The Relic density

DarkSUSY 4.2
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Relic density

simple approach (more advanced in real life)

Decoupling occurs when

$$\Gamma < H$$

We have that

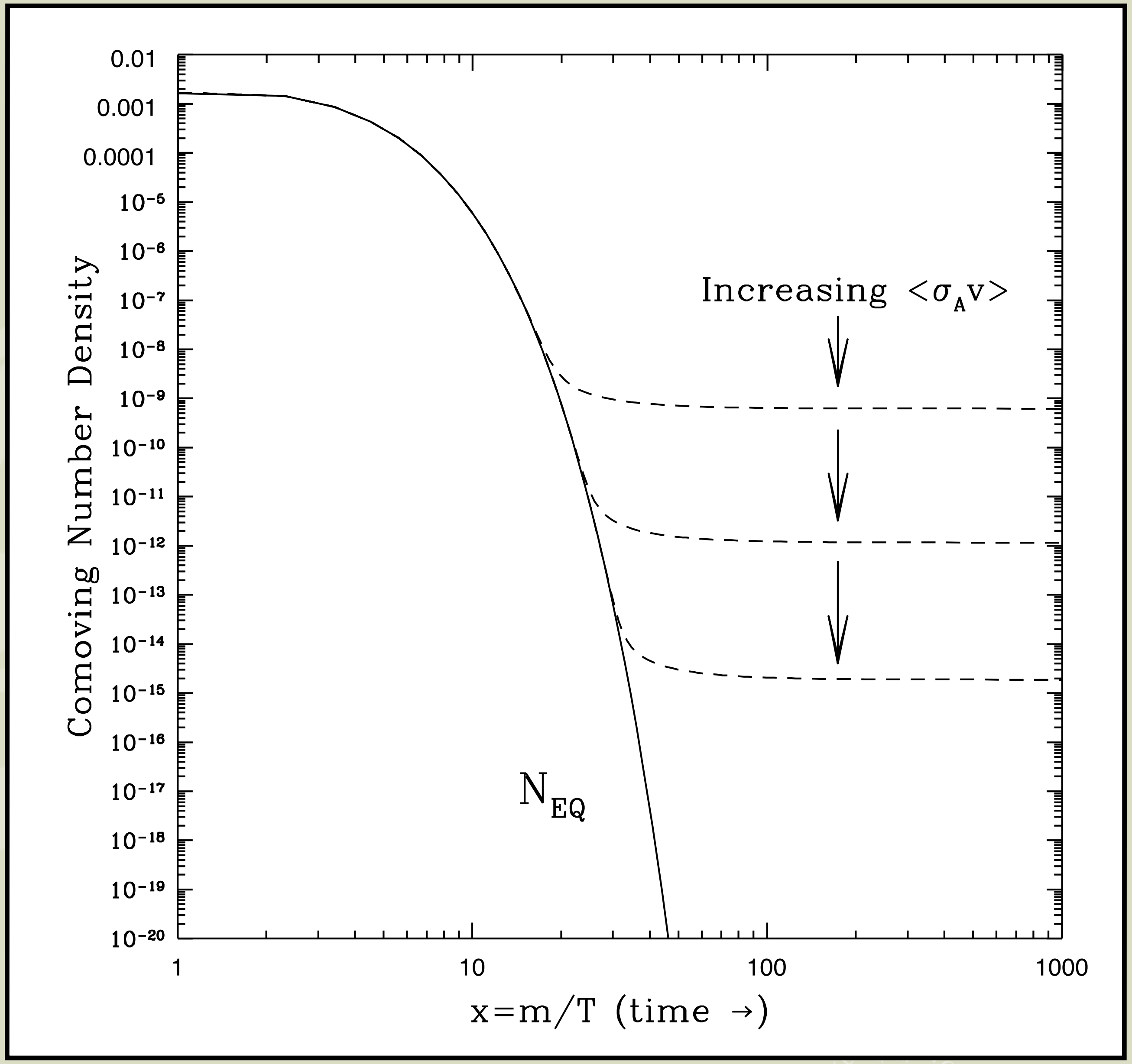
$$\Gamma = \langle \sigma_{\rm ann} v \rangle n_\chi$$

$$n_\chi^{\rm eq} = g_\chi \left(\frac{m_\chi T}{2\pi} \right)^{3/2} e^{-m_\chi / T}$$

$$H(T) = 1.66 g_*^{1/2} \frac{T^2}{m_{\rm Planck}}$$

$$\Gamma \simeq H \quad \Rightarrow \quad T_f \simeq \frac{m_\chi}{20}$$

$$\Omega_\chi h^2 \simeq \frac{3 \times 10^{-27} cm^3 s^{-1}}{\langle \sigma_{\rm ann} v \rangle}$$



$$\langle \sigma_{\rm ann} v \rangle \simeq \langle \sigma_{\rm ann} v \rangle_{WIMP} \quad \Rightarrow \quad \Omega_\chi h^2 \simeq 1$$

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Relic density – accurate approach

- Solve the Boltzmann equation

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\mathrm{ann}} v \rangle \left(n^2 - n_{\mathrm{eq}}^2 \right)$$

