Aspects of QED in 2+1 dimensions

Pierluigi Niro

SISSA (Trieste)

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2410.05366 by T. Dumitrescu, PN, R. Thorngren
[see also 2409.17913 by S. Chester and Z. Komargodski]
WIP by T. Dumitrescu, K. Intriligator, PN, O. Sela

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Motivations

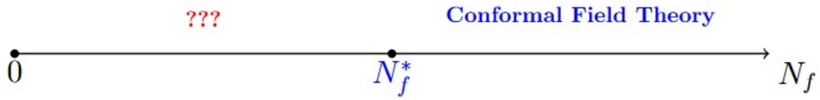
- $(2+1)$-d QFTs (at $T=0$) can describe equilibrium statistical systems in 3 space dimensions, i.e. lattice systems, when correlation length $\gg$ lattice spacing
- $(3+1)$-d QFTs at high T are described by $(2+1)$-d QFTs
- 3d gauge theories are classically strongly coupled, their dynamics is a simpler example of strong coupling than ordinary 4d gauge theories

QED$_3$ $=$ $U(1)$ gauge theory $+$ N_f charge-1 (massless) Dirac fermions ψ^i

$$\mathcal{L} = -\frac{1}{4e^2} f^{\mu\nu} f_{\mu\nu} - i \sum_{i=1}^{N_f} \bar{\psi}_i \gamma^\mu \left(\partial_\mu - i a_\mu\right) \psi^i \,, \qquad [e^2] = 1 \,.$$

No Chern-Simons term: the parity anomaly [Niemi, Semenoff; Redlich] requires N_f to be even, and the theory enjoys time reversal symmetry

IR behavior of QED_3



1. What is N_f^* ?
2. What happens for $N_f < N_f^*$? ✍

➤ Solvable regimes: $N_f = 0$ (pure Maxwell) and large N_f (vector model)

Maxwell theory: dual to the theory of a compact scalar ('dual photon')

$$\widetilde{\mathcal{L}} = -\frac{e^2}{8\pi^2}(\partial_\mu \sigma)(\partial^\mu \sigma)\,, \qquad \sigma \sim \sigma + 2\pi\,.$$

- $U(1)_m$ is the shift symmetry of σ: $j_m = \frac{1}{2\pi} \star f \leftrightarrow \frac{e^2}{(2\pi)^2} d\sigma$
- Monopole operators are $\mathcal{M}_{q_m} = \exp\left(i q_m \sigma\right) \Rightarrow \langle \mathcal{M}_{q_m} \rangle \neq 0$

$$\boxed{U(1)_m \text{ is spontaneously broken by monopole condensation}}$$

3d Coulomb phase = massless photon = S^1 sigma model

IR behavior of QED_3

Large N_f (w/ $\Lambda \equiv e^2 N_f$ fixed): $\langle a_\mu(p) a_\nu(-p)\rangle = -\frac{i\eta_{\mu\nu}}{N_f}\begin{cases}\Lambda/p^2 & \text{UV}\\ 16/|p| & \text{IR}\end{cases}$

$\exists$ non-trivial symmetry-preserving CFT in the IR, amenable to a systematic $1/N_f$ expansion [Appelquist, Nash, Wijewardhana]

What is the fate of massless QED_3 in the IR?

From bootstrap analysis: indications that $\exists$ CFT if $N_f \geq 4$, and that $N_f = 2$ is not a symmetry-preserving CFT [Chester, Pufu; He, Rong, Su; Albayrak, Erramilli, Li, Poland, Xin; Li]

This is also compatible with supersymmetric RG flows [WIP]

In this talk: focus on $N_f = 2$ (can be generalized to any $N_f \in 2\mathbb{Z}$), and assume the theory does not flow to a symmetry-preserving CFT

QED$_3$ with $N_f = 2$: Global Symmetries and Anomalies

$$U(2) = \frac{SU(2)_f \times U(1)_m}{\mathbb{Z}_2}\,, \qquad \mathcal{C}\,, \qquad \mathcal{T}$$

