Binary dynamics of spinning black holes in the Post-Minkowskian regime

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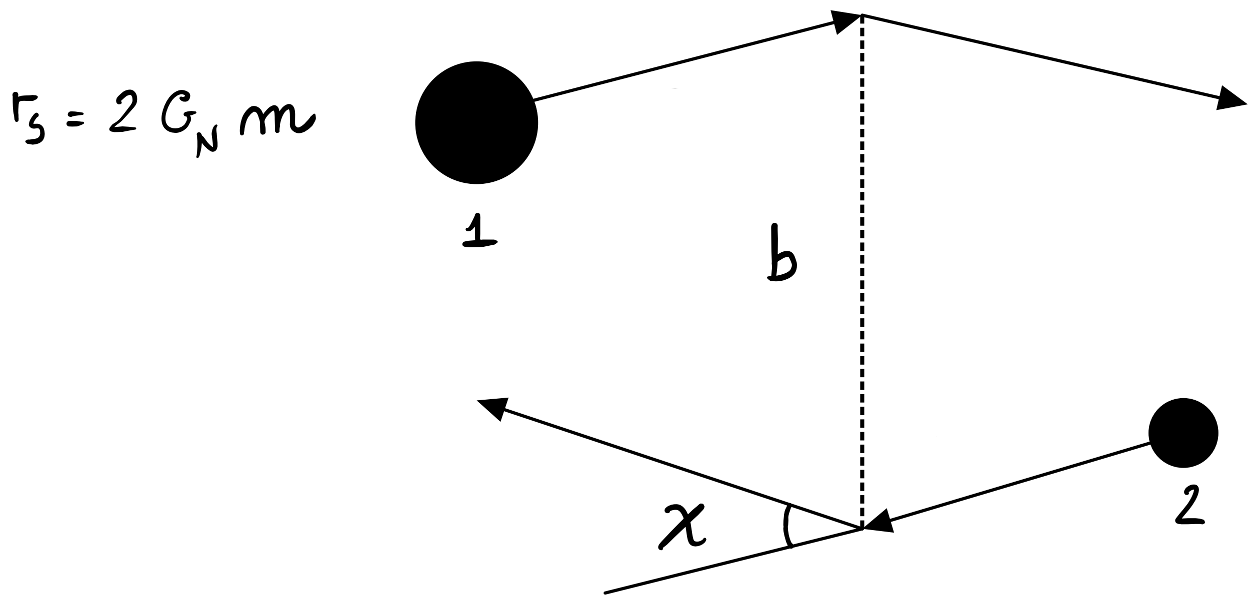
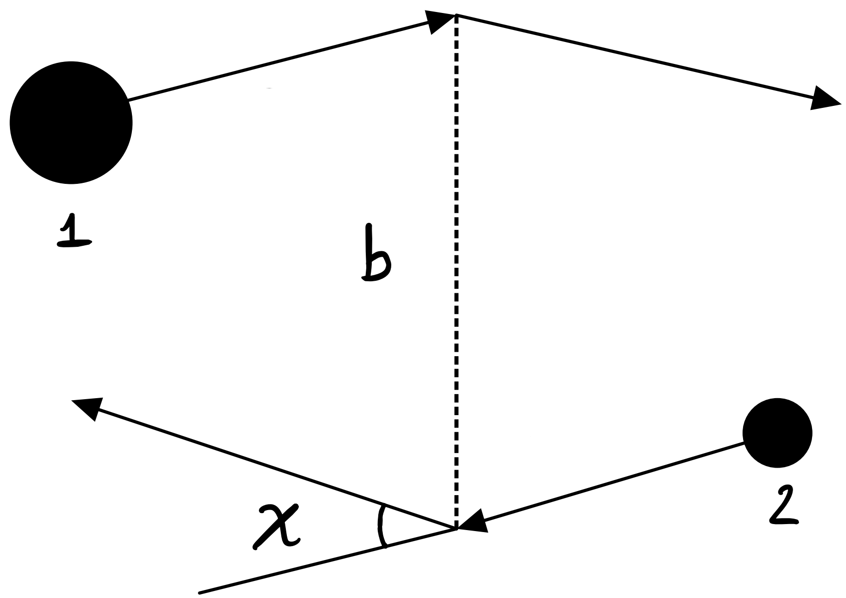
Black-hole scattering in the PM regime:



PM regime: $\quad b \gg r_s \gg \frac{\hbar}{m}$ classical limit

Black-hole scattering in the PM regime:

$r_s = 2\, G_N\, m$



PM regime: $\quad b \gg r_s \gg \dfrac{\hbar}{m}$ $\quad$ classical limit

- Black holes are treated as classical point-particles:

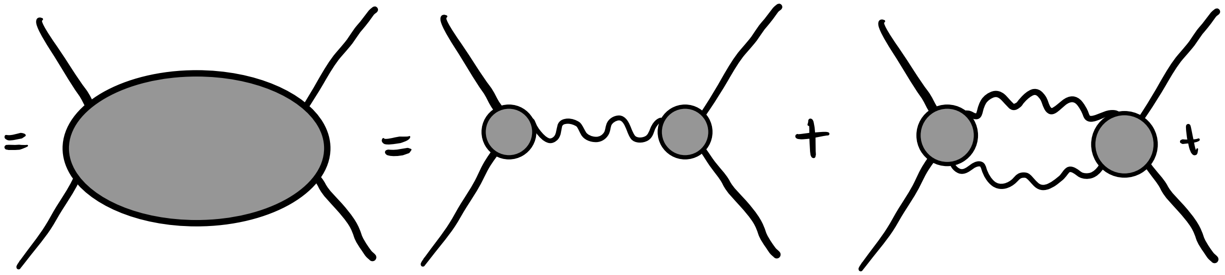
$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2\kappa_N^2} - \frac{1}{2}\sum_{\alpha=1,2}\left(g^{\mu\nu}\partial_\mu\phi_\alpha\,\partial_\nu\phi_\alpha + m_\alpha^2\,\phi_\alpha^2\right)\right]$$

[Bjerrum-Bohr, Donoghue, Vanhove, Damgaard, Holstein, Festuccia, Planté, Bern, Cheung, Roiban, Shen, Solon, Zeng, Herman Parra-Martinez, Di Vecchia, Heissenberg, Russo, Veneziano, Collado, Cristofoli, Kosower, Maybe, O'Connell, ...]

