D-Branes and Gauge/Gravity correspondence:

The 2° String Revolution with Paolo

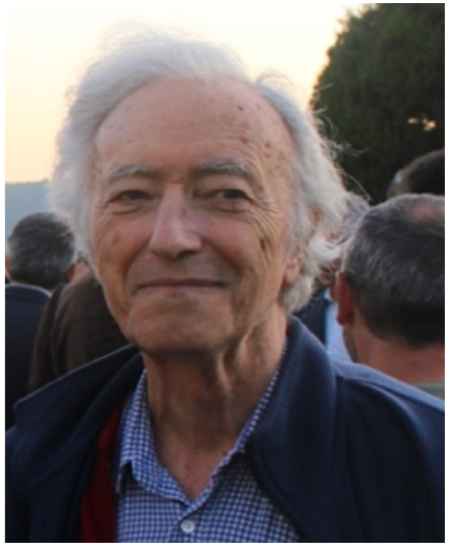
Marialuisa FRAU

Università di Torino and INFN Sez. di Torino

Paolo's fest, Nordita, May 15, 2023

Plan of the talk

- Second String Revolution ⟶
  Boundary State and Classical D-Branes Solutions
- AdS/CFT ⟶ Gauge-Gravity Correspondence
- Later generalizations
- Final remarks



Second String Revolution:
Boundary State and classical D-Branes solutions
(1996-2000)

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Classical p-branes from boundary state ★

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Microscopic string analysis of the D0–D8 brane system and dual R–R states ★

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THE LORENTZ FORCE BETWEEN D0 AND D6 BRANES IN STRING AND M(ATRIX) THEORY*

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(F,Dp) bound states from the boundary state ☆

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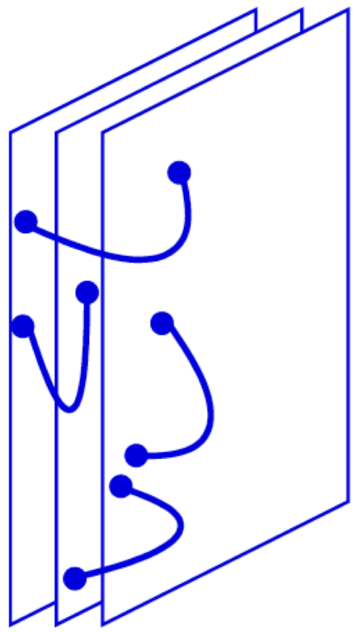
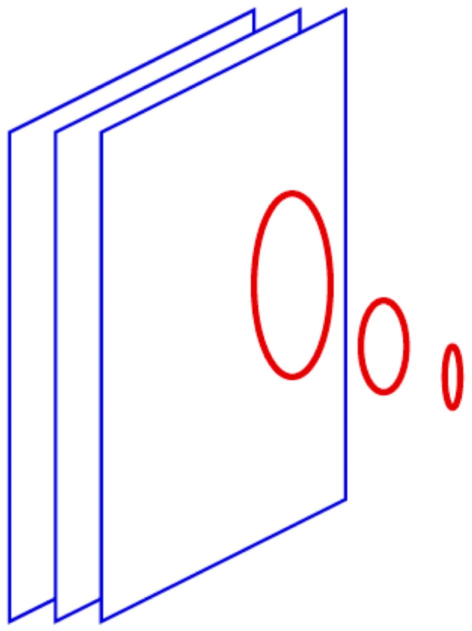
Second String Revolution

- 1995: Polchinski writes «Dirichlet Branes and RR Charges» →

  hyperplanes on which open strings with mixed DD and NN b.c. ends are D-branes, i. e. solitonic solution of SUGRA charged under RR potentials

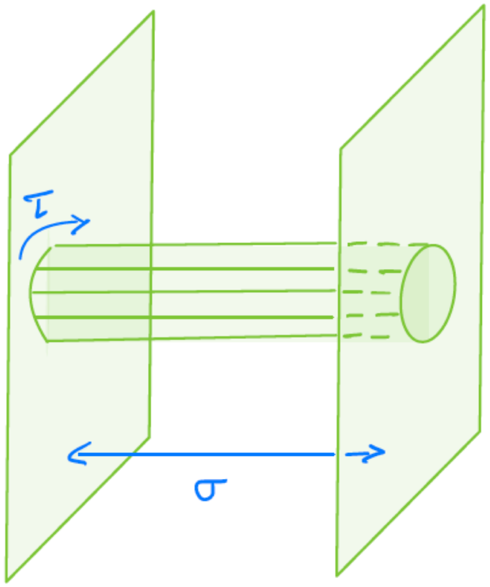




Second String Revolution

- 1995: Polchinski writes «Dirichlet Branes and RR Charges»

  he proves that the description of D-branes in terms of open strings is a concrete calculational tool by computing the interaction between two branes as a 1-loop amplitude of open strings with the corresponding b.c.

