NON-INVERTIBLE SYMMETRIES IN 2d NON-LINEAR SIGMA MODELS

Eurostrings 2025 - Stockholm, 25th August 2025

Guillermo Arias Tamargo - Imperial College London

Based on work with Chris Hull and Max Velásquez Cotini Hutt

2503.20865 , 2508.tomorrow , WIP

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Modern understanding of generalised symmetries

&

Long-standing knowledge of T-duality

GENERALISED GLOBAL SYMMETRIES

- Action on fields $\longrightarrow$ topological operators (Gaiotto, Seiberg, Kapustin, Willet '14)

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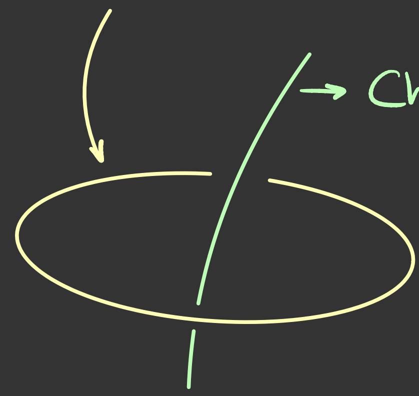
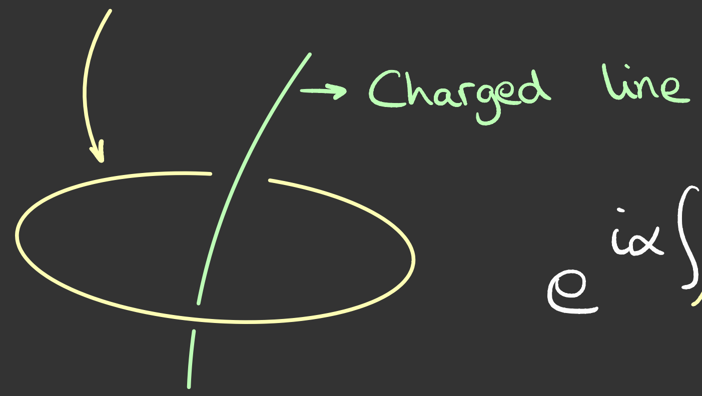


Charged line

$$e^{i\alpha\int_{M} \star j}$$

GENERALISED GLOBAL SYMMETRIES

- Action on fields $\longrightarrow$ topological operators (Gaiotto, Seiberg, Kapustin, Willet '14)
  - Higher form
  - Higher group
  - Non-invertible



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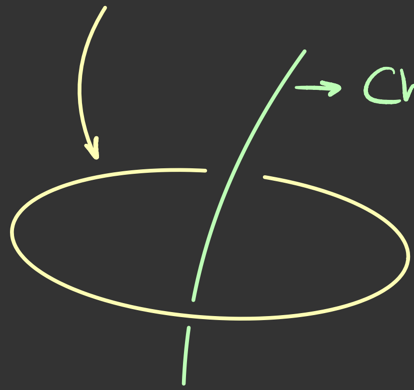

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- Some genuinely new predictions

- More systematic understanding of QFTs

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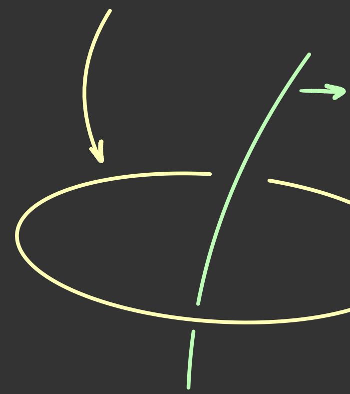

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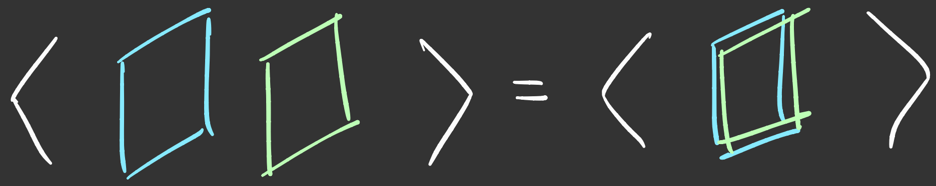
NON-INVERTIBLE SYMMETRIES

- Fusion of topological operators: OPE in correlation function



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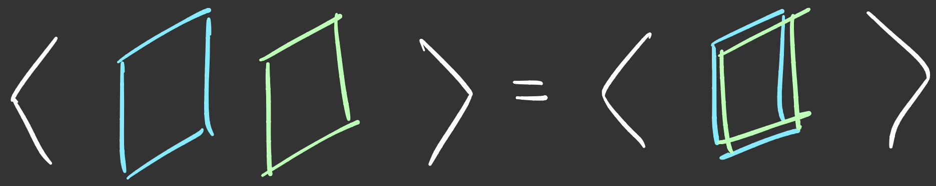

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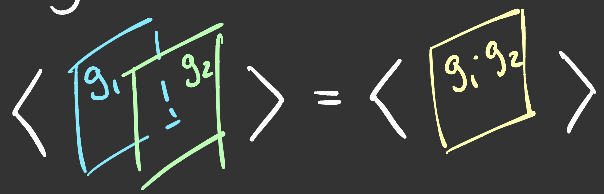

