Computing Yukawa Couplings from String Theory

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based on: arXiv:2402.01615, arXiv:2507.03076, arXiv:2508.nnnn

in collaboration with: Andrei Constantin, Kit Fraser-Taliente, Thomas Harvey, Lucas Leung, Luca Nutricati, Burt Ovrut

Outline

- Introduction: Yukawa couplings - why are they hard to compute?
- The tool: learning solutions to differential equations
- A heterotic line bundle example model
- Results
- Heterotic Froggatt-Nielsen models
- Conclusion

Introduction

Standard models from string theory

- It is now possible to engineer (many) string models with the exact (MS)SM spectrum.

- Models are based on heterotic CY compactifications with general vector bundles.

- Only additional fields are SM singlets (= moduli)

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Low-energy Yukawa couplings

Matter fields: C^I

Moduli fields: ϕ^A

Superpotential:

$$W = Y_{IJK}(\phi) C^I C^J C^K + \cdots$$

Kahler potential:

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For example, up-quark Yukawa couplings:

$$W = Y^u_{ij}(\phi) H^u Q^i U^j + \cdots$$

$$K = K^Q_{ij}(\phi, \bar{\phi}) Q^i \bar{Q}^j + K^u_{ij}(\phi, \bar{\phi}) U^i \bar{U}^j + k(\phi, \bar{\phi}) H^u \bar{H}^u + \cdots$$

Heterotic CY models

Compactification data:

- CY three-fold X with freely acting symmetry group Γ
- `Observable' vector bundle $V \to X$, Γ - equivariant, poly-stable, with structure group $SU(5)$, $S(U(1)^5)$, . . .
- `Hidden' vector bundle $\tilde{V} \to X$, Γ - equivariant, poly-stable, with structure group that embeds into E_8

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Outcome (for appropriate choices): MSSM + moduli + hidden sectors

Specific model classes

- `Standard embedding' models: $V = TX$, $\tilde{V}$ trivial, no 5-branes
  -> analytic formulae for physical Yukawa coupling,
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focus for this talk:

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Calculating Yukawa couplings

matter fields $\leftrightarrow$ bundle-valued harmonic forms 1-forms

$$C^I \qquad \leftrightarrow \quad \nu_I \in H^1(X, U)\,, \quad U = V,\, \wedge^2 V,\, \wedge^2 V^*, \ldots$$

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hol. Yukawa couplings

$Y_{IJK} \sim \int_X \nu_I \wedge \nu_J \wedge \nu_K \wedge \Omega$

invariant under $\nu_I \mapsto \nu_I + \bar{\partial}\sigma_I$,
can be computed analytically
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matter field Kahler metric

$$K^{\rm matter}_{IJ} \sim \int_X \nu_I \wedge (\bar{\star}_U \nu_J)$$

requires harmonic representatives
-> need Ricci-flat CY metric,
HYM connection on bundle,
harmonic bundle-valued forms

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Computing $K^{\rm matter}_{IJ}$ has been the main problem since Ricci-flat metric, HYM connection and harmonic forms are not known analytically.

Need to solve complicated diff. eqs. on CY manifolds numerically.

Machine-learning solutions to differential equations

Neural networks

Function families $F_\theta : R^{d_{\text{in}}} \to R^{d_{\text{out}}}$ constructed by function composition:

$$\boxed{F_\theta = f^{(\delta)}_{\theta_\delta} \circ \cdots \circ f^{(2)}_{\theta_2} \circ f^{(1)}_{\theta_1} : \mathbb{R}^{d_0} \to \mathbb{R}^{d_\delta}}$$

where $\theta = (\theta_1, \ldots, \theta_\delta),\ d_{\text{in}} = d_0,\ d_{\text{out}} = d_\delta$.

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$$\sigma(z) = \theta(z) z$$

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activation fct. weights biases

$$\sigma(z) = \theta(z) z \quad \text{(ReLU)}$$

Solving differential eqs. with neural networks

Differential eq. $p(D^2g(x), Dg(x), g(x), x) = 0 \qquad x \in X$

Idea: Want to train neural net F_θ so that is solves diff. eq., $g \simeq F_\theta$

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- Select training set $\{x_i \in X \,|\, i = 1, \ldots, N\}$ (with known distribution)
- Define loss function $\mathcal{L}(\theta) = \dfrac{1}{N} \sum_{i=1}^{N} |p(D^2 F_\theta(x_i), DF_\theta(x_i), F_\theta(x_i), x_i)|^2 + \cdots$

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- The neural net F_{θ_0} with those parameters is an approximate solution

Solving diff. eqs. on CYs

M. Larfors, A. Lukas, F. Ruehle, R. Schneider, arXiv:2111.01436, cymetric package

- Ricci-flat metric on CY X

diff. eq. (Monge-Ampere)

$$J^3 = \kappa\,\Omega \wedge \bar{\Omega}$$

Ansatz

$$J = J^{(\mathrm{ref})} + \partial\bar{\partial}\phi$$

loss

$$\mathcal{L}_{\mathrm{metric}} = \left\|1 - \frac{1}{\kappa}\frac{J(\phi)\wedge J(\phi)\wedge J(\phi)}{\hat{\Omega}\wedge\bar{\hat{\Omega}}}\right\|_1 + \cdots$$

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diff. eq. (HYM)	Ansatz	loss
$J^2 \wedge \partial\bar{\partial}\ln(H) = 0$	$H = e^{\beta} H^{(\mathrm{ref})}$	$\mathcal{L}_{\mathrm{HYM}} = \|\|\Delta\beta - \rho_\beta\|\|_1 + \cdots$

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Reference quantities are taken to be of `Fubini-Study' type.

A heterotic example model

Compactification data and low-energy spectrum

- Tetra-quadric CY $X \in \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \end{array}\right]$ with symmetry $\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$

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- Line bundle sum $X \leftarrow V = \mathcal{O}_X \begin{array}{c} \begin{array}{ccccc} L_1 & L_2 & L_3 & L_4 & L_5 \end{array} \\ \left[\begin{array}{ccccc} -1 & -1 & 0 & 1 & 1 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 2 & -1 & -1 & 0 \\ 1 & 2 & 0 & -1 & -2 \end{array}\right] \end{array}$

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particle	associated cohomology
$Q^{1,2},\ U^{1,2},\ E^{1,2}$	$H^1(X, L_2)$
$Q^3,\ U^3,\ E^3$	$H^1(X, L_5)$
$D^{1,2},\ L^{1,2}$	$H^1(X, L_2 \otimes L_5)$
$D^3,\ L^3$	$H^1(X, L_2 \otimes L_4)$
H^d	$H^1(X, L_2 \otimes L_5)$
H^u	$H^1(X, L_2^* \otimes L_5^*)$

plus bundle, Kahler and complex structure moduli (all SM singlets)