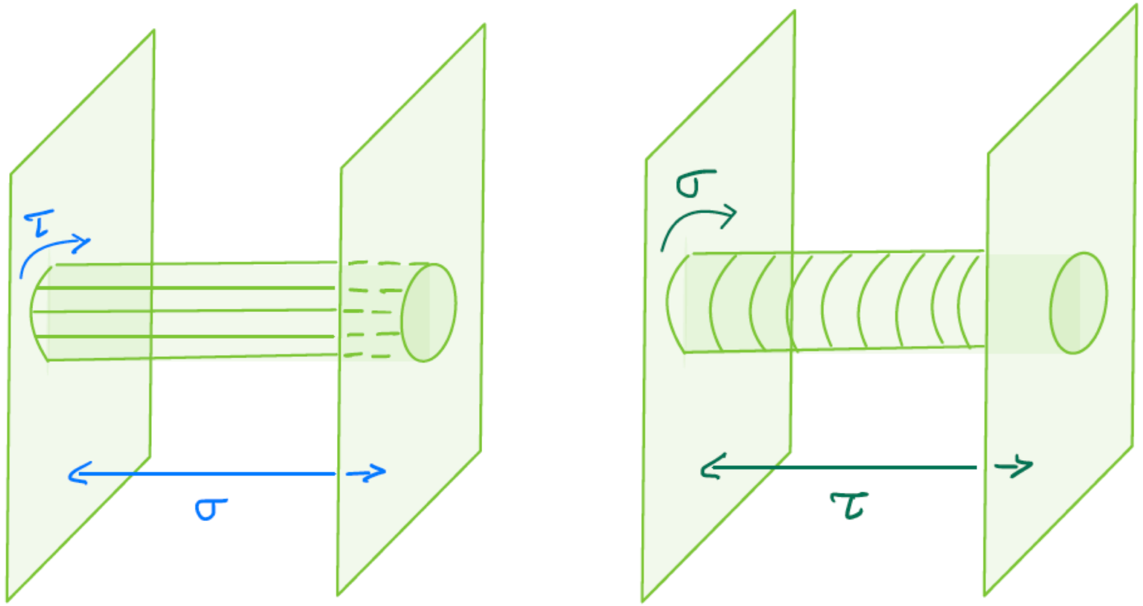


$$\mathcal{A} \propto \mathrm{Tr} \int_0^\infty \frac{dt}{t} \mathrm{e}^{-t\, L_0}$$

whenever the configuration is BPS the amplitude vanishes → there is a no-force condition

Second String Revolution

However, a 1-loop open string amplitude can be read as a closed string tree level process



$$\mathcal{A} \propto \mathrm{Tr} \int_0^\infty \frac{dt}{t} \mathrm{e}^{-t\,L_0}$$

$$\langle B| \frac{1}{L_0 + \bar{L}_0} |B\rangle$$

and can be computed as expectation value of a closed string propagator between two closed string «vacuum states» with the appropriate b.c. the Boundary State $|B\rangle$ (Ademollo et al, Callan et al, Polchinski Cai, etc…)

Second String Revolution

- 1996: Soon after Polchinski's paper we all began to work on this subject
- 1997: Paolo and Stefano realized that in this context the boundary operator, introduced in their paper many years before, could be particularly useful.

  At first we worked independently

  - in Torino we (M.F., Lerda, Pesando, Russo, Sciuto) built the bosonic operator describing the emission of closed strings from a many boundary Riemann surface
  - In Cph Paolo, M. Billò and D. Cangemi constructed a «modern» version of boundary state describing boosted D-branes in superstring models

Second String Revolution

- 1996: Soon after Polchinski's paper we all began to work on this subject
- 1997: Paolo and Stefano realized that in this context the boundary operator, introduced in their paper many years before, could be particularly useful.

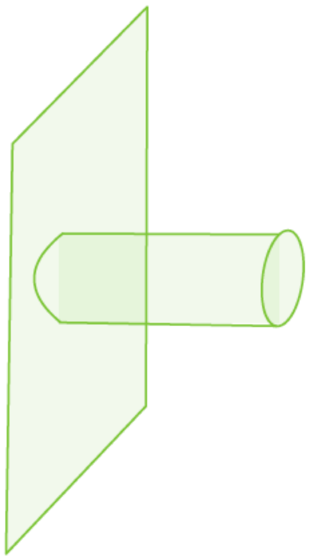
  At first we worked independently … but we soon joined our efforts!

- The results of this collaboration is collected in four joint papers (+ others, and in particular two reviews by Paolo and A. Liccardo) and lead to two substantial results:
  - Precise definition of the Boundary State in Superstring
  - Direct proof that the D-branes description via Boundary State is related to their classical solution ➡ the Boundary State is the source of massless fields whose space-time profile coincide with the leading order of the D-brane solution

Boundary State

- The Boundary state $|B\rangle$ is a BRST invariant closed string state that encodes all the information about the D-brane

i.e. it enforces on the closed string fields the corresponding overlap conditions



$$\partial_\tau X^\alpha|_{\tau=0}\,|B\rangle = 0 \qquad \alpha = 0, 1, \cdots p \qquad \text{for NN directions}$$

$$(X^i - y^i)|_{\tau=0}\,|B\rangle = 0 \qquad i = p+1, \cdots 9 \qquad \text{for DD directions}$$

(and analogously for the world-sheet fermions)

Boundary State

- The Boundary state $|B\rangle$ is a BRST invariant closed string state that encodes all the information about the D-brane

  i.e. it enforces on the closed string fields the corresponding overlap conditions implying that left and right closed string modes are identified

$$\alpha_n^\mu\,|B\rangle = -S^\mu_\nu\,\tilde{\alpha}^\nu_n\,|B\rangle \qquad \text{for world-sheet bosons}$$

$$\psi_n^\mu\,|B\rangle = \pm\,\mathrm{i}\,S^\mu_\nu\,\tilde{\psi}^\nu_n\,|B\rangle \qquad \text{for world-sheet fermions}$$

- S^μ_ν is the matrix of the boundary conditions
- i. e. for a simple D-brane $S = \begin{pmatrix} \eta & 0 \\ 0 & -1 \end{pmatrix}$

Boundary State

- The Boundary state $|B\rangle$ is a BRST invariant closed string state that encode all the information about the D-brane