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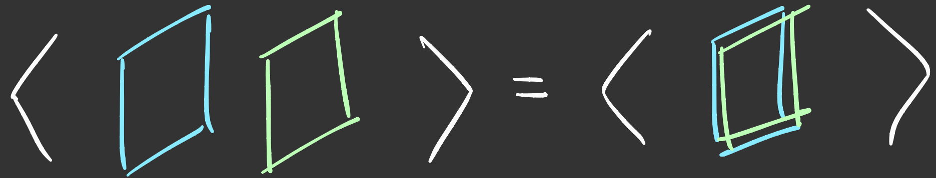

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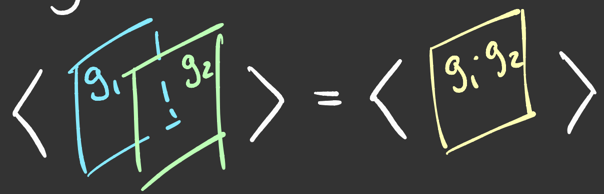

  - Non-invertible symmetry: no reason OPE has only one term

- Many ways to find non-invertible symmetries $\longrightarrow$ Here, half-space gauging

Prototypical example: compact boson in 2d

(Frolich, Fuchs, Runkel, Schweigert '06
Bachas, Brunner, Roggenkamp '12
Chang, Lin, Shao, Wang, Yin '18
Thorngren, Wang '21
Damia, Galati, Hulik, Mancani '24
Bharadwaj, Niro, Roumpedakis '24)

GOAL OF THE TALK

Generalise from the compact boson to

arbitrary* NLSM with Wess-Zumino term

OUTLINE

- Introduction
- Non-invertible defects in NLSMs
- Applications
- Conclusion and outlook

PART II

NON-INVERTIBLE DEFECTS IN NLSMs

HALF-SPACE GAUGING

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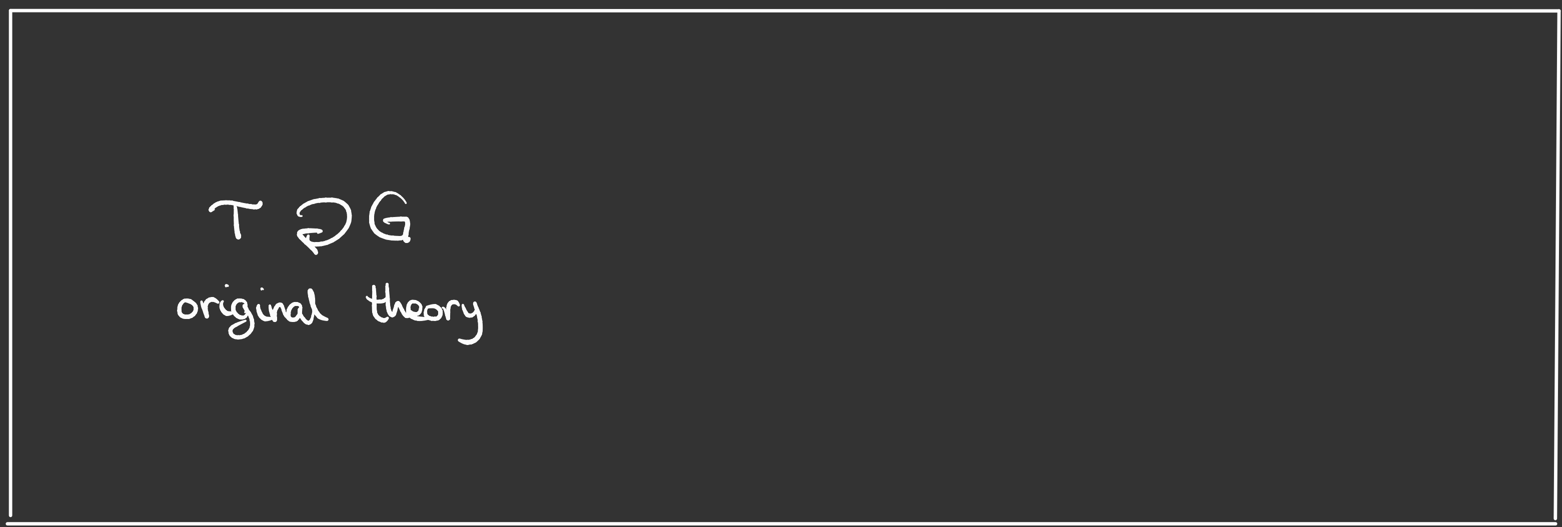
1 - Theory $\mathcal{T}$ in space W with finite global symmetry G

