Classical Higher-Spin Amplitudes and Kerr Black Holes

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[arXiv: 2207.03947] with P. Pichini
[arXiv: 2212.06120] with M. Chiodaroli, H. Johansson, A. Ochirov, P. Pichini and E. Skvortsov

PaoloFest

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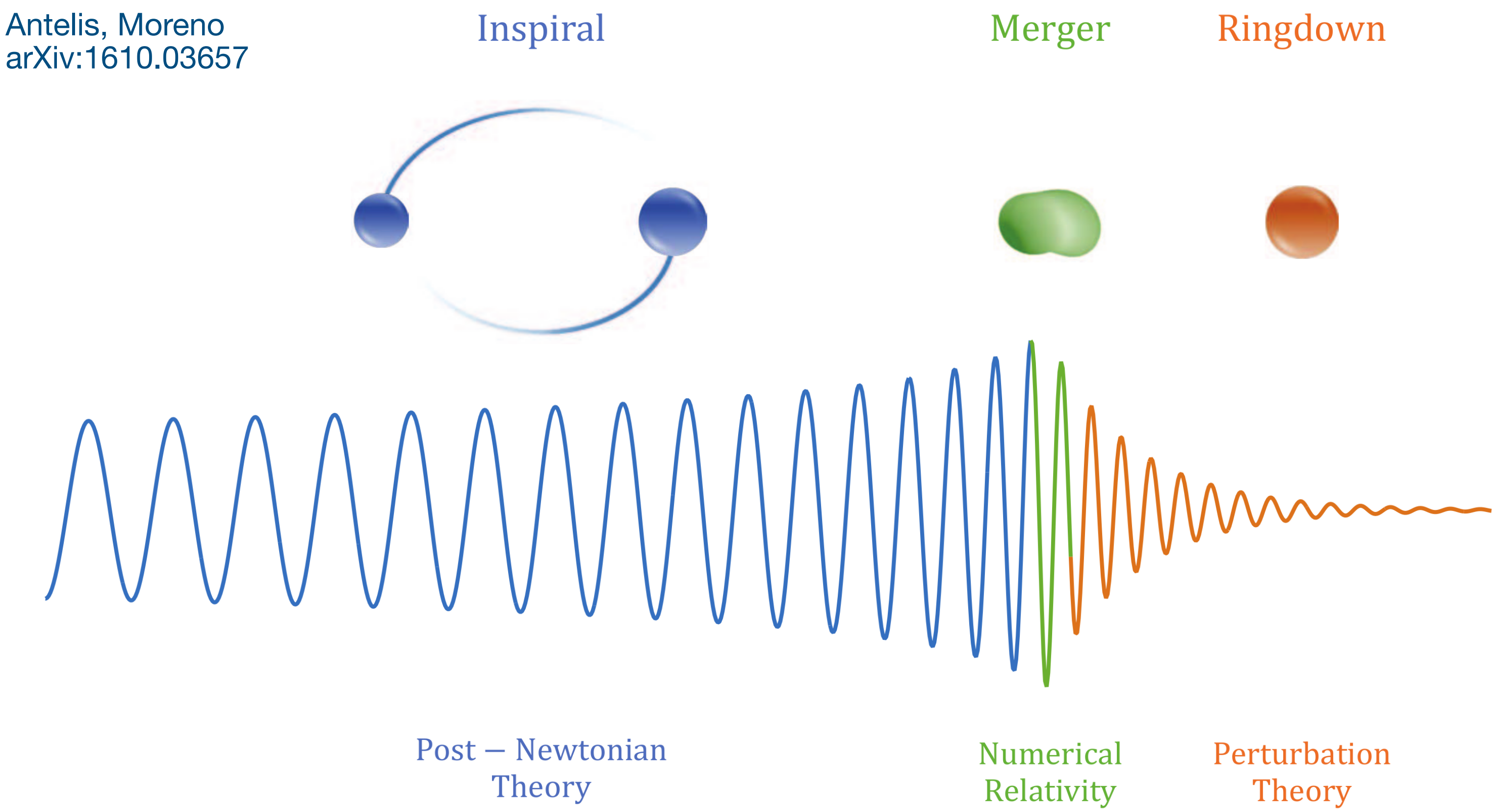
Outline

- Motivation
- Kerr, root-Kerr & string amplitudes @ 3pts *[arXiv: 2207.03947]*
- Constructing Kerr EFTs *[arXiv: 2212.06120]*
  - higher-spin gauge invariance
- Extension to Compton
- Outlook

Antelis, Moreno
arXiv:1610.03657

Motivation



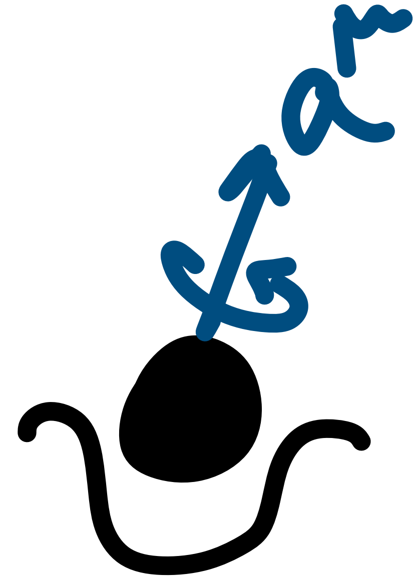
- detection of GWs from binary BH systems

- inspiral —> BHs well-separated — EFT description
  - worldline formalism *[Rothstein, Goldberger; Kälin, Liu, Porto; Levi, Steinhoff; Jakobsen, Mogull, Plefka, Steinhoff; Ledvinka, Schaefer, Bicak; Damour; Blanchet, Fokas …]*
  - quantum scattering amplitudes
    - post-Minkowskian scattering *[Bjerrum-Bohr, Damgaard, Festuccia, Planté; Cristofoli, Vanhove et al.; Damour; Bern, Cheung, Roiban, Shen, Solon, Zeng; Parra-Martinez, Ruf, Zeng; …]*
    - high-energy scattering *[Di Vecchia, Heissenberg, Russo, Veneziano; …]*
    - spin and tidal effect *[Guevara, Ochirov, Vines; Chung, Huang, Kim, Lee; Arkani-Hamed, O'Connell; Aoude, Haddad, Helset; Bern, Luna, Roiban, Shen, Zeng; Alessio, Di Vecchia; Bjerrum-Bohr, Chen, Skowronek; …]*

Motivation

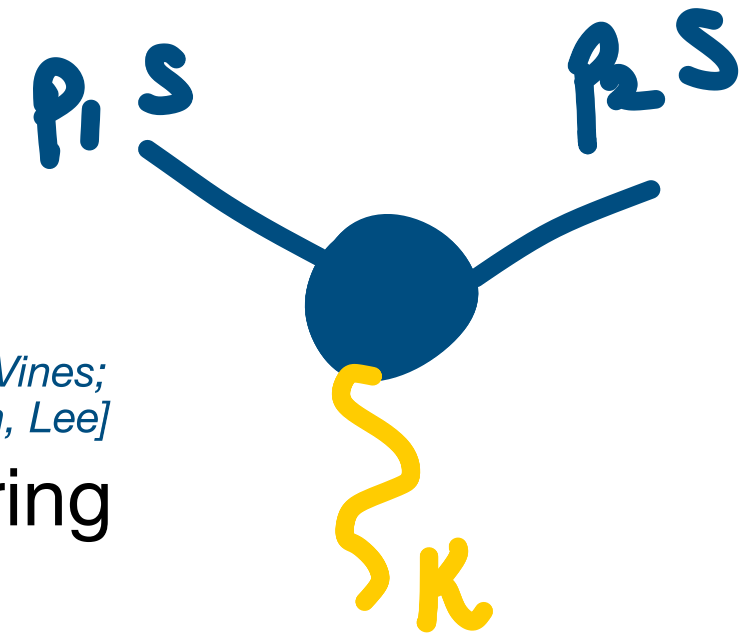
Spinning case

linearised energy-mom. tensor Kerr



[Guevara, Ochirov, Vines; Chung, Huang, Kim, Lee]

