AdS3/CFT2 and defect CFTs

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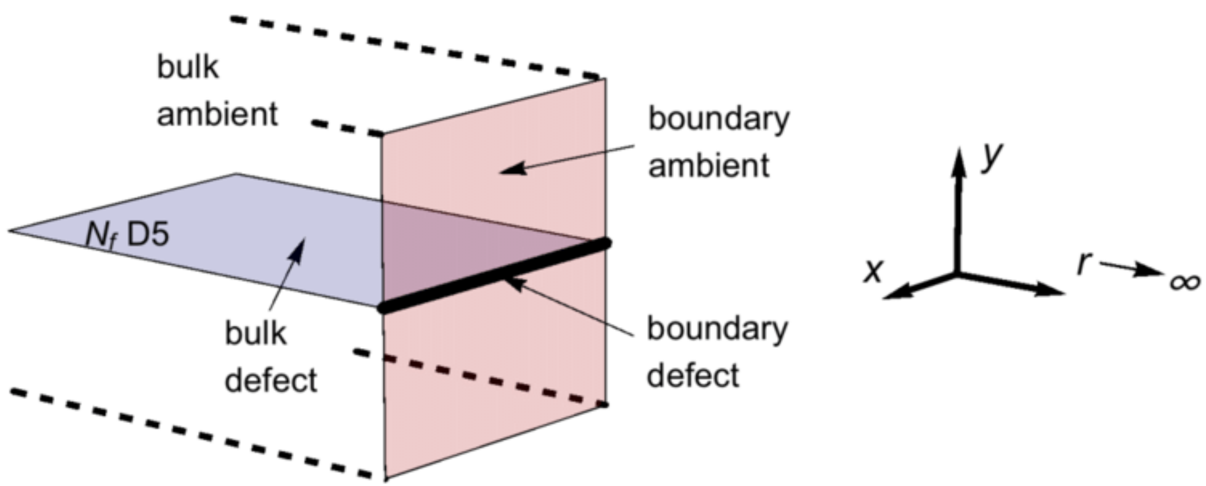
1. Introduction

Low dimensional holography plays a prominent role in black hole physics

Also in the description of conformal defects

These are defects that preserve a subgroup of the superconformal group of the system where they are embedded



Thus, holography provides a very powerful tool for their study

In String Theory, defect conformal field theories can be engineered in terms of brane intersections of defect branes and background branes, where, in the latter, a higher dimensional CFT lives

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This is different from an RG flow, where the low dim AdS would only arise in the IR and the high dim AdS would not have extra fluxes

The presence of the non-compact direction renders the defect field theory ill-defined, but this is interpreted as the need to complete the CFT by the higher dim one away from the defects

In recent years some low dimensional AdS solutions have been proposed as dual to defect CFTs

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In this talk we will focus on the CFT interpretation of some AdS solutions to (massive) Type IIA supergravity and their possible relation to conformal defects

Based on Y.L., Macpherson, Petri, Ramirez'24

Conti, Dibitetto, Y.L., Petri, Ramirez'24

Work in progress with A. Conti and C. Rosen

Classifying low dimensional AdS spaces is especially challenging, due to the many possibilities for geometries and topologies of the internal space, and therefore of supersymmetries

For AdS3:

Partial classifications of (0,8), (4,4), (0,4), (3,3), (0,3), (2,2), (0,2), (1,1)…

Bachas, Chiodaroli, Couzens, D'Hoker, Dibitetto, Estes, Gauntlett, Gutperle, Kim, Krym, Legramandi, Lo Monaco, Lawrie, Lozano, Macpherson, Martelli, Nunez, O Colgain, Passias, Petri, Ramirez, Schafer-Nameki, Tomasiello, Waldram..

More recently: (0,5) and (0,6): Macpherson, Ramirez'23

Outline

1. The class of (0,6) $AdS_3 \times CP^3 \times I$ solutions to massive IIA:

   Surface defects in ABJM/ABJ

2. A class of (0,4) $AdS_3 \times S^3 \times S^2 \times \Sigma_2$ solutions to massive IIA:

   Deconstruction or surface defects in 6d (1,0) CFTs?

2. New AdS_3 solutions to massive IIA with (0,6) SUSY

(Macpherson, Ramirez'23)

$AdS_3 \times CP^3 \times I$ solutions to massive IIA with (0,6) SUSY and OSp(6|2) superconformal group $\rightarrow$ SO(6) R-symmetry

Can be regarded as an extension of ABJM/ABJ to the massive case, in which one of the external directions becomes an energy scale, and generates a flow towards an AdS3 space

$$ds^2 = \frac{h}{\sqrt{2hh'' - h'^2}} ds^2_{AdS_3} + \sqrt{2hh'' - h'^2}\left(\frac{1}{h} dr^2 + \frac{1}{h''} ds^2_{CP^3}\right)$$

$$e^{\phi} = \frac{(2hh'' - h'^2)^{1/4}}{(h'')^{3/2}} \qquad B_2 = 4\pi\left(-(r-l) + \frac{h'}{h''}\right) J$$

Specified by $h(r)$, that satisfies the Bianchi identity:
$h''' = -2\pi F_0$ with F_0 the Romans mass

