Gravitational scattering at high energies

A journey through the world of (string) amplitudes with Paolo

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Queen Mary University of London

Meeting Paolo

I started my collaboration with Paolo in 1995 thanks to Lorenzo Magnea (my MSc advisor) who introduced me to his collaborators*

* Di Vecchia, Lerda, Marotta

They were working on the low-energy limit of loop string amplitudes to understand how gauge theory amplitudes arise from open string theory

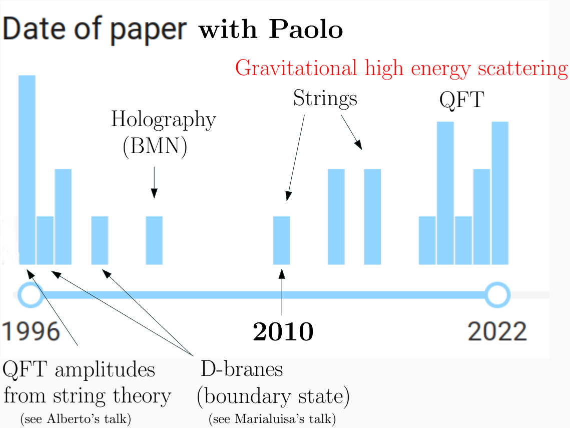
At the time my discussions with Paolo relied on VT220 terminals such as these ones $\Longrightarrow$

My first paper with Paolo appeared in January 1996 (hep-th/9601143), however we met in person only on May 1$^{\text{st}}$ 1996

Together with Alberto Lerda we spent the whole day at "Villa Gualino" in Torino discussing physics

The results are in our second paper hep-th/9607141

My work with Paolo



In this talk I focus on the most recent period (since 2010)

We have been studying the eikonal approach to the gravitational scattering in string theory (2010-2015) and QFT (2018-. . .)

Ultra-Relativistic (UR) scattering

In 2010 I started studying the papers by Amati, Ciafaloni and Veneziano about high-energy scattering in string theory: my aim was to apply their approach to the string-brane scattering.

I invited Gabriele to give a talk in London. Just after that Paolo met Gabriele at the College de France who mentioned this idea to him

It certainly sounded familiar to Paolo: D-branes, string amplitudes and...

Double Photon Emission in $e^{\pm}e^{-}$ Collisions.

P. Di Vecchia and M. Greco

Laboratori Nazionali del CNEN - Frascati

In his first project Paolo studied the the UR scattering (in QED)

(ricevuto il 16 Dicembre 1966)

Summary. — In this work we evaluate, for the process $e^{\pm}+e^{-}\rightarrow$ $\rightarrow e^{\pm}+e^{-}+2\gamma$, the angular distribution of the two photons, their energy spectrum and the total cross-section for emitted photons of energy $\geqslant \varepsilon$, in the extreme relativistic limit.

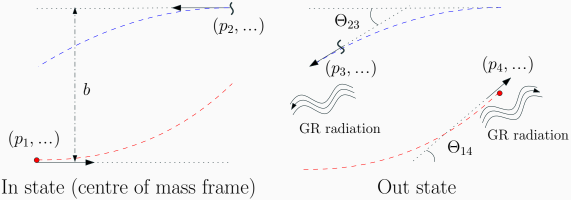
Together with Giuseppe D'Appollonio, we started a fruitful collaboration

The gravitational setup

In the initial state, two objects (strings, D-branes, black holes ...) have a large CoM energy E and large impact parameter b

This regime ($2\delta_0 \sim G_D|p_1 p_2|/b^{D-4} \gg \hbar\ ,\ G_D E/b^{D-3} \ll 1$) is dominated by gravitational interactions. What is the final state ?

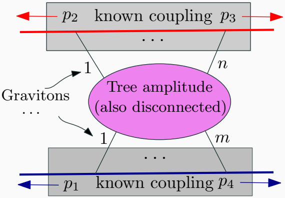
The expectation is sketched below



The aim is to give explicit expressions for the classical observables of the final state: scattering angles, radiation spectrum, tidal excitations ...

