The DRM- String transition and the Supergroup story

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- ✓ 1968: Gabriele Veneziano discovered his four-point amplitude, the first fundamental step of dual resonance model (DRM), a crossing-symmetric, Regge behaved S-matrix with linearly rising trajectories for the scattering process $\pi\,\pi \to \pi\,\omega$
- ✓ this discovery generated an explosion of research activity of world-wide young theorists directed toward discovering and developing the underlying theory of the Veneziano model.
- ✓ essential was the support and encouragement of three masters in the older generation: Sergio Fubini, Stanley Mandelstam and Yoichiro Nambu.
- ✓ soon N-point amplitudes were constructed by K. Bardakçi & H. Ruegg, C.G. Goebel & B. Sakita, Chan Hong-Mo & T.S. Tsun and put in a more symmetric $SL(2,R)$ invariant form by Koba & Nielsen (1969)

- ✓ Fubini and Veneziano, and Bardakci and Mandelstam were the first to prove the necessary factorization properties of the N particle amplitude
- ✓ The vertex involving three arbitrary states was constructed by Stefano Sciuto (1969) and Della Selva and Saito (1970)
- ✓ The version completely symmetric under the permutations of the three states by L. Caneschi, A. Schwimmer and Veneziano (1970)
- ✓ Its generalisation to N legs (N-Reggeon vertex) was first computed by C. Lovelace (1970) and used for computing multiloop amplitudes by means of the sewing procedure [V. Alessandrini, D. Amati, M. Le Bellac and D. Olive (1971)]
  However the correct integration measure was missing
- ✓ This was obtained much later, when a BRST invariant formulation of string theory and the light-cone functional integral could be used for computing multiloops by Paolo Di Vecchia, Marialuisa Frau, Alberto Lerda and Stefano Sciuto (1987) J. L. Petersen, J. R. Sidenius and A.K. Tollsten (1988) and S. Mandelstam (1992)

- ✓ At the end of 1969 Miguel Virasoro discovered that only the model with a massless spin-one particle (i.e. $\alpha_0 = 1$) has an infinite family of Ward-like identities (later identified with the 2d conformal symmetry) that could provide a ghost-free spectrum
- ✓ $\alpha_0 = 1$ was an early disappointing signal that the known dual resonance amplitudes could not give a precise description of hadron spectrum.
- ✓ Fubini and Veneziano (1969) found the algebra of the Virasoro generarors (the central charge was first calculated by Joe Weis) $[L_n, L_m] = (n-m)L_{n+m} + \frac{D}{12}(n^3-n)\delta_{n,-m}$
- ✓ Fubini, Gordon and Veneziano (1969) wrote the N-point amplitude in the form $B_N = \langle 0|\, V\, D\, V \ldots D\, V|0\rangle$ (V= vertex, D=propagator) This opened the way to study the spectrum using the L_n's
- ✓ Emilio Del Giudice and Paolo Di Vecchia (1970) wrote the equations characterizing the physical states:

$$L_n|\lambda\rangle = (L_0 - 1)|\lambda\rangle = 0\,,\ \forall\, n > 0$$

- ✓ The one-loop non-planar diagram of Veneziano model showed the presence of unitarity violating cuts. Claude Lovelace (1971) showed that when $\alpha_0 = 1$ these singularities are factorisable; furthermore, using some bold assumption, argued that those cuts become perfectly allowed additional poles if $D = 26$ (*this looked very crazy at that time*)
- ✓ The vertex operator for an arbitrary physical state was constructed by Campagna, Fubini, Napolitano and Sciuto (1971)
- ✓ Del Giudice , Di Vecchia and Fubini (1972), using the "strong photon" vertex associated with the physical state at $\alpha(s) = 1$, constructed the so called DDF operators $A_{i,m}, (i = 1, \dots D - 2)$, i.e. an infinite family of harmonic oscillators with $[A_{i,m}, A_{j,n}] = n\, \delta_{i,j}\, \delta_{m+n,0}$ which commute with the L_n's, hence generate an infinite family of orthogonal, positive norm, physical states in one-to- one correspondence with the transverse states of the spectrum.