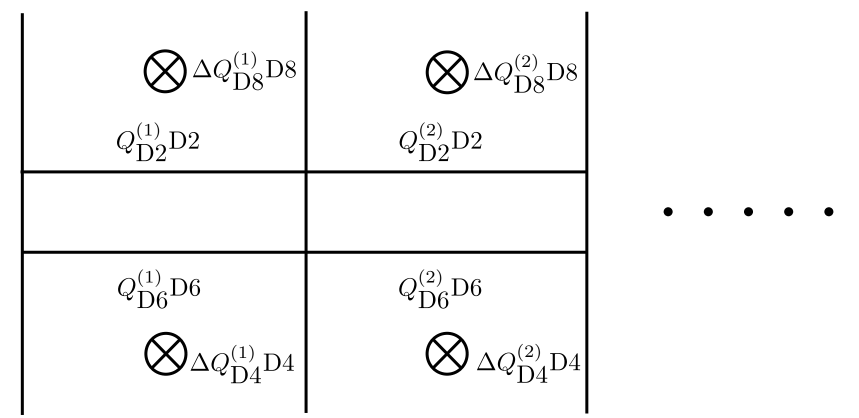
Construct global solutions by glueing local solutions with D8-branes:

$$h_l(r) = Q_2^l - Q_4^l(r-l) + \frac{1}{2}Q_6^l(r-l)^2 - \frac{1}{6}Q_8^l(r-l)^3$$

with Q_p^l the Page charges at $r \in [l, l+1]$

NS5-branes located at $r = l$

Hanany-Witten brane set-up:



The presence of the CP^3 makes it more complicated

Massless limit:

A change of variables $\sinh\mu = \frac{Q_6 r - Q_4}{\sqrt{2Q_2Q_6 - Q_4^2}}$ $\rightarrow$ ABJM

with $ds^2_{AdS_4} = d\mu^2 + \cosh^2\mu\, ds^2_{AdS_3}$

$B_2 = -4\pi\frac{Q_4}{Q_6}J$ Discrete holonomy of ABJ

Brane set-up:

branes	x^0	x^1	r	x^3	x^4	x^5	ψ	x^7	x^8	x^9
$N\ D3$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$\times$	$-$	$-$	$-$
$NS5'$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$
$(1,k)5'$	$\times$	$\times$	$\times$	$\cos\theta$	$\cos\theta$	$\cos\theta$	$-$	$\sin\theta$	$\sin\theta$	$\sin\theta$

When one takes into account the Freed-Witten anomaly and the higher curvature terms (Aharony, Hashimoto, Hirano,Ouyang'09):

$$Q_2 = N + \frac{k}{12} \qquad Q_4 = M - \frac{k}{2} \qquad Q_6 = k$$

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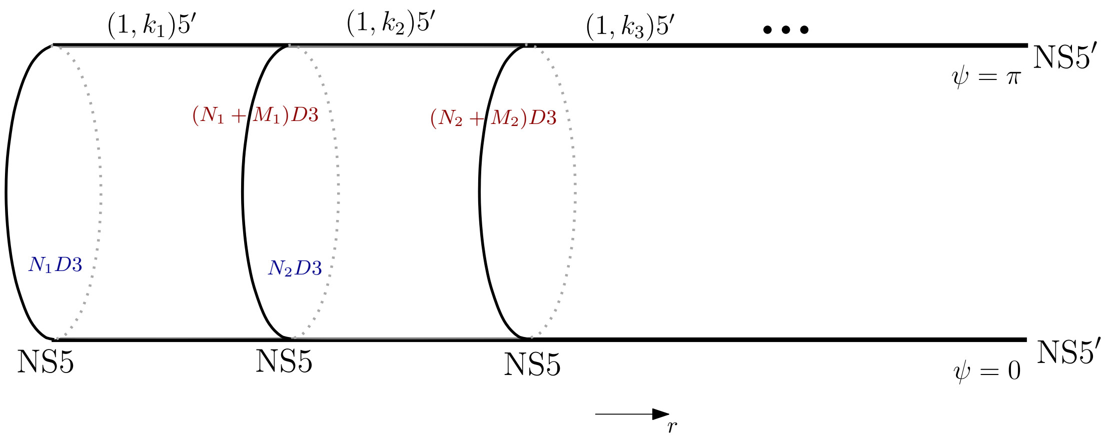
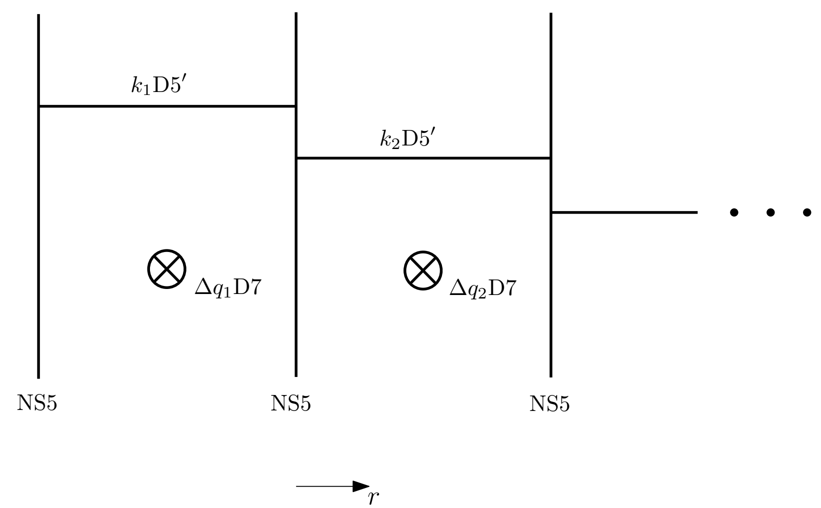
In the massive case:

branes	x^0	x^1	r	x^3	x^4	x^5	ψ	x^7	x^8	x^9
$D3$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$\times$	$-$	$-$	$-$
$NS5'$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$
$(1,k)5'$	$\times$	$\times$	$\times$	$\cos\theta$	$\cos\theta$	$\cos\theta$	$-$	$\sin\theta$	$\sin\theta$	$\sin\theta$
$D7$	$\times$	$\times$	$-$	$\times$	$\times$	$\times$	$-$	$\times$	$\times$	$\times$
$NS5$	$\times$	$\times$	$-$	$-$	$-$	$-$	$\times$	$\times$	$\times$	$\times$

