# Near-extremal Black Holes and their microstates

Sameer Murthy
King's College London

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# Near-extremal BHs

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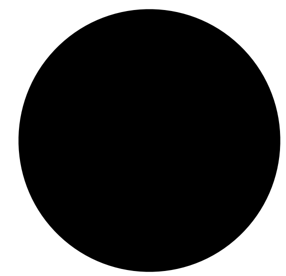

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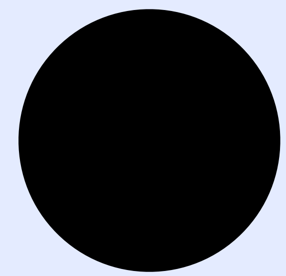

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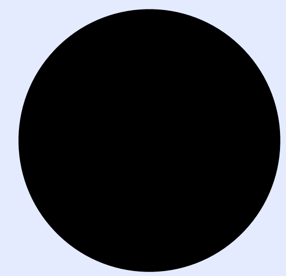

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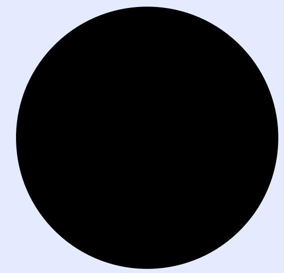

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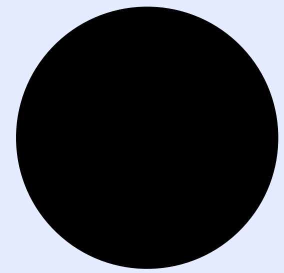

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- Explanation of BH entropy in string theory

# In the semiclassical approximation, a near-extremal BH carries large entropy

- (Naive) Semiclassical picture

$$S_{\mathrm{BH}}(Q,T) \;=\; S_0(Q) + 4\pi Q^3 \ell_{\mathrm{P}} T + \ldots$$

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- Something must go wrong with semi-classical picture at low temperatures. [Preskill, Schwarz, Shapere, Trivedi, Wilczek '91]

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  Note: large quantum effects even for weakly-curved horizons.

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- Revisit BH microstates in string theory
  - decoupling of supersymmetric BHs
  - gravitational index

# The appearance of nearly-gapless modes

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- Correct description is canonical ensemble, includes environment.

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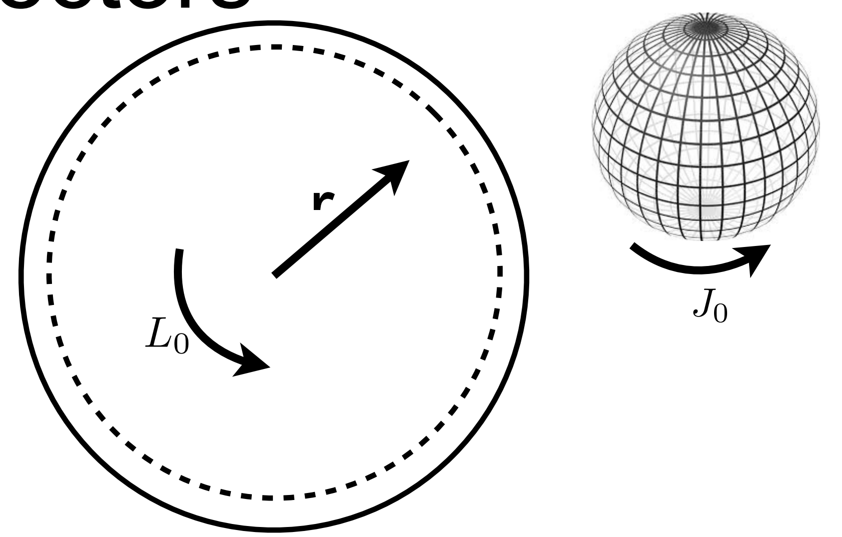


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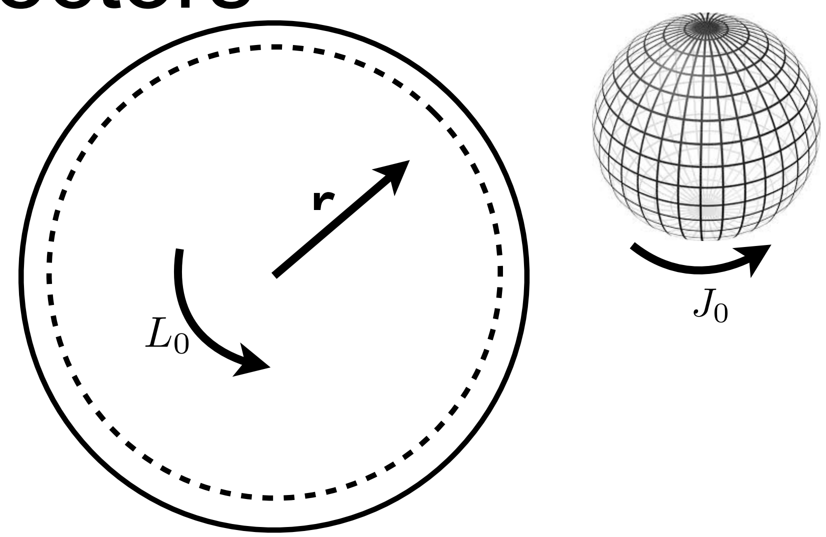


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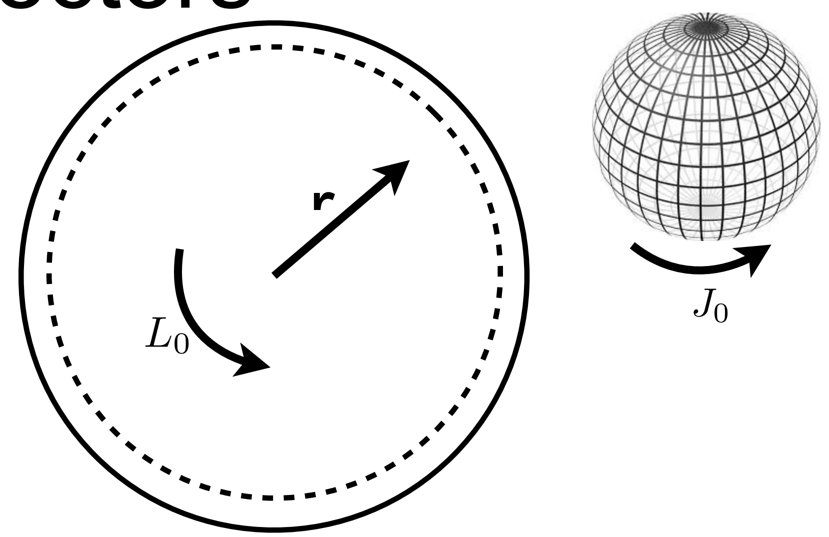


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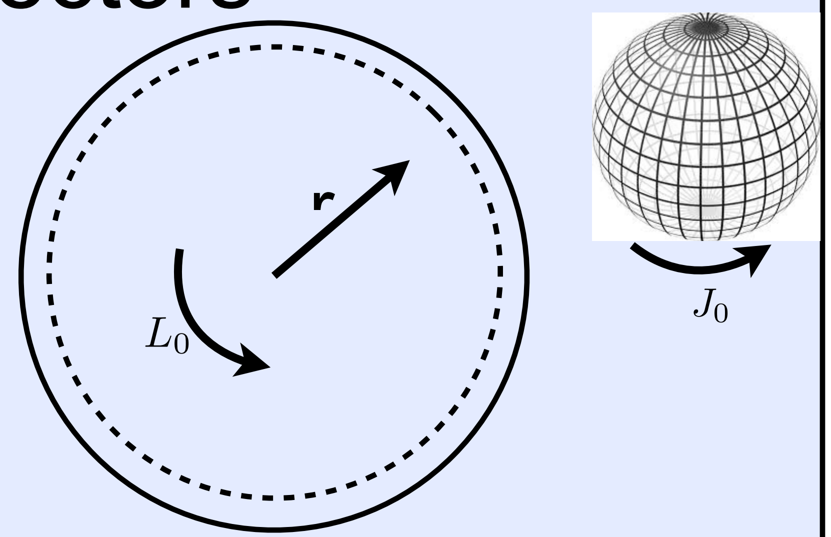


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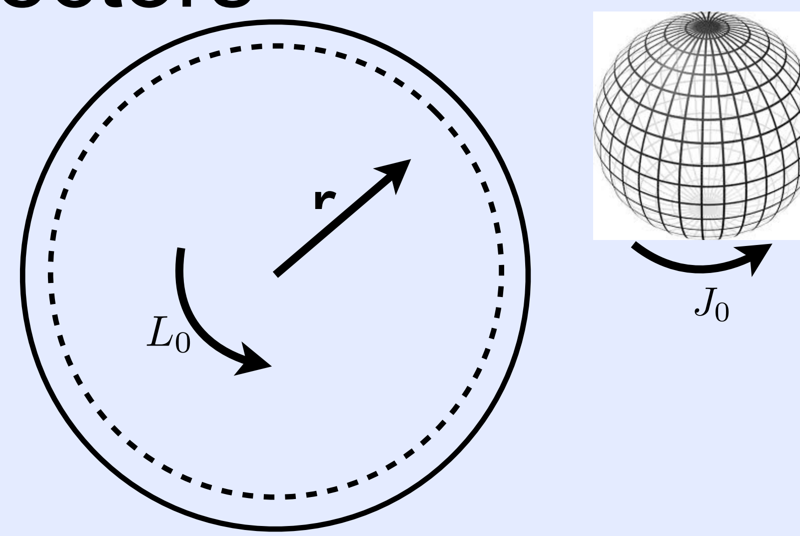


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Is the extremal BH decoupled from the environment at the quantum level?

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- Field fluctuations around $AdS_2 \times S^2$. Non-zero action.
  $\Rightarrow$ Quantum corrections to extremal BH entropy.