- properly taking the thermal average < ·>
- including the full annihilation cross section (including all final states, resonances and thresholds)
- including all so-called coannihilations between all neutralinos, charginos and sfermions

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The Boltzmann equation with coannihilations I

$$
\begin{aligned}
\frac{dn_i}{dt} \quad = \quad & -3Hn_i - \sum_{j=1}^{N} \langle \sigma_{ij} v_{ij} \rangle \left(n_i n_j - n_i^{\mathrm{eq}} n_j^{\mathrm{eq}} \right) \qquad \text{annihilation/coannihilation} \\
& \cancel{- \sum_{j \neq i} \left[\langle \sigma'_{Xij} v_{ij} \rangle \left(n_i n_X - n_i^{\mathrm{eq}} n_X^{\mathrm{eq}} \right) - \langle \sigma'_{Xji} v_{ji} \rangle \left(n_j n_X - n_j^{\mathrm{eq}} n_X^{\mathrm{eq}} \right) \right]} \\
& \qquad \qquad \text{scattering} \\
& \cancel{- \sum_{j \neq i} \left[\Gamma_{ij} \left(n_i - n_i^{\mathrm{eq}} \right) - \Gamma_{ji} \left(n_j - n_j^{\mathrm{eq}} \right) \right]} \qquad \text{decay}
\end{aligned}
$$

Sum over all SUSY particles: $n = \sum_i n_i$

$$
\Rightarrow \quad \frac{dn}{dt} = -3Hn - \sum_{i,j=1}^{N} \langle \sigma_{ij} v_{ij} \rangle \left(n_i n_j - n_i^{\mathrm{eq}} n_j^{\mathrm{eq}} \right)
$$

The Boltzmann equation with coannihilations II

Assume that $\frac{n_i}{n} \simeq \frac{n_i^{\mathrm{eq}}}{n^{\mathrm{eq}}}$ during freeze-out

We then get

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\mathrm{eff}} v \rangle \left(n^2 - n_{\mathrm{eq}}^2 \right)$$

with

$$\langle \sigma_{\mathrm{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{\mathrm{eq}}}{n^{\mathrm{eq}}} \frac{n_j^{\mathrm{eq}}}{n^{\mathrm{eq}}}$$

The same Boltzmann equation as without coannihilations, but with the annihilation cross section replaced with an effective annihilation cross section!

The effective annihilation cross section

$$\sim e^{\frac{m_{\chi_1}-m_j}{T}} \underbrace{\sim}_{T\sim m_{\chi_1}/20} e^{-20\frac{m_j-m_{\chi_1}}{m_{\chi_1}}}$$

$$\langle\sigma_{\mathrm{eff}}v\rangle = \sum_{ij}\langle\sigma_{ij}v_{ij}\rangle\frac{n_i^{\mathrm{eq}}}{n^{\mathrm{eq}}}\frac{n_j^{\mathrm{eq}}}{n^{\mathrm{eq}}}$$

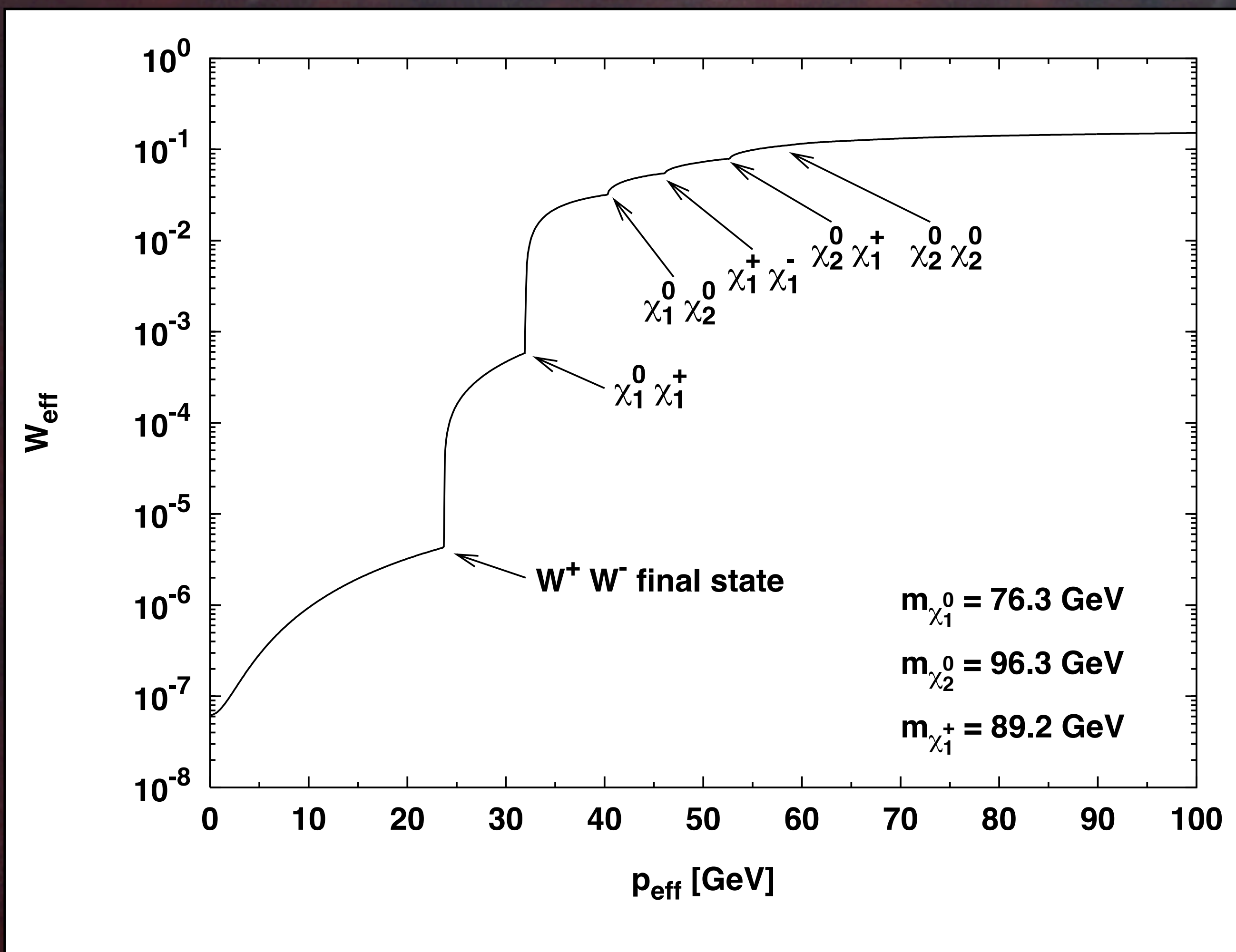
The Boltzmann suppression can be huge!