Extra D7-NS5, or D8-NS5 *defect* branes render the field theory two dimensional and (0,3) supersymmetric

The brane box is (0,3) SUSY, due to the rotations of the 5' branes. This should be enhanced to (0,6) in the IR, as in ABJM

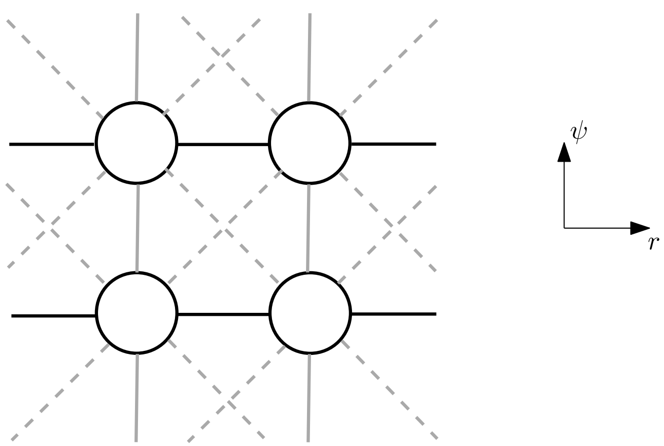


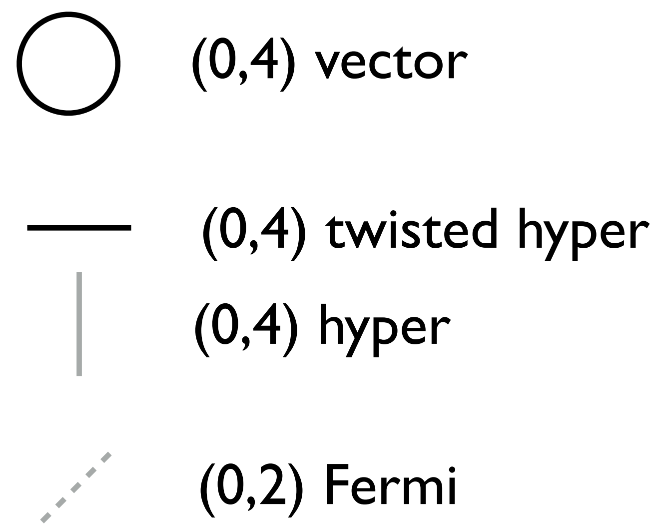


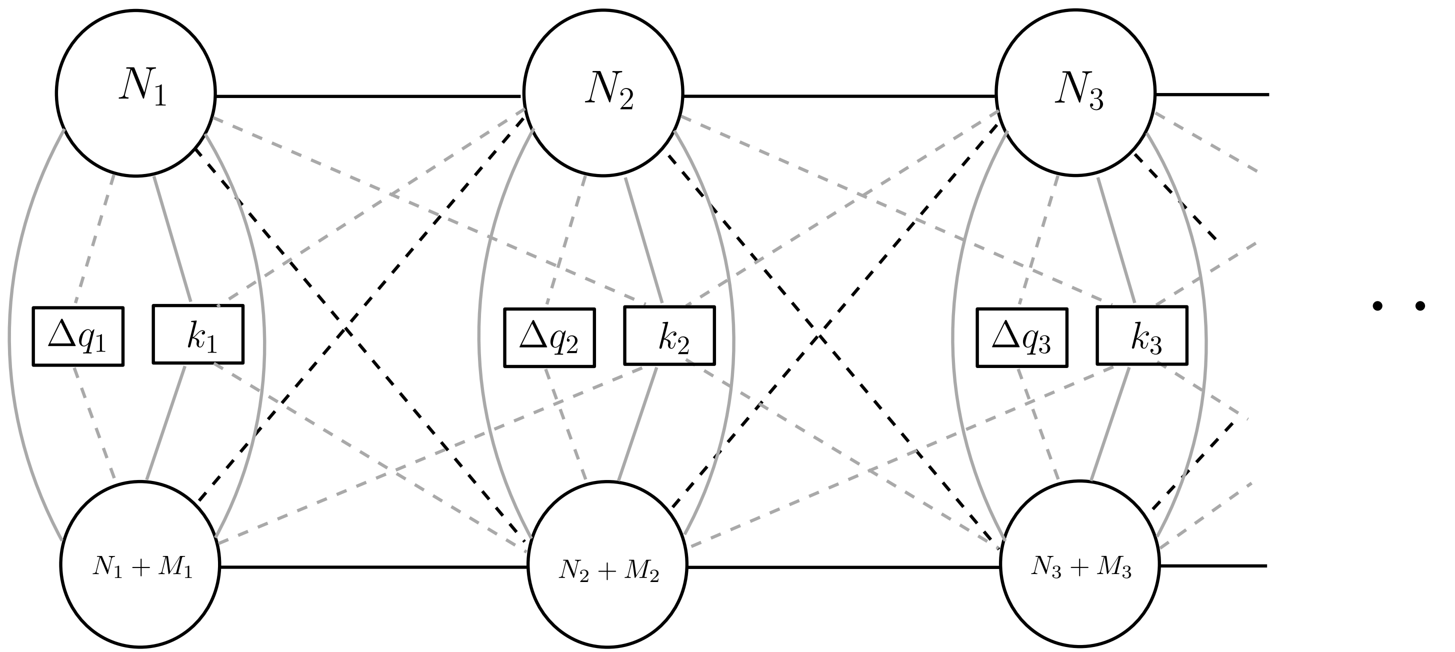
As in 3d, the (0,3) gauge theory living in the D3-branes is expected to have the same field content of a (0,4) gauge theory, except for the deformations introduced by the rotations of the 5'-branes, that will render some multiplets massive