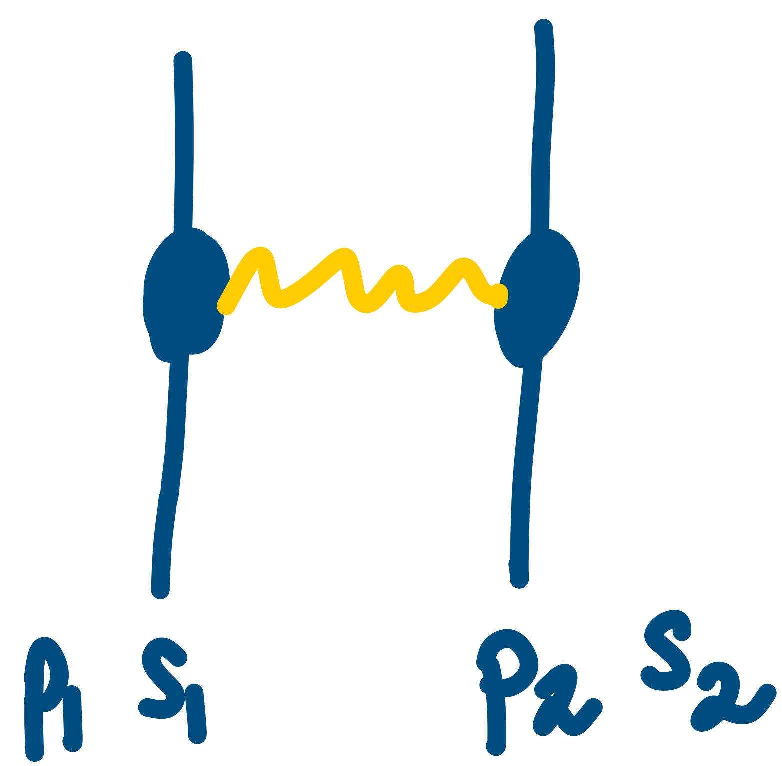
3pt scattering amp.



$$\varepsilon_{\mu\nu,k} T^{\mu\nu}(k) \sim (p_1 \cdot \varepsilon_k^+)^2 e^{k\cdot a}$$

$$\mathcal{M}_{Kerr} \sim (p_1 \cdot \varepsilon_k^+)^2 \langle e^{k\cdot \hat{a}} \rangle$$

- Classical observables *e.g. momentum shift, spin shift, scattering angle*
  - at 1 PM -> 3pt is enough

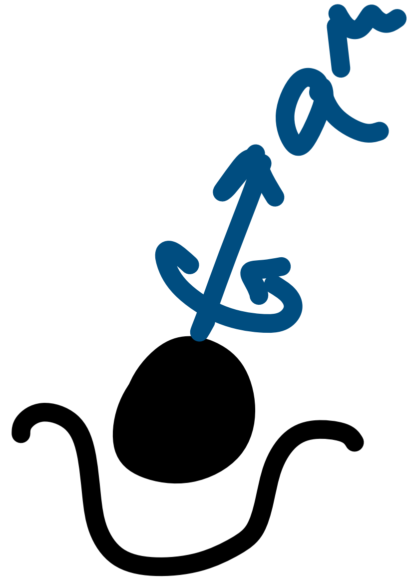


post-Minkowski (PM) expansion:

$$\frac{Gm}{|\mathbf{b}|} \ll 1 \qquad \text{and} \qquad |k| \ll m, p, |k|S$$

Motivation

Spinning case

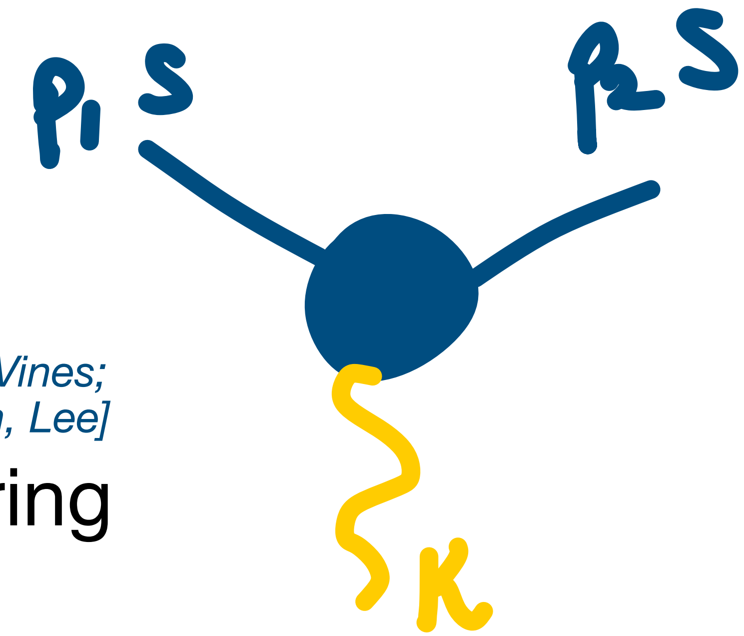


linearised energy-mom. tensor Kerr

$$\varepsilon_{\mu\nu,k}T^{\mu\nu}(k) \sim (p_1 \cdot \varepsilon_k^+)^2 e^{k\cdot a}$$

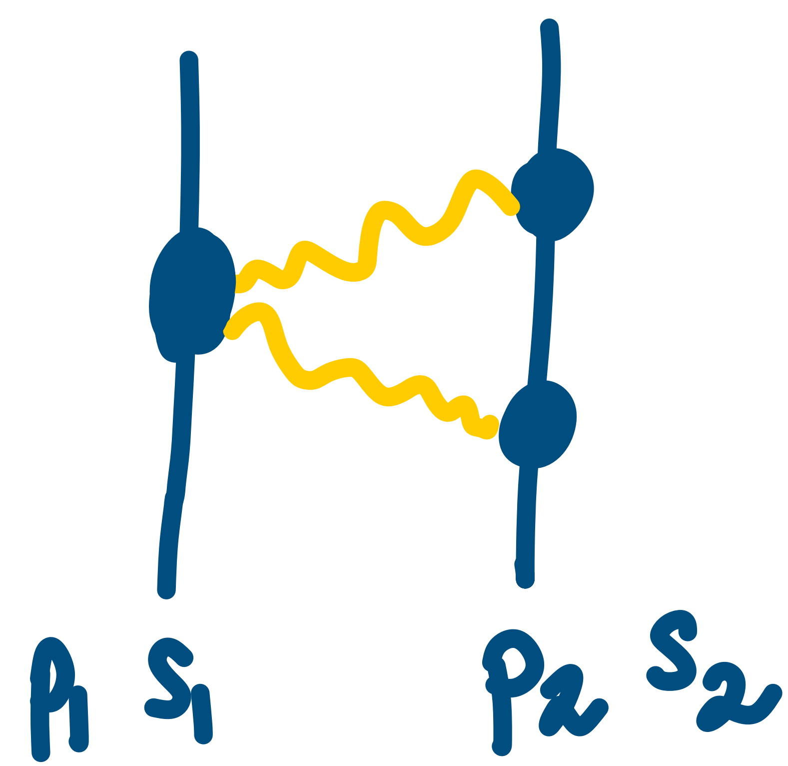
[Guevara, Ochirov, Vines; Chung, Huang, Kim, Lee]

3pt scattering amp.



$$\mathcal{M}_{Kerr} \sim (p_1 \cdot \varepsilon_k^+)^2 \langle e^{k\cdot \hat{a}} \rangle$$

- Classical observables *e.g. momentum shift, spin shift, scattering angle*
  - at 1 PM -> 3pt is enough
  - from 2 PM -> need 4pt Compton + higher



post-Minkowski (PM) expansion:

$$\frac{Gm}{|\mathbf{b}|} \ll 1 \quad \text{and} \quad |k| \ll m, p, |k|S$$

Kerr amplitudes @ 3pt

Spinor Form:

[Arkani-Hamed Huang & Huang]

$$\mathcal{M}_{Kerr} = (p_1 \cdot \varepsilon_k)^2 \left(\frac{\langle \mathbf{12} \rangle}{m} \right)^{2s}$$ Gravity