Perturbative expansion of observables

$1PM \sim \mathcal{O}(G_N)$ $\qquad$ $2PM \sim \mathcal{O}(G_N^2)$

$$\mathcal{M}(q) =$$



$$+ \dots = \sum_{\ell=0}^{\infty} G_N^{\ell+1} \mathcal{M}^{(\ell)}(q)$$

Perturbative expansion of observables

1PM $\sim O(G_N)$ 2PM $\sim O(G_N^2)$

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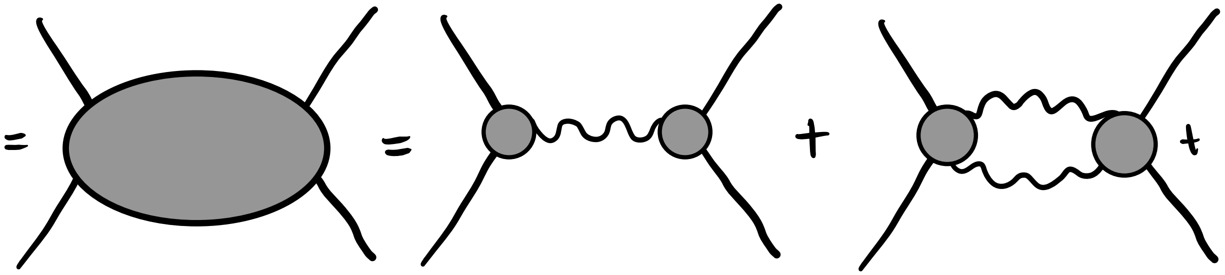


$$+ \ldots = \sum_{\ell=0}^{\infty} G_N^{\ell+1} \mathcal{M}^{(\ell)}(\mathbf{q})$$

Classical observables are encoded in the eikonal phase

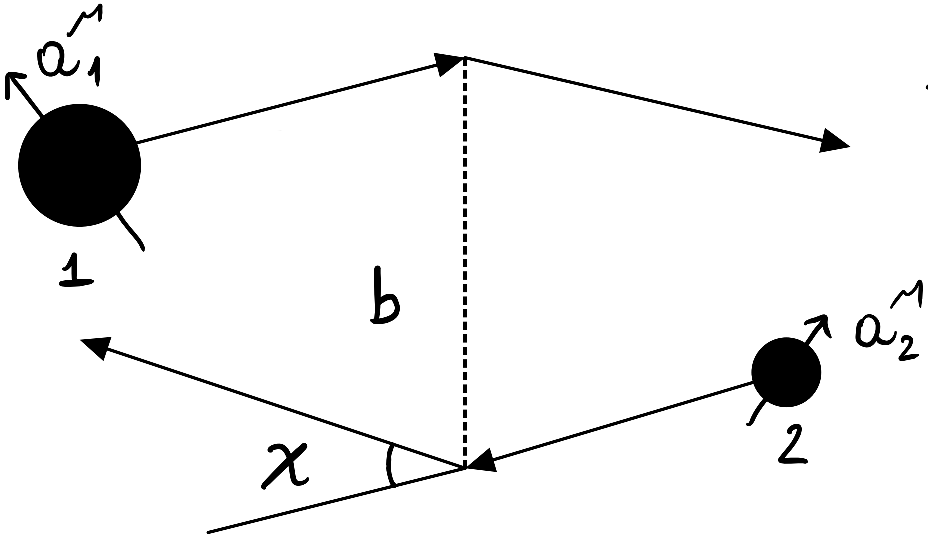
$$\tilde{\mathcal{M}}(\mathbf{b}) = \frac{1}{4E|P|}\int d^2\mathbf{q}\, \mathcal{M}(\mathbf{q})\, e^{i\mathbf{q}\cdot\mathbf{b}} \underset{\hbar\to 0}{\sim} i\left(1 - e^{2i\delta(\mathbf{b})}\right)$$

$$\Downarrow$$

$$\delta(\mathbf{b}) = \sum_{\ell=0}^{\infty} G_N^{\ell+1}\, \delta^{(\ell)}(\mathbf{b})$$

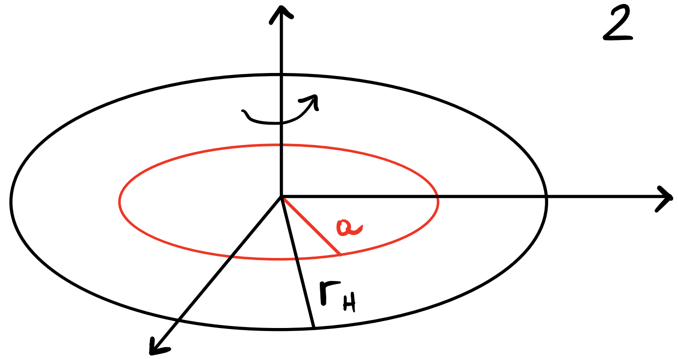
Observables are expanded as $\theta = G_N \theta^{(1)} + G_N^2 \theta^{(2)} + \ldots$ (e.g. χ, T)

Kerr black-hole scattering in the PM regime:



ring radius: $a^\mu = \dfrac{S^\mu}{m}$

event horizon: $r_H = \dfrac{r_s + \sqrt{r_s^2 - 4a^2}}{2}$



PM regime: $b \gg r_H \gg \dfrac{\hbar}{m}$ classical limit

↓

Spin multipoles: $b \gg a \gg \dfrac{\hbar}{m}$ classical limit

- From quantum mechanics: $S^2 = S_\mu S^\mu = \underset{\to 0}{\hbar^2}\, \underset{\to\infty}{s}(s+1)$

- Kerr black holes: $\lim_{s\to\infty}$ (massive s fields)

[Arkani-Hamed, Huang, Huang, Bern, Luna, Roiban, Sen, Zeng, Guevara, Ochirov, Vines, Aoude, Haddad, Helset, Chung, Kim, Chen, Chiodaroli, Pichini Johansson, Jakobsen, Mogull, Plefka, Steinhoff, ...]