- ✓ No ghost theorem at the tree level only if $\alpha_0 = 1$ & $D \leq 26$
  At $D = 26$ the DDF states generate a complete set of positive norm (Peter Goddard and Charles Thorn (1972) , Richard Brower (1972))
- ✓ Trouble with unitarity at one loop for $D \neq 26$
- ✓ Concerning the interaction of yhe DDF states, S. Mandelstam (1974) and Eugéne Cremmer & Jean-Loup Gervais (1974) showed that the on shell three-point amplitude, computed in string theory with the path integral method, was identical to that of three arbitrary DDF states written by Marco Ademollo , Del Giudice, Di Vecchia and Fubini (1974)

- ✓ Out of the blue, Pierre Ramond (1970) discovered a dual resonance model for fermions.
  Shortly after André Neveu and John Schwarz wrote a new DRM "for pions" with a supersymmetric extension of the Virasoro algebra
- ✓ Charles Thorn found a model with two fermions and an arbitrary number of bosons, which was to become the Ramond sector
- ✓ Edward Corrigan and David Olive (1972) constructed the fermion emission vertex making it possible to construct amplitudes with more than two fermions
- ✓ The final merging of the two sectors into the *dual spinor model* (or R-NS model) came with the paper of Lars Brink, Olive, Claudio Rebbi and Joël Scherk (1973)

The string as a mechanical analogue of DRM spectrum

- ✓ Soon after the discovery of Fubini and Veneziano and Bardakci and Mandelstam that the single-particle states of DRM could be consistently described by an infinite collection of harmonic oscillators, Y. Nambu, H. B. Nielsen and L. Susskind formulated independently the conjecture that they could be described as states of a free vibrating string
- ✓ Nambu (unpublished) and Goto (1972) wrote a string action proportional to the area swept by the string in the external target space, however how to quantize this non-linear, constrained action was unknown
- ✓ The Virasoro conditions on the physical states of DRM were directly related to intrinsic geometrical properties of the string world-sheet (Chang & Mansouri 1972, and Mansouri & Nambu 1972).

The Brink & Nielsen mass formula

- In the string picture the Regge intercept α_0 can be seen as (minus) the zero-point energy of a free vibratimg string (m_0 lowest-lying state) $-\alpha_0 = \alpha' m_0^2 = \frac{D-2}{2} \sum_{n\in\mathbb{N}} n\,.$ diverges!

  Lars Brink and Holger Bech-Nielsen (1973) regularized it by renormalizing the speed of phonons along the string. As now seems obvious, a simpler way to do it is to use the ζ-function regularization (FG '76)

$$-\alpha_0 = \frac{D-2}{2}\left(\sum_{n\in\mathbb{N}} \frac{1}{n^z}\right)_{z=-1} = \frac{D-2}{2}\,\zeta(-1) = -\frac{D-2}{24}$$

- Combining this with the fact that the first excited state has only transverse components and hence must be massless $\Rightarrow\ \alpha_0 = 1\,, D = 26$. The same reasonig works for the Ramond Neveu-Schwarz model and gives $D = 10$

1972: the GGRT paper

- The correct treatment and the light-cone quantization of the Nambu-Goto action was performed in the seminal paper of P. Goddard, J. Goldstone,C. Rebbi and Ch. B. Thorn (October, 1972)
- They pointed out the fundamental role of the reparametrization invariance of the string action
- The choice of the orthonormal (or conformal) gauge $\dot{x}^2 + x'^2 = \dot{x} \cdot x' = 0$ implied at once
  - the D'Alembert equation of motion $\ddot{x}_\mu - x''_\mu = 0$
  - at the classical level, the vanishing of 2D energy momentum tensor $T_{++} = T_{--} = 0 \quad [T_{\pm\pm} \equiv (\dot{x} \pm x')^2]$
  - at the quantum level, the Virasoro gauge conditions on the physical states: $L_n|phys\rangle = (L_o - \alpha_0)|phys\rangle = 0 \ \ [L_n = \frac{1}{2i\pi} \oint dz\, z^{n+1}\, T_{++} \ , z = e^{-i\tau}]$
  - no Lorentz anomaly and only transverse degrees of freedom for $D = 26$