The field theory can then be studied using what is known for (0,4) brane box models (Hanany, Okazaki'18):

Building blocks:







Embedding of D8-NS5 defect branes within the ABJM 3d quiver

Gauge anomaly is cancelled with the quantised charges obtained from the solutions

In each interval (Bergman, Lifschytz'10):

$$Q_2 = N + \frac{k}{12} \qquad Q_4 = M - \frac{k}{2} + \frac{q}{12} \qquad Q_6 = k \qquad Q_8 = -q$$

Transformations of charges across intervals (brane creation):

$$N \to N - M + k \qquad M \to M - k \qquad k \to k + q$$

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Extension of Seiberg duality in ABJ for non-vanishing mass!

$$U(N+M)_k \times U(N)_{-k+q} \to U(N)_{k+q} \times U(N-M+k)_{-k}$$

Derived in Bergman, Lifschytz'10 in a non-supersymmetric setting

Seiberg duality as a large gauge transformation (Aharony, Hashimoto, Hirano, Ouyang'09) albeit in one dimension less

When the D8-branes are embedded in the 3d theory in a supersymmetric way, which implies adding as well NS5-branes, one of the 3d field theory directions turns into an energy scale, and generates a flow towards a 2d CFT

Geometrically the backreaction of the D8-NS5 branes gives rise to a (0,6) supersymmetric $AdS_3 \times CP^3$ geometry where r becomes part of the internal space

In this geometric setting it is now possible to perform large gauge transformations along the r direction that precisely realise the extension of Seiberg duality to the massive case proposed by Bergman, Lifschytz

Prediction for the central charge from holography:

$$c_{hol} = \frac{3R}{2G_3} = \int dr \Big(2hh'' - (h')^2 \Big)$$

Recover in field theory from (0,3) multiplets? Extremization?

Interesting to address!

3. New $AdS_3 \times S^3 \times S^2 \times \Sigma_2$ solutions to massive IIA with (0,4) SUSY (small)

SU(1,1|2) superconformal group $\rightarrow$ SU(2) R-symmetry

$$ds^2 = \sqrt{\frac{\alpha}{-\alpha''}}\Big(x^2(ds^2_{AdS_3} + ds^2_{S^3}) + \frac{dx^2}{\sqrt{c+x^4}}\Big) + \frac{\sqrt{c+x^4}}{x^2}\sqrt{\frac{-\alpha''}{\alpha}}\Big(dz^2 + \frac{\alpha^2 x^4}{\Delta} ds^2_{S^2}\Big)$$

$$\Delta = x^4(\alpha'^2 - 2\alpha\alpha'') - 2c\alpha\alpha''$$

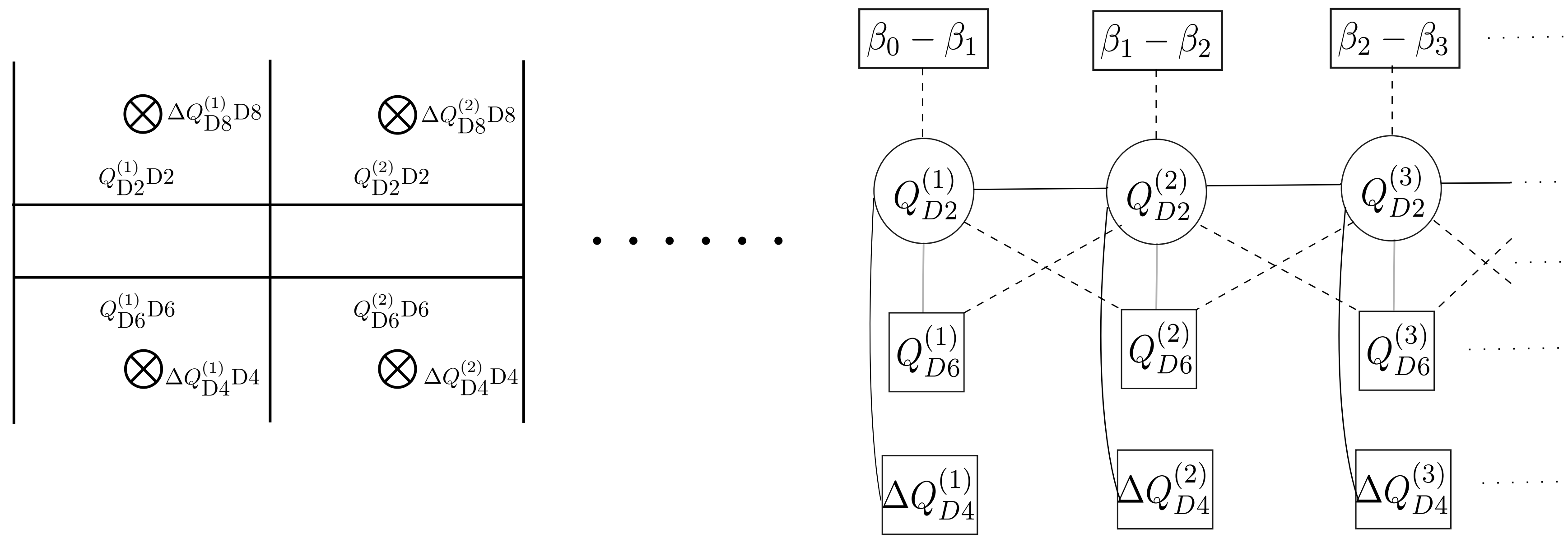
$\alpha(z)$ satisfies the Bianchi identity $\alpha(z)''' = -162\pi^3 F_0$