[Banerjee, Gupta, Sen '10, … See Review Sen'11]

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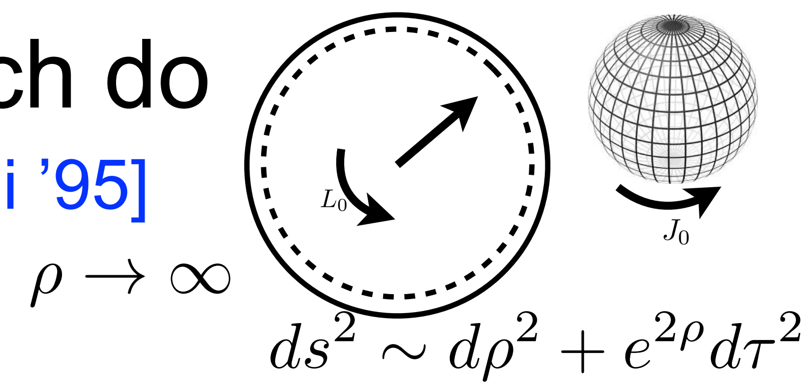

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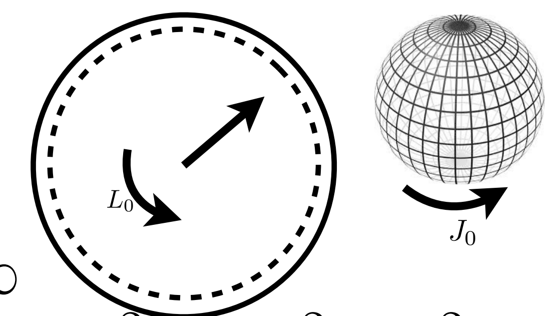


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$ds^2 \sim d\rho^2 + e^{2\rho} d\tau^2$

$$h^{(0)}_{\mu\nu} = \nabla_\mu \zeta_\nu + \nabla_\nu \zeta_\mu \qquad \zeta \sim \sum_{|n|\geq 2} \varepsilon_n \mathrm{e}^{in\tau} \left(in\partial_\rho - \partial_\tau\right)$$

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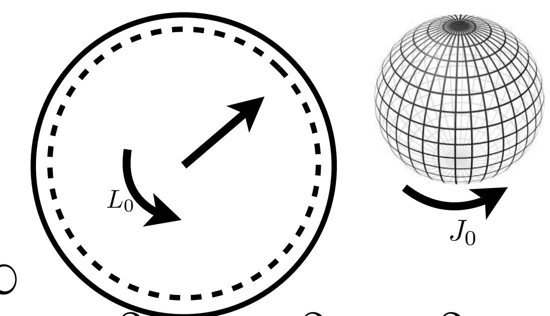


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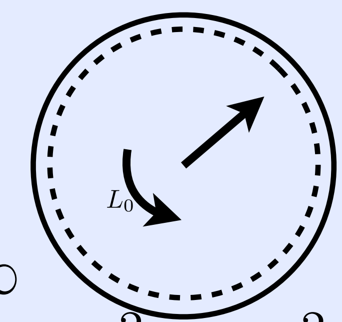


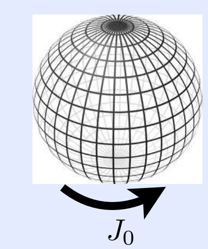


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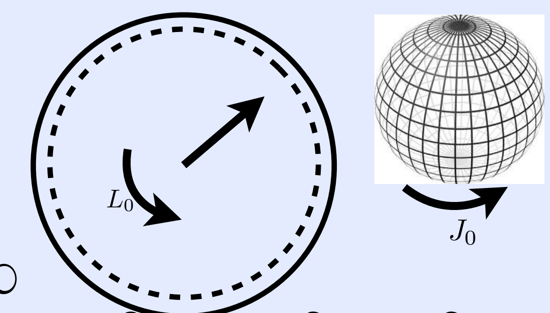


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- Note: zero mode gives important contribution to one-loop logarithmic corrections to extremal entropy
  $Z^{\text{extr}}(Q) = Q^{c_{\log}} \exp\!\left(\pi Q^2 + \dots\right)$ [Sen '11; Jeon, S.M.'18]

# Near-AdS2 (x S2) has nearly-zero modes

- Introduce small T regulator: recouples $\mathrm{AdS}_2$

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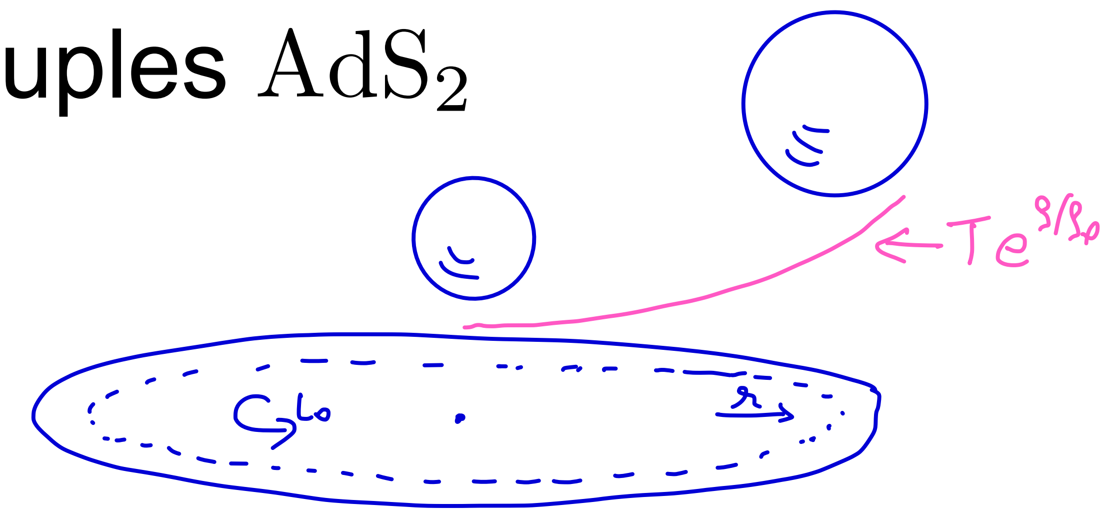

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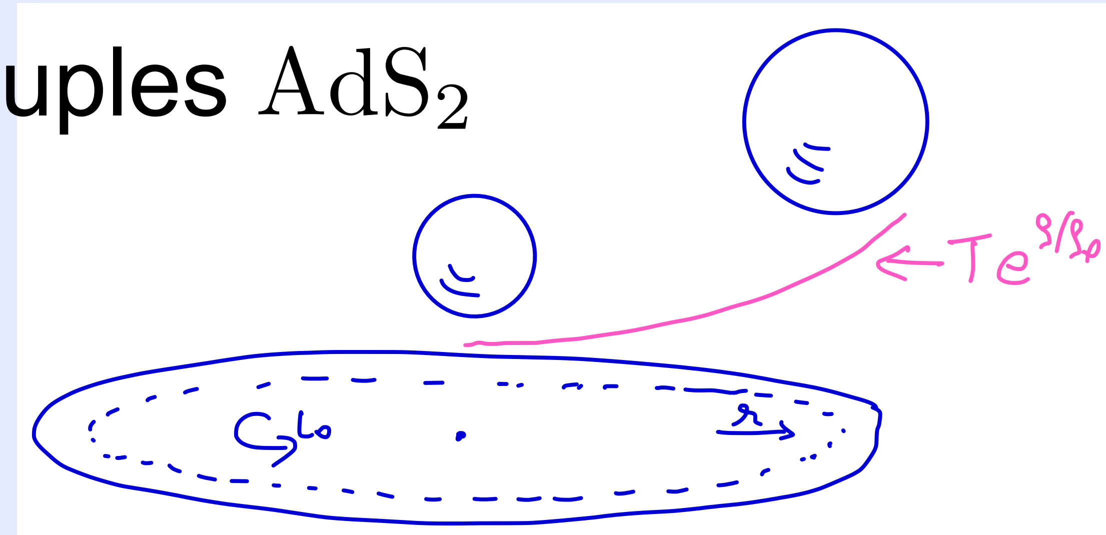

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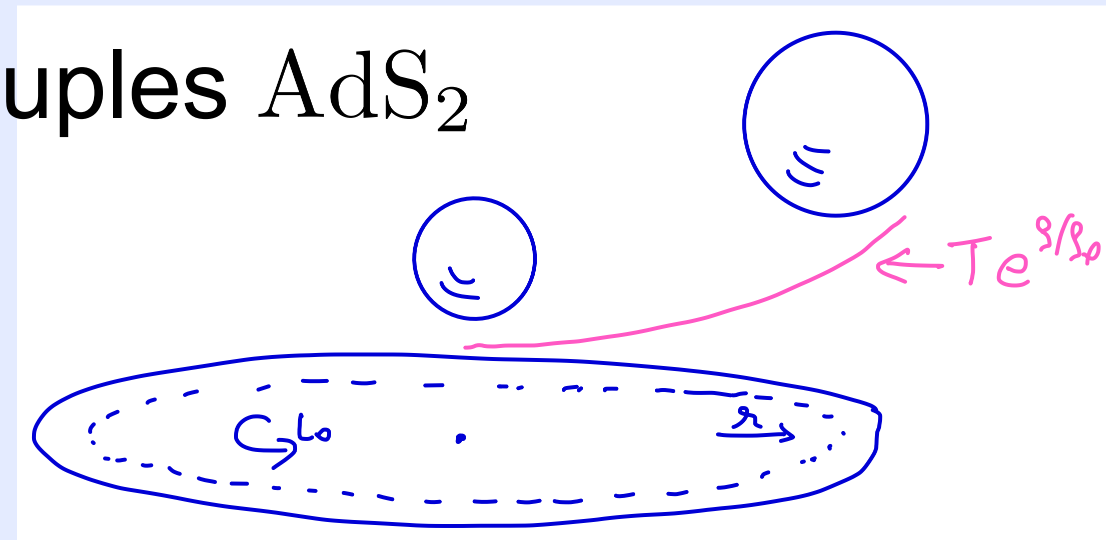

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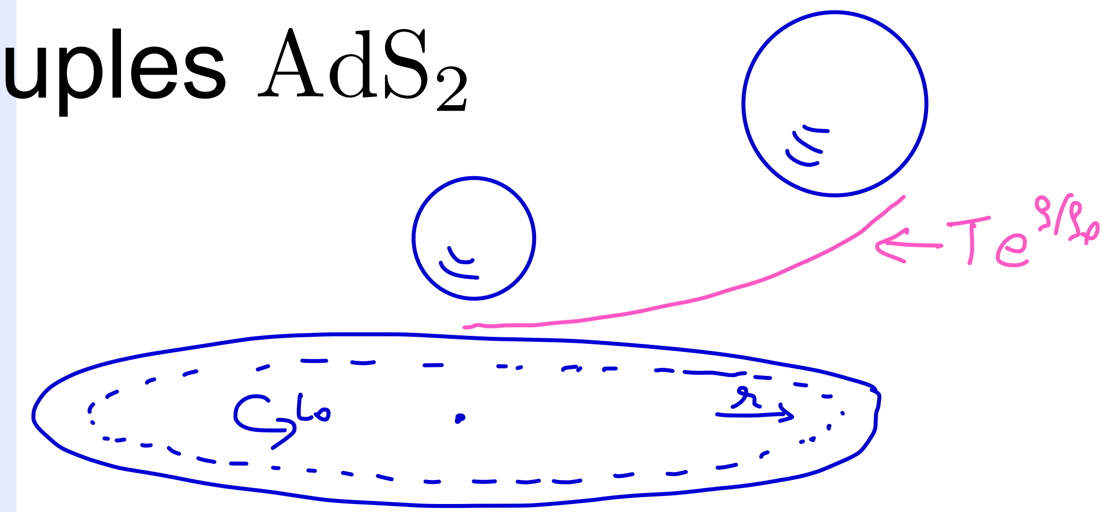

