AdS N-body problem at large spin

Jeremy A. Mann

Based on **[2412.12328]** and W.I.P. with Petr Kravchuk

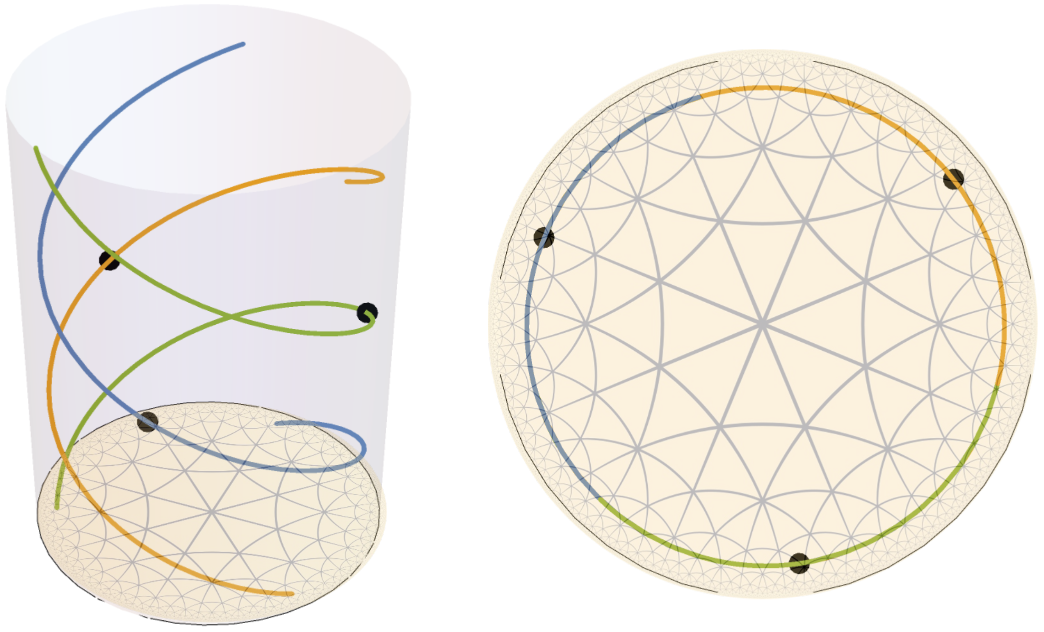
Eurostrings 2025, Stockholm

Introduction

What?

- Unitary & local QFT in AdS_{d+1}
- Lowest-E, N-particle states at spin J

Goal: $E' \coloneqq E - J$ at $J \gg 1$



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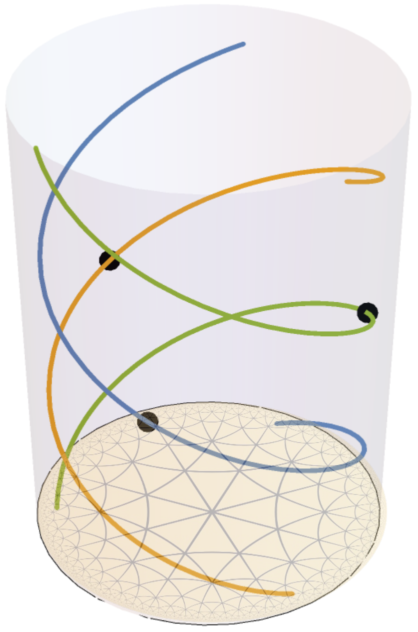