But the annihilation cross section can be even larger!

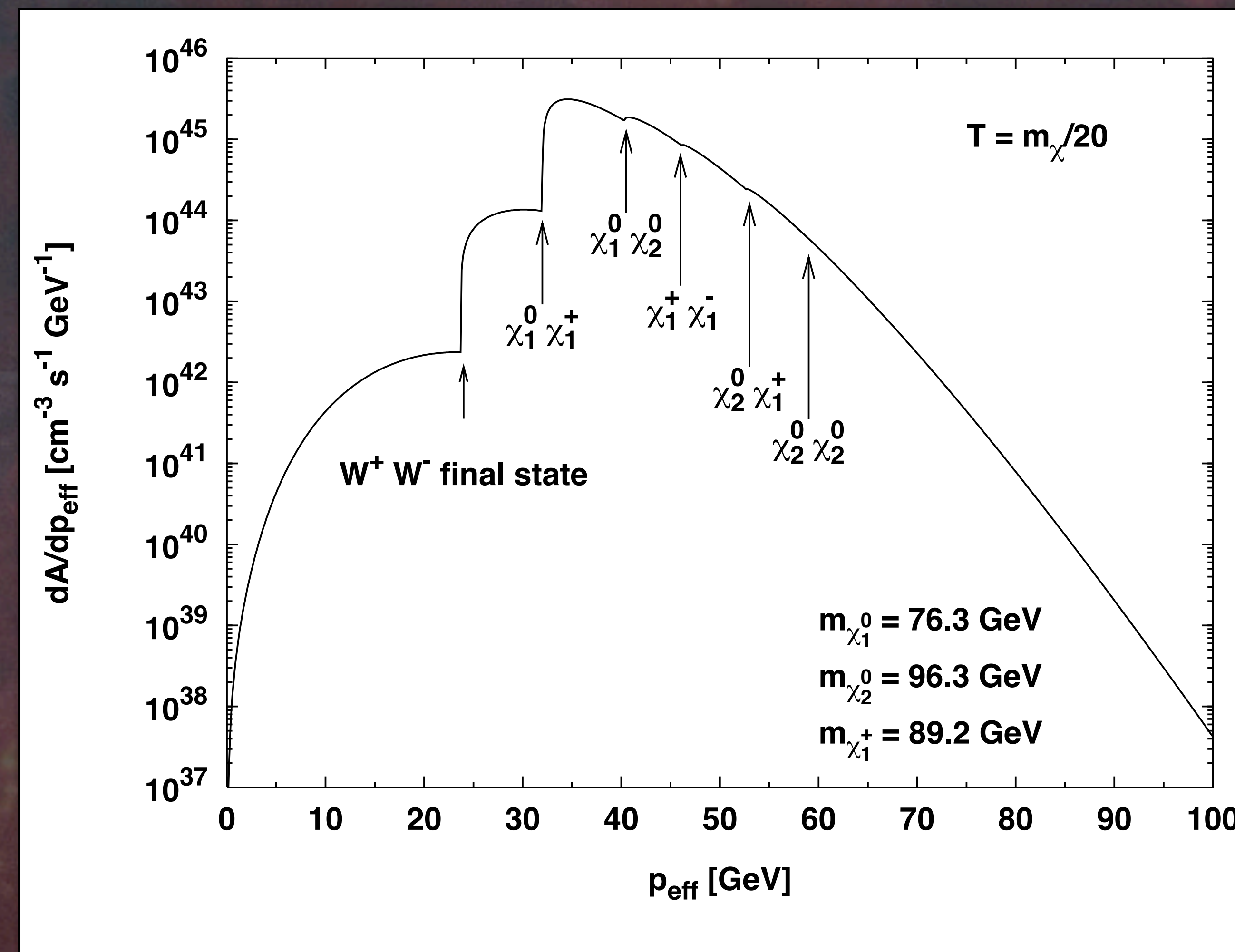
Typically, coannihilations are important if either
a) the mass differences are small, or
b) the annihilation cross sections are large

Example of coannihilation case

W_{eff}



W_{eff} x Boltzmann suppression



Initial state thresholds appear as final state thresholds in the effective annihilation rate!

