STRINGY EFFECTS IN THE $\mathcal{N}=2$ SUPERCONFORMAL QUIVER

Di Vecchia 80 Fest - Nordita (Stockholm)

May 16th, 2023

Work in progress with S. Ashok, M. Frau, A. Lerda

Based on 2206.13582, 2207.08846

and a vast earlier literature

FOREWORD

* Paolo Di Vecchia has been extremely influential for me (since the "Nordic meeting" in Stockholm, 1996...)

* I'm really happy to be here to celebrate and thank him and I'm really grateful to the organizers for their effort.

- I'll talk about a current project with many ties to "old" research discussed yesterday by Marialuisa Frau

INTRODUCTION

- **Big picture:**
  * Non-perturbative QFT (including YM)?
  * QFT & gravity?
- **Strategy**
  * Start from higly symmetric cases
- **Frame of mind**
  * String Theory always offers good inspiration!

THE ARENA: $\mathcal{N}=2$ SYM IN $d=4$

* One of the best laboratories for exact results in a non-trivial QFT
* The theory on S_4 localizes (Pestun, 2007): some observables (part. function, BPS Wilson loops, extremal correlators of chiral op.s) are mapped to an interacting matrix model

Andree Young 2010; Ley Sugama 2011, ...
Baggio et al 2014; Gerkovitz et al 2016; ...

* In conformal cases, results on S_4 $\longrightarrow$ results on $\mathbb{R}^4$
(be careful with mapping of operators Rodriguez-Gomez, Russo 2016, ...)

THE ARENA: $\mathcal{N}=2$ SYM IN $d=4$

* Some conformal cases have a holographic dual

* Perfect context to study the AdS/CFT correspondence in non maximally supersymmetric cases

IN THIS TALK

* Focus on a $\mathcal{N}=2$ quiver SYM theory — Kachru, Silverstein 1998; Oz, Terning 1998, ...

  - Localizes to an interacting multi-matrix model

    Pini et al, 2017; Fiol et al 2020; Zarembo 2020; Galvagno, Preti 2020; ...

  - Conformal

  - Holographic dual: type IIB on $AdS_5 \times S_5 / \mathbb{Z}_M$

IN THIS TALK

* Use localization to resum the perturbative expansion of the structure constants for the single trace chiral/antichiral operators

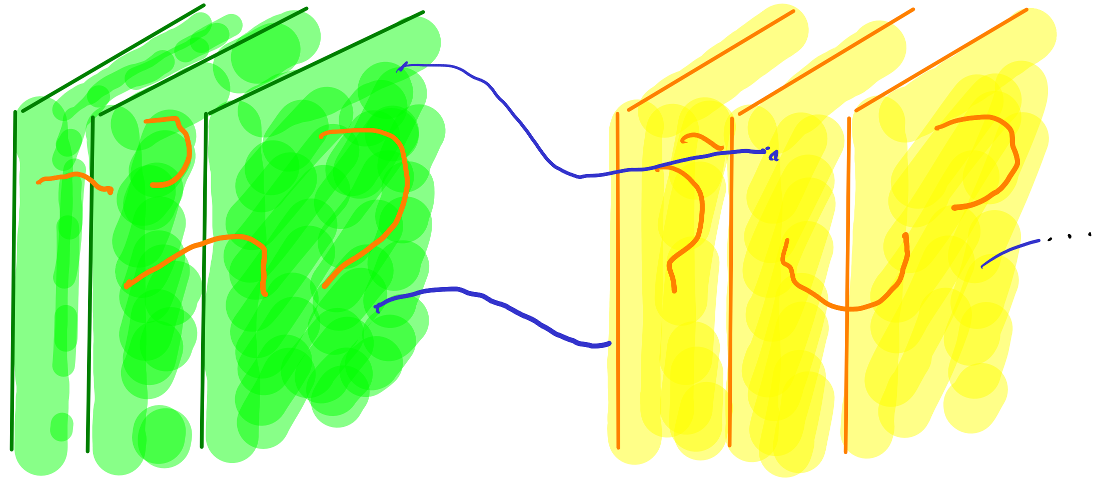
* Obtain their strong coupling behaviour in the large N 't Hooft limit

* Find perfect match with the dual SUGRA description

    * except for structure constants of 3 "twisted" operators these require higher derivatives stringy corrections!

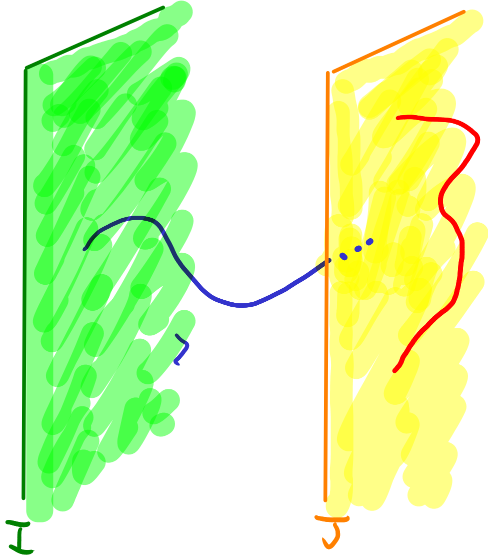
THE SET-UP

fractional D3 branes in a $\mathbb{R}^4 \times \mathbb{C} \times \mathbb{C}^2/\mathbb{Z}_M$ background



Focus on the massless open string sector $\rightarrow$ (quiver) gauge theory

OPEN STRINGS



* M types of fractional D3 (fD3), associated to the irrepses of $\mathbb{Z}_M$: $R_I(\zeta^\alpha) = \zeta^{I\alpha}$ $\quad I, \alpha = 0, \dots, M-1$

