Filling the gaps

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Plan of the talk

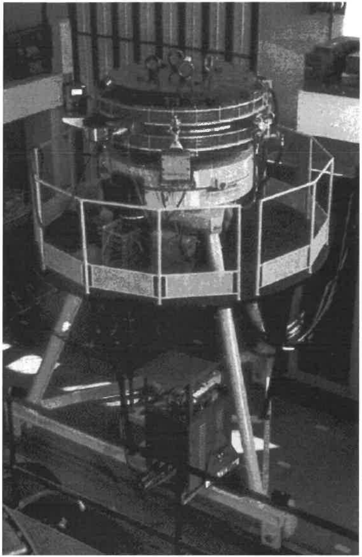
1. Thesis in Rome and job in Frascati
2. Instantons and Monopoles
3. The two-dim WZW model
4. The CP^{N-1} model

Thesis in Rome and job in Frascati

- My master thesis advisor in Rome was Bruno Touschek.
- To use all energy available he proposed the construction of storage rings in alternative to fixed target accelerators.
- The first e^+e^- storage ring, called ADA, was constructed (early sixties) in Frascati and brought to Orsay to reach higher luminosity.
- As a thesis he asked me to compute the cross-section of double Bremsstrahlung

$$e^- + e^+ \to e^- + e^+ + 2\gamma$$

- He was thinking of using it as a monitor for the luminosity of ADONE (big ADA) that was under construction in Frascati.
- After many months of work we (together with Mario Greco in 1966) finally finished this calculation publishing a paper on the total cross-section as a function of the frequencies of the two photons.
- Although all the group in Frascati was involved in the calculation of radiative corrections of various processes to be measured in ADONE, I decided to leave QED and move to S-matrix theory.



ADA (Anello Di Accumulazione), 1961-1964

- I was the first assistant professor appointed at Nordita and I started to work in Copenhagen in May 1974.
- In the following I present the work from 1976 to 1978 with Lars Brink and with Alessandro D'Adda who unfortunately are not anymore with us.
- In spring 1976 Stanley Deser came to give a seminar on the newly found supergravity
[Deser and Zumino, Phys. Lett. B **62** (1976) 335].
- Together with Lars Brink and Paul Howe we were trying to write a non-linear Lagrangian for the superstring that generalises the Nambu-Goto Lagrangian of the bosonic string.
- As a simpler exercise we were trying to write a generalisation of the Polyakov action for a spin $\frac{1}{2}$ particle.

- After Deser's seminar we sat down and we wrote immediately the non-linear action, describing the spin $\frac{1}{2}$ massless Dirac particle:

$$S = \frac{1}{2}\int d\tau \left[\frac{\dot{x}^2}{e} - i\psi\dot{\psi} - \frac{i}{e}\chi\psi\cdot\dot{x}\right]$$

that is invariant under the local supersymmetry transformations:

$$\delta x^\mu = i\alpha\psi^\mu \;\; ; \;\; \delta\psi^\mu = \frac{\alpha}{e}\left(\dot{x}^\mu - \frac{i}{2}\chi\psi^\mu\right) \; ; \; \delta e = i\alpha\chi \; ; \; \delta\chi = -2\dot{\alpha}$$

[Brink, Deser, Di Vecchia, Howe and Zumino, Phys. Lett. **64** B (1976) 435.]

- For the massive case one gets:

$$S = \int d\tau \left[m\sqrt{\left(\dot{x} - \frac{i}{m}\dot{\psi}_5\psi\right)^2} - \frac{i}{2}\psi\dot{\psi} - \frac{i}{2}\psi_5\dot{\psi}_5\right]$$

[Casalbuoni, Phys. Lett **62** B (1976) 49]
[Brink, Di Vecchia and Howe, Nucl. Phys. B **118** (1977) 76]

- In the quantum theory $\psi^\mu \to \gamma^\mu$ and $\psi_5 \to \gamma_5$.

