Non-Gaussianities from primordial quantum diffusion

Vincent Vennin

vincent.vennin@ens.fr

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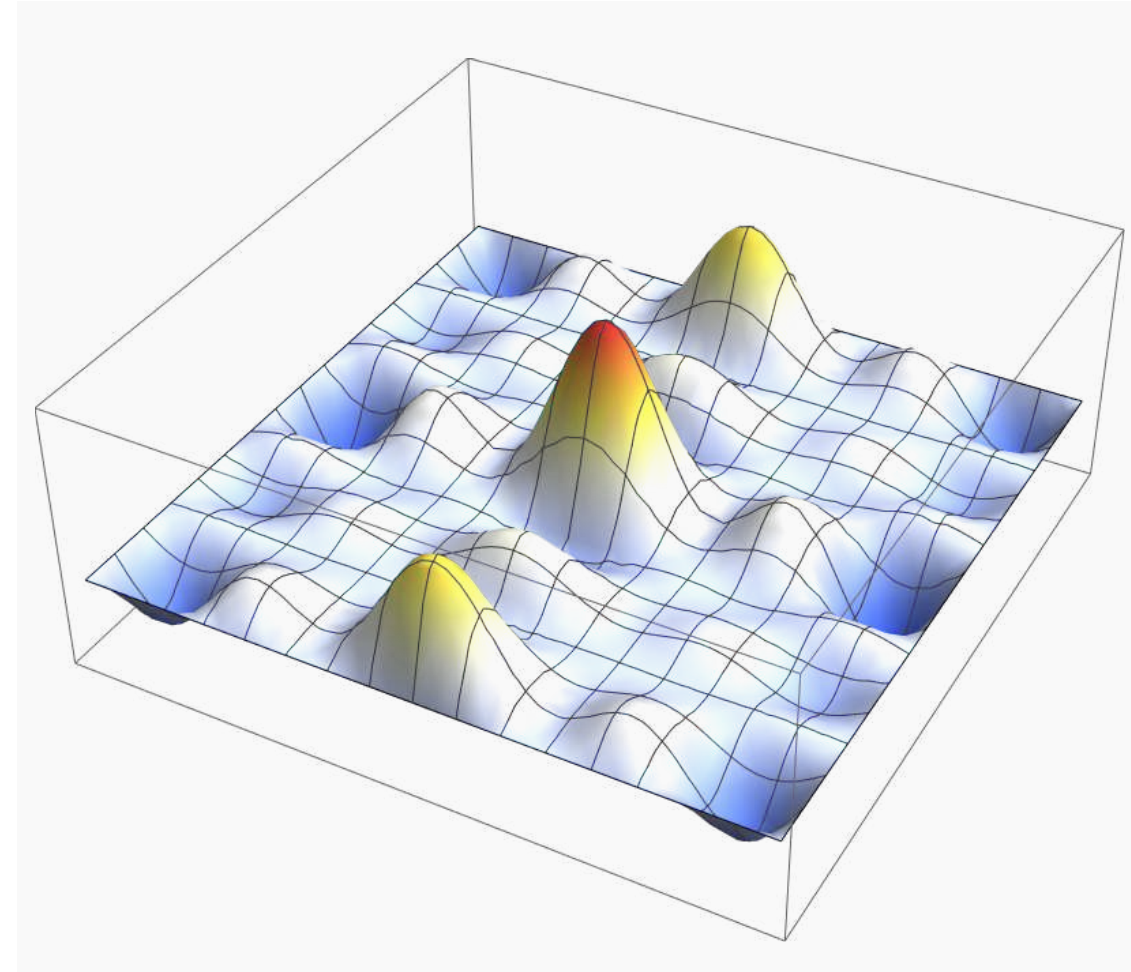
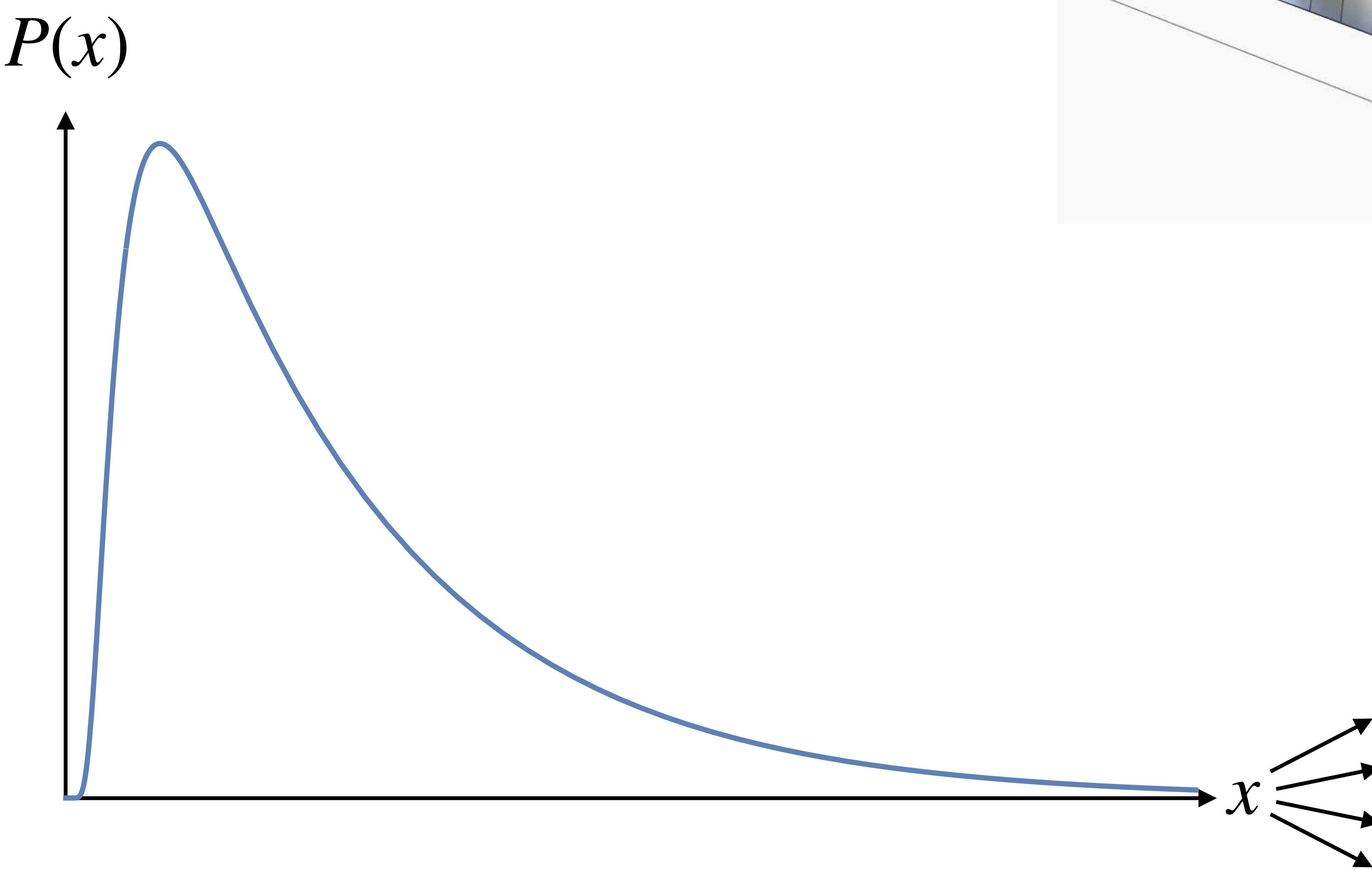
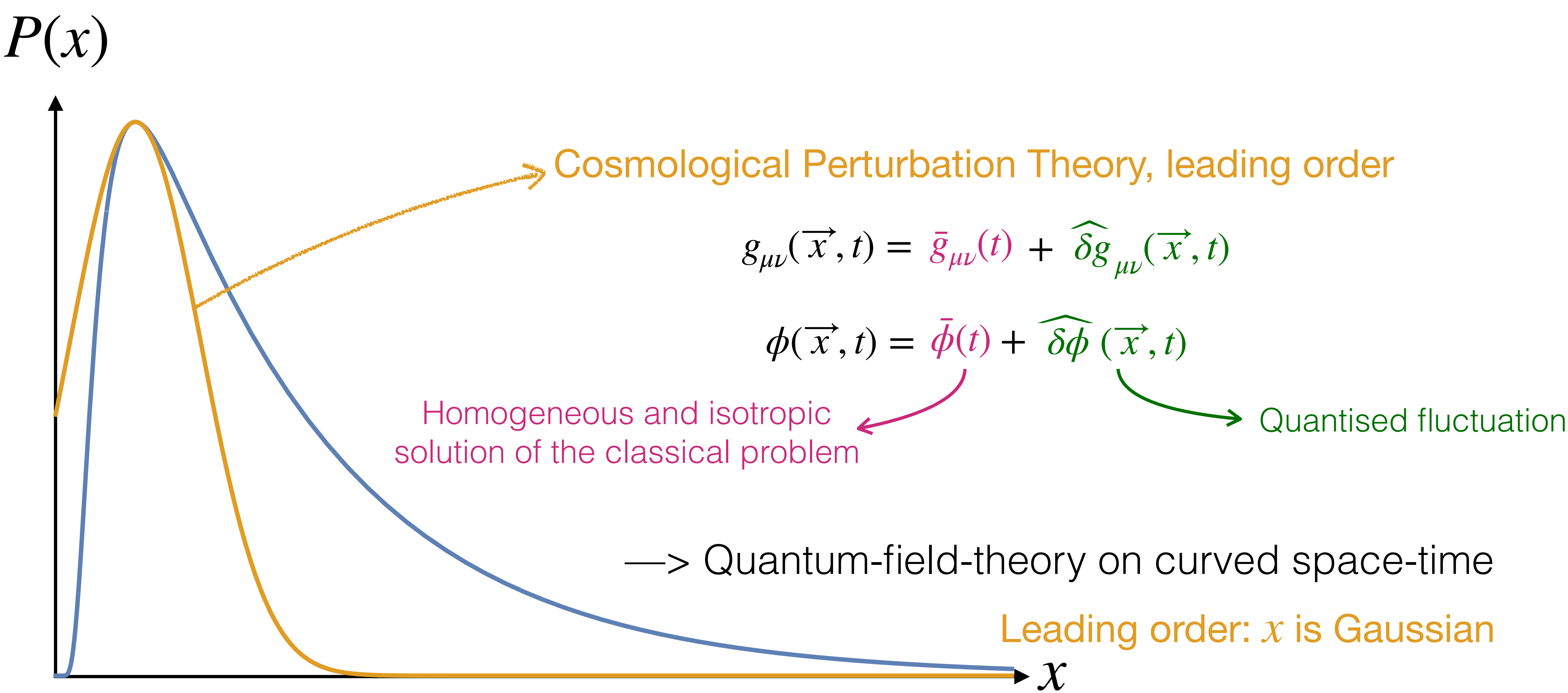
Oskar Klein Center, Stockholm

How likely is it to form a given cosmological structure?