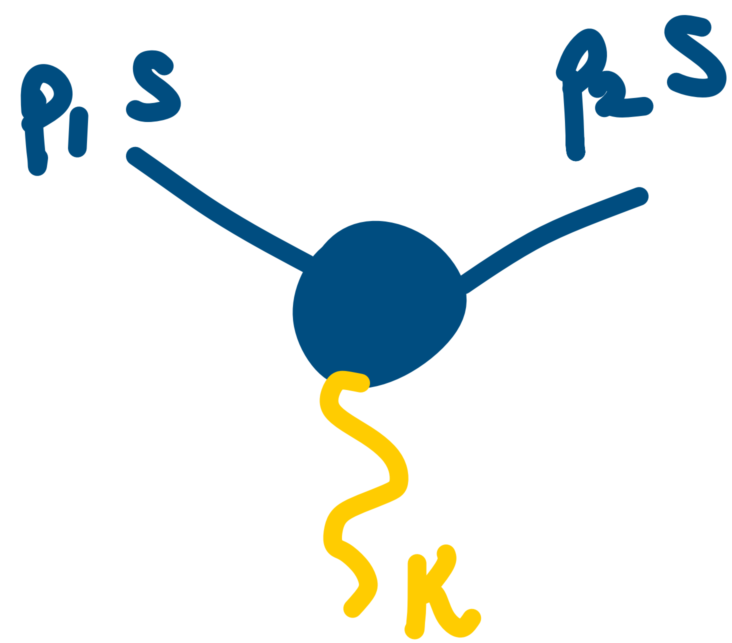
$$\mathcal{A}_{\sqrt{Kerr}} = (p_1 \cdot \varepsilon_k) \left(\frac{\langle \mathbf{12} \rangle}{m} \right)^{2s}$$ Gauge theory

Massive Weyl Spinors

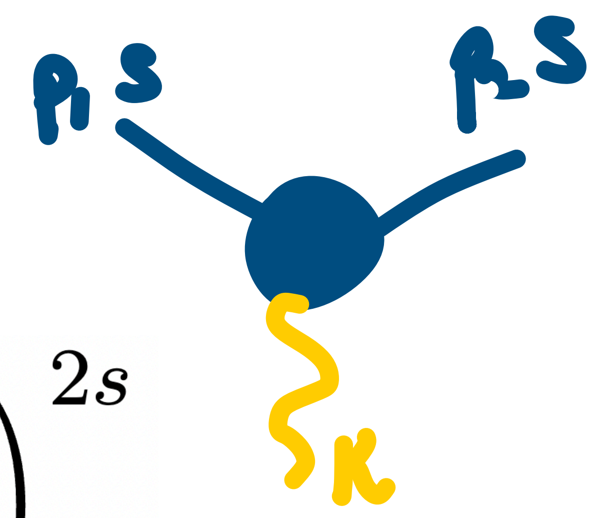
$$p_i \cdot \sigma |\boldsymbol{i}] = m |\boldsymbol{i}\rangle \qquad \varepsilon_i^\mu = \frac{\langle \boldsymbol{i} | \sigma^\mu | \boldsymbol{i}]}{\sqrt{2} m}$$

$$|\mathbf{1}\rangle := |1^a\rangle z_a \qquad |\bar{\mathbf{1}}\rangle := |1^a\rangle \bar{z}_a$$

$$|\mathbf{2}\rangle = |\bar{\mathbf{1}}\rangle + \frac{1}{2m} (k \cdot \sigma) |\bar{\mathbf{1}}]$$



Kerr amplitudes @ 3pt



Spinor Form:

[Arkani-Hamed Huang & Huang]

$$\mathcal{M}_{Kerr} = (p_1 \cdot \varepsilon_k)^2 \left(\frac{\langle \mathbf{12} \rangle}{m} \right)^{2s}$$

Exponential Form:

[Guevara, Ochirov, Vines; Chung, Huang, Kim , Lee; Aouade, Haddad, Helset]

$$\mathcal{M}_{Kerr} = (p_1 \cdot \varepsilon_k)^2 \left\langle \sum_{n=0}^{2s} \frac{1}{n!} (k \cdot \hat{a})^n \right\rangle$$

$$= (p_1 \cdot \varepsilon_k)^2 \langle \exp(k \cdot \hat{a}) \rangle$$

Massive Weyl Spinors

$$p_i \cdot \sigma |\boldsymbol{i}] = m |\boldsymbol{i}\rangle \qquad \boldsymbol{\varepsilon}_i^\mu = \frac{\langle \boldsymbol{i} | \sigma^\mu | \boldsymbol{i}]}{\sqrt{2}m}$$

$$|\mathbf{1}\rangle := |1^a\rangle z_a \qquad |\bar{\mathbf{1}}\rangle := |1^a\rangle \bar{z}_a$$

$$|\mathbf{2}\rangle = |\bar{\mathbf{1}}\rangle + \frac{1}{2m} (k \cdot \sigma) |\bar{\mathbf{1}}]$$

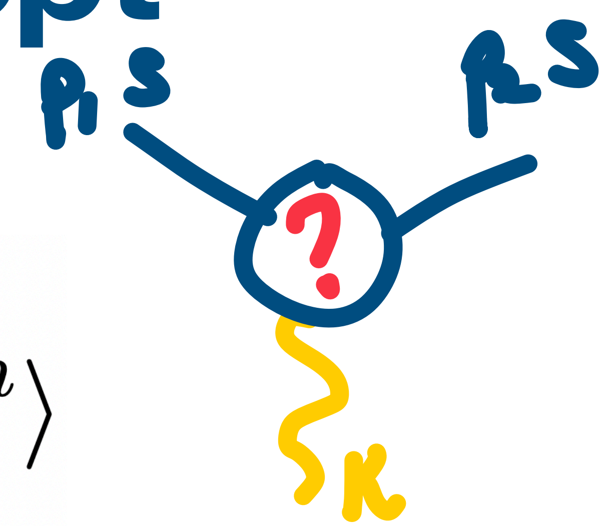
Pauli Lubanski spin operator

$$\hat{S}^\mu_{(s)} = \frac{1}{2m} \epsilon^{\mu\nu\rho\sigma} p_{1\nu} M_{(s)\rho\sigma} = m \hat{a}^\mu$$

satisfies SU(2) algebra $[\hat{S}^\mu, \hat{S}^\nu] = \frac{i}{m} \epsilon^{\mu\nu\rho\sigma} p_{1\rho} \hat{S}_\sigma$

expectation value $\langle \hat{S}^\mu_{(s)} \rangle = \dfrac{1}{\langle \bar{\mathbf{1}} \mathbf{1} \rangle^{2s}} \langle \bar{\mathbf{1}} |^{2s} \hat{S}^\mu_{(s)} | \mathbf{1} \rangle^{2s}$

Leading Regge strings @ 3pt



General 3pt amp

$$\mathcal{M}_3 = (p_1 \cdot \varepsilon_k)^2 \sum_{n=0}^{2s} c_n^{(s)} \langle (k \cdot \hat{a})^n \rangle$$

[LC, Pichini]	Gravity $(p_1 \cdot \varepsilon_k)^2$		Gauge Theory $(p_1 \cdot \varepsilon_k)$	
	Kerr	**Closed Superstring** Leading Regge States	**root-Kerr**	**Open Superstring** Leading Regge States
monopole $c_0^{(s)}$	1	1	1	1
dipole $c_1^{(s)}$	1	1	1	1 *Di Vecchia et al.*
quadrupole $c_2^{(s)}$	$\frac{1}{2}$	$\frac{4s^2-7s+4}{2s(2s-1)}$	$\frac{1}{2}$	$\frac{3s^2-7s+8}{2s(2s-1)}$
octupole $c_3^{(s)}$	$\frac{1}{6}$	$\frac{2s-3}{2(2s-1)}$	$\frac{1}{6}$	$\frac{3s^2-12s+14}{2(2s-1)(2s-2)}$

Leading Regge strings @ 3pt

General 3pt amp $\mathcal{M}_3 = (p_1 \cdot \varepsilon_k)^2 \sum_{n=0}^{2s} c_n^{(s)} \langle (k \cdot \hat{a})^n \rangle$

$\lim_{s\to\infty} \langle \hat{a}^\mu \rangle = a^\mu$

[LC, Pichini]	Gravity $(p_1 \cdot \varepsilon_k)^2$		Gauge Theory $(p_1 \cdot \varepsilon_k)$	
	Kerr	**Closed Superstring** Leading Regge States	**root-Kerr**	**Open Superstring** Leading Regge States
monopole $c_0^{(s)}$	1	1	1	1
dipole $c_1^{(s)}$	1	1	1	1
quadrupole $c_2^{(s)}$	$\frac{1}{2!}$	1	$\frac{1}{2!}$	$\frac{3}{4}$
octupole $c_3^{(s)}$	$\frac{1}{3!}$	$\frac{1}{2}$	$\frac{1}{3!}$	$\frac{3}{8}$
$\lim_{s\to\infty} \sum_{n=0}^{2s} c_n^{(s)} \langle (k \cdot \hat{a})^n \rangle$	$e^{k\cdot a}$	$[I_0(k\cdot a) + I_1(k\cdot a)]^2$	$e^{k\cdot a}$	$I_0\left(\frac{k\cdot a}{2}\right) + I_1\left(\frac{k\cdot a}{2}\right)$