Ademollo *et al. : an unusual collaboration*
"the Supergroup"

Shortly after the appearance of the GGRT paper (30 October 1972) a group of former students and young collaborators of Sergio Fubini and Tullio Regge in Florence,Naples, Rome and Turin decided to join their efforts to understand the dual resonance model in the light of this new mechanical model.

There was no recognised leader inside the group and the ideas circulated freely (by ordinary mail and/or extemporaneous meetings) without any care of priority questions (May 1968 was not too far!)

Ideally, they prosecuted the line of thought of the Fubini-Veneziano collaboration which was concluded that year, combining it with the new physical insight coming from the string picture

The initial collaboration was composed by Marco Ademollo from Florence, Alessandro D'Adda, Riccardo D'Auria, Ernesto Napolitano and Stefano Sciuto from Turin, Renato Musto and Francesco Nicodemi from Naples, Paolo Di Vecchia and myself who were at that time both fellows at CERN

Roberto Pettorino and Emilio Del Giudice, both from Naples, joined us later

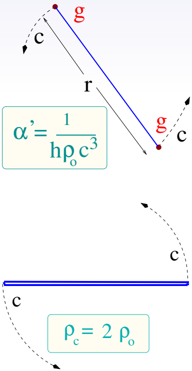
We also collaborated with Sergio Ferrara, Lars Brink and John Schwarz

We wrote six papers in four years

We began to study the motion of an open string in a background external field interacting with point-like charges at the free ends

1973 : The interacting string

- “..if the relativistic string theory is more than an analogue model for the spectrum of DRM, it can be used to obtain informations on the couplings.. ”



- Leading Regge trajectory of the open string

$$J = \frac{\pi}{2} c\rho_o r^2 = \frac{c}{2\rho_o\pi} m^2 = \hbar\alpha' m^2 c^2 + \alpha_0$$

$m = \pi\rho_o r$ (ρ_o mass density in the string frame)

- The gyromagnetic ratio is G=2, like in the coupling of the “strong photon ” in DRM
- α' of the open string is twice that of the closed string, according to the spectrum of the “Pomeron ” sector calculated by Olive and Scherk (1973)
- $\alpha_o^P = 2\alpha_o^R$

the open string in an external electromagnetic field

- $S = \int_{\tau_i}^{\tau_f} d\tau \int_0^{\pi} d\sigma\, \mathcal{L}(x, \dot{x}, x')$; $\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$
- $\mathcal{L}_{int} = \frac{1}{c}\rho(\sigma)\dot{x}_\mu A^\mu(x)$; $\rho(\sigma) = g_o\delta(\sigma) + g_\pi\delta(\sigma)$
- $A_\mu(x) = \epsilon_\mu\, e^{ik\cdot x}$
- Reparametrization invariance of the string world-sheet required

$$k^2 = \epsilon \cdot k = 0$$

i.e. the external field had to be the massless photon state of DRM

- Under these conditions it turned out that the interacting open string had the same mass spectrum of the free case

- The amplitude for the emission of a number of photons from an initial string state to a final one coincided exactly with the corresponding N-point DRM amplitude
- This argument was extended also to excited external fields: reparametrization invariance implied the conformal invariance of the excited vertex. Denoting by L_f the generator of the infinitesimal transformation $\delta\tau = \epsilon\, f(\tau)$, we obtained

$$i\,[L_f, V] = \frac{d}{d\tau}\,(f(\tau)\,V)$$

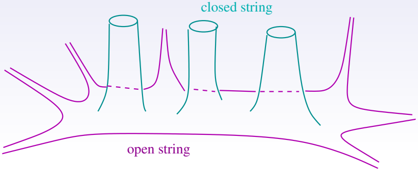
- this established in turn a one-to-one correspondence between the excited vertices and the open string states at D=26.