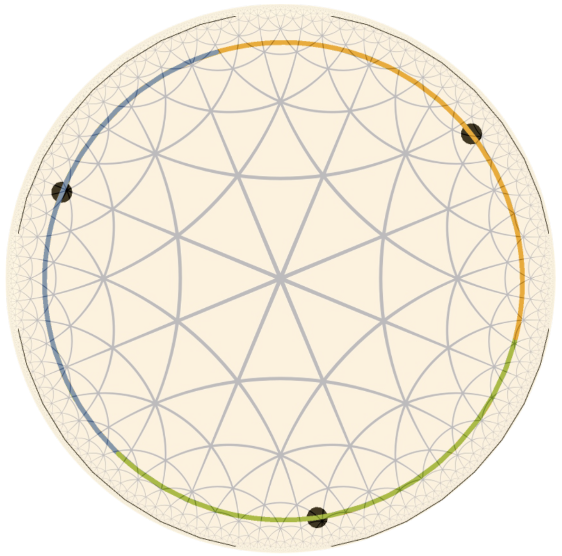

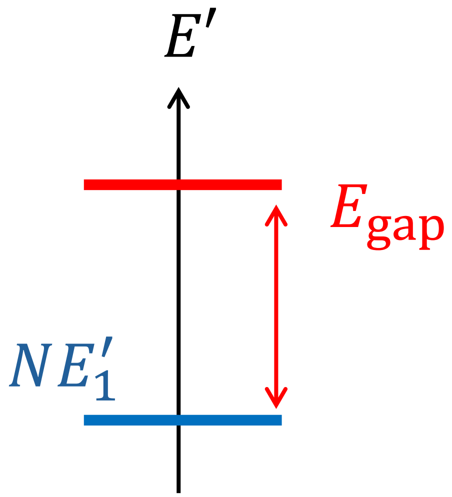

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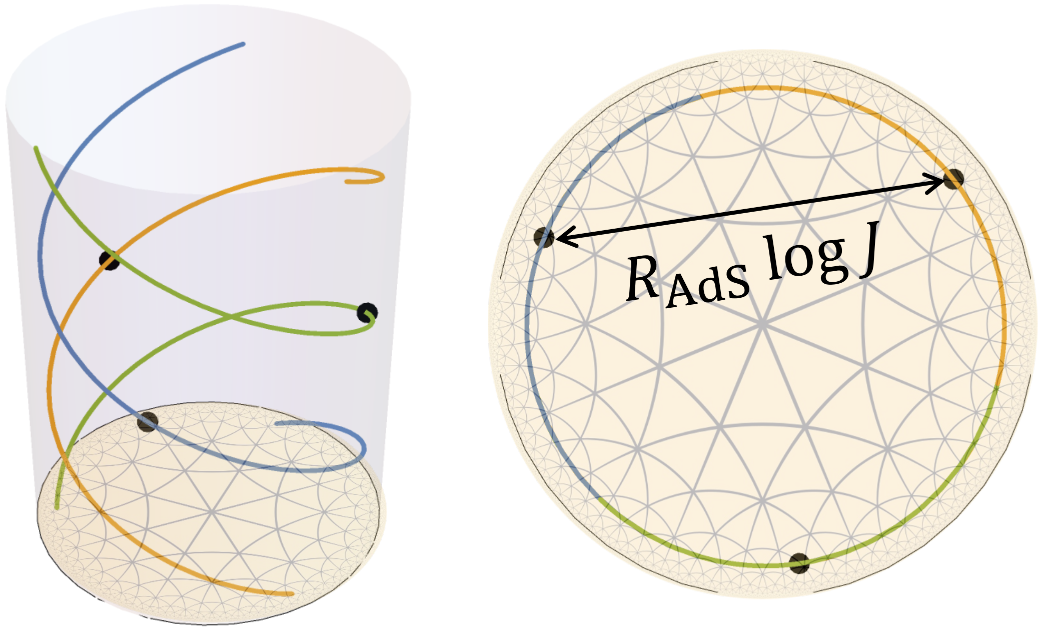
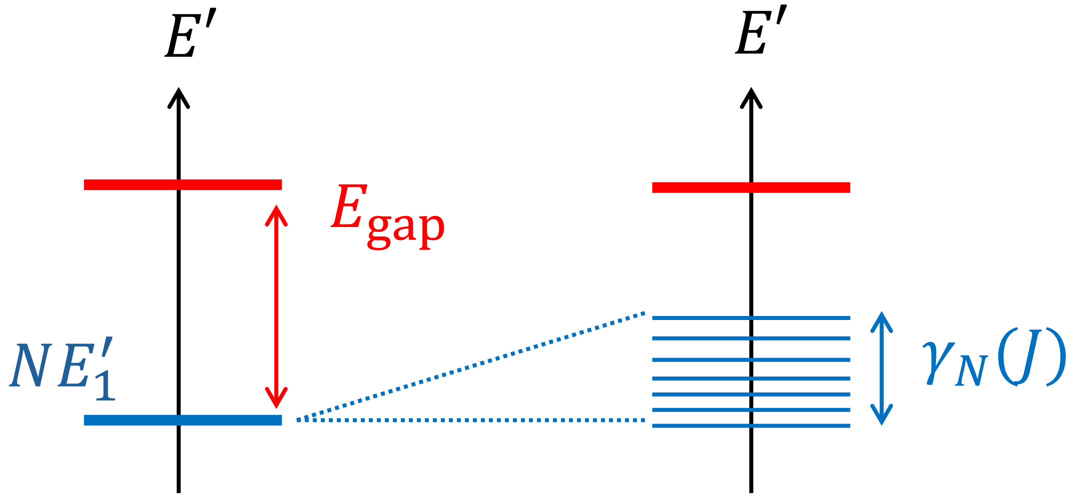
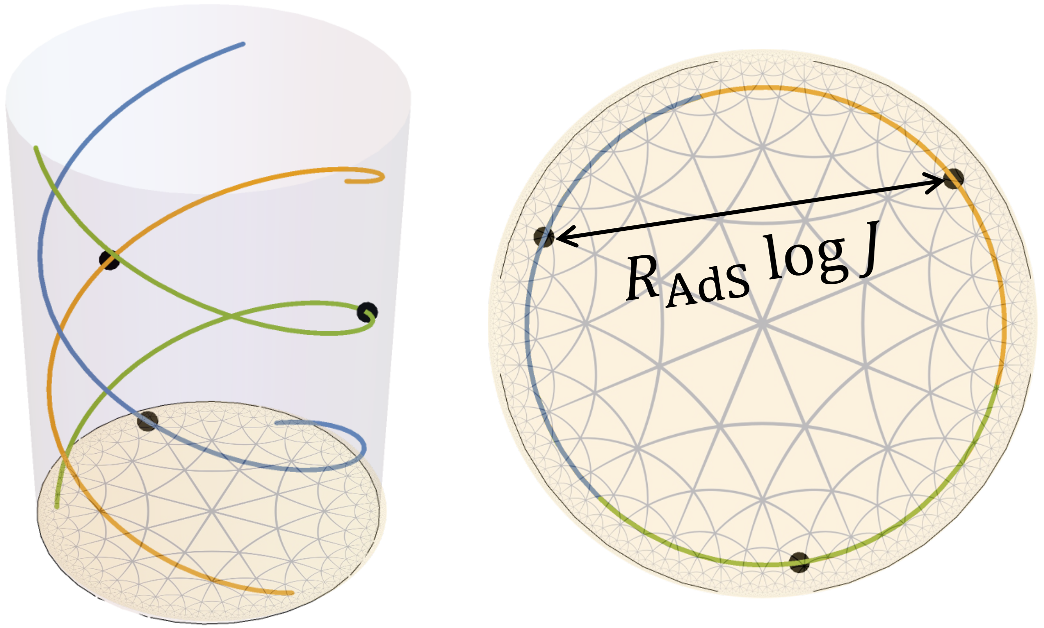




Introduction

Main result:

Most of the large-J spectrum is quantum mechanics with $\hbar = 1/J$



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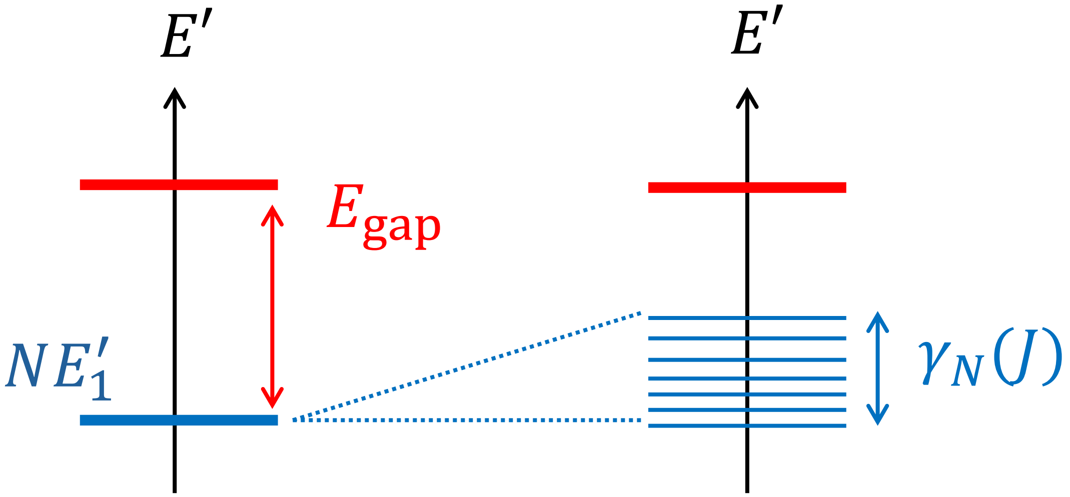

Motivation: “Coarse” holography [FKPS-D’13,…,FFL’24]

EFT in $\mathbf{AdS}_{d+1}$	**$\mathbf{CFT}_d$**
distance, $1/E' \gg R_{\mathrm{AdS}}$	Lightcone limits
locality	**crossing**
N-particle states $E' = NE_1' + \gamma_N(J)$	N-twist primaries $\Delta - J = N\Delta_\phi + \gamma_N(J)$

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Conjecture:

Same N-body problem at large J for $\mathbf{CFT}_d$

Evidence: Lightcone bootstrap of $\langle \phi \ldots \phi \bar{\phi} \ldots \bar{\phi} \rangle$

Plan

1. States

2. Tree-level interactions

3. $J = 1/\hbar$

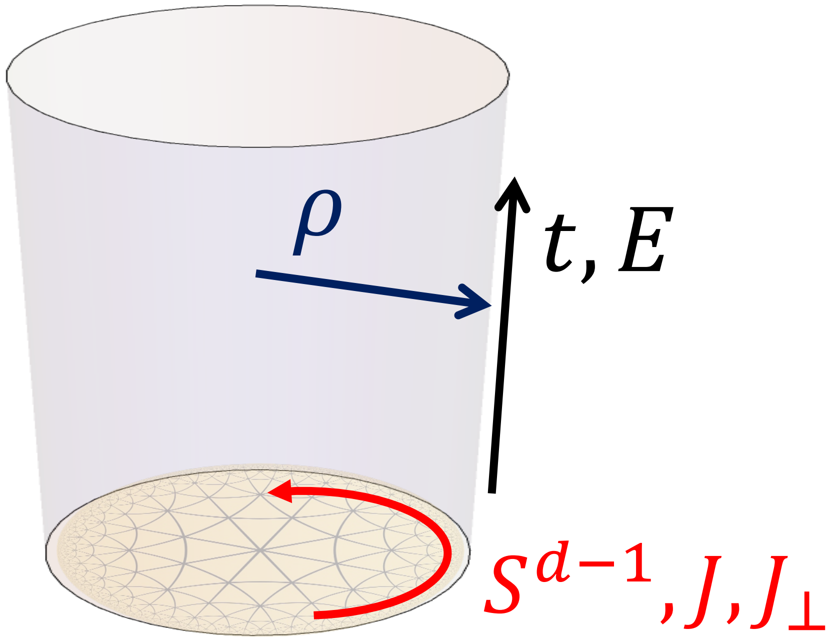
4. Lightcone bootstrap

1 States

- Free scalar Φ, $m_\Phi^2 = \Delta_\phi(\Delta_\phi - d)$

- **Here:** $\min E' = N\Delta_\phi,\ J_\perp = 0 \Rightarrow$

  $\langle 0|\bar{\Phi}(x_1)\ldots\bar{\Phi}(x_N)|\Psi\rangle = (\text{prefactor}) \times \psi(\alpha_1, \ldots, \alpha_N)$