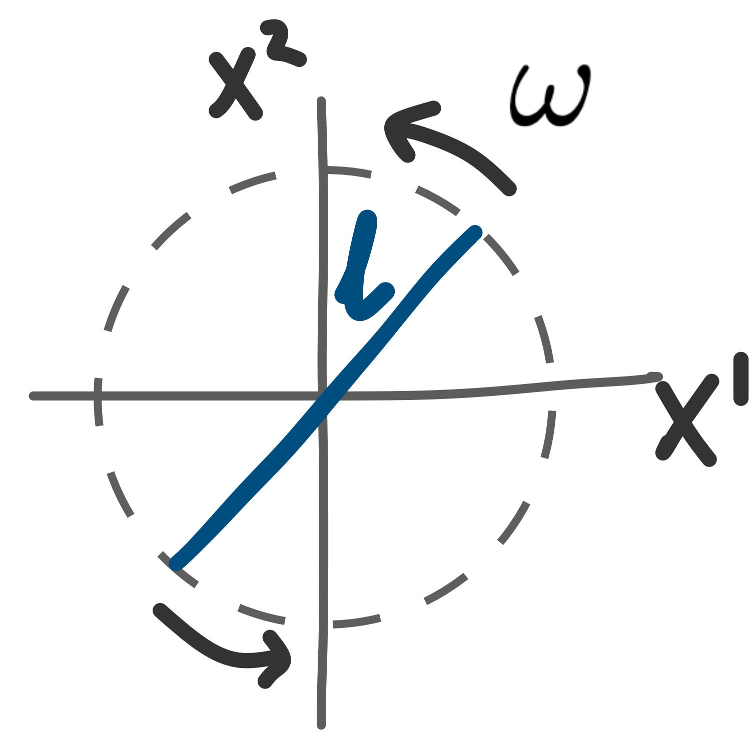
Classical Solutions @ 3pt

[LC, Pichini]	Gravity $(p_1 \cdot \varepsilon_k)^2$		Gauge Theory $(p_1 \cdot \varepsilon_k)$	
	Kerr	**Closed Superstring** Leading Regge States	**root-Kerr**	**Open Superstring** Leading Regge States
Classical limit of 3pt amplitude	$e^{k\cdot a}$	$\left[I_0(k\cdot a) + I_1(k\cdot a)\right]^2$	$e^{k\cdot a}$	$I_0\left(\frac{k\cdot a}{2}\right) + I_1\left(\frac{k\cdot a}{2}\right)$
$\varepsilon_{\mu\nu,k}T^{\mu\nu}(k)$	$e^{k\cdot a}$	$\left[I_0(2\,l\,E_k)+I_1(2\,l\,E_k)\right]^2$	$e^{k\cdot a}$	$I_0(l\,E_k) + I_1(l\,E_k)$

Rotating rigid folded string of length $2\,l$,

angular velocity $\omega = 1/l$

$$J = 2\alpha' E^2$$



What are the underlying QFTs that produce Kerr amplitudes?

Kerr EFTs

Known for low quantum spin

EFTs	$s = {}^1\!/_2$	$s = 1$	$s = {}^3\!/_2$	$s = 2$	$s = {}^5\!/_2$	$s \geq 3$
Kerr	Major.	Proca	Rar.-Sch.	KK grav.	HS	HS
$\sqrt{\text{Kerr}}$	Dirac	W-boson	gravitino	HS	HS	HS

[Chiodaroli, Johansson, Pichini]
[LC, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov]

Kerr EFTs $s \leq \frac{5}{2}$

root-Kerr EFTs $s \leq \frac{3}{2}$

uniquely determined by

(CC) current constraint

$$p_1 \cdot \mathcal{J} = \mathcal{O}(m)$$

Kerr EFTs

Known for low quantum spin

EFTs	$s = {}^1\!/_2$	$s = 1$	$s = {}^3\!/_2$	$s = 2$	$s = {}^5\!/_2$	$s \geq 3$
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$\sqrt{\text{Kerr}}$	Dirac	W-boson	gravitino	HS	HS	HS

[Chiodaroli, Johansson, Pichini]
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Kerr EFTs $s \leq \frac{5}{2}$

root-Kerr EFTs $s \leq \frac{3}{2}$

uniquely determined by

(CC) current constraint

$$p_1 \cdot \frac{\partial}{\partial \epsilon_1} V_{\Phi\overline{\Phi}A}\big|_{(2,3)} = \mathcal{O}(m)$$

root-Kerr EFT: Spin 1

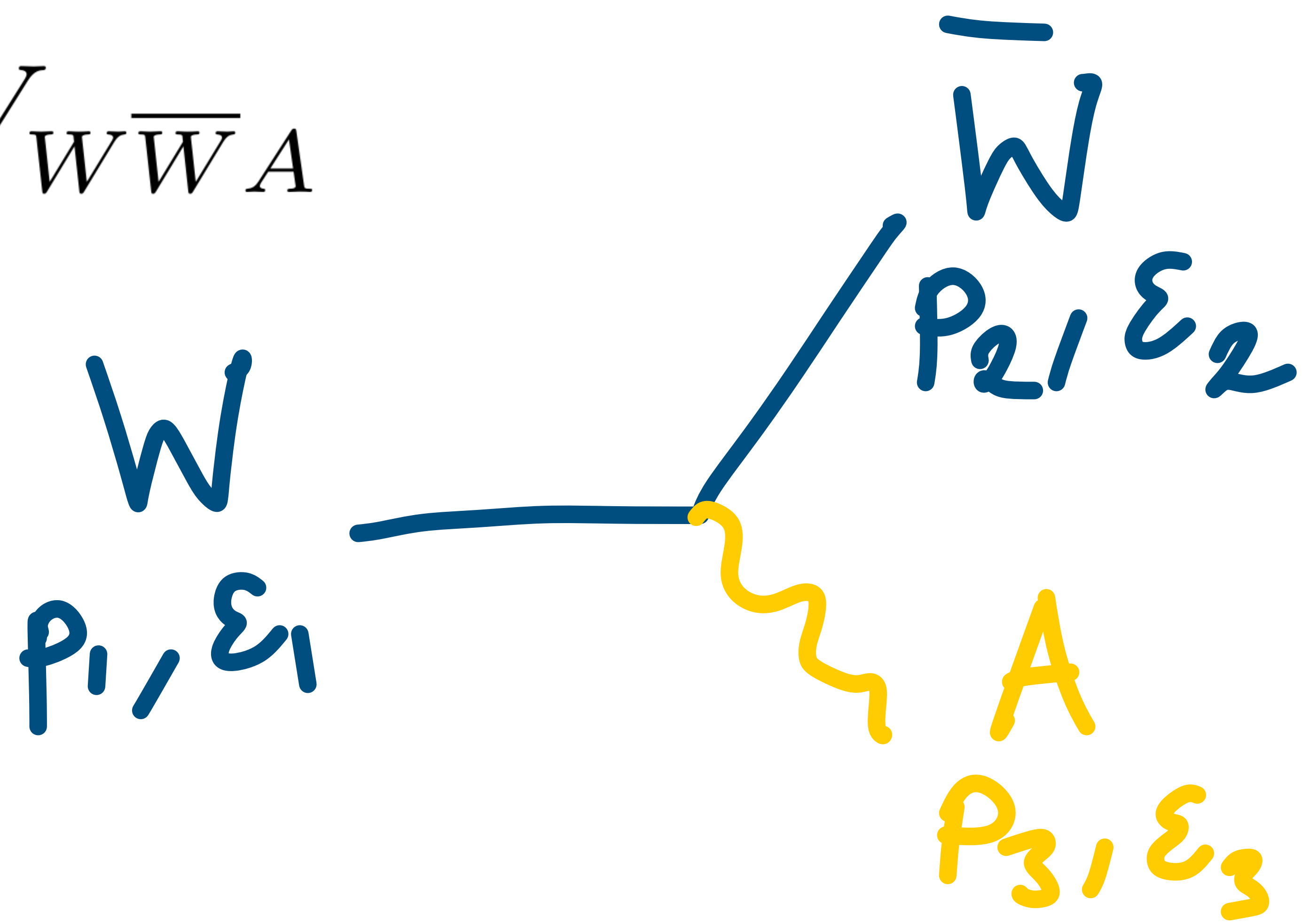
via current constraint

non-min. interaction

$$\mathcal{L} = -2|D_\mu W_\nu|^2 + |mW|^2 - \alpha\, ieF_{\mu\nu}W^\mu \bar{W}^\nu$$

kinetic term — minimal coupling

off-shell 3pt vertex $V_{W\overline{W}A}$



root-Kerr EFT: Spin 1

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non-min. interaction

$$\mathcal{L} = -2|D_\mu W_\nu|^2 + |mW|^2 - \alpha\, ieF_{\mu\nu}W^\mu \bar{W}^\nu$$

kinetic term — minimal coupling

3pt current:

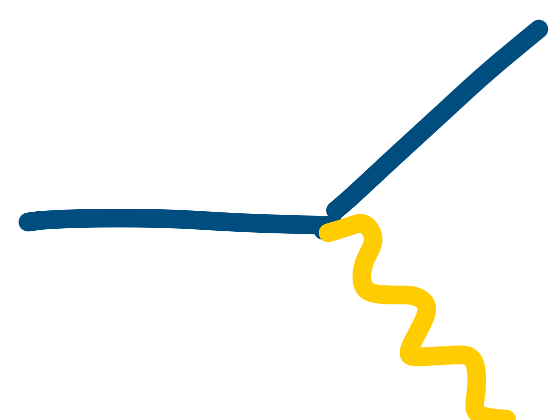
$$V_{W\overline{W}A}\big|_{(2,3)} = \varepsilon_1 \cdot \varepsilon_2 \varepsilon_3 \cdot (p_2 - p_1) - \varepsilon_2 \cdot \varepsilon_3 \varepsilon_1 \cdot p_1 - \alpha\, \varepsilon_2 \cdot f_3 \cdot \varepsilon_1$$

Current constraint:

$$p_1 \cdot \frac{\partial}{\partial \epsilon_1} V_{W\overline{W}A}\big|_{(2,3)} = \mathcal{O}(m)$$

$\cdot\ \varepsilon_2$ *on shell*

$p_1 \cdot$



$\cdot\ \varepsilon_3$ *on shell*

$\Rightarrow \quad \alpha = 1$ matches W-bosons in spontaneously broken YM!

Higher-spin Kerr EFTs

via massive gauge symmetry

- higher-spin EFTs **not** uniquely determined by ***(CC)***

(CC) current constraint

$$p_1 \cdot \frac{\partial}{\partial \epsilon_1} V_{\Phi\overline{\Phi}A}\big|_{(2,3)} = \mathcal{O}(m)$$

Our proposal: *arXiv: 2212.06120*

Higher-spin gauge symmetry is a central guiding principle for constraining Kerr dynamics

root-Kerr EFT: Spin 1

revisited with massive gauge symmetry

$\alpha = 1$ required value for W-bosons in spontaneously broken YM!

$$\mathcal{L}_{SSB} = -\frac{1}{4}F^2 - 2|D_\mu W_\nu|^2 + |mW - D\phi|^2 - ieF_{\mu\nu}W^\mu \bar{W}^\nu + \ldots$$

massive gauge transformations:

$$\delta W_\mu = \partial_\mu \xi + \ldots$$

$$\delta \varphi = m\xi + \ldots$$

(WI) Ward Identities:

$$mV_{\varphi \overline{W} A} - ip_1^\mu \frac{\partial}{\partial \epsilon_1^\mu} V_{W\overline{W}A}\Big|_{(2,3)} = 0.$$

root-Kerr EFT: Spin 1

revisited with massive gauge symmetry

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bottom up approach

$$\mathcal{L}_{Stückelberg} + \mathcal{L}_{F^2} = -\frac{1}{4}F^2 - 2|D_\mu W_\nu|^2 + |mW - D\phi|^2$$

+ ansatz constrained by ***(WI)***

Constructing Kerr EFTs

Ingredients

[Zinoviev]

- tower of symm. tensor fields $\Phi^k := \Phi^{\mu_1\mu_2\cdots\mu_k}$ $k = 0, 1, 2, \ldots, s$

  + traceless gauge param's $\xi^k := \xi^{\mu_1\mu_2\cdots\mu_k}$ $k = 0, 1, 2, \ldots, s-1$

  + massive gauge transform's

$$\delta\Phi^k = \partial^{(1}\xi^{k-1)} + m\alpha_k\xi^k + m\beta_k\eta^{(2}\xi^{k-2)}$$

Constructing Kerr EFTs

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$$\delta\Phi^k = \partial^{(1}\xi^{k-1)} + m\alpha_k\xi^k + m\beta_k\eta^{(2}\xi^{k-2)}$$

- free Lagrangian for spin-s field *[arXiv:2212.06120]*

$$\mathcal{L}_0 = -\sum_{k=0}^{s}\frac{(-1)^k}{2}\left[\Phi^k(\Box+m^2)\Phi^k - \frac{k(k-1)}{4}\tilde{\Phi}^k(\Box+m^2)\tilde{\Phi}^k\right] + \frac{1}{2}\sum_{k=0}^{s-1}(-1)^k(k+1)G^kG^k$$

$$G^k = \partial\cdot\Phi^{k+1} - \frac{k}{2}\partial^{(1}\tilde{\Phi}^{k+1)} + m\left(\alpha_k\Phi^k - \gamma_k\tilde{\Phi}^{k+2} - \delta_k\eta^{(2}\tilde{\Phi}^{k)}\right)$$

Constructing Kerr EFTs

Ingredients

- tower of symm. tensor fields $\Phi^k := \Phi^{\mu_1\mu_2\cdots\mu_k}$ $\quad k = 0, 1, 2, \ldots, s$

  \+ traceless gauge param's $\xi^k := \xi^{\mu_1\mu_2\cdots\mu_k}$ $\quad k = 0, 1, 2, \ldots, s-1$

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- 3pt vertex

$$V_{\Phi^k\Phi^s A} = V^{\text{min.}}_{\Phi^k\Phi^s A} + V^{\text{non-min.}}_{\Phi^k\Phi^s A}$$

ansatz with power-counting ***(PC)*** $k+s-1$

Constructing Kerr EFTs

Constraints

[arXiv:2212.06120]

(MC) minimal coupling extension of $\mathcal{L}_0$

(PC) power-counting bound

$$V^{\text{non-min.}}_{\Phi^{s_1}\Phi^{s_2}A} \sim \partial^{s_1+s_2-2}\,(F_{\mu\nu})$$

(ND) near-diagonal interactions

$V_{\Phi^{s_1}\Phi^{s_2}A} = 0$ for $|s_1 - s_2| > 1$

(WI) Ward identities

$$V_{\xi^k\Phi^s A}\Big|_{(2,3),\epsilon_1^2\to 0} = 0$$

(CC) current constraint

$$p_1\cdot\frac{\partial}{\partial\epsilon_1}V_{\Phi^s\Phi^s A}\Big|_{(2,3),\epsilon_1^2\to 0} = \mathcal{O}(m)$$

$$V_{\xi^k\Phi^s A} := m\alpha_k V_{\Phi^k\Phi^s A} - \tilde{\gamma}_{k+1}p_1\cdot\frac{\partial}{\partial\epsilon_1}V_{\Phi^{k+1}\Phi^s A} + m\tilde{\beta}_{k+2}\frac{\partial}{\partial\epsilon_1}\cdot\frac{\partial}{\partial\epsilon_1}V_{\Phi^{k+2}\Phi^s A}$$

Constructing root-Kerr EFTs

Three points

[arXiv:2212.06120]

(MC) + ***(PC)*** + ***(WI)***

$$\mathcal{A}(\Phi_1^s\,\overline{\Phi}_2^s\,A_3^+) = \mathcal{A}_0 \frac{\langle \mathbf{12}\rangle^{2s}}{m^{2s}} \left\{ 1 + \sum_{k=1}^{s-1} c_k \left(\frac{[\mathbf{12}]^k}{\langle \mathbf{12}\rangle^k} - 1 \right) \right\}$$

+ ***(CC)*** + ***(ND)*** constrains coefficients uniquely to $\mathcal{A}_{\sqrt{Kerr}}$

Uniquely predict root-Kerr 3pt amplitudes!