Up-quark Yukawa couplings

Structure of up-quark Yukawas (due to $S(U(1)^5)$ symmetry):

$$Y^u = \begin{pmatrix} 0 & 0 & \lambda_1 \\ 0 & 0 & \lambda_2 \\ \lambda_3 & \lambda_4 & 0 \end{pmatrix} \qquad K^u = \begin{pmatrix} \mathcal{K}^u & & 0 \\ & & 0 \\ 0 & 0 & k^u \end{pmatrix} \qquad K^Q = \begin{pmatrix} \mathcal{K}^Q & & 0 \\ & & 0 \\ 0 & 0 & k^Q \end{pmatrix}$$

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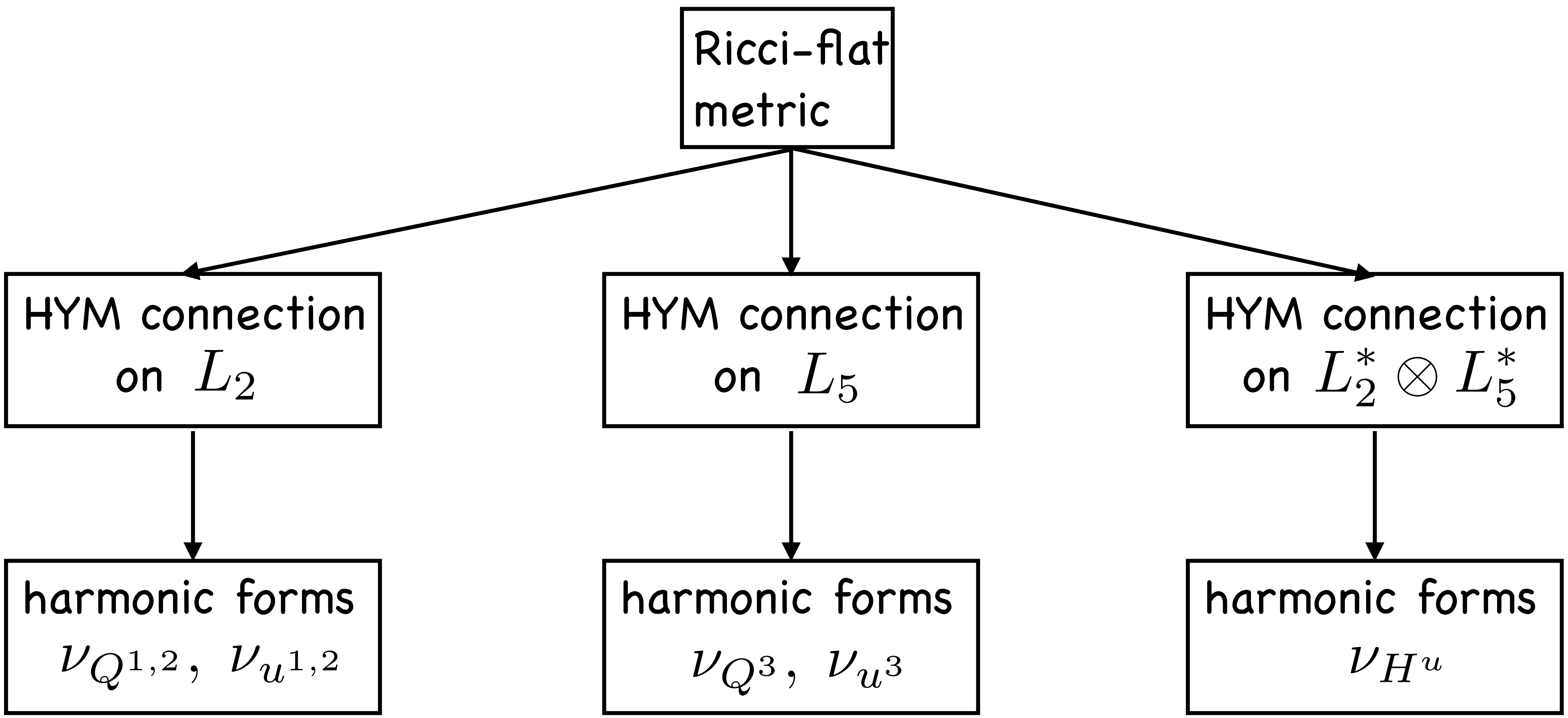
The calculation, schematically:



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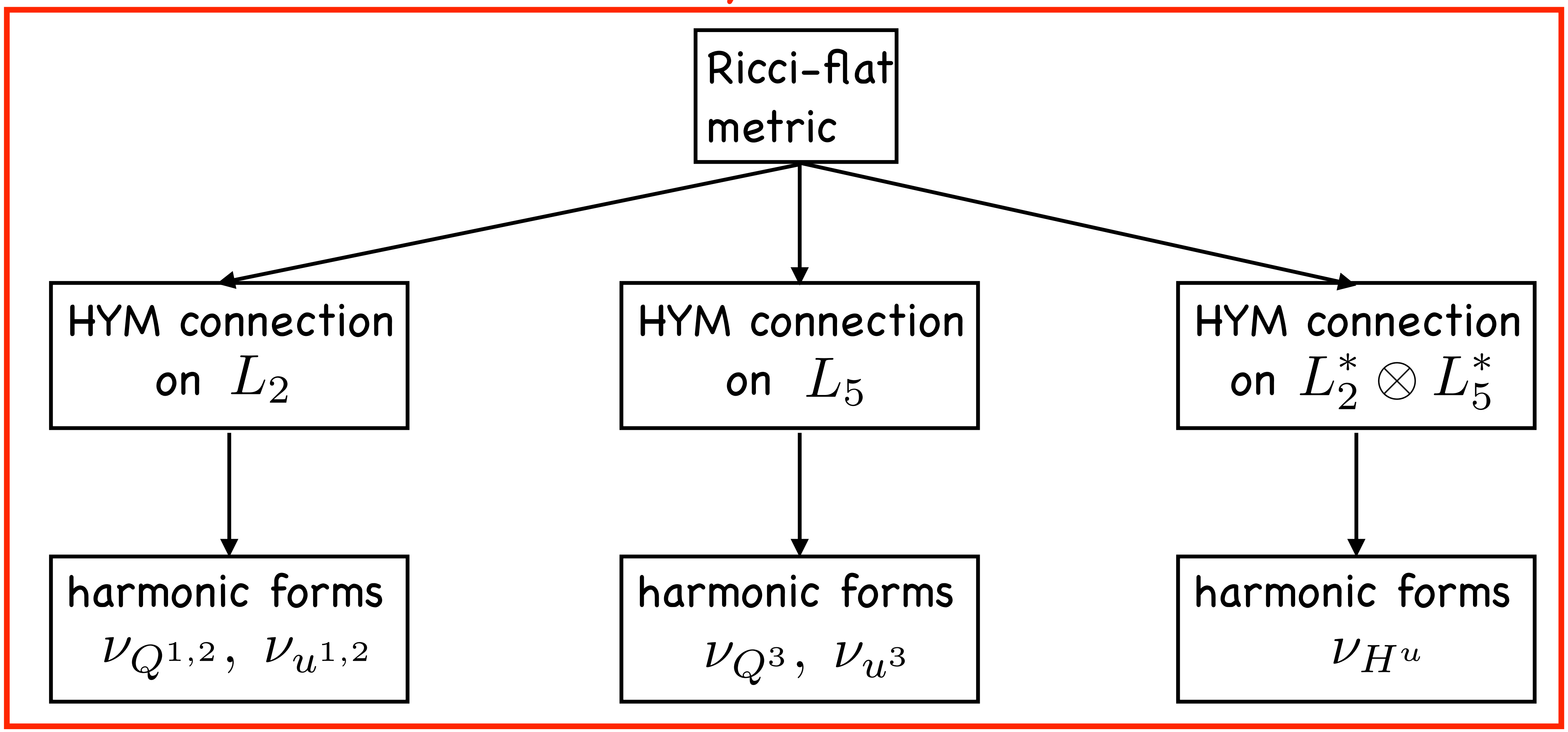