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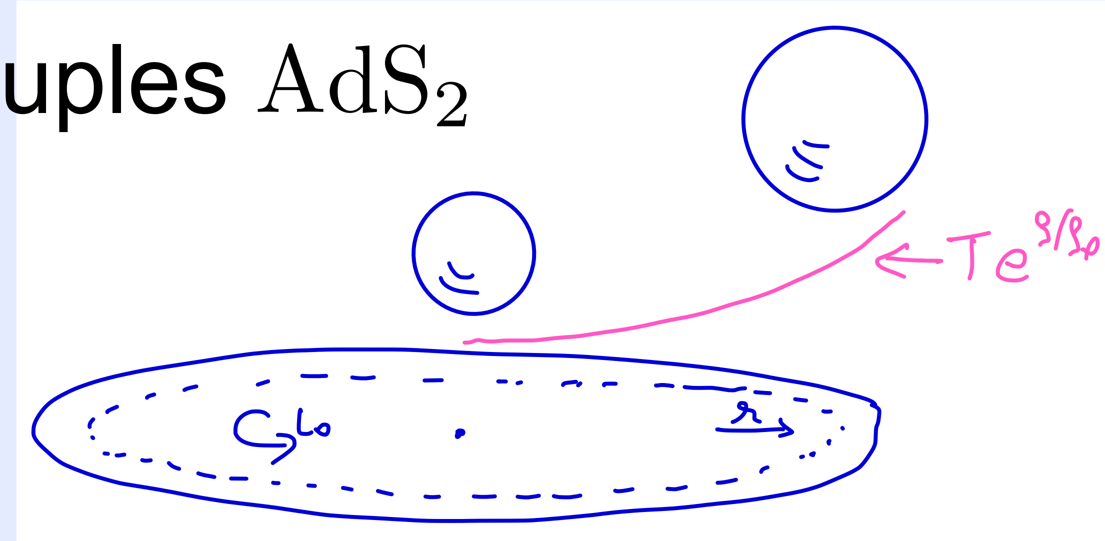


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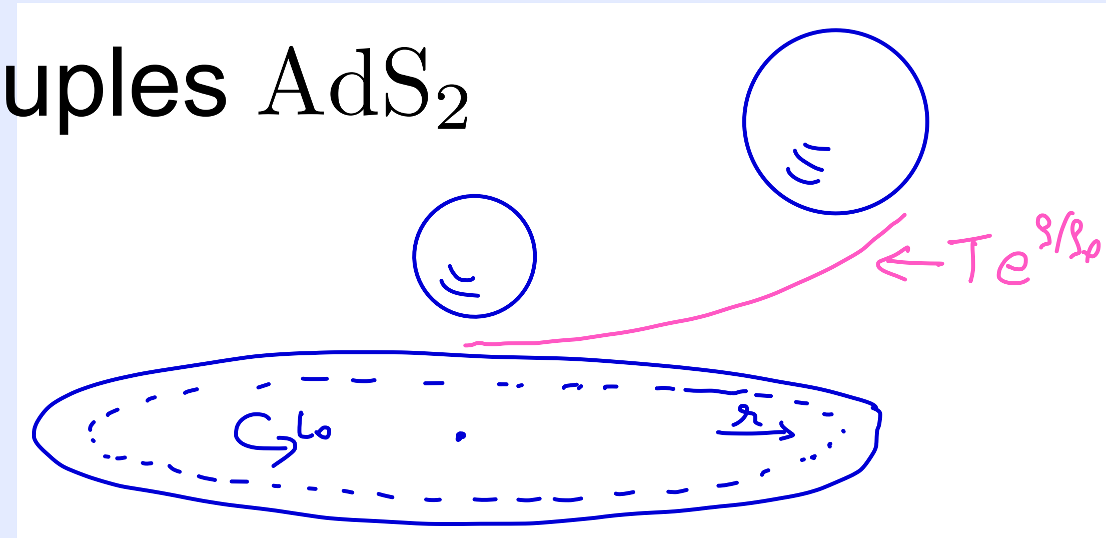

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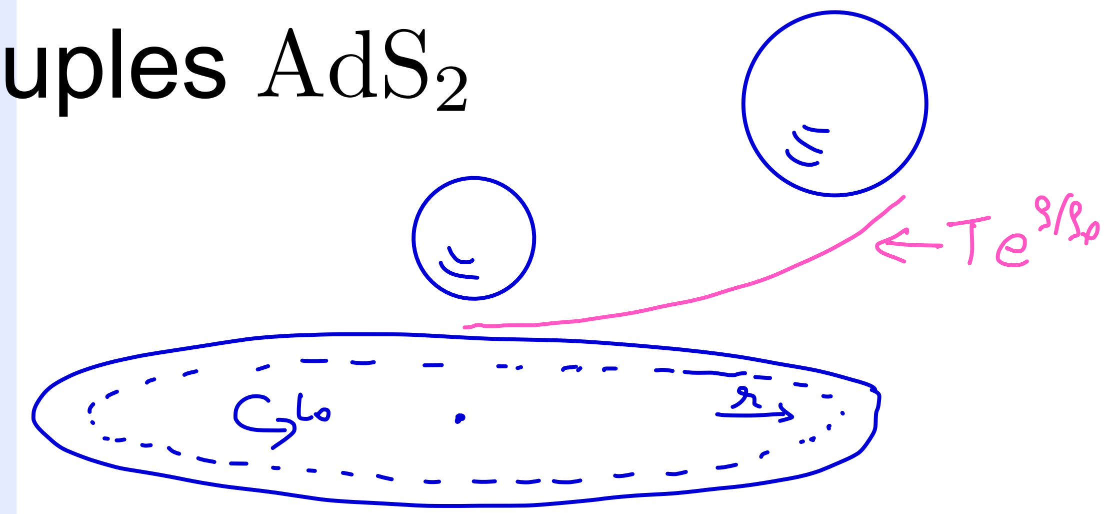


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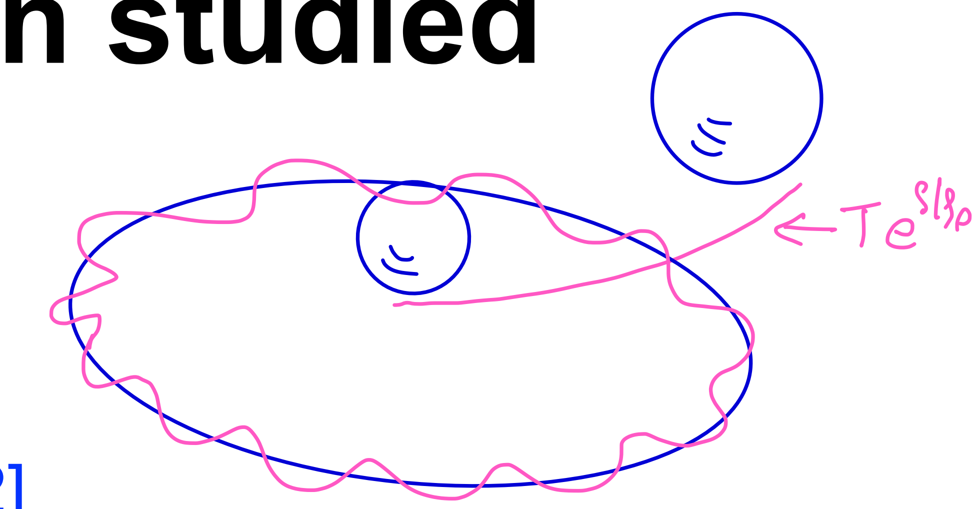
controls coupling of BH to asymptotic region

[Iliesiu, S.M. Turiaci, '22]

# Schwarzian theory has been studied intensively in recent years

- Change of coordinates to equivalent presentation

[Iliesiu, S.M. Turiaci ’22]



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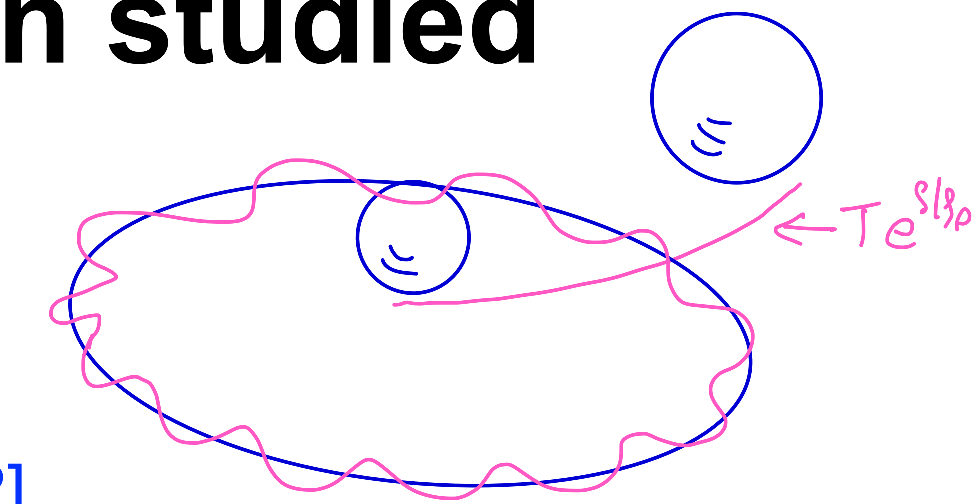


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[Iliesiu, S.M. Turiaci '22]

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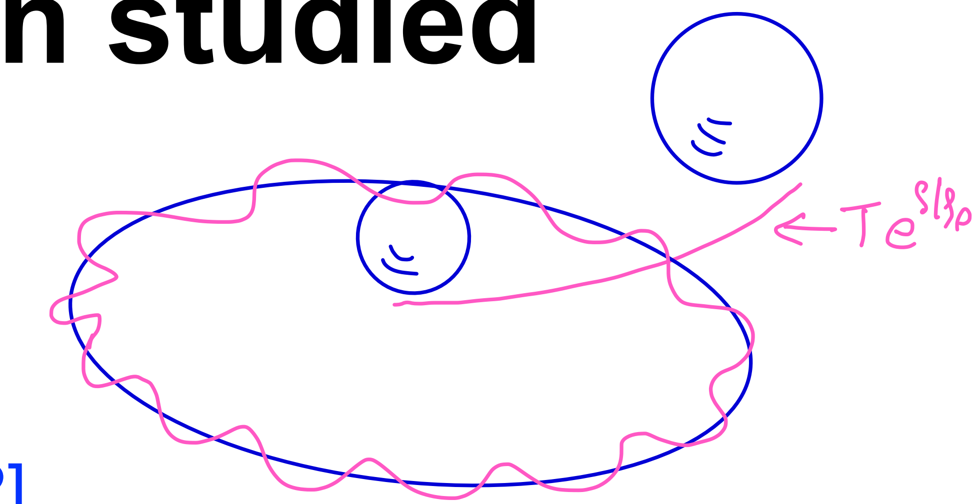