Perturbative expansion of observables

Classical observable $\mathcal{O}$:

$$\mathcal{O} = G_N \left[1 + c_1^{(1)} \frac{a}{b} + c_2^{(1)} \left(\frac{a}{b}\right)^2 + \dots \right] \mathcal{O}^{(1)} + G_N^2 \left[1 + c_1^{(2)} \frac{a}{b} + c_2^{(2)} \left(\frac{a}{b}\right)^2 + \dots \right] \mathcal{O}^{(2)} + \dots$$

1PM Spin multipoles

2PM Spin multipoles

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1 PM Spin multipoles 2 PM Spin multipoles

Finite spin s : $\sum_{n=0}^{2s} c_n^{(i)} \left(\frac{a}{b}\right)^n$ Infinite spin $\sum_{n=0}^{\infty} c_n^{(i)} \left(\frac{a}{b}\right)^n$

Perturbative expansion of observables

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Classical transfer momentum/spin kick :

- $$Q^\mu = - \frac{\partial}{\partial b_\mu} \mathcal{M}_4(a,b)$$

- $$\Delta a_a^\mu = \frac{1}{m_a^2} \left(p_a^\mu a_a^\nu \frac{\partial}{\partial b^\nu} - \epsilon^{\mu\nu\rho\sigma} p_{a\nu} a_{a\rho} \frac{\partial}{\partial a_a^\sigma} \right) \mathcal{M}_4(a,b)$$

Q: Is it possible to describe Kerr black holes with an effective Lagrangian depending on the classical spin a^μ and therefore carrying, as a built-in feature, an infinite spin-multipole expansion?

M: Purely theoretical interest. Simplicity of BHs:

Black holes $\longleftrightarrow$ elementary particles

- Schwarzschild
- Kerr

- minimally coupled scalars
- minimal coupling?

What can GR teach us about QFT?

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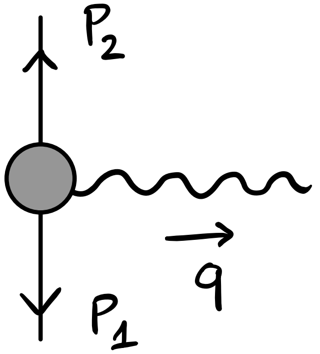
What can GR teach us about QFT?

M: A simple Lagrangian would allow to increase the precision in analytical computations (e.g. nPM $n>1$)

3-point amplitudes

- $P = P_1 - P_2$
- $\Lambda^{\rho}{}_{\mu}(a,q) = \exp\{ i \epsilon^{\rho}{}_{\mu\alpha\beta}\, a^{\alpha} q^{\beta} \}$

$a^\mu = 0$



$$= \begin{cases} i g P^{\mu} \\ \kappa_N P^{(\mu} P^{\nu)} \end{cases}$$

Gauge theory (scalar QCD)

Gravity (Schwarzschild)

3-point amplitudes

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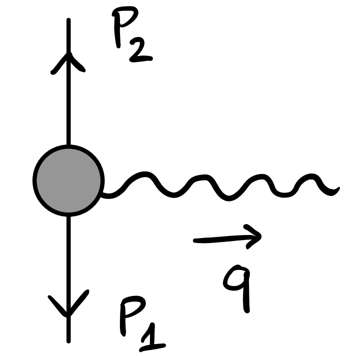
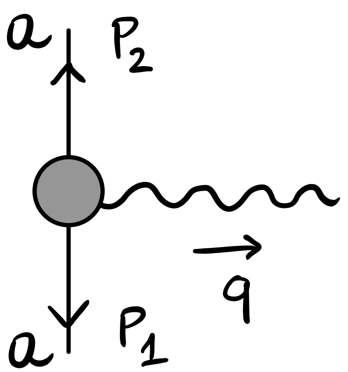


$$= \begin{cases} i\,g\,p^\mu & \text{Gauge theory (scalar QCD)} \\ \kappa_N\, p^{(\mu} p^{\nu)} & \text{Gravity (Schwarzschild)} \end{cases}$$

$a^\mu \neq 0$



$$= \begin{cases} i\,g\,\Lambda^{\mu\sigma}(a,q)\,p_\sigma & \text{Gauge theory } (\sqrt{\text{Kerr}}) \\ \kappa_N\, p_\sigma \Lambda^{\sigma(\mu}(a,q)\, p^{\nu)} & \text{Gravity (Kerr)} \quad \text{[Vines, 2017]} \end{cases}$$

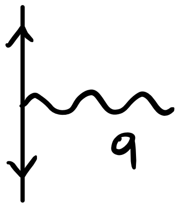
"Minimal coupling" [Arkani-Hamed, Huang, Huang, 2017]

Higher point are not known in full generality!

Minimal coupling with classical spin [F.A.]

$$\mathcal{L}_{free} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\alpha=1,2} (\partial_\mu \phi_\alpha)^* (\partial^\mu \phi_\alpha) - m_\alpha^2 \phi_\alpha^* \phi_\alpha$$

- Assuming minimal coupling (QFT-like)



$$= i g \Lambda^{\mu\sigma}(a,q) P_\sigma \implies D_\mu^{(a)} = \partial_\mu - i g \exp\left\{\epsilon_{\mu\sigma\alpha\beta} a^\alpha \partial^\beta\right\} A^\sigma$$

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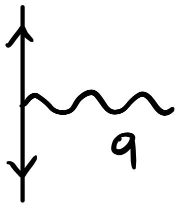
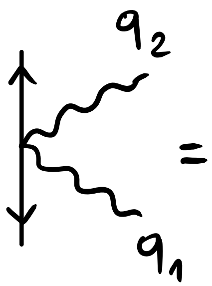


$$= i g \Lambda^{\mu\sigma}(a,q) P_\sigma \implies D_\mu^{(a)} = \partial_\mu - i g \exp\{\epsilon_{\mu\sigma\alpha\beta} a^\alpha \partial^\beta\} A^\sigma$$

- It is a "good" covariant derivative: $D_\mu^{(a)} \phi_\alpha \longrightarrow \mathcal{U} D_\mu^{(a)} \phi_\alpha$!!!