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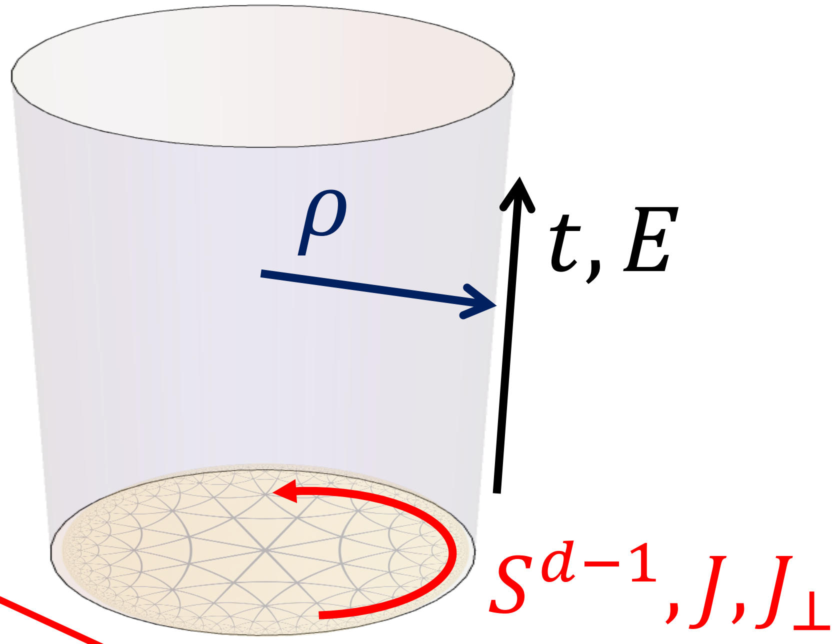

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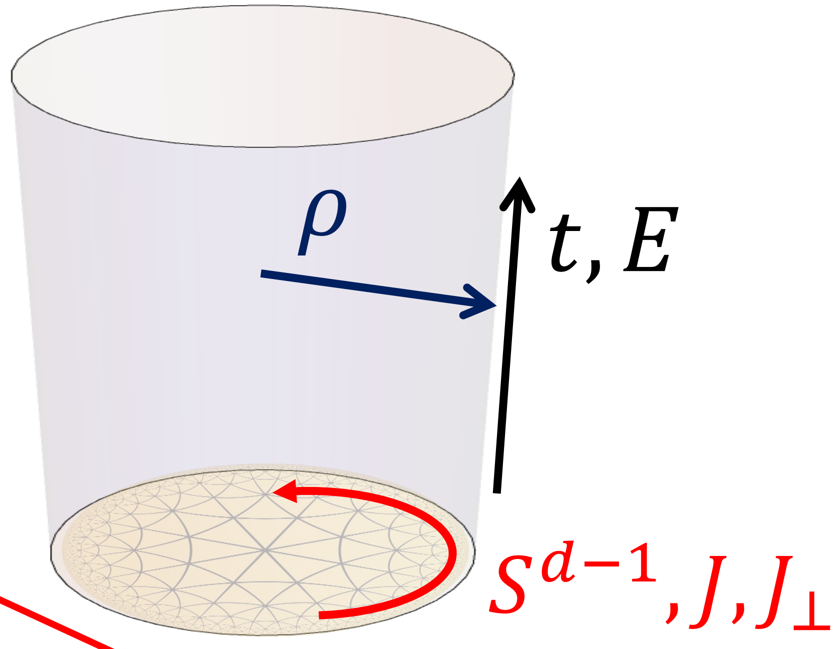

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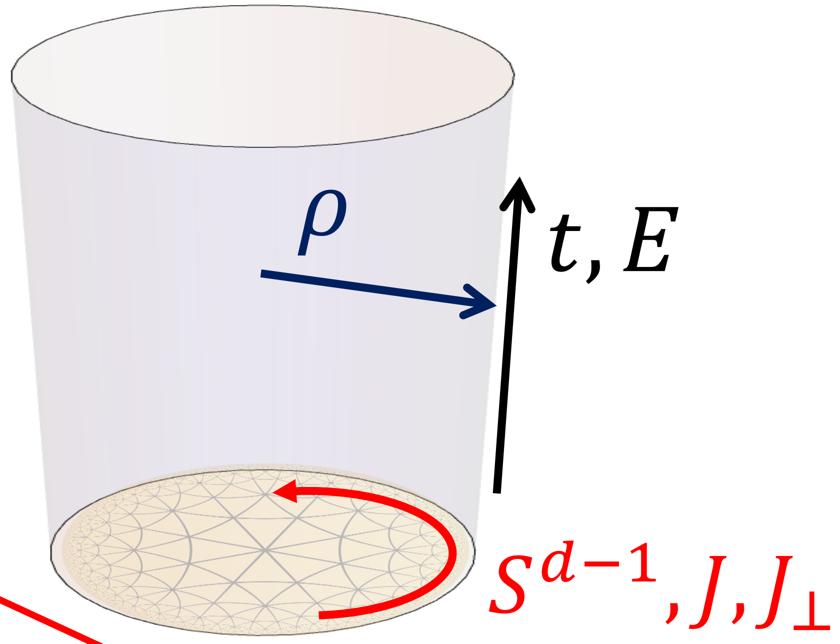

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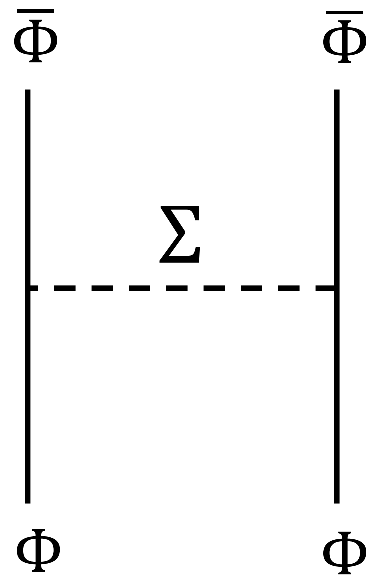
- $\mathrm{Iso}(\mathbb{D}) = SU(1,1)$ **lowest-weight states**:

$$\boxed{\psi(\lambda\alpha_i + \beta) = \lambda^J\, \psi(\alpha_i), \qquad \dim \mathcal{H}_{N,J} = \frac{J^{N-2}}{N!\,(N-2)!} + \cdots}$$

2 Tree-level interactions

- $\mathcal{L}_{\text{int}} = \Lambda\, \bar{\Phi}\Phi\Sigma$, integrate out Σ



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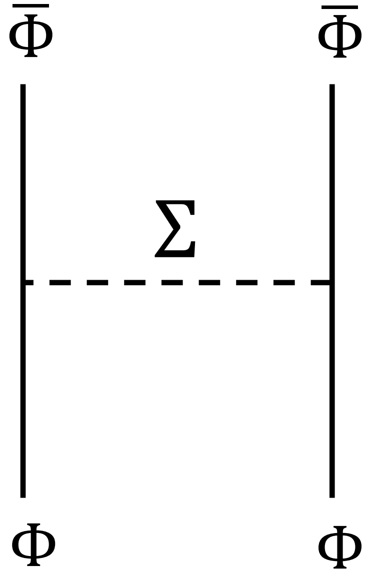
$$\langle \Psi_1 | \gamma_N^{\text{tree}} | \Psi_2 \rangle = \langle \Psi_1 | V_{\text{int}} | \Psi_2 \rangle = \langle \psi_1(\alpha) | U_N(\alpha, \bar{\alpha}) | \psi_2(\alpha) \rangle$$