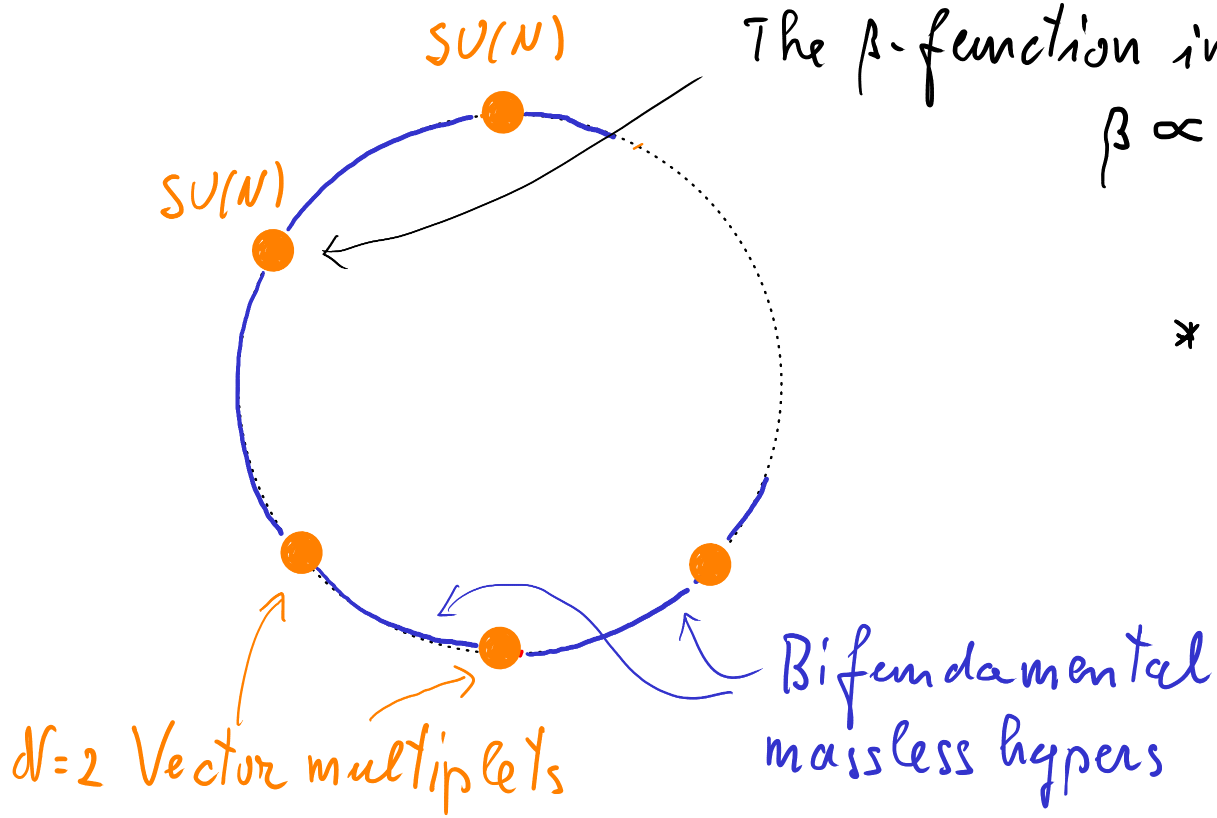
($\zeta = \exp(2\pi i/M)$ from now on)

* Endpoints of open strings attached to fD3's transform in $R_I \otimes \overline{R}_J$

* This plus the geometric action fixes the invariant spectrum

* At the massless level $\rightsquigarrow$ $\mathcal{N}=2$ quiver gauge theory

QUIVER GAUGE THEORY

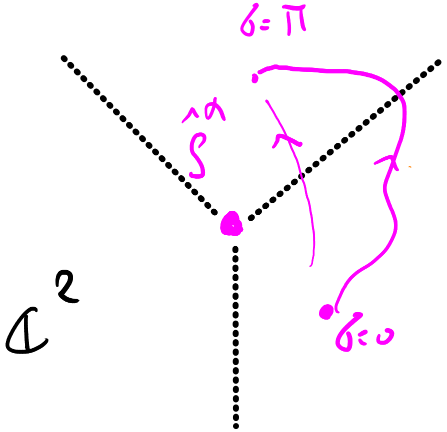


The β-function in each node vanishes:

$$\beta \propto 2N - 2N = 0$$

* We take all gauge couplings equal to g

CLOSED STRINGS



* The string close up to the $\mathbb{Z}_M$ orbifold action:

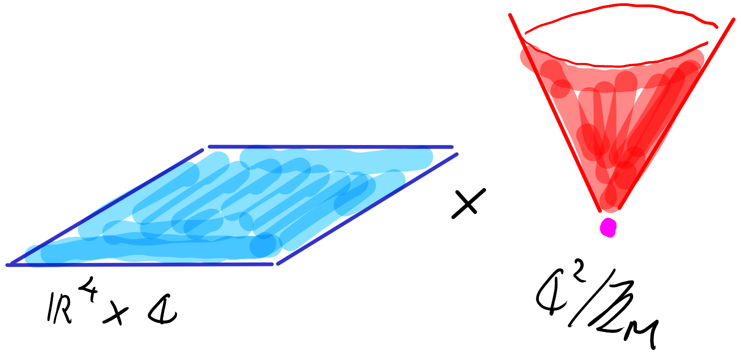
$$X^M(\tau,\sigma+2\pi)=(\hat{\jmath}^{\alpha}X)^M(\tau,\sigma)$$

action as rotation in $\mathbb{C}^2$

* Sectors $\longleftrightarrow$ conjugacy classes of $\mathbb{Z}_M$

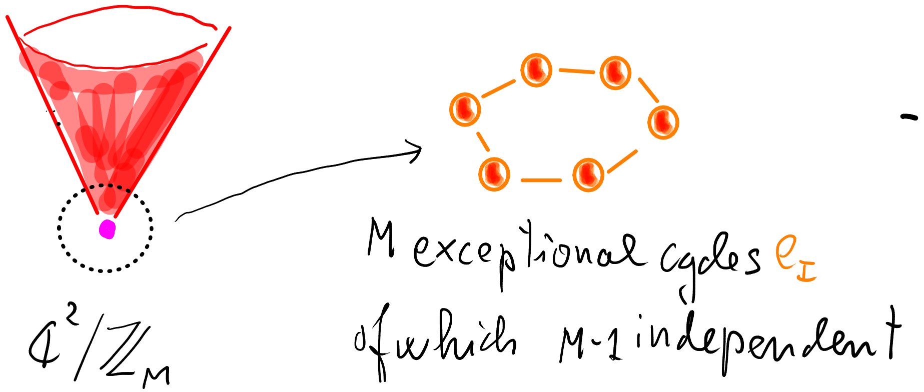
- untwisted ($\alpha=0$): lives in 10d
- twisted sectors ($\alpha=1,\dots M-1$): stuck at the orbifold fixed point

TWISTED FIELDS



* The twisted fields are six-dimensional fields

* At the SUGRA level one works with the resolved orbifold

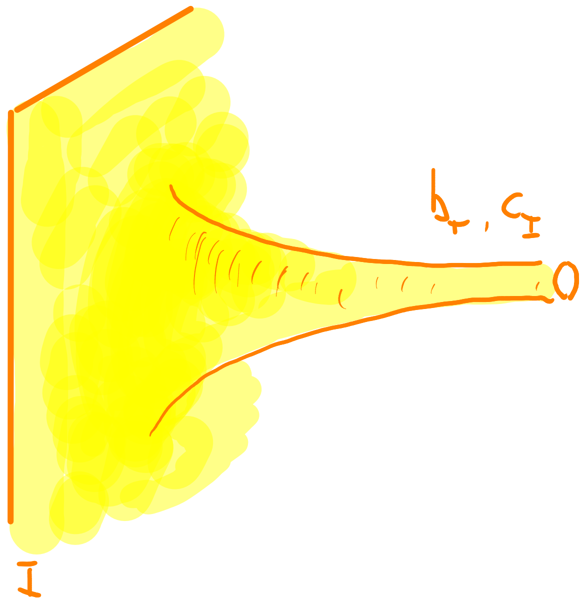


- the quiver adjacency matrix is the intersection of these cycles

OPEN-CLOSED INTERPLAY

* Fractional D3's ~ solitons of IIB SUGRA on the (resolved) orbifold

* ~ D5 wrapped on the exceptional cycles e_I



* They emit scalars b_I, c_I

  ~ NS-NS and R-R 2-forms $B_{(2)}$, $C_{(2)}$

  wrapped on e_I

DISCRETE FOURIER TRANSFORM

- Fields b_I, c_I emitted by a fD3 of type I $\longleftrightarrow$ irreps of $\mathbb{Z}_M$

