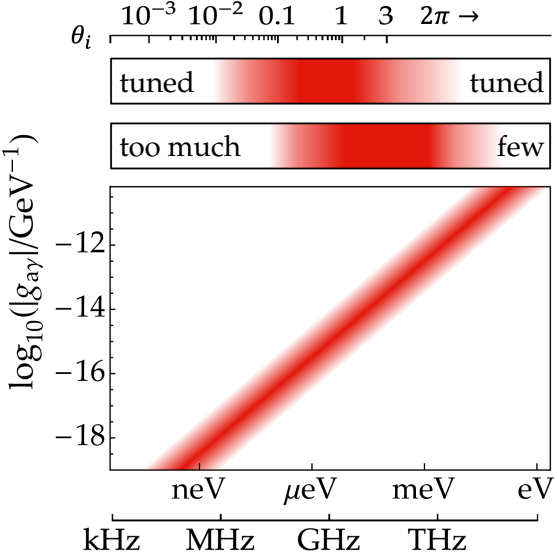
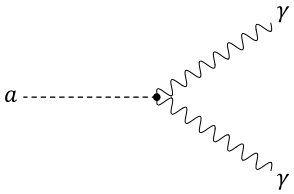


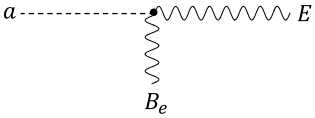
NARROW SIGNAL $Q \sim 10^6$
ORDERS OF MAGNITUDE

A CLASSIC APPROACH



Coherent oscillations
Macroscopic wave behavior

$$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma}}{4} a\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$



A CLASSIC APPROACH

$$\epsilon \nabla \cdot \boldsymbol{E} = \rho - g_{a\gamma} \boldsymbol{B}_{\mathrm{e}} \cdot \nabla a$$
$$\nabla \times \boldsymbol{H} - \dot{\boldsymbol{E}} = \boldsymbol{j} + g_{a\gamma} (\boldsymbol{B}_{\mathrm{e}} \dot{a} - \boldsymbol{E} \times \nabla a)$$
$$\ddot{a} - \nabla^2 a + m_a^2 a = g_{a\gamma} \boldsymbol{E} \cdot \boldsymbol{B}_{\mathrm{e}}$$

Signature

EM radiation in vacuum in the presence of a magnetic field

A CLASSIC APPROACH

$$\epsilon \nabla \cdot \boldsymbol{E} = \rho - g_{a\gamma} \boldsymbol{B}_{\mathrm{e}} \cdot \nabla a$$
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$$\ddot{a} - \nabla^2 a + m_a^2 a = g_{a\gamma} \boldsymbol{E} \cdot \boldsymbol{B}_{\mathrm{e}}$$

$$|\boldsymbol{E}| = \frac{g_{a\gamma} B_{\mathrm{e}} a_0}{\epsilon} e^{-i m_a t}$$
$$|\boldsymbol{E}| = 1.3 \cdot 10^{-12} \, |C_{a\gamma}| \left(\frac{B_{\mathrm{e}}}{10\,\mathrm{T}} \right) \left(\frac{0.3\,\mathrm{GeV\,cm^{-3}}}{\rho_a} \right)^{1/2} \frac{\mathrm{V}}{\mathrm{m}}$$

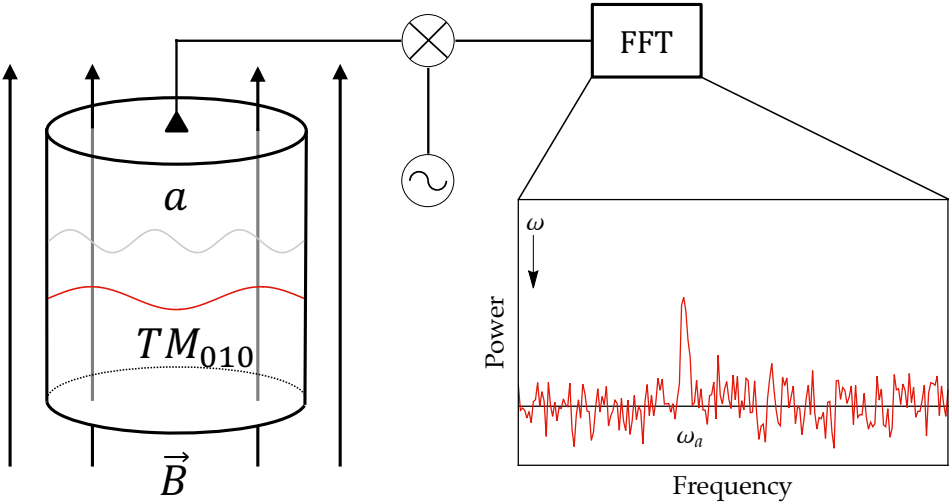
A CLASSIC APPROACH

SIKIVIE'S HALOSCOPE

$$|\boldsymbol{E}| = g_{a\gamma} B_{\mathrm{e}} a_0 \left(1 - \frac{\omega_p}{\omega_a^2 - i\omega_a^2 \Gamma_p}\right)^{-1}$$

[3] P. Sikivie, Phys. Rev. Lett. 51, 1415 (1983).

SIKIVIE'S HALOSCOPE



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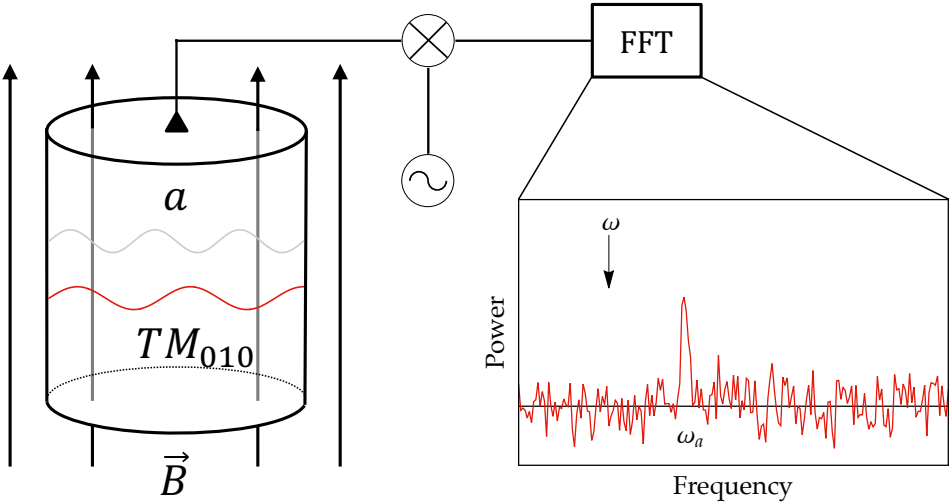