W



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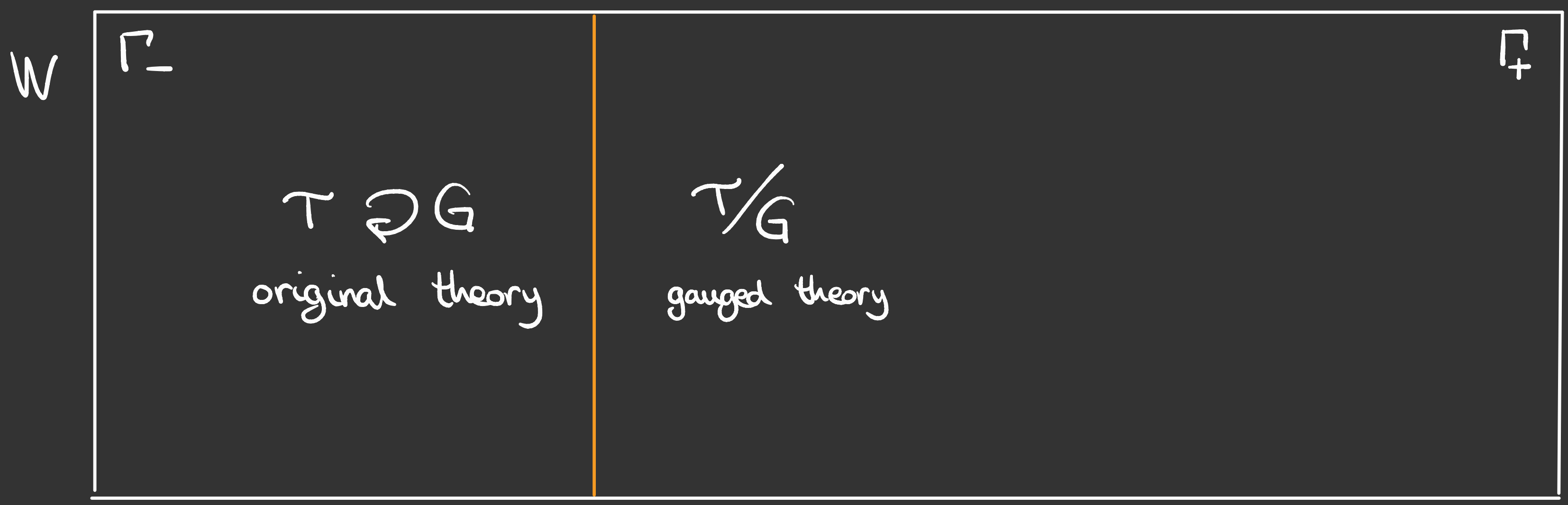

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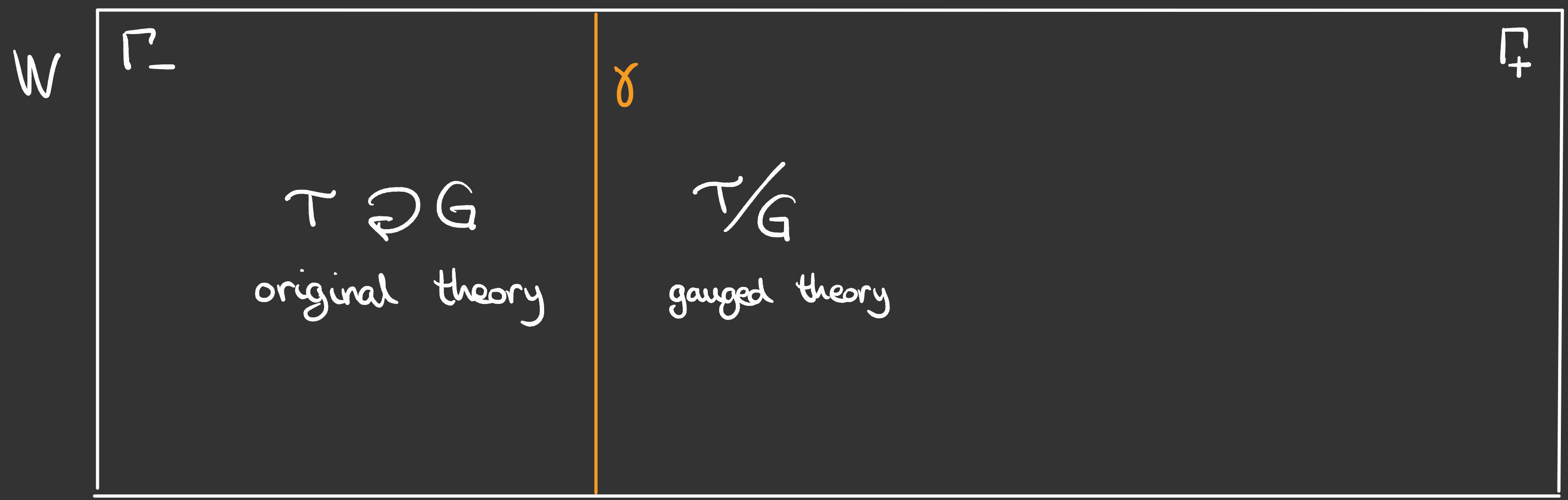