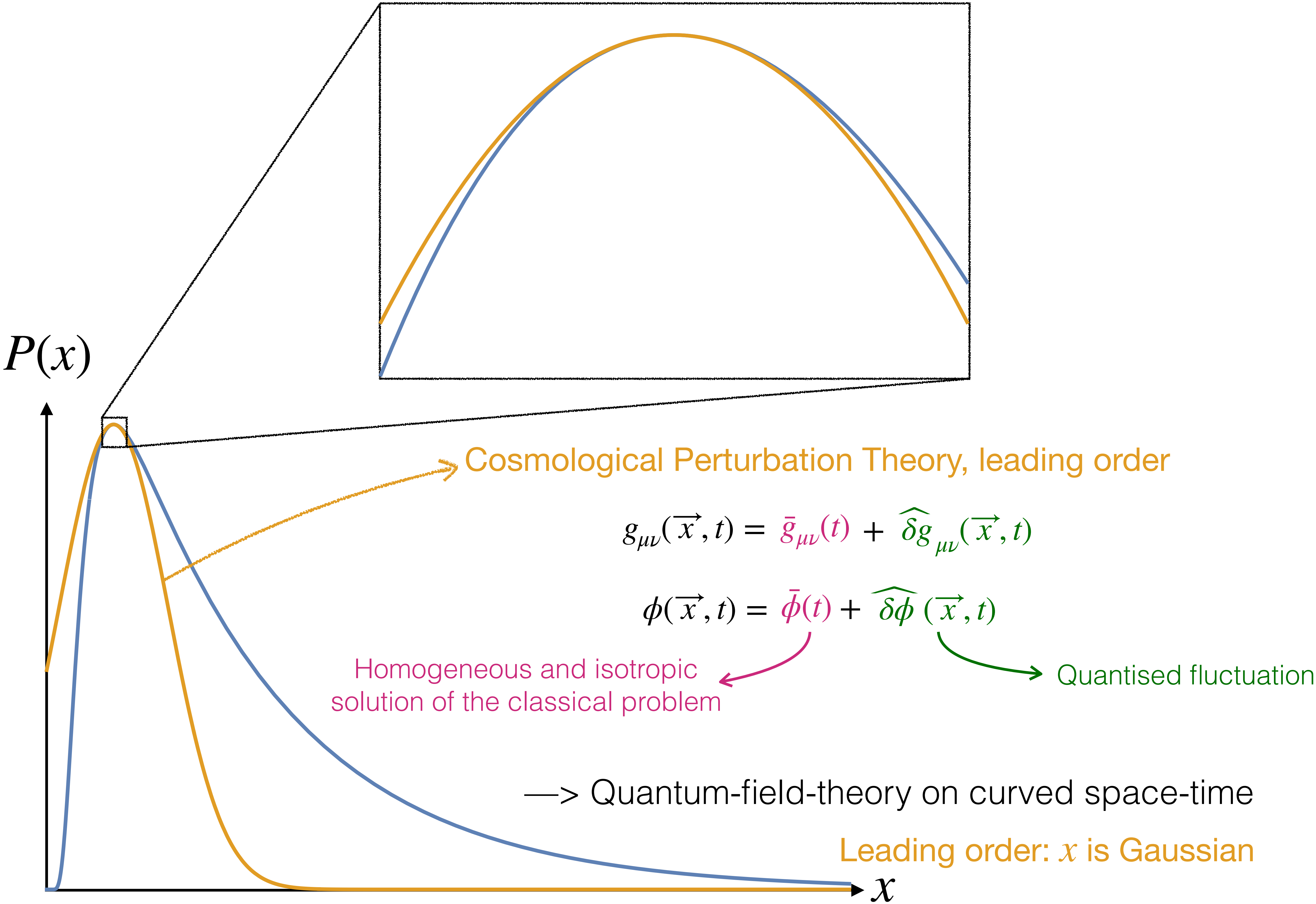




- Local curvature
- energy density
- maximum compaction
- etc



$P(x)$
Cosmological Perturbation Theory, leading order
$g_{\mu\nu}(\overrightarrow{x}, t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\overrightarrow{x}, t)$
$\phi(\overrightarrow{x}, t) = \bar{\phi}(t) + \widehat{\delta\phi}(\overrightarrow{x}, t)$
Homogeneous and isotropic solution of the classical problem
Quantised fluctuation
—> Quantum-field-theory on curved space-time
Leading order: x is Gaussian
x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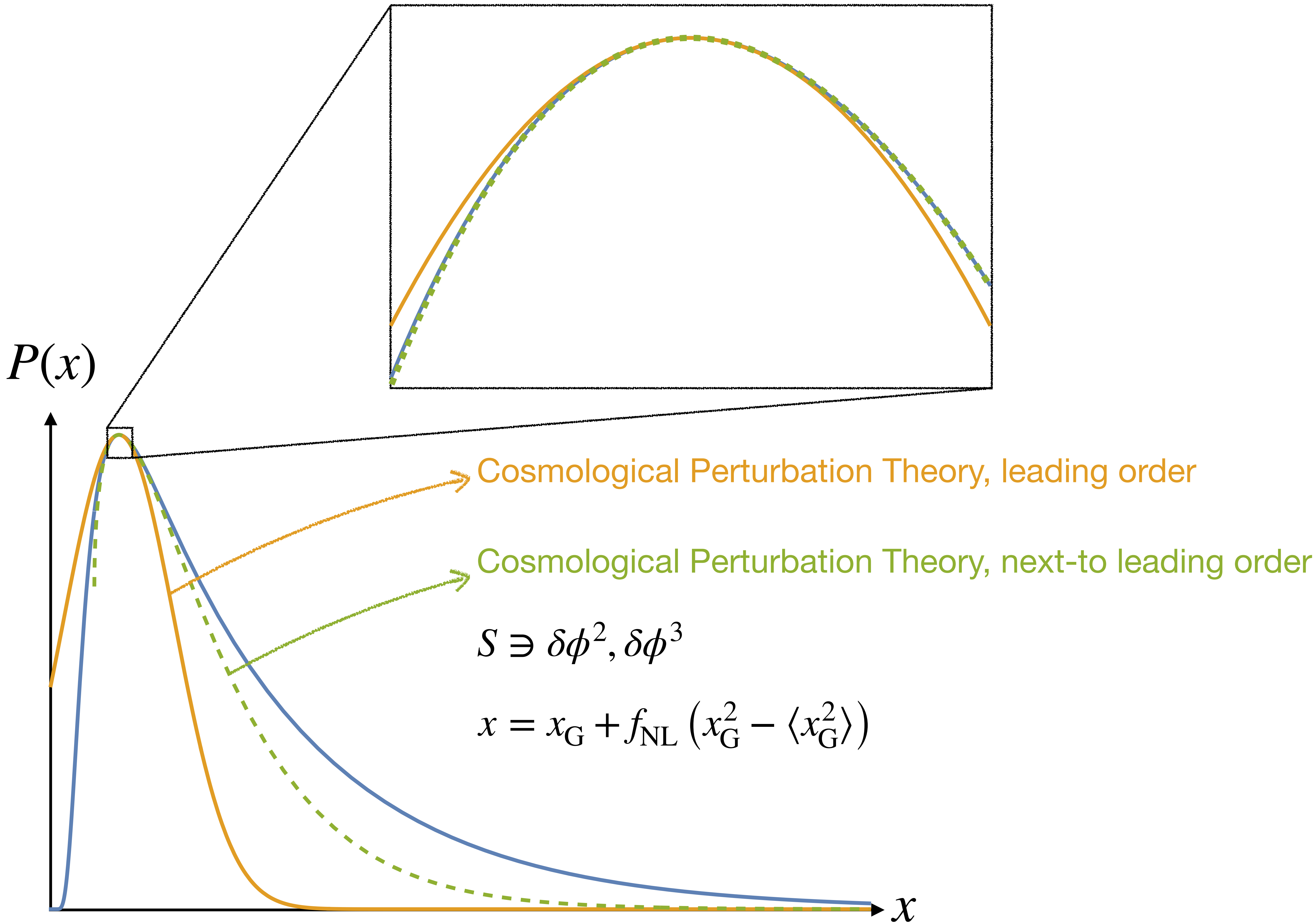
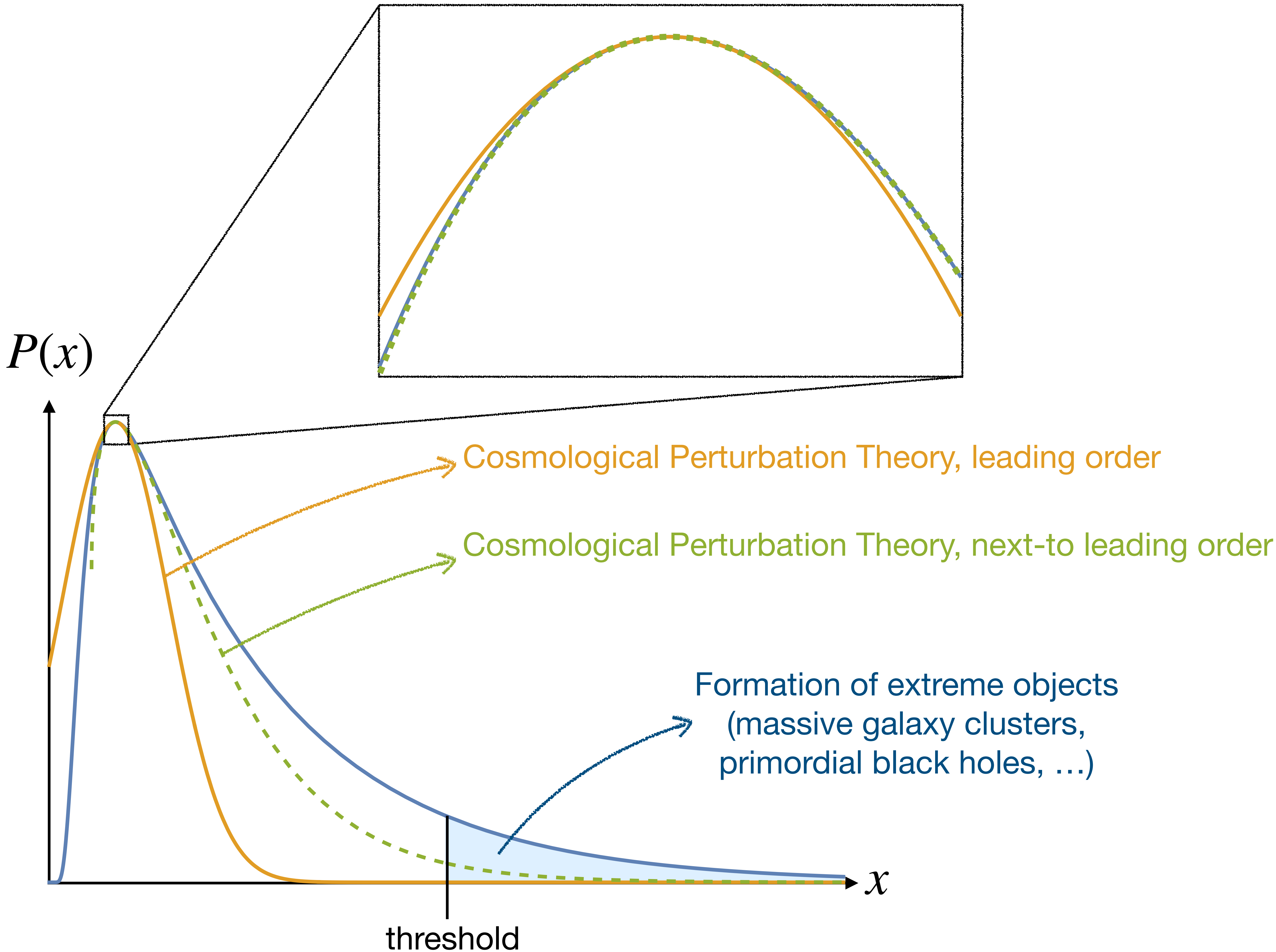
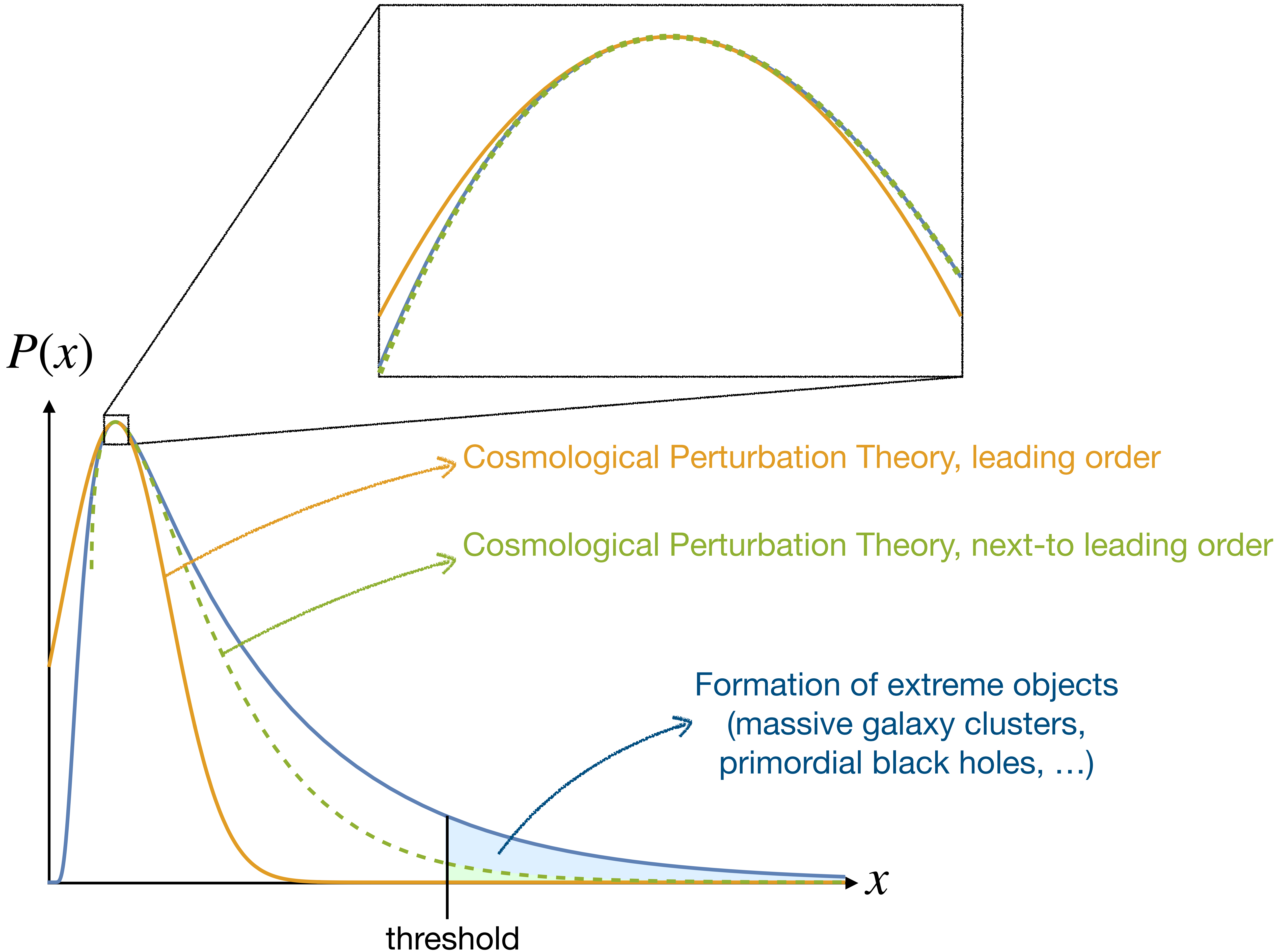

$P(x)$
x
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
$S \ni \delta\phi^2, \delta\phi^3$
$x = x_G + f_{NL}\left(x_G^2 - \langle x_G^2 \rangle\right)$


$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes, …)
x
threshold



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Formation of extreme objects
(massive galaxy clusters,
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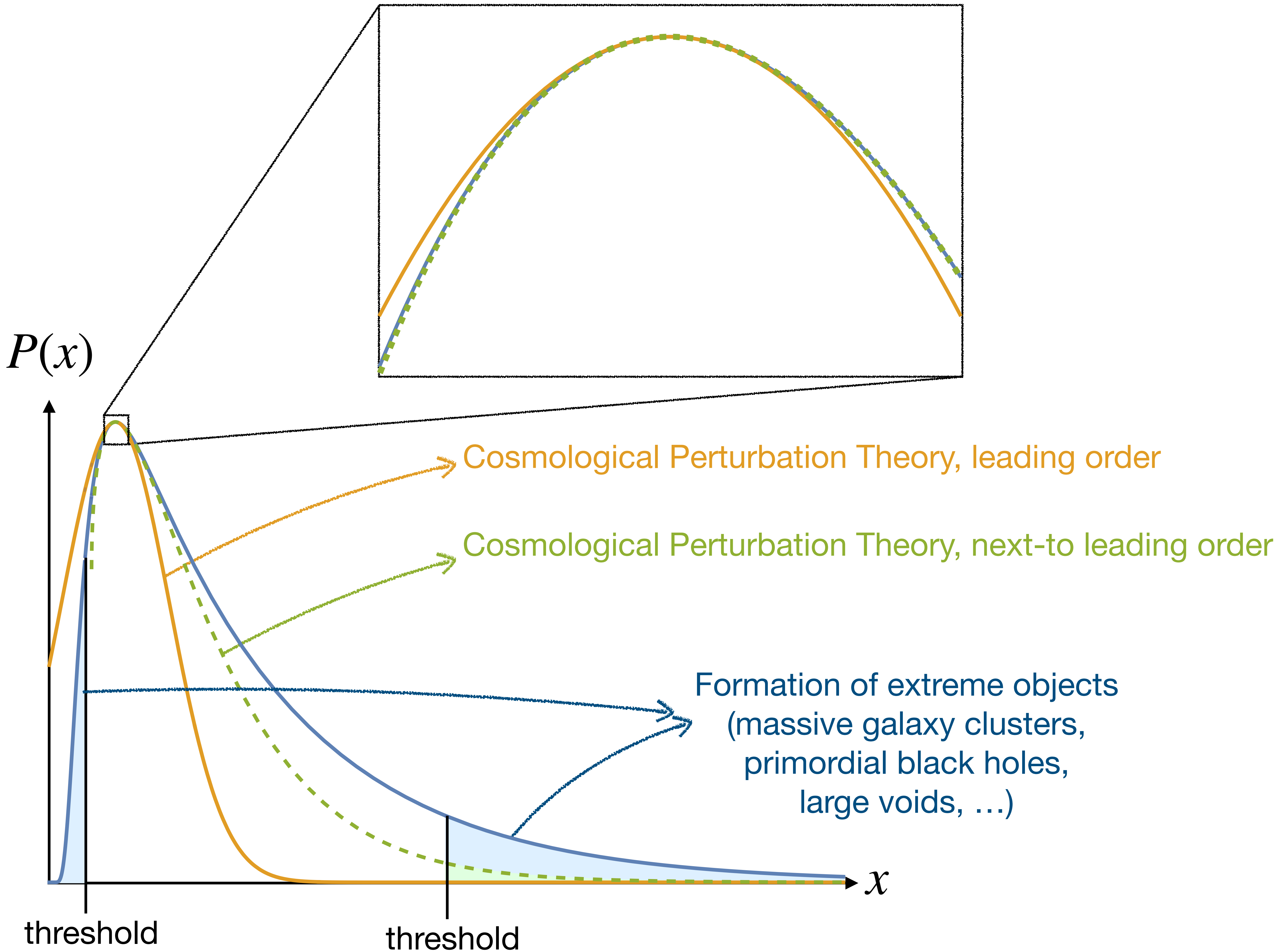
x
threshold



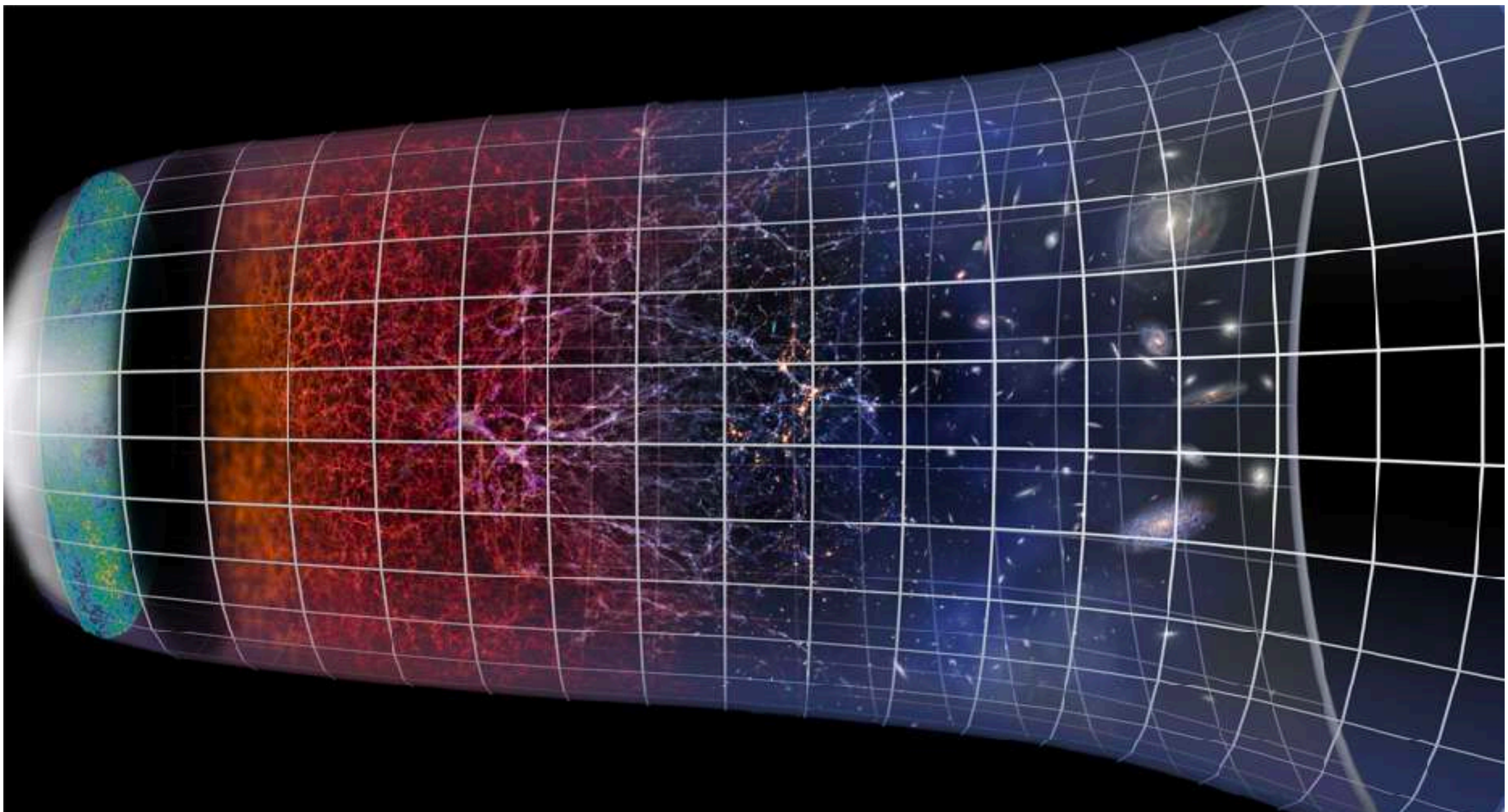
$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes,
large voids, …)
x
threshold
threshold

Outline

- Inflationary Cosmlogy
- Stochastic Inflation
- First-Passage-Time Analysis
- Illustration: Primordial Black Holes and other extreme objects

Cosmic Inflation

$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2$ with $\ddot{a} > 0$ and $(10\,\mathrm{MeV})^4 < \rho < \left(10^{16}\,\mathrm{GeV}\right)^4$