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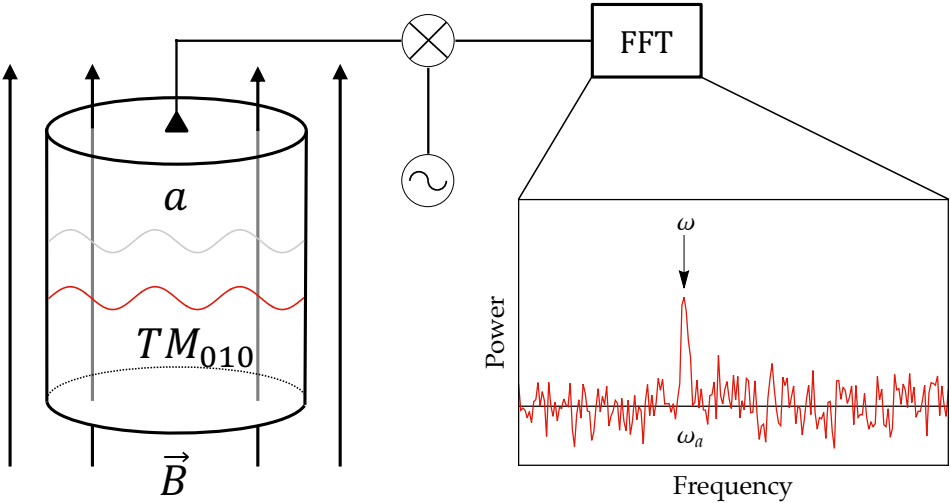
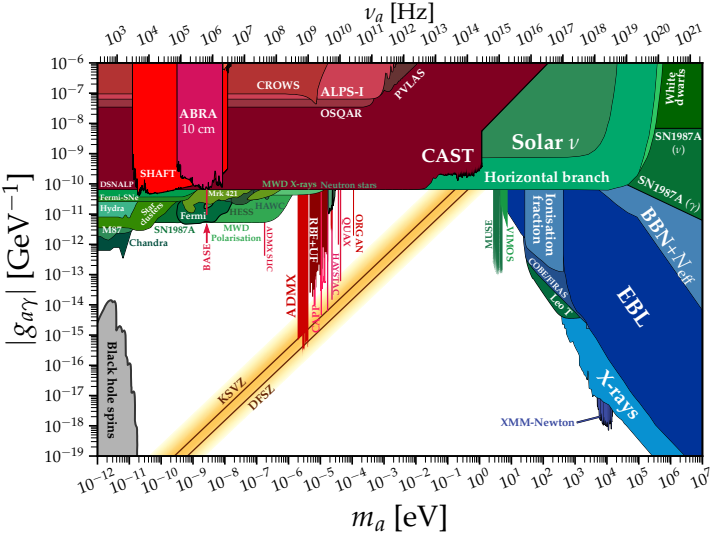


CURRENT LIMITS

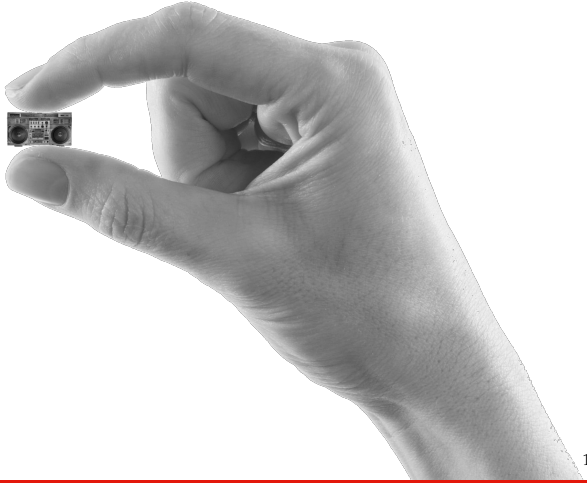


[4] O' Hare, `cajohare/AxionLimits:AxionLimits`.

MATCHING WAVELENGTHS

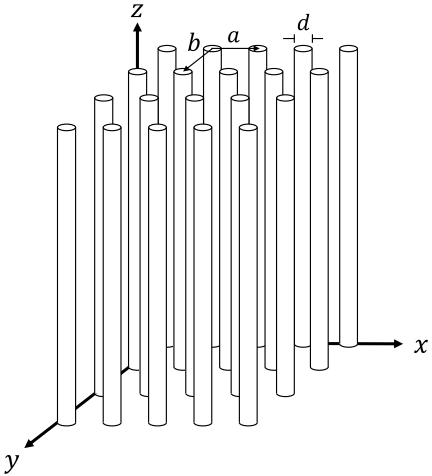
Desiderata

- Cryogenic temperature
- Tunability
- Large volume
- “Low” plasma frequency



WIRE METAMATERIALS

Metamaterials
Composite materials with different properties than their single parts

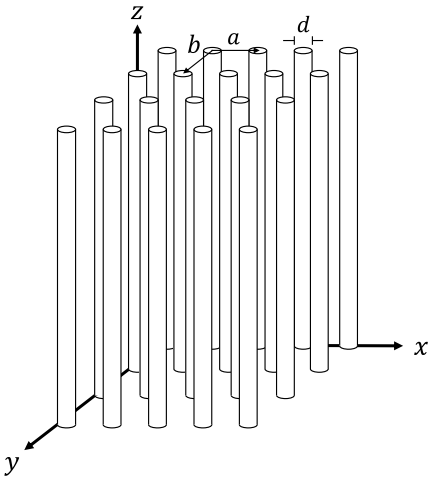


[5] P.A. Belov *et al.*, J. Electromagn. Waves. Appl. 16, 8 (2002).

WIRE METAMATERIALS

$$\frac{\omega_p^2}{c^2} = \frac{2\pi}{ab}\left[\log\left(\frac{\sqrt{a\,b}}{\pi\,d}\right) + F\left(\frac{a}{b}\right)\right]^{-1}$$

$$F(u) = -\frac{\log u}{2} + \sum_{n=1}^{\infty}\left(\frac{\coth(\pi n u) - 1}{n}\right) + \frac{\pi u}{6}$$



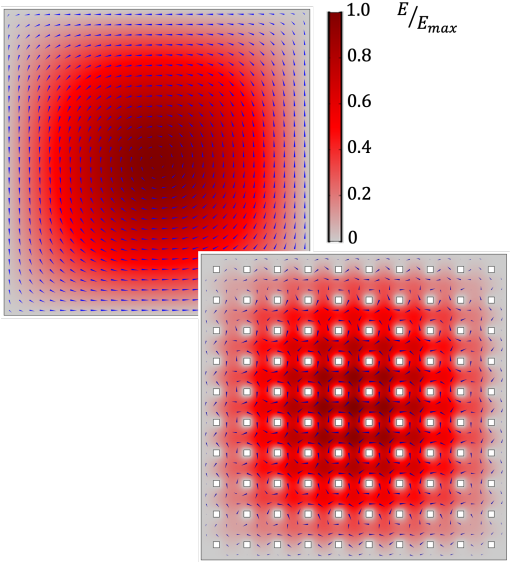
[5] P.A. Belov *et al.*, J. Electromagn. Waves. Appl. 16, 8 (2002).

WIRE METAMATERIALS

TM_{110} mode structure

Behavior as an effective medium

Field distortion near wires might complicate readout



[6] A. Millar *et al.*, arXiv:2210:00017.

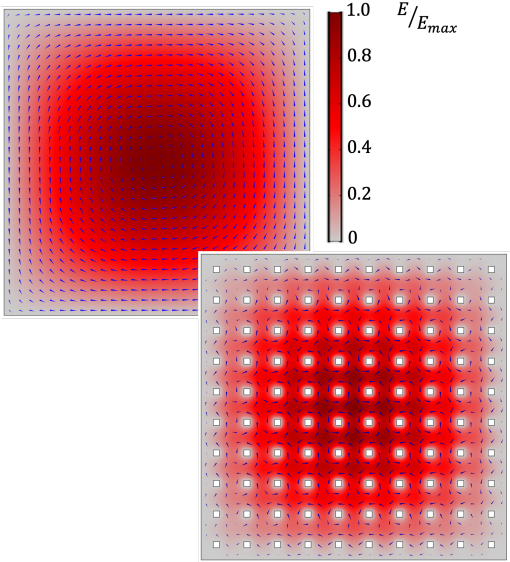
WIRE METAMATERIALS

PROs

- Cryogenic
- Solenoidal magnet
- Much larger volume than cavities

CONs

- Behind on R&D



[6] A. Millar *et al.*, arXiv:2210:00017.