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$$\psi_n^\mu \,|B\rangle = \pm \mathrm{i}\, S^\mu_\nu \,\tilde{\psi}_n^\nu \,|B\rangle \qquad \text{for world-sheet fermions}$$

- S^μ_ν is the matrix of the boundary conditions
- but with more general b.c. (bound states of different branes, boosted branes etc) the conditions are the same with different a S^μ_ν

Boundary State

- The Boundary state of a Dp-brane configuration is

$$|B\rangle = |B\rangle_{NS} + |B\rangle_{R} \qquad \text{with} \qquad |B\rangle_{NS,R} = N\,|B_{X,bc}\rangle\,|B_{\psi,\beta\gamma}\rangle^{GSO}_{NS,R}$$

- The normalization and the bosonic component are similar to the ones that can be derived in the bosonic string (normalization only from factorization!)

$$N\,|B_{X,bc}\rangle = \frac{T_p}{2}\,\delta^{9-p-1}(q_i - y_i)\exp\left[\sum_{n=1}^{\infty} -\frac{1}{n}\alpha^{\mu}_{-n}S^{\mu}_{\nu}\tilde{\alpha}^{\nu}_{-n}\right]|0\rangle|\tilde{0}\rangle|B_{bc}\rangle$$

$$T_p = \sqrt{2}\,(4\pi^2\,\alpha'^2)^{\frac{3-p}{2}}$$ is the brane tension

- The ghost part $|B_{bc}\rangle$ is essential to decouple unphysical states....

Boundary State

- The Boundary state of a Dp-brane configuration is

$$|B\rangle = |B\rangle_{NS} + |B\rangle_{R} \qquad \text{with} \qquad |B\rangle_{NS,R} = N\,|B_{X,bc}\rangle\,|B_{\psi,\beta\gamma}\rangle^{GSO}_{NS,R}$$

- The fermionic component has a structure very similar to the bosonic one:

    - coherent state like term, imposing the non-zero modes identifications
    - in the R case, non trivial fermionic zero modes term, whose structure determines the couplings with the massless states

Boundary State

- The Boundary state of a Dp-brane configuration is

$$|B\rangle = |B\rangle_{NS} + |B\rangle_{R} \qquad \text{with} \qquad |B\rangle_{NS,R} = N\,|B_{X,bc}\rangle\,|B_{\psi,\beta\gamma}\rangle^{GSO}_{NS,R}$$

- The fermionic component has a structure very similar to the bosonic one, but the superghost sector has a relevant peculiarity
- The charge carried by the superghost vacuum in $|B_{\psi,\beta\gamma}\rangle^{GSO}_{NS,R}$ has to saturate the disk superghost number anomaly («picture» = -2)
- In the NS sector this leads to a simple and symmetric (-1, -1) distribution between left and right vacuum ⟶ $|B_{\psi,\beta\gamma}\rangle^{GSO}_{NS}$ couples to the standard NS-NS states in the (-1, -1) picture

Boundary State

- The Boundary state of a Dp-brane configuration is

$$|B\rangle = |B\rangle_{NS} + |B\rangle_{R} \qquad \text{with} \qquad |B\rangle_{NS,R} = N\,|B_{X,bc}\rangle\,|B_{\psi,\beta\gamma}\rangle^{GSO}_{NS,R}$$

- The fermionic component has a structure very similar to the bosonic one, but the superghost sector has a relevant peculiarity
- The charge carried by the superghost vacuum in $|B_{\psi,\beta\gamma}\rangle^{GSO}_{NS,R}$ has to saturate the disk superghost number anomaly («picture» = -2)
- In the R sector there must be an asymmetric (-1/2, -3/2) charge distribution between left and right vacuum $\longrightarrow$ $|B_{\psi,\beta\gamma}\rangle^{GSO}_{R}$ does not couple to the standard R-R states, that are in the (-1/2, -1/2) picture

Boundary State

- The Boundary state of a Dp-brane configuration is

$$|B\rangle = |B\rangle_{NS} + |B\rangle_{R} \qquad \text{with} \qquad |B\rangle_{NS,R} = N\,|B_{X,bc}\rangle\,|B_{\psi,\beta\gamma}\rangle^{GSO}_{NS,R}$$

- This structure is completely general and can be adapted to any D-brane configuration
  - Boosted branes
  - D-branes bound states
  - (F, D_p) bound states
  - Fractional branes on different orbifolds (with some modification…)
  - …

Boundary State

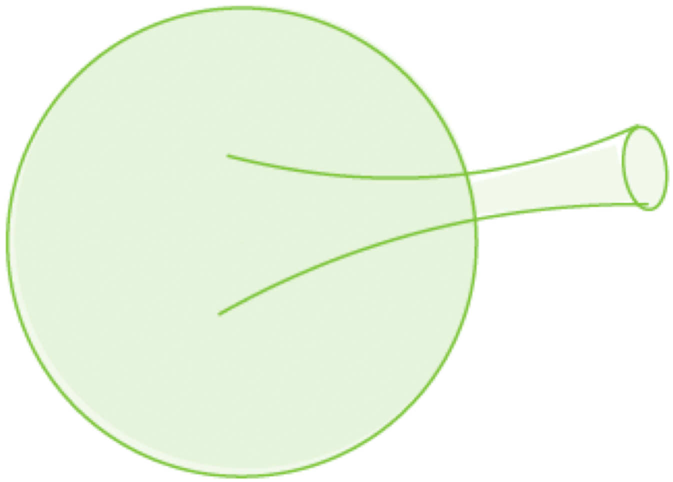
- The Boundary state of a Dp-brane configuration is

$$|B\rangle = |B\rangle_{NS} + |B\rangle_{R} \qquad \text{with} \qquad |B\rangle_{NS,R} = N\,|B_{X,bc}\rangle\,|B_{\psi,\beta\gamma}\rangle^{GSO}_{NS,R}$$