Non-vanishing B_2 such that: one NS5 at $z = l$

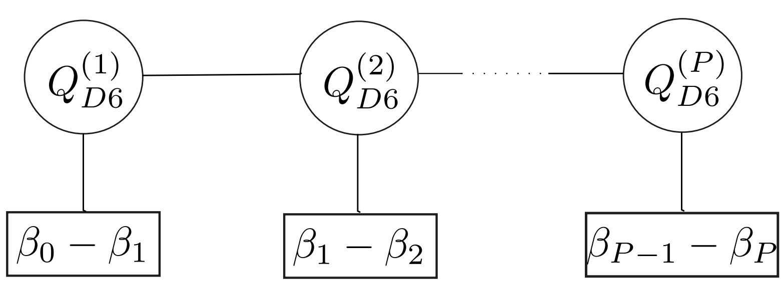
D2-D4 *defect* branes in 6d (1,0) theories living in NS5-D6-D8 brane intersections?

The solutions asymptote locally to $AdS_7 \times S^2 \times I$

Hanany-Witten brane set-up and associated quiver:



Explicit embedding of D2-D4 branes in 6d quivers:



However, computation of central charge:

$$c^{2d}_{hol} = \int dz(-\alpha\alpha'') \int_0^{\tilde{\Lambda}} dx x^3 \qquad \text{and} \qquad c^{6d}_{hol} = \int dz(-\alpha\alpha'') \int_{\Lambda}^{\frac{\pi}{2}} d\beta \frac{\cot^3\beta}{\sin^2\beta}$$

with $$ds^2_{AdS_7} = \frac{1}{\sin^2\beta}\Big(ds^2_{\mathrm{AdS}_3} + \cos^2\beta ds^2_{S^3} + d\beta^2\Big)$$

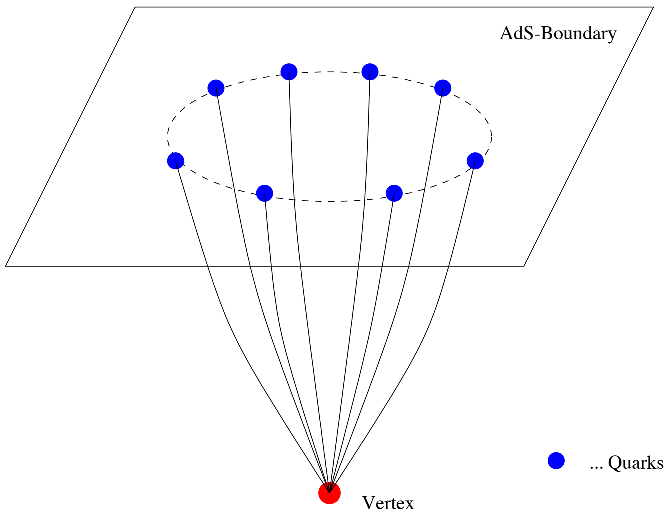
exactly agree (in some regularisation scheme)

Same happens with other configurations, such as baryon vertices and giant gravitons