2. ALPHA

Axion Longitudinal Plasma HAloscope

A. Millar *et al.*, arXiv:2210.00017, accepted by PRD

ALPHA CONSORTIUM

Fermilab	**Stockholm University & OKC**
IIT Chicago	**UC Berkeley**
IIT Kanpur	UC Davis
ITMO University	UCL London
MIT Cambridge	University of Maryland
Niels Bohr Institute	University of Oxford
Oak Ridge National Labs	Uppsala University

POWER IN THE DETECTOR

$$P_s = \left(\frac{\rho_a}{m_a} g_{a\gamma}^2\right) \kappa B_e^2 Q \mathcal{G} V$$

Quality factor Q

System dampening

$$Q = \frac{\omega \mathcal{U}}{P_{\text{loss}}} = \frac{\omega}{c\,\Gamma_p}$$

Geometry factor $\mathcal{G}$

Normalization of stored energy

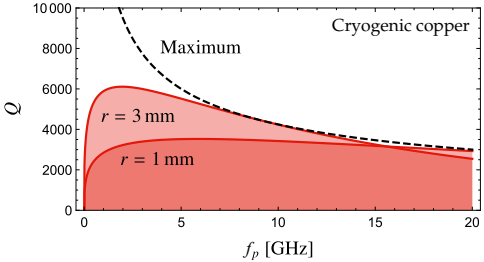
$$\mathcal{G} = \frac{1}{a_0^2 g_{a\gamma}^2 B_e^2 V Q^2} \int |\boldsymbol{E}|^2 \,\mathrm{d}V$$

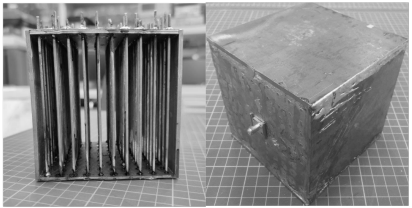
Volume V

QUALITY FACTOR

- Tightly bound to design
  ↪ (material, shape, volume…)
- Theory ✓ Simulation ✓ Experiment
- $Q \sim$ some 1000s
  ↪ $\times 10$ than expected
- Wire losses $>$ surface
  ↪ Asymptotically $P_s \propto V$

SINGLE CAVITY MODE BREAKS DOWN





[7] R. Balafendie *et al.*, PRB 106, 075106 (2022).

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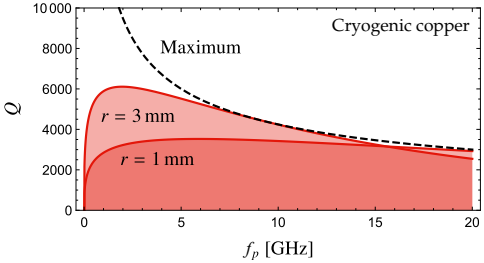
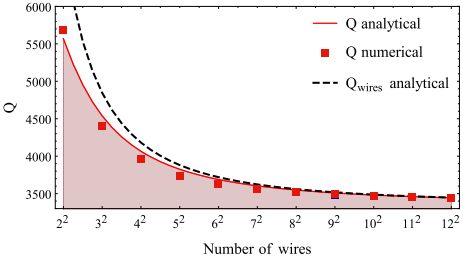




SINGLE CAVITY MODE BREAKS DOWN

[7] R. Balafendie *et al.*, PRB 106, 075106 (2022).

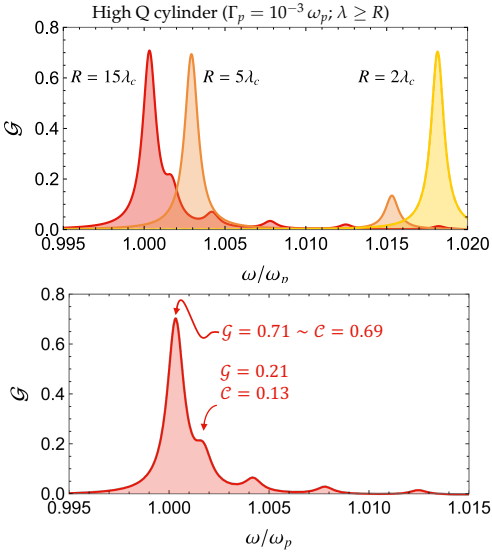
GEOMETRY FACTOR

For single-mode systems

$$\mathcal{G} = \mathcal{C} = \frac{1}{\overline{B}_e^2 V^2} \left(\int B_e \mathcal{E}_i \, \mathrm{d}V \right)^2$$

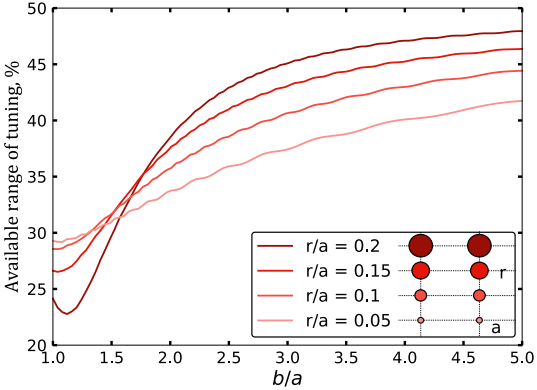
For multi-mode systems

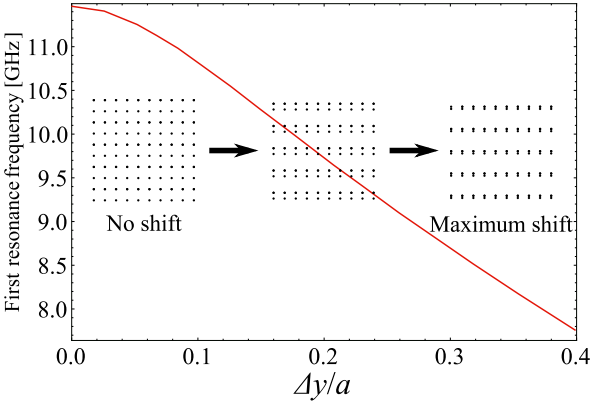
MORE THAN ONE ANTENNA NEEDED



[6] A. Millar *et al.*, arXiv:2210:00017.

TUNING





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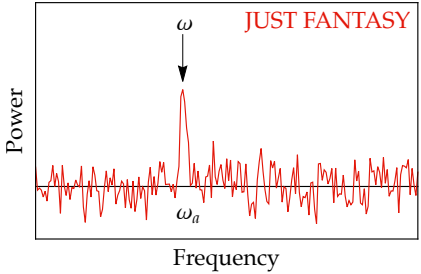
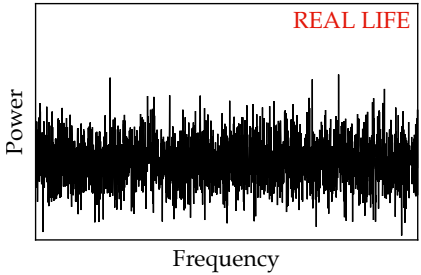
3. Statistical inference

Searching strategies and sensitivity projections

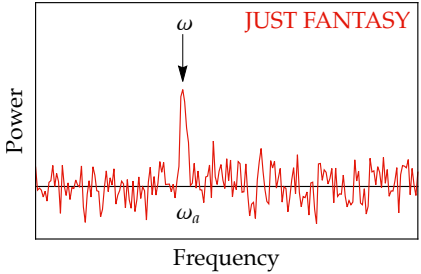
A. Gallo Rosso *et al.*, arXiv:2210.16095