- This structure is completely general and can be adapted to any D-brane configuration
- The Boundary State tecnique has been used to solve a variety of different problems concerning D-brane physics

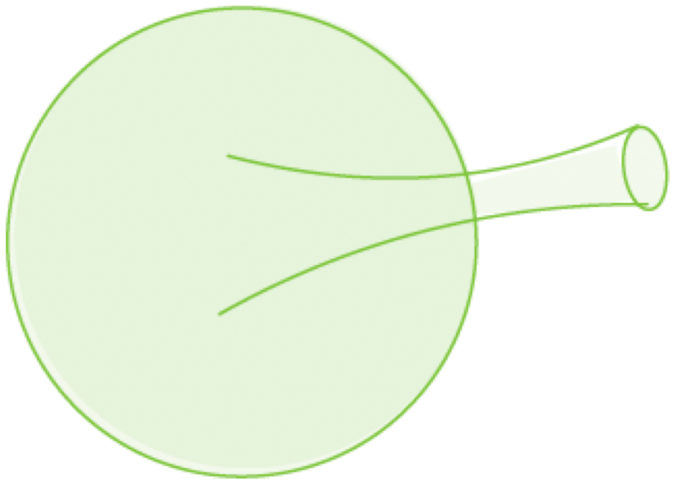
→ Paolo + Bertolini, Billò, Cangemi, MF, Gallot, Lerda, Liccardo, Marotta, Pesando, Pezzella, Russo, Sciuto ... in different permutations!

Classical Solution from Boundary State



- The boundary state is the source of the closed strings emitted by the D-brane
- At long distance from the brane only massless fields survive

Classical Solution from Boundary State



- We may expect that
  - propagating the emitted closed string state far away from the brane and
  - projecting the resulting state on a given massless representation

$$\langle \mathcal{P} | D | B \rangle$$

- We should get information about the long distance behaviour of the emitted field!

Classical Solution from Boundary State

- In the NS-NS case, $|B\rangle$ emits massless states in the standard (-1,-1)-picture, so that we have

$$\langle \mathcal{P}^{\mu\nu}_{(-1,-1)} | D | B \rangle_{NS} = T_p \frac{V_{p+1}}{2k_\perp^2} S^{\mu\nu}$$

$$\Downarrow$$

$$h^{\mu\nu} = \sqrt{2}\,\mu_p \frac{V_{p+1}}{k_\perp^2} \mathrm{diag}(-A, A, \cdots, B, \cdots B) \qquad A = -\frac{7-p}{16}, \quad B = -\frac{p+1}{16}$$

$$B^{\mu\nu} = 0$$

$$\phi = \frac{3-p}{4} \mu_p \frac{V_{p+1}}{k_\perp^2}$$

Classical Solution from Boundary State

- In the R-R case, $|B\rangle$ emits massless states in the (-1/2,-3/2)-picture

What are they?

- R-R massless states in the (-1/2,-1/2)-picture:

$$|V\rangle_{-1/2,-1/2} = F_{\dot{\alpha}\dot{\beta}}|\dot{\alpha}\rangle_{-1/2}|\widetilde{\dot{\beta}}\rangle_{-1/2}$$

- Virasoro constraints $\Rightarrow$ $dF = d * F = 0$
- F is a (p+2) field strength

Classical Solution from Boundary State

- In the R-R case, $|B\rangle$ emits massless states in the (-1/2,-3/2)-picture

What are they?

- R-R massless states in the (-1/2,-3/2)-picture have a highly non-trivial structure

$$|V\rangle_{-1/2,-3/2} = A_{\dot{\alpha}\beta}|\dot{\alpha}\rangle_{-1/2}|\widetilde{\beta}\rangle_{-3/2} + \cdots$$

but only the fist term is relevant for the coupling with the D-brane

- Virasoro constraints ➡ $\Box A = 0$

A is potential, as required for coupling to the D-brane!

Classical Solution from Boundary State

- In the R-R case, $|B\rangle$ emits massless states in the (-1/2,-3/2)-picture and they turn out to be (p+1)-form potential longitudinal to the D-brane volume:

$$\langle \mathcal{P}^{01\ldots p}_{(-1/2,-3/2)}|D|B\rangle_R = \pm\mu_p \frac{V_{p+1}}{2k_\perp^2} \qquad\qquad \mu_p = \sqrt{2}\,T_p$$

Classical Solution from Boundary State

- Taking the Fourier transform in the trasverse space, one can recover the spacetime profile of the emitted fields

$$h^{\mu\nu} = 2\,\frac{Q_p}{r^{7-p}}\,\mathrm{diag}(-A, A, \cdots, B, \cdots B)$$

$$\phi = \frac{3-p}{4}\,\frac{Q_p}{r^{7-p}}$$

$$A^{01\ldots p} = \pm\frac{Q_p}{r^{7-p}}$$

- and see that it coincides with the leading non trivial term in the large distance expansion of the Dp-brane solution!

Classical Solution from Boundary State

- The connection between Boundary State and Classical Solution has been verified in many known cases

  - Bound states of D-branes with different dimensions
  - Bound states of D-branes and fundamental strings
  - ….

Classical Solution from Boundary State

- The connection between Boundary State and Classical Solution has been verified in many known cases

- But one could also think of using the information contained in the Boundary State to actually construct new classical solutions corresponding to given brane systems!