Flavor $SU(2)_f$, Magnetic $U(1)_m$, Quotient by $\mathbb{Z}_2$: $(-\mathbb{I}_2, -1)$

- All gauge-invariant operators are bosons
- Non-monopole operators ($q_m = 0$) are in reps of $SO(3)_f$, e.g. $\vec{O} = i\bar{\psi}\vec{\sigma}\psi$ is in the adjoint of $SU(2)_f$
- Monopole operators ($q_m \neq 0$) are in reps of $SU(2)_f$, e.g. $\mathcal{M}^i(x)$ is in the fundamental of $U(2)$ [Borokhov, Kapustin, Wu]

Mixed 't Hooft anomaly between $U(2)$ and $\mathcal{T}$ [Benini, Hsin, Seiberg]

$$S_{\text{anomaly}} = \pi \int_{X_4} c_2(U(2)) = \frac{\pi}{8\pi^2} \int_{X_4} \left[\mathrm{tr}\mathcal{F} \wedge \mathrm{tr}\mathcal{F} - \mathrm{tr}(\mathcal{F} \wedge \mathcal{F})\right]$$

$\boxed{\text{There must be non-trivial IR dynamics to match the anomaly}}$

Our Proposal: Symmetry Breaking Scenario

$\langle \mathcal{M}^i \rangle \neq 0$: Spontaneous Symmetry Breaking $U(2) \to U(1)_{\text{unbroken}}$, via the condensation of the $q_m = 1$ monopole (as Higgsing in SM)

$$\Rightarrow 3 \text{ NGBs parametrizing } U(2)/U(1) = S^3$$

Hopf Map: given $v^2 \equiv \langle \mathcal{M}_i^\dagger \rangle \langle \mathcal{M}^i \rangle$, construct the map $\pi : S^3 \to S^2$

$$\mathcal{M}^i \to \vec{n} = \frac{\mathcal{M}^\dagger \vec{\sigma} \mathcal{M}}{v^2}\,, \qquad \vec{n}^2 = 1$$

Given $\vec{n}$ (triplet of $SU(2)_f$ and singlet of $U(1)_m$), one gets

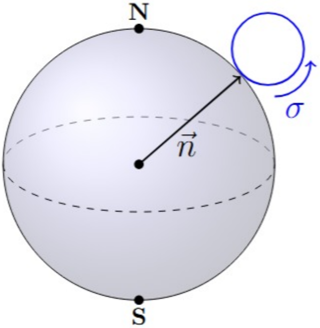
$$\mathcal{M}^i(\vec{n}, \sigma) = v\, \xi^i(\vec{n}) e^{i\sigma}\,, \qquad \sigma \sim \sigma + 2\pi\,, \quad \xi^\dagger \vec{\sigma} \xi = \vec{n}$$

σ parametrizes the S^1 fiber over each point of the S^2 base

$$ds^2(S^3) = R^2 d\vec{n} \cdot d\vec{n} + \frac{e_{\text{eff}}^2}{8\pi^2}(d\sigma - \alpha)^2\,, \qquad \int_{S^2} \frac{d\alpha}{2\pi} = 1$$

$$\boxed{3 \text{ NGBs: } \vec{n} \in S^2 \text{ (triplet breaking) and } \sigma \in S^1 \text{ (dual photon)}}$$

Fermion Bilinear and Small Triplet Mass



Example: $\vec{n} = \pm\hat{z}$, preserving $U(1)_f$

$U(1)_\pm \equiv \frac{1}{2}\left(U(1)_m \pm U(1)_f\right)$

- **N** pole: $\mathcal{M}^i(+\hat{z},\sigma) = ve^{i\sigma}(1\,0)^t$
  $q_+ = 1$ and $q_- = 0$
- **S** pole: $\mathcal{M}^i(-\hat{z},\sigma) = ve^{i\sigma}(0\,1)^t$
  $q_+ = 0$ and $q_- = 1$