Tested up to $s \leq 6$

(MC) minimal coupling

(WI) Ward identities

(CC) current constraint

(PC) power-counting bound

(ND) near-diag. interactions

Constructing root-Kerr EFTs

Three points

[arXiv:2212.06120]

(MC) + ***(PC)*** + ***(WI)***

$$\mathcal{A}(\Phi_1^s\,\overline{\Phi}_2^s\,A_3^+) = \mathcal{A}_0\frac{\langle\mathbf{12}\rangle^{2s}}{m^{2s}}\left\{1+\sum_{k=1}^{s-1} c_k\left(\frac{[\mathbf{12}]^k}{\langle\mathbf{12}\rangle^k}-1\right)\right\}$$

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(MC) minimal coupling

(WI) Ward identities

(CC) current constraint

(PC) power-counting bound

(ND) near-diag. interactions

Constructing Kerr EFTs

Three points

[arXiv:2212.06120]

(MC) + ***(PC)*** + ***(WI)***

$$\mathcal{M}(\Phi_1^s \, \Phi_2^s \, h_3^+) = M_0 \frac{\langle \mathbf{12} \rangle^{2s}}{m^{2s}} \left\{ 1 + \left(1 - \frac{[\mathbf{12}]}{\langle \mathbf{12} \rangle} \right)^2 \sum_{k=0}^{s-4} c'_k \frac{[\mathbf{12}]^k}{\langle \mathbf{12} \rangle^k} \right\}$$

+ ***(CC)*** constrains coefficients uniquely to $\mathcal{M}_{Kerr}$

(MC) minimal coupling

(WI) Ward identities

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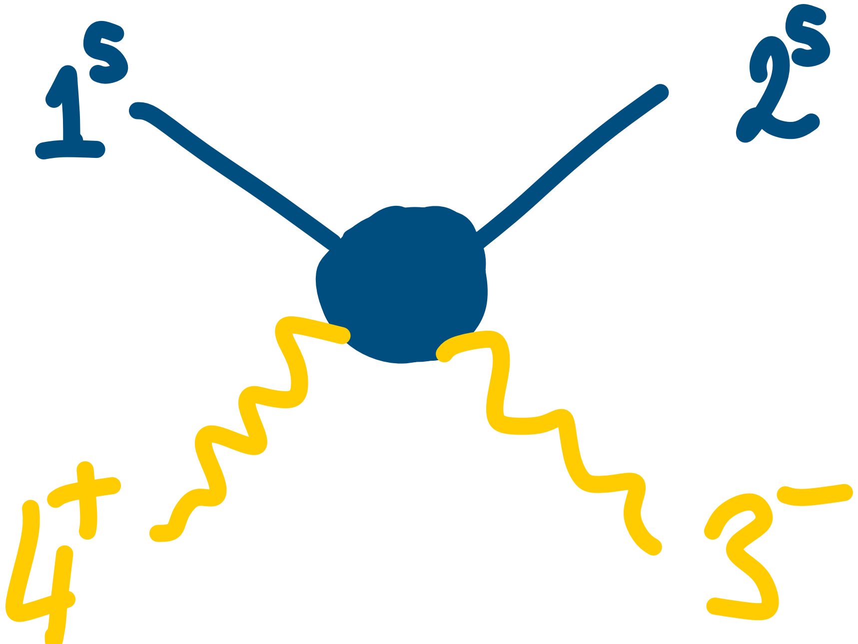
Uniquely predict Kerr 3pt amplitudes!

Tested up to $s \leq 6$

Extension to Compton



Method: *[arXiv:2212.06120]*

- For low spin $s \leq 2$
  - off-shell 3pt fns + propagator
  - identify best variables to use

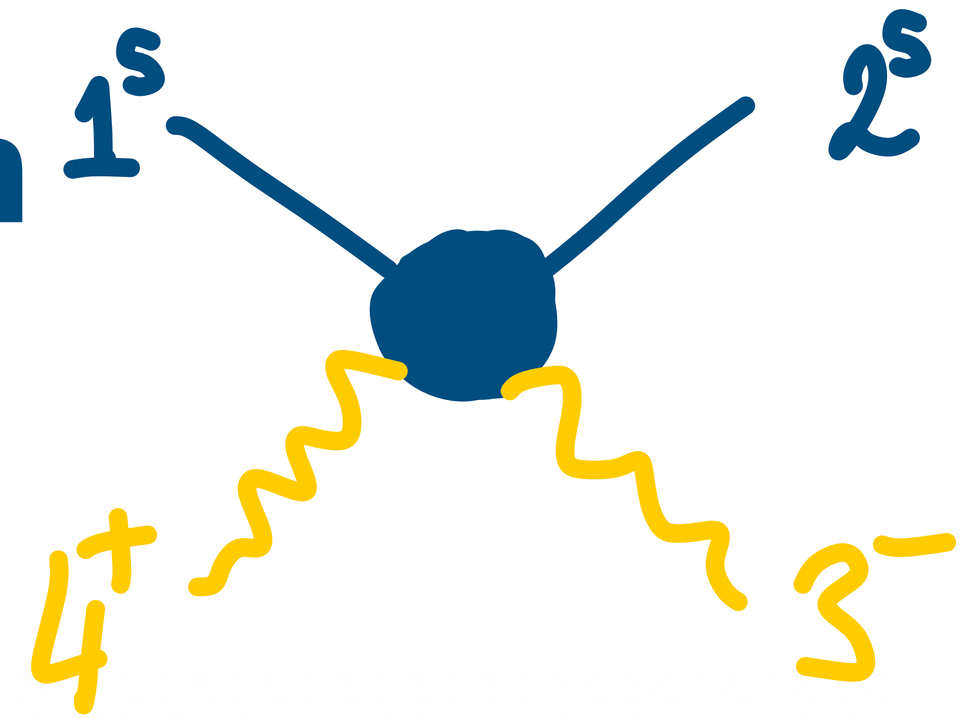
$$\Delta(\epsilon,\bar{\epsilon}) = \sum_{s=0}^{\infty} (\epsilon)^s \cdot \Delta^{(s)} \cdot (\bar{\epsilon})^s = \frac{1}{p^2 - m^2 + i0} \frac{1 - \frac{1}{4}\epsilon^2\bar{\epsilon}^2}{1 + \epsilon\cdot\bar{\epsilon} + \frac{1}{4}\epsilon^2\bar{\epsilon}^2}$$

$$W_\pm = \frac{m}{2}\left(\langle \mathbf{12}\rangle \pm [\mathbf{12}]\right), \quad V = -\frac{1}{2}\big(\langle \mathbf{1}|3|\mathbf{2}] + \langle \mathbf{2}|3|\mathbf{1}]\big), \quad U = -\frac{1}{2}\big(\langle \mathbf{1}|3|\mathbf{2}] - \langle \mathbf{2}|3|\mathbf{1}]\big) - m\,\langle \mathbf{12}\rangle$$

- for high spin $s \leq 10$
  - factorisation properties

- Checked against chiral approach *[Ochirov, Skvortsov]*

Extension to Compton



Gauge Theory Analysis:

Abelian amplitude *[arXiv:2212.06120]*

match known Comptons for $s \leq \frac{3}{2}$

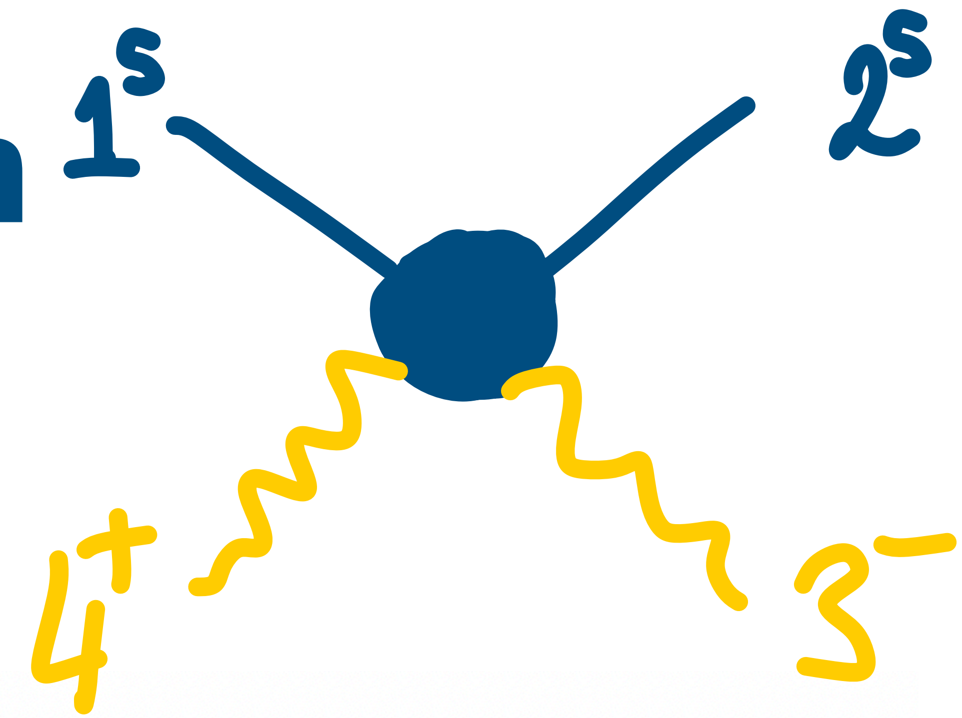
$$\mathcal{A}(\mathbf{1}^s,\mathbf{2}^s,3^-,4^+) = \frac{\langle 3|1|4]^2 (U+V)^{2s}}{m^{4s} t_{13} t_{14}} - \frac{\langle 3|1|4]\,\langle \mathbf{13}\rangle\,[4\mathbf{2}]}{m^{4s} t_{13}} P_{2s} + \frac{\langle \mathbf{13}\rangle\,[4\mathbf{2}][\mathbf{1}4]\,\langle 3\mathbf{2}\rangle}{m^{4s}} P_{2s-1} + C_s$$

$$V = -\frac{1}{2}\left(\langle \mathbf{1}|3|\mathbf{2}] + \langle \mathbf{2}|3|\mathbf{1}]\right),\ W_{\pm} = \frac{m}{2}\left(\langle \mathbf{12}\rangle \pm [\mathbf{12}]\right)$$

$$U = -\frac{1}{2}\left(\langle \mathbf{1}|3|\mathbf{2}] - \langle \mathbf{2}|3|\mathbf{1}]\right) - m\,\langle \mathbf{12}\rangle,\ t_{ij} = 2p_i \cdot p_j$$

$$P_k = \frac{1}{2V}\left[(U+V)^k - (U-V)^k\right]$$

Extension to Compton



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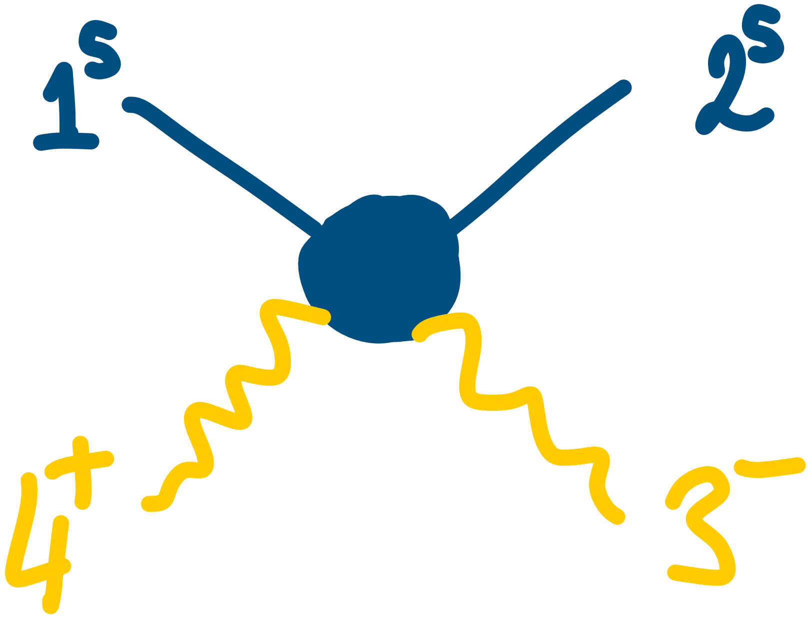
implications of massive gauge invariance:

$s \leq \frac{3}{2}$ $\quad C_s = 0$ no freedom in contact terms	$s = 2$ $\quad$ 3 free contact terms $C_2 = \frac{\langle \mathbf{1}3\rangle[4\mathbf{2}][\mathbf{1}4]\,\langle 3\mathbf{2}\rangle}{m^6}(c_1 W_+^2 - c_2 W_-^2)$

Non-Abelian Compton

Upcoming Work



$$\mathcal{A}(\mathbf{1}^s, \mathbf{2}^s, 3^-, 4^+) = \frac{\langle 3|\mathbf{1}|4]^2 (U+V)^{2s}}{m^{4s} s_{12} t_{14}} - \frac{\langle 3|\mathbf{1}|4] \langle \mathbf{1}3\rangle [4\mathbf{2}] P_{2s}}{m^{4s} s_{12}} + \frac{\langle \mathbf{1}3\rangle [4\mathbf{2}][\mathbf{1}4] \langle 3\mathbf{2}\rangle}{m^{4s} s_{12}} (t_{13} Q_{2s} - 2V R_{2s}) + \text{contacts}$$

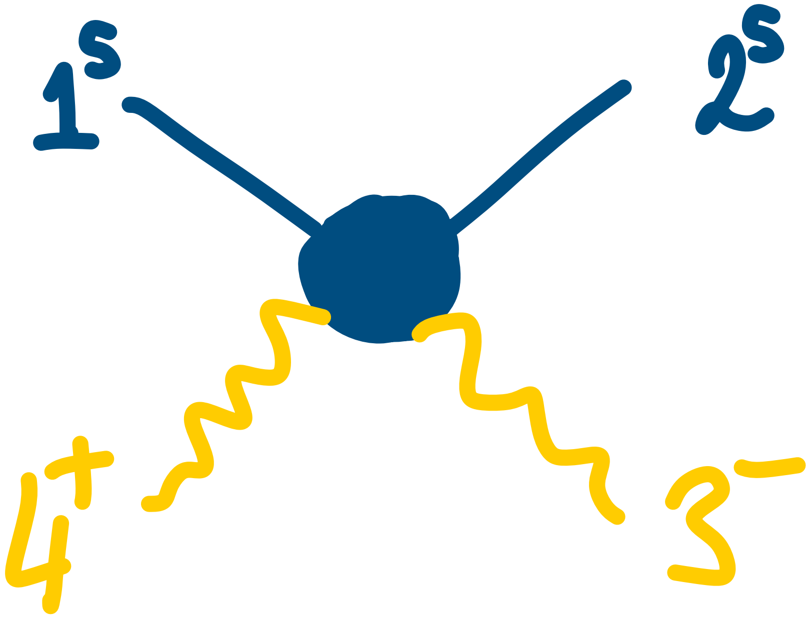
matches known Comptons for $s \leq \frac{3}{2}$

first orders in the polynomials that appear

s	0	$^1/_2$	1	$^3/_2$	2	$^5/_2$
P_{2s}	0	1	$2U$	$3U^2+V^2$	$4U(U^2+V^2)$	$5U^4+10U^2V^2+V^4$
Q_{2s}	0	0	1	$2U$	$3U^2+V^2+W_-^2-W_+^2$	$4U(U^2+V^2)+2(W_-^2-W_+^2)(U+W_+)$
R_{2s}	0	0	0	1	$2(U+W_+)$	$3U^2+V^2+W_-^2+4UW_++3W_+^2$