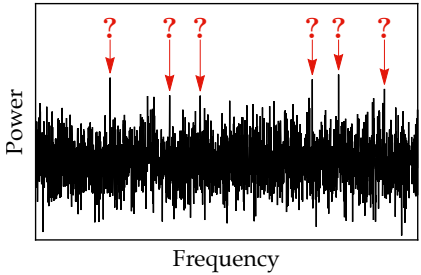
TUNING IN


JUST FANTASY
ω
ω_a
Power
Frequency


REAL LIFE
Power
Frequency

TUNING IN


JUST FANTASY
ω
ω_a
Power
Frequency


?
?
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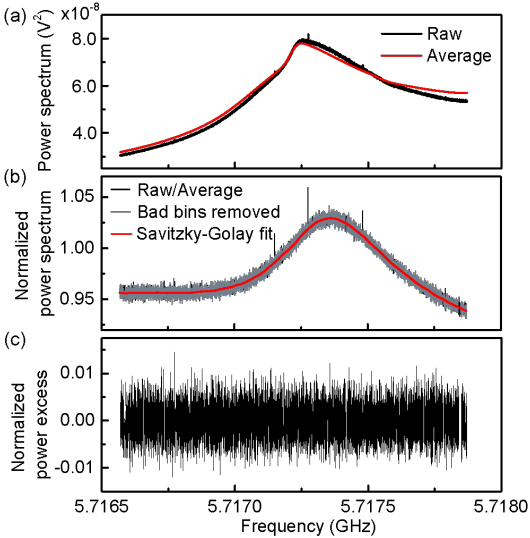
?
?
?
Power
Frequency

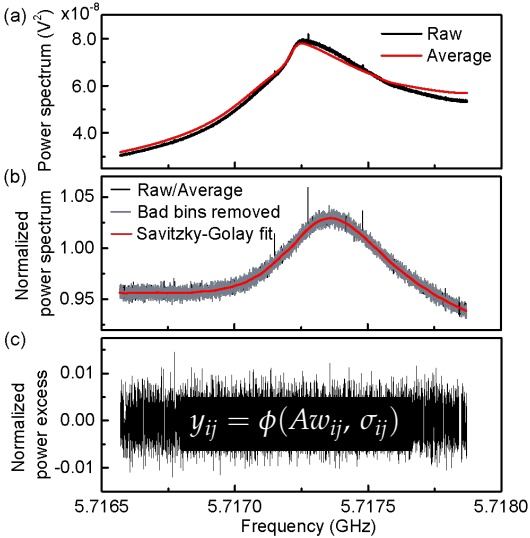
SCANNING STRATEGY

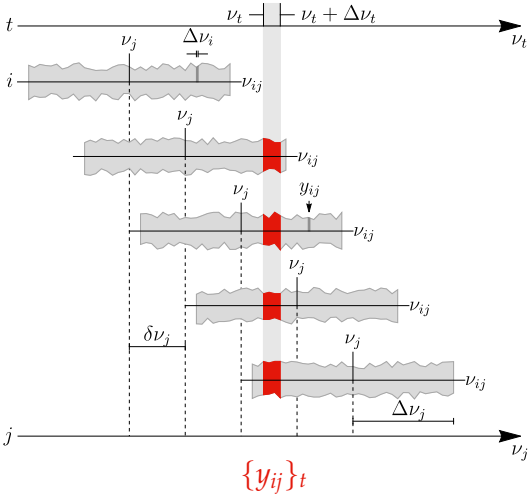
	1	2	3	4
v_1	✘			
v_2	✘			
v_3	✔	✔	✘	
v_4	✘			
v_5	✔	✘		
v_6	✔	✔	✔	✔
v_7	✘			

SCANNING STRATEGY

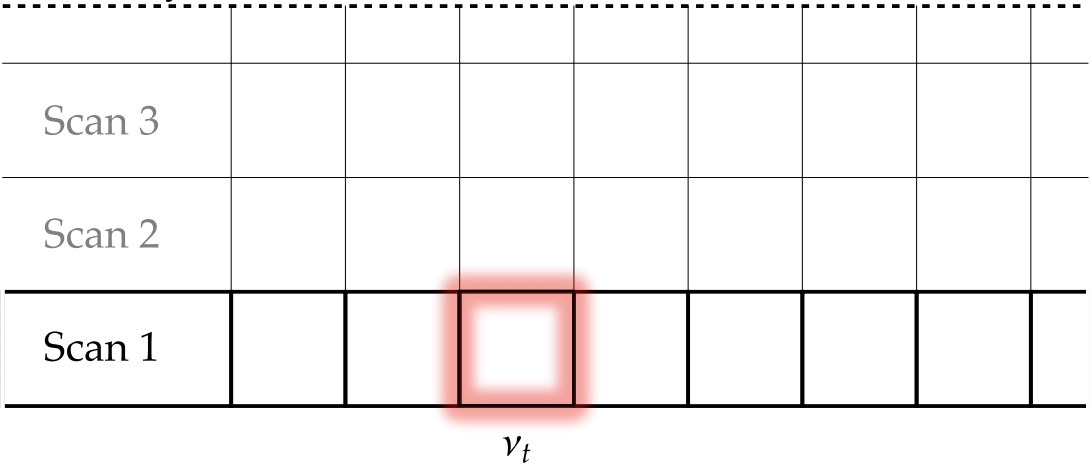


SCANNING STRATEGY





Discovery



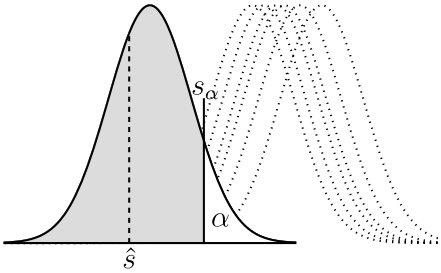
ONE FREQUENCY, ONE SCAN

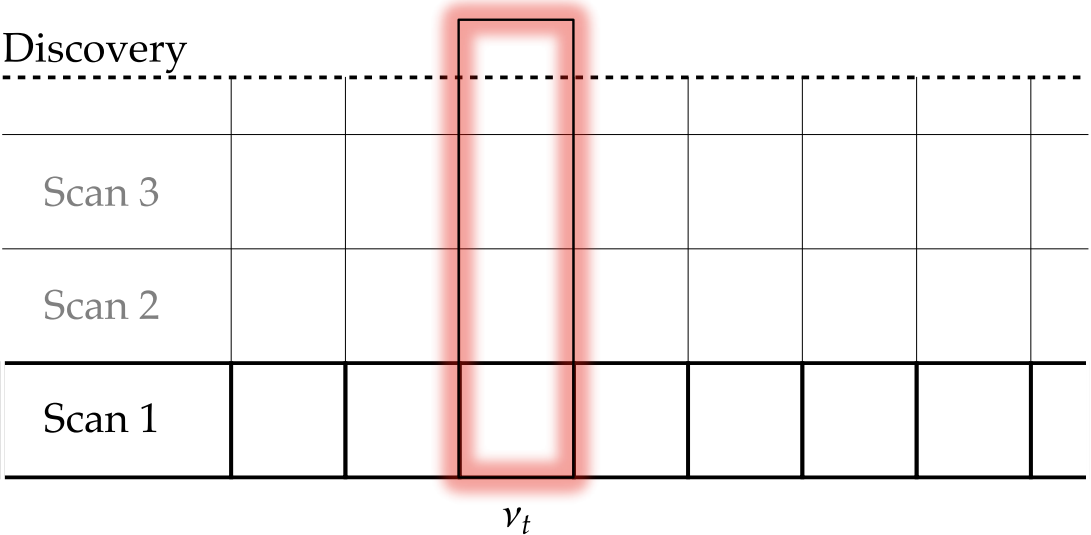
Data $\{y_{ij}\}_t$

Fluctuations $$x = \sum_{i \in \mathcal{I}} \sum_{j=1}^{N_i} \frac{w_{ij} y_{ij}}{\sigma_{ij}^2}$$