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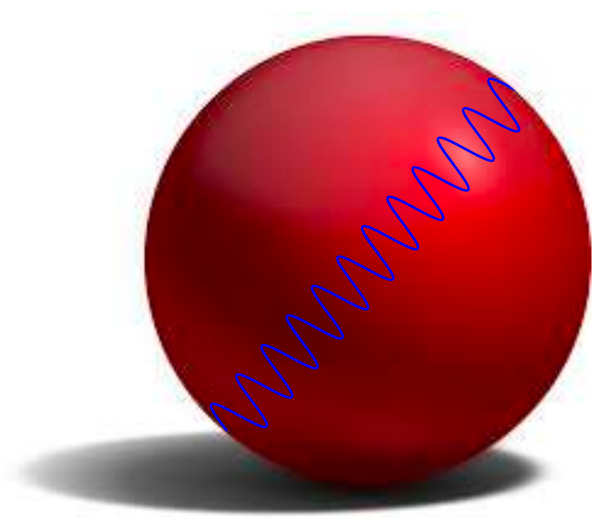
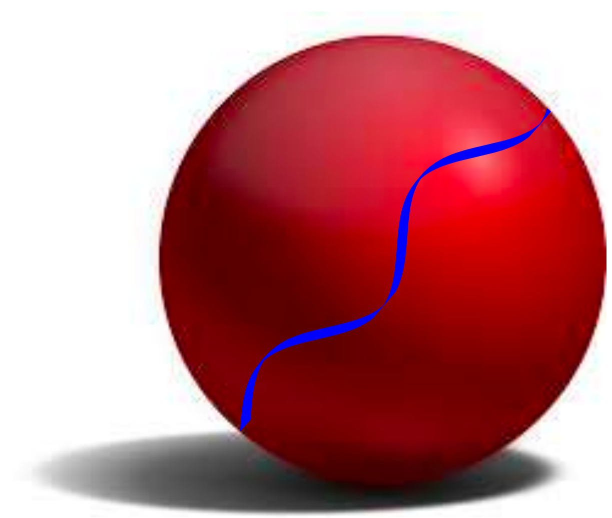
Hubble parameter $H = \dot{a}/a$

H^{-1} : characteristic time scale, or length scale ($c = 1$), of the expansion



$\lambda \ll H^{-1}$

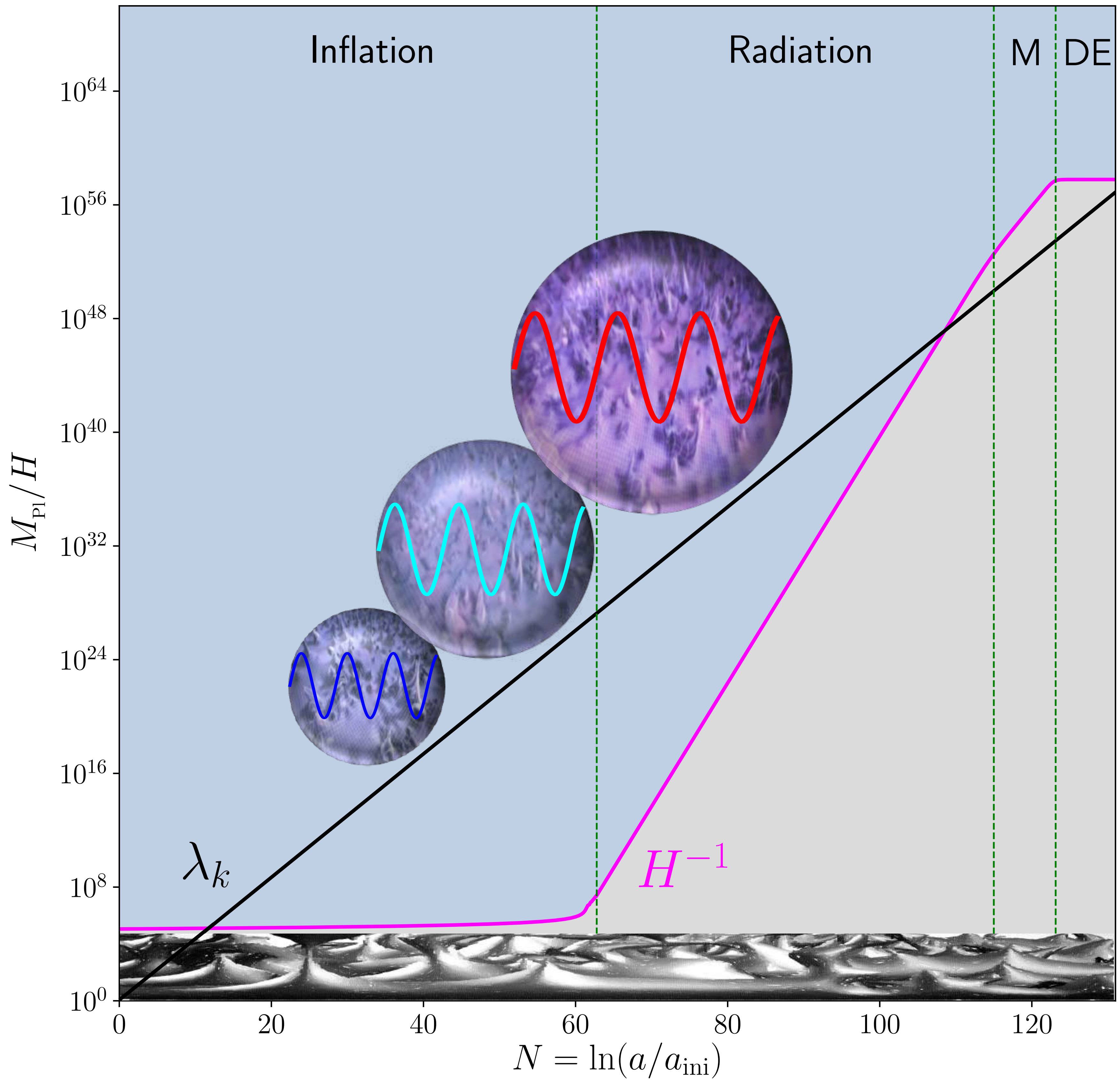
Insensitive to space-time curvature



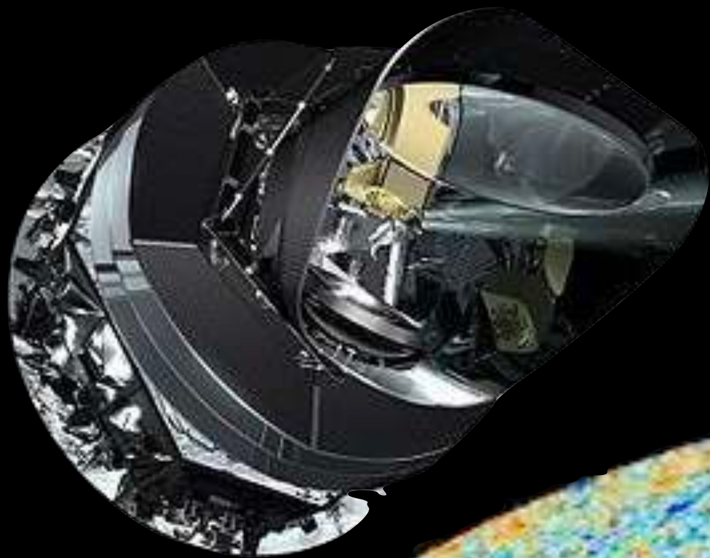
$\lambda \gtrsim H^{-1}$

Feels space-time curvature

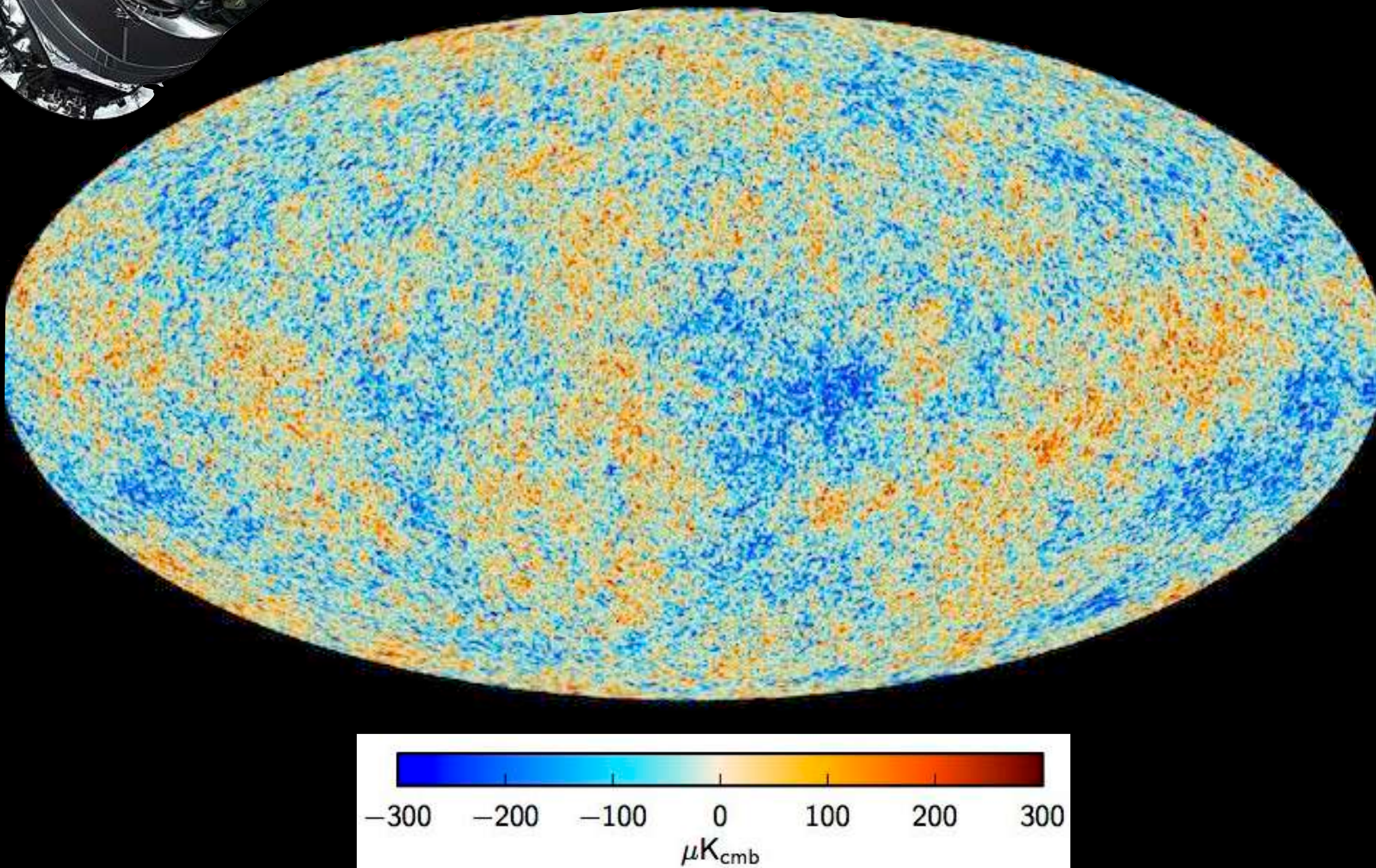
Cosmic Inflation



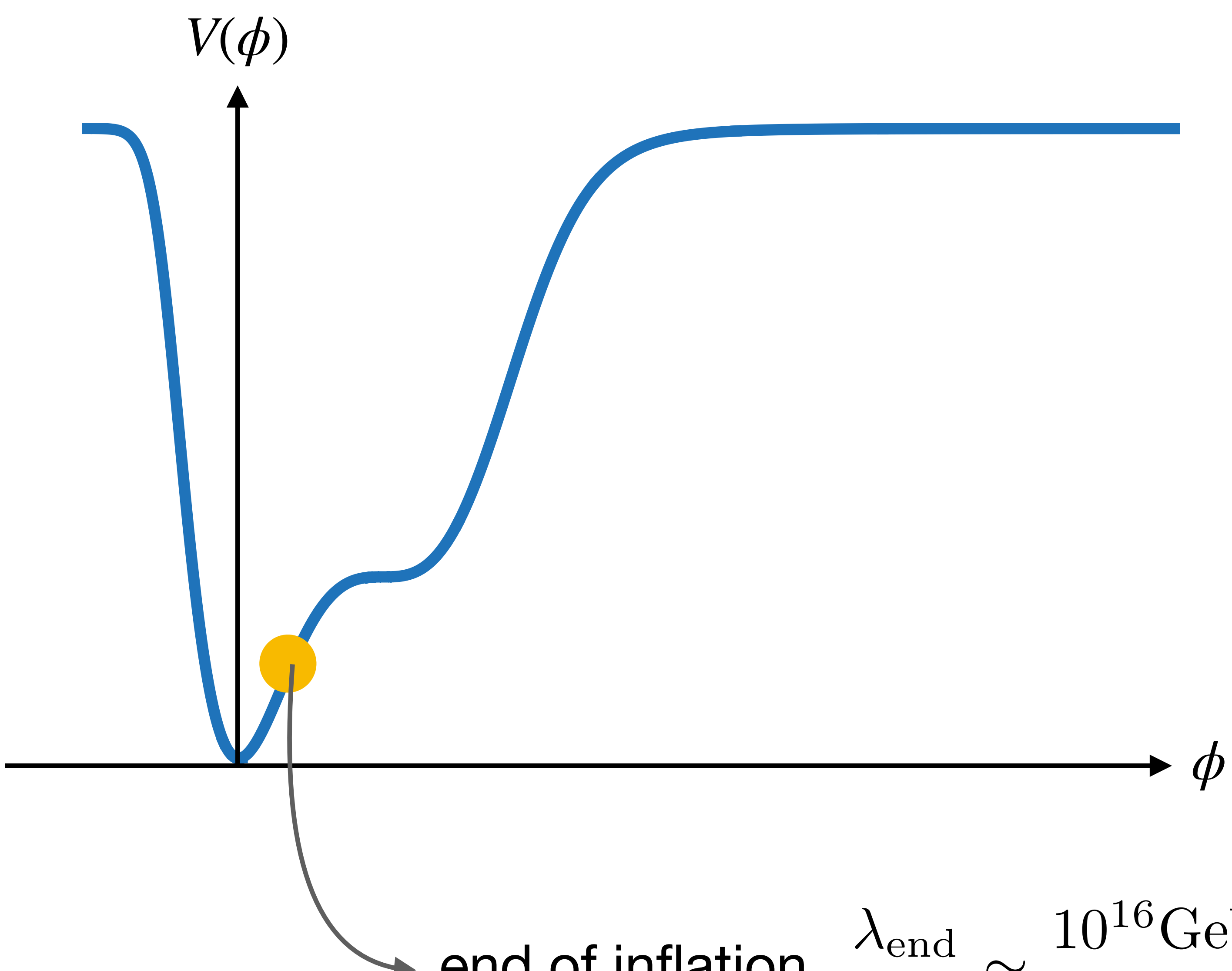
Planck satellite



$$\frac{\delta T}{T} \sim 10^{-5} \ll 1$$

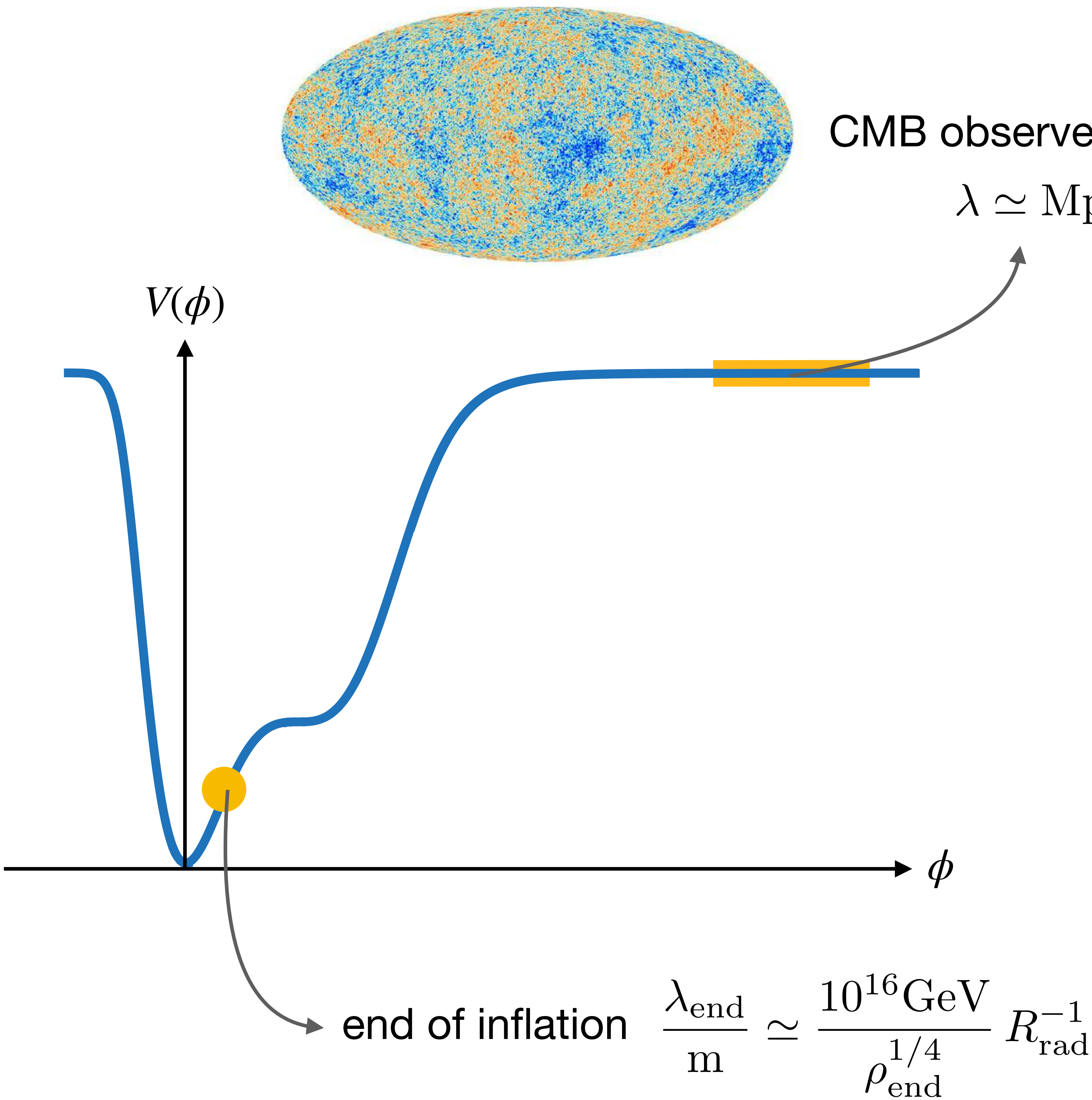


Probing the end of inflation



end of inflation $\frac{\lambda_{\rm end}}{\rm m} \simeq \frac{10^{16}\,{\rm GeV}}{\rho_{\rm end}^{1/4}}\, R_{\rm rad}^{-1}$