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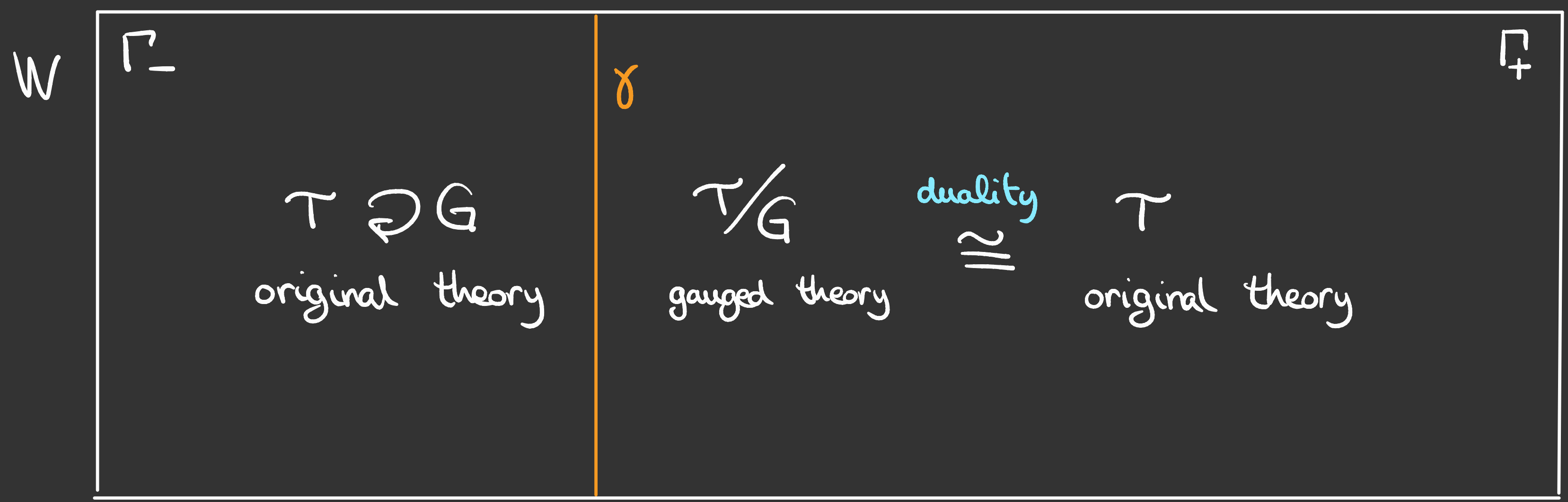

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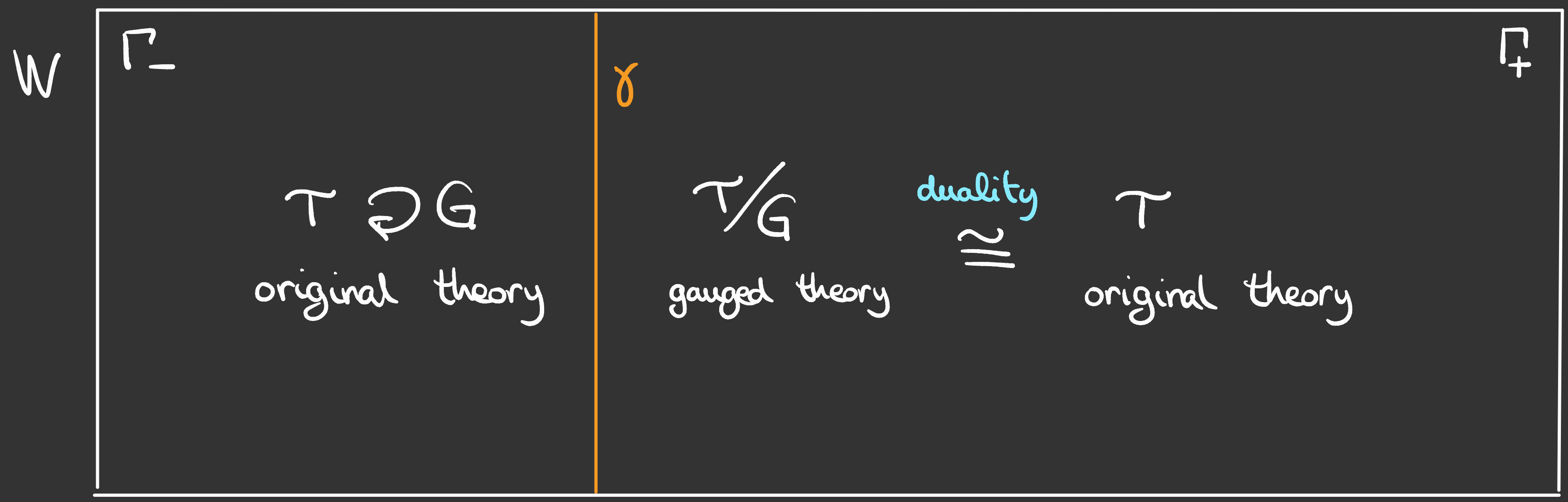
5 - Then, we have defined a topological defect of $\mathcal{T}$, $\mathcal{N}(\gamma)$

(Choi, Córdova, Hsin, Lam, Shao '21
Kaidi, Ohmori, Zheng '21)

$$\mathcal{N} \times \mathcal{N}^\dagger = \sum_{g \in G} U_g \longrightarrow \text{Non-invertible}$$