A diagrammatic approach

A "particle physics" approach to the problem requires to evaluate these diagrams $\Longrightarrow$



A general key feature: the number of states exchanged is large ($n, m \sim \delta_0/\hbar$) and a resummation is needed

At leading order the final state is shifted by a phase $e^{2i\delta_0}$ with respect to the initial one: the exponential form follows from the eikonal resummation

The LO scattering angle is $\Theta_{14}^{\mathrm{LO}} = \Theta_{23}^{\mathrm{LO}} = -\frac{1}{|\vec{p}|}\partial_b(2\delta_0)$

Beyond the first order the details depends on the process and the regime under analysis with several interesting new effects

We will explore some of them below

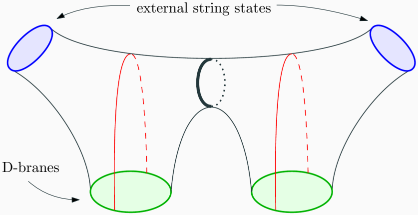
String-brane scattering

We first focused on the following setup: a light string state of energy E_s scattering off a stack of N Dp-branes

We worked in the regime $N G_D M_{\mathrm{D-brane}} \gg G_D E_s \to 0$ (no radiation)

We provided evidence of the eikonal exponentiation at the string level

1008.4773, following ACV's approach



Leading high energy contribution from worldsheets with $h+1$ boundaries

$$\frac{\mathcal{A}_h^L}{2E_s} = \frac{i^h}{(h+1)!} \prod_{i=1}^{h+1} \int \frac{d^{d_\perp}\mathbf{k}_i}{(2\pi)^{d_\perp}} \int_0^{2\pi} \frac{d\sigma_i}{2\pi} \frac{\mathcal{A}_0(E_s,\mathbf{k}_i)}{2E_s}$$
$$\delta^{d_\perp}\left(\sum_{i=1}^{h+1} \mathbf{k}_i - \mathbf{q}\right) \langle 0| : e^{i\mathbf{k}_i \hat{X}(\sigma_i)} : |0\rangle$$

$\Leftarrow$ a worldsheet with two boundaries

We checked that the results from amplitudes are consistent with the propagation of a light string in the backreaction of the Dp-branes

including string effects

Unitarity in string-brane scattering: closed string sector

A novelty of the string case is that the probe can be tidally excited

Amati, Ciafaloni, Veneziano; Giddings

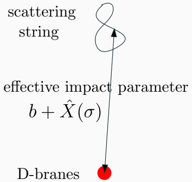
At large E_s this happens at threshold b_t parametrically larger than the curvature scale thanks to an enhancement by $\frac{E_s}{T}$ (T is the string tension)

$$b_t^{D-2} \sim \frac{E_s}{T} G_D M_{\mathrm{D-branes}}$$

These tidal excitations are described by an eikonal operator $e^{2i\hat{\delta}_0}$

$$2\hat{\delta}_0 = \int_0^{2\pi} \frac{d\sigma}{2\pi} : 2\delta_0(b + \hat{X}) :$$

$$\hat{X}^j = i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \left(\frac{A_{n,j}}{n} e^{in\sigma} + \frac{\bar{A}_{n,j}}{n} e^{-in\sigma} \right)$$



where δ_0 is the leading eikonal and the $A_{n,j}$, $\bar{A}_{n,j}$ are the DDF oscillators; this identification made it possible a microscopic analysis of the eikonal operator

Del Giudice, Di Vecchia, Fubini; DDRV 1310.1254

Causality in string-brane scattering

Initially we focused on type II string theories (with maximal susy)

In absence of susy the 3-graviton coupling receives higher derivative corrections (weighted by ℓ_{hd}): when they are relevant δ_0 becomes an operator in the massless sector already in a QFT setup

When $b \lesssim \ell_{hd}$, δ_0 always develops a negative eigenvalue: this yields negative time delays (and repulsive GR force)

a breakdown of classical causality: Camanho, Edelstein, Maldacena, Zhiboedov

However this pathological behaviour should be absent in string theory

We checked this in the case of bosonic string where $\ell_{hd} \sim \ell_s$ DDRV: 1502.01254

- the whole leading Regge trajectory contributes in this case
- when $b \sim \ell_s \sqrt{\ln(\ell_s^2 E_s^2)}$, δ_0 becomes a constant

The tachyonic instability does not play any role in this analysis ...

Unitarity in string-brane scattering: open string sector

Naively the picture above holds till when the scattering string touches the D-branes, i.e. $b \sim \ell_s$? (for simplicity we focus on the bosonic case)

Actually already when $b \sim \ell_s \sqrt{\ln(\ell_s^2 E_s^2)}$ the leading eikonal develops a new imaginary part related to the closed-open string transition

The scattering string is captured

At leading order a heavy open string can be produced and in 1510.03837 we provided a microscopic description of this state DDRV

- it's a coherent state constructed with the open string oscillators $a_{n,j}$
- in an appropriate light-cone gauge, there exists an efficient perturbative (in the number of open oscillators) description

How does the eikonal exponentiation work in this sector? A hard problem!