Probing the end of inflation



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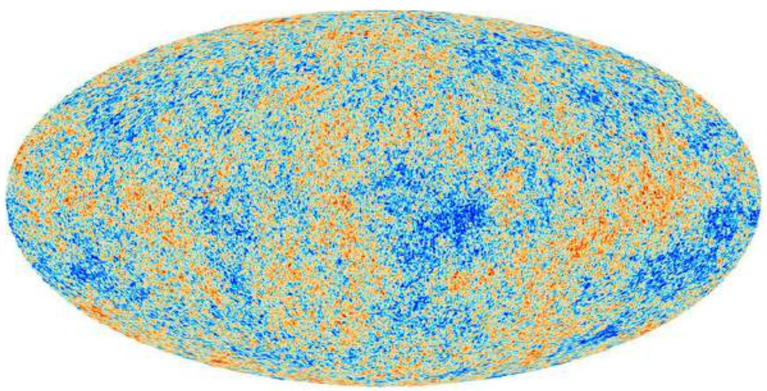
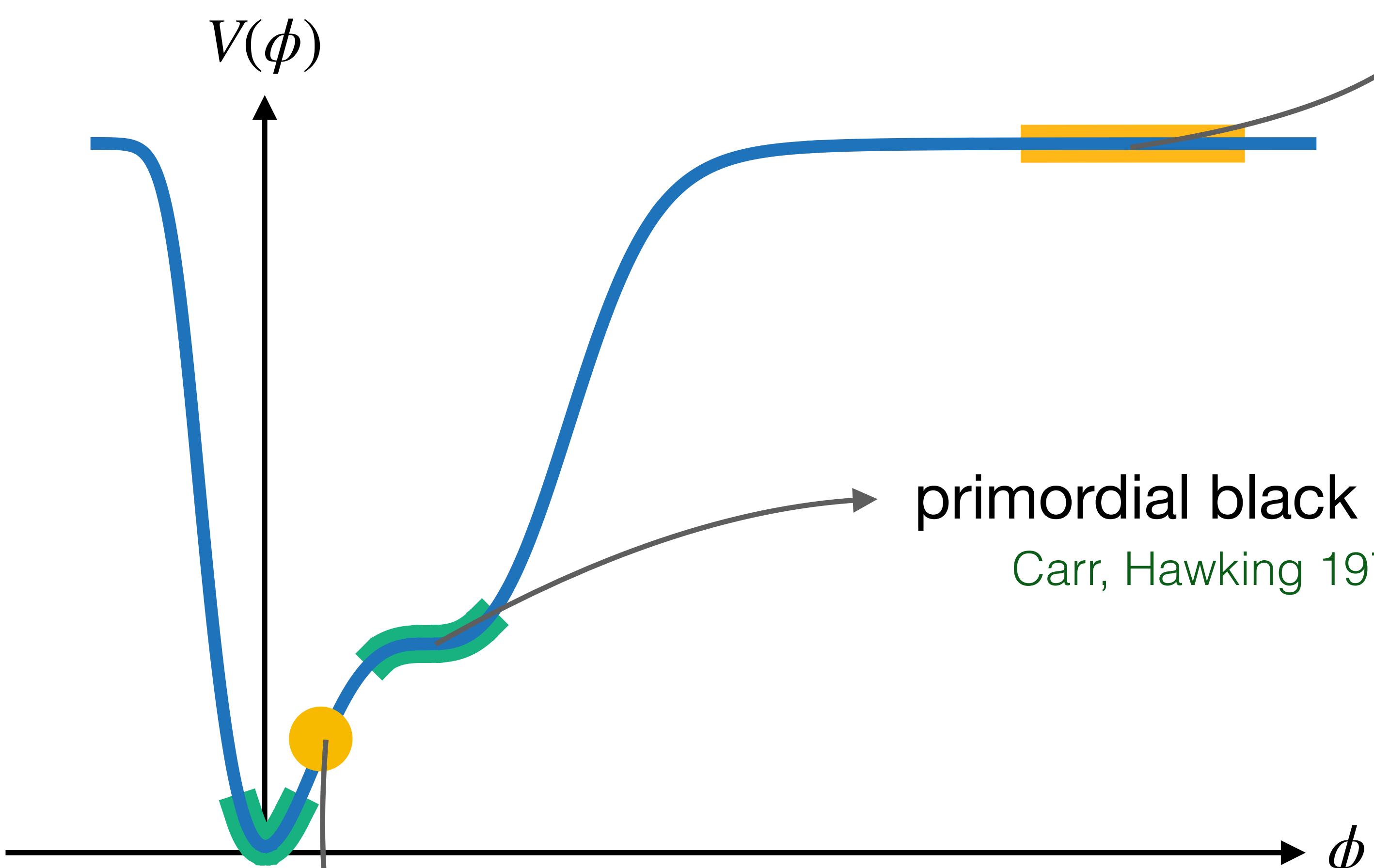


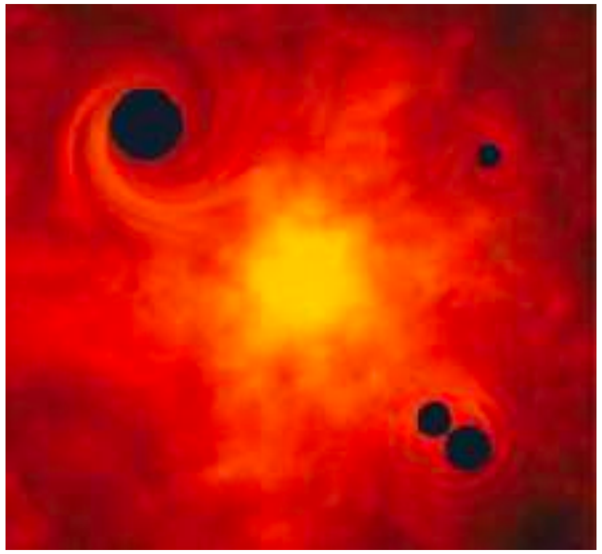
CMB observed scales

$\lambda \simeq \mathrm{Mpc}$



primordial black holes?

Carr, Hawking 1974



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Primordial black holes

- Could constitute part or all of dark matter Chapline 1975
  $M = 10^{16} - 10^{17}\mathrm{g},\ 10^{20} - 10^{24}\mathrm{g},\ 10 - 10^3 M_\odot$
- Could provide progenitors for the LIGO/VIRGO events
  $M = 10 - 100\ M_\odot$
- Could provide seeds for cosmological structures
  $M > 10^3 M_\odot$
  Mészáros 1975
  Afshordi, McDonald, Spergel, 2003
- Could provide seeds for supermassive black holes in galactic nuclei
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  Carr, Rees 1984
  Bean, Magueijo 2002

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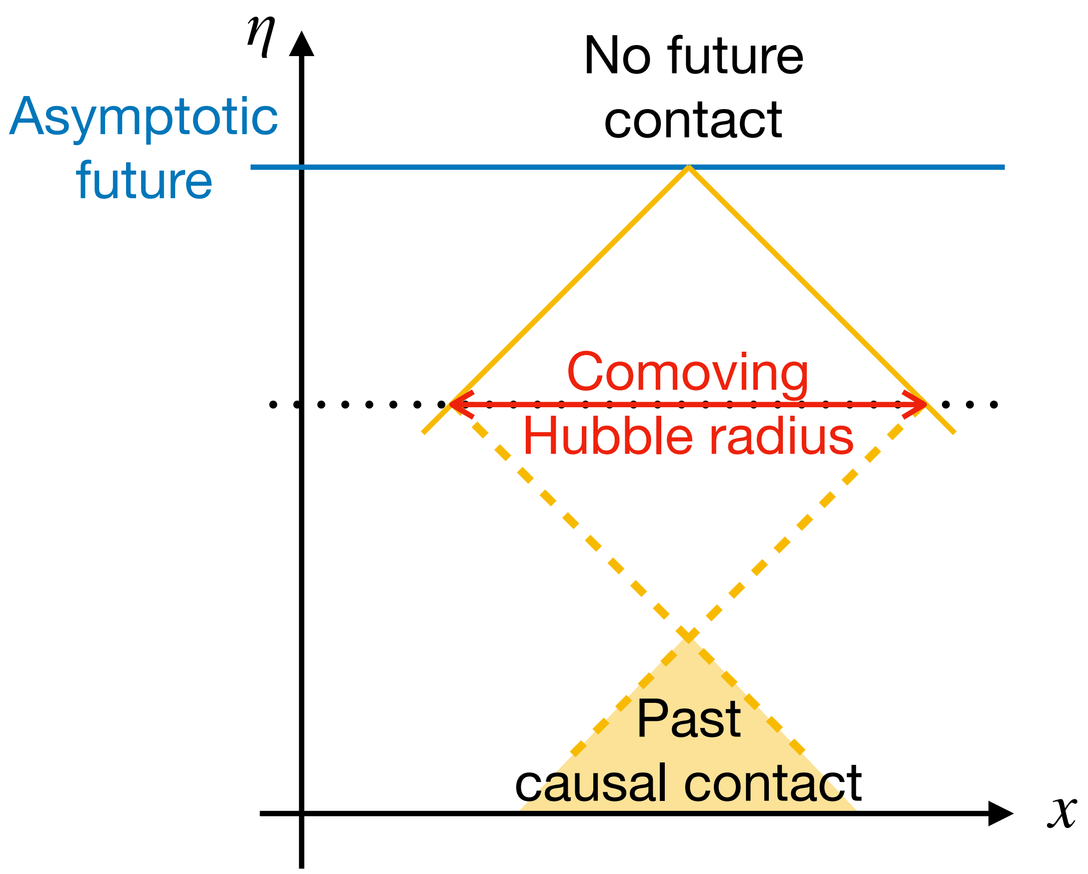
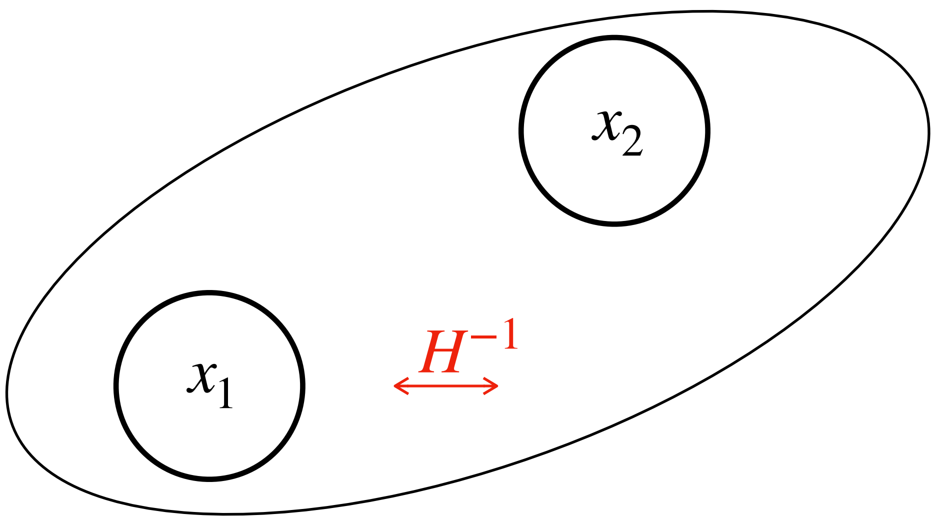
For hints in favour of PBH existence, see *e.g.* García-Bellido and Clesse (1711.10458)

Separate Universe

$$\mathrm{d}s^2 = a^2 \left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right)$$

de-Sitter universe: $a = -1/(H\eta), \quad -\infty < \eta < 0$



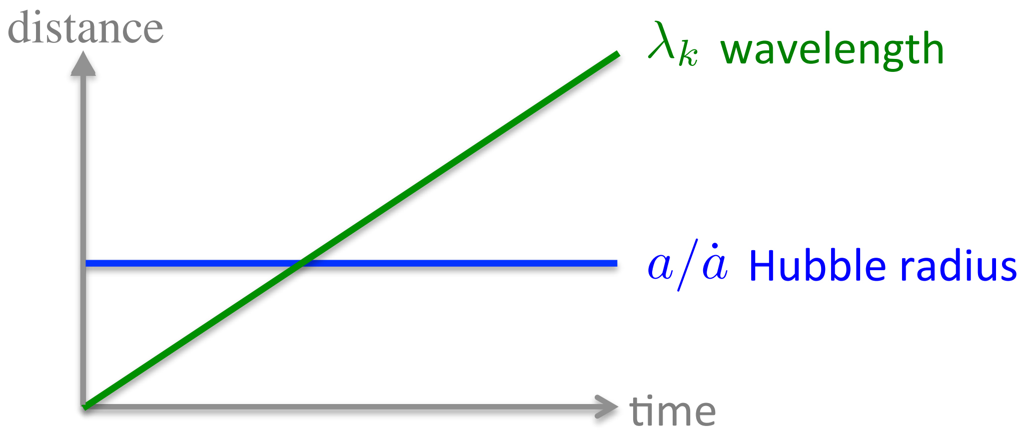


If a large fluctuation develops at $\boldsymbol{x}_1$, this cannot affect the local geometry at $\boldsymbol{x}_2$

Separate universe: On large scales, the universe can be described by an ensemble of independent, locally homogeneous and isotropic patches

Salopek & Bond; Sasaki & Stewart; Wands, Malik, Lyth & Liddle

Stochastic Inflation



super-Hubble scales —> stochastic inflation

sub-Hubble scales —> CPT

Coarse-grained field $\hat{\mathbf{\Phi}}_{\rm cg}(N, \vec{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\vec{k} \left[\mathbf{\Phi}_{\vec{k}}(N) e^{-i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} + \mathbf{\Phi}^{\star}_{\vec{k}}(N) e^{i\vec{k}\cdot\vec{x}} \hat{a}^{\dagger}_{\vec{k}} \right]$

$N = \ln(a)$

Quantum fluctuations source the **background**

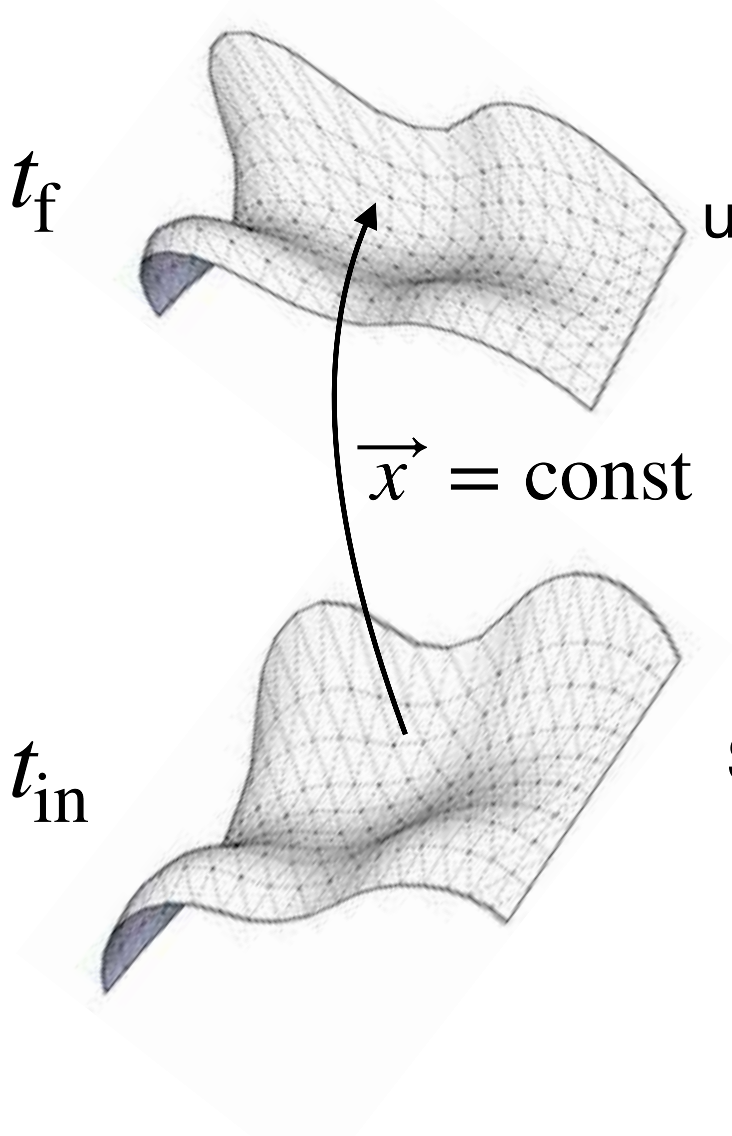
Equation of motion $\frac{\mathrm{d}}{\mathrm{d}N}\mathbf{\Phi}_{\rm cg} = \mathscr{F}_{\rm background}(\mathbf{\Phi}_{\rm cg}) + \xi$ Starobinsky, (1982) 1986

Over one e-fold: $\frac{\Delta\phi_{\rm quant}}{\Delta\phi_{\rm classical}} \sim \zeta_{\rm classical}$

What about far from the classical regime?

What about tail effects?