Thermal averaging

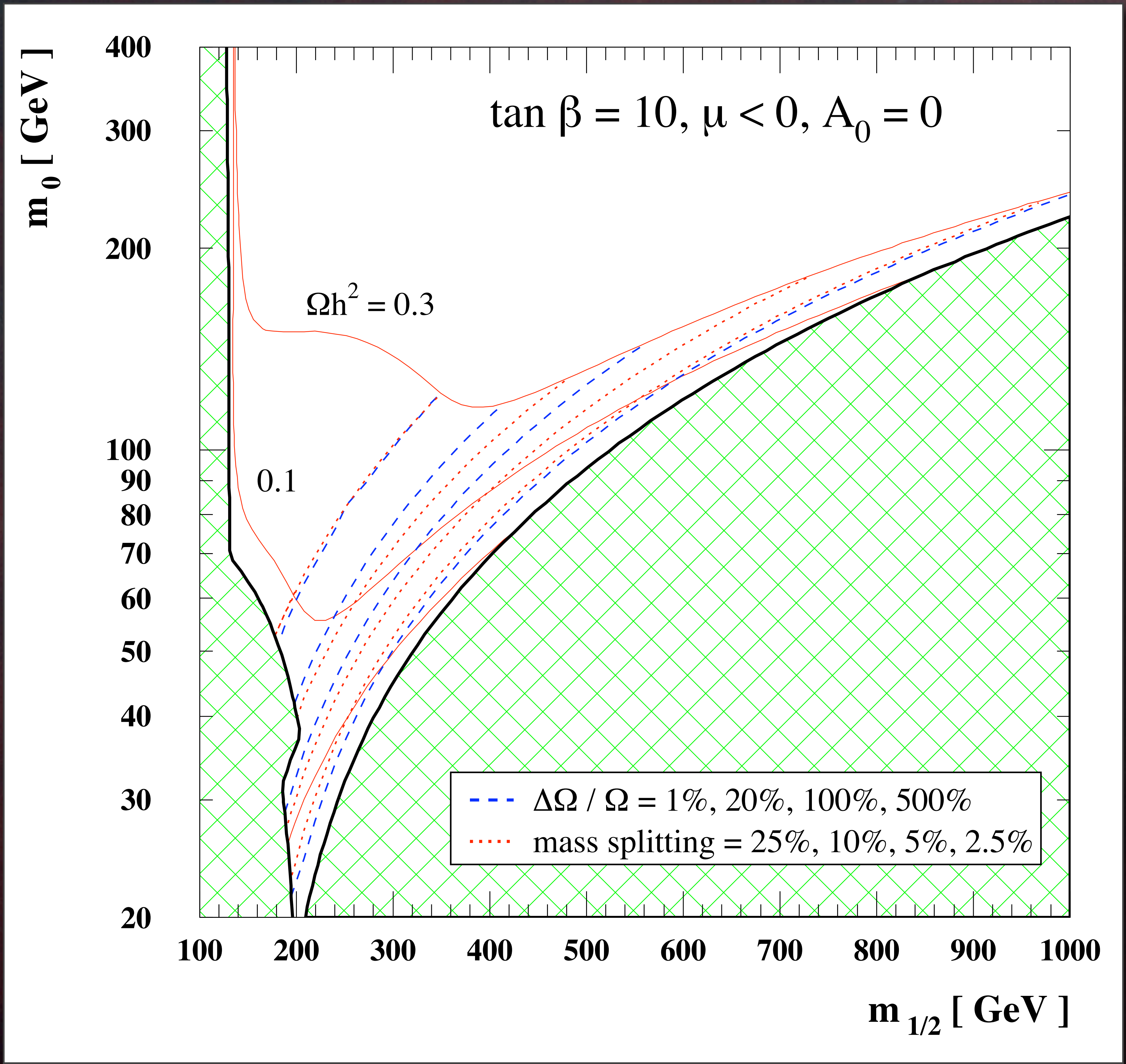
As the SUSY particles are in thermal equilibrium (due to the vast amount of SM particles), we can insert a thermal velocity distribution and calculate the thermally averaged annihilation cross section

$$\langle \sigma_{\rm eff} v \rangle = \frac{\int_0^\infty dp_{\rm eff}\, p_{\rm eff}^2 W_{\rm eff} K_1\left(\frac{\sqrt{s}}{T}\right)}{m_1^4 T \left[\sum_i \frac{g_i}{g_1}\frac{m_i^2}{m_1^2} K_2\left(\frac{m_i}{T}\right)\right]^2}$$

$$W_{\rm eff} = \sum_{ij} \frac{p_{ij}}{p_{11}} \frac{g_i g_j}{g_1^2} W_{ij} \quad ; \quad W_{ij} = 4E_1 E_2 \sigma_{ij} v_{ij}$$

See Edsjö and Gondolo, PRD 56 (1997) 1879 for details.