- Strategy: solve the bulk field equations in presence of the source terms dictated by the boundary state.
- Useful for the study of the gauge/gravity correspondence!

AdS/CFT:
Gauge/Gravity correspondence
(2000-2002)

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Is a classical description of stable non-BPS D-branes possible? ☆

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Fractional D-branes and their gauge duals*

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$\mathcal{N} = 2$ gauge theories on systems of fractional D3/D7-branes

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More anomalies from fractional branes

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AdS/CFT

- 1997: AdS/CFT correspondence:

  $\mathcal{N} = 4$ SU(N) SYM theory $\longleftrightarrow$ Type IIB String Theory on $AdS_5 \times S_5$

- All the community started to work on this subject
  - looking for evidences to confirm the duality
  - trying to extend it to less supersymmetric and possibly non conformal gauge theories

- It was clear that, in order to find the putative gravity dual of non maximally supersymmetric and non conformal gauge theories, one had to study branes configurations whose world-volume theory had these features

AdS/CFT

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  $\mathcal{N} = 4$ SU(N) SYM theory $\longleftrightarrow$ Type IIB String Theory on $AdS_5 \times S_5$

- All the community started to work on this subject
  - looking for evidences to confirm the duality
  - trying to extend it to less supersymmetric and possibly non conformal gauge theories

(Douglas, Moore, Gomis, Diaconescu…)

- Our idea was to consider systems of fractional branes on orbifolds and look for the corresponding classical solution, using the technology of the Boundary State

Branes on orbifolds

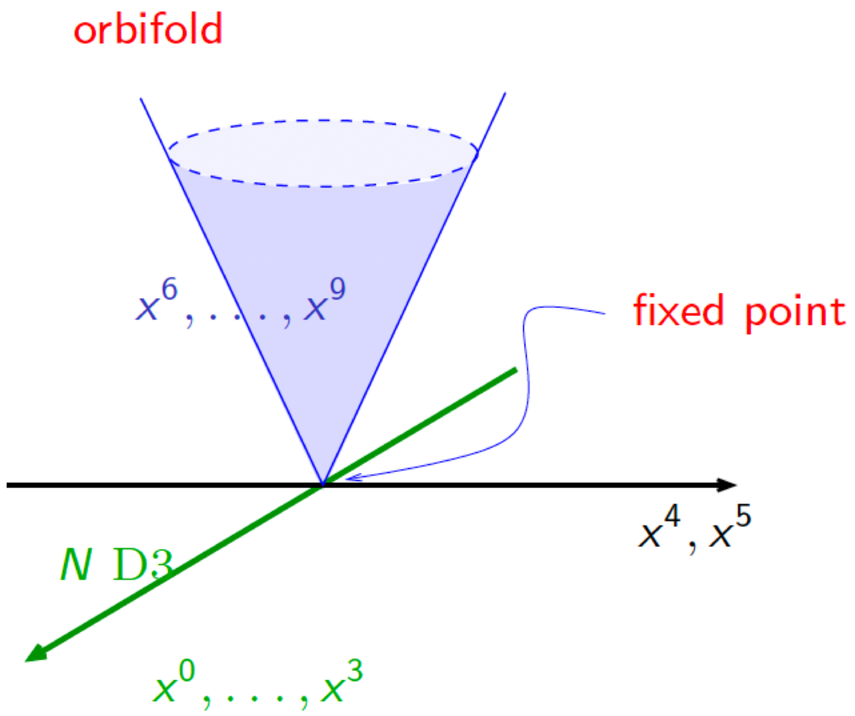
- To construct gauge theories in 4d with reduced SUSY one must change the 10d geometry and use non trivial internal 6d manifolds:

$$\mathbb{R}^{1,3} \times \mathbb{R}^6 \rightarrow \mathbb{R}^{1,3} \times \mathbb{X}^6$$