The QFT case

In 2018 Thibault Damour presented the work below at the conference "50 Years of the Veneziano Model: From Dual Models to Strings, M-theory and Beyond" we organised at the GGI

1710.10599

PHYSICAL REVIEW D **97,** 044038 (2018)

High-energy gravitational scattering and the general relativistic two-body problem

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(Received 29 October 2017; published 26 February 2018)

A technique for translating the classical scattering function of two gravitationally interacting bodies into a corresponding (effective one-body) Hamiltonian description has been recently introduced [Phys. Rev. D **94**, 104015 (2016)]. Using this technique, we derive, for the first time, to second-order in Newton's constant (i.e. one classical loop) the Hamiltonian of two point masses having an arbitrary (possibly relativistic) relative velocity. The resulting (second post-Minkowskian) Hamiltonian is found to have a tame high-energy structure which we relate both to gravitational self-force studies of large mass-ratio binary systems, and to the ultra high-energy quantum scattering results of Amati, Ciafaloni and Veneziano. We derive several consequences of our second post-Minkowskian Hamiltonian: (i) the need to use special phase-space gauges to get a tame high-energy limit; and (ii) predictions about a (rest-mass independent) linear Regge trajectory behavior of high-angular-momenta, high-energy circular orbits. Ways of testing these predictions by dedicated numerical simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the two-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian.

The massive eikonal

The paper by ACV (1990) mentioned above focused on the high-energy scattering of massless scalars:

- it provided an explicit expression for the sub-subleading eikonal δ_2
- it showed that at this order radiation is unavoidable ($\text{Im}\delta_2 \neq 0$)

Damour's suggestion is that δ_2 contains interesting information about the 2-body problem in GR and so about black hole binaries

In the context of the Ligo/Virgo discovery all this was very interesting!

We took a two-pronged approach

- we introduced masses in the QFT eikonal (up to NLO) 1904.02667
- we used the most recent data to check the eikonal exponentiation in $\mathcal{N} = 8$ (massless) sugra to N^3-leading order 1908.05603, 1911.11716

A puzzle at 3PM

In just 13 months Damour's question was answered!

Bern Cheung, Roiban, Solon, Zeng 1901.04424

Their fully covariant result passed all low velocity checks and provided the first new PM result for BH scattering since the eighties!

However it did not smoothly connect in the UR limit to ACV90

Here I present a simplify version of this puzzle in $\mathcal{N} = 8$ sugra (with external massive states) . . . and its solution

DHRV: 2008.12743

radiation reaction

$$2\delta_2^{\mathcal{N}=8} = \frac{16 m_1^2 m_2^2 G^3 \sigma^6}{b^2(\sigma^2-1)^2} - \frac{16 m_1^2 m_2^2 \sigma^4 G^3}{b^2(\sigma^2-1)} \cosh^{-1}(\sigma) \left[1 - \frac{\sigma(\sigma^2-2)}{(\sigma^2-1)^{\frac{3}{2}}} \right]$$

$[\ldots] \to 0$ as $\sigma \to \infty$

Parra-Martinez, Ruf Zeng: 2005.04236

$$\sigma = -\frac{p_1 p_2}{m_1 m_2}, \ \text{PN limit } v \to 0, \quad \sigma^2 - 1 = v^2(1-v^2)^{-1} \sim v^2 , \quad \cosh^{-1}(\sigma) \sim v$$

The term in blue comes from "potential" gravitons (captured by Bern et al.). The terms in green are a Radiation-Reaction (RR) contribution

The same mechanism works in GR

Damour 2010.01641; DHRV: 2101.05772,2104.03256

Outlook I: QFT side

In the UR regime the 3PM eikonal is universal (the same results is recovered in GR and various susy extensions)

A satisfying picture. . . but does it hold beyond 3PM?

- the full 4PM result is now available!
  Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng; Dlapa, Kälin, Liu, Neef, Porto
- it has again a bad UR behaviour . . .
- there is also a puzzle in the eikonal exponentiation at 4PM for the massless scattering in $\mathcal{N} = 8$ sugra DNRVW: 1911.11716
- one needs to incorporate radiation (a QFT eikonal operator)
  See Carlo Heissenberg's talk touching also on some recent papers with Paolo

New 5PM results will soon be available (I think). . . I am sure that the high energy regime will continue to provide important conceptual and technical checks

Outlook II: string side

The string problem is richer, but comparatively much less developed. The QFT side suggests that the following extensions would be very interesting

- include masses (as done in 1904.02667 on the QFT side)
- derive the 2PM string eikonal $\hat{\delta}_1$
  it requires studying a corner of loop string amplitudes not explored in detail so far!
- include radiation (for instance when $NG_D M_{\mathrm{D-brane}} \gg G_D E_s \neq 0$)
- use the recent developments about the gravitational eikonal in a holographic context Cornalba, Costa, Penedones

What to do?