- Then Lars Brink came again to Copenhagen in summer 1976 and we worked for one week writing the Lagrangian for superstring that was invariant under both reparametrisations and local supersymmetry transformations:

$$L = e\Big[-\frac{1}{2}\partial_\alpha x \cdot \partial_\beta x \, g^{\alpha\beta} - \frac{i}{2}\bar{\psi}\gamma^\alpha \cdot \partial_\alpha \psi + \frac{i}{2}\bar{\chi}_\alpha \gamma^\beta \gamma^\alpha \psi \cdot \partial_\beta x + \frac{1}{8}(\bar{\chi}_\alpha \gamma^\beta \gamma^\alpha \psi)(\bar{\chi}_\beta \psi)\Big]$$

  [Brink, Di Vecchia and Howe, Phys. Lett. **65** B (1976) 471]
  (Rec. 23.09.1976)
  [Deser and Zumino, Phys. Lett. **65** B (1976) 369]
  (Rec. 24.09.1976)
- At the end of the week we managed to show that the Lagrangian transformed as a total derivative under the local supersymmetry transformations. First papers where the Polyakov action appears.
- No more string theory.

Instantons and Monopoles

- Around 1976 practically nobody (except Green and Schwarz) was still working on string theory.
- If you wanted to get a job, you better work on something else.
- Pierre Ramond went almost out of physics and only in the last moment he got one year at Yale.
- Gell-Mann writes: ... I set up at Caltech a nature reserve for endangered superstring theorists. I brought J. H. Schwarz and P. Ramond at Caltech and encouraged A. Neveu to visit. [Gell-Mann, in the Birth of String Theory, edited by A. Cappelli and al].
- We profited from the fact that Polyakov was visiting the NBI in Copenhagen for three months in the fall of 1976.
- In this period he gave several lectures on the newly found instanton solutions and on their use in Yang-Mills theory.
- At the end of November 1976 we organised a Conference in Copenhagen that was attended by many people that, although were working on different aspects of particle physics, were all very interested in the latest developments.

- The instanton is a non-perturbative solution of the self-duality equation $F^{\mu\nu} \pm \tilde{F}^{\mu\nu} = 0$ where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$:

$$A_\mu^a = \frac{2}{g} \frac{\eta^a_{\mu\nu}(x - x_0)^\nu}{(x - x_0)^2 + \lambda^2} \;\; ; \;\; \eta^{a\mu\nu} = \epsilon^{a\mu\nu 4} - \delta^{\mu 4}\delta^{\nu a} + \delta^{\mu a}\delta^{4\nu}$$

  [Belavin, Polyakov, Schwartz and Tyupkin, Phys. Lett. B **59** (1975) 85] and it is of course also a solution of the YM equation of motion.
- Its contribution to the action is given by:

$$\frac{1}{4}\int d^4x\, F^2 = \frac{1}{4}\int d^4x\, F\tilde{F} = \frac{8\pi^2}{g^2}$$

- The integrand in the previous quantity is a total derivative and its integral is a topological invariant that takes integer values

$$\frac{g^2}{32\pi^2}\int d^4x F\tilde{F} = n$$

- For the instanton $n = 1$.
- The discovery of a non-perturbative solution of YM theory gave hope that one could use it to show confinement and chiral symmetry breaking that cannot be studied in perturbation theory.

- Compute the first quantum correction by expanding the action around the instanton classical solution.
- The result consists of a part containing integrals over the five zero modes x_0^μ, λ and a part containing a product of infinite determinants for the gluon, ghost, fermions and scalars.

- Using the $SO(5)$ invariance of the instanton solution we computed all eigenvalues of the various determinants and then, using the ζ-function regularisation, we obtained:

$$(2\pi)^6 2^8 \int \frac{d\lambda}{\lambda^5}\frac{d^4x_0}{g^8}(\lambda m)^{C(T)} e^{-\frac{8\pi^2}{g^2}+A\log(\mu\lambda)+B}$$

where $C(T) = \frac{2T(T+1)(2T+1)}{3}$ is the number of fermion zero modes in the field of the instanton and

$$A = \frac{22}{3} - \frac{1}{3}\sum_T 2N_f(T)C(T) + \sum_T \frac{1}{12}N_s(T)C(T)$$

is given by the zero modes of the gluon, ghost, fermions and scalars in the representation T of the gauge group $SU(2)$ [Chadha, D'Adda, Di Vecchia and Nicodemi, Phys. Lett. **72** B (1977) 103]