11 neural networks

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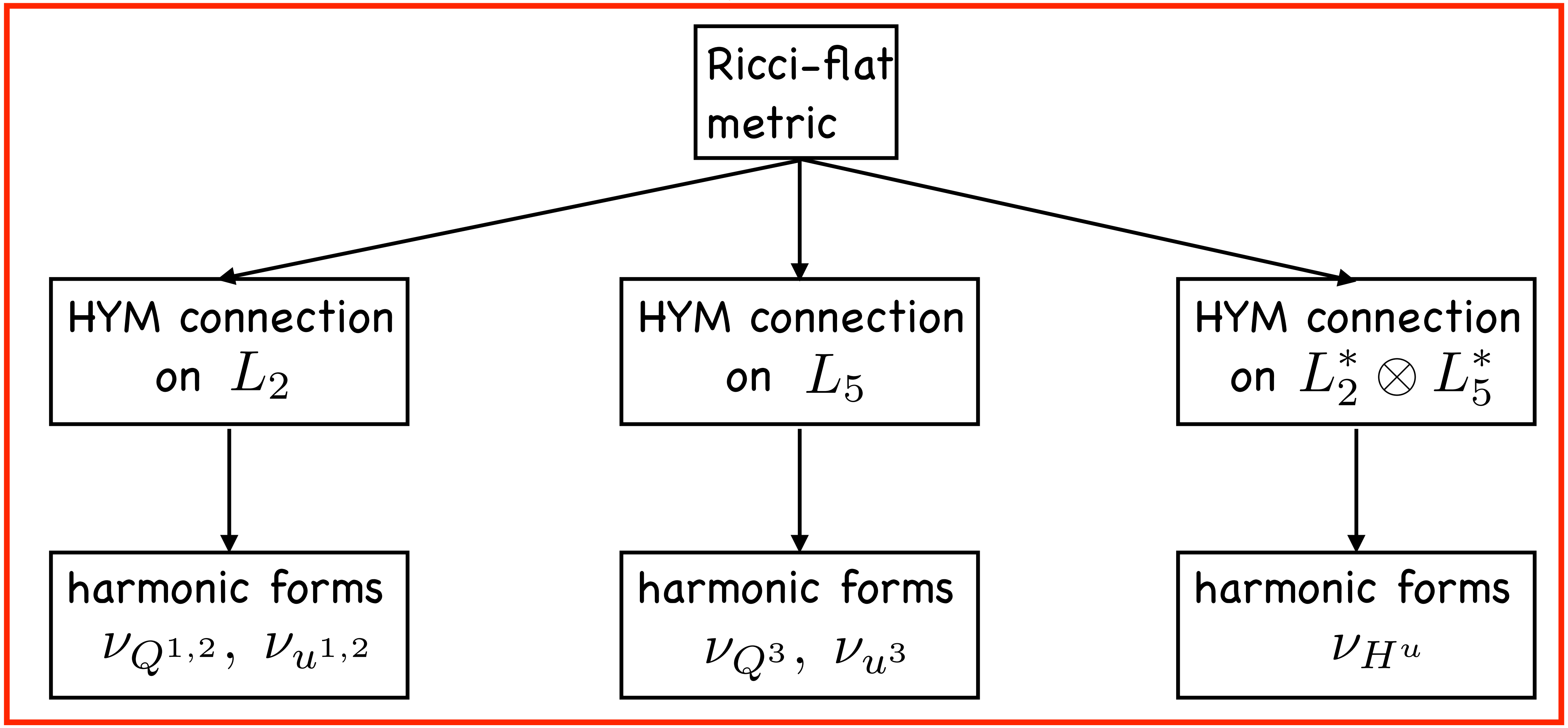

11 neural networks

Then use Monte-Carlo integration on X to evaluate

$$Y_{IJK} \sim \int_X \nu_I \wedge \nu_J \wedge \nu_K \wedge \Omega \qquad K_{IJ}^{\text{matter}} \sim \int_X \nu_I \wedge (\bar{\star}_U \nu_J)$$

Results

Training

- training set: 300000 points $x_i \in X$ (Fubini-Study distribution)
- NN: fully-connected, width 128, depth 4, GeLU activation, 100 epochs.
- training: 100 epochs, batch size 64, learning rate $\lambda = 0.001$