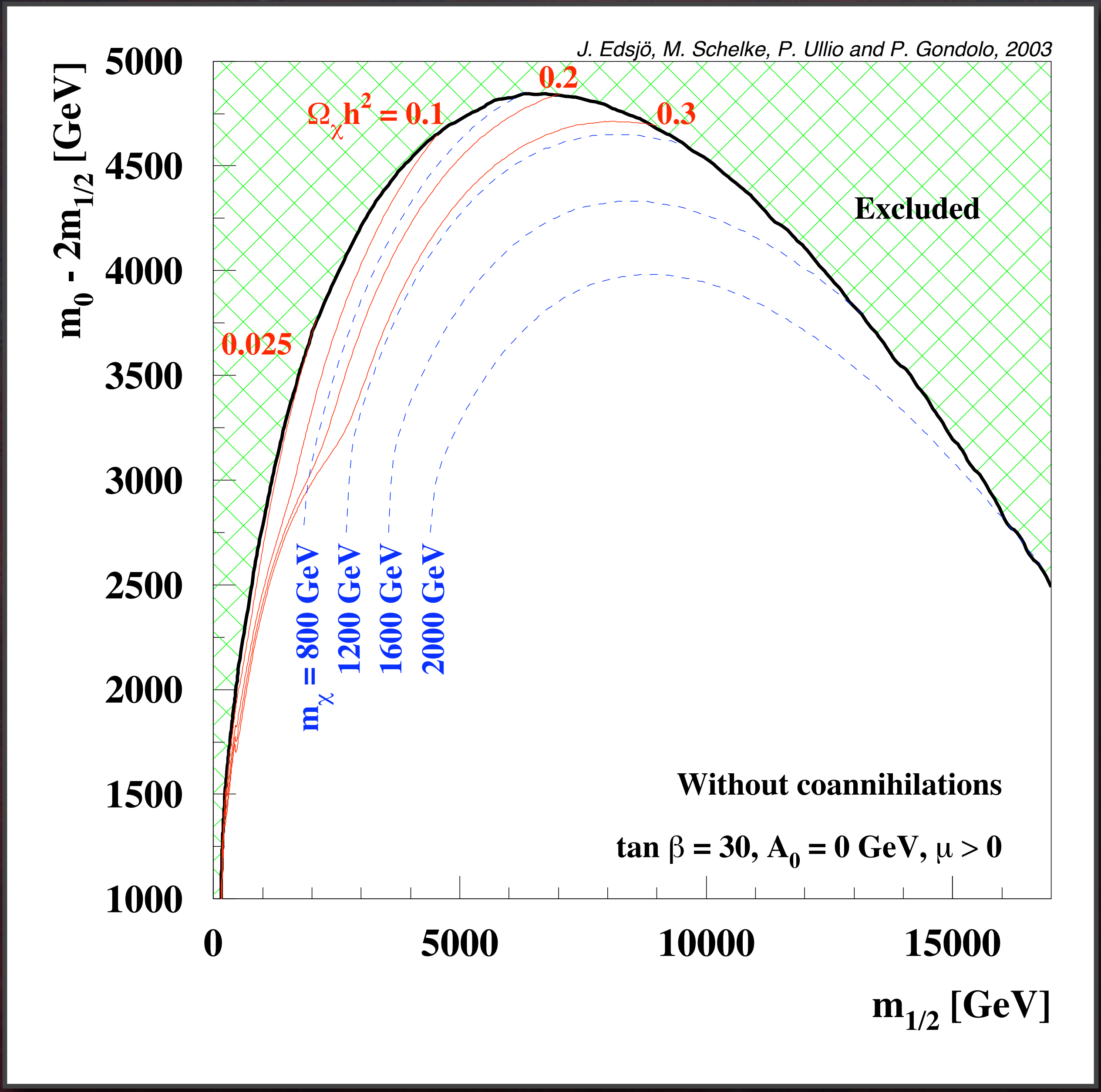
Example: stau coannihilation in mSUGRA



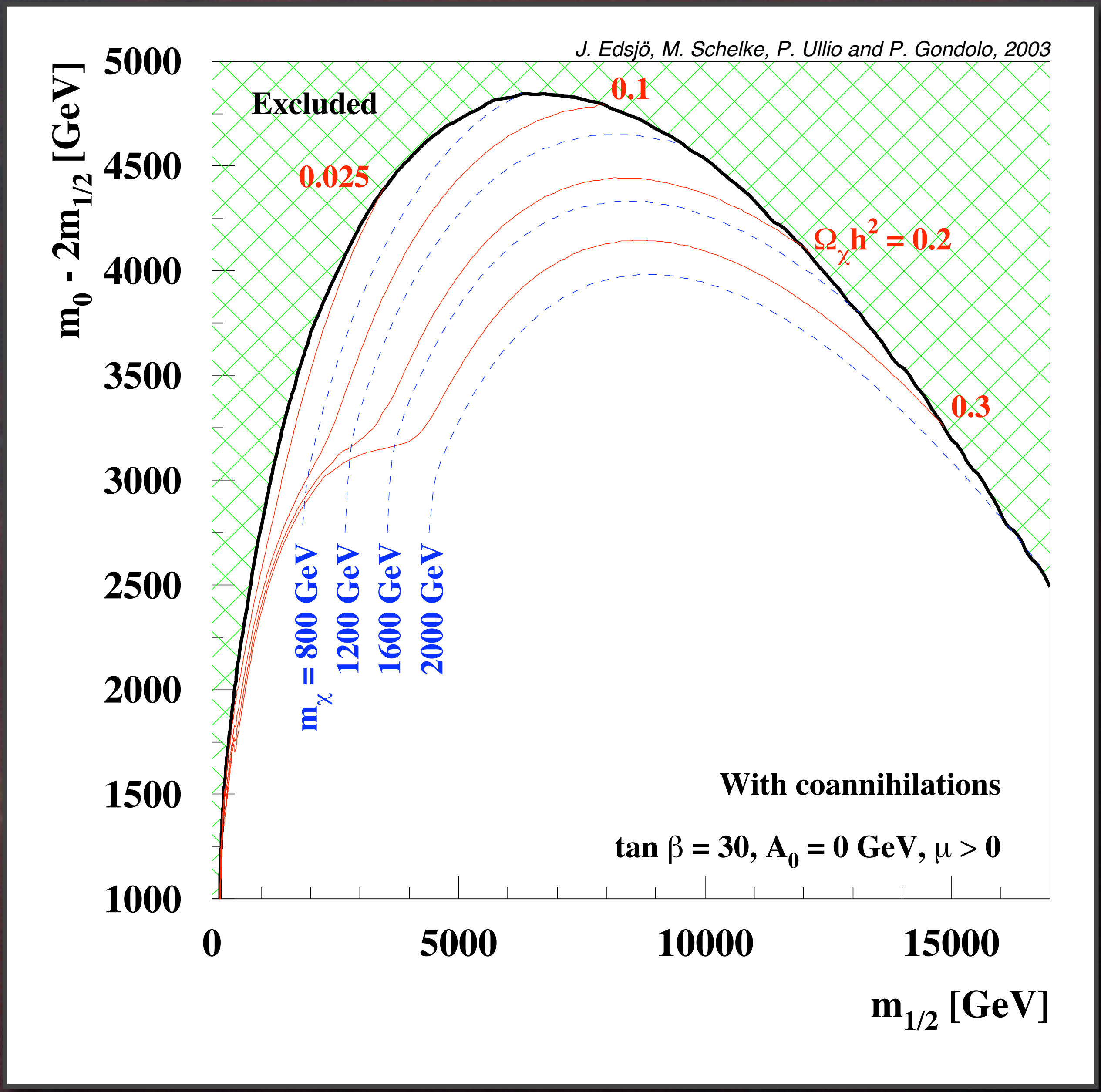
Coannihilations can change the relic density by more than 1000%!