Stochastic-δ*N* formalism



uniform density slice

$\delta\rho = 0$

Spatially flat slice

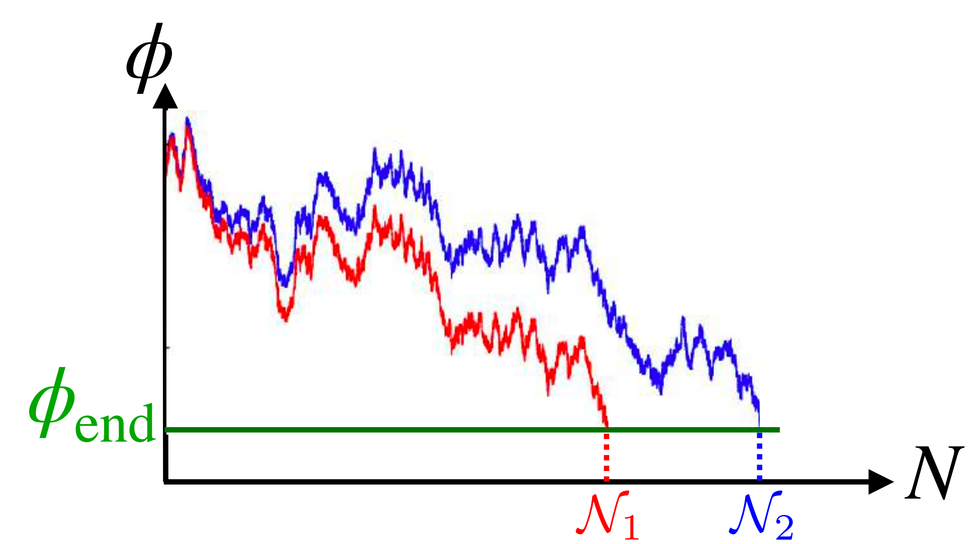
$\psi = 0$

$$\zeta(t, x) = N(t, x) - N_0(t) \equiv \delta N$$

Lifshitz, Khalatnikov (1960)
Starobinsky (1983)
Wands, Malik, Lyth, Liddle (2000)

The realised number of e-folds is a stochastic quantity:

$$\zeta_{\text{coarse grained}} = \mathcal{N} - \langle \mathcal{N} \rangle$$



Enqvist, Nurmi, Podolsky, Rigopoulos (2008) ; Fujita, Kawasaki, Tada, Takesako (2014); VV, Starobinsky (2015)

Stochastic-δN formalism

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N) \longrightarrow \frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right) = \mathscr{L}_\phi \cdot P$$

Langevin equation

Fokker-Planck equation

Equation for the PDF of the first passage time

VV, Starobinsky (2015)
Pattison, VV, Assadullahi, Wands (2017)

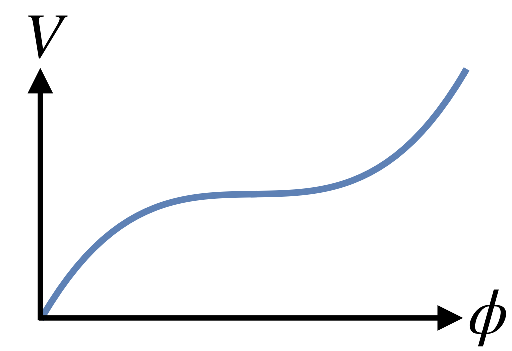
$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\mathrm{FPT}}(\mathcal{N};\phi) = \mathscr{L}_\phi^\dagger \cdot P_{\mathrm{FPT}}$$

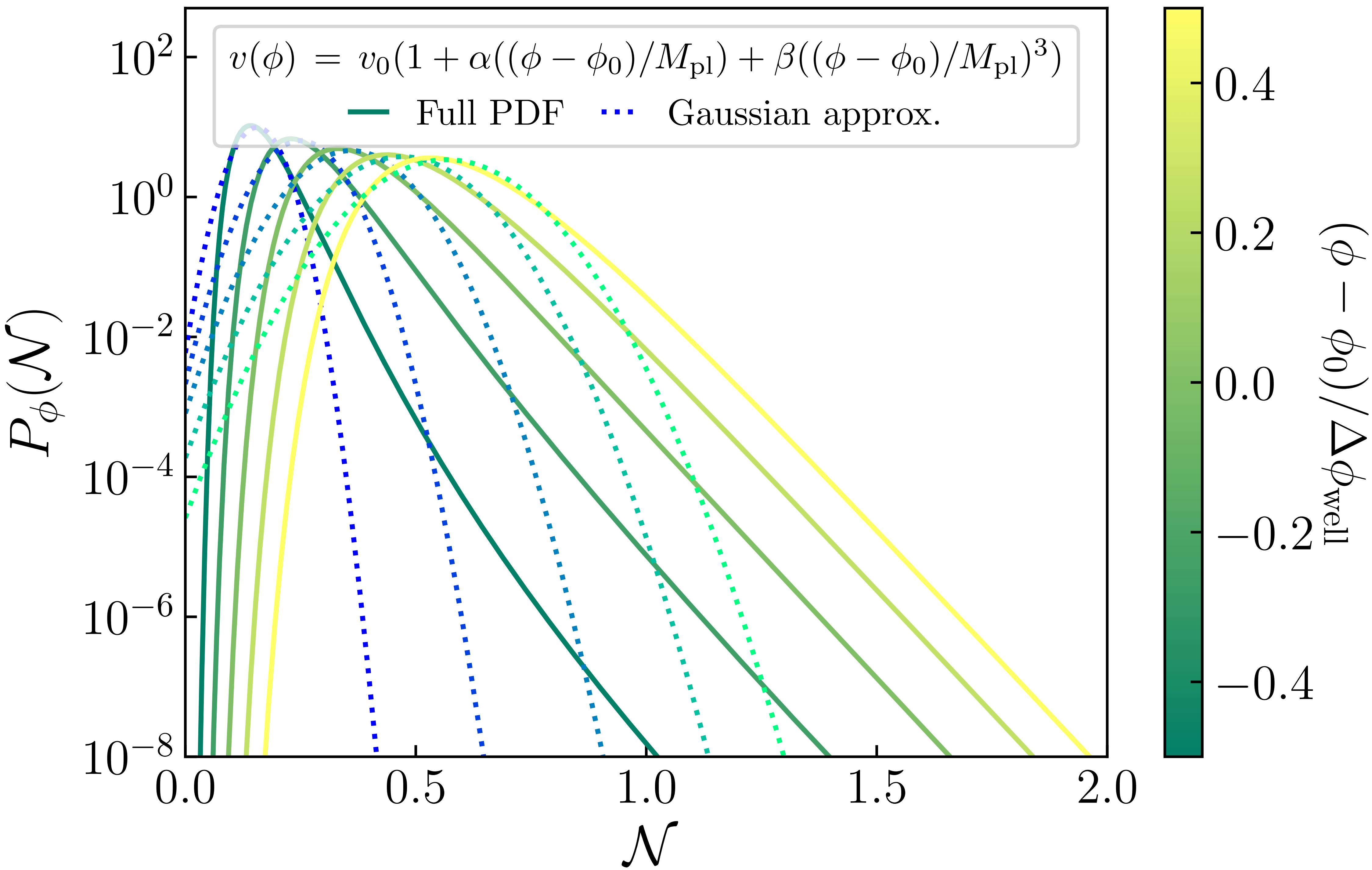
Computational program:

- Solve the first-passage-time problem
- This gives the one-point PDF of curvature perturbation coarse-grained at H_{end}
- Extract cosmologically relevant quantities (power spectrum, mass functions, compaction function, etc)

Exponential tails

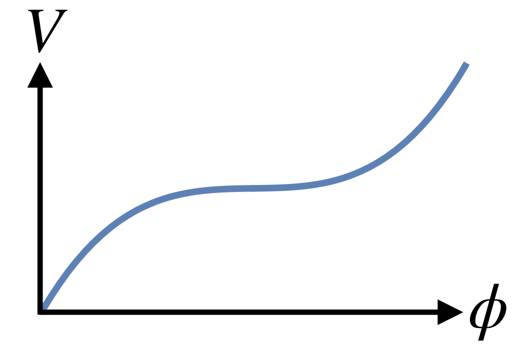
Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)

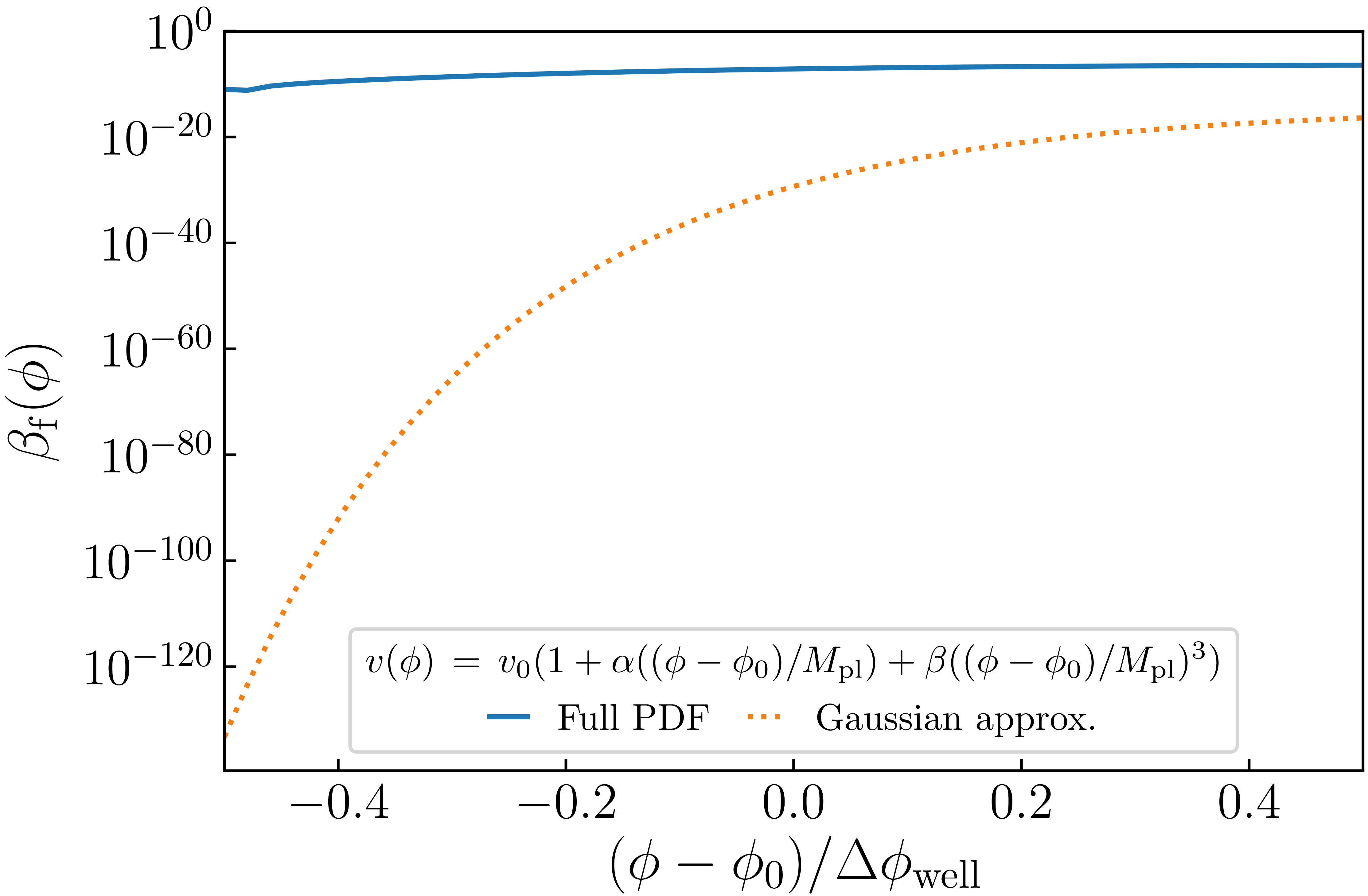




Impact on PBHs

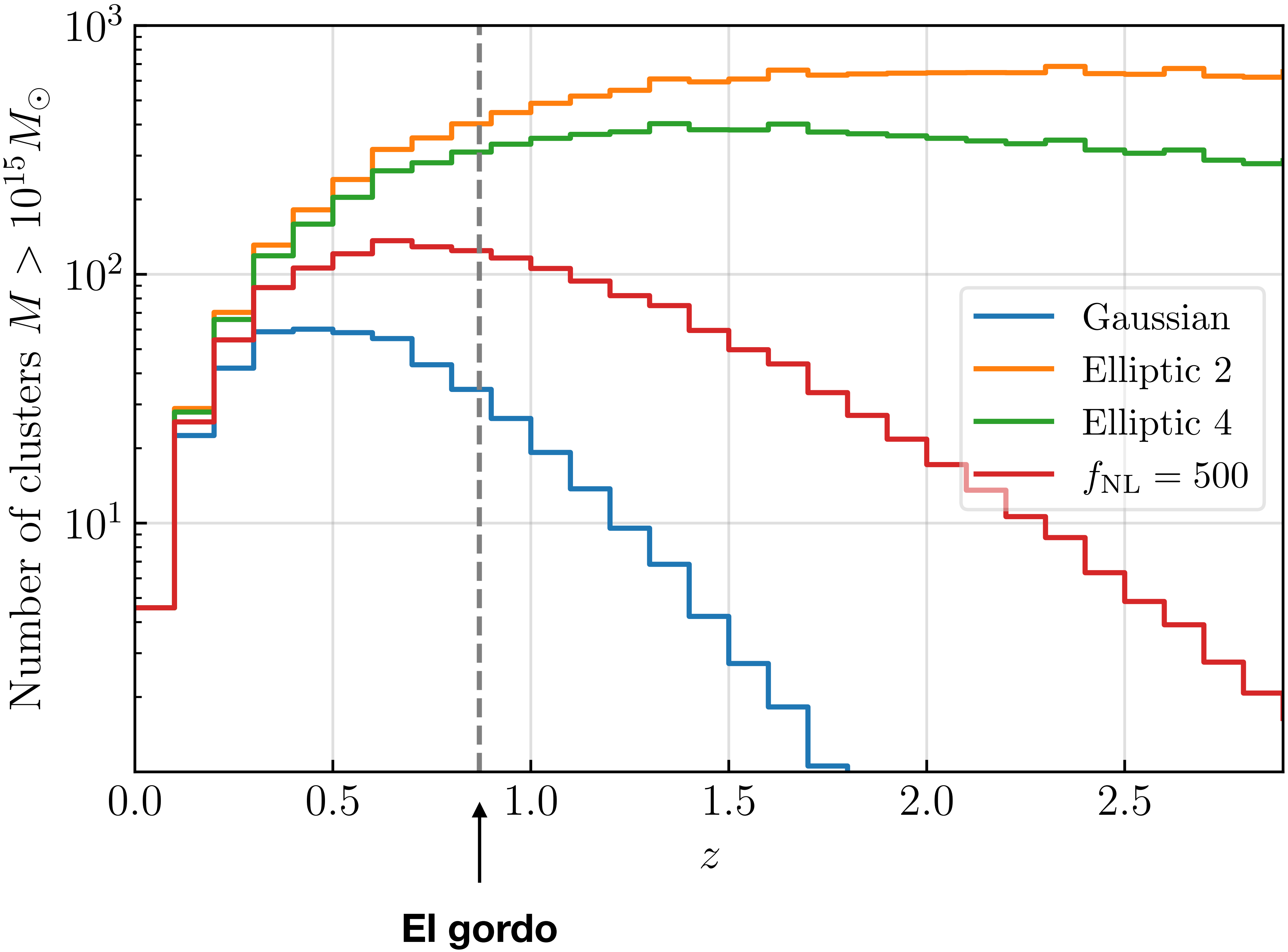
Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)





Impact on LSS

Ezquiaga, Garcia-Bellido, VV (2022)



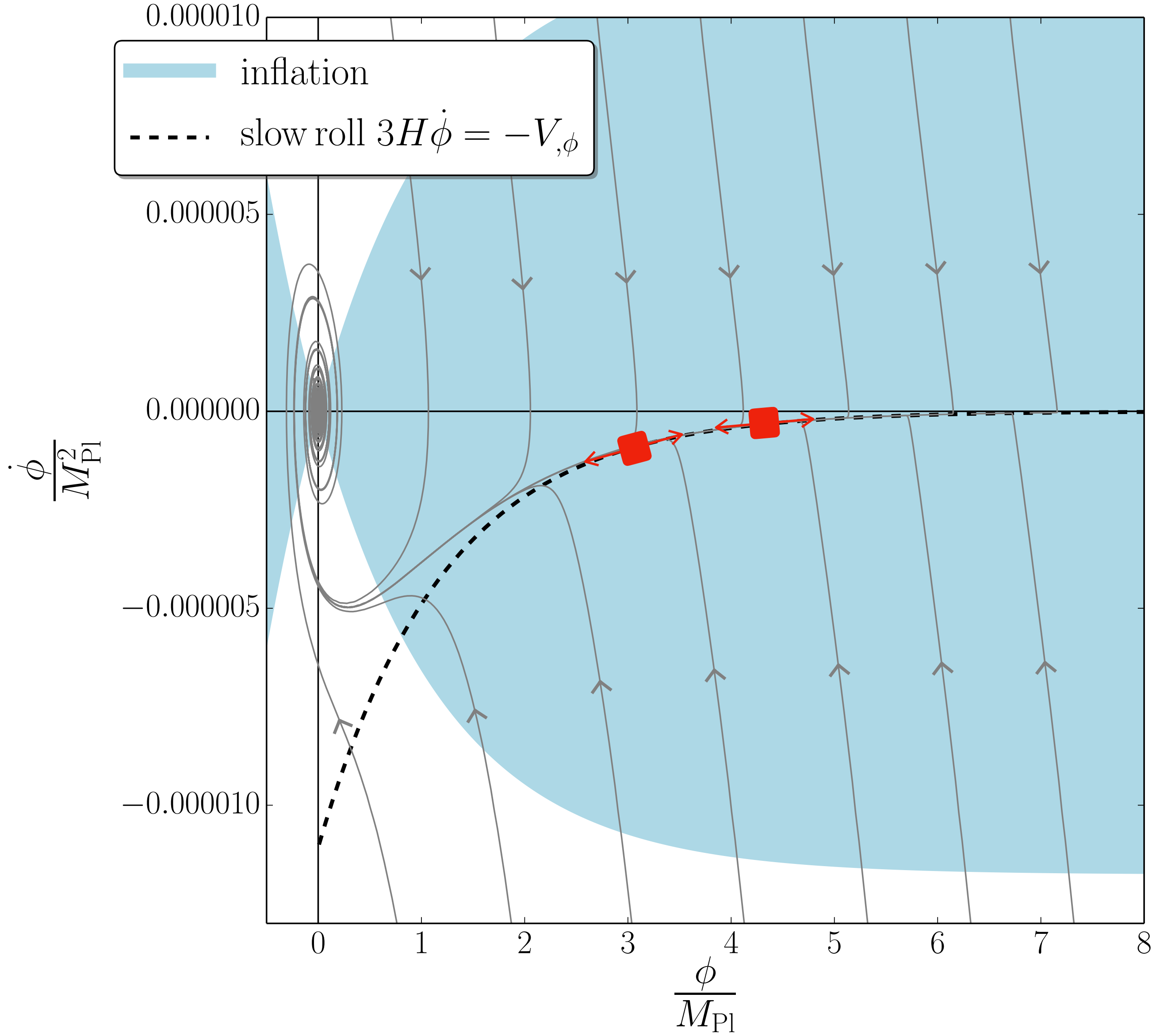
Beyond slow roll

Grain, VV (2017): Formulation of stochastic inflation in phase space

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = \pi + \xi_\phi$$

$$\frac{\mathrm{d}\pi}{\mathrm{d}N} = -\left(3 - \frac{\pi^2}{2M_{\mathrm{Pl}}^2}\right)\pi - \frac{V'}{H^2} + \xi_\pi$$