- For example, one can consider the $\mathbb{Z}_2$-orbifold

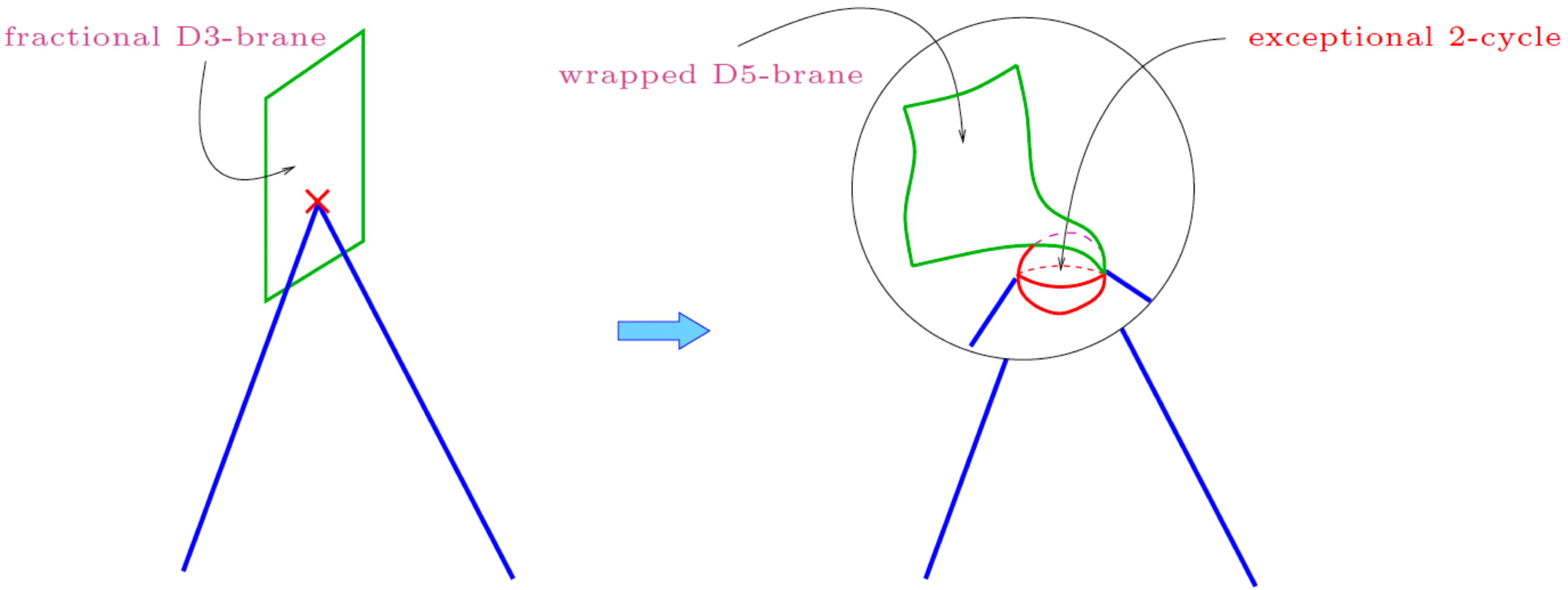
$$\mathbb{R}^{1,3} \times \mathbb{R}^2 \times \mathbb{R}^4/\mathbb{Z}^2 \quad \text{with}$$

$$\mathbb{Z}^2 \ : \ \{x_6, \ldots, x_9\} \rightarrow \{-x_6, \ldots, -x_9\}$$



Branes on orbifolds

- In this background, $\mathcal{N}=2$ SU(N) SYM is the world-sheet theory of a stack of N Fractional D3-branes
- They can be interpreted as D5-branes wrapped around the (singular) exceptional 2-cycle of the $\mathbb{Z}_2$-orbifold (Douglas, Moore)



Closed Strings Couplings of Fractional Branes

- Factorizing the 1-loop amplitude of open string stretched between fractional D3-branes

$$Z = \int_0^\infty \frac{ds}{s} \, \mathrm{Tr} \left[P_{GSO} \left(\frac{1+g}{2} \right) \mathrm{e}^{2\pi s (L_0 - a)} \right]$$

one see that , due to the orbifold projection, fractional branes interacts not only exchanging

- ordinary (untwisted) closed string states, but also via
- new twisted closed string states
- Their boundary states have new twisted components $|B\rangle^T_{NS}$ $|B\rangle^T_R$ responsible for the emission of extra closed string excitations (Billò, Craps, Roose)

Closed Strings Couplings of Fractional Branes

Fractional D3-branes couples to the

- Untwisted closed string sector
  - Metric
  - Dilaton
  - RR 4-form potential with self-dual field strength
- Twisted closed string sector
- NS-NS twisted scalar $$B_2 = b\,\omega_2 \quad \Rightarrow \quad b = \int_{\mathcal{S}_2} B_2$$
- R-R twisted scalar $$C_2 = c\,\omega_2 \quad \Rightarrow \quad c = \int_{\mathcal{S}_2} C_2$$

Classical Solution for Fractional D3-branes

The classical solution associated to this brane system can be obtained by solving the Field Equations derived from

$$S_{IIB} + S_b$$

where

- the sugra action S_{IIB} contains
  - a 10d part, describing the dynamics of the untwisted fields, and
  - a 6d part, describing the dynamics of the twisted fields that are fixed at the orbifold fixed point
- S_b describes the (linearized) couplings of the sugra field with the brane, as encoded by the boundary state

Classical Solution for Fractional D3-branes

The classical profiles of the untwisted fields have the standard D3-branes structure:

- Metric $$ds^2 = H^{-\frac{1}{2}}\, dx^\mu dx_\mu + H^{\frac{1}{2}}\Big[(d\rho)^2 + \rho^2\,(d\theta)^2 + dx^a dx_a\Big]$$
- Dilaton $$\mathrm{e}^{\varphi} = g_s$$
- RR self-dual 5-form field-strength

$$F_5 = d\Big(H^{-1}\, dx^0 \wedge \ldots \wedge dx^3\Big) + {}^*d\Big(H^{-1}\, dx^0 \wedge \ldots \wedge dx^3\Big)$$

with a non trivial warp factor $H(r,\rho)$, where $z = \rho\,\mathrm{e}^{\mathrm{i}\theta}$ is a variable parametrizing the complex plane trasverse to the branes and to the orbifold

Classical Solution for Fractional D3-branes

The field equations for the twisted fields are very simple:

- For instance for the NS-NS scalar we have a standard quadratic 6d bulk action:

$$S_{IIB}^{(b)} = -\frac{(\pi^2\,\alpha')^2}{\kappa^2}\int d^6x\left(\frac{1}{2}\,\partial b\cdot\partial b+\cdots\right)$$

and a 4d boundary action, that can be rewritten as

$$S_b^{(b)} = -\frac{\sqrt{\pi}\,N}{2\kappa}\int d^6x\,b\,\delta^2(z)$$

so the equation is $\Box\, b = 4\,N\,g_s\,\delta^2(z)$

Classical Solution for Fractional D3-branes

- The solution for the NS-NS twisted scalar is therefore a simple function of ρ :

$$b = b_0 + \frac{2\,N\,g_s}{\pi}\,\log\frac{\rho}{z_0}$$

- while the R-R scalar turn out to be related to θ :

$$c = -2\,\frac{N}{\pi}\,\theta$$

- so the complex scalar $t = c + \frac{\mathrm{i}}{g_s} b$ turns out to be an analytic function of z :

$$t = c + \frac{\mathrm{i}}{g_s} b = t_0 + \mathrm{i}\,\frac{2\,N}{\pi}\,\log\frac{z}{z_0}$$