$D_\mu^{(a)} \equiv$ classical spin connection $\rightarrow \mathcal{L}^{(a)} = \mathcal{L}_{free}(\partial_\mu \longrightarrow D_\mu^{(a)})$ [F.A.]

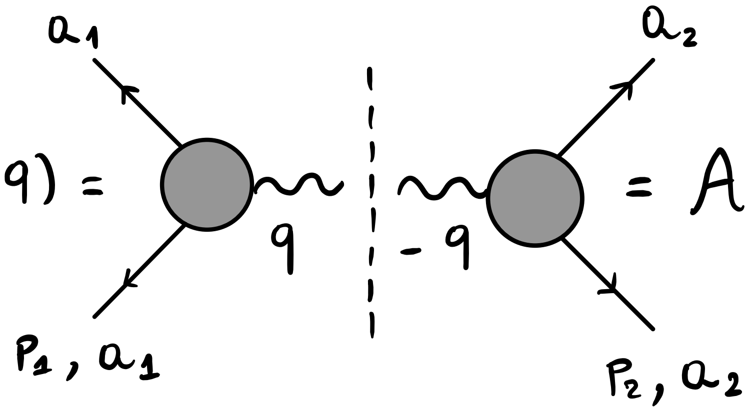


$$= i g^2 \Lambda^\nu{}_\rho (a, q_1) \Lambda^{\mu\rho}(a, q_2)$$

Contact terms are generated by the same mechanism that selects them in the spinless case. Schwarzschild is minimal.

Q : Are Kerr black-holes minimal?

Kerr three-point and tree-level (1PM)

$A(a,q) =$ 

$= A^3_\mu(a,q)\,\Pi^{\mu\nu} A^3_\nu(a,-q)$

- Double copy: $\mathrm{Kerr}(a) = \sqrt{\mathrm{Kerr}}(a) \otimes \sqrt{\mathrm{Kerr}}(a=0)$

$$\tilde{M}(a,q) \sim q^2 A(a,q) A(q) = M(a,q) + M_a(a,q) + M_\phi(a,q)$$

$$M(a,q) = \frac{\kappa_N^2}{q^2} m_1^2 m_2^2 \sigma^2 \sum_{\pm} (1 \pm v)^2 \exp\left\{ \pm i \frac{\epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu a^\rho q^\sigma}{m_1 m_2 \sigma v} \right\}$$

[Guevara, Ochirov, Vines, 2018]

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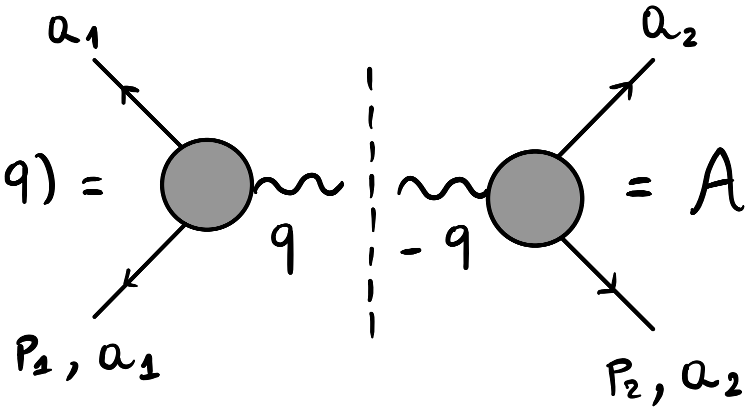
$$A(a,q) =$$ 

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[Guevara, Ochirov, Vines, 2018]

- Observables $\rightarrow$ eikonal

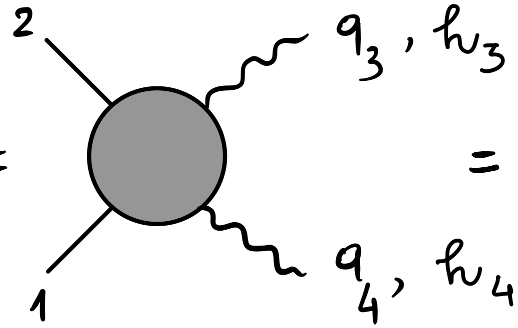
$$\tilde{\mathcal{M}}(a,b) = \frac{1}{4E|p|}\int \frac{d^{2-2\epsilon}q}{(2\pi)^{2-2\epsilon}} \mathcal{M}(q)\, e^{iq\cdot b} = i\left(1 - e^{2i\delta(b)}\right)$$

$$\propto \sum_{\pm} \frac{1}{|b \pm ia|^{-2\epsilon}}$$

Newman-Janis shift

Kerr Compton amplitude & 1-loop amplitude (2PM)