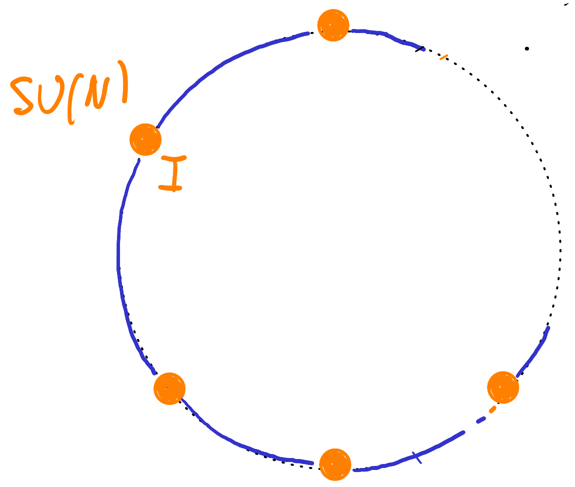
- Fields b_α, c_α in the twisted sector α $\longleftrightarrow$ conj. classes of $\mathbb{Z}_M$

$$\boxed{b_\alpha = \frac{1}{2\sqrt{M}} \sum_{I=0}^{M-1} \underbrace{\varsigma^{-\alpha I}}_{\text{character table of } \mathbb{Z}_M} b_I}$$

(Discrete Fourier Transform)

* b_α, c_α have a diagonal Gd action (see later)

CHIRAL OPERATORS IN THE $\mathcal{N}=2$ QUIVER GAUGE THEORY



$$\boxed{O_n^{(I)}(x) = \mathrm{Tr}\, \Phi_I^n(x)}$$

complex scalar in the vector multiplet of node I

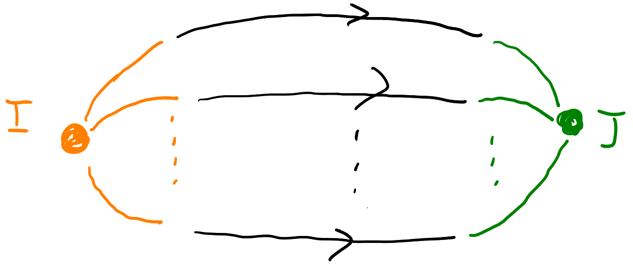
- Gauge invariant, BPS
- Dimension $n \leftrightarrow U(1)_R$ charge
- Form a chiral ring (including multi-traces)

* Consider the 2-point correlator as a function of g

$$\boxed{\langle O_n^{(I)}(x)\, \bar{O}_n^{(J)}(y) \rangle = \frac{G_n^{(IJ)}}{|x-y|^{2n}}}$$

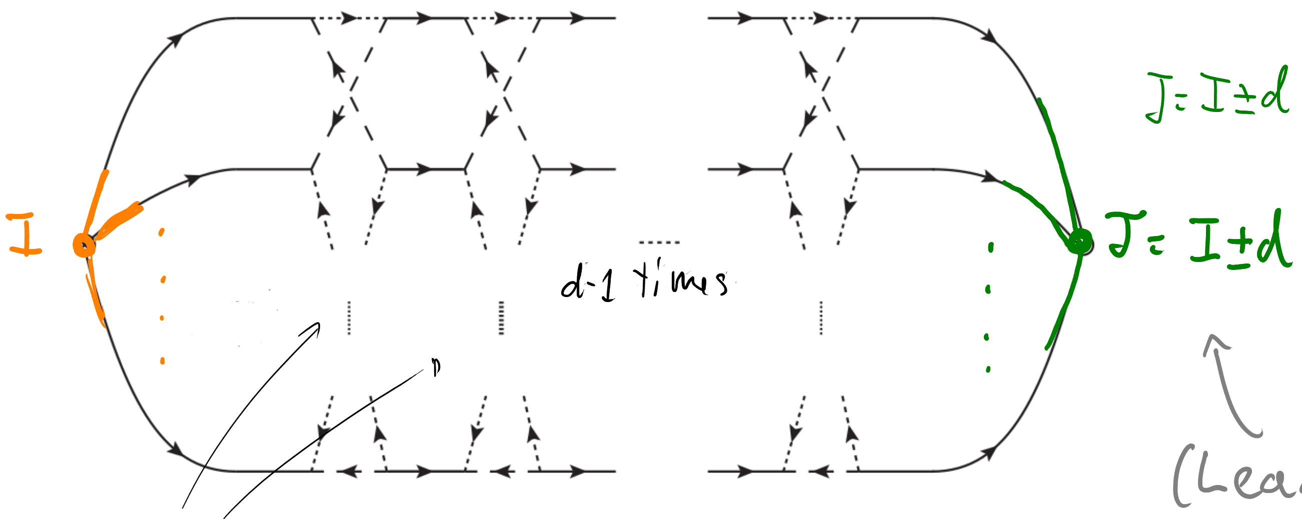
TWO POINT FUNCTION

* At tree level it is diagonal:



$$G_n^{IJ}(g=0) = n\left(\frac{N}{2}\right)^n \delta^{IJ}$$

* Perturbative corrections spoil this:



matter hypermultiplet loops

$$G_n^{(I,\, I\pm d)} \propto \lambda^{nd}\left[\zeta(2n-1)\right]^d$$

(Leading terms in the 't Hooft limit)

UNTWISTED AND TWISTED OPERATORS

* As suggested by the string embedding via fD3 branes on the orbifold, change basis by D.F.T, i.e set

$$U_n(\vec{x}) = \frac{1}{\sqrt{M}}\left(\mathcal{O}_n^{(0)}(\vec{x}) + \mathcal{O}_n^{(1)}(\vec{x}) + \dots + \mathcal{O}_n^{(M-1)}(\vec{x})\right)$$ untwisted

$$T_{\alpha,n}(\vec{x}) = \frac{1}{\sqrt{M}}\sum_{I=0}^{M-1} \rho^{-\alpha I}\, \mathcal{O}_n^{(I)}(\vec{x}) \,, \qquad \alpha = 1, \dots M-1$$ twisted

DIAGONAL TWO-POINT FUNCTIONS

* The untwisted and twisted sectors are orthogonal:

$$\boxed{\langle U_n(x)\, \bar{U}_m(y)\rangle = \frac{G_n}{|x-y|^{2n}}\, \delta_{n,m}\,, \qquad \langle T_{\alpha\, n}(\vec{x})\, T_{\beta,\, m}(\vec{0})\rangle = \frac{G_{\alpha\, n}}{|x-y|^{2n}}\, \delta_{n,m}\, \delta_{\alpha,\beta}}$$

non-trivial functions of g, N.