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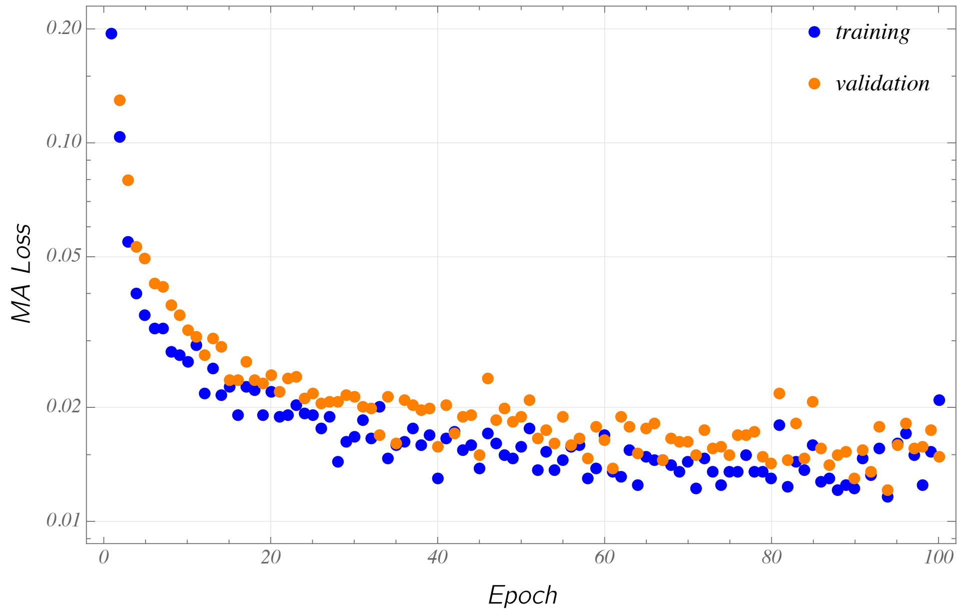
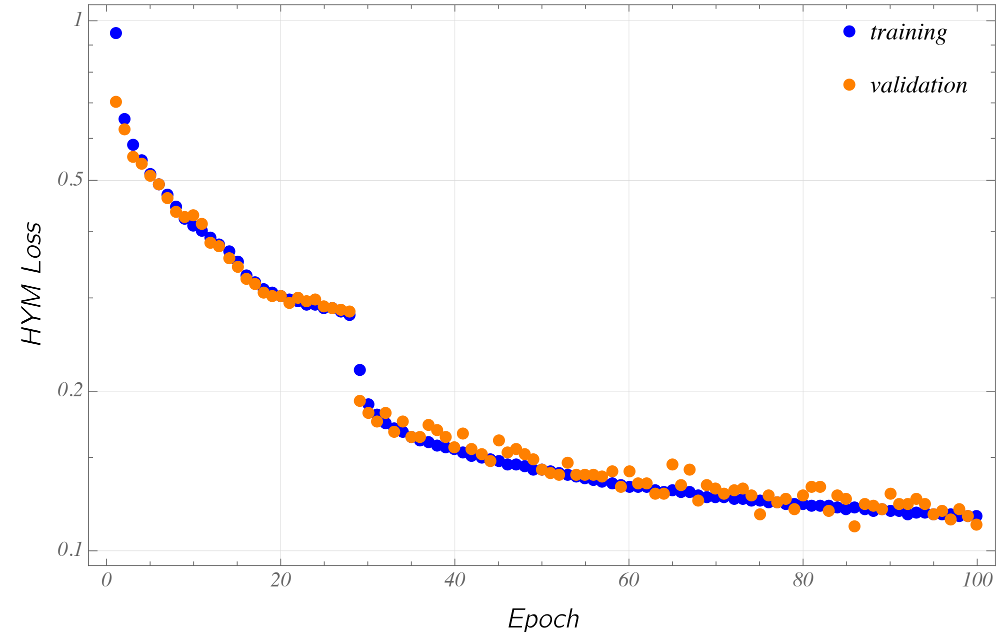
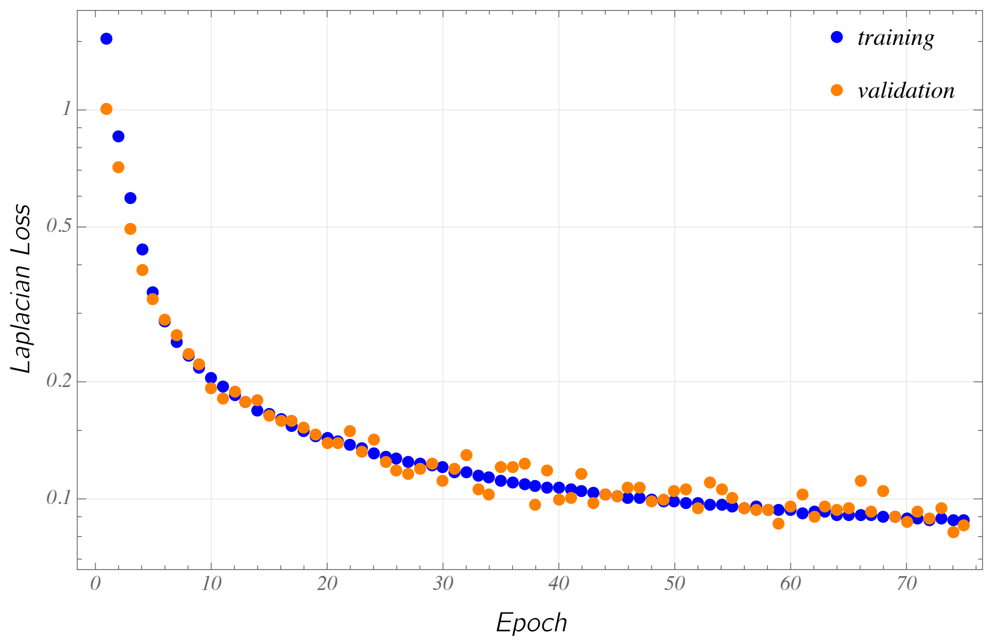
$\mathcal{L}_{\mathrm{metric}}$