$$\mathcal{M}^{(a)}_{(h_3,h_4)} =$$



$$= e^{-a\cdot(h_3 q_3 + h_4 q_4)}\,\mathcal{M}^{(a=0)}_{(h_3,h_4)}$$

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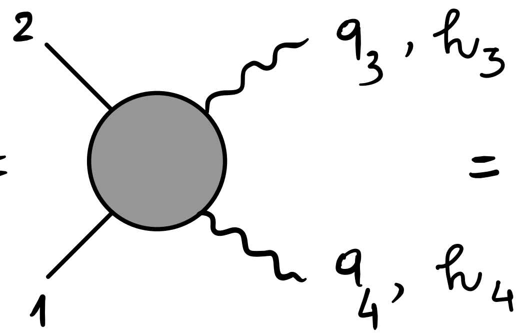
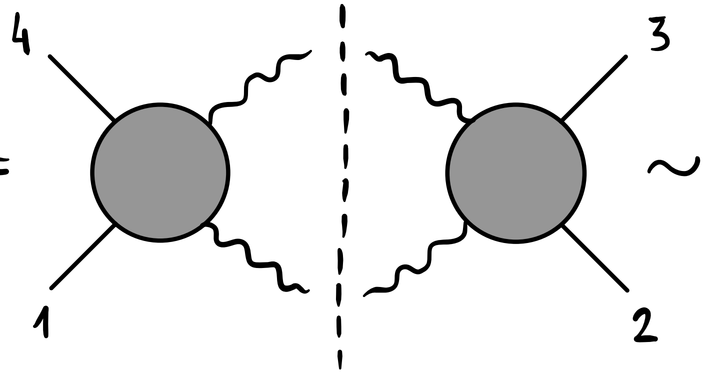


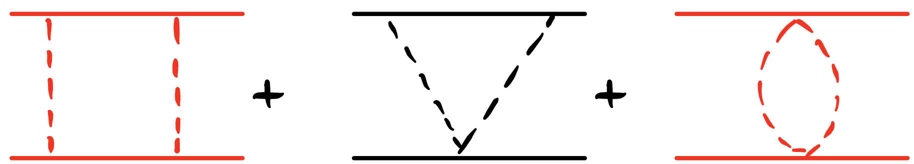
$$= e^{-a\cdot(h_3 q_3 + h_4 q_4)}\, \mathcal{M}^{(a=0)}_{(h_3,h_4)}$$

$$\Downarrow$$

$$\mathcal{M}_{1\text{-loop}}(a) =$$



$$\sim$$



$$= \kappa_N^4 \Big[c_\triangleright I_\triangleright(a) + (c_\triangleright)_{\mu\nu} I^{\mu\nu}_\triangleright(a) + (d_\triangleright)_\mu J^\mu_\triangleright(a) + (m_1 \leftrightarrow m_2) \Big] \quad [\text{F. A.}]$$

$$I^{\mu_1\cdots\mu_n}_\triangleright(a) = \int \frac{d^4\ell}{(2\pi)^4} \frac{\ell^{\mu_1}\cdots\ell^{\mu_n}\cosh(2a\cdot\ell + a\cdot q)}{\ell^2 (\ell+q)^2[(\ell+P_1)^2 - m_1^2]} \qquad J^{\mu_1\cdots\mu_n}_\triangleright(a) = \int \frac{d^4\ell}{(2\pi)^4} \frac{\ell^{\mu_1}\cdots\ell^{\mu_n}\sinh(2a\cdot\ell + a\cdot q)}{\ell^2 (\ell+q)^2[(\ell+P_1)^2 - m_1^2]}$$

Radiation reaction (RR) effects

- At 3PM the eikonal develops an imaginary part:

$$2\delta_2(b) = \mathrm{Re}\, 2\delta_2(b) + i\, \mathrm{Im}\, 2\delta_2(b)$$

  - $\mathrm{Re}\, 2\delta_2(b) = 2\delta_2^{\mathrm{con}}(b) + 2\delta_2^{\mathrm{rr}}(b)$

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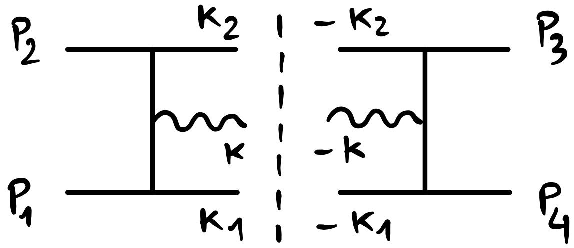

$$= \frac{1}{2}\int_{k,k_1,k_2} \left|\tilde{M}_5\right|^2 \overset{\epsilon\to 0}{=} \frac{1}{2}\int_{k,k_1,k_2} \left| \kappa_N \sum_{i=1}^{4} \frac{p_i^\mu p_i^\nu}{p_i\cdot k}\, \tilde{M} \right|^2$$

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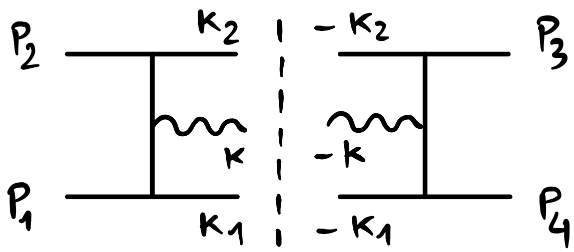
$\mathrm{Im}\, 2\delta_2(b) =$ 

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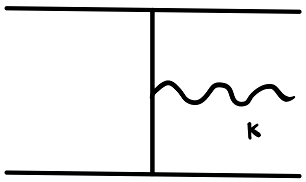
- By knowing the 1PM amplitude we can compute $2\delta_2^{rr}(b) \Rightarrow \chi_2^{rr}(b)$

$$\chi_2^{rr}(b) = -\frac{1}{p}\frac{\partial\, \mathrm{Re}\, 2\delta_2^{rr}(b)}{\partial b}$$

Radiation reaction (RR) effects for Kerr

1) Use Weinberg soft graviton theorem:

$$\mathcal{M}_5^{\mu\nu}(a,q,k) =$$



$$\overset{k \to 0}{=} \kappa_N \sum_{i=1}^{4} \frac{p_i^\mu p_i^\nu}{p_i \cdot k} \mathcal{M}(a,q)$$

with:

$$\mathcal{M}(a,q) = \frac{\kappa_N^2}{q^2} m_1^2 m_2^2 \sigma^2 \sum_{\pm} (1 \pm v)^2 \exp\left\{ \pm i \, \frac{\epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu a^\rho q^\sigma}{m_1 m_2 \sigma v} \right\}$$