EXTREMAL 3-POINT FUNCTIONS

* For instance, with 3 untwisted operators,

$$\langle U_k(x)\, U_\ell(y)\, \bar{U}_p(z)\rangle = \frac{G_{k,\ell;p}}{|x-z|^{2k}\,|y-z|^{2\ell}} \qquad (p = k+\ell)$$

* Similarly we have twisted/untwisted and purely twisted cases:

$$G_{k,\alpha\ell;\alpha p} \qquad G_{\alpha k,\, M-\alpha\,\ell;\, p} \qquad G_{\alpha k,\beta\ell;\gamma p} \qquad (\gamma = \alpha+\beta)$$

STRUCTURE CONSTANTS

* To eliminate the dependence on the normalizations, define

$$\boxed{C_{\kappa,\ell;p} = \frac{G_{\kappa,\ell;p}}{\sqrt{G_\kappa G_\ell G_p}}, \qquad C_{\kappa,\alpha\ell;\alpha p} = \frac{G_{\kappa,\alpha\ell;\alpha p}}{\sqrt{G_\kappa G_{\alpha\ell} G_{\alpha p}}}, \quad \ldots}$$

* These structure constants form an important part of the CFT data of the quiver theory

STRUCTURE CONSTANTS

* Our aim is to study them at large N, where the single traces form a closed subsector

* We want to find their exact dependence on the 't Hooft coupling λ and extract their (leading) behaviour for large λ

* Holographically, this regime is mapped to the dual string theory at tree level (lowest order in g_s) and at lowest order in α'

LOCALIZATION TO A MATRIX MODEL

* To go beyond lowest perturbative orders, use localization (Pestun 2007)

* On the sphere S_4, consider observables closed w.r.t a BRS charge Q (formed from SUSY): the path integral localizes to a matrix model

  - Applies to partition function, BPS Wilson loop, chiral/antichiral vev, ...

* For conformal cases S_4 results $\longrightarrow$ $\mathbb{R}^4$ (conformally equivalent)

  Be careful with the operator map!

THE QUIVER MATRIX MODEL

* $N\times N$ traceless hermitean matrices a_I $(I=0,1,\dots M-1)$

$$Z=\int\prod_I\left(da_I\, e^{-\mathrm{tr}\, a_I^2}\right)\left|Z_{1\text{-loop}}\; Z_{\text{inst}}\right|^2$$

Instantons suppressed: $Z_{\text{inst}}\to 1$ in the 't Hooft limit

$$=\int\prod_I\left(da_I\quad e^{-\mathrm{tr}\, a_I^2}\right)\, e^{-S_{\text{int}}}=\left\langle e^{-S_{\text{int}}}\right\rangle_0$$

* S_{int} is complicated (see later), but its vev is taken in a gaussian (multi)-matrix model

THE FULL LIE ALGEBRA APPROACH

* For such free matrix correlators, we developed a "full Lie Algebra" approach:

    - do not reexpress in terms of the eigenvalues

    - use recursive relations that simplify in the large-N limit

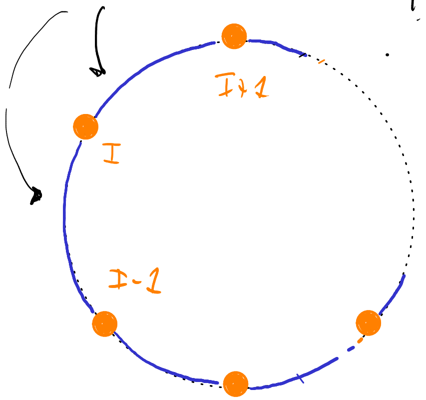
THE INTERACTION ACTION

Galvagno, Preti 2020

$$S_{int} = \sum_{I=0}^{M-1} \sum_{m=2}^{\infty} \sum_{k=2}^{2m} (-1)^{m+k} \left(\frac{g}{8\pi^2}\right)^{2m} \binom{2m}{k} \frac{\zeta(2m-1)}{2m} \left(\operatorname{tr} a_I^{2m-k} - \operatorname{tr} a_{I+1}^{2m-k}\right)\left(\operatorname{tr} a_I^{k} - \operatorname{Tr} a_{I+1}^{k}\right)$$

diagonalized by introducing untwisted and twisted combinations of the operators, just as in the gauge theory (Discrete Fourier Transform)

Nearest-neighbour interactions in the quiver



CHIRAL OPERATORS

* Also the correlators of chiral op.s are mapped into matrix model correlators. The map is

$$\mathcal{O}_n^{(I)}(x) = \mathrm{tr}\, \Phi_I^n(x) \longrightarrow \,:\mathrm{tr}\, a_I^n:$$

$\Phi_I(x)$ chiral
no self-contractions

a_I hermitean
self-contractions have to be subtracted

* Thus, for instance,

$$\langle \mathcal{O}_n^{(I)}(x)\, \bar{\mathcal{O}}_m^{(J)}(y) \rangle = \frac{G_n^{(IJ)}}{|x-y|^{2n}}\,\delta_{nm} \longleftrightarrow \langle :\mathrm{tr}\, a_I^n: \; :\mathrm{tr}\, a_J^m: \rangle = G_n^{(IJ)}$$

in the interacting matrix model

FREE MATRIX MODEL (at large N)

* Construct combinations of traces which represent in the matrix model the untwisted and twisted chiral operators at $\lambda = 0$

$$\{\mathrm{tr}\, a_I^n\} \xrightarrow{\text{Discrete Fourier transform}} \{P_{\alpha, n}\}$$

Normal ordering in the free model

choice of normaliz.

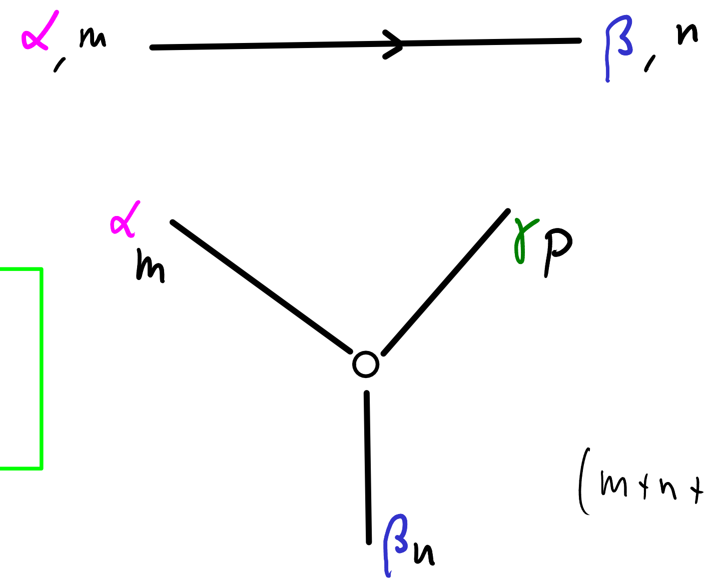
($\alpha = 0$ untwisted)

FREE MATRIX MODEL (at large N)

* For these matrix operators

- $\boxed{\langle P_{\alpha,m} \; P^{\dagger}_{\beta,n} \rangle_0 = \delta_{\alpha,\beta} \; \delta_{mn}}$

- $\boxed{\langle P_{\alpha,m} \; P_{\beta,n} \; P^{\dagger}_{\gamma,p} \rangle_0 = C_{mnp} \; \delta_{\alpha+\beta,\gamma}}$