The bosonic string in an external gravitational field

- This game was extended also to the case of gravitons by coupling an external gravitational field to the target-space metric of the string
  - Reparametrization Invariance yielded the right vertex operator of the “strong graviton”
  - as in the case of the photon, the "strong graviton" was required to be on shell, i.e. the massless closed string state
- such a consistency condition appears to be a precursor of the equations of motion obtained much later requiring the vanishing of the β-function in the σ-model formulation of the string action (Lovelace,1984)
- A simple but important observation made in the conclusions of our first paper iwas that the external gravitational field does not distinguish between open and closed strings. This anticipated our second paper:

"Unified model for interacting open and closed strings"

A general recipe for the tree amplitudes containing both open and closed string states, known today as disk amplitudes



- The mixed amplitude could also be factorized in a closed string channel (here is the first example of what nowadays is called boundary state formalism)
- The closed string factor coincided with the Shapiro-Virasoro model
- A new feature: closed-open string transition represented by a double pole whenever this transition is kinematically possible

- The decay amplitude of a off-mass shell ground state pomeron in N ground state reggeons coincided with the (non-planar) amplitude for one scalar current that Neveu and Scherk obtained the same year factorizing the orientable non-planar one-loop dual graph
- In particular, the pomeron decay in three ground state reggeons $k \to p_1 + p_2 + p_3$ can be written in a closed form

$$B \propto \frac{\Gamma(-\frac{1}{2}\alpha_R(k^2))\Gamma(-\frac{1}{2}\alpha(s))\Gamma(-\frac{1}{2}\alpha(t))\Gamma(-\frac{1}{2}\alpha(u))}{\Gamma(-\frac{1}{2}\alpha(s)-\frac{1}{2}\alpha(t))\Gamma(-\frac{1}{2}\alpha(t)-\frac{1}{2}\alpha(u))\Gamma(-\frac{1}{2}\alpha(u)-\frac{1}{2}\alpha(s))}$$

with $\alpha_R = 1 - k^2$, $\alpha(s) = 1 - (p_1 + p_2)^2$, $\alpha(t) = 1 - (p_2 + p_3)^2$, $\alpha(u) = 1 - (p_3 + p_1)^2$,

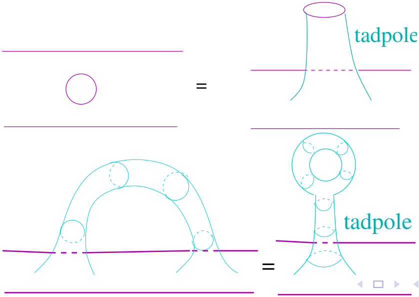
- Complete agreement between Pomeron-Reggeon amplitudes of DRM with the unfied model for interacting closed and open strings at $D = 26$

this unified formulation was the main ingredient of our third paper(1975) on a general way of resumming a class of divergent contributions due to unitarity corrections of string tree amplitudes

These can be expressed as a sum over topologically inequivalent surfaces

The topology of a (orientable) surface is determined by the number of holes and handles

the divergent part (besides the tachyon singularity) can be thought of as the contribution of the on-shell soft dilaton decaying into the vacuum