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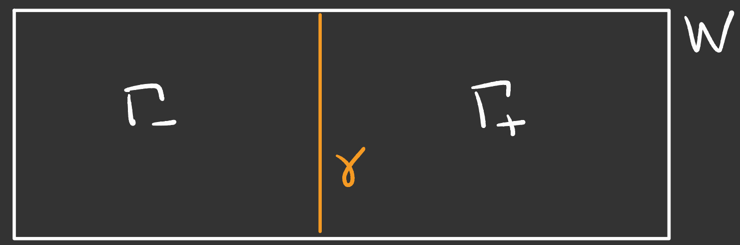

- In Γ_-: $\quad S_{\Gamma_-} = \frac{R^2}{2}\int_{\Gamma_-} d\phi \wedge * d\phi$

Shift symmetry $\phi \to \phi + \text{const.}$ $\quad U(1) \supset \mathbb{Z}_p$

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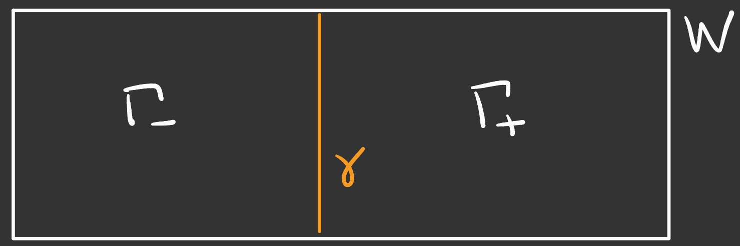

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- Needed: Shift symmetry + T-duality

NEXT : THE 3-SPHERE

- In Γ_- : S^3 (as Hopf fibration) + WZ

$$S_{\Gamma_-} = \frac{R^2}{8}\int_{\Gamma_-} (2d\phi - \cos\theta\, d\psi)^2 + d\Omega^2_{S^2} - \frac{k}{4\pi}\int_{\Gamma_-} \cos\theta \; d\psi \wedge d\phi$$

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- Self-duality conditions:

$$\boxed{R^2 = \frac{p}{2\pi} \qquad p = k}$$

$\rightarrow$ Non-invertible symmetry

$SU(2)_p$ WZW ?

NON-LINEAR SIGMA MODELS WITH WZ TERM

- Theory of maps $\Phi : W \longrightarrow M$. Choose local coordinates X^i and write action

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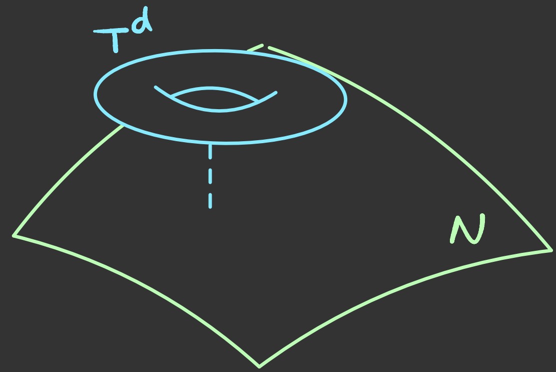

- $U(1)^d$ isometry of $M \longrightarrow$ fibration $T^d \to M \to N$

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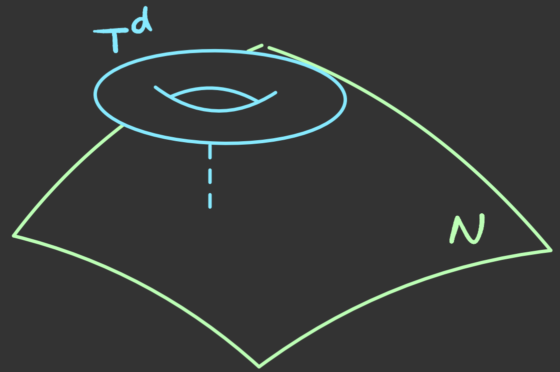

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- Two main characters: topological data of the fibration (Hull '06)

1. Connection associated with the metric $A^m \to \frac{1}{2\pi}\int dA^m$: Chern classes

2. $\iota_{K_m} H = \frac{1}{2\pi} dv_m$: also a connection 1-form $\longrightarrow \frac{1}{2\pi}\int dv_m$: H-classes

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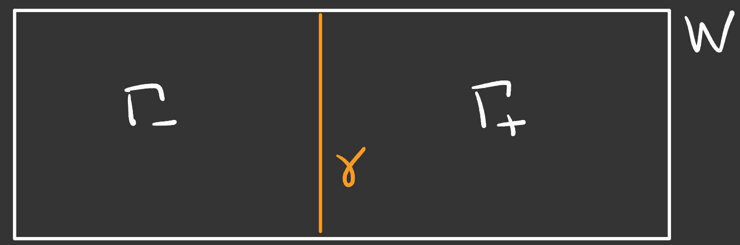

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SEE MAX'S POSTER

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     - Norm of Killing vectors $\quad G_{mn} = g_{ij}\, k^i_m k^j_n$
     - b-field along fibre $\quad B_{mn} = \hat{c}_m \hat{v}_n$