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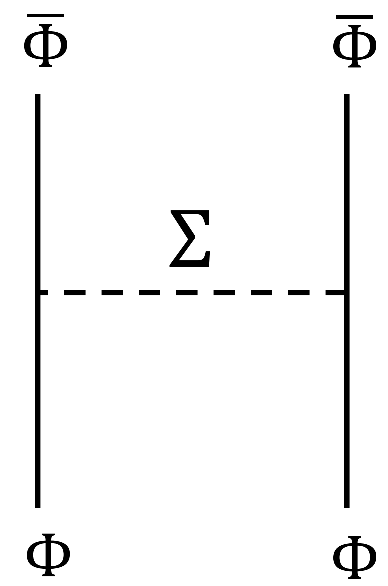

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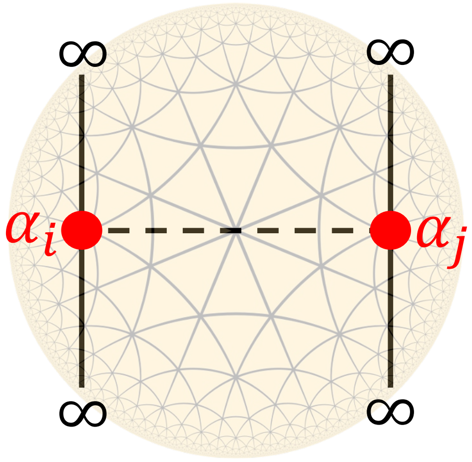
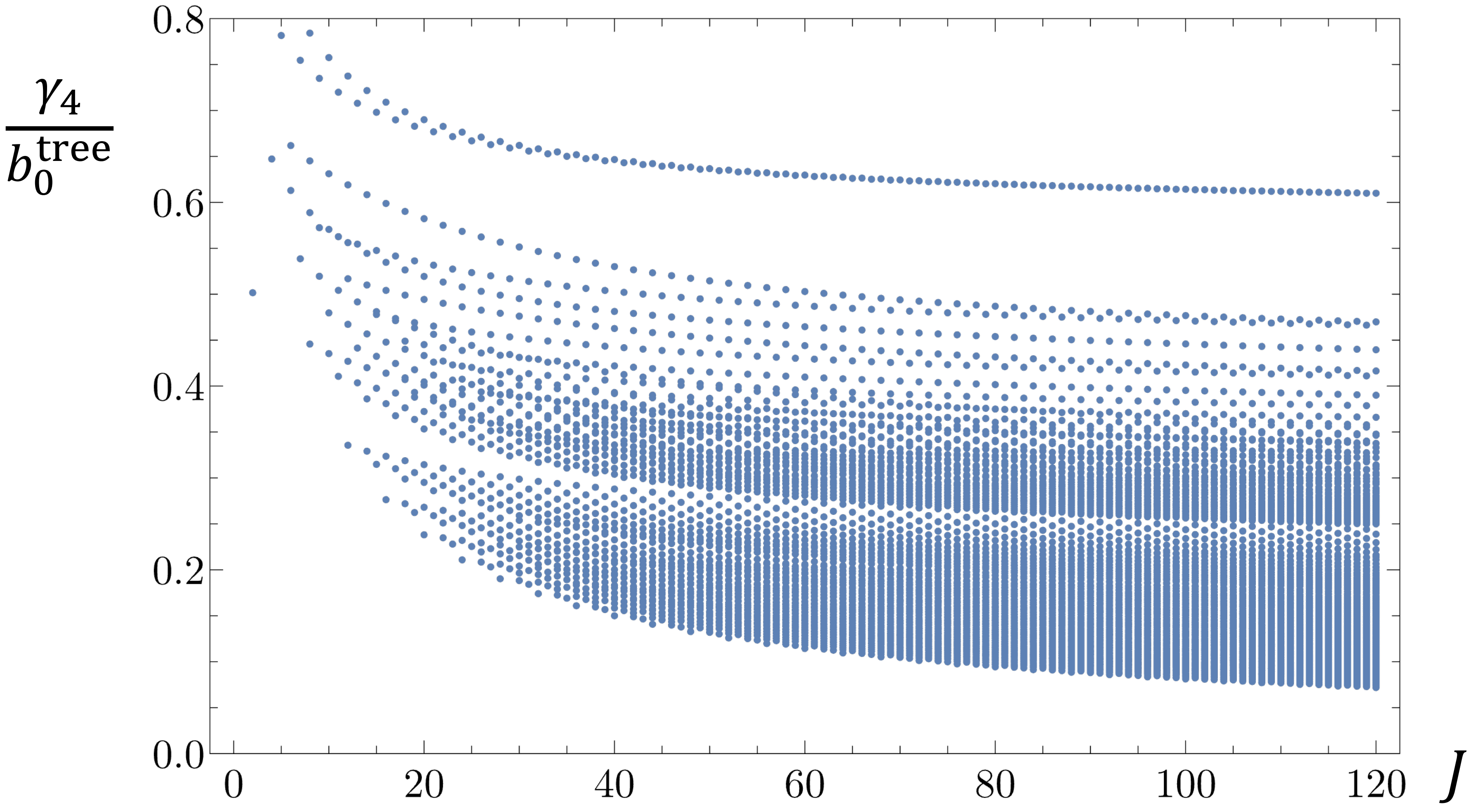
$$U_N = \sum_{i<j}$$



$$\sim b_0^{\mathrm{tree}} \sum_{i<j} \left(\frac{(1 - |\alpha_i|^2)(1 - |\alpha_j|^2)}{|\alpha_i - \alpha_j|^2} \right)^{\frac{\Delta_\sigma}{2}}$$

2 Tree-level interactions

- Example: $N = 4,\ J \leq 120,\ \dim \mathcal{H} \leq 331,\ (\Delta_\phi, \Delta_\sigma) = (1.234, 0.6734)$