Gauge-Gravity correspondence

- The complex field t plays the role of the complexified gauge coupling in the brane action:

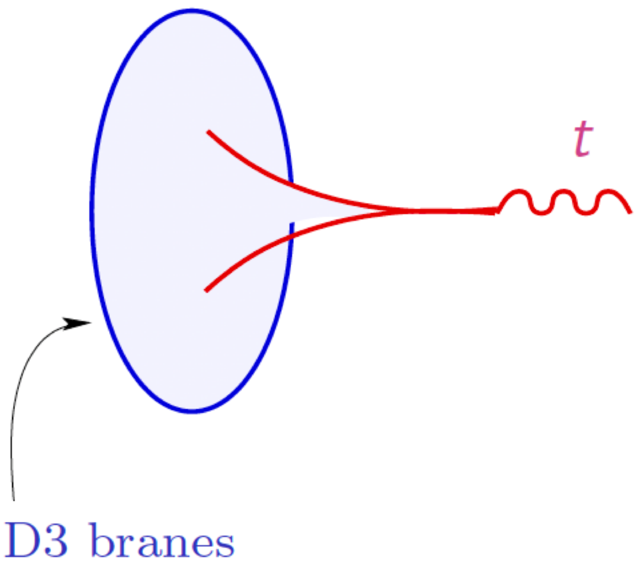
$$S_b(F) \propto \int d^4x\, t\, \mathrm{Tr} F^2 \qquad t \equiv \tau_{YM} = \frac{\theta_{YM}}{\pi} + \mathrm{i}\, \frac{8\pi}{g_{YM}^2}$$

- And indeed, writing $z = (2\pi\alpha')\,\mu$, we see that t has the correct logaritmic running for a $\mathcal{N}=2$ SU(N) SYM theory:

$$\tau_{YM} \equiv t = \frac{\mathrm{i}}{g_s} + \mathrm{i}\,\frac{2\,N}{\pi}\,\log\frac{\mu}{\mu_0} = \mathrm{i}\,\frac{2\,N}{\pi}\,\log\frac{\mu}{\Lambda}$$

where $\Lambda = \mu_0\, \mathrm{e}^{-\frac{\pi}{2N g_s}}$ is the dynamical generated scale of the theory

Gauge-Gravity correspondence



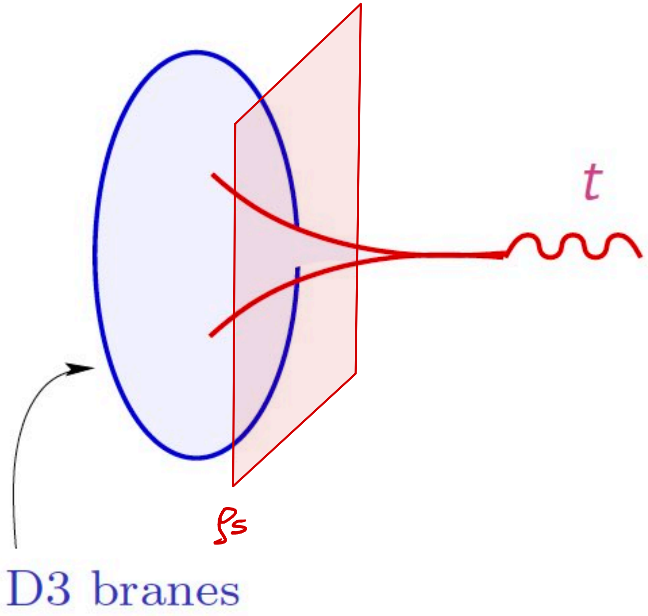
In this correspondence, the point z at which we evaluate $t(z)$

is identified

with the (complexified) energy scale at which we compute τ_{YM}

The quantum perturbative properties of the gauge theory can be recovered from the classical geometry!

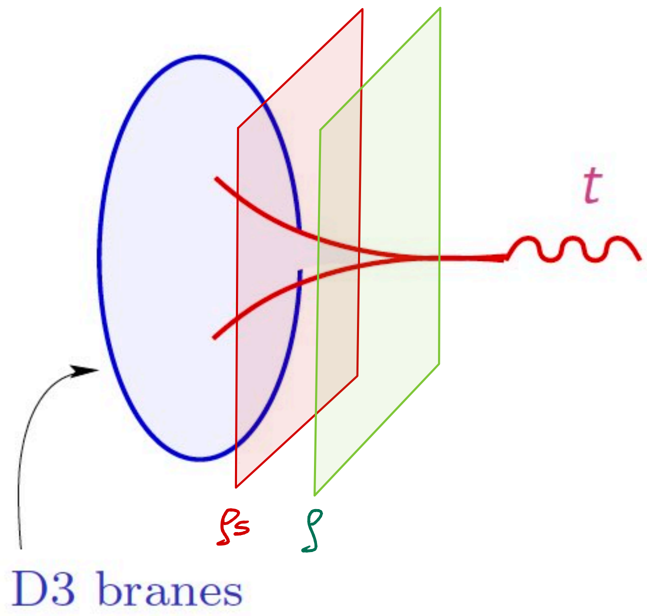
Gauge-Gravity correspondence



The fractional branes solution has problems:

- it has a short distance (IR) singularity at $|z| = \rho_s$ where $H(\rho_s) = 0$

Gauge-Gravity correspondence



The fractional branes solution has problems:

- at $|z| = \rho_e = (2\pi\alpha')\,\Lambda \geq \rho_s \;\Rightarrow\; \tau_{YM} = 0$
  the YM coupling diverges and massive probes become tensionless : Enhançon

- ρ_e is the scale at which gauge theory becomes strogly coupled, but new degrees of freedom (instantons!) came into play, so we cannot use this solution it to explore the IR regime!