Roles of $U(1)_\pm$ reversed $\Rightarrow$ fibration

- Fermion bilinear is aligned with $U(1)_f$ singled out by $\langle\mathcal{M}^i\rangle$:
  $\langle i\bar{\psi}\vec{\sigma}\psi\rangle \xrightarrow{\text{RG}} \vec{n}$ (no further symmetry breaking)
- Triplet Mass ($\mathcal{T}$-invariant) :
  $\mathcal{L}_{\vec{m}} = i\vec{m}\cdot\bar{\psi}\vec{\sigma}\psi \xrightarrow{\text{RG}} \vec{m}\cdot\vec{n} \quad \Rightarrow \quad \vec{n} \parallel \vec{m}$
  Mass perturbation selects a single point on S^2: at low energies we get a Coulomb phase, parametrized by the dual photon $\sigma \in S^1$

't Hooft Anomaly Matching

- $\mathcal{C}$ and $\mathcal{T}$ are unbroken ($\mathcal{T}$ follows from Vafa-Witten theorem)
- $\mathcal{T}/U(2)$ anomaly needs to be matched in the S^3 sigma model: it admits a conventional theta term, since $\pi_3(S^3) = \mathbb{Z}$

$$S_\theta = \frac{\theta}{24\pi^2} \int_{\mathcal{M}_3} \mathrm{Tr} \left(U^{-1} dU\right)^3 , \qquad U \in U(2)/U(1)$$

$\mathcal{T}$ allows only $\theta = 0, \pi$: $\theta = \pi$ matches the anomaly

Technically, coupling to $\mathcal{A} \in U(2)$: $S_\theta[\mathcal{A}] = \theta \int_{X_4} c_2(U(2))$

- Symmetry breaking scenario discussed in the literature [Pisarski] mainly focuses on $\langle i\bar{\psi}\vec{\sigma}\psi\rangle \neq 0$, leading to $SU(2)_f \to U(1)_f$ and 2 NGBs parametrizing S^2, but this is *incompatible* with anomaly matching! (S^2 can be lifted by the $U(1)_f \times U(1)_m$-preserving mass)

Perturbative Regime: Large Triplet Mass

Couple to $U(1)_f$ with $J_f^\mu = \bar{\psi}\gamma^\mu\sigma_z\psi$ and $U(1)_m$ with $\star J_m = f/2\pi$

$$\mathcal{L} = -\frac{1}{4e^2}f^{\mu\nu}f_{\mu\nu} - i\bar{\psi}_i\left[(\not{\partial} - i\not{a})\delta^i_j - i\not{A}_f(\sigma_z)^i_j\right]\psi^j + \frac{1}{2\pi}da\wedge A_m$$

- Add $\vec{m} = m\,\hat{z}$: $\quad \mathcal{L}_{\vec{m}} = im\,\bar{\psi}\sigma_z\psi = im\left(\bar{\psi}_1\psi^1 - \bar{\psi}_2\psi^2\right)$
- Integrate out fermions at $|m| \gg e^2$: Coulomb phase (1 NGB)

$$\mathcal{L}_{IR} = -\frac{1}{4e_m^2}f^{\mu\nu}f_{\mu\nu} + \cdots + \frac{1}{2\pi}da\wedge\left(A_m + \mathrm{sign}(m)A_f\right)$$

$$\begin{cases} m > 0 : \text{condensing } \mathcal{M}^1 \text{ and unbroken } U(1)_- \\ m < 0 : \text{condensing } \mathcal{M}^2 \text{ and unbroken } U(1)_+ \end{cases}$$

In our proposal, large and small mass regimes are continuously connected! One evidence for this is the existence of a massless photon for any non-zero triplet mass (because all electrically charged dof's decouple)

Non-Perturbative Constraints on Symmetry Breaking

- The Vafa-Witten theorem imposes constraints on the allowed patterns of symmetry breaking in $\mathcal{T}$-invariant theories [Vafa, Witten]
- Subtlety for QED$_3$: naively, $U(1)_f \subset SU(2)_f$ is unbroken, but there is a mixing between $U(1)_f$ and $U(1)_m$

Applying Vafa-Witten arguments to QED$_3$:

1. Time-reversal $\mathcal{T}$ is unbroken
2. If no monopole operator condenses, $U(1)_f \times U(1)_m$ is unbroken
3. If a (q_m, q_f) monopole operator condenses, the linear combination $U(1) = q_m U(1)_f - q_f U(1)_m$ is unbroken (explains alignment); anomaly matching further requires q_m to be odd

Our proposal realizes 1) and 3) with $q_m = 1$.