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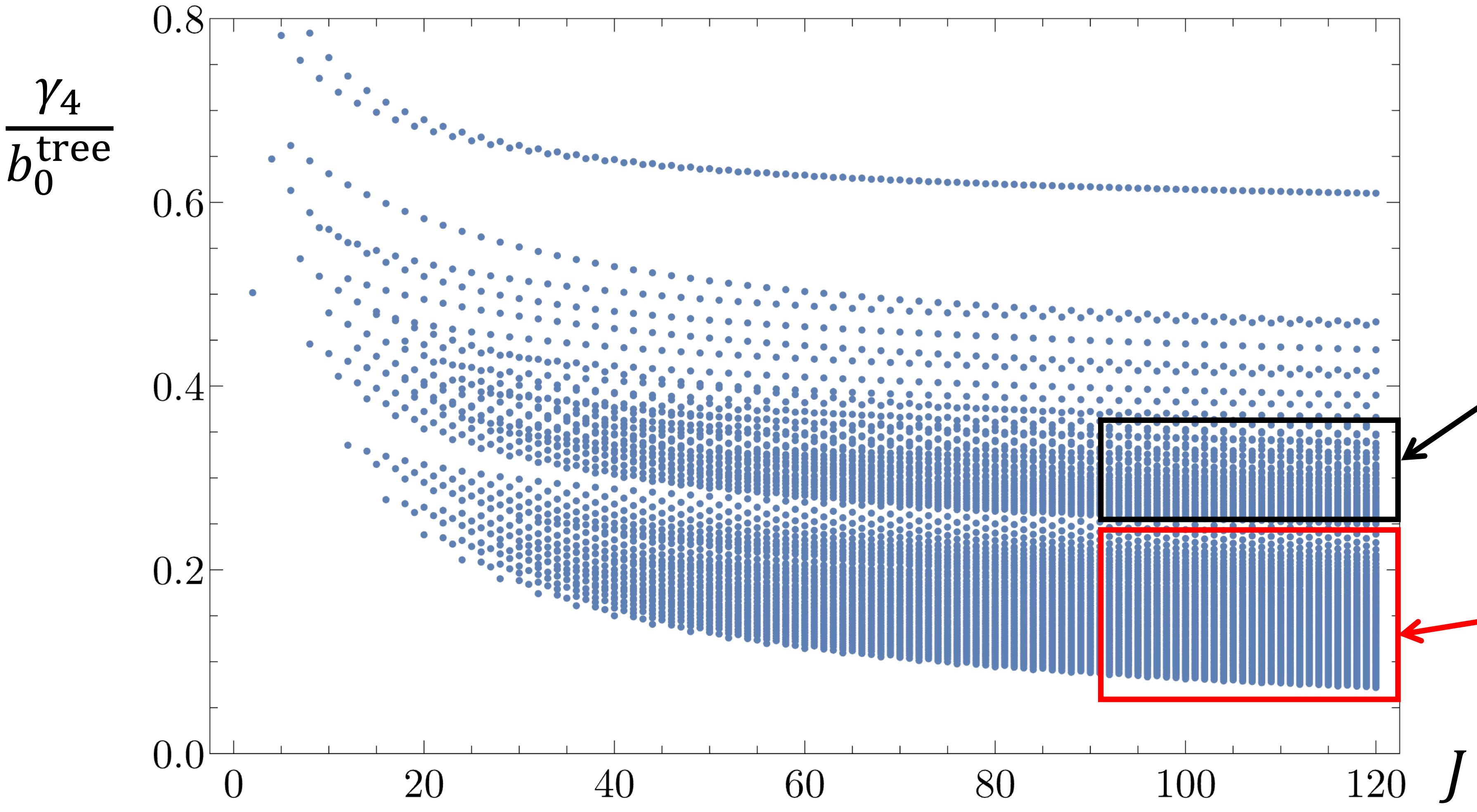

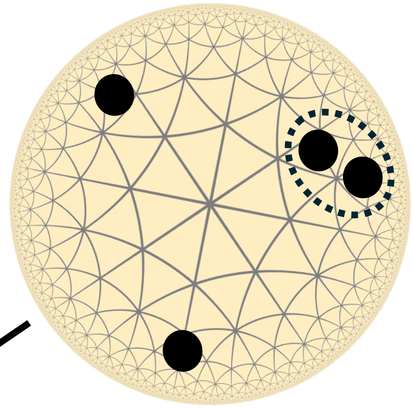

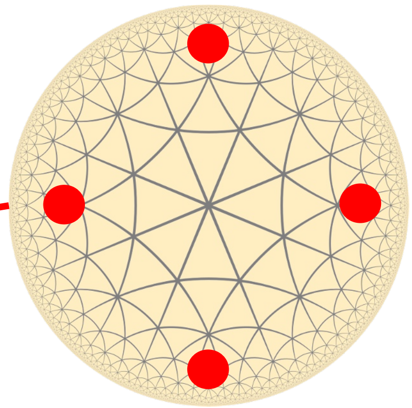

3 $J = 1/\hbar$

Berezin-Toeplitz quantization **[Berezin 1980]**

1. Holomorphic $\tilde{\psi}(z)$ on $\mathbb{C}P^{N-2}$
2. Measure $d\mu_J \sim e^{-J\boldsymbol{\mathcal{K}}} J^{\#}(\ldots)$
3. "Toeplitz" operator: $P_{\mathrm{hol}}\, \mathcal{U}_J(z,\bar{z}),\ \ \mathcal{U}_J \sim J^{\#}\boldsymbol{H} + \cdots$

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$\Rightarrow$ $\boxed{\mathcal{M}_{\mathrm{class}} = \boldsymbol{\mathcal{K}}\text{ähler}, \qquad \omega = i\partial\bar{\partial}\boldsymbol{\mathcal{K}}/2, \qquad \boldsymbol{H} = \text{Hamiltonian}}$

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“Momentum space”

- $\psi(\alpha) \to \tilde{\psi}(z), \partial_\alpha \to z,\;\; HL^2(\mathbb{D}) \to HL^2(\mathbb{C})$
- $SU(1,1)$ lowest weight $\to z_1 + \cdots + z_N = 0,\;\; \tilde{\psi}(\lambda z) = \lambda^J \tilde{\psi}(z)$

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Classical limit:

- $\mathcal{M}_{\text{class}} = \text{Gr}^+(2, N)/\mathbb{Z}_N$

- $\boldsymbol{\mathcal{K}} = 2 \log \sum_i |z_i|$

- $\boldsymbol{H} = \# \sum_{i<j} \left(\frac{e^{\mathcal{K}/2}}{\text{Im} \sqrt{z_j \bar{z}_i}} \right)^{\Delta_\sigma}$

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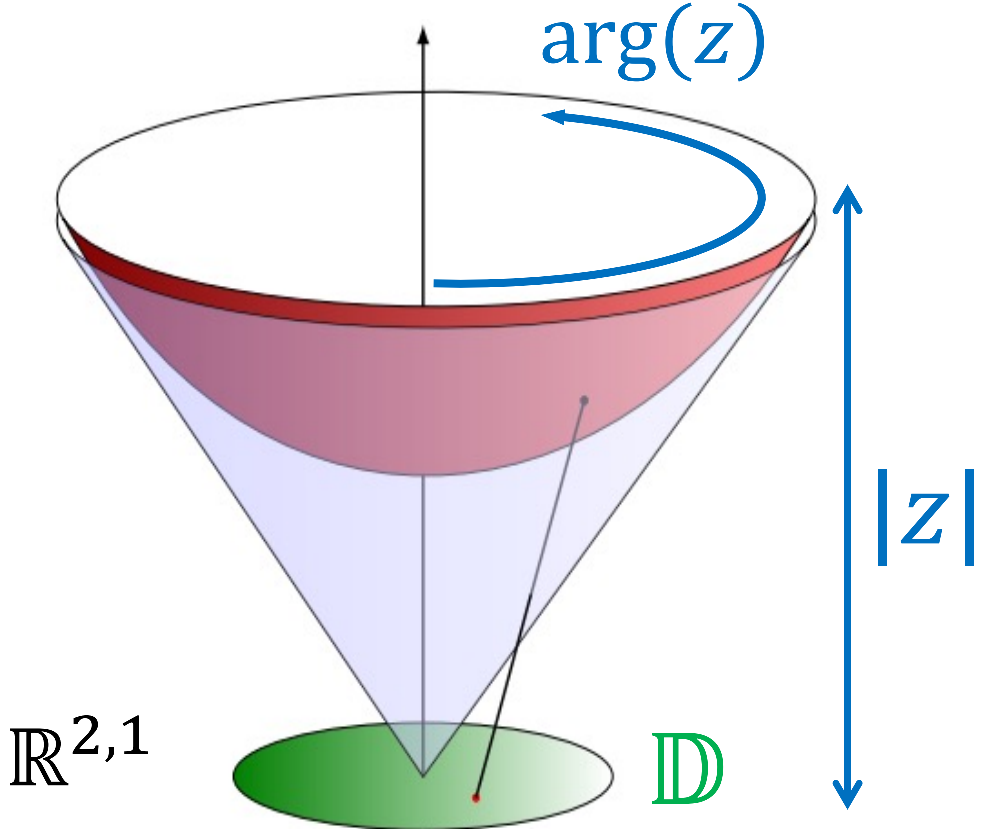