Gauge-Gravity correspondence

Add matter:

- Systems of D3/D7 branes on orbifolds
  - (Bertolini, Di Vecchia, M.F., Lerda, Marotta)
  - (Grana, Polchinski)
- Quiver gauge theories
  - (Billò, Gallot, Liccardo)

$\mathcal{N}=1$ Gauge Theories:

- Orbifolds
  - (Bertolini, Di Vecchia, Ferretti, Marotta)
- Wrapped Branes on CY or Conifold
  - (Maldacena, Nunez)
  - (Di Vecchia, Lerda, Merlatti)
  - (Klebanov et al.)

General feature:

Singular solution $\Longrightarrow$ perturbative properties

Regular solution $\Longrightarrow$ also non perturbative properties

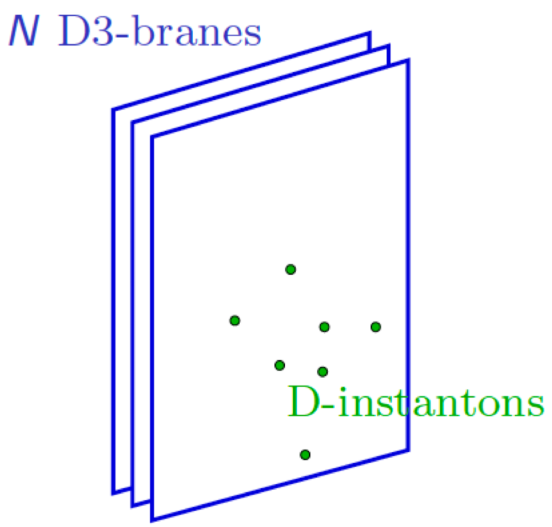
Later generalizations

Instantons in String Theory

The results of our work on the gauge/gravity correspondence lead us to study the string description of instantonic effects

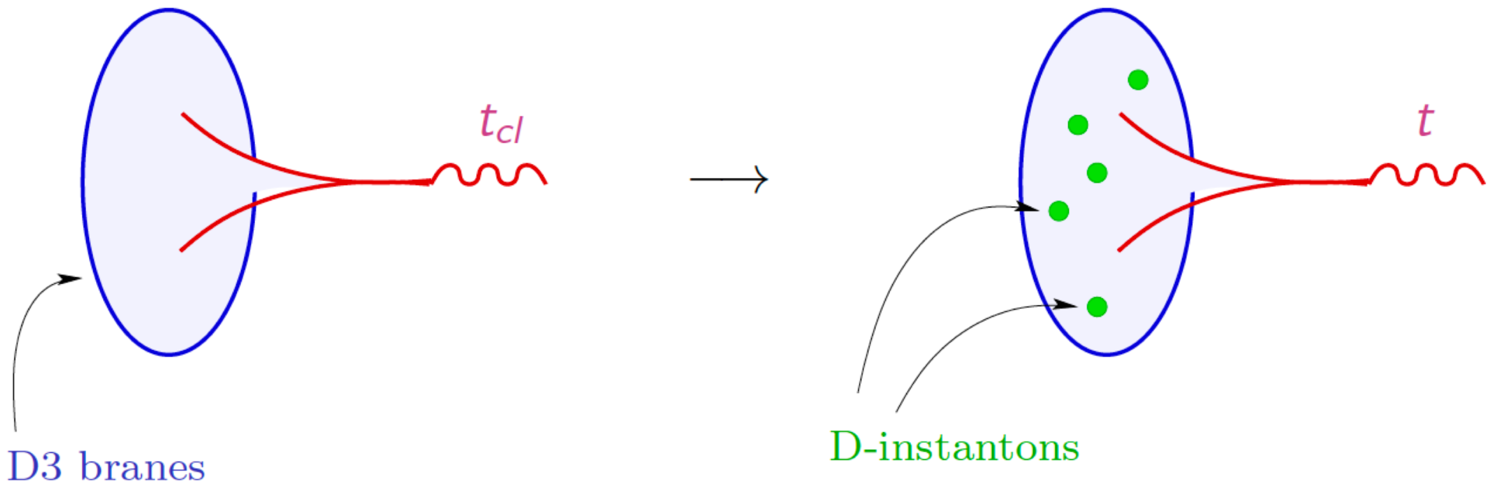
- Instanton-charge k solutions of SU(N) gauge theories correspond to k D- instantons inside N D3 branes.

  (Witten 1995, Douglas 1995, Dorey 1999, ...)



Instantons corrected classical solution

- Knowing how to represent instantons in string theory one can compute the instanton corrected t profile:



- The t field does not coincide with the exact gauge coupling, but still contains all the information about it
- For instance, in the SU(2) case one has $t = \mathrm{i}\frac{16}{\pi}\,\frac{\eta(4\tau)^8}{\eta(\tau)^8}$

(Billò, MF, Fucito, Giacone, Lerda, Morales, Ricci-Pacifici)

Final Remarks


Grazie Paolo!