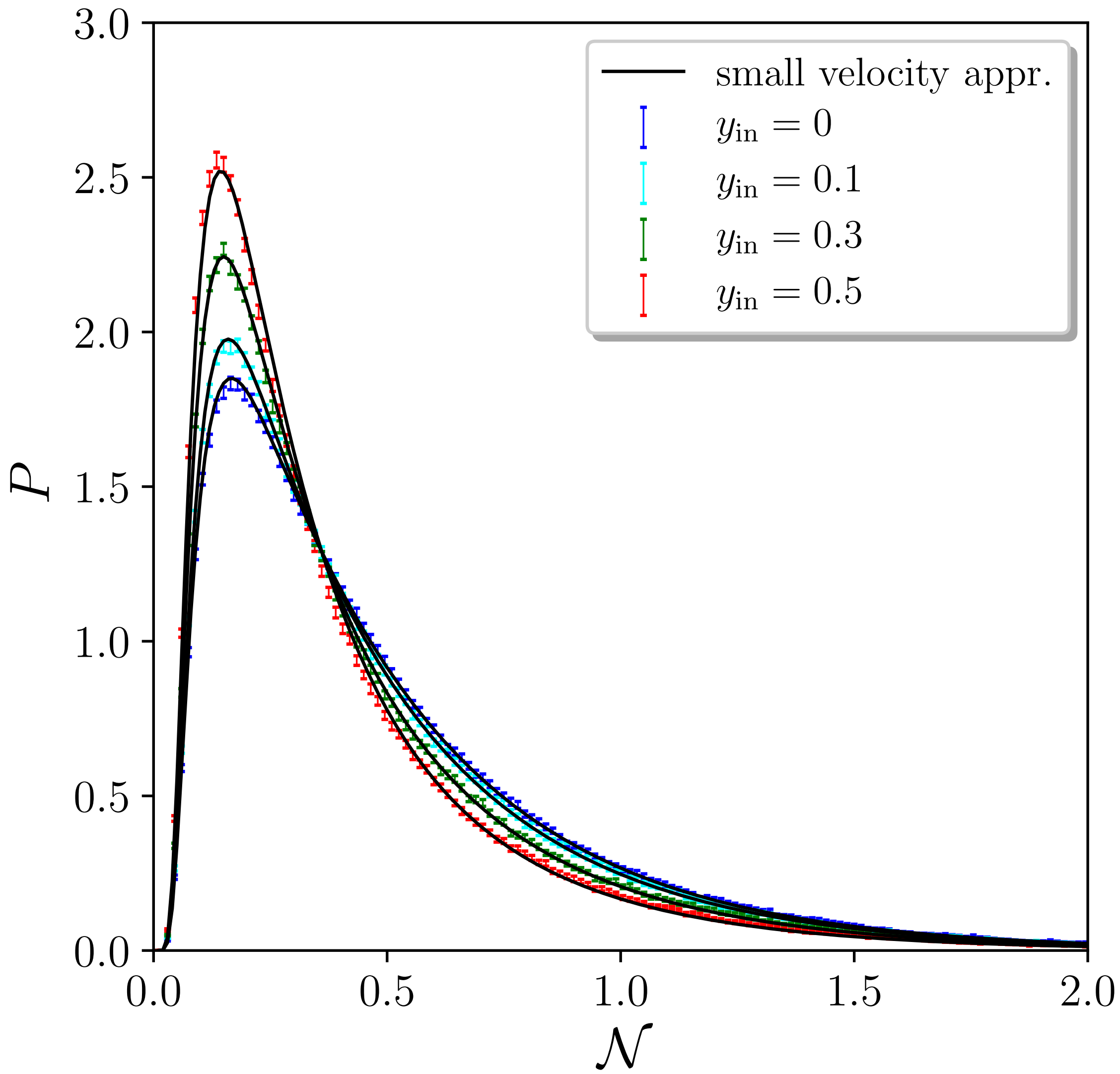
→ Slow roll is a stochastic attractor

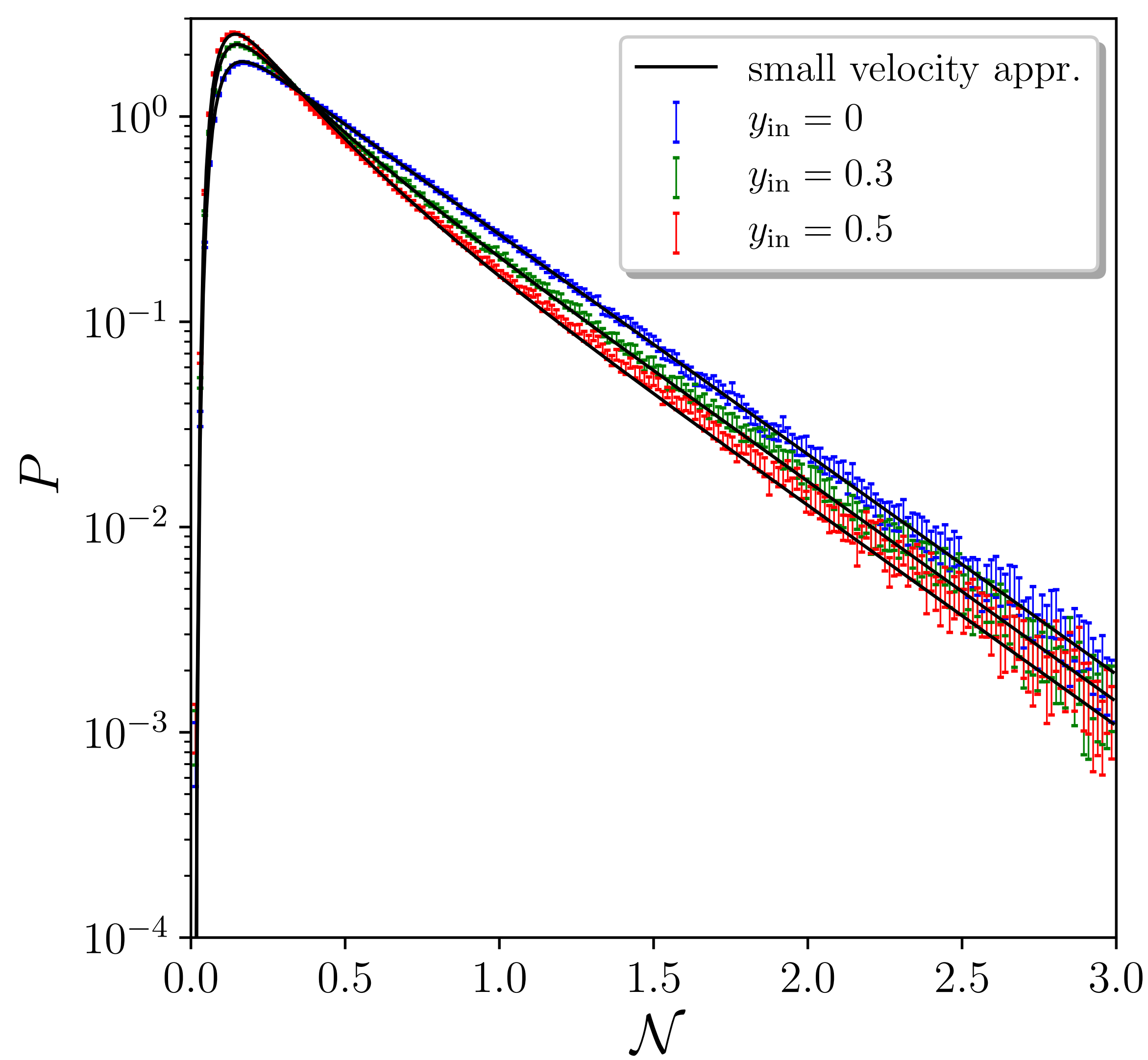


Starobinsky model

Exponential tails
in ultra slow roll models

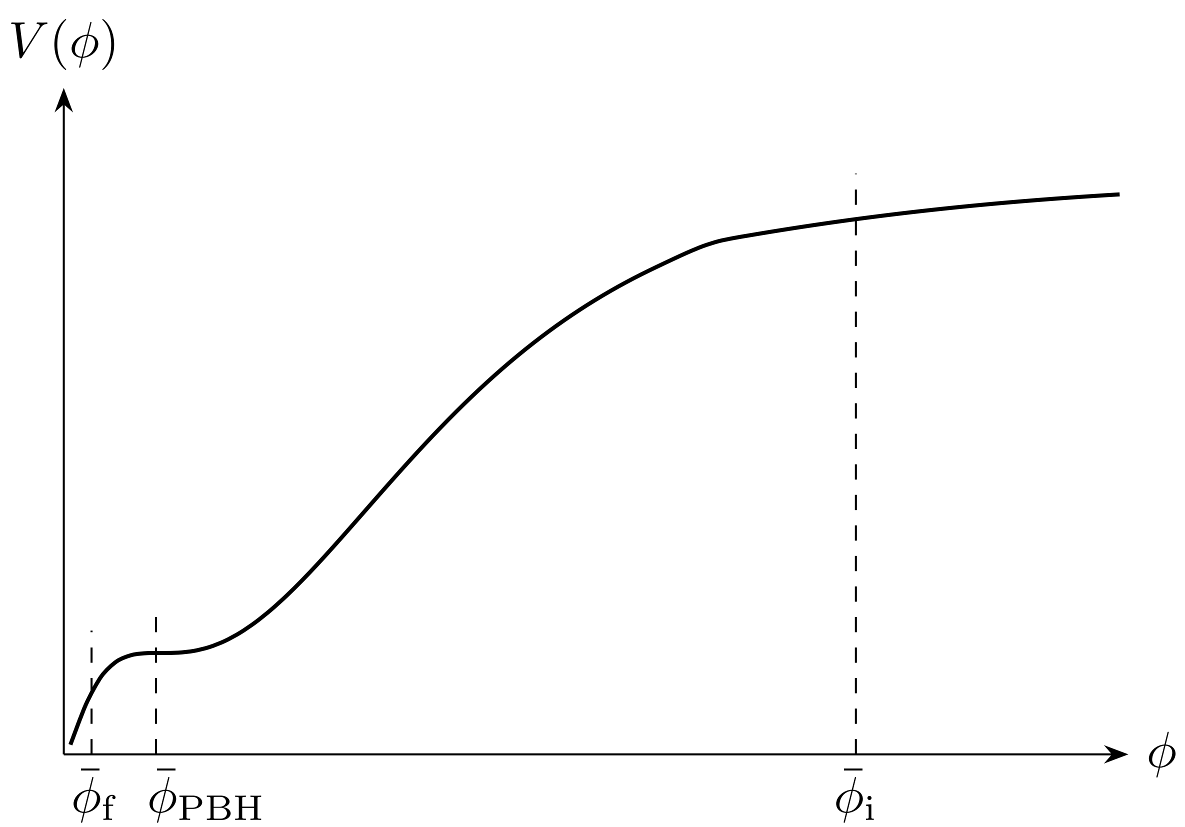
Pattison, Vennin, Wands, Assadullahi (2021)

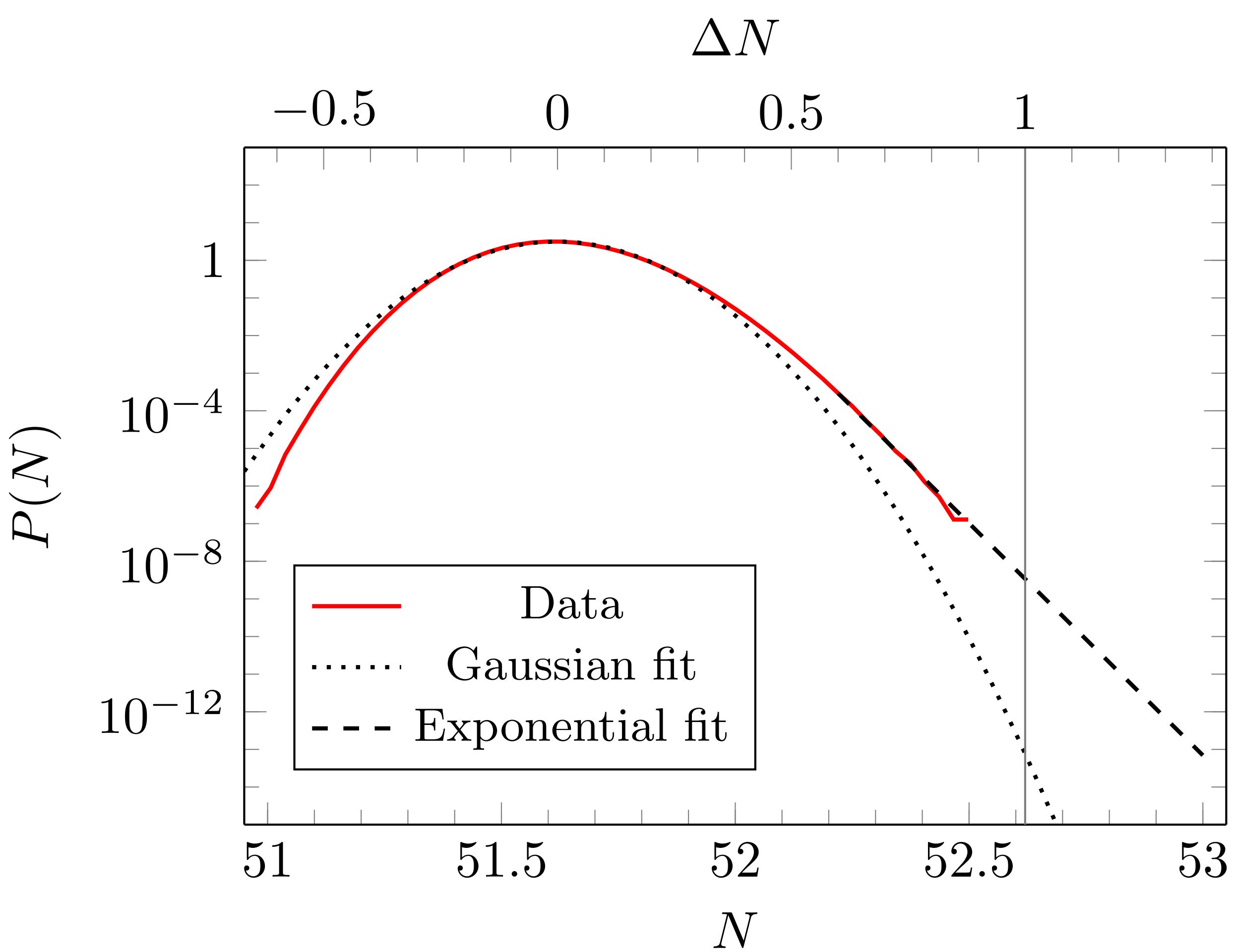




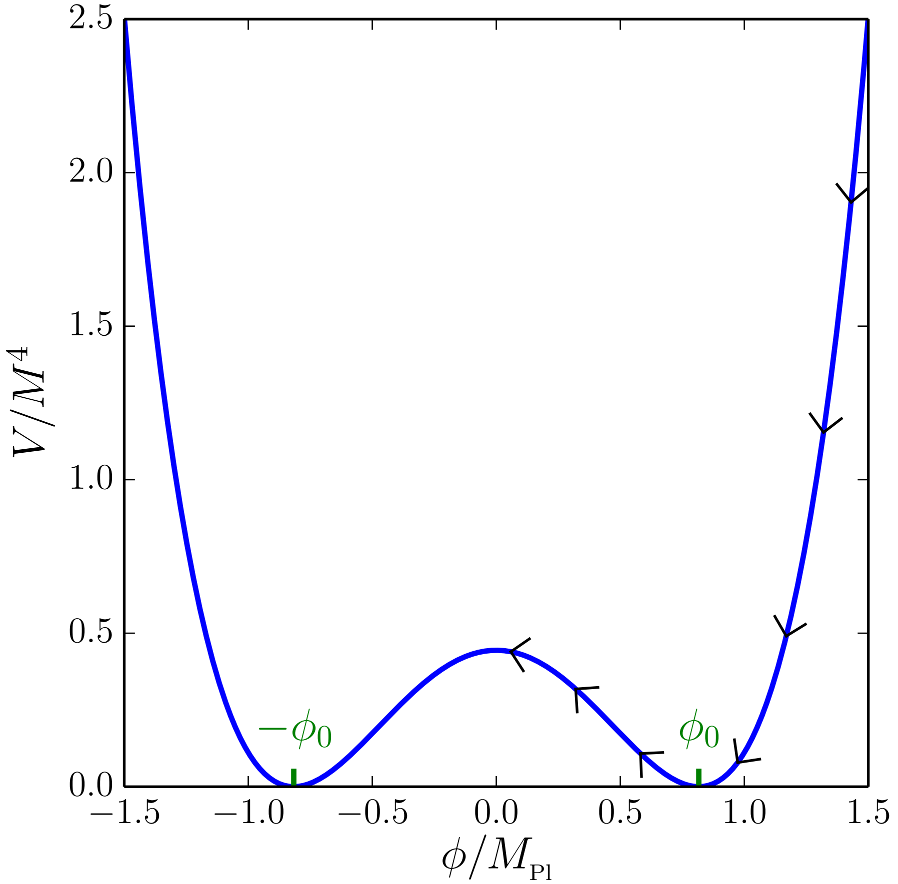
Exponential tails
in ultra slow roll models