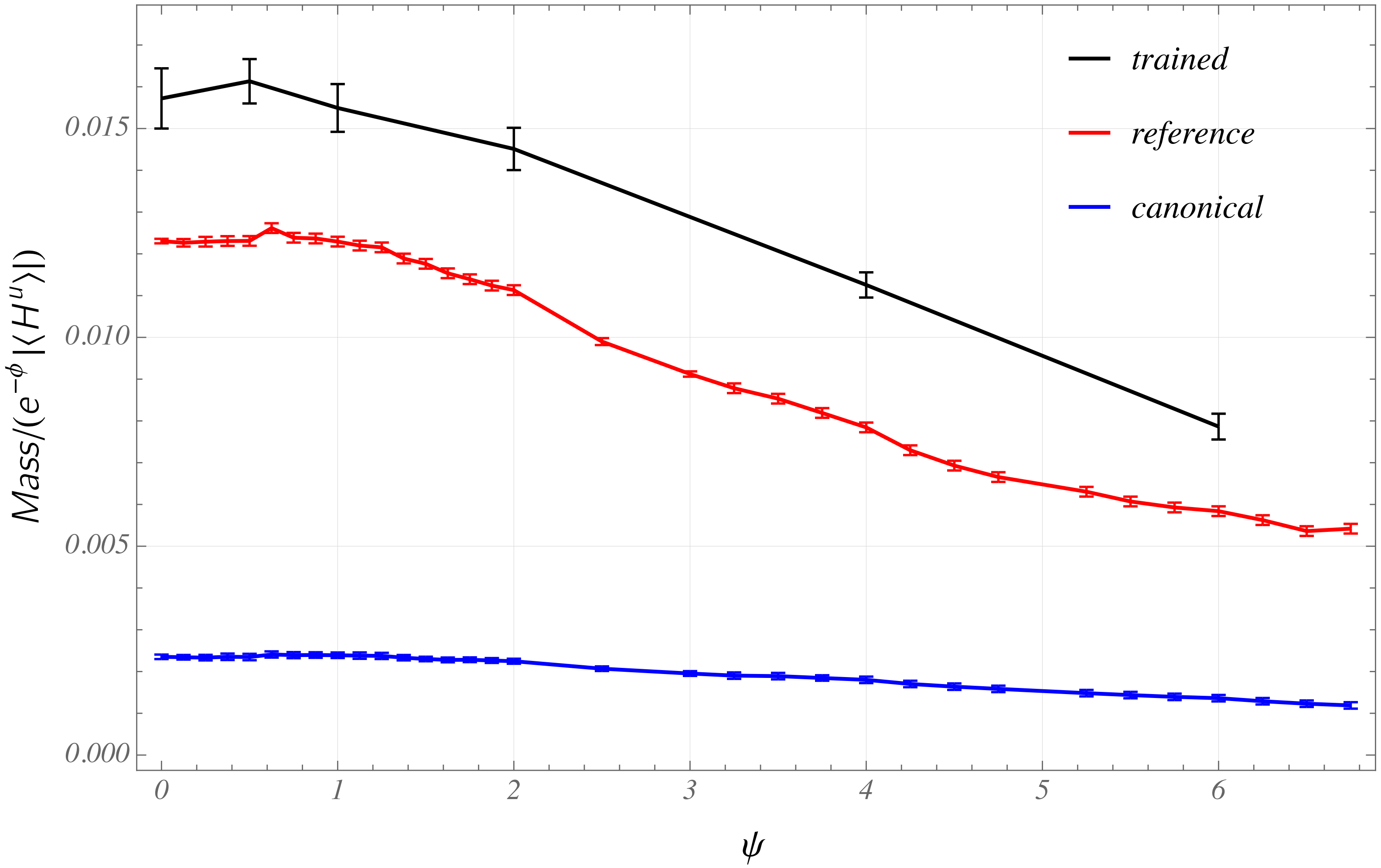
$\mathcal{L}_{\mathrm{HYM}}$



$\mathcal{L}_{\mathrm{harmonic}}$



Numerical Yukawa couplings



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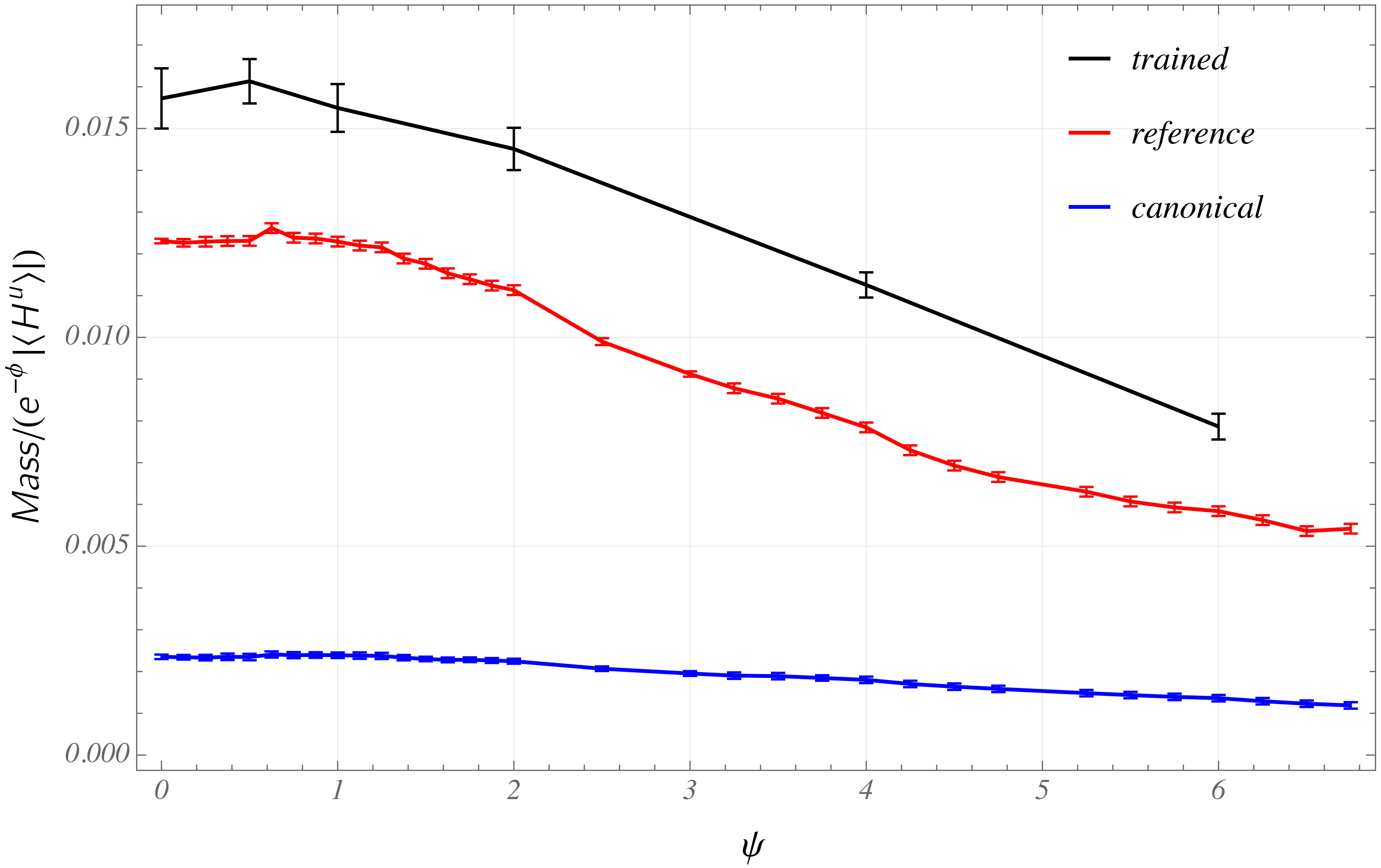