A soft dilaton theorem

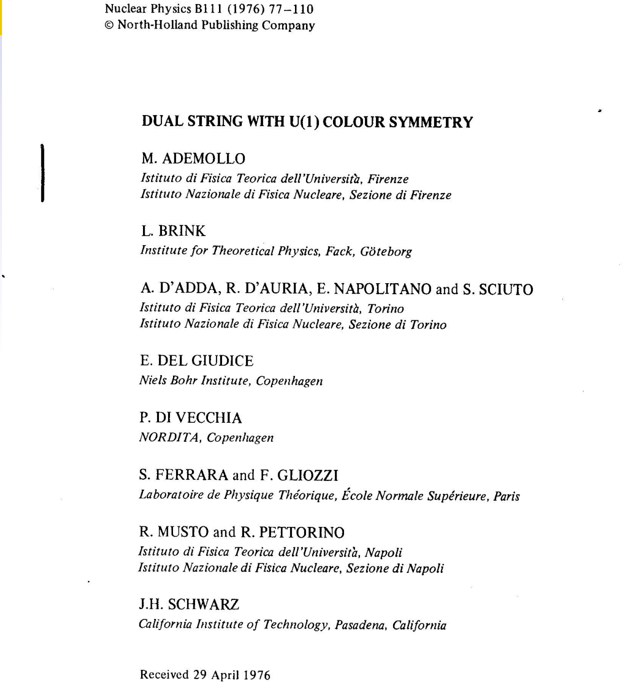
- ✻ The soft dilaton limit of the amplitudes is particularly simple when all the physical states of open and/or closed strings are massless:

$$\lim_{k\to 0} T(k, p_1, p_2, \ldots, p_n) =$$
$$\pi g_c \alpha'^{\frac{d-2}{2}} \left[\sqrt{\alpha'}\frac{\partial}{\partial\sqrt{\alpha'}} - \frac{d-2}{2}\left(\frac{1}{2} g \frac{\partial}{\partial g} + g_c \frac{\partial}{\partial g_c}\right)\right] T(p_1, p_2, \ldots, p_n)$$

- ➩ The sum over all the possible soft dilaton insertions can be explicitly performed
- ➩ the net effect is simply a renormalization of the slope α' , of the open string (g) and the closed string (g_c) couplings
- ➩ $\alpha'_{\mathcal{R}} = Z\alpha'$; $g_{\mathcal{R}} = Z^{\frac{2-d}{8}} g$; $g_{c\mathcal{R}} = Z^{\frac{2-d}{4}} g_c$

A feedback from Supersymmetry

- 1971: Gervais & Sakita reformulate the R-NS algebra in terms of local supersymmetric transformations in a 2D field theory and introduce for the first time anticommuting parameters
- 1973: Wess& Zumino introduce and develop the concept of global supersymmetry in D=4 space-time
- 1974: Salam & Strathdee introduce the notion of superfield and extended supersymmetry
- 1975: Ademollo *et al.* extend the R-NS algebra to $\mathcal{N} = 2$ and $\mathcal{N} = 4$ superconformal algebras using D=2 free superfields (fully appreciated only after the first String Revolution)



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DUAL STRING WITH U(1) COLOUR SYMMETRY

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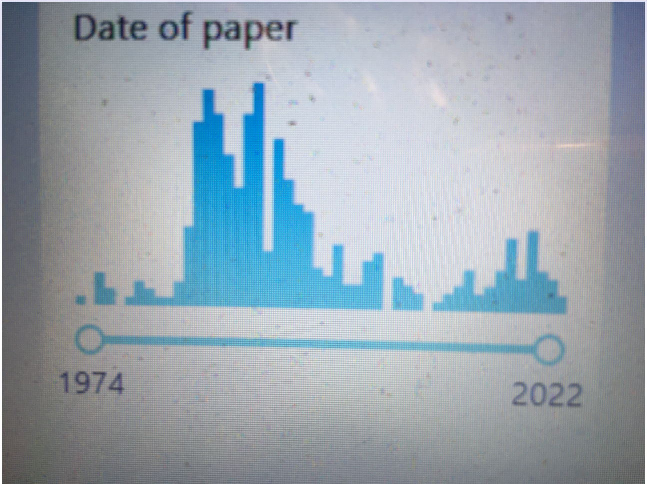
Date of paper
1974
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Virasoro

$$[L_m, L_n] = (m-n)\, L_{m+n} + \frac{2b+f}{24} D n(n^2-1)\delta_{m,-n}$$

N S–R

$$\{G^j_r, G^j_s\} = 2L_{r+s} + \frac{(b+f)D}{16}(4r^2-1)\,\delta_{r,-s}$$

$$[L_n, G^j_r] = \left(\frac{n}{2} - r\right) G^j_{n+r}$$

U(1)