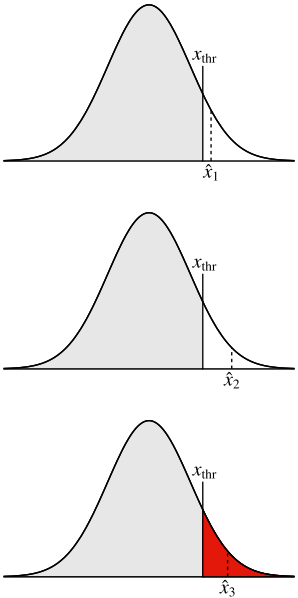
Configurations $$u = \sum_{i \in \mathcal{I}} \sum_{j=1}^{N_i} \frac{w_{ij}^2}{\sigma_{ij}^2}$$

Test statistics $$s = \frac{x}{\sqrt{u}}$$




Discovery
Scan 3
Scan 2
Scan 1
ν_t

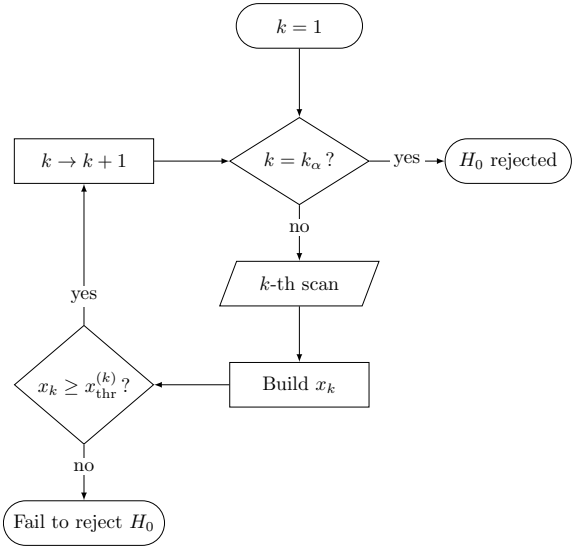
GEOMETRIC TEST



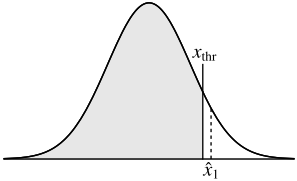
$\hat{x}_1 \geq x_{\text{thr}}$

$\hat{x}_2 \geq x_{\text{thr}}$

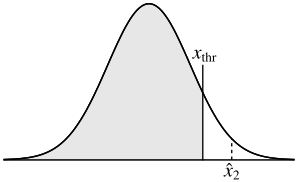
$\boxed{\hat{x}_3 \geq x_{\text{thr}}}$



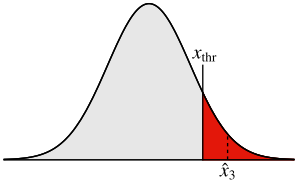
GEOMETRIC TEST



$$\hat{x}_1 \geq x_{\text{thr}}$$



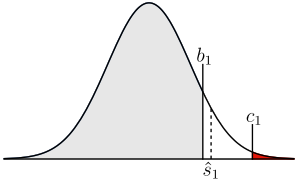
$$\hat{x}_2 \geq x_{\text{thr}}$$



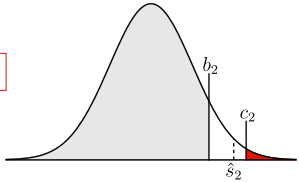
$$\boxed{\hat{x}_3 \geq x_{\text{thr}}}$$

LIKELIHOOD BASED

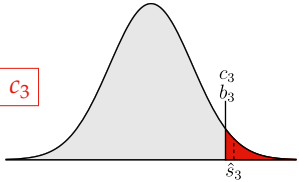
$$\hat{s}_1 = \frac{\hat{x}_1}{\sqrt{u_1}} \begin{cases} \boxed{\geq c_1} \\ \geq b_1 \end{cases}$$



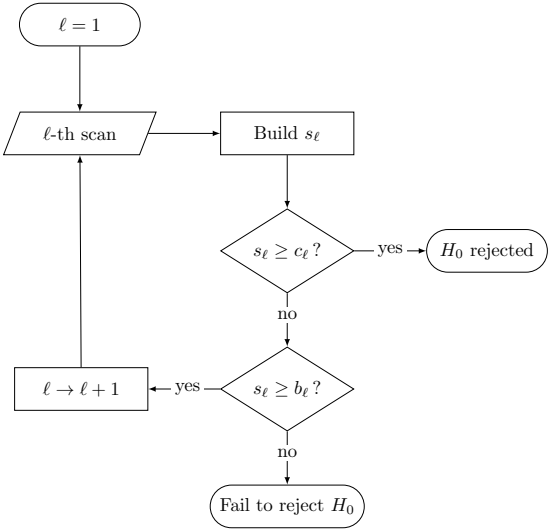
$$\hat{s}_2 = \frac{\hat{x}_1 + \hat{x}_2}{\sqrt{u_1 + u_2}} \begin{cases} \boxed{\geq c_2} \\ \geq b_2 \end{cases}$$



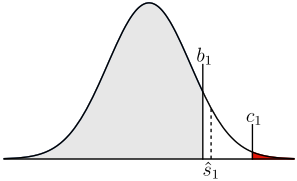
$$\hat{s}_3 = \frac{\hat{x}_1 + \hat{x}_2 + \hat{x}_3}{\sqrt{u_1 + u_2 + u_3}} \boxed{\geq c_3}$$



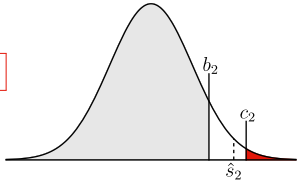
LIKELIHOOD BASED



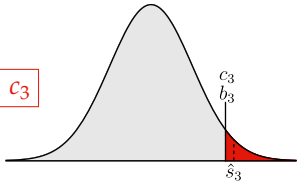
$$\hat{s}_1 = \frac{\hat{x}_1}{\sqrt{u_1}} \begin{cases} \boxed{\geq c_1} \\ \geq b_1 \end{cases}$$



$$\hat{s}_2 = \frac{\hat{x}_1 + \hat{x}_2}{\sqrt{u_1 + u_2}} \begin{cases} \boxed{\geq c_2} \\ \geq b_2 \end{cases}$$



$$\hat{s}_3 = \frac{\hat{x}_1 + \hat{x}_2 + \hat{x}_3}{\sqrt{u_1 + u_2 + u_3}} \boxed{\geq c_3}$$

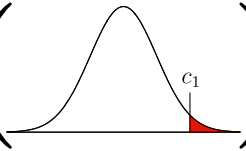


GEOMETRIC TEST

$$\underbrace{\left[P\left(x_\ell > x_{\text{thr}} \middle| H_0\right)\right]}_{\text{Rescan probability}}^{k_\alpha - 1} \leq \alpha$$