The baryon vertex in $AdS_5 \times S^5$

Gauge invariant coupling of N external quarks

Through AdS/CFT external quarks are regarded as endpoints of F-strings in AdS

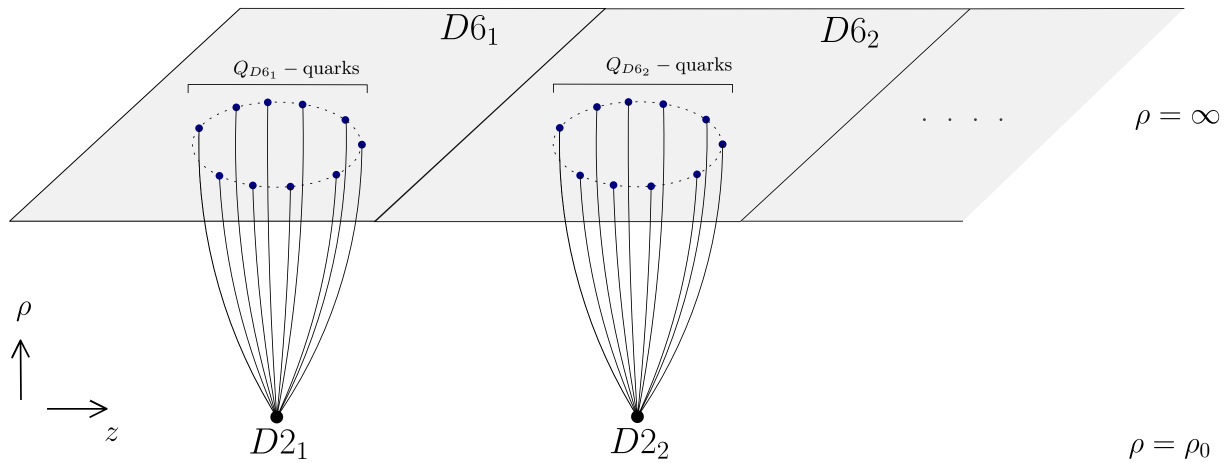


D5-brane:

$$S_{D5} = T_5 \int_{\mathbb{R}\times S^5} F_5 \wedge A = N T_{F1} \int_{\mathbb{R}} dt A_t$$

Cancel this charge with the charge induced by the endpoints of N open F-strings stretching between the D5 and the boundary

Baryon vertex in the 6d theory:



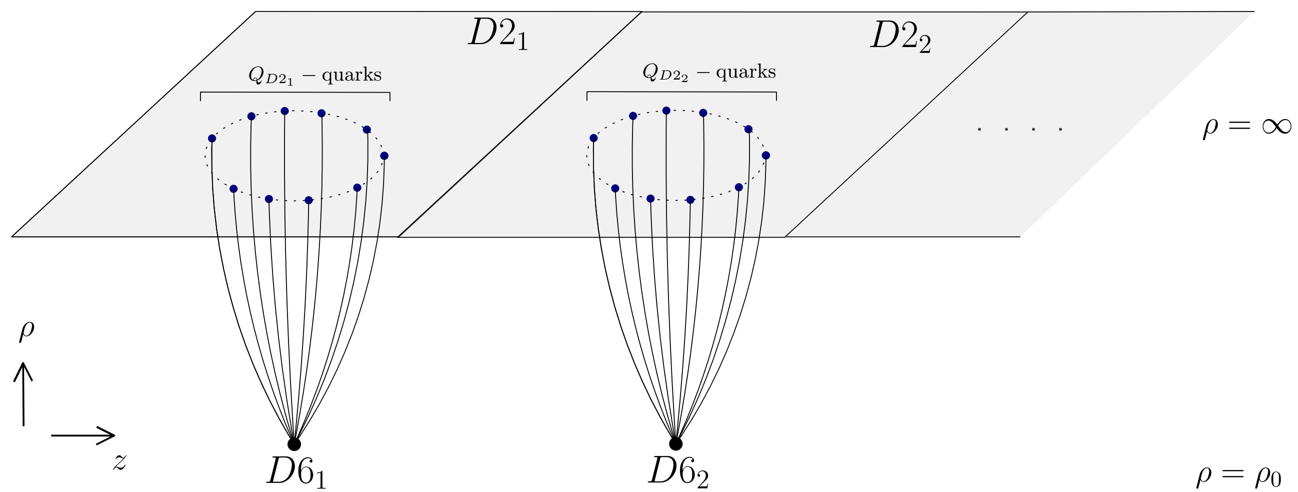
Coupling: $S_{D2} = T_2 \int_{\mathbb{R}\times S^2} \hat{F}_2 \wedge A = Q_{D6} T_{F1} \int_{\mathbb{R}} dt A_t$

Analyse the stability in ρ :

$$S = S_{D2} + S_{Q_{D6}F1}$$

Minimising: Size and energy of the configuration

In the 2d theory:



Coupling: $$S_{D6} = T_6 \int_{\mathbb{R}, I_x, S^3, S^2} \hat{F}_6 \wedge A = Q_{D2} T_{F1} \int_{\mathbb{R}} dt A_t$$

Minimise $$S = S_{D6} + S_{Q_{D2}F1}$$

Same size and energy as in AdS7!

Our interpretation:

The AdS3 solution describes a 6d (1,0) CFT deconstructed in terms of 2d (0,4) degrees of freedom

The 2d quivers are explicit realisations of this

Provide further checks of this proposal

5. Conclusions

Field theory interpretation of AdS3 solutions with a non-compact direction as surface defects

- With (0,6) supersymmetry: Extension of ABJM/ABJ to the massive case, in which one of the external directions becomes an energy scale, and generates a flow towards an AdS3 space

  Realisation of Seiberg-duality as a large gauge transformation

- With (0,4) supersymmetry: Defect interpretation linked to the existence of a non-compact direction that becomes part of AdS_7 when it approaches infinity

  No contribution from defects to the degrees of freedom
  $\rightarrow$ Deconstructed 6d (1,0) theories

THANKS!!