Supersymmetric RG Flows: SQED with 8 Supercharges

3d $\mathcal{N} = 4$ SQED with 1 hyper of charge 1: $(a_\mu, \varphi^{(ij)}, \lambda_\alpha^{ii'}; h^{i'}, \psi^i)$

$$\{Q_\alpha^{ii'}, Q_\beta^{jj'}\} \sim \epsilon^{ij}\epsilon^{i'j'}\gamma^\mu_{\alpha\beta}P_\mu\,; \quad (i,i') \in SU(2)_L \times SU(2)_R\,; \quad U(1)_m$$

$\Rightarrow$ 1 hypermultiplet $\supset$ 2 charge-1 Dirac fermions ψ^i, doublet of $SU(2)_L$

- There is a Coulomb branch of vacua parametrized by the dual photon σ + the real scalars $\varphi^{(ij)} = (\varphi_1, \varphi_2, \varphi_3)$ [Seiberg, Witten], it is a smooth, singularity-free σ-model with target space metric

$$ds^2 = V(r)\left(d\varphi_1^2 + d\varphi_2^2 + d\varphi_3^2\right) + V^{-1}(r)\left(\frac{d\sigma}{2\pi} + \frac{1}{4\pi}\cos\theta d\phi\right)^2$$

  where $V(r) \equiv \frac{1}{e^2} + \frac{1}{4\pi r}$, and (r,θ,ϕ) are polar coordinates of φ_i

- By mirror symmetry, this can be dualized to a free (twisted) hypermultiplet $\widetilde{\Phi} = (\mathcal{M}^i, \Psi^{i'}) \Rightarrow$
  vev's of monopole operators $\langle \mathcal{M}^i \rangle$ parametrize the Coulomb branch
- $U(2) = \frac{SU(2)_L \times U(1)_m}{\mathbb{Z}_2}$ acting on the Coulomb branch (as in QED_3)

RG Flow to non-SUSY QED$_3$

- Deform the theory to give mass to $(\varphi^{ij}, \lambda_\alpha^{ij'}, h^{i'})$, but not (a_μ, ψ^i)
- Use components of multiplets that can be reliably tracked in the IR [Cordova, Dumitrescu] (e.g. the stress-tensor multiplet)
- At low energies SQED + deformation flows to non-SUSY QED$_3$ with $N_f = 2$ massless Dirac fermions
- Tracking the UV deformation to the IR, we get a non-trivial scalar potential on the Coulomb branch $\mathcal{V}(r)$
- The deformation lifts the degeneracy of the Coulomb branch, as the minimum of $\mathcal{V}(r)$ is at $\langle r\rangle \neq 0$:
  the target space metric at $r = const.$ is the S^3 sigma model
- We recover $U(2) \to U(1)$ via monopole condensation, $\langle \mathcal{M}^i \rangle \neq 0$

Outlook

- Symmetry breaking in QED_3 is driven by monopoles, which carry both magnetic and flavor quantum numbers, and not only by fermion bilinears (as in 4d QCD)
- Using Vafa-Witten theorem, 't Hooft anomaly matching, and the large mass regime, we gave evidence that if the theory does not flow to a CFT, then

$$U(2) \to U(1) \qquad\qquad \text{via } \langle \mathcal{M}^i \rangle \neq 0$$

  giving rise to an S^3 sigma model with $\theta = \pi$, matching the anomaly
- One of the NGBs is (for any $N_f \in 2\mathbb{Z}$) the dual photon
- Consistent with RG flows from $\mathcal{N} = 4$ SQED with $N_{hyper} = 1$
- For $N_{hyper} \geq 2$ (corresponding to even $N_f \geq 4$) the Coulomb branch has a singularity at the origin where new massless dof's appear: Hints for a different (CFT-like) behavior [WIP]

THANK YOU FOR YOUR ATTENTION !