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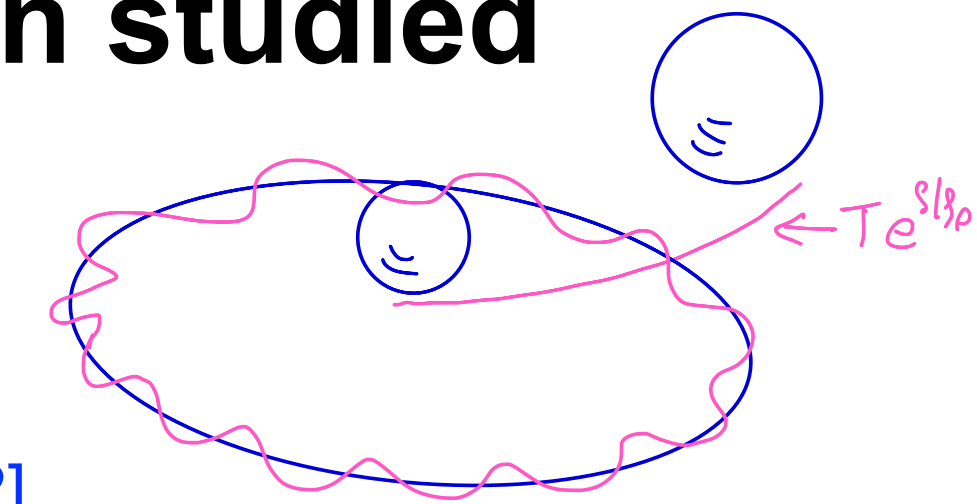

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- Different methods to solve the theory, cf JT gravity (partition function, density of states, correlators,…)

[Sachdev '15; Almheiri, Kang '16; Maldacena, Stanford, Yang '16; Yang '18, Moitra, Trivedi, Vishal '18; Ghosh, Maxfield,Turiaci '19; Iliesiu,Turiaci, '20; Mertens, Turiaci, Verlinde '20; Heydeman, Iliesiu, Turiaci, Zhao '20;…]

[See Review Mertens-Turiaci '22]

# Quantum fluctuations change the near-extremal density of states of extremal BHs

- Partition function

$$Z_{\mathrm{BH}}(Q,T) = (Q^3T)^{\frac{3}{2}}\ Q^{c_{\log}} \exp\Big(\pi Q^2 + 4\pi^2 Q^3 T + \dots\Big)$$

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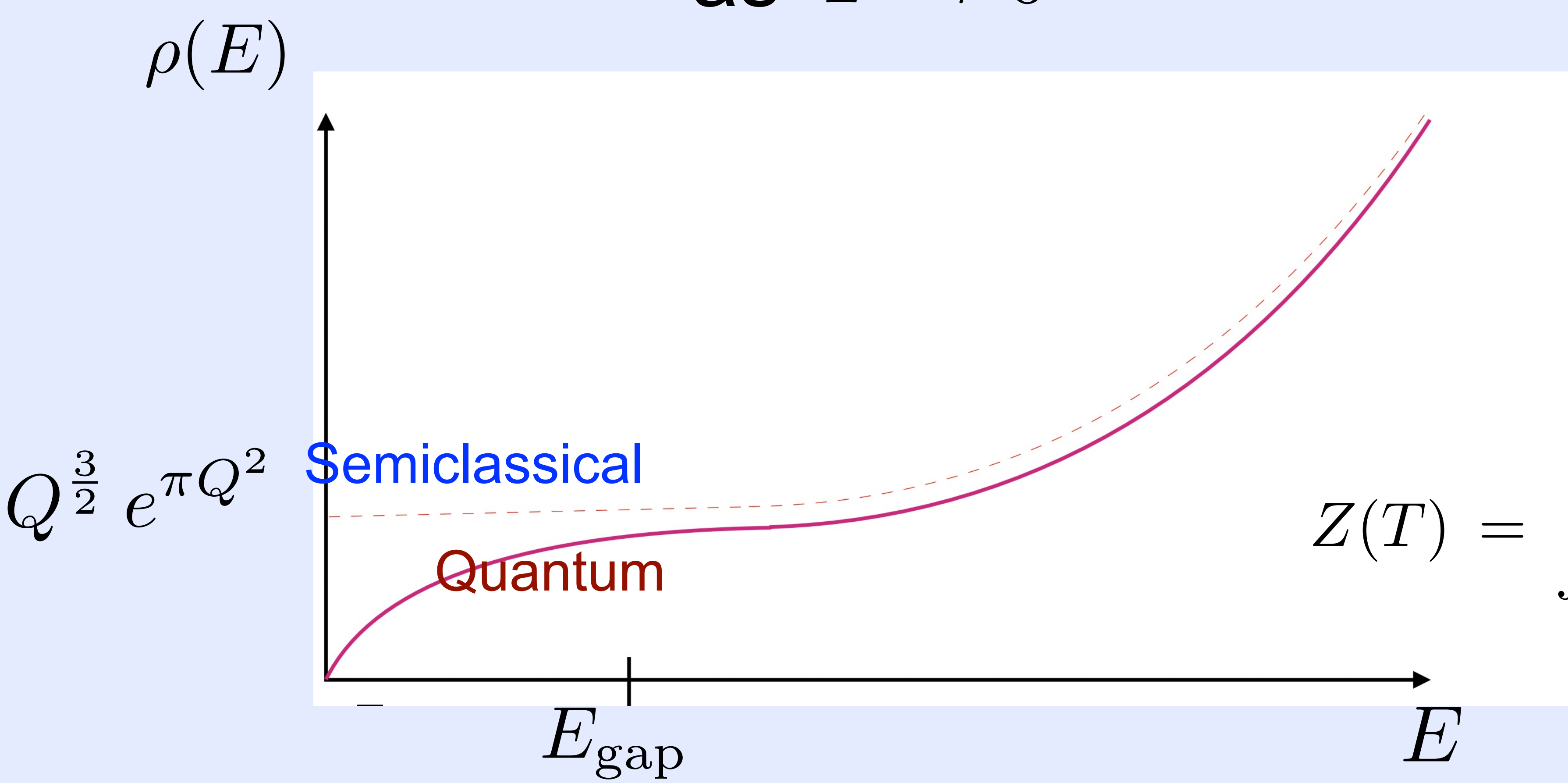


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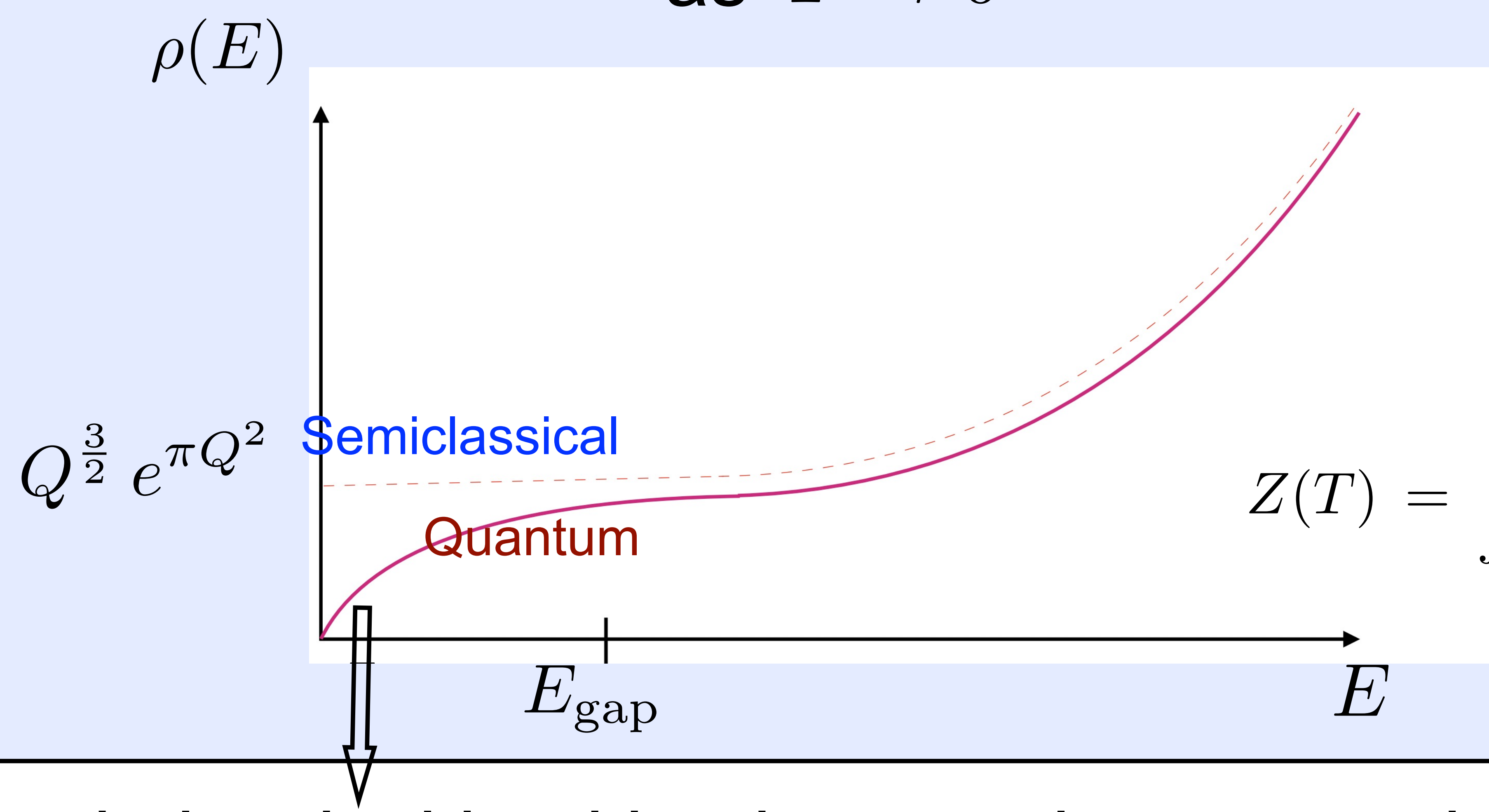


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- Semi-classical intuition is strongly corrected

[Iliesiu, Turiaci ’20; Iliesiu, S.M. Turiaci ’22]