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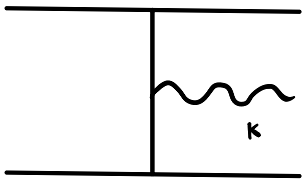

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2) Fourier transform:

$$\tilde{\mathcal{M}}_5^{\mu\nu}(a,b,k) \overset{k\to 0}{=} \sum_{\pm} \int \frac{d^{2-2\epsilon} q}{(2\pi)^{2-2\epsilon}} \left(\dots \right)^{\mu\nu} \frac{e^{i q \cdot \left(\underbrace{b \pm i a}_{b_{\pm}}\right)}}{q^2}$$

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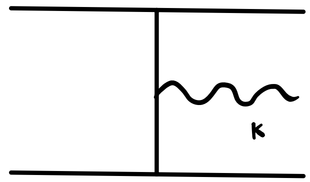
$$\mathcal{M}_5^{\mu\nu}(a,q,k) =$$  $$\overset{k\to 0}{=} \kappa_N \sum_{i=1}^{4} \frac{p_i^\mu p_i^\nu}{p_i \cdot k} \mathcal{M}(a,q)$$

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$$\mathcal{M}(a,q) = \frac{\kappa_N^2}{q^2} m_1^2 m_2^2 \sigma^2 \sum_{\pm} (1 \pm v)^2 \exp\left\{ \pm i \frac{\epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu a^\rho q^\sigma}{m_1 m_2 \sigma v} \right\}$$

2) Fourier transform:

$$\tilde{\mathcal{M}}_5^{\mu\nu}(a,b,k) \overset{k\to 0}{=} \sum_{\pm} \int \frac{d^{2-2\epsilon} q}{(2\pi)^{2-2\epsilon}} (\dots)^{\mu\nu} \frac{e^{i q\cdot \overbrace{(b \pm i a)}^{b_\pm}}}{q^2}$$

3) Real analyticity, crossing symmetry and unitarity:

$$2\delta_2^{rr}(b) = -\frac{\pi}{2} \lim_{\epsilon \to 0} \epsilon \int_{k,k_1,k_2} |\tilde{\mathcal{M}}_5|^2 = 2\delta_2^{rr}(b)\Big|_{a=0} f^2 , \quad f^\mu = \frac{\sigma^2 b}{2(2\sigma^2-1)} \sum_{\pm} \frac{(1\pm v)^2 b_\pm^\mu}{b_\pm^2}$$

Radiation reaction observables

- $$\chi_2^{rr}(b) = \chi_2^{rr}(b)\Big|_{a=0} \frac{\left(1 + \frac{2\sigma\sqrt{\sigma^2-1}}{2\sigma^2-1}\frac{a}{b}\right)\left[1 + \frac{4\sigma\sqrt{\sigma^2-1}}{2\sigma^2-1}\frac{a}{b} + \left(\frac{a}{b}\right)^2\right]}{\left[1-\frac{a}{b}\right]^3}$$

[F.A. Di Vecchia 2203.13272]

$$= \chi_2^{rr}(b)\Big|_{a=0}\left[1 + \frac{6\sigma\sqrt{\sigma^2-1}}{2\sigma^2-1}\frac{a}{b} + \left(\frac{a}{b}\right)^2 + O(a^3)\right]$$

[Jakobsen, Mogull 2201.07778]

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"San Gennaro's miracle"

Kerr scattering from string theory

Q : "Where is" Kerr in string theory?

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M : (super) String $\supset$ Gravity $\supset$ Kerr;

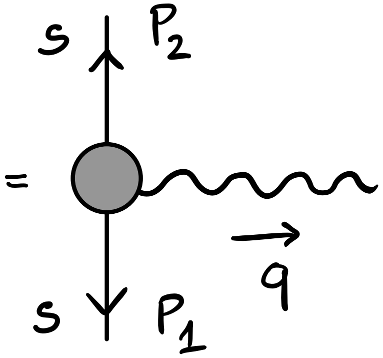
M : Strings are consistent; Unique n- point amplitudes;

M : Strings contain naturally an infinite tower of massive higher spins.

3-point amplitudes

- leading Regge Trajectory : $\alpha' m^2 = s - 1$

$$A_3^{(\text{string})}(s) \sim \langle 0 | \hat{V}_s(p_1)\, \hat{V}_{\text{gluon}}(q)\, \hat{V}_s(\hat{p}_2) | 0 \rangle =$$



[Schlotterer]

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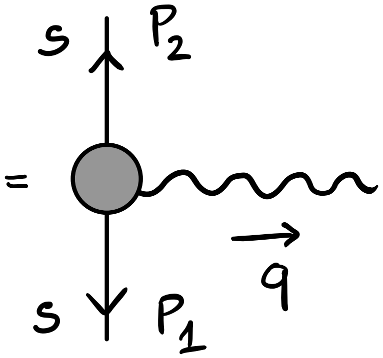

[Schlotterer]

$$\Big\downarrow \quad \text{classical limit } \; s \to \infty$$

- $A_3^{(\text{string})}(a) \sim \varepsilon_{(h)} \cdot p \left[I_0(2a\cdot q) + I_1(2a\cdot q) \right] \neq A_3^{(\sqrt{\text{Kerr}})}(a)$

- $M_3^{(\text{string})}(a) \sim \left(A_3^{(\text{string})}(a) \right)^2 \neq M_3^{(\text{Kerr})}(a)$

[Cangemi- Pichini]

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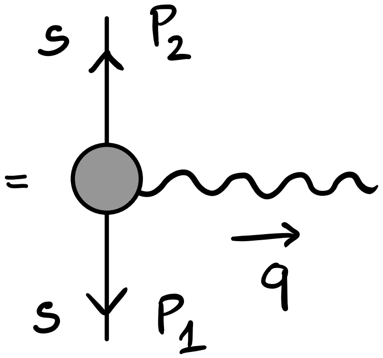

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Note that $A_3^{(\mathrm{string})}(a)$ and $M_3^{(\mathrm{string})}(a)$ used are α' exact!