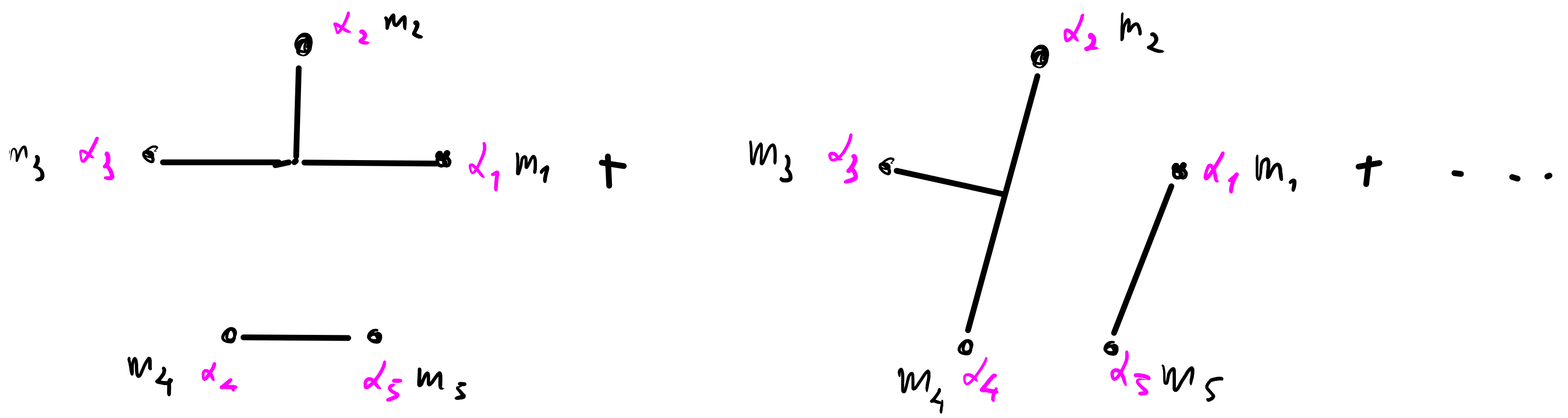
($m+n+p$ even)

with $\quad C_{mnp} = \dfrac{1}{\sqrt{M}} \cdot \dfrac{\sqrt{mnp}}{N}$

result in the $\mathcal{N}=4$ theory

"WICK PROPERTY"

* At large N, higher point correlators of the $P_{\alpha,n}$ in the free matrix model come from all possible contractions with the above propagator (and one cubic vertex for odd # of insertions)



INTERACTING MATRIX MODEL

* In the $\{P_{\alpha,n}\}$ basis the interactions are diagonal in the sectors α

$$\boxed{S_{int} = -\frac{1}{2}\sum_{\alpha=1}^{M-1} s_\alpha\, P_\alpha^\dagger \cdot X \cdot P_\alpha}$$

$$s_\alpha = \sin^2 \frac{\pi\alpha}{M}$$

Eigenvalues of the adjacency matrix

Only twisted operators appear!

$$\boxed{X_{r,s} = -8\sqrt{rs}\sum_{p=0}^{\infty}(-1)^p C_{r,s,p}\,\frac{\zeta(r+s+2p-1)}{r+s+2p}\left(\frac{\lambda}{16\pi^2}\right)^{\frac{r+s+2p}{2}}}$$

for r,s both even or both odd, otherwise zero

$$= \frac{(r+s+2p)!^2}{p!(r+p)!(s+p)!(r+s+p)!}$$

RESUMMING THE X MATRIX

$$X_{r,s} = \pm 8 \, (-)^{\frac{r+s}{2}} \sqrt{rs} \int_0^\infty \frac{dt}{t} \frac{e^t}{(e^t-1)^2} J_r\left(\frac{t\sqrt{\lambda}}{2\pi}\right) J_s\left(\frac{t\sqrt{\lambda}}{2\pi}\right)$$

+ or − if r, s both odd or even

* perturbative series: expand the Bessels for small λ then integrate over t

* strong coupling behaviour: use Mellin to find an asymptotic expansion

* Very similar "Bessel Kernels" appear in several corners of $\mathcal{N}=4$ and $\mathcal{N}=2$ (e.g. for the cusp anomalous dimension via BES equation Beisert et al 2006)

CORRELATORS IN THE INTERACTING MODEL

* In each twisted sector we compute free matrix correlators of $P_{\alpha,n}$ operators with the insertion of $e^{-S_{int}(\alpha)}$

* $S_{int}(\alpha)$ is quadratic in in the $P_{\alpha,n}$ with "mass matrix" X

* At large N the Wick rule holds

$\Rightarrow$ find exact expressions of the correlators in terms of the X matrix

EXACT RESULTS

* Partition function in the 'α'th twisted sector

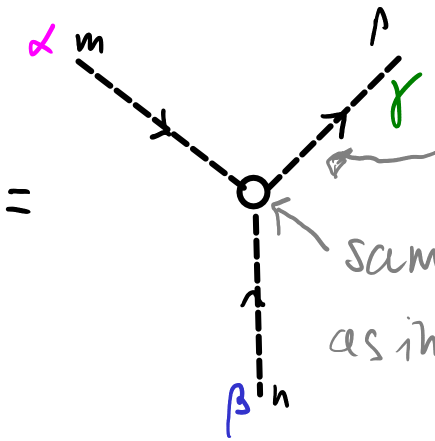
$$Z_{(\alpha)} = \langle 1 \rangle = \langle e^{-S_{int(\alpha)}} \rangle_0 = \det{}^{-1/2}\left(\mathbb{1} - S_\alpha X\right)$$

* 2-point functions

$$\langle P_{\alpha,m} P^\dagger_{\beta,n} \rangle = \left(\frac{1}{\mathbb{1} - S_\alpha X} \right)_{m,n}$$

* 3-point functions

$$\langle P_{\alpha,m} P_{\beta,n} P^\dagger_{\gamma,p} \rangle =$$



interacting propagators

same vertex as in the free case

INTERACTING NORMAL-ORDERING

* Due to interactions, the $P_{\alpha,n}$ are not normal-ordered

* Apply Gram-Schmidt : $\{P_{\alpha,n}\} \xrightarrow[G.S.]{} \{\mathcal{G}_{\alpha,n}\}$

* $\mathcal{G}_{\alpha,n}$ = counterparts of the (normalized) twisted chiral op.s in the gauge theory

* Their correlators capture the (coordinate independent part of) the 2- and 3-point functions in the gauge theory → structure constants

STRONG COUPLING BEHAVIOUR

* In 2211.11795 we found exact expressions for these quantities in terms of the $\mathbb{X}$ matrix (always in the large-N limit)

* In this talk I focus just on their expression at the leading order in the strong-coupling expansion for large λ

STRONG COUPLING BEHAVIOUR

* Expression of $\mathbb{X}$ as convolution of Bessel functions $\longrightarrow$ Mellin transform
  $\rightarrow$ leading behaviour $\boxed{\mathbb{X} \underset{\lambda\to\infty}{\sim} -\lambda\, S' + \dots}$ 3-diagonal matrix

* The normal ordering simplifies: $\mathcal{T}_{\alpha,k} \underset{\lambda\to\infty}{\sim} \mathcal{P}_{\alpha,k} - \sqrt{\frac{k}{k-2}}\, \mathcal{P}_{\alpha,k-2}$