# Supersymmetric BHs exhibit a mass gap and quantum decoupling

- Large quantum fluctuations of bosonic and fermionic super-Schwarzian modes

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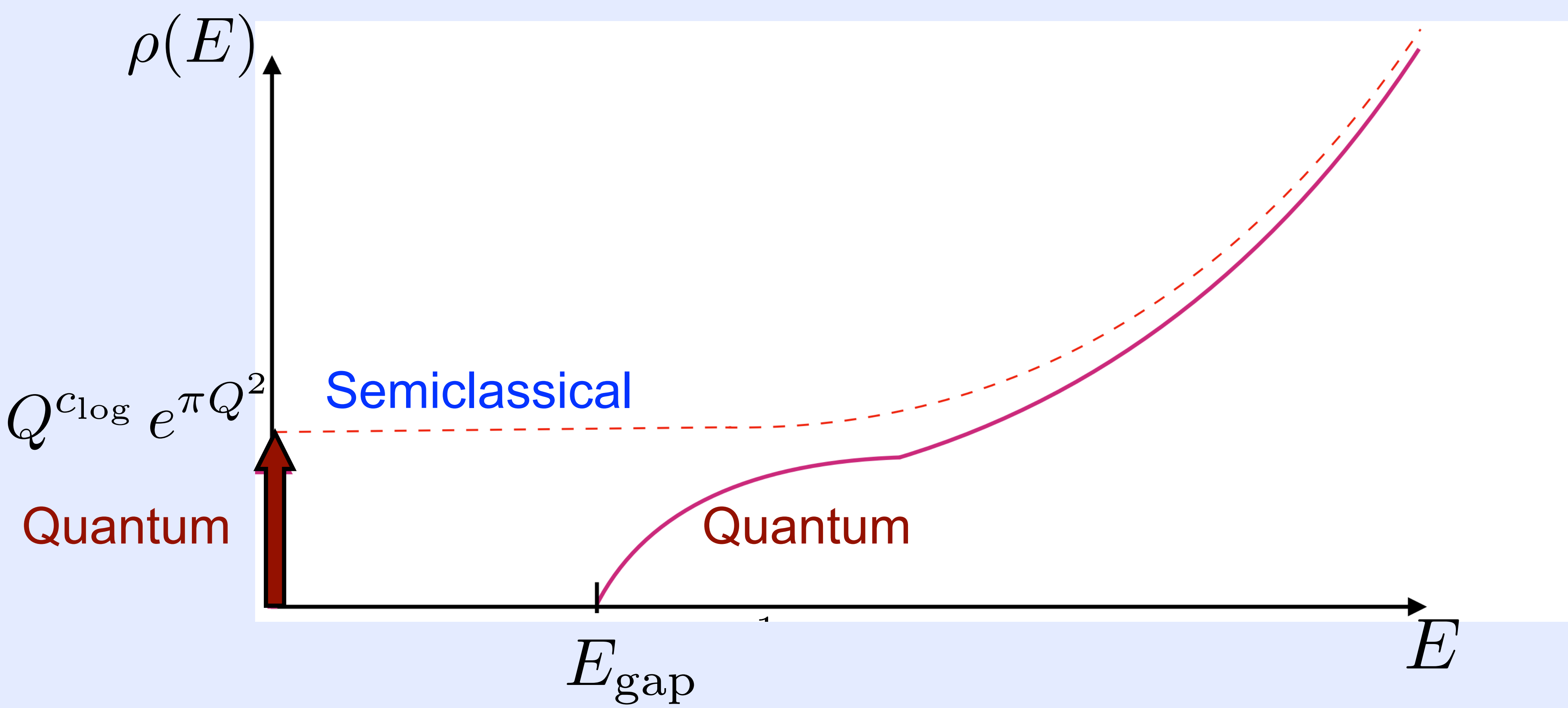


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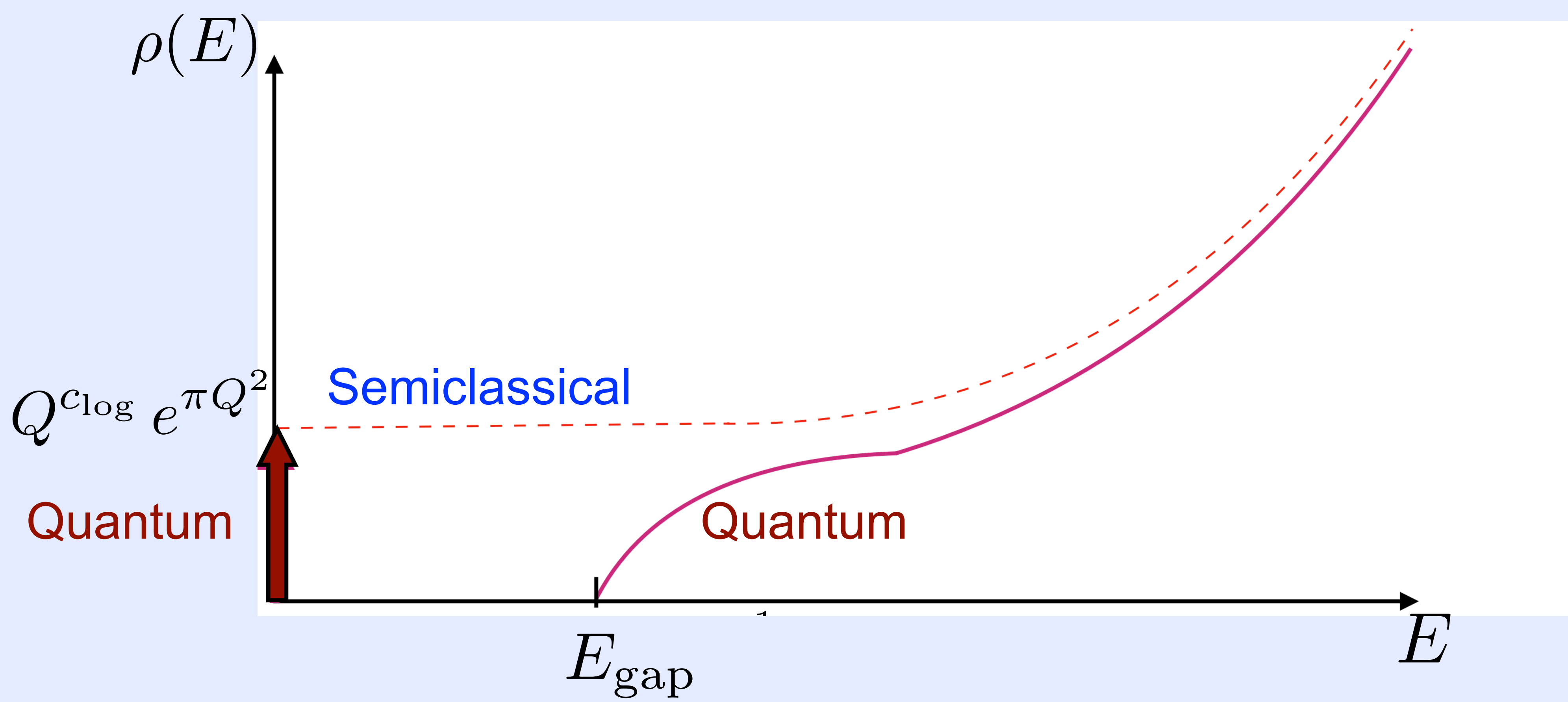


- Note: decoupling justifies microscopic counting of BPS states in string theory.

[Iliesiu, Turiaci ’20; Iliesiu, S.M. Turiaci ’22]

# Applications of quantum near-extremal physics

# 1a. Exact entropy of supersymmetric BHs

- Exact formulas from localization in sugra (e.g. ⅛-BPS BHs in N=8 string theory). [Dabholkar, Gomes, S.M.'11-'14]

[microscopic formula from analytic Number theory: Hardy-Ramanujan-Rademacher]

$$d_{\mathrm{micro}}(Q) = \sum_{c=1}^{\infty} \frac{1}{c} \, K_c(Q) \, \frac{1}{c^{7/2}} \, \widetilde{I}_{7/2}(\pi Q/c)$$

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all-order quantum fluctuations of bulk modes

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- Volume of space of super-Schwarzian modes on orbifold

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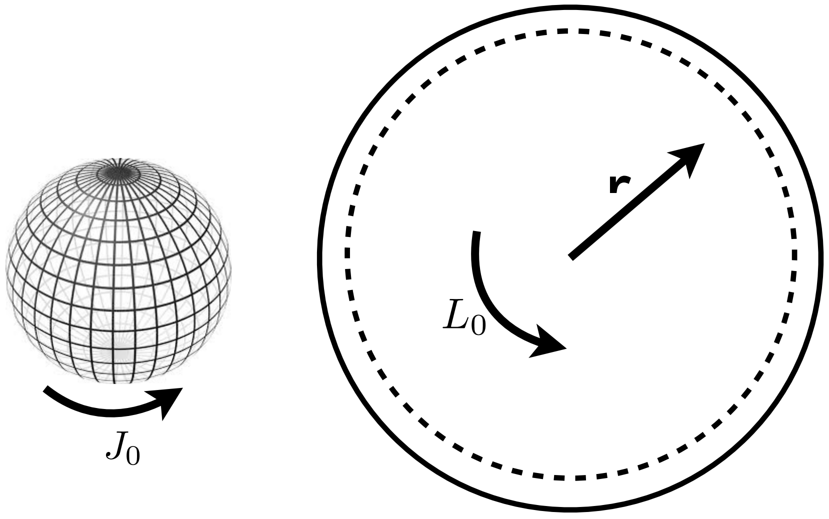
# 1b. Black hole degeneracy = index

- 4d supersymmetric black holes are spherically symmetric and therefore have zero net angular momentum $J_0 = F$



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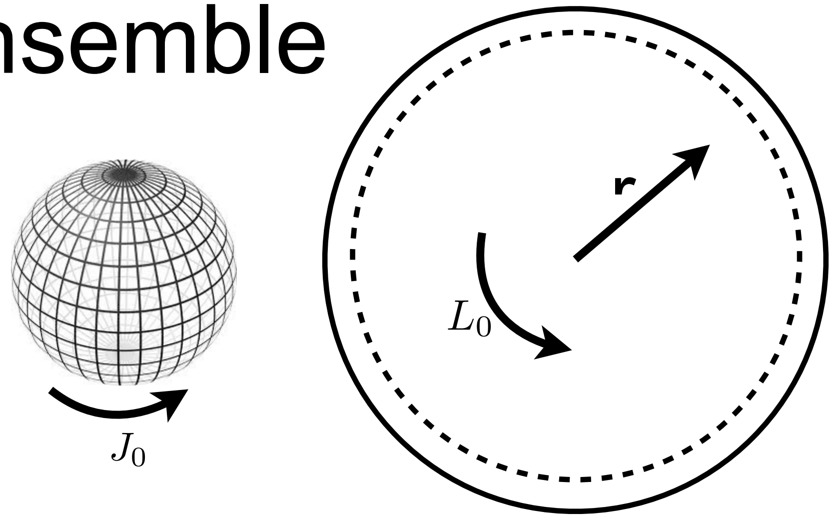

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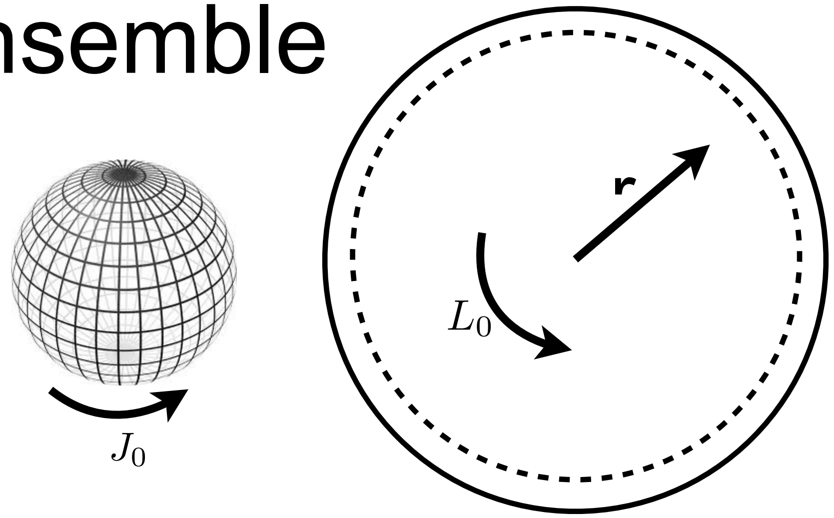