$k_\alpha =$ number of scans

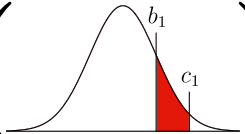
LIKELIHOOD BASED APPROACH

$P\Bigg($  $\Bigg) = \alpha / k_\alpha$

- ✓ c_1
- ✓ b_1
- ✓ c_2
- ✓ b_2

$k_\alpha =$ number of scans $\qquad q_\ell =$ rescan probability

LIKELIHOOD BASED APPROACH

$P\Big($  $\Big) = q_1$

✓ c_1

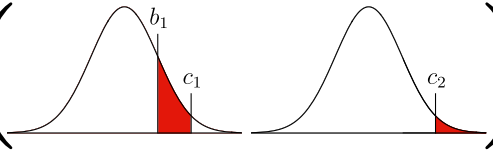
✓ b_1

✓ c_2

✓ b_2

$k_\alpha =$ number of scans $\qquad q_\ell =$ rescan probability

LIKELIHOOD BASED APPROACH

$P\Bigg($ 

$\Bigg) = \alpha / k_\alpha$

✓ c_1

✓ b_1

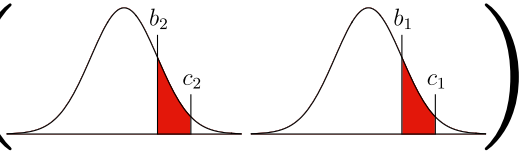
✓ c_2

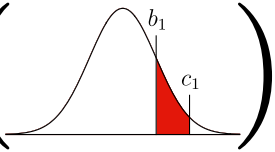
✓ b_2

$k_\alpha =$ number of scans $\qquad q_\ell =$ rescan probability

LIKELIHOOD BASED APPROACH

$$\frac{P\left(\text{[illustration]}\right)}{P\left(\text{[illustration]}\right)} = q_2$$





✓ c_1

✓ b_1

✓ c_2

✓ b_2

$k_\alpha =$ number of scans $\qquad q_\ell =$ rescan probability

LIKELIHOOD BASED APPROACH

$$\frac{\alpha}{k_\alpha} = P(D_\ell | H_0) =$$

$$P(s_\ell \geq c_\ell, b_{\ell-1} \leq s_{\ell-1} < c_{\ell-1}, \ldots, b_1 \leq s_1 < c_1) =$$

$$\int_{c_\ell}^{\infty} \mathrm{d}s_\ell \int_{b_{\ell-1}}^{c_{\ell-1}} \mathrm{d}s_{\ell-1} \cdots \int_{b_1}^{c_1} \mathrm{d}s_1 \, \mathcal{N}_\ell(\boldsymbol{s} | \, \boldsymbol{0}, \boldsymbol{\Sigma_s})$$

$$P\left(C_\ell^c \cap B_\ell \middle| \bigcap_{h=1}^{\ell-1} \{C_h^c \cap B_h\}, H_0 \right) =$$

$$P\left(\bigcap_{h=1}^{\ell} \{C_h^c \cap B_h\} \middle| H_0 \right) \Big/ P\left(\bigcap_{h=1}^{\ell-1} \{C_h^c \cap B_h\} \middle| H_0 \right) =$$

$$\frac{\int_{b_\ell}^{c_\ell} \mathrm{d}s_\ell \cdots \int_{b_1}^{c_1} \mathrm{d}s_1 \, \mathcal{N}_\ell(\boldsymbol{s} | \boldsymbol{\mu_s}, \boldsymbol{\Sigma_s})}{\int_{b_{\ell-1}}^{c_{\ell-1}} \mathrm{d}s_{\ell-1} \cdots \int_{b_1}^{c_1} \mathrm{d}s_1 \, \mathcal{N}_{\ell-1}(\boldsymbol{s} | \boldsymbol{\mu_s}, \boldsymbol{\Sigma_s})} = q_\ell$$

✓ c_1

✓ b_1

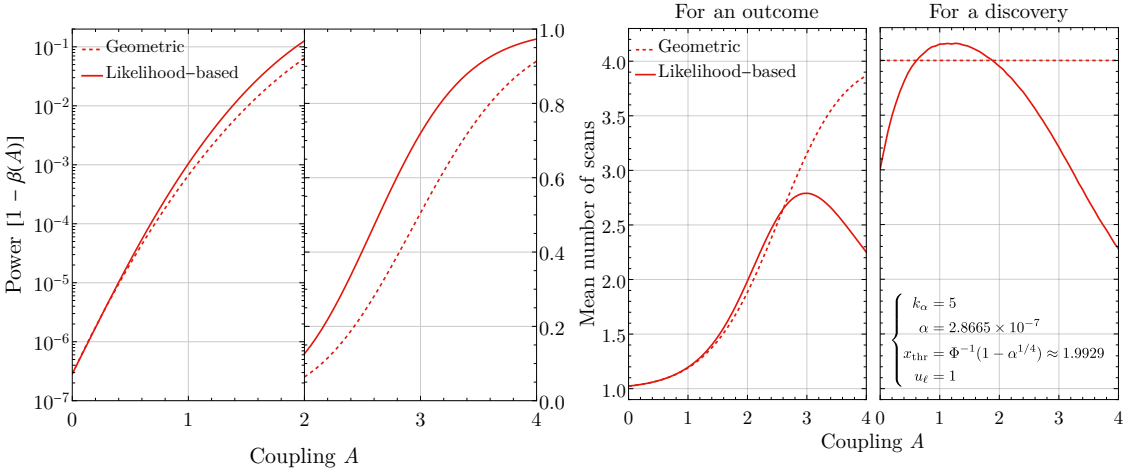
✓ c_2

✓ b_2

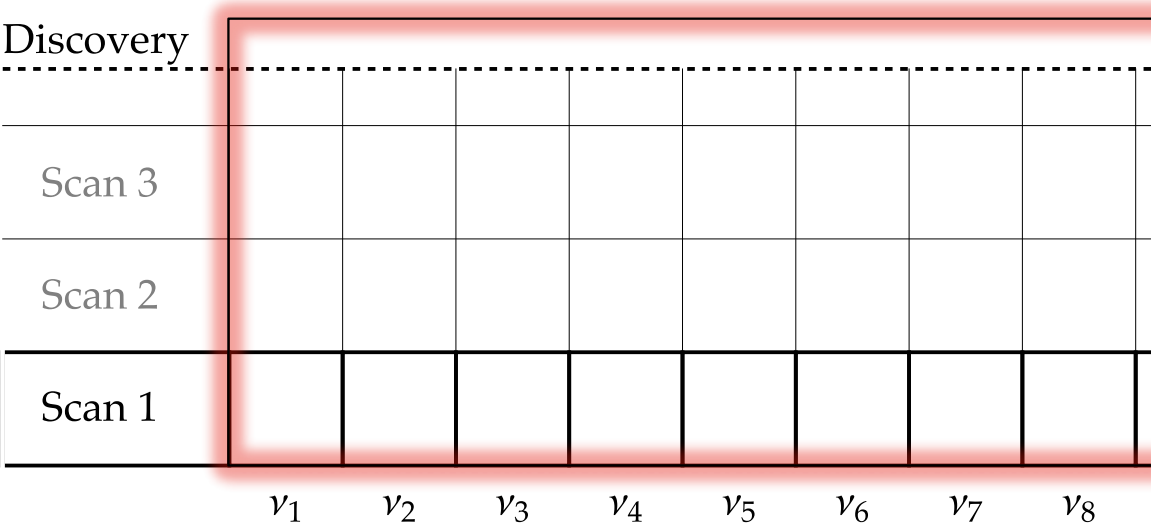
✓ $\ldots$

$k_\alpha =$ number of scans $\qquad q_\ell =$ rescan probability

ONE FREQUENCY, MANY SCANS



[8] A. Gallo Rosso *et al.*, arXiv:2210.16095.