Example: focus point region in mSUGRA

Without coannihilations

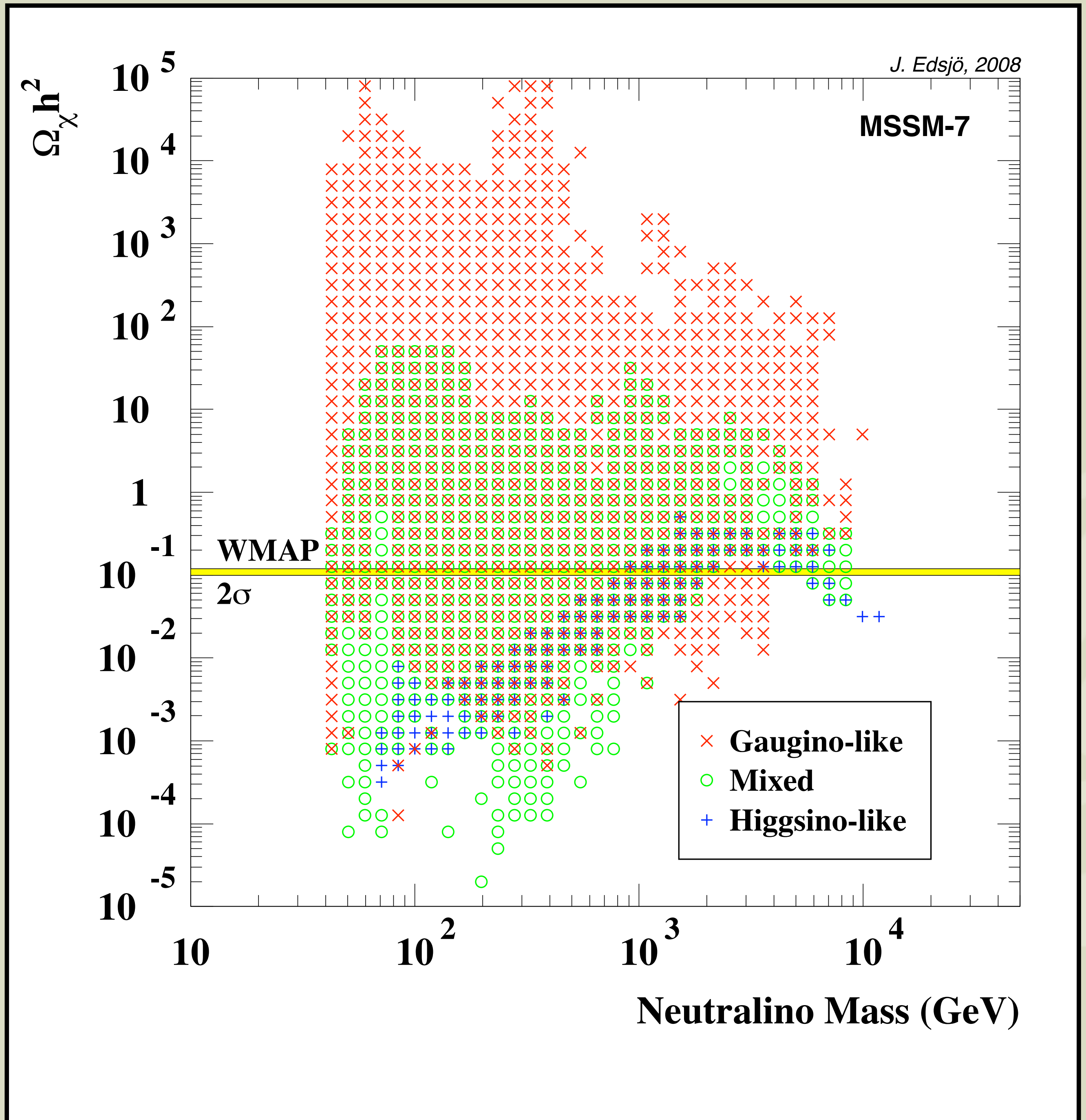


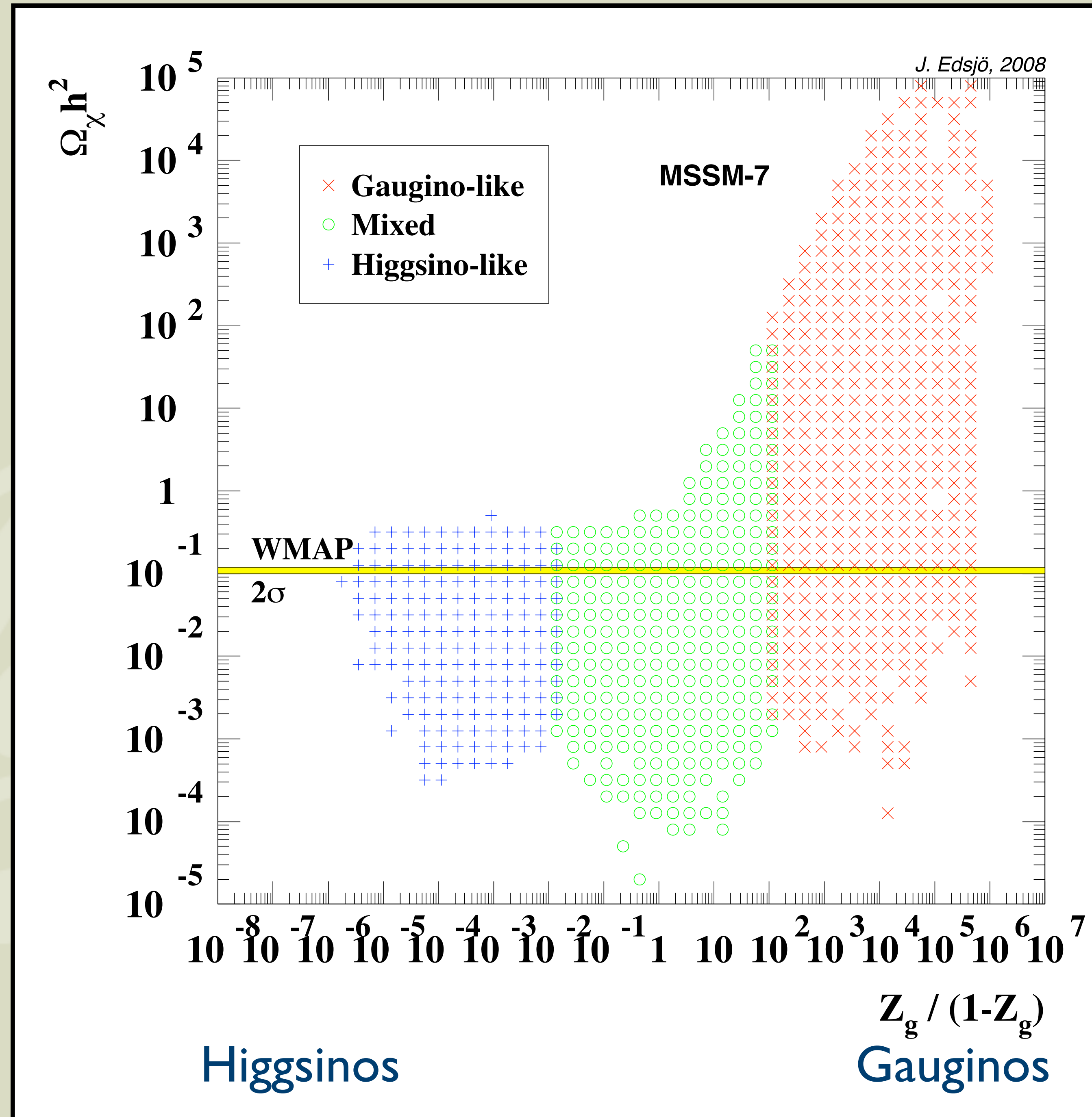
With coannihilations



In the focus point region coannihilations between neutralinos and charginos are important.