Probably Paolo sees the QFT side as a warming up for the string analysis!

String theory may highlight mechanisms that are relevant also for black hole physics (now an unorthodox view, but I think it's shared by Paolo)

Thanks!

Thanks Paolo for the wonderful collaboration over almost three decades and for the many scientific initiatives you contributed to organise (PhD schools, RTN networks, journal clubs . . .). For instance, a school that was very important for me (and I think many others) was

NORDITA
Nordic Institute for Theoretical Physics

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LIST OF NORDITA SCHOOLS

1998

Nordic Graduate Student Program
7 - 18 September 1998
Nordita / NBI / TAC, Copenhagen
Organizer: H. Heiselberg, M. Asplund

Summer School: "Physics of Molecular Biology"
19 - 28 August 1998
Krogerup Højskole, Humlebæk, Denmark
Organizers: K. Sneppen, J. Hertz, E. Aurell, H. Flyvbjerg, M.H. Jensen

Nordic Course: "Duality in String and Field Theories"
4 - 14 August 1998
Nordita, Copenhagen
Organizers: P. Di Vecchia

Extra slides

GR at 3PM

From the 2-loop amplitude we extract 3PM data Bern, Cheung, Roiban, Shen, Solon, Zeng; Herrmann, Parra-Martinez, Ruf, Zeng; DHRV; Bjerrum-Bohr, Damgaard, Plante, Vanhove; Brandhuber, Chen, Travaglini, Wen

$$2\delta_2 = \frac{4G^3 m_1^2 m_2^2}{b^2}\Bigg\{ \frac{(2\sigma^2-1)^2(8-5\sigma^2)}{6(\sigma^2-1)^2} - \frac{\sigma(14\sigma^2+25)}{3\sqrt{\sigma^2-1}}$$
$$+ \frac{s(12\sigma^4-10\sigma^2+1)}{2m_1 m_2(\sigma^2-1)^{\frac{3}{2}}} + \cosh^{-1}\sigma\left[\frac{\sigma(2\sigma^2-1)^2(2\sigma^2-3)}{2(\sigma^2-1)^{\frac{5}{2}}} + \frac{-4\sigma^4+12\sigma^2+3}{\sigma^2-1}\right]\Bigg\}$$
$$+ i\,\frac{2m_1^2 m_2^2 G^3}{\pi b^2}\frac{(2\sigma^2-1)^2}{(\sigma^2-1)^2}\Bigg\{-\frac{1}{\epsilon}\left[\frac{8-5\sigma^2}{3} - \frac{\sigma(3-2\sigma^2)}{(\sigma^2-1)^{\frac{1}{2}}}\cosh^{-1}(\sigma)\right]$$
$$+ \left(\log(4(\sigma^2-1)) - 3\log(\pi b^2 e^{\gamma_E})\right)\left[\frac{8-5\sigma^2}{3} - \frac{\sigma(3-2\sigma^2)}{(\sigma^2-1)^{\frac{1}{2}}}\cosh^{-1}(\sigma)\right]$$
$$+ (\cosh^{-1}(\sigma))^2\left[\frac{\sigma\left(3-2\sigma^2\right)}{(\sigma^2-1)^{\frac{1}{2}}} - 2\frac{4\sigma^6-16\sigma^4+9\sigma^2+3}{(2\sigma^2-1)^2}\right]$$
$$+ \cosh^{-1}(\sigma)\left[\frac{\sigma\left(88\sigma^6-240\sigma^4+240\sigma^2-97\right)}{3(2\sigma^2-1)^2(\sigma^2-1)^{\frac{1}{2}}}\right]$$
$$+ \frac{\sigma(3-2\sigma^2)}{(\sigma^2-1)^{\frac{1}{2}}}\mathrm{Li}_2(1-z^2) + \frac{-140\sigma^6+220\sigma^4-127\sigma^2+56}{9(2\sigma^2-1)^2}\Bigg\}$$

radiation reaction (agrees with Damour's 2010.01641)

potential gravitons Bern, Cheung, Roiban, Shen, Solon, Zeng 1901.04424

probe limit

A consequence of analyticity and crossing

$$z = \sigma - \sqrt{\sigma^2-1}$$

$$2\delta_2 \xrightarrow{UR} \frac{4G^3 m_1^2 m_2^2}{b^2}\left[\sigma^2\left(-\frac{10}{3} - \frac{14}{3} + 12\right)\right] = \frac{16 G^3 m_1^2 m_2^2}{b^2}\sigma^2$$