- We found agreement with a slightly corrected version of the calculation by ['t Hooft, Phys. Rev. D **14** (1976) 3432]

- A is the coefficient of the β-function of YM theory with gauge group $SU(2)$ and with Dirac fermions and real scalars in arbitrary representations of $SU(2)$.
- B is given by the contribution of the non-zero modes of the various determinants.
- In a supersymmetric theory one gets:

$$B = 0 \ ; \ \ A = 8 - 4N_f$$

  where N_f is the number of Dirac fermions.
- In a supersymmetric theory there is no contribution from the non-zero modes of the various determinants [D' Adda and Di Vecchia, Phys. Lett. **73** B (1978) 162]

- We constructed a supersymmetric $\mathcal{N} = 2$ generalisation of the 't Hooft-Polyakov monopole and of the Julia-Zee dyon obtaining its classical mass given by

$$M = v\sqrt{Q^2 + G^2} \;\; ; \;\; G = -\frac{4\pi}{g} \;\; ; \;\; Q = -\frac{4\pi}{g}\alpha \;\; ;$$

  where v is related to the vev of the two scalar fields.
- Then we computed the first quantum correction:

$$M = M_{cl} + \frac{1}{2}\left[6\sum \omega_B - 4\sum \omega_F - 2\sum \omega_G\right]$$

  where the ωs are the eigenvalues of the various fluctuation equations.
- In a supersymmetric theory they are equal and then we got zero in a $\mathcal{N} = 2$ supersymmetric gauge theory [D'Adda, Di Vecchia and Horsley, Phys. Lett. **76** B (1978) 298]
- But we missed the fact that the various contributions are divergent and must be regularised, as a consequence they do not cancel.
- They cancel only in an $\mathcal{N} = 4$ theory [Osborn, Phys. Lett. **83** B (1979) 321]

The two-dim WZW model

- We computed the two-dim WZW model using the procedure proposed by Wess and Zumino by means of a finite gauge transformation.
- We got

$$
\begin{aligned}
WZW(U, A_\mu) = & \frac{i}{4\pi} \int d^2 x \epsilon_{\mu\nu} F^i_{\mu\nu} \theta^i \\
& + \frac{1}{2\pi} \int d^2 x (D_\mu \theta)^j \left(\frac{1 - \cos(T^i \theta^i)}{(T^i \theta^i)^2} \right)_{jk} (D_\mu \theta)^k \\
& + \frac{1}{2\pi} \int d^2 x \epsilon_{\mu\nu} (D_\mu \theta)^j \left(\frac{\sin(T^i \theta^i) - T^i \theta^i}{(T^i \theta^i)^2} \right)_{jk} (D_\nu \theta)^k
\end{aligned}
$$

where $(D_\mu \theta)^k = \partial_\mu \theta^k - f^{ijk} A^i_\mu \theta^j$ and $U = e^{iT^i \theta^i}$
[D'Adda, Davis and Di Vecchia, Phys. Lett. **121** B (1983) 335]

- For $A_\mu = 0$ it reduces to the two-dim WZW action:

$$WZW(U, A=0) = \frac{1}{8\pi}\int d^2x \mathrm{Tr}\left(\partial_\mu U^{-1}\partial_\mu U\right)$$
$$-\frac{i\epsilon^{ABC}}{12\pi}\int d^3x\left((U^{-1}\partial_A U)(U^{-1}\partial_B U)(U^{-1}\partial_C U)\right)$$

The CP^{N-1} model

- The $1/N$ expansion is difficult in QCD because the gauge field $(A_\mu)^i{}_j$ is a matrix of $SU(N)$.
- It can be explicitly done in vector models.
- Can we find a vector model as close as possible to QCD?
- Can we then study its properties and get a hint on what happens in QCD?
- The two-dimensional model closest to QCD is the CP^{N-1} model with fermions described by the following Lagrangian:

$$L = \overline{D_\mu z^i} D^\mu z^i + \bar{\psi}(\not{D} - M_B)\psi + \frac{g}{2N_f}\left[(\overline{\psi}\lambda^i\psi)^2 + (\overline{\psi}\lambda^i\gamma_5\psi)^2\right]$$

  where $z^\alpha = (z^1, z^2 \dots z^N)$ with $|z|^2 = \frac{N}{2f}$; $D_\mu z^\alpha = \partial_\mu z^\alpha + \frac{i}{\sqrt{N}} A_\mu z^\alpha$
  $\psi^{A;a}$ has both a colour and a flavour index.
- Large N_c expansion is performed on the colour index A, keeping fixed the flavour index a.