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Degeneracy

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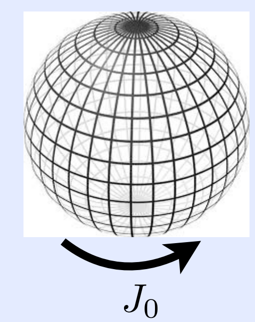


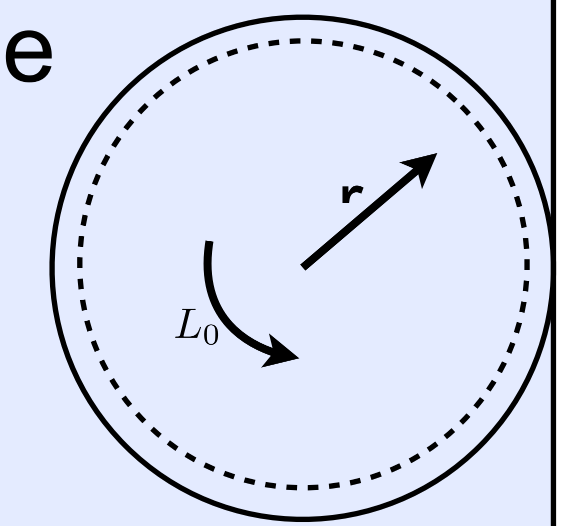


[Sen ’10; Dabholkar, Gomes, S.M., Sen ’10]

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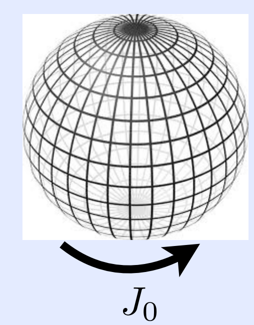
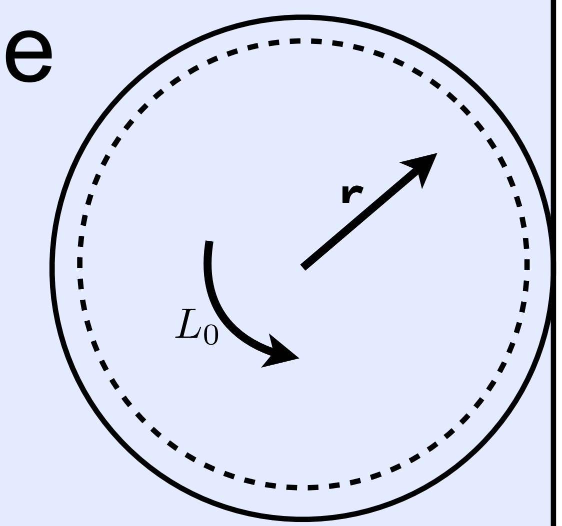


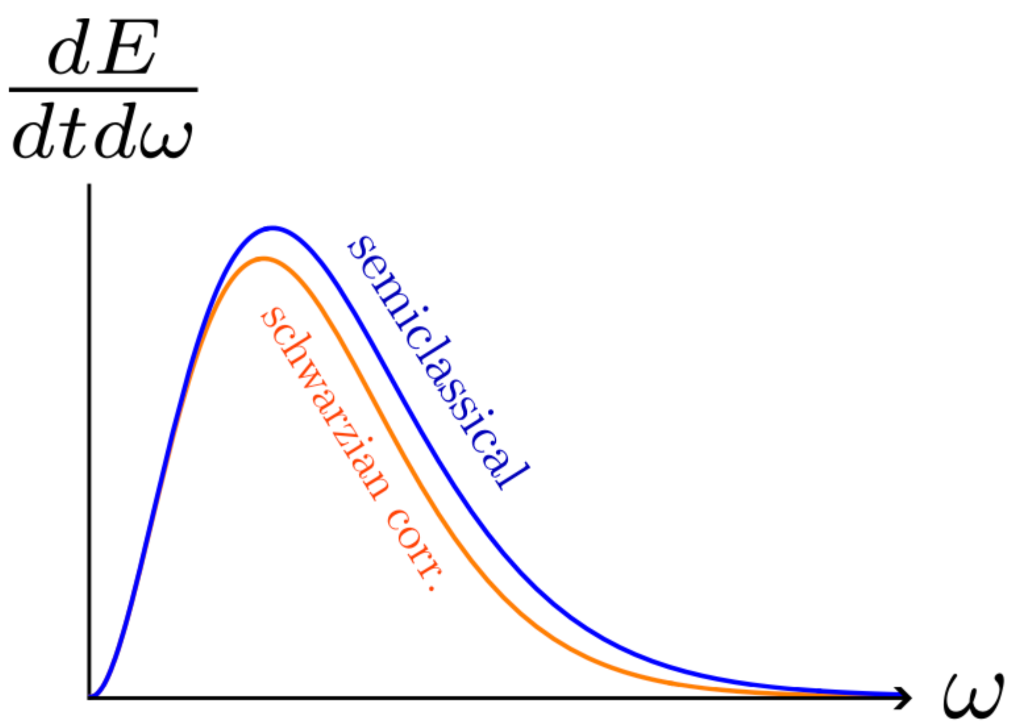




[Sen '10; Dabholkar, Gomes, S.M., Sen '10]

- Argument extended to Schwarzian modes. The result is that, again, only $J_0 = 0$ (bosonic) states contribute.

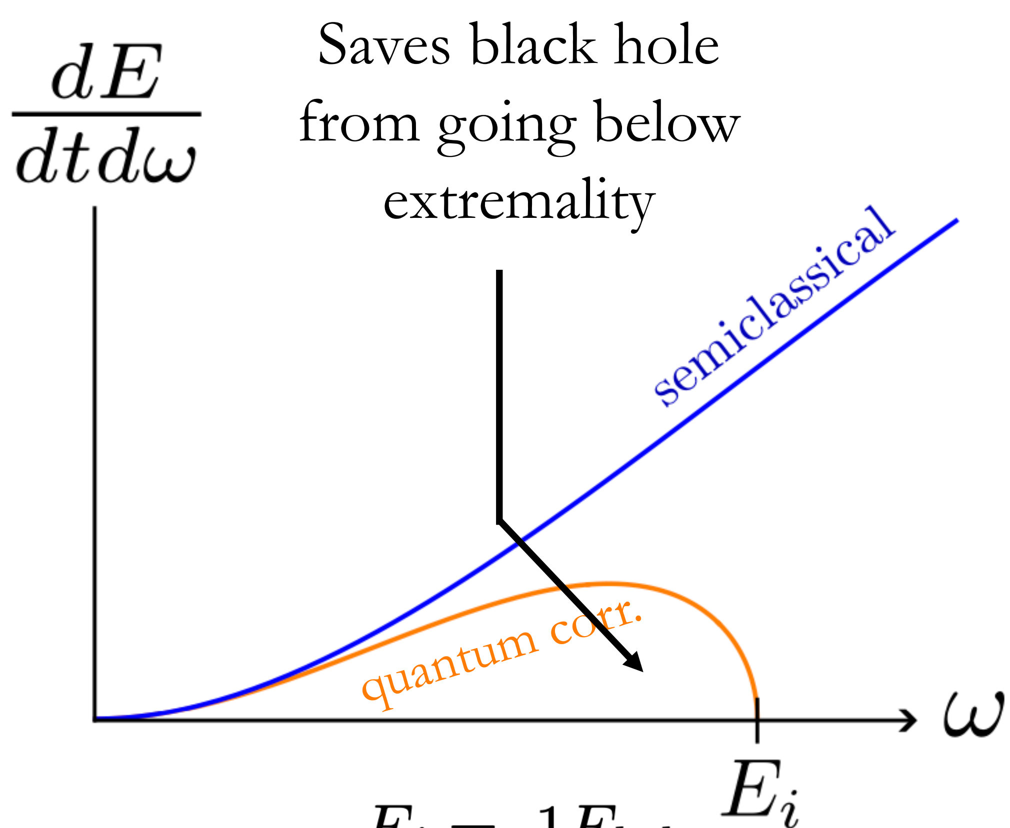
[Iliesiu, Kologlu,Turiaci '19; Iliesiu, S.M., Turiaci '22]

# 2a. Hawking radiation from near-extremal BHs




$E_i = 100 E_{\text{brk.}}$

Semi-classical regime

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Quantum regime

[Brown, Iliesiu, Penington, Usatyuk '24]