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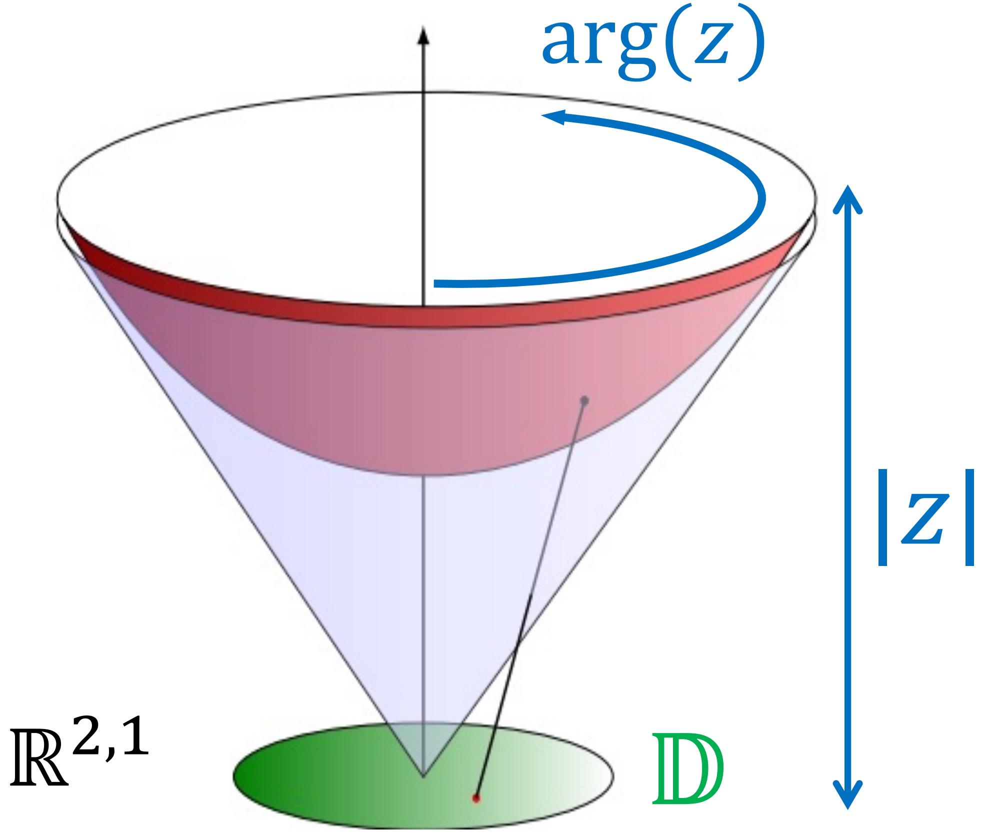

$\boldsymbol{H} = \boldsymbol{H}_{\text{min}}$ $\qquad$ $\boldsymbol{H} \to \infty$

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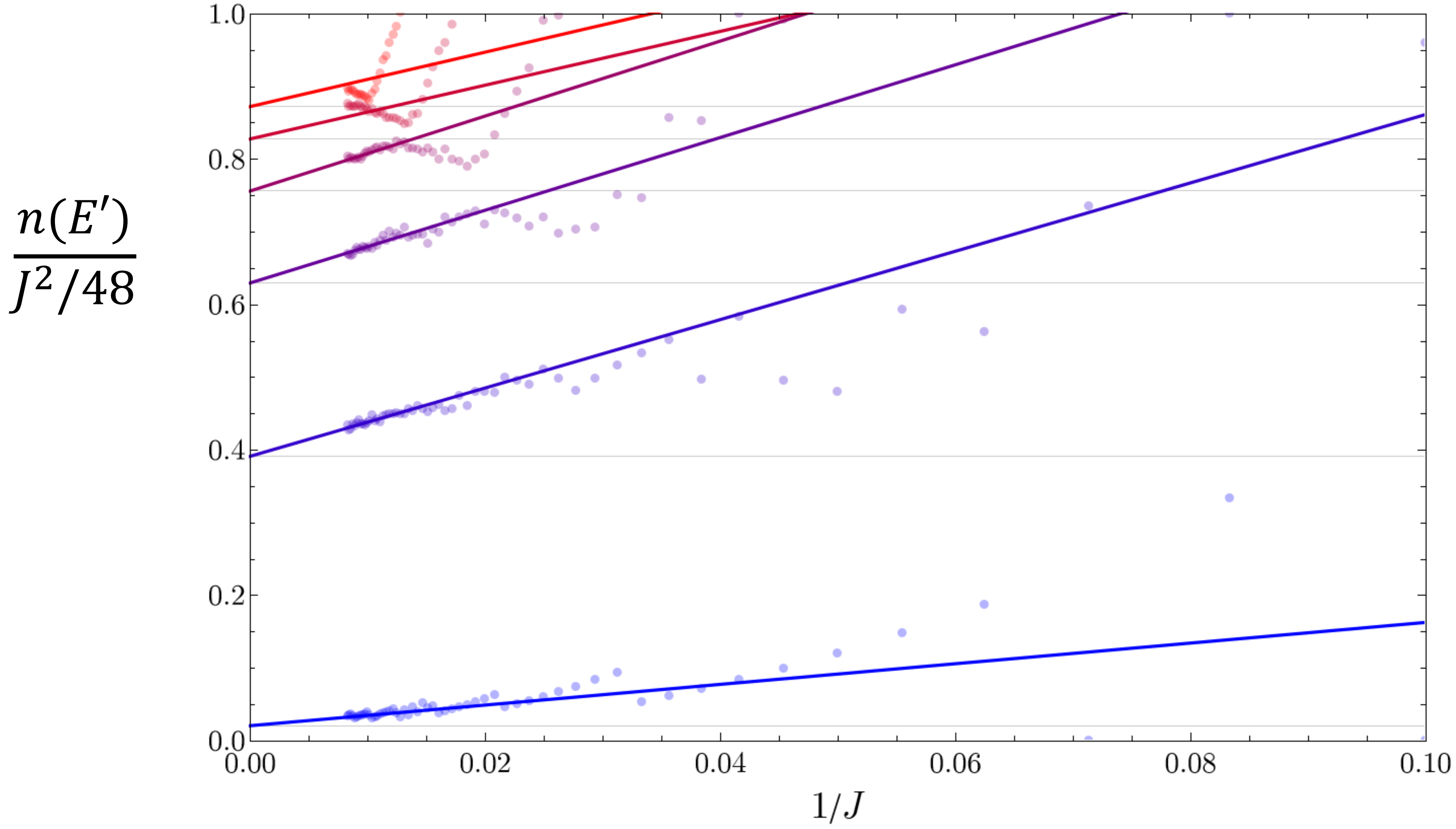
Density of states: $\frac{n\left(E'_\psi \leq E'\right)}{\dim \mathcal{H}} = \frac{\mathrm{Vol}(H \leq E')}{\mathrm{Vol}\,\mathcal{M}_{\mathrm{class}}} + O(\hbar)$