- Properties shared with QCD
- Conformal invariance at classical level.
- Existence of a topological charge

$$Q = \frac{1}{2\pi\sqrt{N}} \int d^2x \epsilon^{\mu\nu} \partial^\mu A^\nu$$

- Instanton classical solutions:

$$D_\mu z^i = \pm i \epsilon^{\mu\nu} D^\nu z^i$$

- $U(N_f) \times U(N_f)$ chiral invariance at the classical level.
- $U(1)$ chiral anomaly:

$$\partial^\mu \left(\overline{\psi} \gamma_5 \gamma_\mu \psi \right) = 2N_f q \;\; ; \;\; q = \frac{1}{2\pi\sqrt{N}} \epsilon^{\mu\nu} \partial_\mu A_\nu$$

- Confinement at the quantum level.
- Dimensional transmutation and asymptotic freedom.

- Unlike in QCD, in the CP^{n-1} model one can explicitly perform the large N_c expansion integrating the fundamental fields (playing the role of quarks and gluons) and obtaining an effective Lagrangian for the composite meson fields.
- At quadratic level one gets the kinetic term for the composite fields (mesons):

$$L_{eff} = \frac{1}{2}\left[(\partial\Pi^i)^2 + m_\pi^2(\Pi^i)^2\right] + \frac{1}{2}\left[(\partial\sigma^i)^2 + (m_\pi^2 + 4M^2)(\sigma^i)^2\right]$$
$$+\frac{\alpha^2}{8\pi m^2} + \frac{F^2}{24\pi m^2} + i\sqrt{\frac{2N_f}{N}}F_\pi FS \;\; ; \;\; F_\pi = \frac{1}{\sqrt{2\pi}} \; ; \; F = \epsilon_{\mu\nu}\partial_\mu A_\nu$$

- At the classical level there is no kinetic term for the gauge field A_μ.
- A kinetic term is generated in the quantum theory.
- A confining linear potential is generated in the quantum theory:

$$V(R) = \sigma R \;\; ; \;\; \sigma = \frac{12\pi m^2}{N}$$

- Confinement in two dimensions does not teach much about confinement in QCD.
- But the study of the CP^{N-1} model was important to solve the $U(1)$ problem in QCD, as discussed by Gabriele.
- Because of the extra coupling of the singlet of the flavour group S with the topological charge density

  $$q(x) = \frac{\epsilon_{\mu\nu}\partial_\mu A_\nu}{2\pi\sqrt{N}}$$

  the singlet gets an extra term in its mass

  $$M_S^2 = m_\pi^2 + \frac{2\chi N_f}{F_\pi^2} \;\; ; \;\; \chi = \langle q(x) \int d^2 y q(y) \rangle = \frac{3m^2}{\pi N}$$

  χ is the topological susceptibility in the theory without fermions.
- The same formula is valid also in QCD and is called the Witten-Veneziano relation.
- The study of the CP^{N-1} model, discussed here, was done by [D'Adda, Di Vecchia and Lüscher, Nucl. Physics B **146** (1978) 63 and **152** (1979) 125] [Witten, Nucl. Phys. B **149** (1979) 285]

- The log term with the topological charge was also extracted from the effective Lagrangian of the CP^{n-1} model by [Riva, Nuovo Cimento A **61** (1981) 69].
  See also [P. Di Vecchia, Phys. Lett. B **85** (1979) 357]
- My position was for three years and, after long discussions in the particle physics community in the Nordic countries, it was renewed for another one and a half year.
- I wanted to stay in Denmark and I applied to two jobs in Denmark.
- Fortunately Daniele Amati was in the committee of the job at NBI.
- He came out of the committee meeting and came to my office to tell me: Paolo nobody wants you here, apply to Cern as a visitor for one year.
- I did and I went back to Cern in the fall 1978 where I found R. Crewther, S. Coleman, Gabriele and E. Witten and I started to work on the $U(1)$ problem in QCD.