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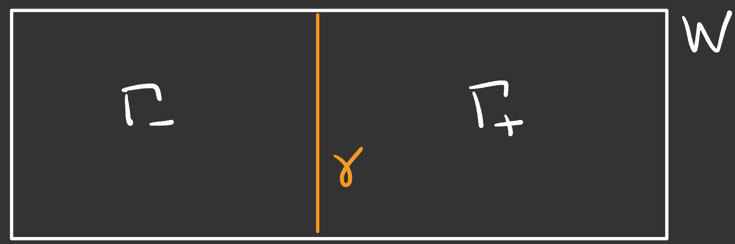

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- Examples:
  - Torus
  - Odd spheres and Lens spaces
  - General Wess-Zumino-Witten models: $SU(N)_k$, $Spin(N)_k$
  - Nilmanifold (not conformal)

PART III

APPLICATIONS

WARD IDENTITIES

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- Derived by nucleating defect and sweeping operator insertions

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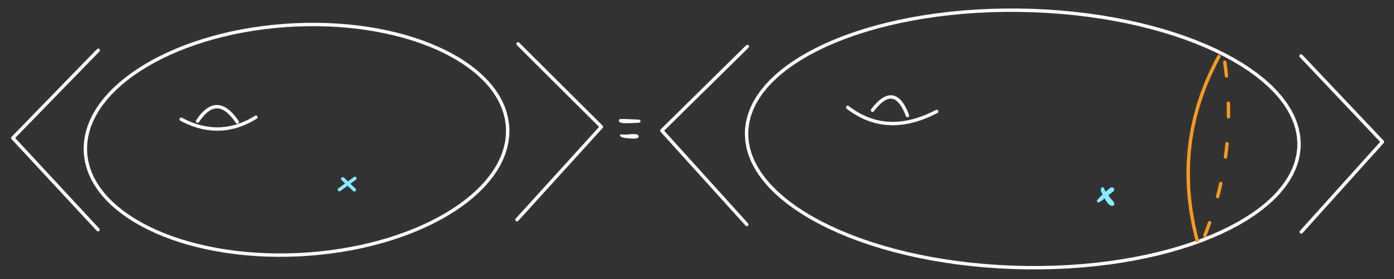

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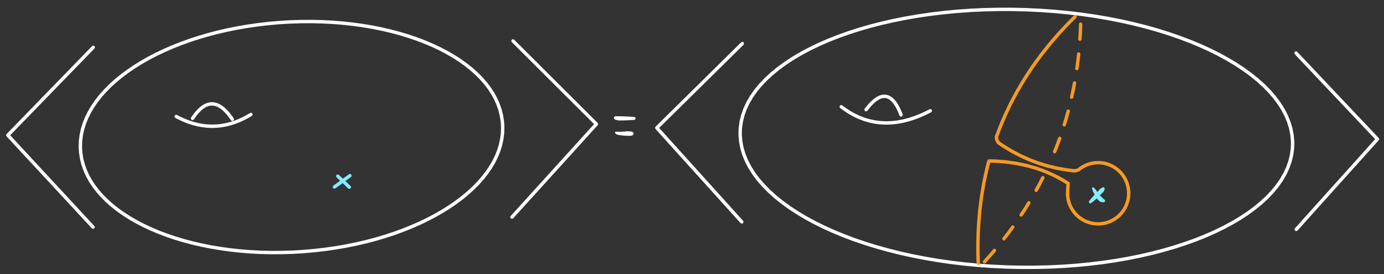

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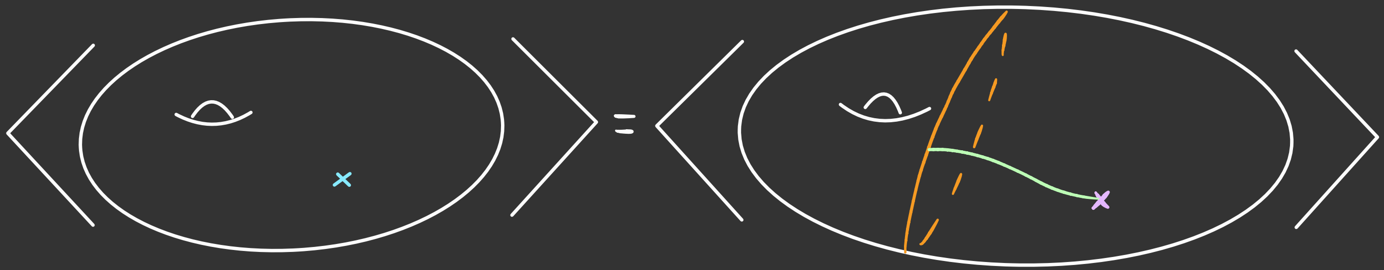