* The final result for the structure constants are very simple and depends on the untwisted/twisted content

RESULTS FOR THE STRUCTURE CONSTANTS at large N

$\lambda = 0 \longrightarrow \lambda \to \infty$

	$\lambda = 0$		$\lambda \to \infty$
$C_{k,\ell;p}$	$\dfrac{\sqrt{k\ell p}}{\sqrt{M}\; N}$	… λ-independent … Lee et al 1998 …	$\dfrac{\sqrt{k\ell p}}{\sqrt{M}\; N}$
$C_{k,\alpha\ell;\alpha p}$	//	… λ-dependent …	$\dfrac{\sqrt{k(\ell-1)(p-1)}}{\sqrt{M}\; N}$
$C_{\alpha k, M-\alpha\,\ell;p}$	//	//	$\dfrac{\sqrt{(k-1)(\ell-1)p}}{\sqrt{M}\; N}$
$C_{\alpha k,\beta\ell;\gamma p}$	//	//	$\dfrac{\sqrt{(k-1)(\ell-1)(p-1)}}{\sqrt{M}\; N}$

HOLOGRAPHIC DESCRIPTION

* IIB string theory on the near-horizon limit of the fD_3 solution

  * untwisted sector: $AdS_5 \times S_5/\mathbb{Z}_M$ (AdS$_5$ radius: R) 10d

  * twisted sectors: $AdS_5 \times S_1$ fixed under the orbifold action 6d

HOLOGRAPHIC DESCRIPTION: parameters

$$\boxed{4\pi g_s = \frac{\lambda}{MN}} \qquad \boxed{\alpha' = \frac{R^2}{\sqrt{\lambda}}}$$

Set $R=1$ afterwards

$$\left\{ \begin{array}{ll} \text{'t Hooft large } N: & g_s \to 0 \\ \text{large } \lambda: & \alpha' \to 0 \end{array} \right. \sim \boxed{\text{sugra}}$$

But...

* Untwisted, 10d SUGRA action $\propto \frac{1}{2\kappa_{10}^2} = \frac{1}{(2\pi)^7 g_s \alpha'^4} = \frac{4(MN)^2}{(2\pi)^5}$

* Twisted, 6d action $\propto \frac{1}{2\kappa_6^2} = \frac{1}{M} \frac{(2\pi\alpha')^2}{2\kappa_{10}^2}$

orbifolding

wrapping on exceptional cycles

HOLOGRAPHIC DESCRIPTION: operators

* To compute correlators by Witten diagrams we need the explicit map

$$\boxed{\text{chiral CFT operators} \longleftrightarrow \text{bulk modes in } AdS_5}$$

* Untwisted $\quad U_k(x) \longleftrightarrow S_k(z)$: combination of KK modes on $S_5/\mathbb{Z}_M$ of the 10d $\begin{cases} \text{metric } h_{mn} \\ \text{RR 4-form } C_4 \end{cases}$

(Just as in $\mathcal{N}=4$ Lee et al 1998) rather complicated

* Twisted $\quad T_{\alpha,k}(x) \longleftrightarrow \eta_{\alpha,k}(z)$: KK modes on S^1_1 of the 6d field $c_\alpha - i\, b_\alpha$

(Gukov, 1998)

2-POINT FUNCTIONS

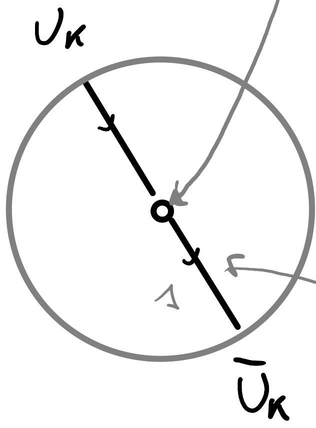
* To evaluate the Witten diagrams we need the quadratic action:

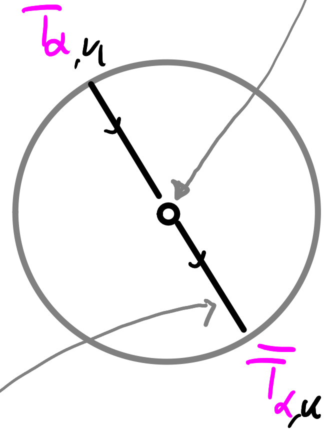
untwisted s_k modes

Lee et al 1998

twisted $\eta_{\alpha,k}$ modes

Gava 1998





Bulk-to-boundary propagation: follow Freedman et al 1998
D'Hoker et al 1999
...

2-POINT FUNCTIONS

* Results:

  * Untwisted: $$G_k = \frac{M N^2 \, k(k-1)^2(k-2)^2}{2^{k-4}\pi^4 (k+1)^2}$$

  * Twisted: $$G_{\alpha,k} = \frac{M N^2 (k-1)(k-2)^2}{S_\alpha \lambda \pi^4}$$

due to the extra α'^2 in the 6d action

3-POINT FUNCTIONS

* We need the cubic action for the s_k and $\eta_{\alpha,u}$ modes

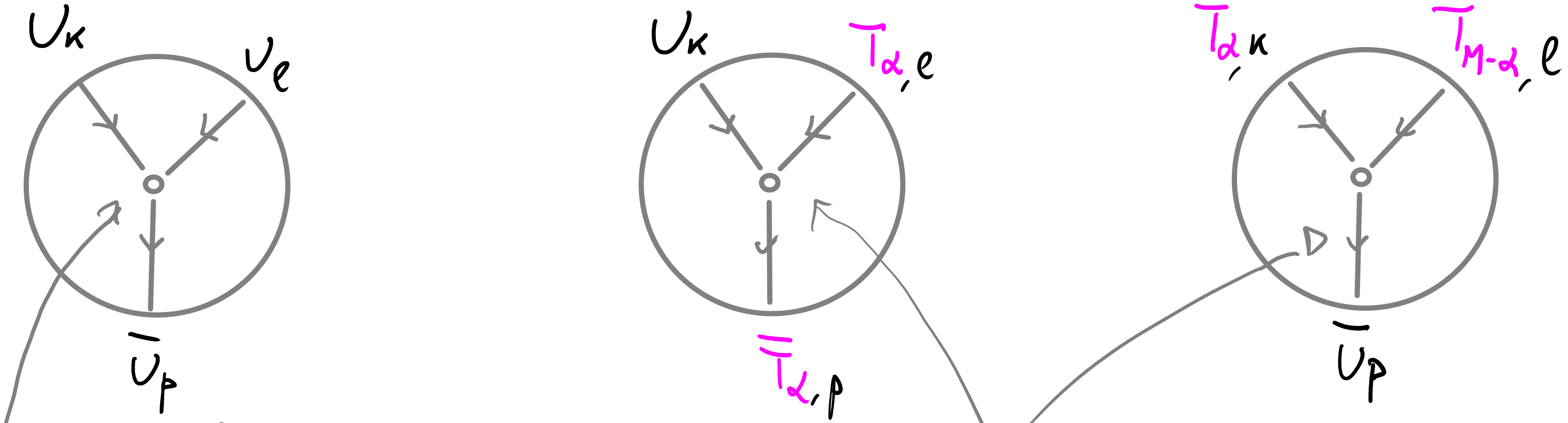
  * the $(s)^3$ terms come from the 10d action reduced on S_5 Lee et al, 1998