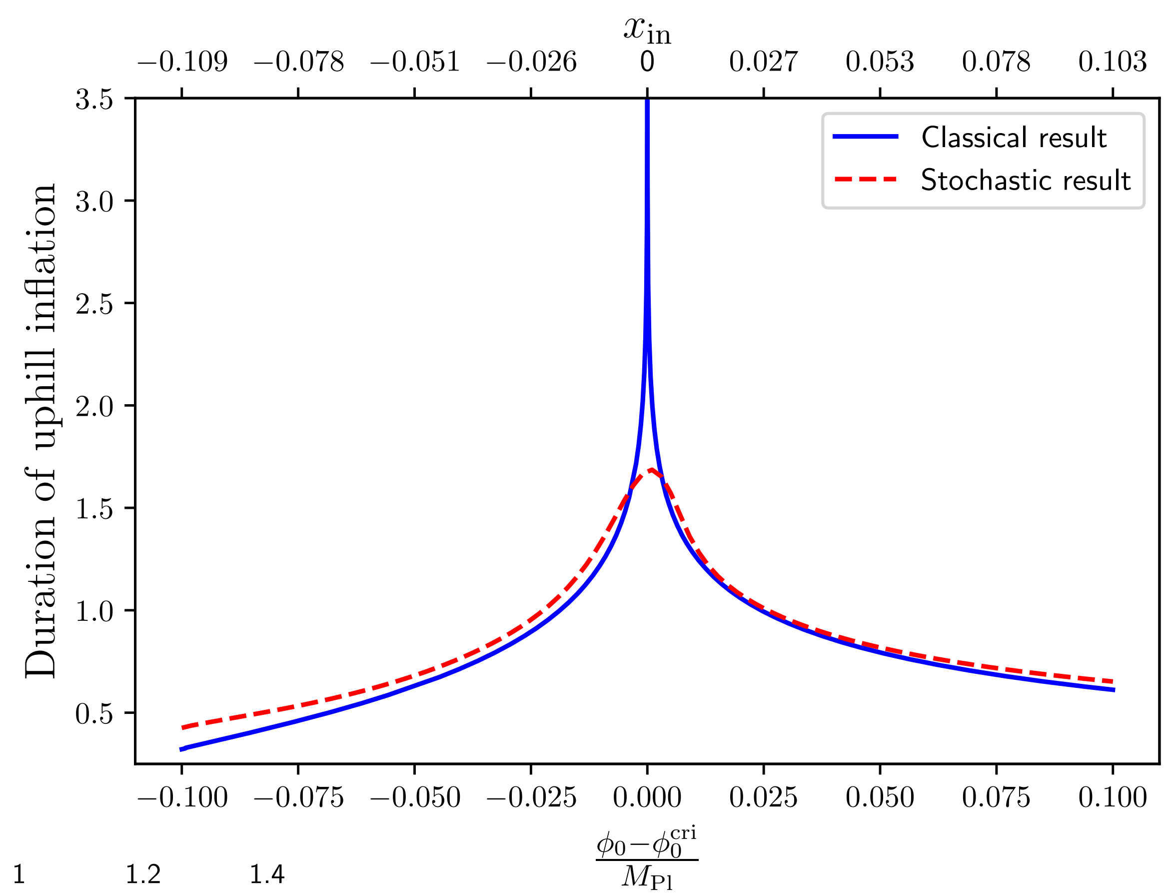
D. Figueroa, S. Raatikainen, S. Räsänen, E. Tomberg (2020)

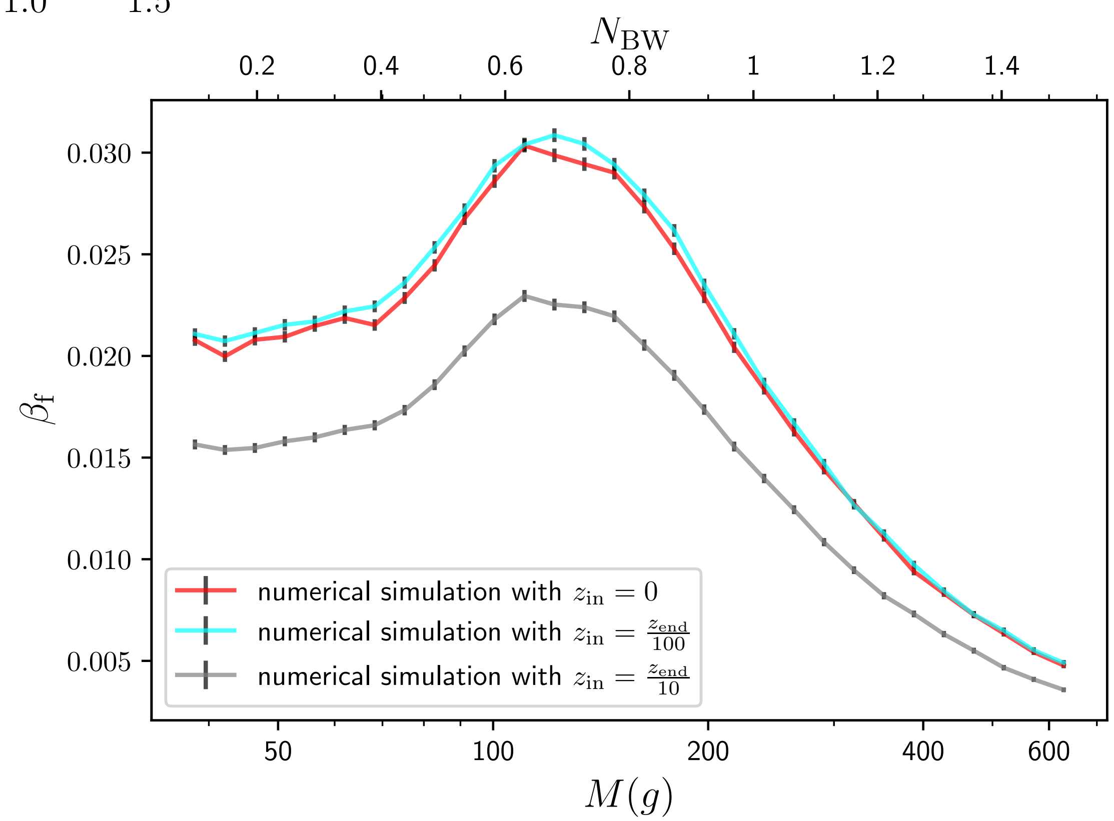




Exponential tails
in non-slow-roll models







Vadim Briaud, VV (2023)

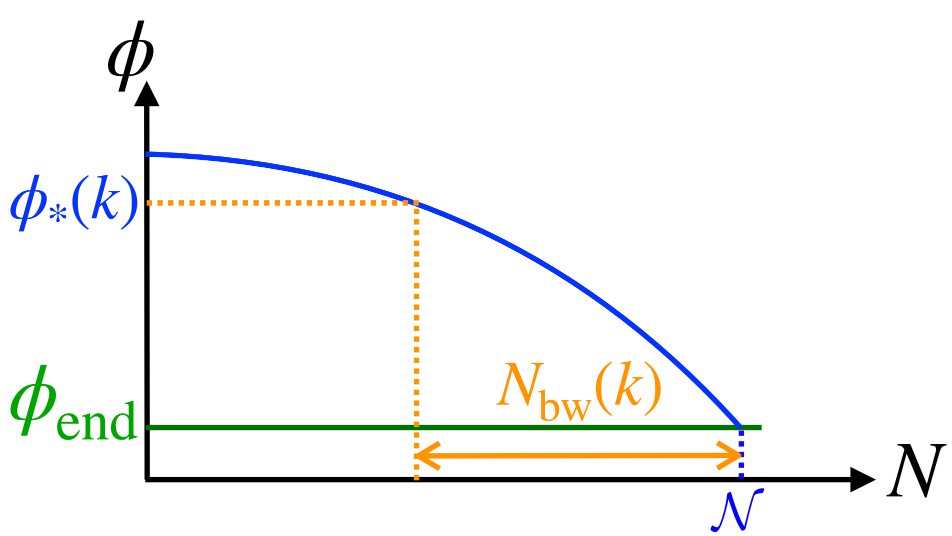
Extracting cosmological observables

Scale k → Hubble-crossing time → Hubble-crossing field ϕ_*

$$N_* = N_{\rm end} - N_{\rm bw}(k)$$

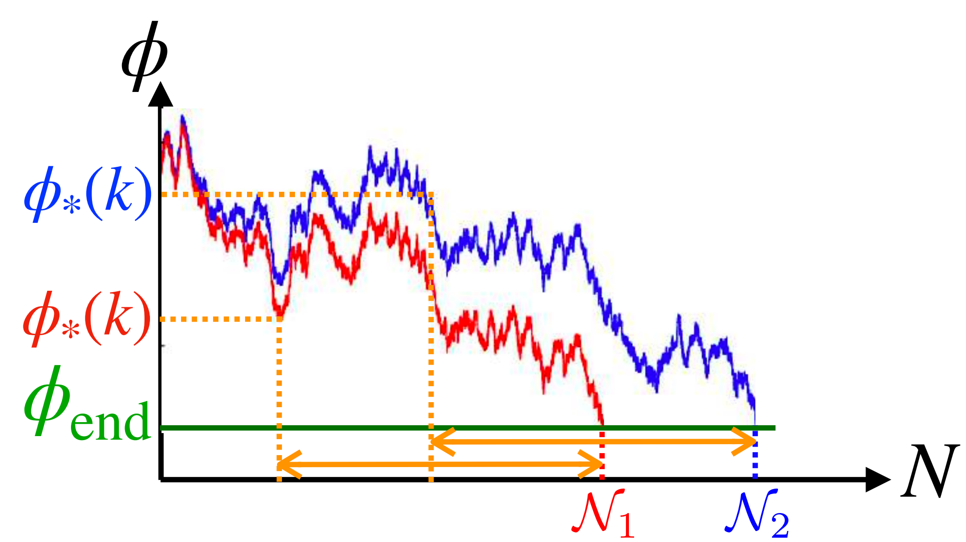
$$N_{\rm bw}(k) = \ln(a_{\rm end}H/k)$$

<u>Classical picture</u>



Observables (power spectrum etc) at scale k depend on <u>**local properties**</u> of the potential at location $\phi_*(k)$

<u>Stochastic picture</u>



$$P_{\rm bw}(\phi_*; k) = P_{\rm FPT}\left[N_{\rm bw}(k); \phi_*\right] \frac{\int_0^\infty P(\phi_*; N)\, \mathrm{d}N}{\int_{N_{\rm bw}(k)}^\infty P_{\rm FPT}(\mathcal{N}; \phi_{\rm in})\, \mathrm{d}\mathcal{N}}$$

Kenta Ando, VV (2020)

Observables at scale k depend on the <u>**whole potential**</u> and on the <u>**initial condition**</u>

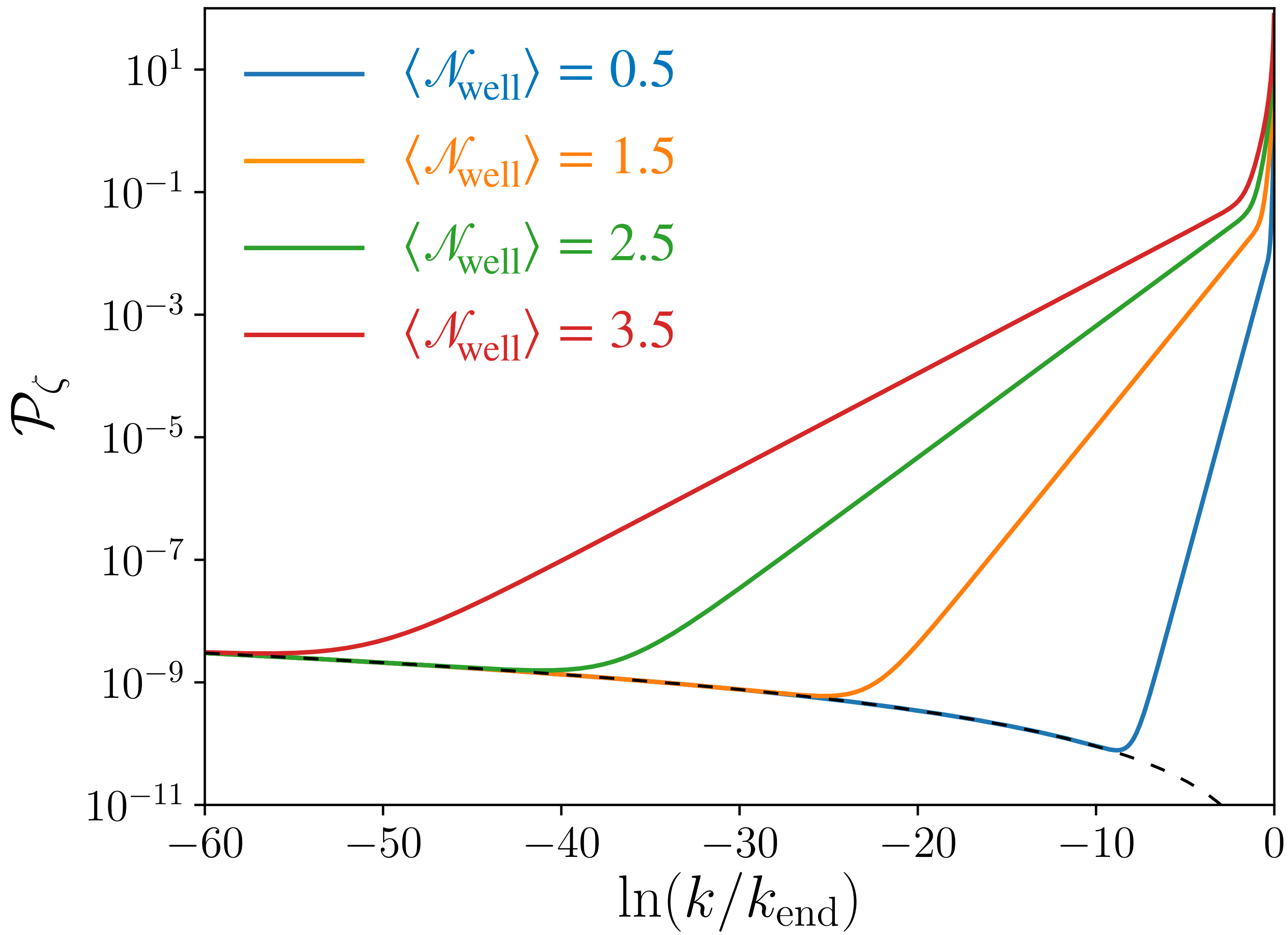
Extracting cosmological observables

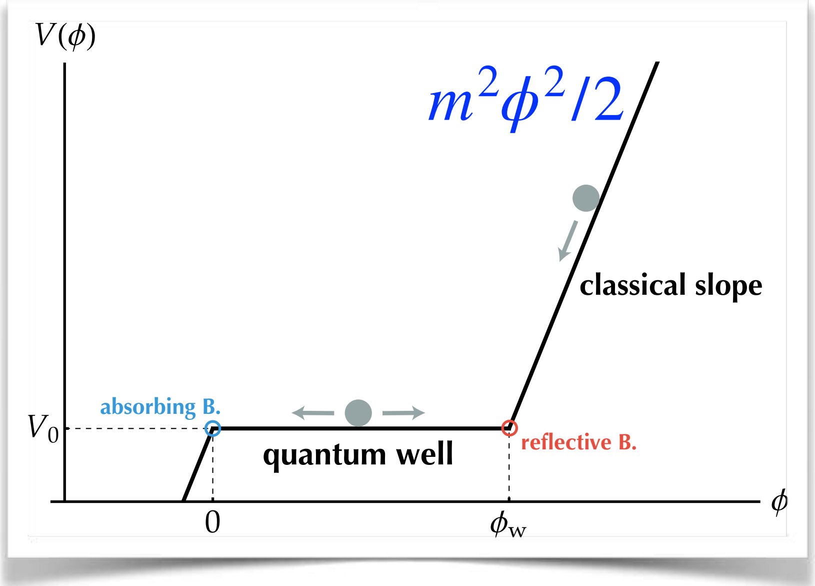
Power Spectrum

Kenta Ando, VV (2020)

$$\mathscr{P}_\zeta(k) = -\int_\Omega \mathrm{d}\mathbf{\Phi}_* \left.\frac{\partial P_{\mathrm{bw}}(\mathbf{\Phi}_*; N_{\mathrm{bw}})}{\partial N_{\mathrm{bw}}}\right|_{N_{\mathrm{bw}}(k)} \left\langle \delta\mathcal{N}^2(\mathbf{\Phi}_0 \to \mathbf{\Phi}_*)\right\rangle$$