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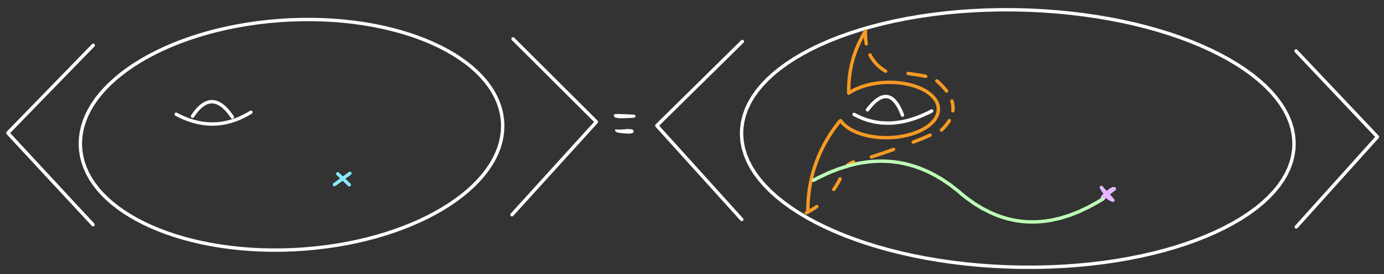

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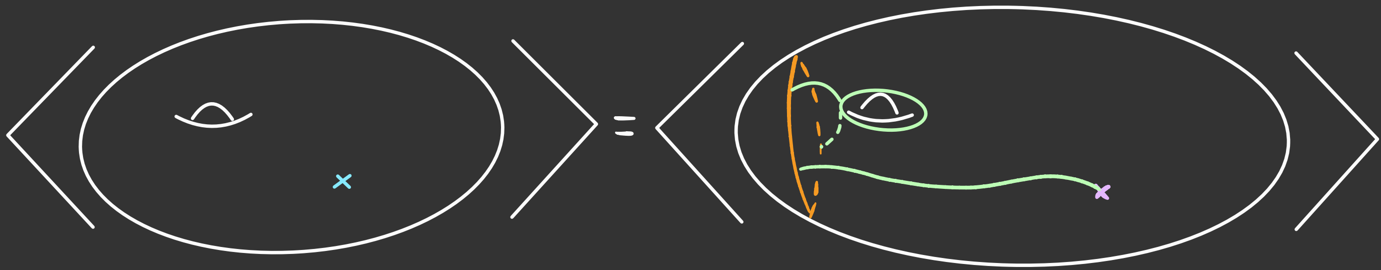

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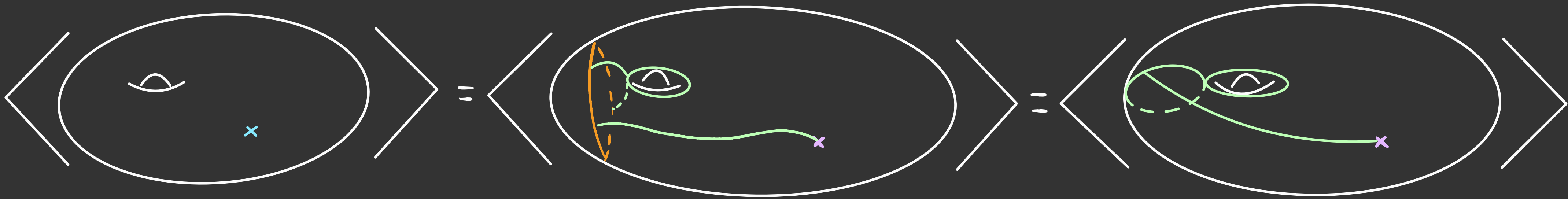

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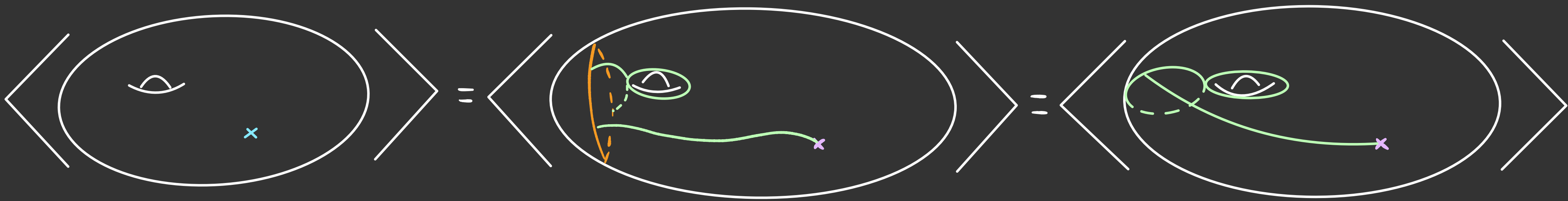

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Compact boson: momentum $\leftrightarrow$ winding

General NLSM: WIP

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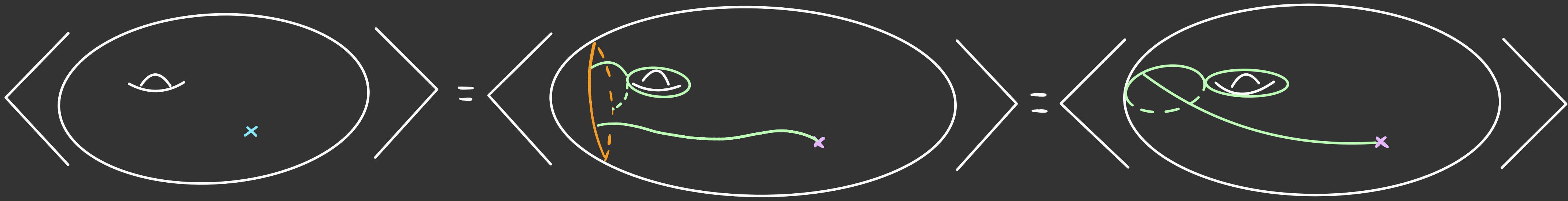