  * the $(s\,\eta^2)$ terms come from the 6d action – we derived them in 2207.08846

$$S_6 = \frac{(2\pi\alpha')^2}{2u_{10}^2 M} \sum_{\alpha=1}^{M-1} \frac{1}{2 s_\alpha} \left[\int d^6x \sqrt{G} \left(\nabla b_\alpha^* \cdot \nabla b_\alpha + \nabla c_\alpha^* \cdot \nabla c_\alpha \right) + \gamma \int C_4 \wedge d b_\alpha^* \wedge d c_\alpha \right]$$

$AdS_5 \times S_1$ bkg ; Expand in S_1 harmonics

3-POINT FUNCTIONS



Cubic coupling of the $(s)^3$ type

Cubic $(s\eta^2)$ couplings

3-POINT FUNCTIONS

* Results

$$G_{\kappa,\ell;p} = MN^2 \frac{\kappa(\kappa-1)(\kappa-2)}{2^{\kappa/2-2}\pi^2(\kappa+1)} \frac{\ell(\ell-1)(\ell-2)}{2^{\ell/2-2}\pi^2(\ell+2)} \frac{p(p-1)(p-2)}{2^{p/2-2}\pi^2(p+1)}$$

$$G_{\kappa,\alpha\ell;\alpha p} = \frac{MN^2}{S_\alpha(\lambda)} \frac{\kappa(\kappa-1)(\kappa-2)}{2^{\kappa/2-2}\pi^2(\kappa+1)} \frac{(\ell-1)(\ell-2)}{\pi^2} \frac{(p-1)(p-2)}{\pi^2}$$

due to extra α'^2 in S_6

$G_{\alpha\kappa, M-\alpha\ell;p} = \ldots$ (mutatis mutandis)

STRUCTURE CONSTANTS

* From the above results we get

$$C_{\kappa,e;p} = \frac{G_{\kappa,e;p}}{\sqrt{G_\kappa G_e G_p}} = \frac{1}{\sqrt{M}\,N}\sqrt{\kappa e p} \quad \checkmark$$

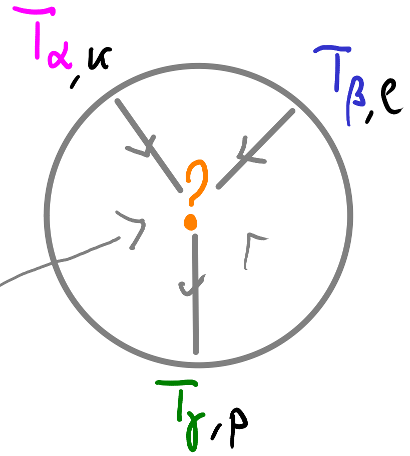
(same as for $\lambda \to 0$)
Lee et al 1998

$$C_{\kappa,\alpha e;\alpha p} = \frac{G_{\kappa,\alpha e;\alpha p}}{\sqrt{G_\kappa G_{\alpha e} G_{\alpha p}}} = \frac{1}{\sqrt{M}\,N}\sqrt{\kappa(e-1)(p-1)} \quad \checkmark$$

Both numerator and denominator are proportional to $1/\lambda$
that cancels out

3-POINT FUNCTIONS: MISSING

* A class of Witten diagrams is missing from the previous description



¡ There is no purely twisted cubic coupling $(\eta)^3$ in the 6d SUGRA action obtained by wrapping on the exceptional cycles the 10d action that was quadratic in $B_{(2)}$ and $C_{(2)}$!

WHAT WOULD WE NEED?

* From the gauge theory side, the 3-twist structure constants

$$C_{\alpha k,\beta l;\gamma p} = \frac{\sqrt{(k-1)(l-1)(p-1)}}{MN}$$

are on the same ground as the others

* To reproduce them on the holographic side we would need

$$G_{\alpha k,\beta l;\gamma p} = C_{\alpha k,\beta l;\gamma p}\sqrt{G_{\alpha k}\, G_{\beta l}\, G_{\gamma p}}$$

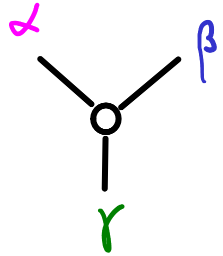
known

WHAT WOULD WE NEED?

* We would need the following result from Witten diagrams:

$$G_{\alpha k,\,\beta \ell;\,\gamma p} = \frac{M N^2}{\sqrt{S_\alpha S_\beta S_\gamma}} \; \frac{1}{\lambda^{3/2}} \; \frac{(k-1)(k-2)}{\pi^2} \; \frac{(\ell-1)(\ell-2)}{\pi^2} \; \frac{(p-1)(p-2)}{\pi^2}$$

* Thus we need a cubic 6d bulk coupling of the type $\eta_\alpha \, \eta_\beta \, \bar{\eta}_\gamma$



with an extra $(\alpha')^3$ w.r.t. the untwisted couplings

WE NEED THE STRING!

* The 6d bulk action that, put on $AdS_5 \times S_1$, yields these couplings:

$$S'_{(6)} \sim \frac{(\alpha')^3}{2\kappa_{10}} \cdot \frac{1}{M} \int d^6x \, \mathcal{L}'(G, C_4; \eta)$$

- $[\mathcal{L}'] = L^{-4}$ $\longrightarrow$ 4-derivative action

(G, components of C_4 and η are dimensionless)

- $\mathcal{L}'$ cubic in the twisted fields

WE NEED THE STRING!

* $S'_{(6)}$ has a power of α' and two derivatives more than the 6d SUGRA action $S_{(6)}$, and is cubic in the twisted fields

* It is natural to think it comes from world-sheet string corrections

* These correction represent the leading terms in α' that give a 3-twist coupling $\Rightarrow$ are needed to match these CFT data

STRATEGY

* We cannot compute directly the string on the $AdS_5 \times S_5/\mathbb{Z}_M$ bkg

* The "flat" bkg $\mathbb{R}^4 \times \mathbb{C} \times \mathbb{C}^2/\mathbb{Z}_M$ is a solvable world-sheet CFT

* We construct the space-time action from string scattering amplitudes, in the flat bkg, covariantizing it