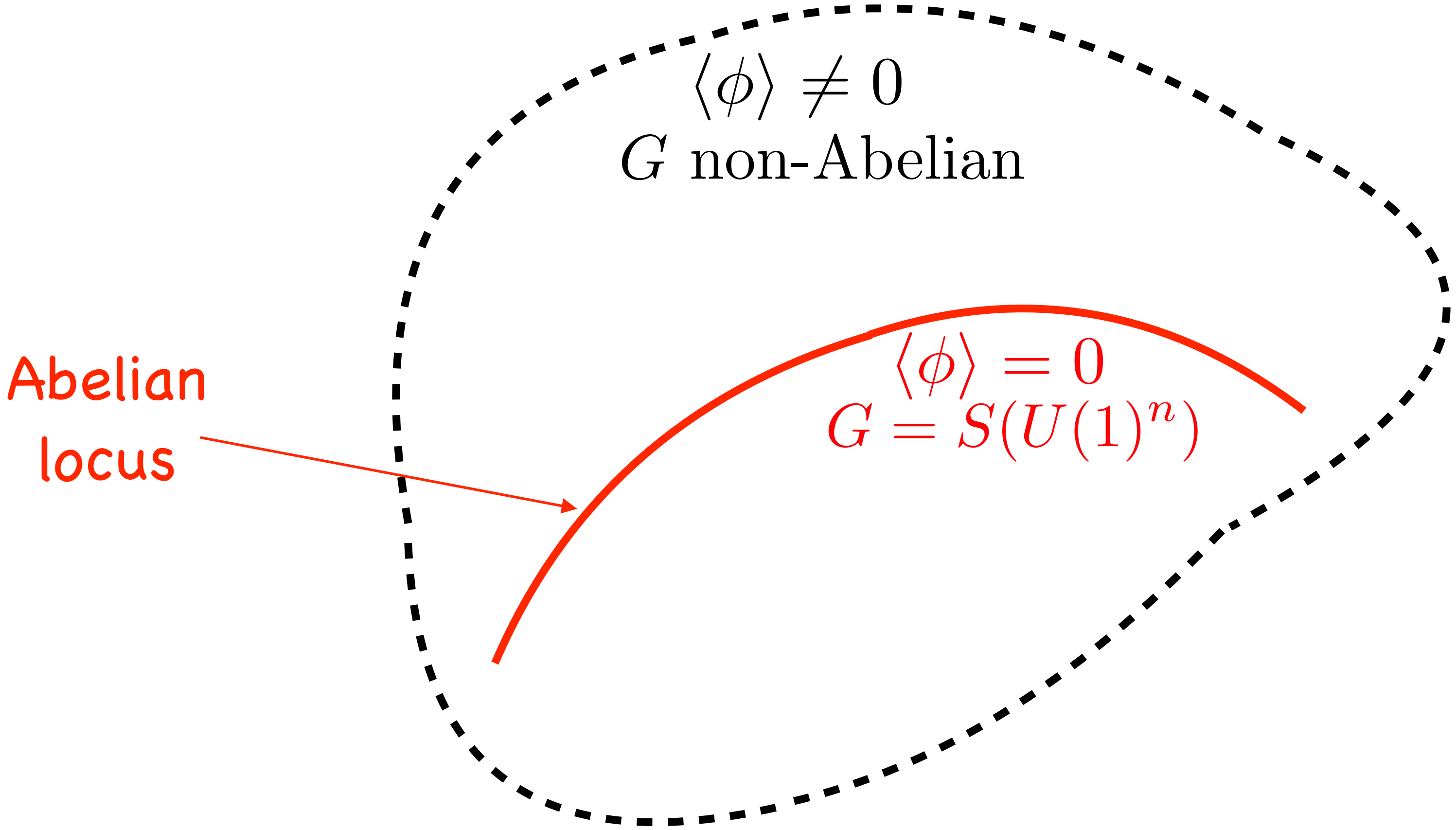
Remarks:

- Additional symmetry means two massive families are degenerate.
- By exploring compl. str. moduli space we can generate a hierarchy.
- Calculation with reference quantities is `not bad'.
- Assuming canonical normalisation gives result far off.

Heterotic Froggatt Nielsen models

Non-Abelian models from line bundle models

sketch of bundle moduli space $\{\langle\phi\rangle\}$:



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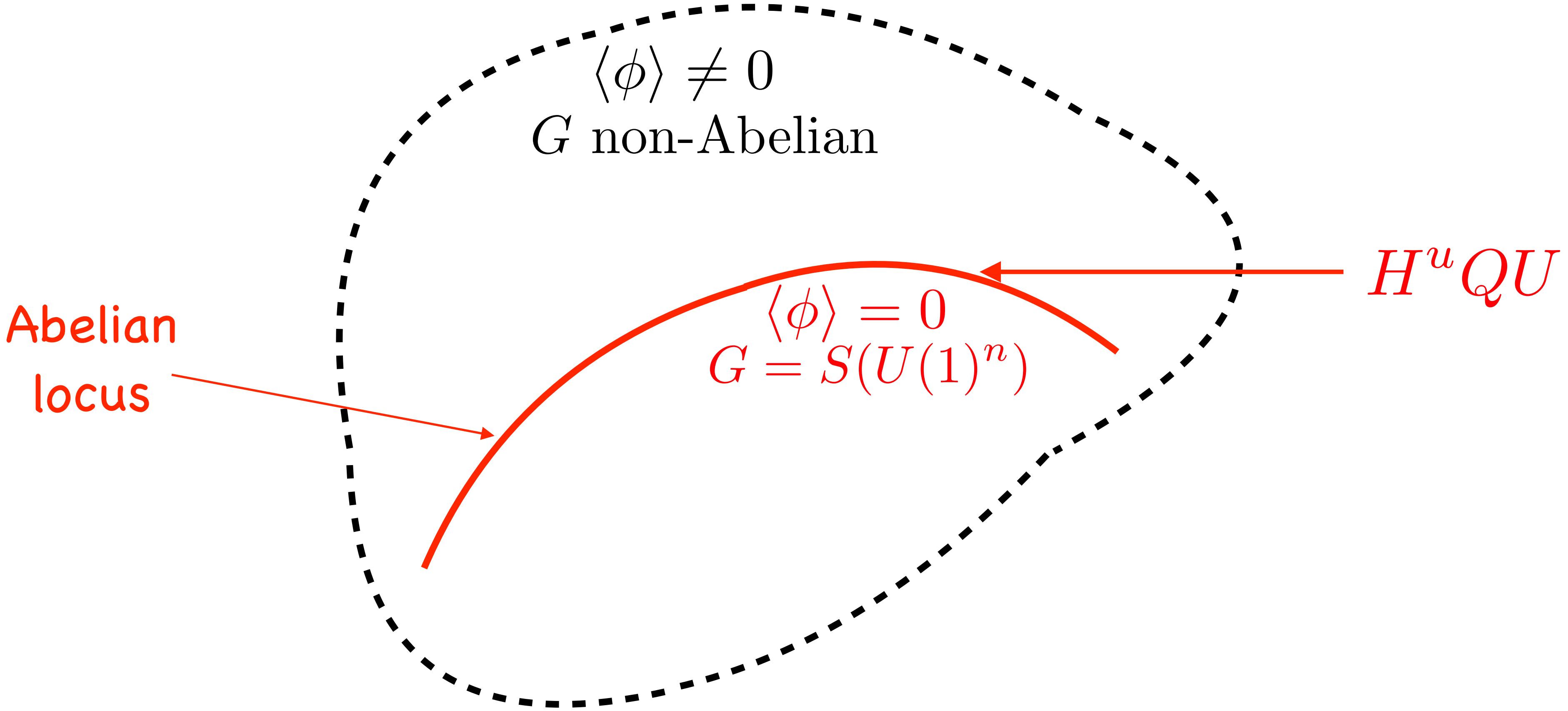