N=2

$$\{G^o_r, G^j_s\} = 2i(r-s)\,T^j_{r+s}$$

$$[L_m, T^j_n] = -n\, T^j_{m+n}$$

$$[T^j_m, T^j_n] = \frac{(b+f)D}{16}\,\delta_{m,-n}$$

$$[T^j_m, G^o_r] = \frac{i}{2} G^j_{m+r} \; ; \; [T^j_m, G^j_r] = \frac{-i}{2} G^o_{m+r}$$

	b	f
VM	1	0
NS–R	1	1
N=2	2	2
N=4	4	4

SU(2)

N=4

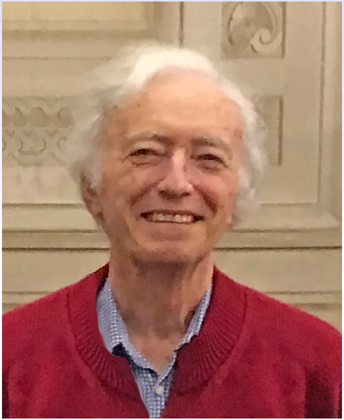
$$\{G^i_r, G^j_s\} = 2i\,\varepsilon_{ijk}(r-s)\,T^k_{r+s}$$

$$\boxed{[T^i_m, T^j_n] = i\,\varepsilon_{ijk}\,T^k_{m+n}} \; ; \; [T^i_m, G^j_r] = \frac{i}{2}\varepsilon_{ijk}G^k_{m+r} - \frac{i}{2}\delta^{ij}G^o_{m+r}$$

Kac–Moody

- ➪ 1976: We felt bitterly disappointed when we realized that the $\mathcal{N} = 2$ string turned out to have $D_{crit} = 2$ (or $2 + 2$, as realized much later) and only one physical state and the $\mathcal{N} = 4$ string contained ghosts for any D
- ➪ No room for a realistic string theory of strong interactions
- ✻ After the discovery of asymptotic freedom in 1973 and charm in 1974, QCD took over
- ✻ Need of a re-interpretation of string theory as a short-distance modified theory of fundamental interactions rather than an hadronic theory with the wrong spectrum (J. Scherk and J.H. Schwarz, 1974)
- ✻ 1976: New suggestions from the discovery of Supergravity: L.Brink, P. Di Vecchia and P. Howe first in collaboration with S. Deser and B. Zumino constructed a supersymmetric world line action describing a Dirac particle and then obtained a locally supersymmetric and reparametrization invariant world-sheet action which describes the R-NS string

- ✓ Eliminating the NS sector having half-integral $(\mathrm{mass})^2$ (among which is the tachyon) and taking the Ramond spinor to be Majorana Weyl (i.e. performing the GSO projection) it turned out that at each level the number of bosonic physical states coincided with that of the fermionic ones both in open and closed string sectors, a necessary ingredient for supersymmetry in the target space in critical dimensions
- ✓ In particular the massless closed string sector formed a ten-dimensonal realization of pure supergravity, hence a four-dimensional supergravity coupled to matter
- ✓ The remaining resonance spectrum, after GSO projection, is ghost-free and tachyon free and includes a massless spin 2 boson, the amplitudes are Regge behaved, but one had to wait until the first string revolution of 1984 for a full appreciation of these results, when string theory had become again a very active field of research as the theory of everything



Happy Birthday, Paolo!