* Only at this point we insert the AdS bkg

STRING DIAGRAMS

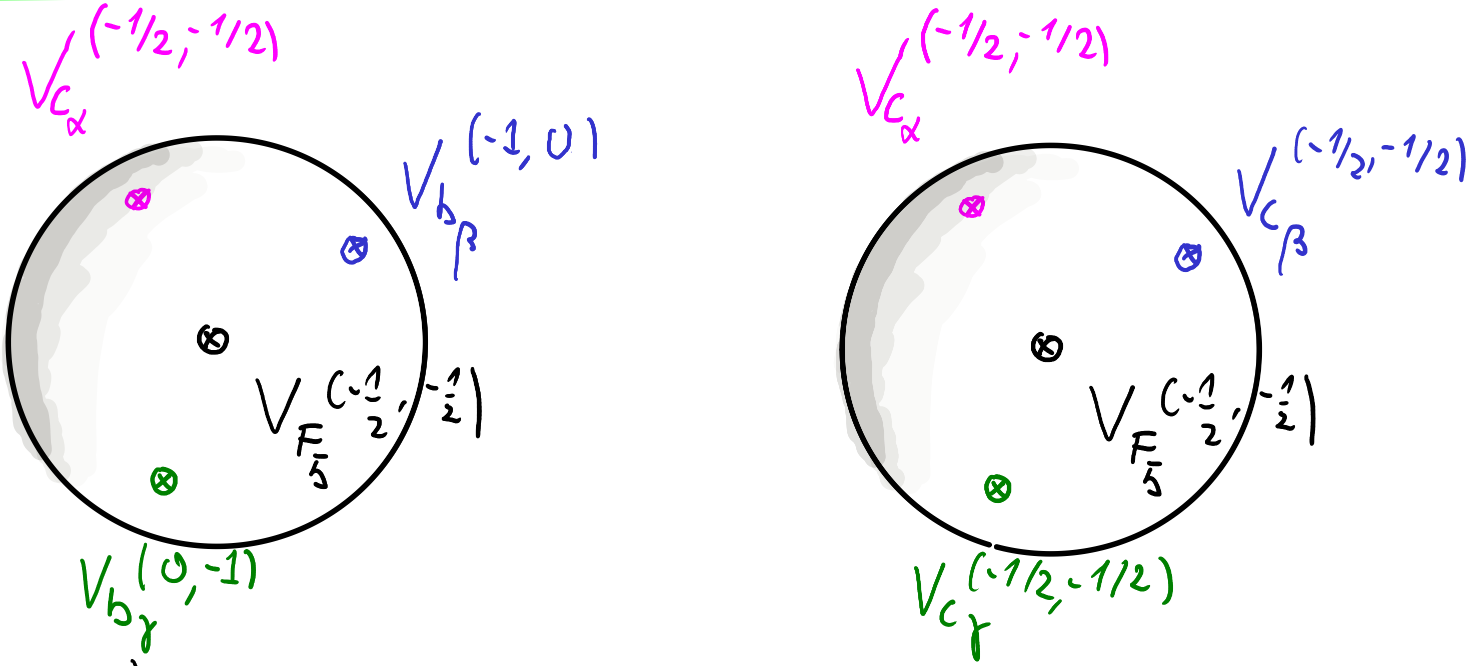
* Type IIB on $\mathbb{R}^6 \times \mathbb{C}^2/\mathbb{Z}_M$ at tree level (sphere diagrams)

* Vertex op.s for twisted scalars b_α (NS-NS) and c_α (R-R) known

* The needed 4-derivative couplings arise from 4-point diagrams
  - 3 twisted fields (b_α or c_α)
  - one untwisted field: C_4 RR form / metric

  appearing via F_5 or curvature $\longrightarrow$ constant on the $AdS_1 \times S_1$ bkg

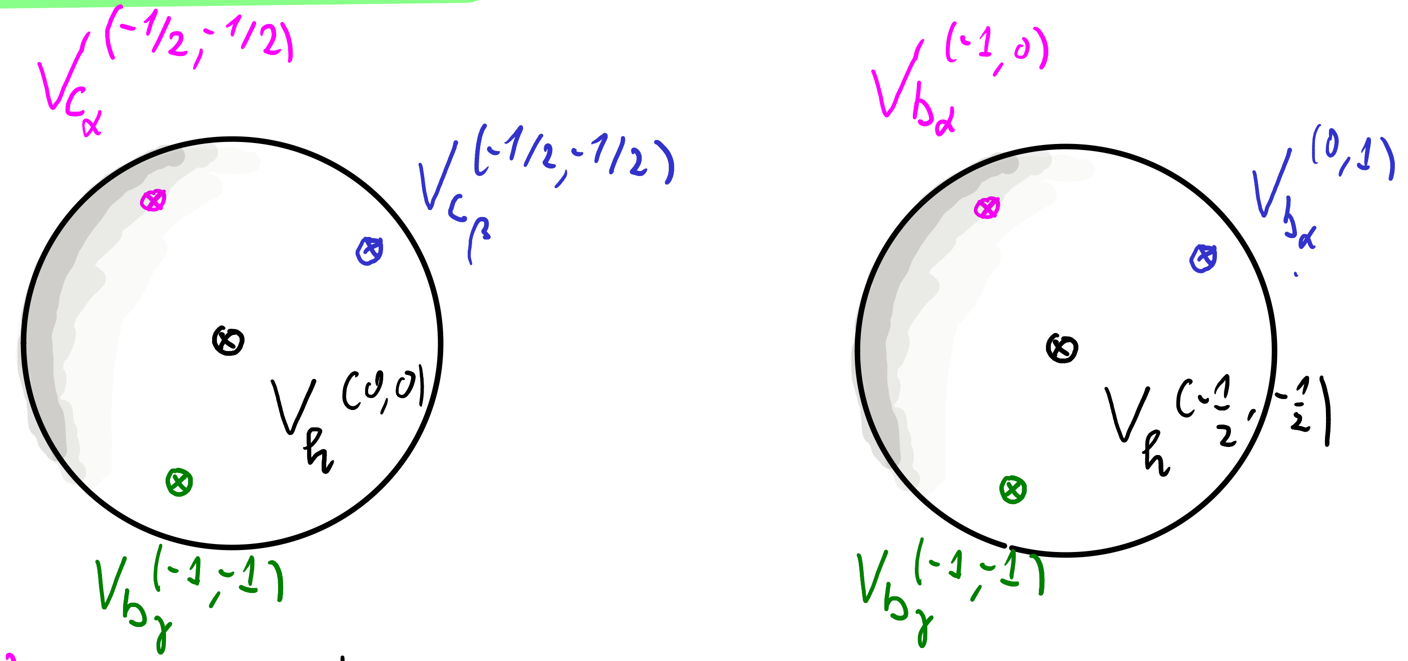
STRING DIAGRAMS with F_5



$$\propto \frac{(\alpha')^3}{2\kappa_{10}^2}\,\frac{1}{\sqrt{s_\alpha s_\beta s_\gamma}}\left\{ (\partial c_\alpha\cdot\partial b_\beta)\,(F_5\wedge db_\gamma) + (\partial c_\alpha\cdot\partial c_\beta)\,(F_5\wedge dc_\gamma)\right\}$$

Exactly the right structure when put on the $AdS_5\times S_5$ bkg

STRING DIAGRAMS with the metric under analysis!



$$\to \frac{(\alpha')^3}{2\kappa_{10}^2}\frac{1}{\sqrt{s_\alpha s_\beta s_\gamma}}\left\{R_{mn}\,\partial^m c_\alpha\,\partial^n c_\beta\, b_\gamma + R_{mn}\,\partial^m b_\alpha\,\partial^n b_\beta\, b_\gamma\right\}$$

Preliminary!

The right structure when put on $AdS_3 \times S_1$

CONCLUSIONS

* We better finish now the computation and check whether the 3-twist structure constants are really reproduced

* In this project, as in many others, enter so many things we all learned along the years working and discussing with Paolo!

* This has not only been useful and instructive but also extremely pleasant at the personal and intellectual level