Integration over the full inflating domain





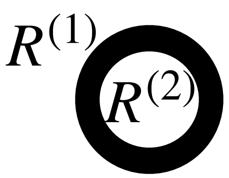
Extracting cosmological observables

One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

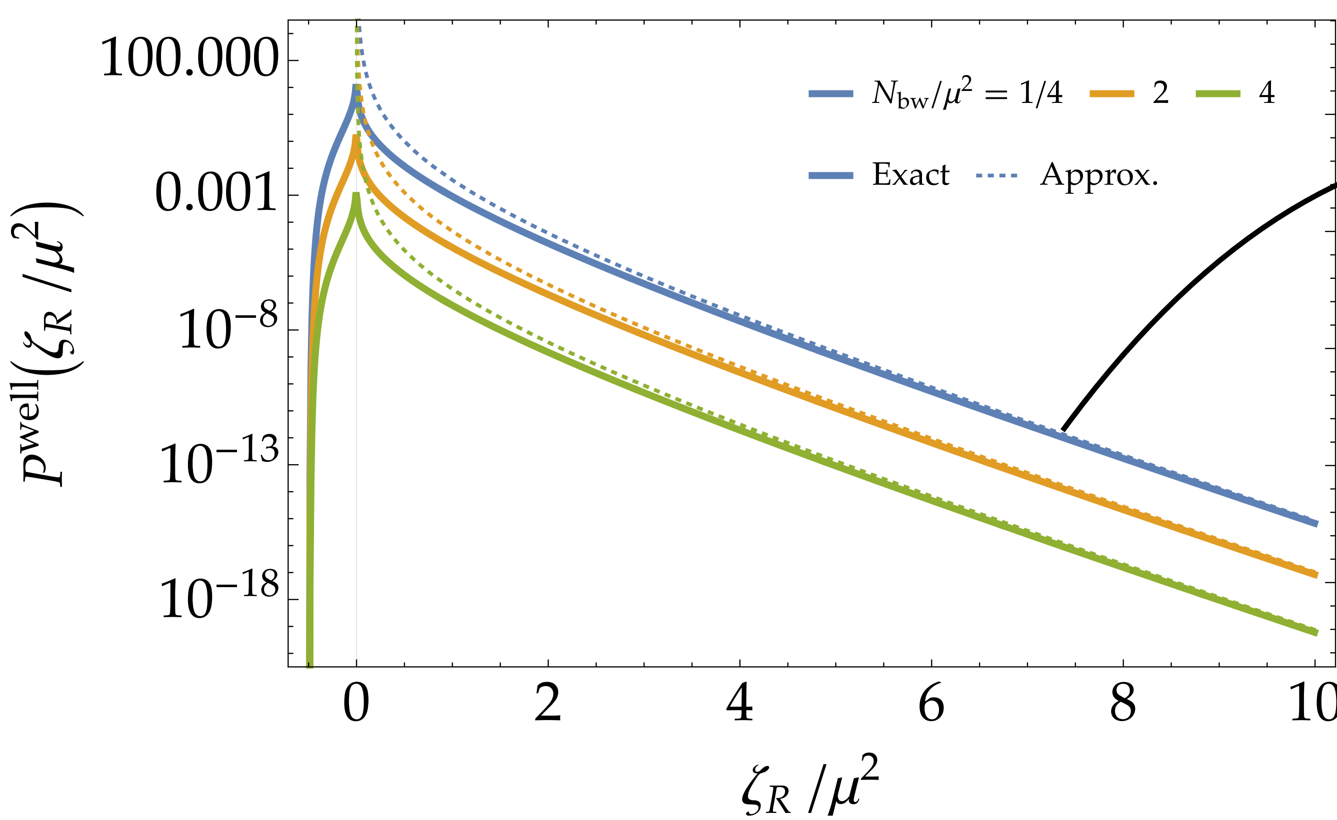
$$P\left(\zeta_R\right) = \int_\Omega \mathrm{d}\mathbf{\Phi}_* P_{\mathrm{bw}}\left[\mathbf{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT}, \mathbf{\Phi}_0 \to \mathbf{\Phi}_*}\left[\zeta_R - \langle \mathcal{N}(\mathbf{\Phi}_*)\rangle + \langle \mathcal{N}(\mathbf{\Phi}_0)\rangle\right]$$

$$P(\Delta\zeta) = \int_\Omega \mathrm{d}\mathbf{\Phi}_*^{(1)} \mathrm{d}\mathbf{\Phi}_*^{(2)} P_{\mathrm{bw}}\left(\mathbf{\Phi}_*^{(1)}, \mathbf{\Phi}_*^{(2)} \middle| N_{\mathrm{bw}}^{(1)}, N_{\mathrm{bw}}^{(2)}\right) \delta\left[\Delta\zeta + \left\langle \mathcal{N}\left(\mathbf{\Phi}_*^{(1)}\right)\right\rangle - \left\langle \mathcal{N}\left(\mathbf{\Phi}_*^{(2)}\right)\right\rangle - \ln(1+\beta)\right]$$



→ Comoving density contrast

→ Compaction function



$$P\left(\zeta_R\right) \propto \frac{e^{-\frac{\pi^2}{4}\frac{\zeta_R}{\mu^2}}}{\left(\zeta_R/\mu^2\right)^3}$$

Quasi-exponential tail

Extracting cosmological observables

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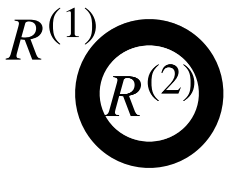
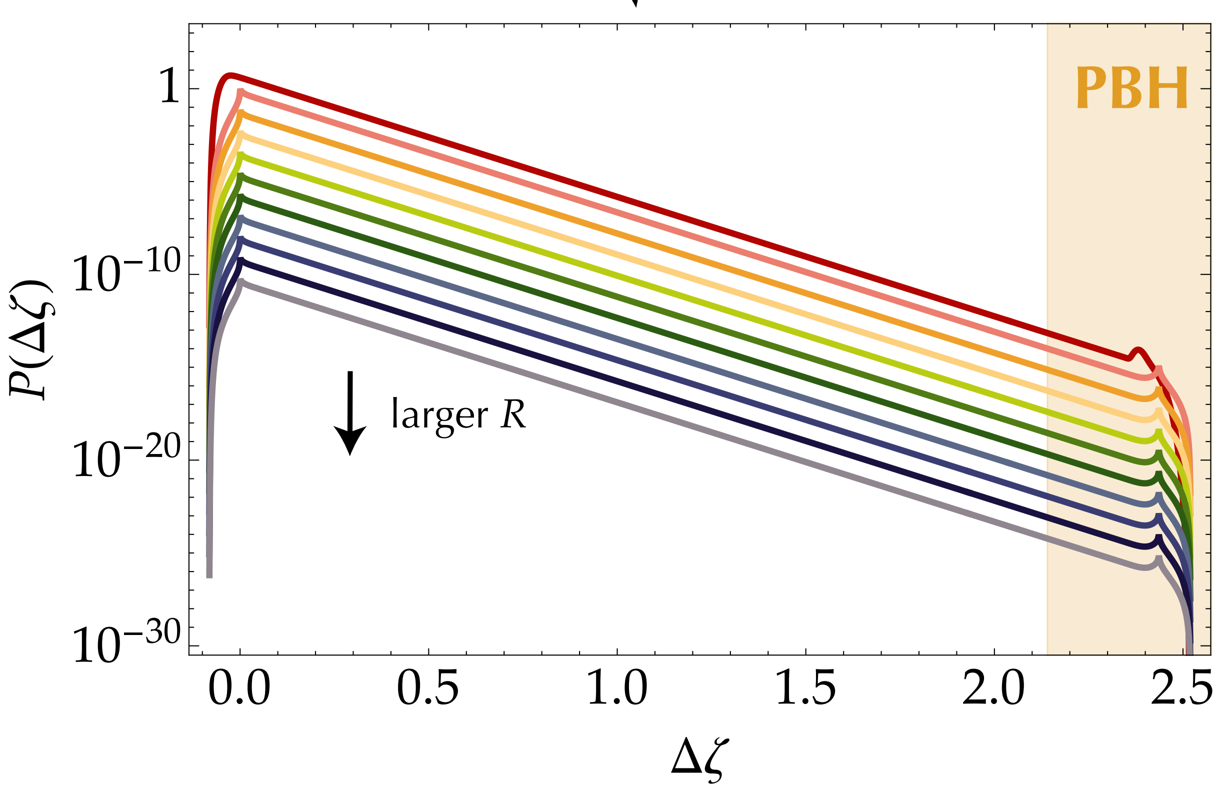
Yuichiro Tada, VV (2021)

$$P\left(\zeta_R\right) = \boxed{\int_\Omega \mathrm{d}\boldsymbol{\Phi}_*} P_{\mathrm{bw}}\left[\boldsymbol{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT},\boldsymbol{\Phi}_0\to\boldsymbol{\Phi}_*}\left[\zeta_R - \langle \mathcal{N}(\boldsymbol{\Phi}_*)\rangle + \langle \mathcal{N}(\boldsymbol{\Phi}_0)\rangle\right]$$

$$P(\Delta\zeta) = \boxed{\int_\Omega \mathrm{d}\boldsymbol{\Phi}_*^{(1)} \mathrm{d}\boldsymbol{\Phi}_*^{(2)}} P_{\mathrm{bw}}\left(\boldsymbol{\Phi}_*^{(1)}, \boldsymbol{\Phi}_*^{(2)} \middle| N_{\mathrm{bw}}^{(1)}, N_{\mathrm{bw}}^{(2)}\right) \delta\left[\Delta\zeta + \left\langle \mathcal{N}\left(\boldsymbol{\Phi}_*^{(1)}\right)\right\rangle - \left\langle \mathcal{N}\left(\boldsymbol{\Phi}_*^{(2)}\right)\right\rangle - \ln\left(1+\beta\right)\right]$$

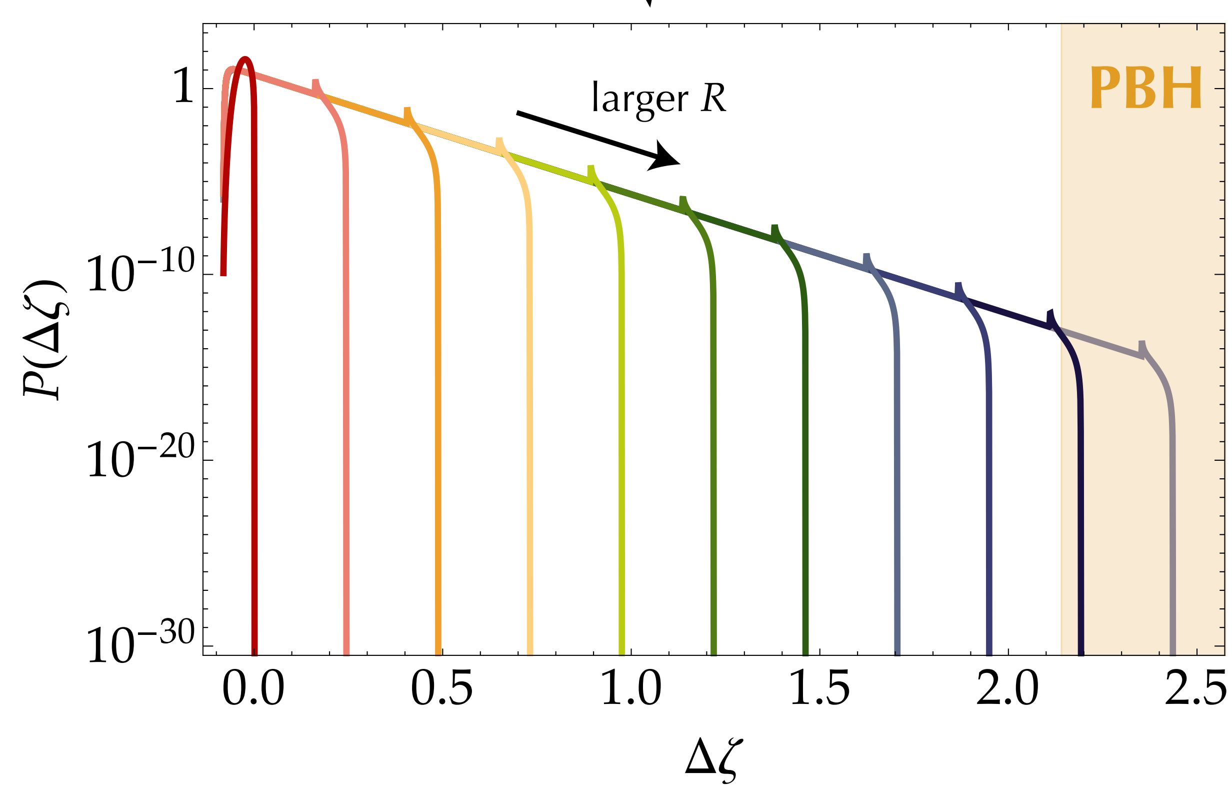


$\mu = \frac{1}{\sqrt{6}}, N_{\mathrm{bw}}^{(2)} > 0$



R_2 exits within the quantum well

$\mu = \frac{1}{\sqrt{6}}, N_{\mathrm{bw}}^{(2)} < 0$



R_2 exits below the quantum well

Extracting cosmological observables

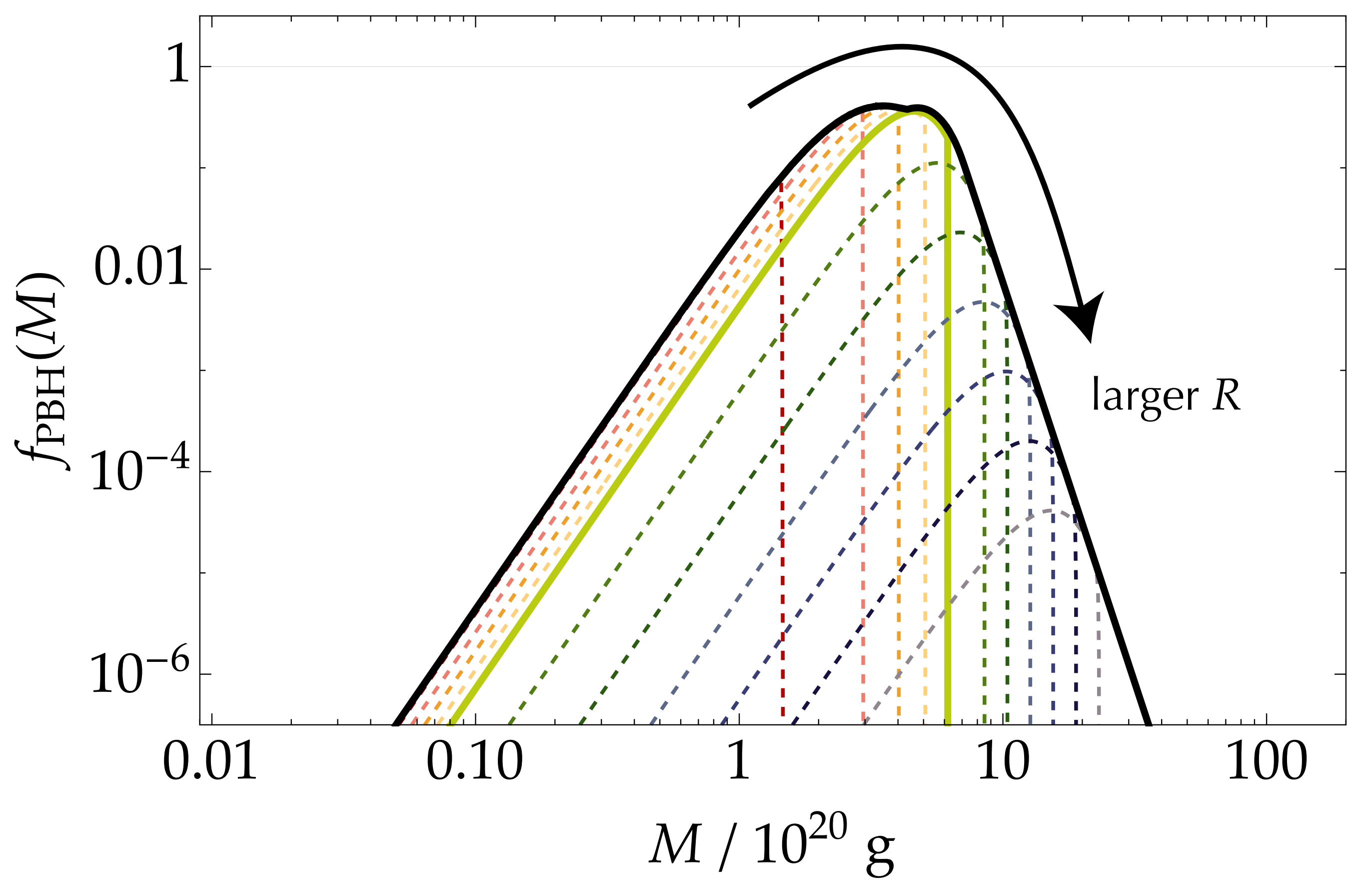
One-point function at arbitrary scale

Yuichiro Tada, VV (2021)

$$P\left(\zeta_R\right) = \int_\Omega \mathrm{d}\boldsymbol{\Phi}_* P_{\mathrm{bw}}\left[\boldsymbol{\Phi}_* \mid N_{\mathrm{bw}}(R)\right] P_{\mathrm{FPT},\boldsymbol{\Phi}_0\to\boldsymbol{\Phi}_*}\left[\zeta_R - \langle\mathcal{N}(\boldsymbol{\Phi}_*)\rangle + \langle\mathcal{N}(\boldsymbol{\Phi}_0)\rangle\right]$$

$$P(\Delta\zeta) = \int_\Omega \mathrm{d}\boldsymbol{\Phi}_*^{(1)}\mathrm{d}\boldsymbol{\Phi}_*^{(2)} P_{\mathrm{bw}}\left(\boldsymbol{\Phi}_*^{(1)}, \boldsymbol{\Phi}_*^{(2)} \middle| N_{\mathrm{bw}}^{(1)}, N_{\mathrm{bw}}^{(2)}\right) \delta\left[\Delta\zeta + \left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*^{(1)}\right)\right\rangle - \left\langle\mathcal{N}\left(\boldsymbol{\Phi}_*^{(2)}\right)\right\rangle - \ln\left(1+\beta\right)\right]$$

$$\mu = \frac{1}{\sqrt{6}}$$



Conclusions

- The back-reaction of vacuum quantum fluctuations on the background dynamics can be incorporated within the formalism of stochastic inflation
- This is necessary to describe regimes leading to large fluctuations, such as those yielding primordial black holes
- Quantum diffusion leads to exponential tails: non-perturbative breakdown of Gaussian statistics
- Most cosmological observables can be reconstructed from first-passage time analysis (power spectrum, mass functions, n-point functions?)
- Quantum diffusion makes the CMB probe the whole potential: models leading to PBHs are constrained by the CMB, even if those two sets of scales are well separated
- **What is the best strategy to look for exponential tails in the data?**

Thank you for your attention