4- point amplitudes

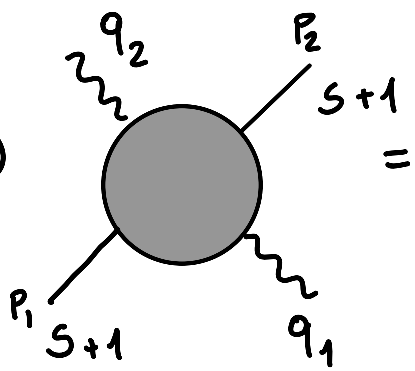
- Higher spin vertices in open superstring:

$$\hat{V}^{\nu_1\dots\nu_n}_{s+1}(p_1) = \frac{e^{-\phi}}{\sqrt{s!}}\, i\,\varepsilon(p_1)\cdot\psi\,\big(i\,\varepsilon(p_1)\cdot\partial_z X\big)^s \underbrace{\big(\partial_z X^{\nu_1}\cdots\partial_z X^{\nu_n}\big)} e^{i\sqrt{2\alpha'}\,p_1\cdot X}$$

$\nu_1\dots\nu_n$ are indices along compact dimensions

Mass on-shell : $-p_1^2 = m^2 = \dfrac{s+n}{\alpha'}$ (spin 1 (s=0) is massive)

- QED Compton :

$A_4^{(\text{string})}(s+1)$ 

$= \langle 0|\,\hat{V}_{s+1}(p_1)\,\hat{V}_{\text{gluon}}(q_1)\,\hat{V}_{s+1}(p_2)\,\hat{V}_{\text{gluon}}(q_2)\,|0\rangle$

- spin 0 $A_4^{(0)} = - \frac{p_1 \cdot F_1 \cdot F_2 \cdot p_1}{p_1 \cdot q_1 \; p_1 \cdot q_2}$ $\qquad F_i^{\mu\nu} = \varepsilon_i^\mu q_i^\nu - \varepsilon_i^\nu q_i^\mu$

- spin 0 $A_4^{(0)} = -\frac{p_1 \cdot F_1 \cdot F_2 \cdot p_1}{p_1 \cdot q_1 \ p_1 \cdot q_2}$ $\qquad F_i^{\mu\nu} = \varepsilon_i^\mu q_i^\nu - \varepsilon_i^\nu q_i^\mu$

spin 1 $A_4^{(1)} = (\tilde{\varepsilon}_1 \cdot \tilde{\varepsilon}_2) A_4^{(0)} - \Big(p_1 \cdot F_1 \cdot p_2 \ \tilde{\varepsilon}_1 \cdot F_2 \cdot \tilde{\varepsilon}_2 + p_1 \cdot F_2 \cdot p_1 \ \tilde{\varepsilon}_1 \cdot F_1 \cdot \tilde{\varepsilon}_2$

$$+ p_1 \cdot q_2 \ \tilde{\varepsilon}_1 \cdot F_1 \cdot F_2 \cdot \tilde{\varepsilon}_2 + p_1 \cdot q_1 \ \tilde{\varepsilon}_1 \cdot F_2 \cdot F_1 \cdot \tilde{\varepsilon}_2 \Big) \frac{1}{p_1 \cdot q_1 \ p_1 \cdot q_2}$$

- Matches [A-H, H, H] up to a^2! For $s > 1$?

- spin 0 $\quad A_4^{(0)} = - \dfrac{p_1 \cdot F_1 \cdot F_2 \cdot p_1}{p_1 \cdot q_1 \; p_1 \cdot q_2}$ $\qquad F_i^{\mu\nu} = \varepsilon_i^\mu q_i^\nu - \varepsilon_i^\nu q_i^\mu$

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$$+ p_1 \cdot q_2 \; \tilde{\varepsilon}_1 \cdot F_1 \cdot F_2 \cdot \tilde{\varepsilon}_2 + p_1 \cdot q_1 \; \tilde{\varepsilon}_1 \cdot F_2 \cdot F_1 \cdot \tilde{\varepsilon}_2 \Big) \frac{1}{p_1 \cdot q_1 \; p_1 \cdot q_2}$$

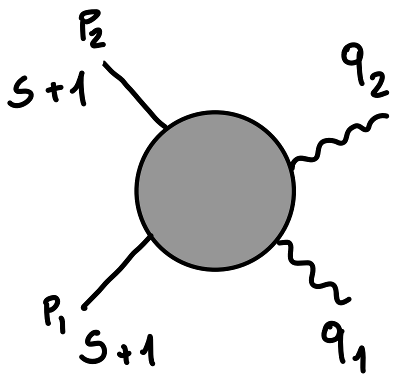
- Matches [A-H, H, H] up to a^2! For $s > 1$?

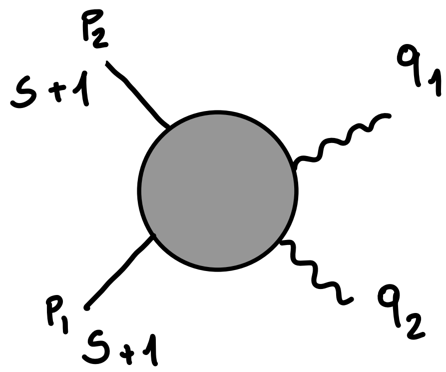
- spin 2 $\quad A_4^{(2)} = (\tilde{\varepsilon}_1 \cdot \tilde{\varepsilon}_2) A_4^{(1)} - 2 \dfrac{p_1 \cdot q}{p_1 \cdot q_1 \; p_1 \cdot q_2} \; \tilde{\varepsilon}_1 \cdot F_1 \cdot \tilde{\varepsilon}_2 \; \tilde{\varepsilon}_1 \cdot F_2 \cdot \tilde{\varepsilon}_2$

$$- \frac{(\tilde{\varepsilon}_1 \cdot \tilde{\varepsilon}_2)}{p_1 \cdot q_1 \; p_1 \cdot q_2} \Big(p_1 \cdot F_1 \cdot p_2 \; \tilde{\varepsilon}_1 \cdot F_2 \cdot \tilde{\varepsilon}_2 + p_1 \cdot F_2 \cdot p_1 \; \tilde{\varepsilon}_1 \cdot F_1 \cdot \tilde{\varepsilon}_2$$