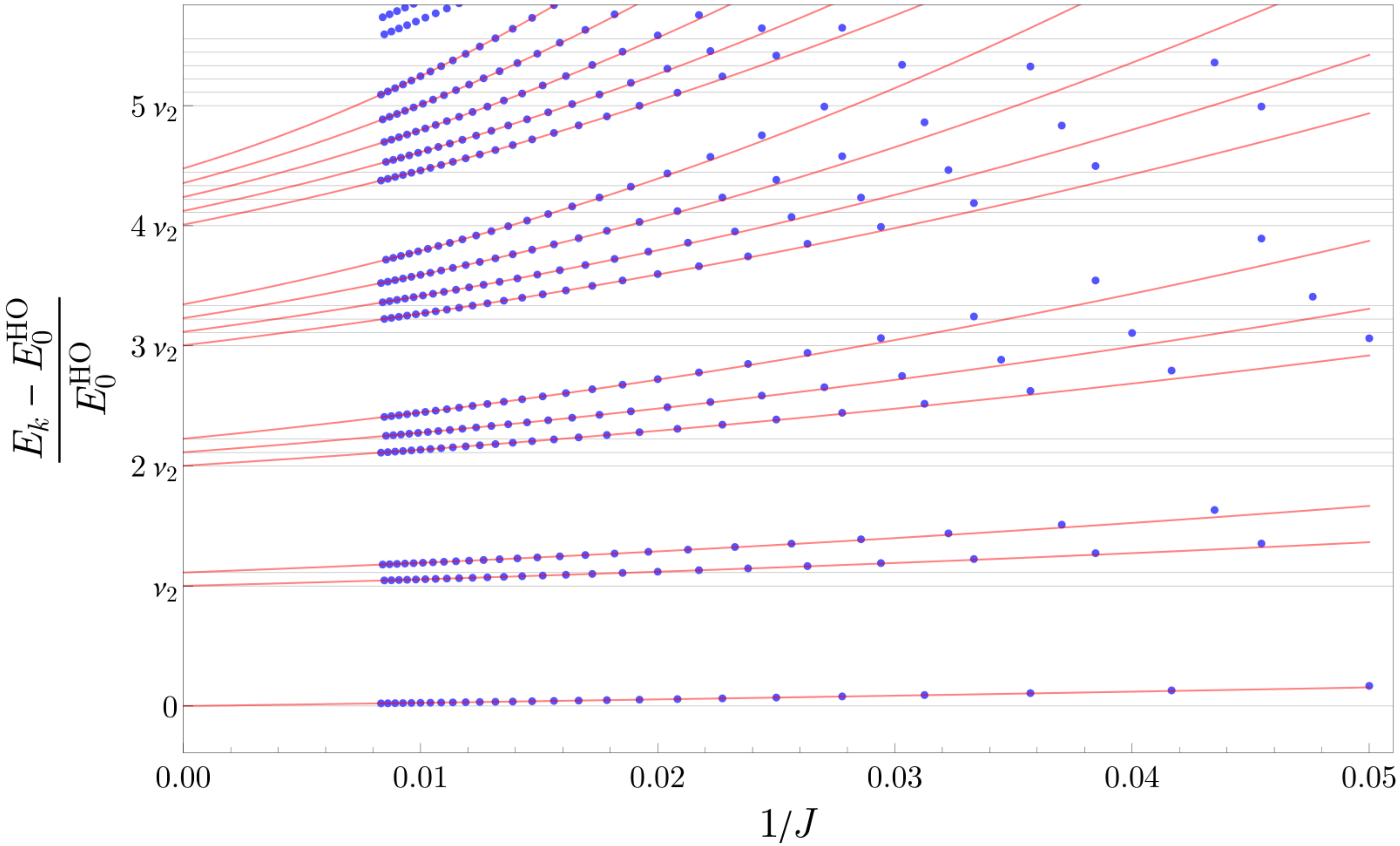


- $N = 4,\ J \leq 120$
- $\mathrm{Vol}(H \leq E')$ from Monte Carlo (10^5 sample points)

3 $J = 1/\hbar$

Lowest energies: harmonic oscillator

$$E_k = \boldsymbol{H}_{\min} + \hbar\left(E_0^{(1)} + \omega_1 k_1 + \cdots + \omega_{N-2} k_{N-2}\right) + O(\hbar^2)$$



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- $\omega_{1,2} = \boldsymbol{H}_{\min}\, \nu_{1,2}$

4 Lightcone bootstrap

1. $\mathbf{CFT}_d$ on $\mathbb{R}_t \times S^{d-1}$

2. **Weyl transformation** [AM'07]:

$\mathbb{R}\times S^{d-1} \cong \mathrm{AdS}_3 \times S^{d-3}$ with $\alpha_{\mathrm{AdS}_3} \equiv \sin\theta\, e^{i(\varphi-t)}$

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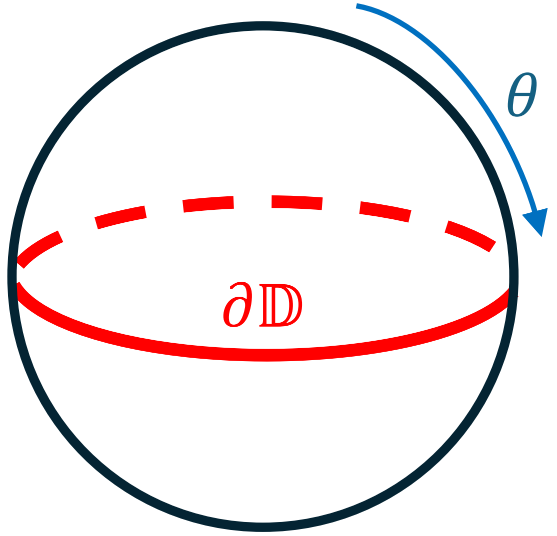

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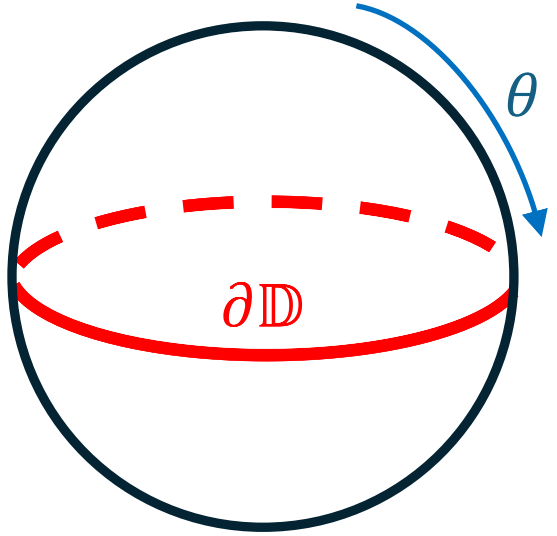

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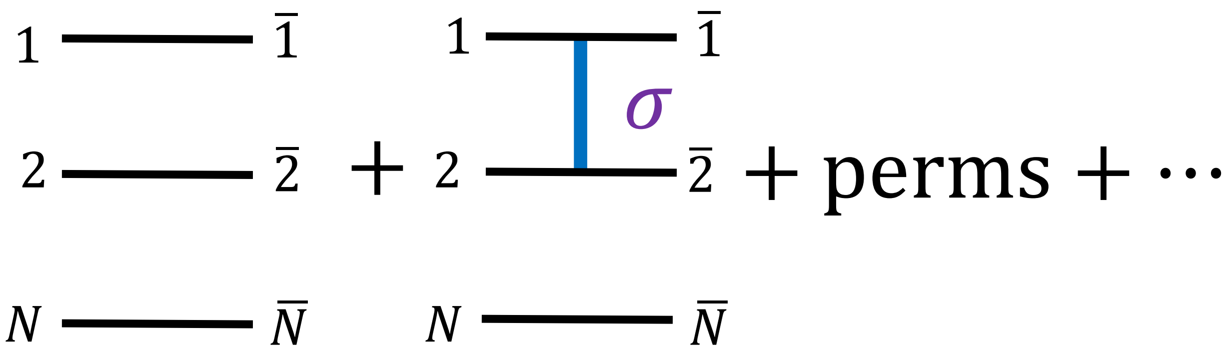