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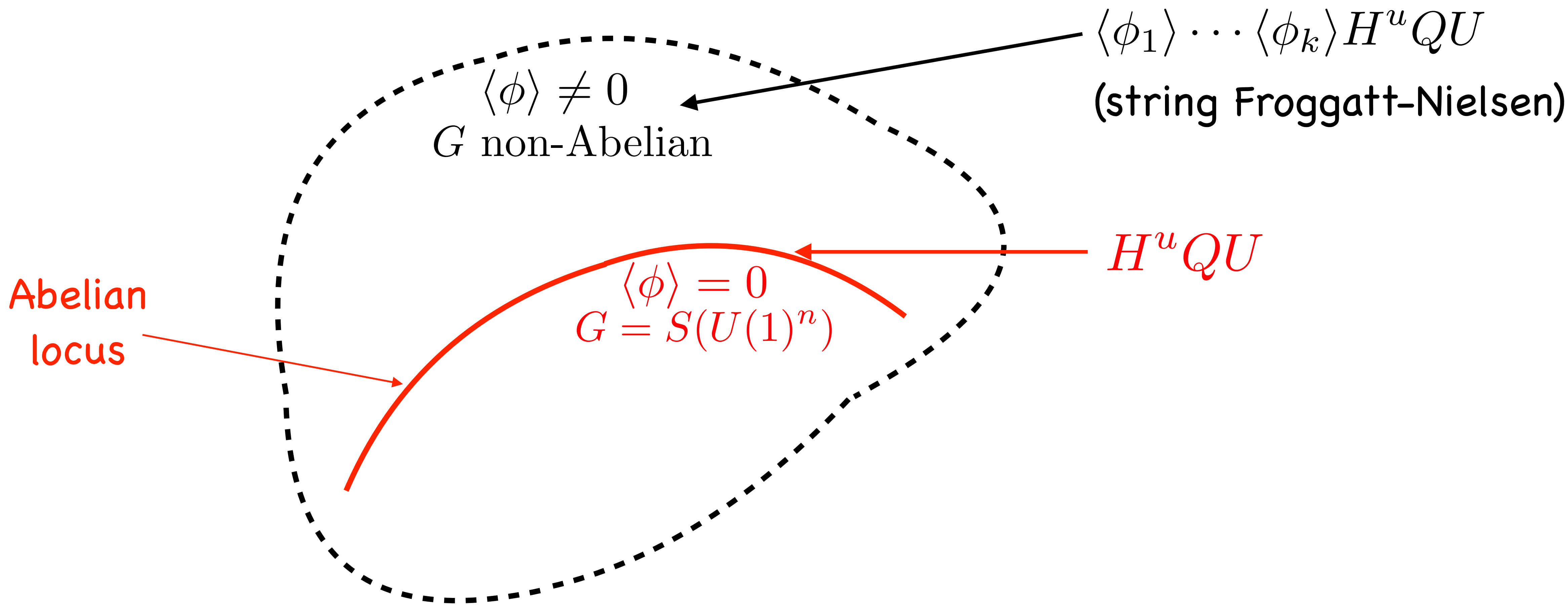

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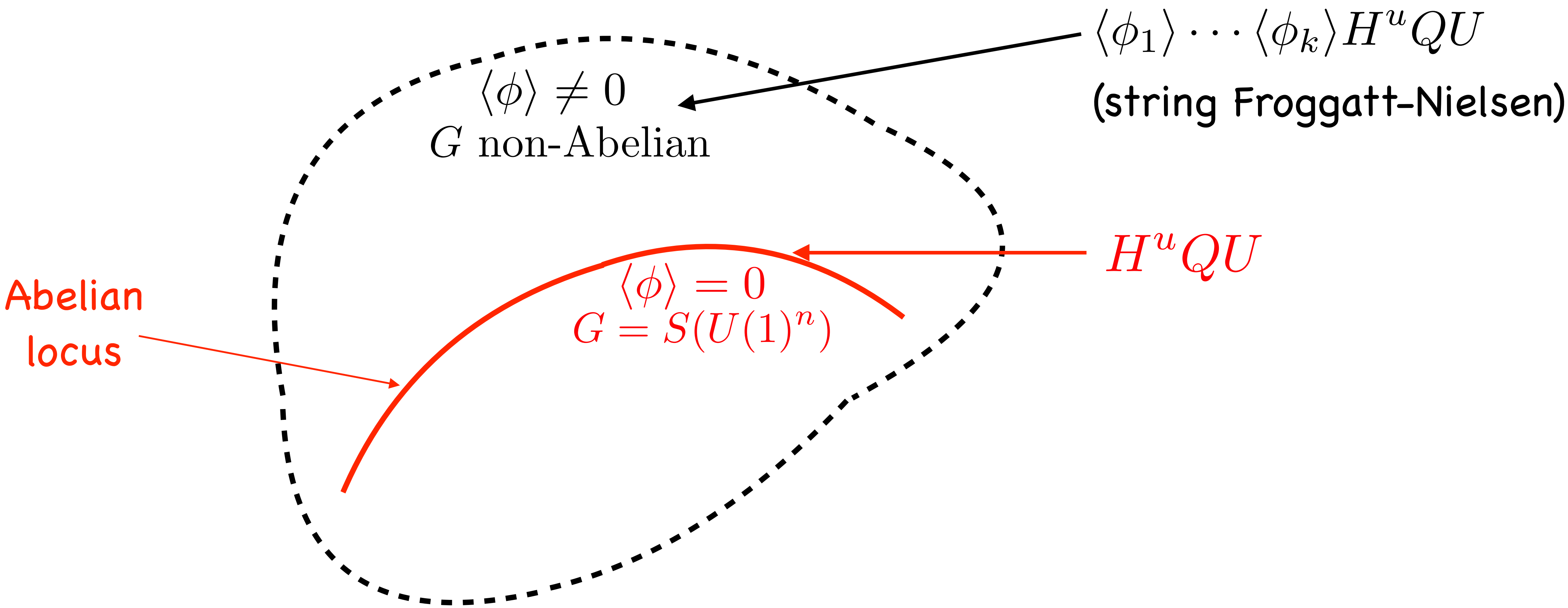

Realistic mass models at the line bundle locus in moduli space unlikely, due to $S(U(1)^5)$ symmetry.