Discovery
Scan 3
Scan 2
Scan 1
v_1
v_2
v_3
v_4
v_5
v_6
v_7
v_8

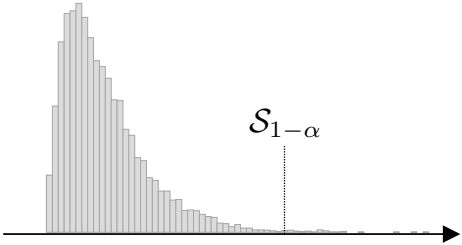
MANY FREQUENCIES, MANY SCANS

	1	2	3	4		
v_1	✘ 1				$\Rightarrow$	$s_1^1 \quad c_1^1$
v_2	✘ 1				$\Rightarrow$	$s_1^2 \quad c_1^2$
v_3	✔	✔	✘ 3		$\Rightarrow$	$s_3^3 \quad c_3^3$
v_4	✘ 1				$\Rightarrow$	$s_1^4 \quad c_1^4$
v_5	✔	✘ 2			$\Rightarrow$	$s_2^5 \quad c_2^5$
v_6	✔	✔	✔	✔ 4	$\Rightarrow$	$s_4^6 \quad c_4^6$
v_7	✘ 1				$\Rightarrow$	$s_1^7 \quad c_1^7$

$$\max_{t=1,\ldots,T} \left\{ s_L^t - c_L^t \right\} = \mathcal{S}$$

MANY FREQUENCIES, MANY SCANS

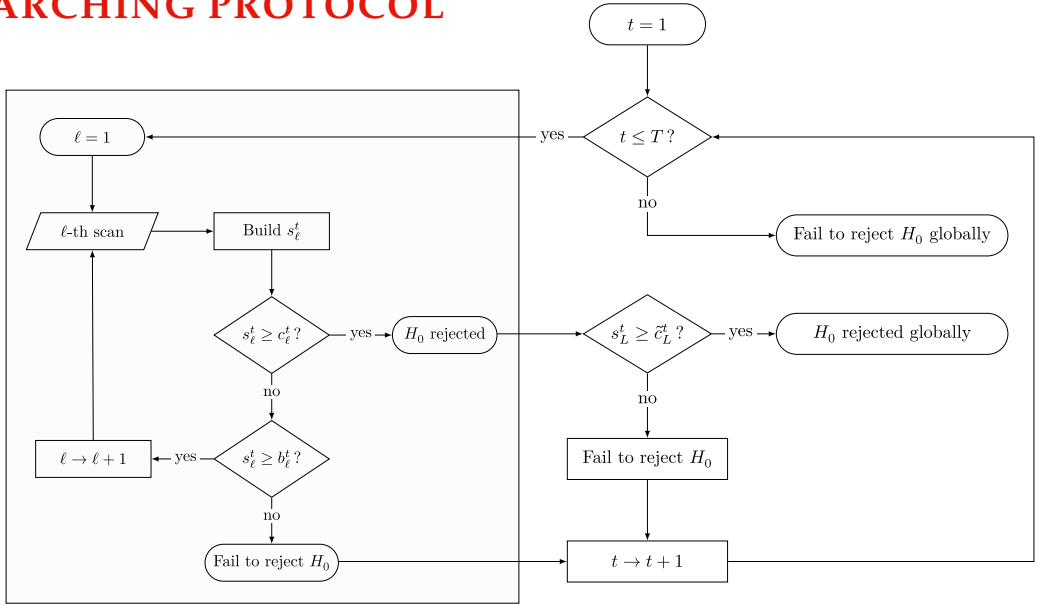
	1	2	3	4		
v_1	✘ 1				$\Rightarrow$	$s_1^1 \quad c_1^1$
v_2	✘ 1				$\Rightarrow$	$s_1^2 \quad c_1^2$
v_3	✔	✔	✘ 3		$\Rightarrow$	$s_3^3 \quad c_3^3$
v_4	✘ 1				$\Rightarrow$	$s_1^4 \quad c_1^4$
v_5	✔	✘ 2			$\Rightarrow$	$s_2^5 \quad c_2^5$
v_6	✔	✔	✔	✔ 4	$\Rightarrow$	$s_4^6 \quad c_4^6$
v_7	✘ 1				$\Rightarrow$	$s_1^7 \quad c_1^7$



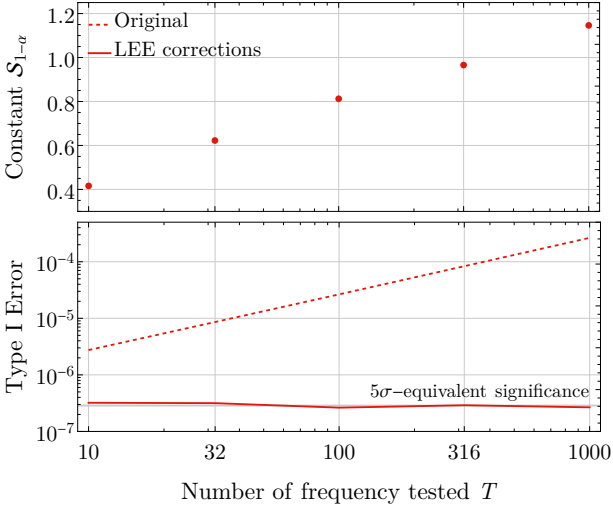
$$\max_{t=1,\ldots,T} \left\{ s_L^t - c_L^t \right\} = \mathcal{S}$$

$$\tilde{c}_L^t = c_L^t + \mathcal{S}_{1-\alpha}$$

SEARCHING PROTOCOL

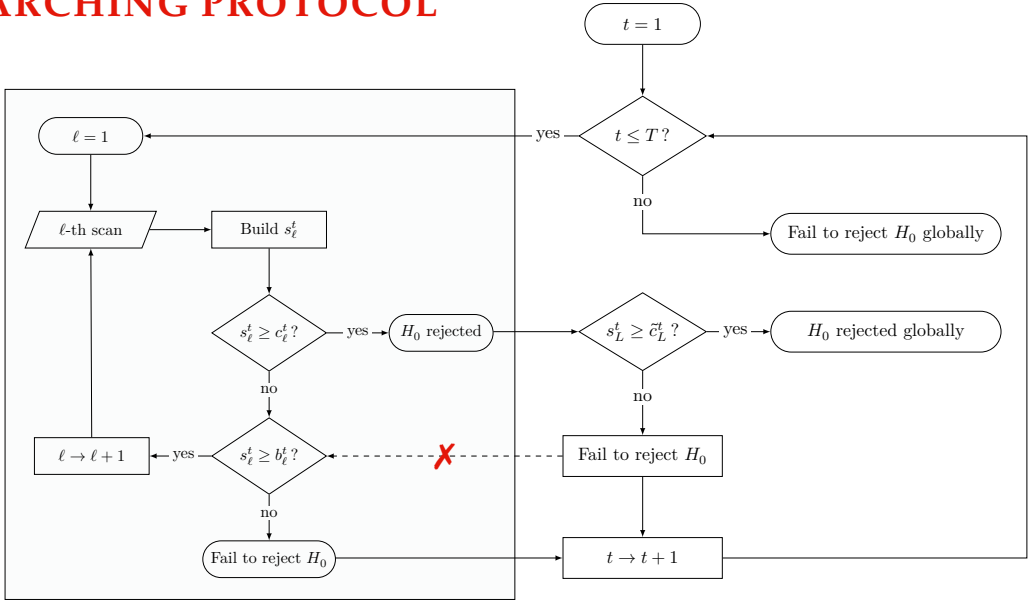


IT WORKS



[8] A. Gallo Rosso *et al.*, arXiv:2210.16095.