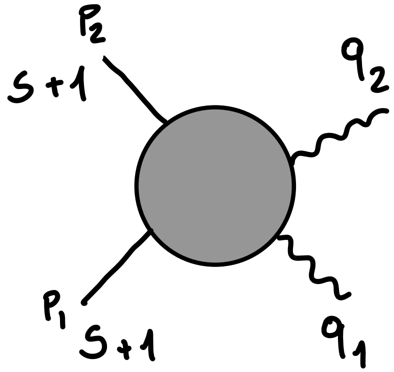
$$+ p_1 \cdot q_2 \; \tilde{\varepsilon}_1 \cdot F_1 \cdot F_2 \cdot \tilde{\varepsilon}_2 + p_1 \cdot q_1 \; \tilde{\varepsilon}_1 \cdot F_2 \cdot F_1 \cdot \tilde{\varepsilon}_2 \Big)$$

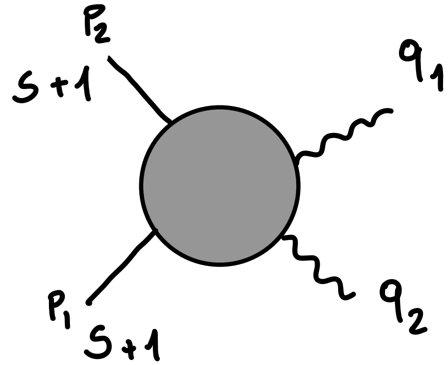
OPEN QUESTIONS

- Compute also 

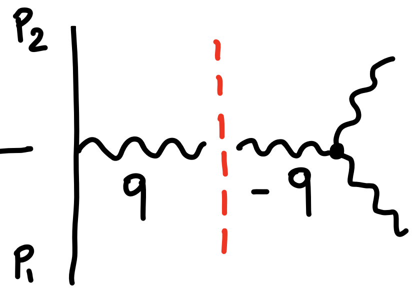
and 

OPEN QUESTIONS

- Compute also 

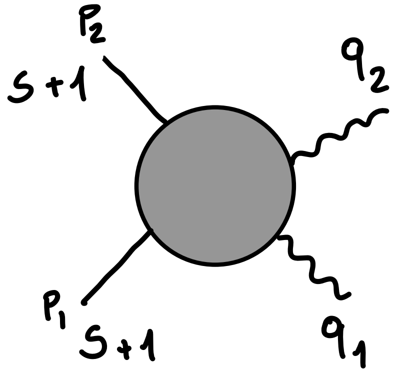
and 

massles pole ($q \to 0$)

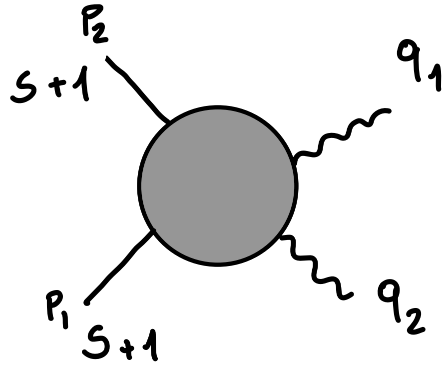
read the 3 point ← 

OPEN QUESTIONS

- Compute also

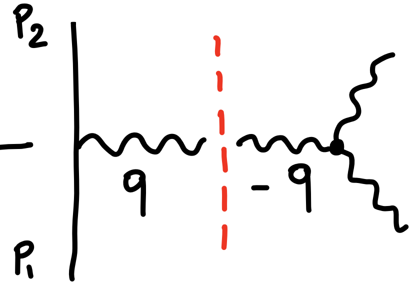


and



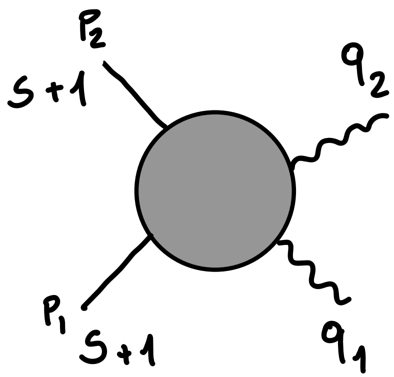
$\downarrow$ massless pole ($q \to 0$)

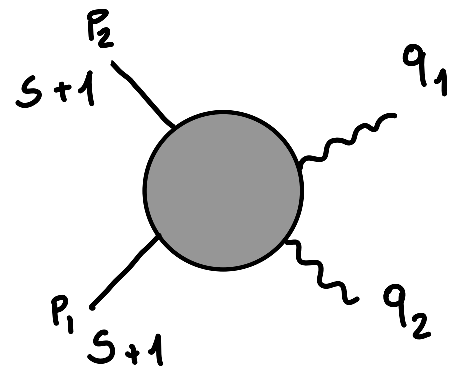
read the 3 point $\leftarrow$



- Double copy $\to$ Kerr amplitudes

(SOME) OPEN QUESTIONS

- Compute also 

and 

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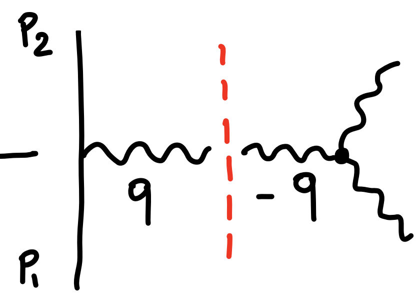
read the 3 point ← 

- Double copy → Kerr amplitudes

- Take $s \to \infty$