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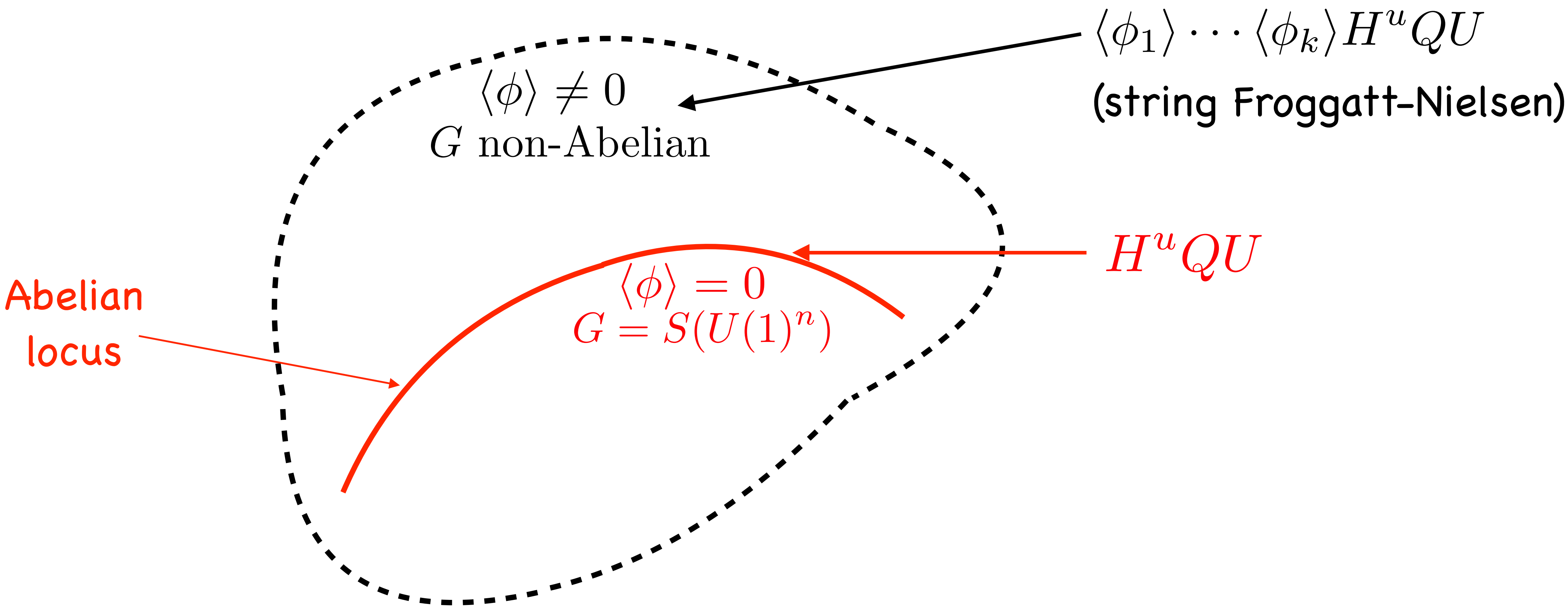

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Two options: 1) Construct non-Abelian bundles and generalise Yukawa calculations to this case.

2) Deform line bundle models by switching on VEVs for bundle moduli ϕ .

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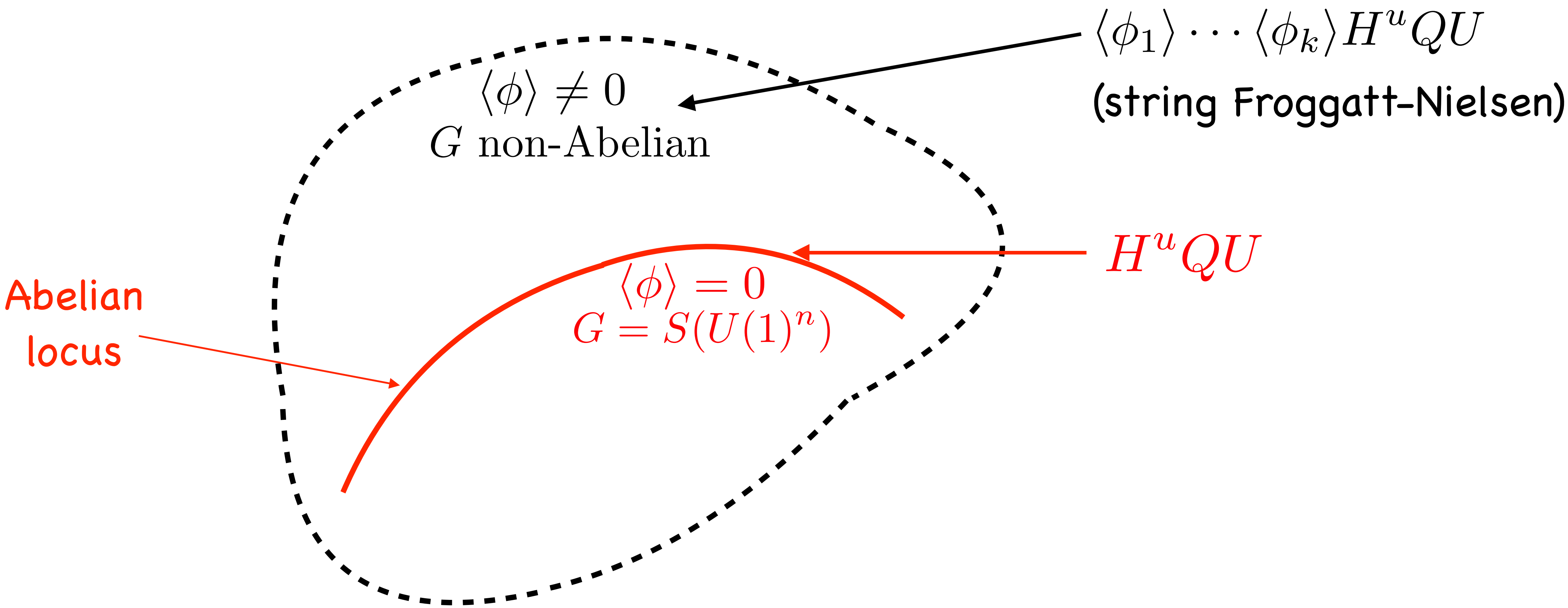

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Non-Abelian models from line bundle models

at Abelian locus

complex
structure

$$W = Y_{IJK}(\underset{\uparrow}{Z})C^I C^J C^K + \cdots$$

Non-Abelian models from line bundle models

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$$W = Y_{IJK}(Z)C^I C^J C^K + \cdots$$

$\longrightarrow$

away from Abelian locus

e^{-T}

$$W = Y_{IJK}(Z, \underbrace{\phi, \Phi)C^I C^J C^K}_{} + \cdots$$

Non-Abelian models from line bundle models

at Abelian locus

$$W = Y_{IJK}(\underset{\substack{\uparrow\\ \text{complex structure}}}{Z})C^I C^J C^K + \cdots$$

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Consider operators $a(Z)\langle\phi_1\rangle \cdots \langle\phi_k\rangle\langle\Phi_1\rangle \cdots \langle\Phi_l\rangle C^3$ as allowed by $S(U(1)^5)$

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For example: $U \leftrightarrow \nu_u \in H^1(X, L_a)$ means $q(U) = e_a$, $q(\Phi^i) = (k_1^i, \ldots, k_5^i)$

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-> Heterotic Froggatt Nielsen models

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This model allows for viable quark masses, charged lepton masses and CKM matrix for suitable VEVs $\langle\phi\rangle$ and $\langle\Phi\rangle$.

Conclusions

- Calculating Yukawa couplings in geometric string models is hard - it requires the Ricci-flat CY metric and related quantities.
- For heterotic line bundle models, we have shown that these quantities and the resulting Yukawa couplings can be calculated numerically, using ML techniques.
- The resulting Yukawa couplings are not yet realistic. This requires moving away from the line bundle locus to non-Abelian bundles.
- We have taken some steps in this direction by studying heterotic Froggatt-Nielsen models.
- We have identified a few viable models of this kind.

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- We need to understand how to perform the numerical computation for non-Abelian bundles.

- We need to consider moduli stabilisation!

Thanks