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  General NLSM: WIP

- Ward identities depend on the genus of the worldsheet

- In String Theory, the non-inverible is broken by the genus expansion

(Heckman, McNamara, Montero, Sharon, Vafa, Valenzuela '24
Kaidi, Tachikawa, Zhang '24)

THE DEFECT IN TARGET SPACE

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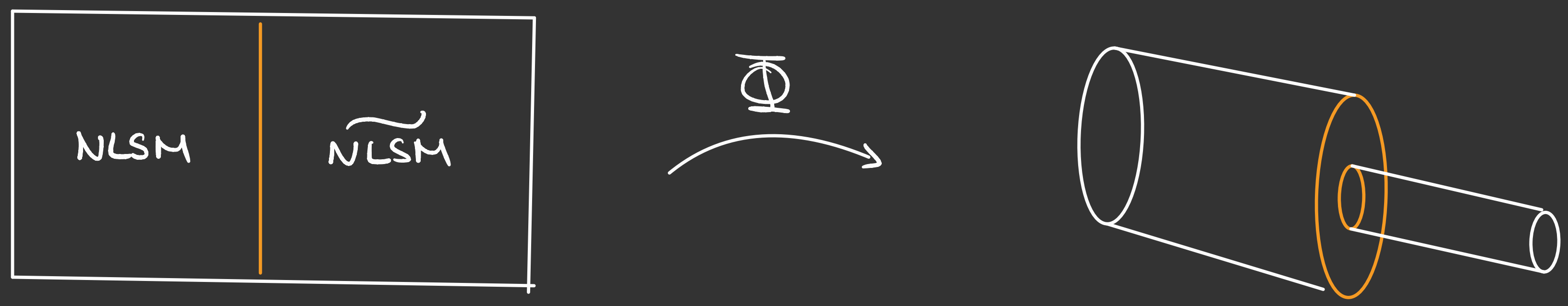
- Similar object: T-fold ~ T-duality in half space



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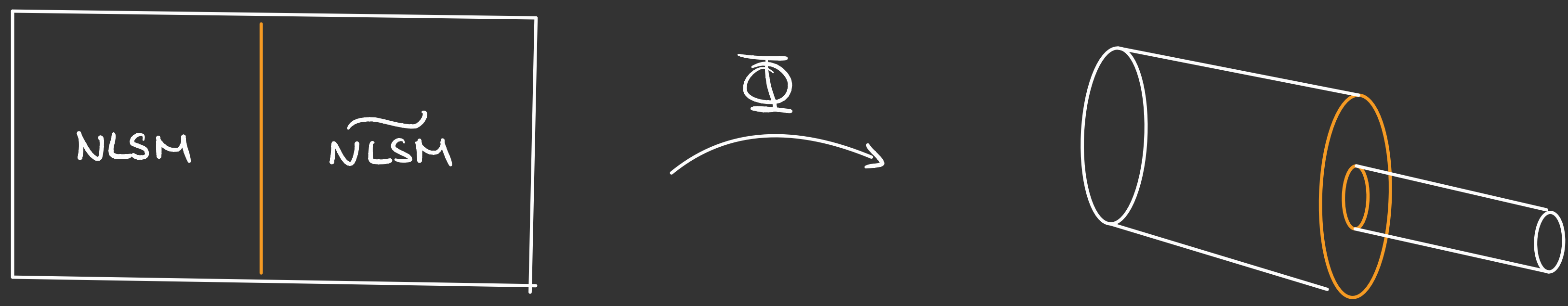

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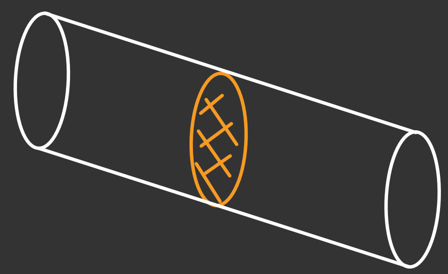
- Non-invertible defect ⟶ Not a T-fold but similar idea

1) Self-duality conditions ⇝ same "radius" on both sides



2) Non-trivial action on vertex operators on the worldsheet ⇝ Not transparent

NON-CONFORMAL THEORIES AND RG FLOWS

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- Conjecture : self-duality condition preserved by the flow

- True in the one tested example (nilfold)

PART IV

CONCLUSION AND OUTLOOK

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- Field configuration corresponding to non-invertible defect
  - Any orientable worldsheet W
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- Also holds for $\mathcal{N}=(1,1)$ SUSY NLSMs (use superfields)

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- Ward identities and No Global Symmetries
- Constraints on RG flows

OUTLOOK

- Ward identities in full generality
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- Ward identities in full generality
- Target space picture
- Self-duality conditions under RG
- All topological operators? Gauge subgroup of winding
- Other string dualities? e.g. Mirror symmetry
- Constraints on IR phases after symmetry preserving deformations
- Many more...

THANK YOU FOR

YOUR ATTENTION!