With large susy: New class of solutions to massive IIA that asymptote locally to $AdS_7 \times S^2 \times I$:

OSp(4|2) $\rightarrow$ SO(4) R-symmetry

$$ds^2 = \sqrt{\frac{\alpha}{-\alpha''}} X^{-1/2} ds_7^2 + X^{5/2} \sqrt{\frac{-\alpha''}{\alpha}} \Big(dz^2 + \frac{\alpha^2}{\alpha'^2 - 2\alpha\alpha'' X^5} ds^2_{S^2} \Big)$$

$$ds_7^2 = \frac{1}{\sin^2\beta} \Big(\frac{1}{(1-\lambda)^2 X^2} ds^2_{AdS_3} + \frac{\cos^2\beta}{(1+\lambda)^2 X^2} ds^2_{S^3} + X^8 d\beta^2 \Big)$$

$$ds_7^2 \sim \frac{dx^2_{1,1} + du^2 + u^2 (1-\lambda)^2 (1+\lambda)^{-2} ds^2_{S^3}}{u^2} + d\beta^2$$

Conical singularity in the $\mathbb{R}^4$ parametrised by u and S^3

Non-vanishing contribution of the defects to the central charge

Our interpretation:

The AdS3 solution describes 6d (1,0) CFTs deconstructed in terms of 2d (0,4) degrees of freedom.

The 2d quivers are explicit realisations of this

In order to describe 2d defects we need a genuine 2d theory

$$ds_7^2 = x^2\Big(\frac{dx_{1,1}^2 + du^2 + u^2 ds_{S^3}^2}{u^2}\Big) + \frac{dx^2}{\sqrt{c+x^4}} = x^2\frac{dx_{1,5}^2}{u^2} + \frac{dx^2}{\sqrt{c+x^4}},$$

$\Rightarrow$ Localised 6d theory

But both configurations have the same size and energy!

Same thing happens with giant graviton configurations:

In AdS7:

$$ds^2_{\mathrm{AdS}_7} = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 ds^2_{S^5}$$

D6 wrapped on (t, S^5, z) and propagating on a circle in S^2

$H = P_\phi$ for finite r

In AdS3:

$$ds^2_{\mathrm{AdS}_3} = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\chi^2$$

D6 wrapped on (t, χ, z, S^3, x)

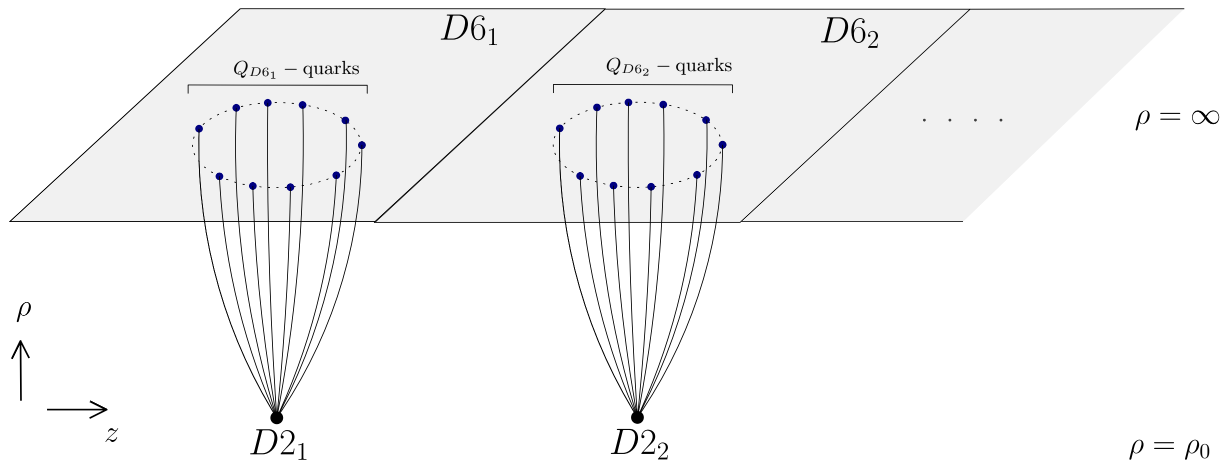
4. New $AdS_3 \times S^3 \times S^2 \times \Sigma_2$ solutions to massive IIA with (0,4) SUSY (large)

$$ds^2 = \sqrt{\frac{\alpha}{-\alpha''}} X^{-1/2} ds_7^2 + X^{5/2} \sqrt{\frac{-\alpha''}{\alpha}} \left(dz^2 + \frac{\alpha^2}{\alpha'^2 - 2\alpha\alpha'' X^5} ds_{S^2}^2 \right)$$

$$ds_7^2 = \frac{1}{\sin^2\beta} \left(\frac{1}{(1-\lambda)^2 X^2} ds_{AdS_3}^2 + \frac{\cos^2\beta}{(1+\lambda)^2 X^2} ds_{S^3}^2 + X^8 d\beta^2 \right)$$

$$X = (1 + \lambda \sin^2\beta)^{-1/5}$$

Baryon vertex in the 6d theory:



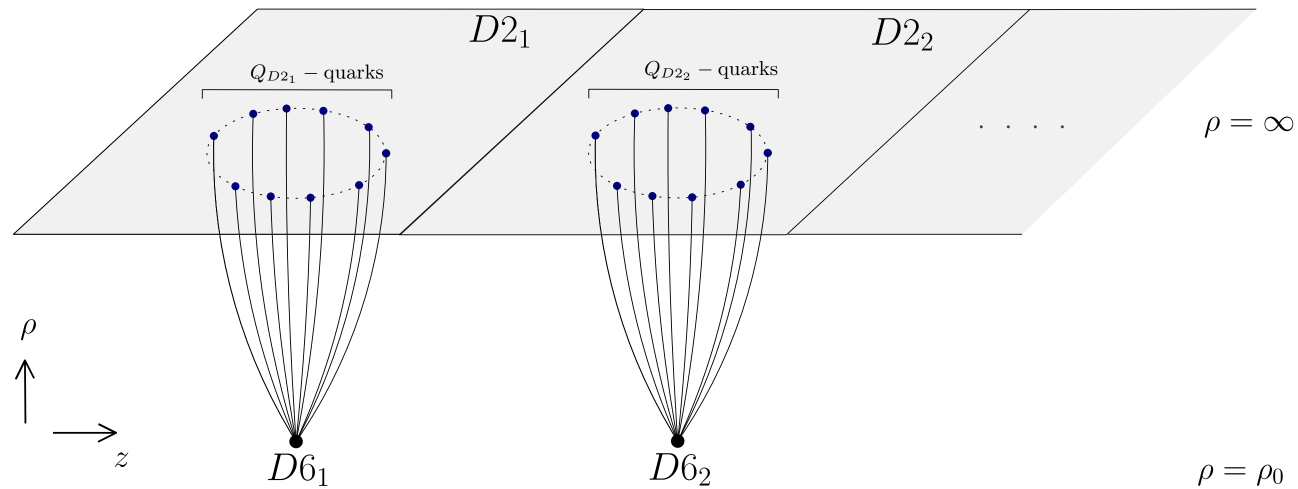
$$\frac{\rho^4}{\sqrt{\dot{\rho}^2+\rho^4}} = \rho_0^2\sqrt{\frac{15}{16}} \equiv \rho_0^2\beta$$

Size: $$\ell = \frac{\beta}{3\rho_0}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4},\frac{7}{4};\beta^2\right)$$

Energy: $$E_{\text{bin(string)}} = -\frac{f_k}{\ell}$$

$$f_k = \frac{2^{5/2}}{3}\beta\sqrt{\frac{\alpha_k}{-\alpha_k''}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4},\frac{7}{4};\beta^2\right)\left({}_2F_1\left(\frac{1}{2},-\frac{1}{4},\frac{3}{4};\beta^2\right)-\frac{1}{4}\right) = 1.26776\sqrt{\frac{\alpha_k}{-\alpha_k''}}$$

In the 2d theory:



$$\frac{\rho^4}{\sqrt{\dot\rho^2+\rho^4}} = \rho_0^2\sqrt{1-\frac{1}{16}\Big(1-\frac{c}{\tilde\Lambda^2\sqrt{c+\tilde\Lambda^4}}\,\mathrm{arcsinh}(\frac{\tilde\Lambda^2}{\sqrt{c}})\Big)^2} \equiv \rho_0^2\,\beta$$

$\tilde\Lambda \to \infty \;\; \to$ Same size and energy per string

Asymptote locally to $AdS_7 \times S^2 \times I$ but

$$ds_7^2 \sim \frac{dx_{1,1}^2 + du^2 + u^2(1-\lambda)^2(1+\lambda)^{-2}\cos^2\beta ds_{S^3}^2}{u^2} + d\beta^2$$

Conical singularity in the $\mathbb{R}^4$ parametrised by u and S^3

Non-vanishing contribution of the defects to the central charge

Baryon vertex configuration with size and energy λ dependent

N F-strings connecting the D5-brane to the boundary of AdS behave as fermions

Dual configuration on the CFT side: N Wilson lines ending on an epsilon tensor $\rightarrow$ Bound state of N quarks

Stability in the AdS direction (Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98):

Use the probe brane approximation

Consider: $S = S_{D5} + S_{NF1}$

$$S_{NF1} = -NT_{F1}\int dtdy\sqrt{\frac{16\rho^4}{L^4}+\dot{\rho}^2}$$

$\rho=\rho(y)$: position in AdS

Bulk equation of motion: $$\frac{\rho^4}{\sqrt{\frac{16\rho^4}{L^4}+\dot{\rho}^2}} = a$$

Boundary equation of motion: $$\frac{\dot{\rho}}{\sqrt{\frac{16\rho^4}{L^4}+\dot{\rho}^2}} = \frac{2T_5}{LNT_{F1}}$$

The two equations can be combined into:

$$\frac{\rho^4}{\sqrt{\frac{16\rho^4}{L^4}+\dot{\rho}^2}} = \frac{1}{4}\,\beta\rho_0^2\,L^2$$ where $$\beta = \sqrt{\frac{15}{16}}$$

Integrating: Size of the configuration:

$$\ell = \int_0^\ell dy = \frac{L^2}{4\rho_0}\int_1^\infty d\hat{\rho}\frac{\beta}{\hat{\rho}^2\sqrt{\hat{\rho}^4-\beta^2}}$$

On-shell energy:

$$E = E_{D5} + E_{NF1} = NT_{F1}\rho_0 \Big(\sqrt{1-\beta^2} + \int_1^\infty d\hat{\rho} \frac{\hat{\rho}^2}{\sqrt{\hat{\rho}^4 - \beta^2}} \Big)$$

Subtracting the energy of the constituents (when the brane is located at $\rho_0 = 0$ the strings become radial and correspond to free quarks) $\rightarrow$ Binding energy:

$$E_{\rm bin} = NT_{F1}\rho_0 \Big(\sqrt{1-\beta^2} + \int_1^\infty d\hat{\rho} \Big[\frac{\hat{\rho}^2}{\sqrt{\hat{\rho}^4 - \beta^2}} - 1 \Big] - 1 \Big) = -\frac{f}{l}$$

$f_{\rm string} = 0.035 > 0 \Rightarrow$ The configuration is stable