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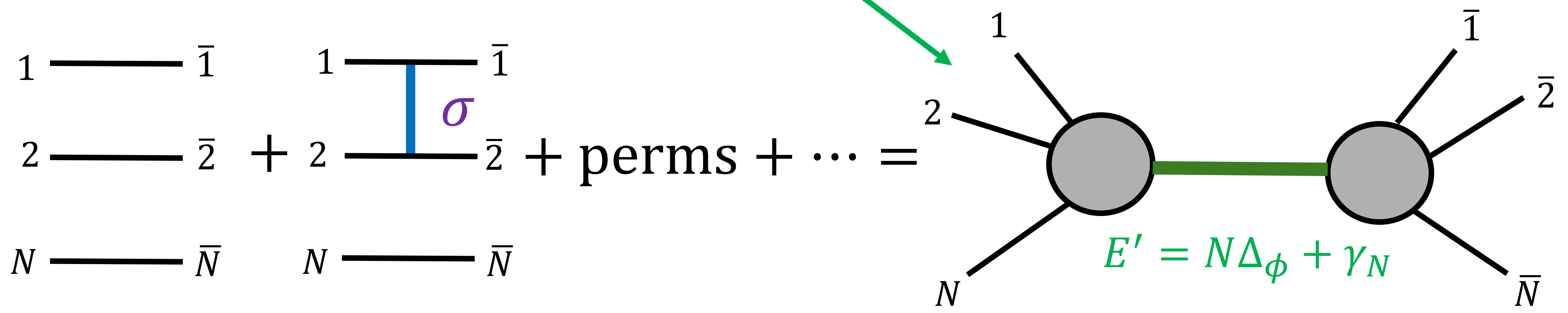

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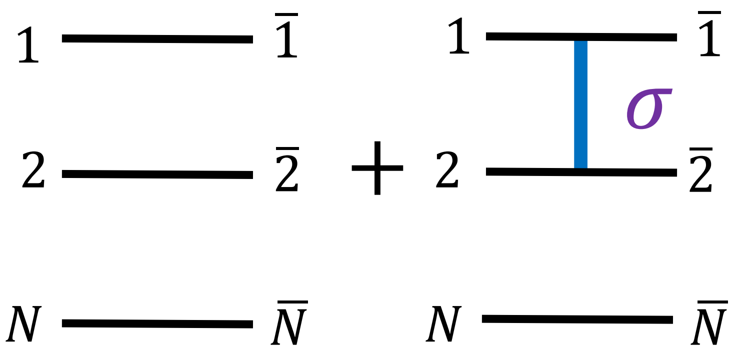
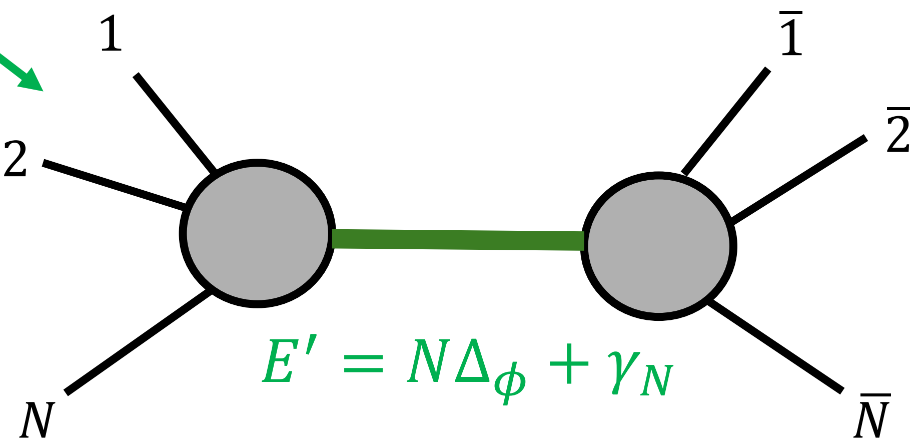


$+$ perms $+\cdots=$



$$\# \frac{\langle e_\alpha|\left(\frac{(1-|\alpha_i|^2)(1-|\alpha_j|^2)}{|\alpha_i-\alpha_j|^2}\right)^{\frac{\Delta_\sigma}{2}}|e_\alpha\rangle}{\langle e_\alpha|e_\alpha\rangle} = \frac{\langle \phi\ldots\phi\, P_{\mathcal{H}}\gamma_N\,\phi\ldots\phi\rangle}{\langle \phi\ldots\phi P_{\mathcal{H}}\phi\ldots\phi\rangle}$$

Outlook

- **Prove** $J = 1/\hbar$ for $\gamma_N(J)$ in $\mathbf{CFT}_{\boldsymbol{d}}$
- Lightcone bootstrap as **large-$\boldsymbol{J}$/low-$\boldsymbol{E'}$ $\mathbf{AdS}_{\boldsymbol{3}}$ EFT**
  → *c.f.* planar conformal gauge theory **[BGK‘03,…,AEKMS’10,…]**
  → *c.f.* **large-$\boldsymbol{J}$/near-threshold** for **S-matrix** **[CSZ‘22]**

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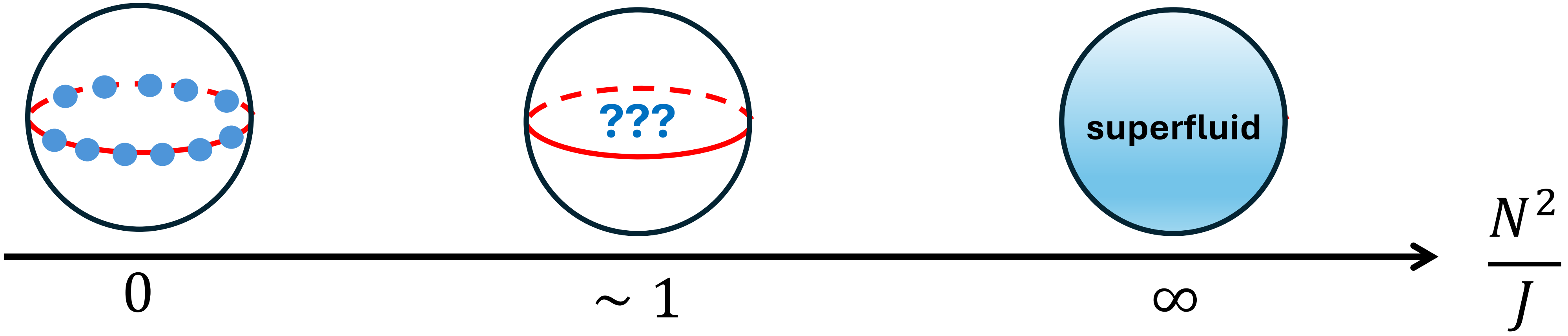

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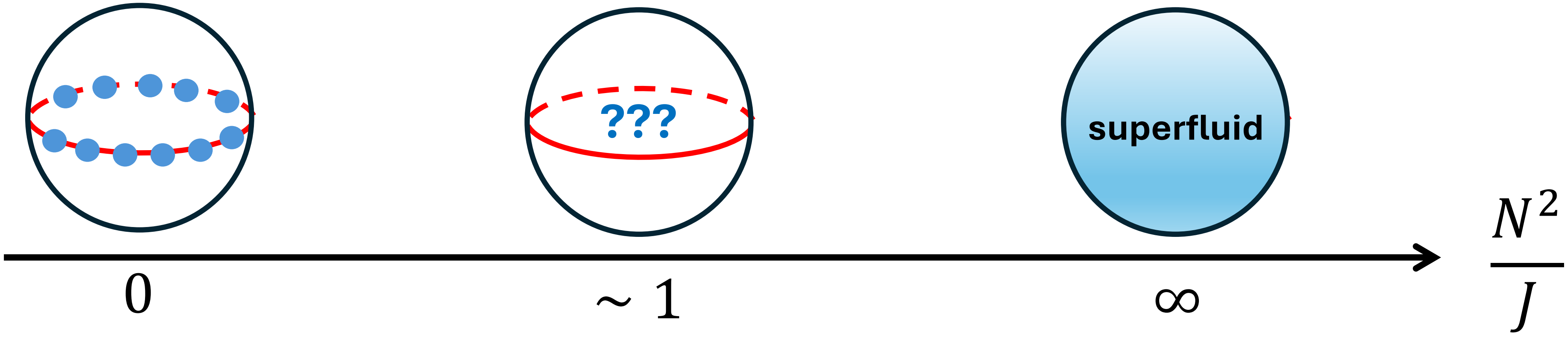

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