Relic density versus mass and composition





The neutralino is cosmologically interesting for a large range of parameter values

(No sfermion coannihilations in this plot yet)

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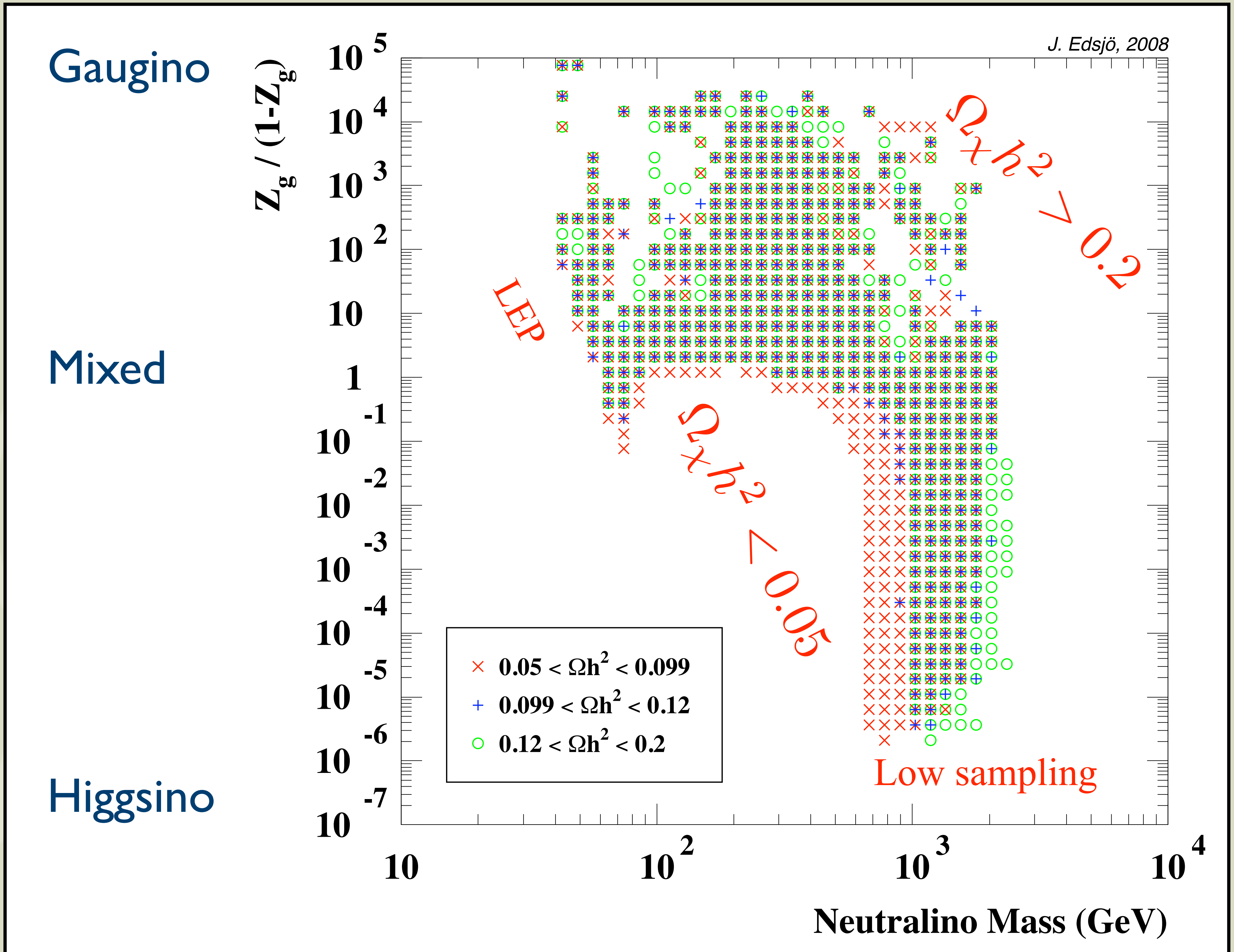
The $m_\chi - Z_g$ parameter space



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Relic density routines

- The main routine for SUSY neutralinos is dsrdomega that calculates the relic density of neutralinos
- However, the relic density routines are more general than that and can be used for any WIMP with a call to dsrdens.

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How to call dsrdens for general WIMPs

Call

dsrdens(wrate,npart,mgev,dof,nrs,rm,rw,nt,tm,oh2,tf,ierr,iwar)

where you have to supply

- wrate - invariant effective annihilation rate (function)
- npart - number of coannihilating particles
- mgev - mass of these
- dof - internal degrees of freedom of these
- nrs - number of resonances
- rm - mass of resonances
- rw - width of resonances
- nt - number of thresholds
- tm - equivalent mass of thresholds

The routine then returns

- oh2 - omega h^2
- tf - freeze-out temperature

Note: All this is taken care of for neutralinos in dsrdomega

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