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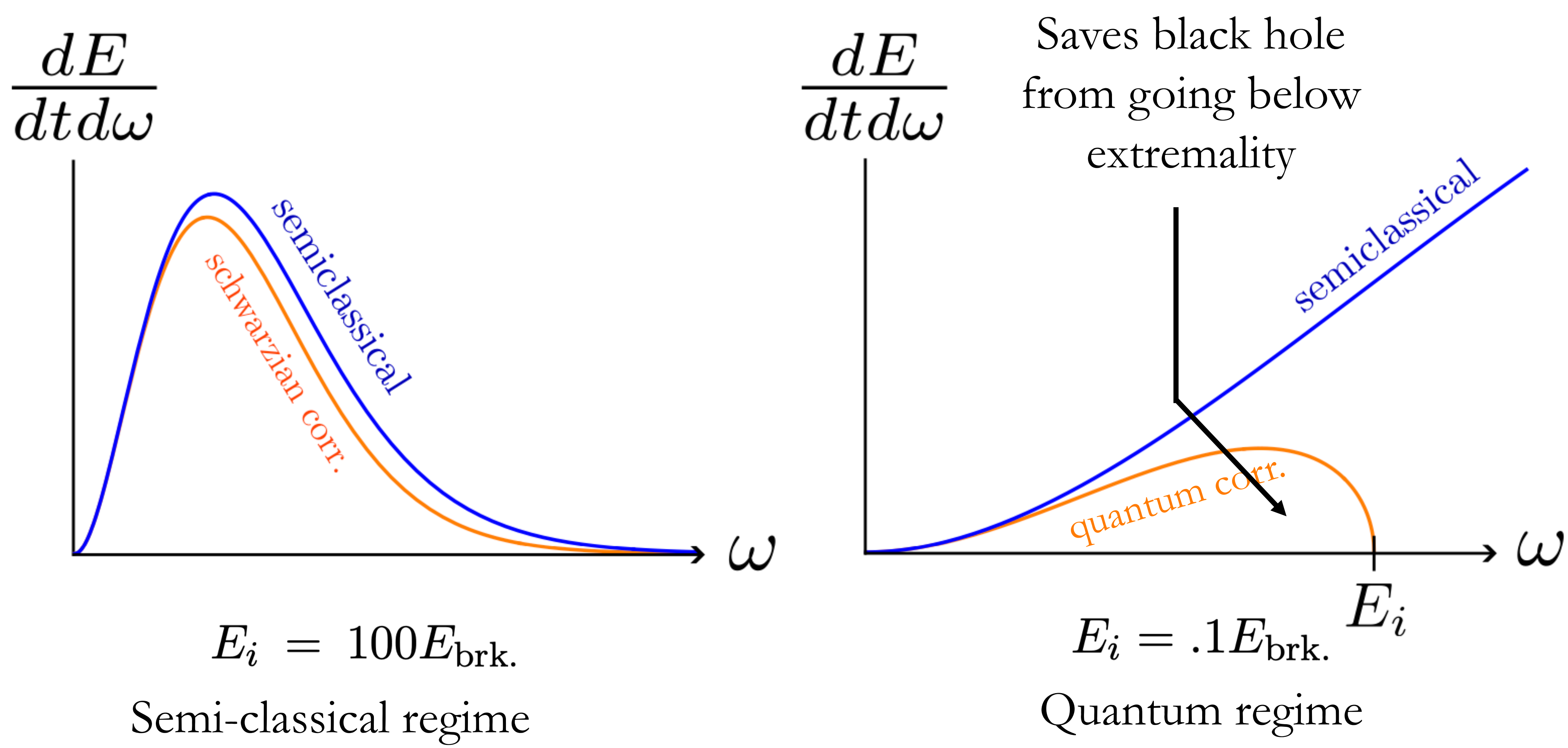
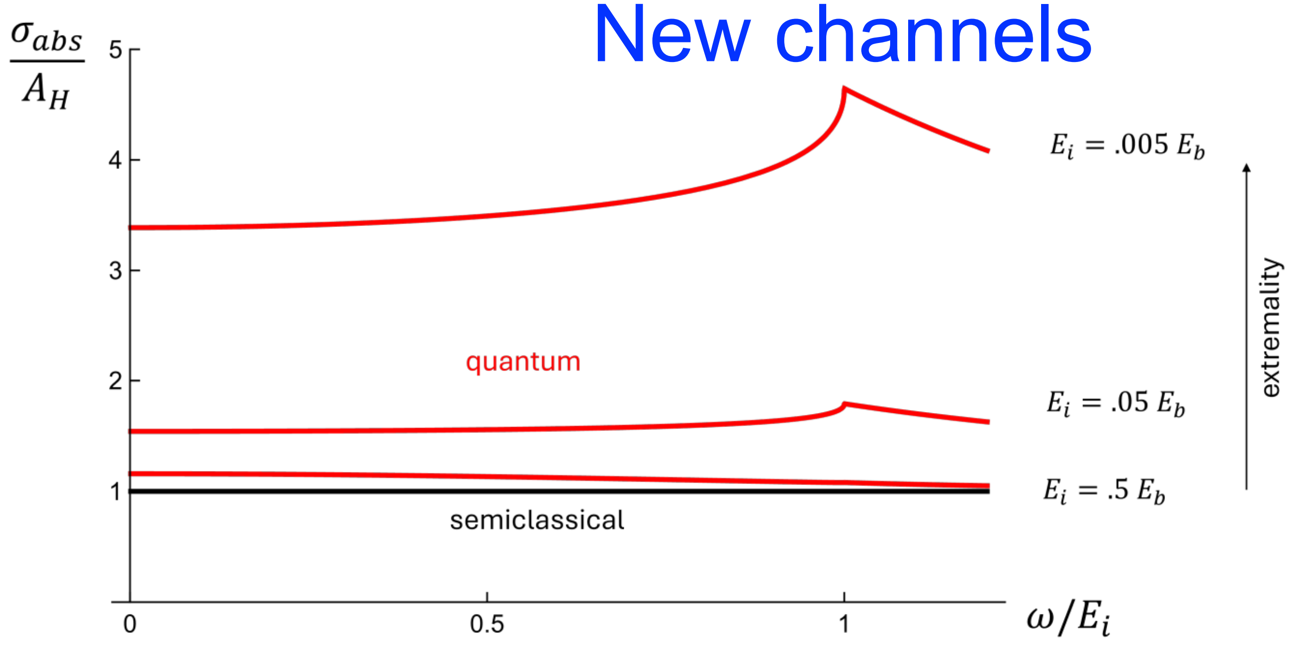



[Brown, Iliesiu, Penington, Usatyuk '24]

- Quantum effects alter the spectrum of radiation at low temperatures, and resolve apparent paradoxes posed in [Preskill, Schwarz, Shapere, Trivedi, Wilczek '91]

# 2b. Scattering from near-extremal BHs

New channels




Scattering off near-extremal BHs in GR

[Emparan ’25; Biggs ’25; Emparan, Trezzi ’25; Lin, Iliesiu, Utasyuk; ’25]

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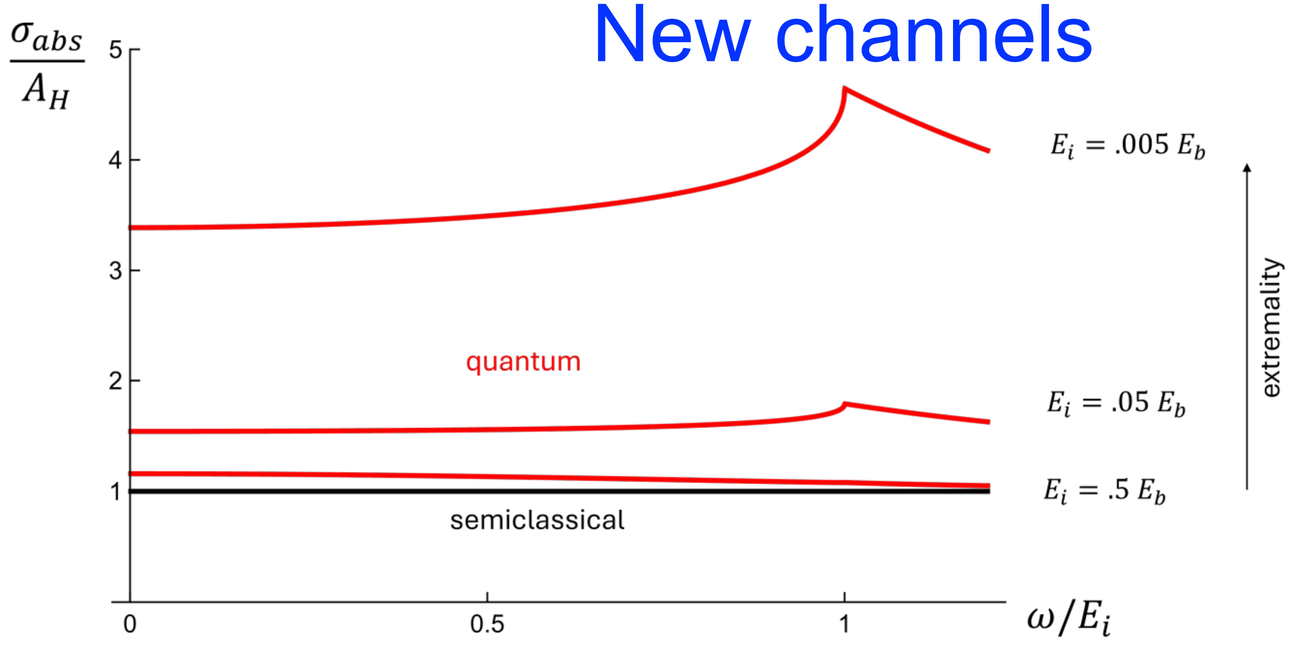


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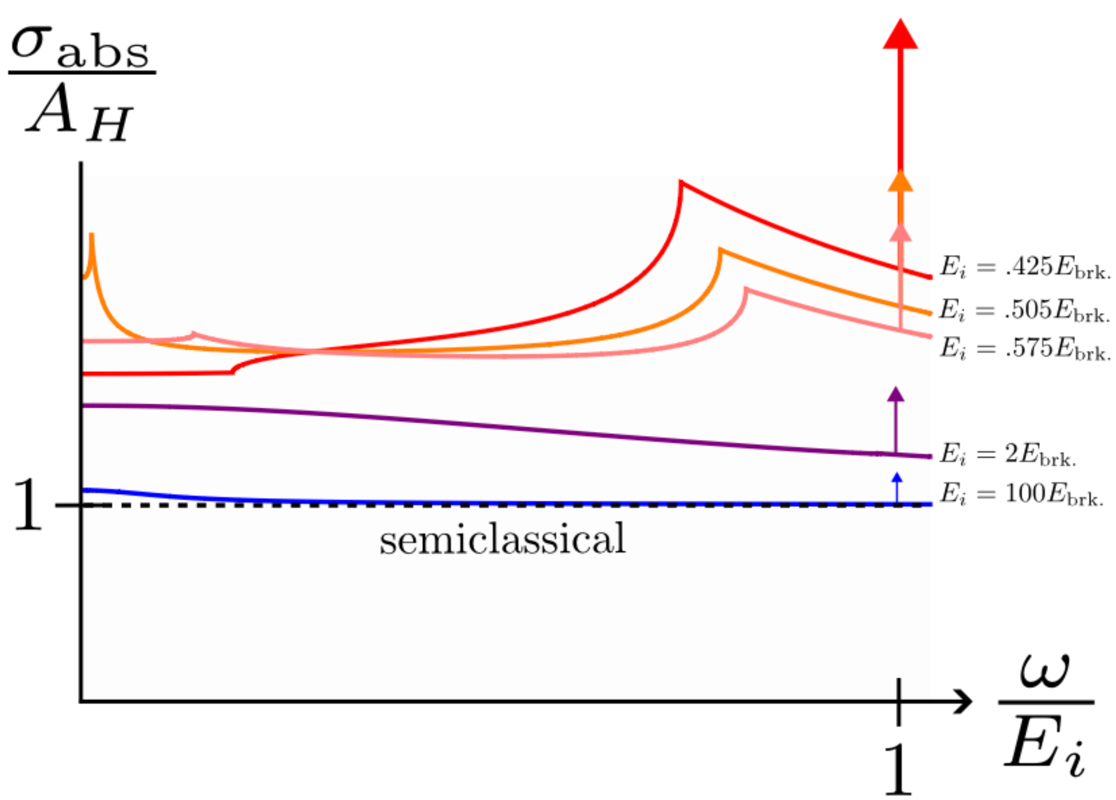