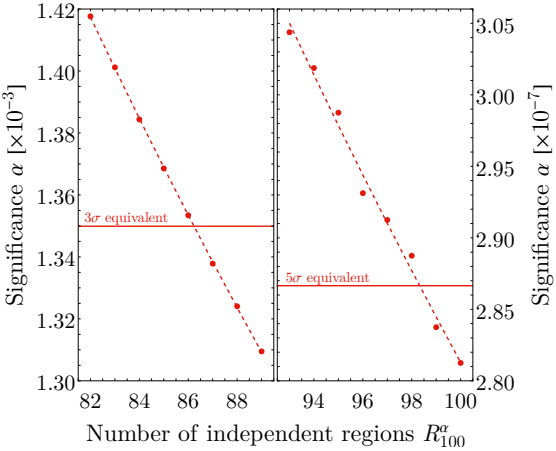
SEARCHING PROTOCOL



LOOK ELSEWHERE CORRECTIONS

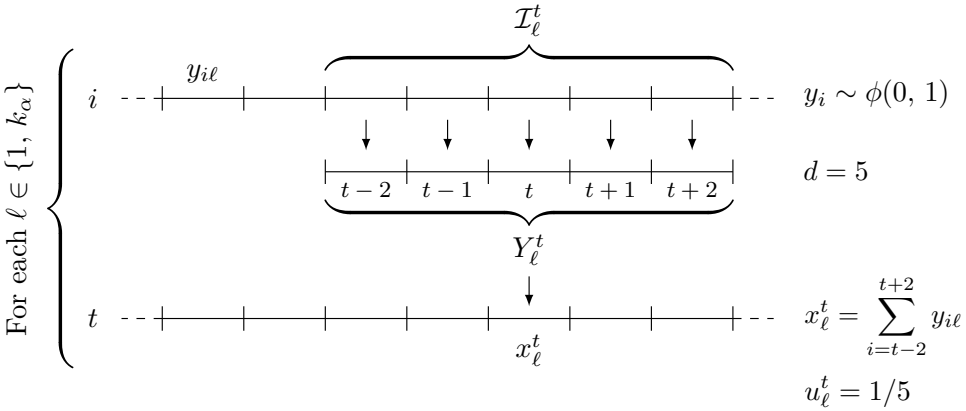
INDEPENDENT REGIONS

$$P(D_{t\ell}|H_0) = \frac{\alpha}{R_T^{\alpha} k_{\alpha}}$$



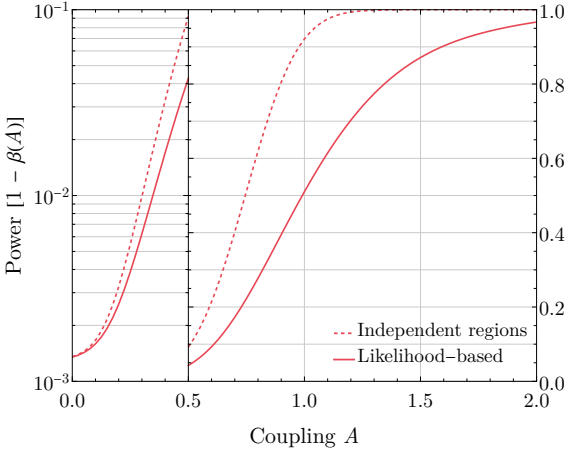
[8] A. Gallo Rosso *et al.*, arXiv:2210.16095.

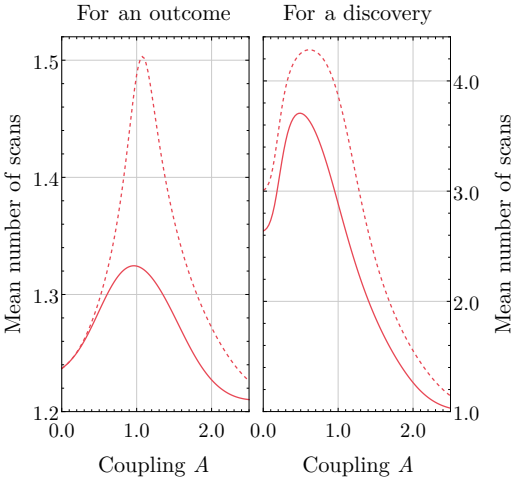
A SIMPLE EXAMPLE



[8] A. Gallo Rosso *et al.*, arXiv:2210.16095.

A SIMPLE EXAMPLE





[8] A. Gallo Rosso *et al.*, arXiv:2210.16095.

DISCOVERY REACH

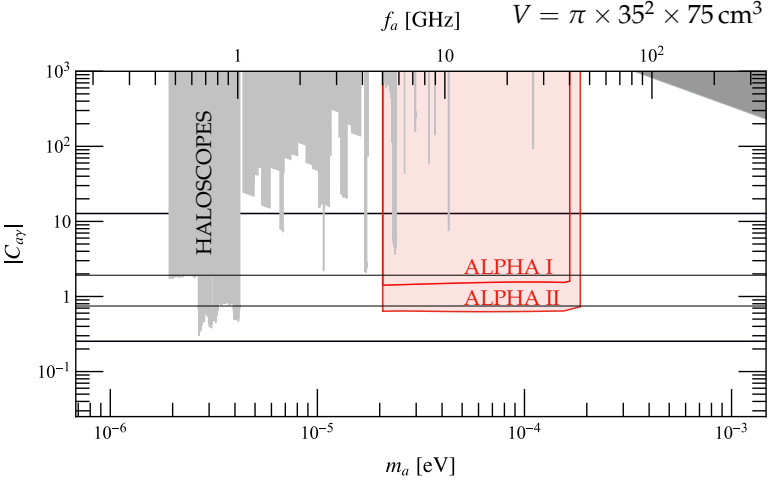
$Q \sim (1 \div 3) \cdot 10^4$
$B = 13\,\mathrm{T}$
$V = \pi \times 35^2 \times 75\,\mathrm{cm}^3$

ALPHA PHASE I

- 2 years run
- $(5 \div 40)$ GHz
- HEMT amplifiers
- Single scan mode

ALPHA PHASE II

- 2 years run
- $(5 \div 45)$ GHz
- Quantum noise
- Single scan mode



[7] A. Millar *et al.*, arXiv:2210:00017.

CONCLUSIONS

ALPHA

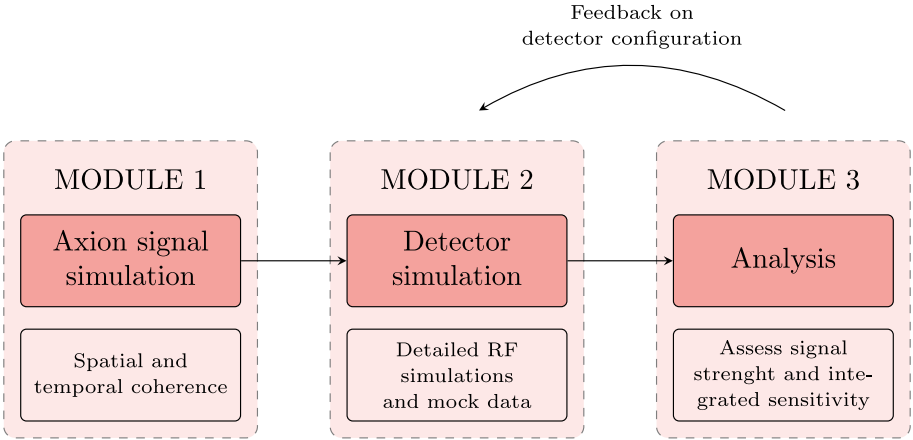
- Overall design validation
  - ✓ Theory/simulation/experiment
  - ✓ Q $\times 10$ than previously expected
  - ✓ Feasible tuning
- KSVZ and DFSZ at reach

DAQ & ANALYSIS

- Framework for inference on sequential tests
  - ✓ One frequency: Closed form for power
  - ✓ Multiple frequencies: Monte Carlo for LEE
- Protocol and computational optimizations

IMPROVE SYNERGY FOR SIMULATION & ANALYSIS

CONCLUSIONS



IMPROVE SYNERGY FOR SIMULATION & ANALYSIS