Classical Compton

General Analysis



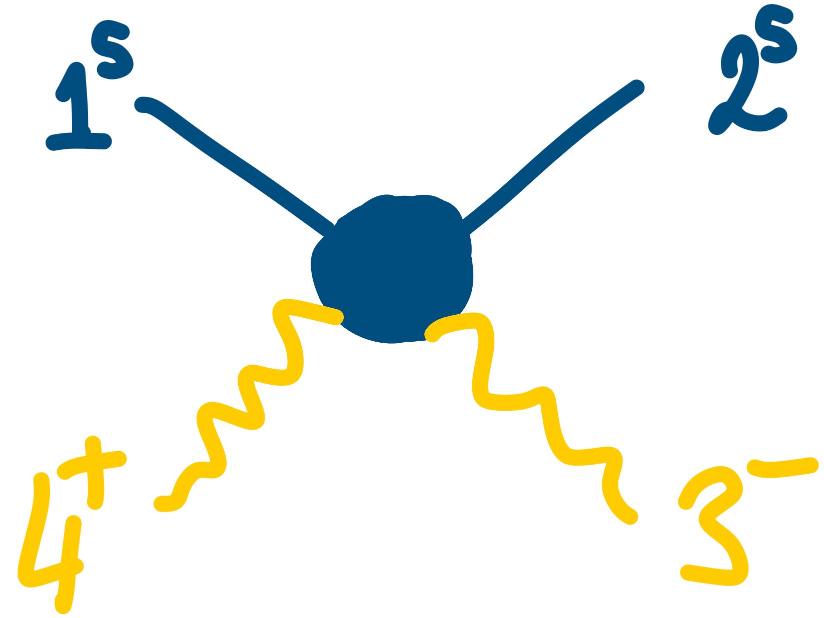
- finite spin analysis
  - relate two massive momenta $p_2 = \Lambda p_1 = p_1 + q$
  - soft massless momenta $k_3, k_4 \sim \mathcal{O}(\hbar)$ and $a \sim \mathcal{O}\left(\hbar^{-1}\right)$
  - simultaneously take $\begin{cases} \hbar \to 0 \\ s \to \infty \end{cases}$ while keeping $\hbar s \sim 1$
- we identify 'nice' variables

$$
\begin{aligned}
q^\mu &= (k_3 + k_4)^\mu & x &= -q_\perp \cdot a & z &= |a| p \cdot q_\perp \\
q_\perp^\mu &= (k_3 - k_4)^\mu & y &= -q \cdot a & w &= \frac{\chi \cdot a}{\chi \cdot p_1} p \cdot q_\perp
\end{aligned}
$$

- Coherent spin formalism *[Aoude, Ochirov]*

Classical Compton

Example Term



- First pole term in $\mathcal{A}(\mathbf{1}^s, \mathbf{2}^s, 3^-, 4^+)$:
    - scalar amp $\times$ $(U+V)^{2s}$
    - in spinors: $U + V = -\langle \mathbf{1}|3|\mathbf{2}] - m\,\langle \mathbf{12}\rangle$

$$(U+V)^{2s} \approx (1 + q_\perp \cdot \hat{a})^{2s} = \sum_{k=0}^{2s} \binom{2s}{k} \langle q_\perp \cdot \hat{a}\rangle^k$$

Classical Compton

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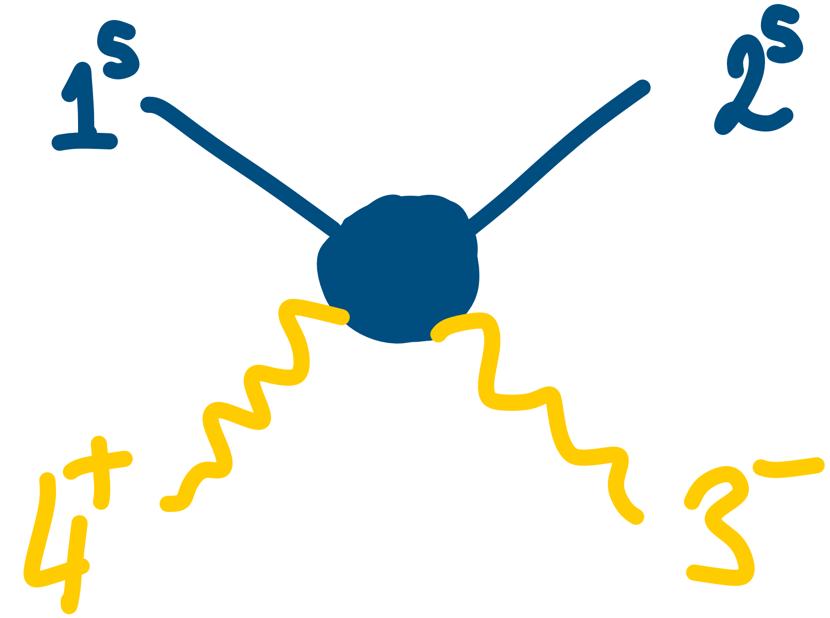

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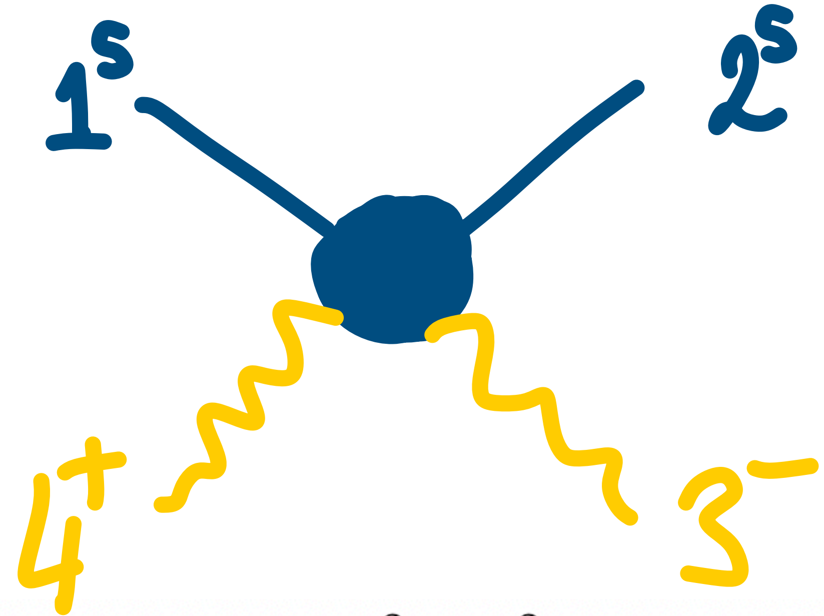
$$= \sum_{k=0}^{2s} \frac{\langle (q_\perp \cdot \hat{a})^k \rangle}{k!} + \mathcal{O}(a^2)$$

$$= \left\langle e^{q_\perp \cdot a} \right\rangle + \mathcal{O}(a^2)$$

first term in known four pt amplitude

Classical Compton

Gauge Theory Analysis



$$\mathcal{A}(\mathbf{1},\mathbf{2},3^-,4^+) = -(p\cdot\chi)^2\left(\frac{[T^{c_3},T^{c_4}]}{q^2\,p{\cdot}q_\perp} + \frac{\{T^{c_3},T^{c_4}\}}{(p{\cdot}q_\perp)^2}\right)\left(e^x\cosh z - w\,e^x\text{sinhc}\,z + \frac{w^2-z^2}{2}E(x,y,z)\right)$$

$$+\,(p\cdot\chi)^2\frac{[T^{c_3},T^{c_4}]}{q^2\,(p{\cdot}q_\perp)^2}\frac{w^2-z^2}{2}i\epsilon(q_\perp,p,q,a)\mathcal{E}(x,y,z)\quad+\text{contacts}$$

- E and $\mathcal{E}$ are **entire functions** in x, y, z
- amplitude is **spurious pole free**
- second line **breaks BCJ** relations
- **no contact terms** expected for non-Ab. sector

$$x = -q_\perp{\cdot}a \qquad z = |a|p{\cdot}q_\perp$$

$$y = -q{\cdot}a \qquad w = \frac{\chi{\cdot}a}{\chi{\cdot}p_1}p{\cdot}q_\perp$$

$$E(x,y,z) = \frac{e^y - e^x\cosh z + (x-y)e^x\,\text{sinhc}\,z}{(x-y)^2 - z^2} + (y\to -y)$$

$$\frac{\partial}{\partial\lambda}\lambda^3\mathcal{E}(\lambda x,\lambda y,\lambda z)\Big|_{\lambda=1} = E(x,y,z)$$

Our proposal:

Higher-spin gauge symmetry is a central guiding principle for constraining Kerr dynamics

Checks:

- reproduce known
  - 3pt amplitudes for Kerr & root-Kerr
  - 4pt amplitudes for root-Kerr s<2
- generate non-trivial 4pt constraints
- predict root-Kerr Compton for s=2

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In progress:

- extending analysis to Compton Kerr
- analysis of classical limit
- comparison to Teukolsky eqn (BH-PT)

Future Directions

- implications for quantum BHs include absorption and emission