Scattering in sugra

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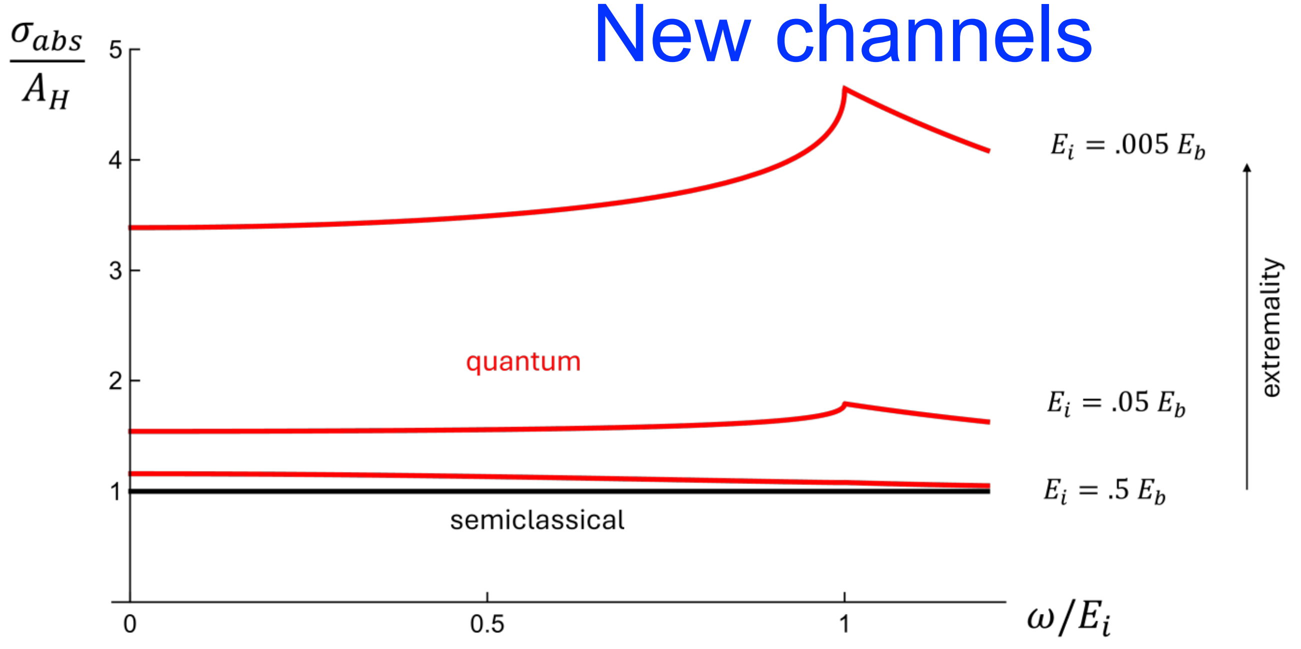


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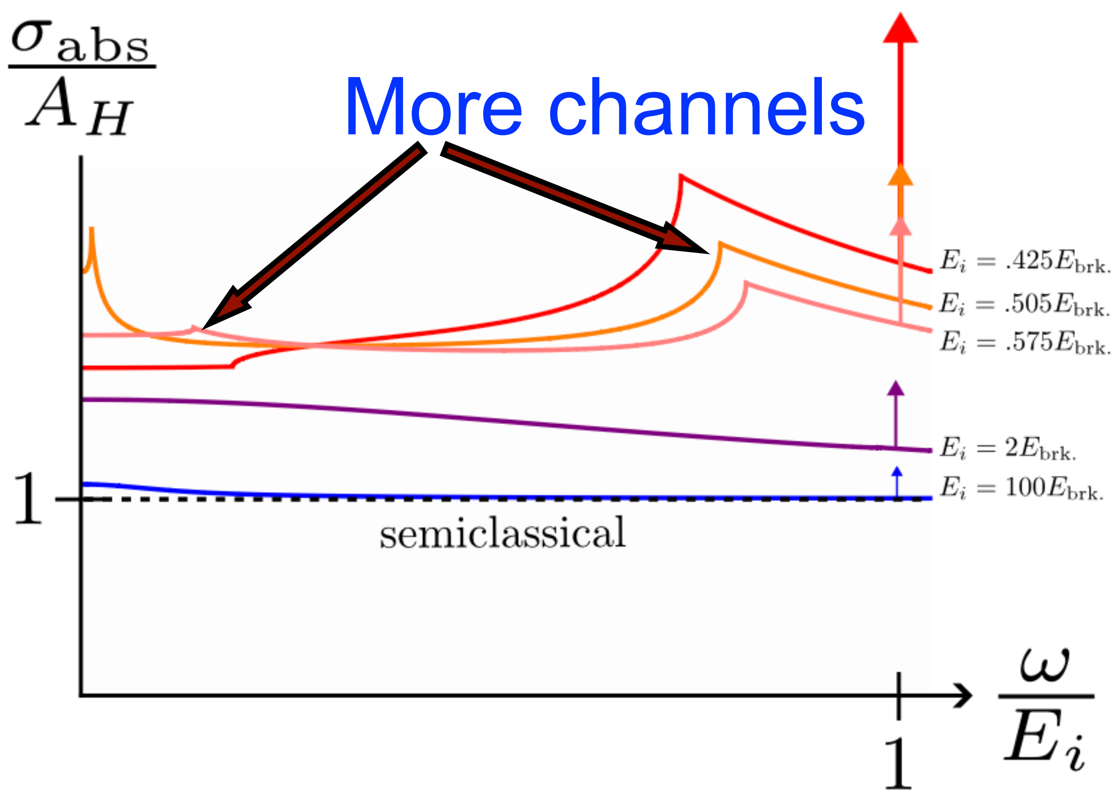
More channels

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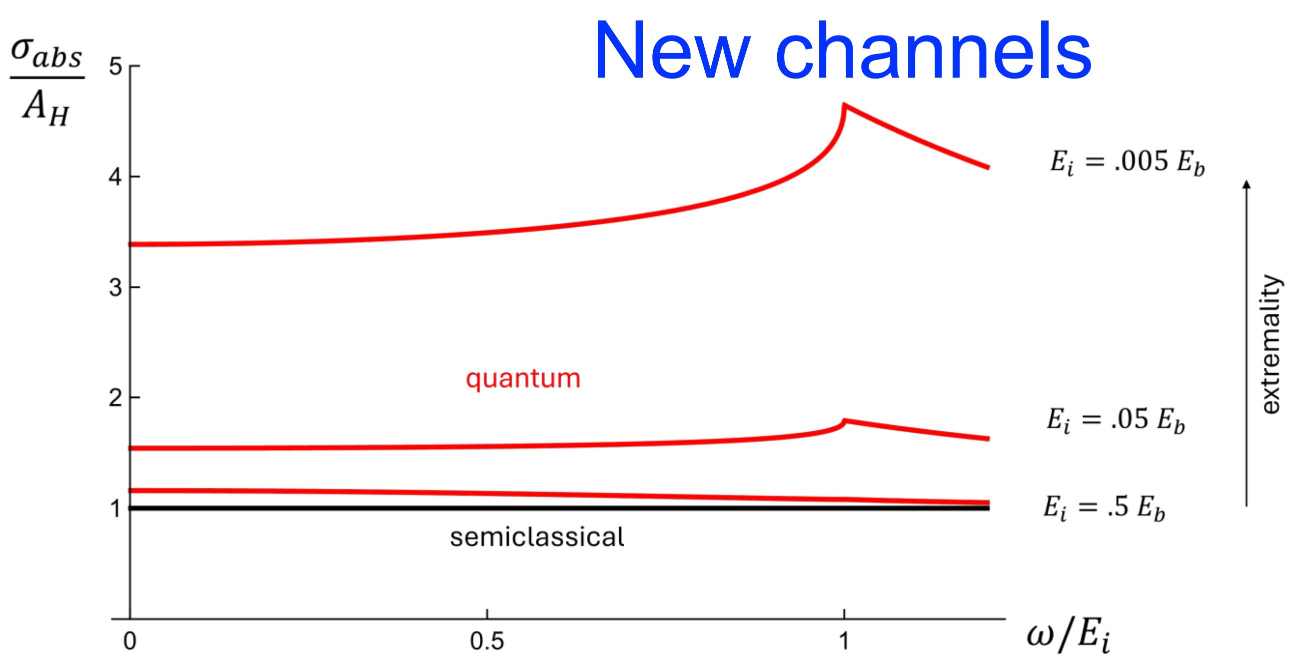


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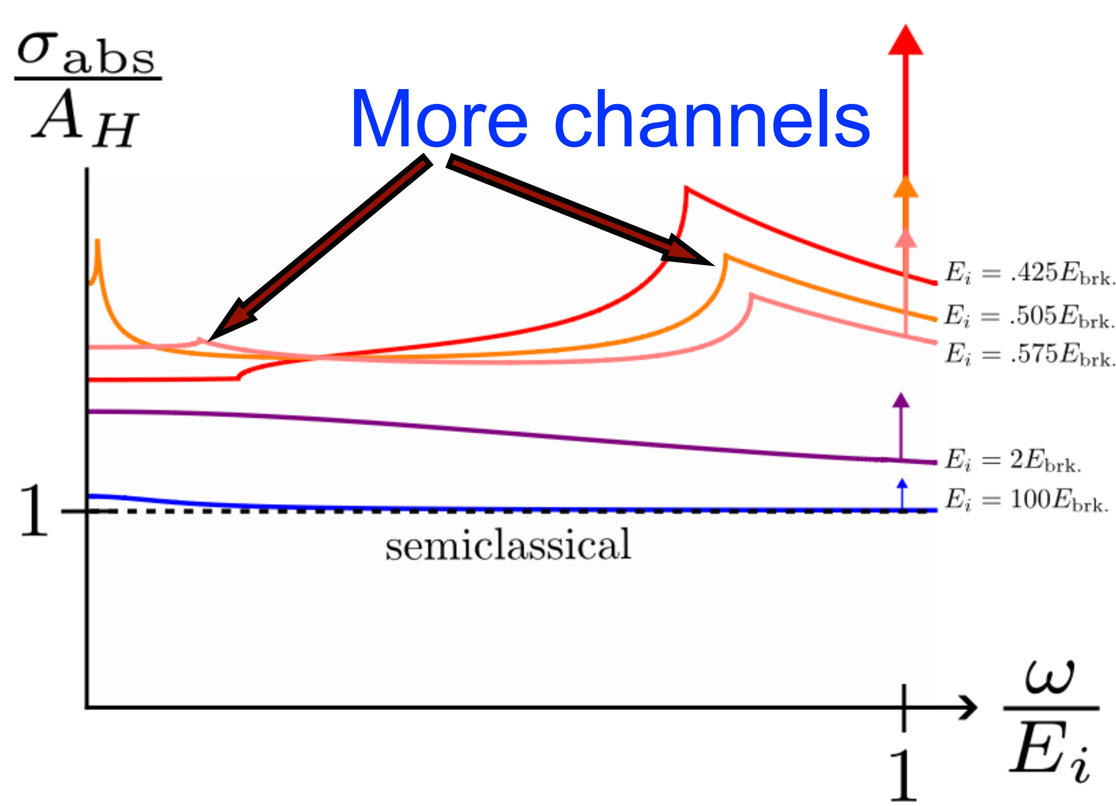


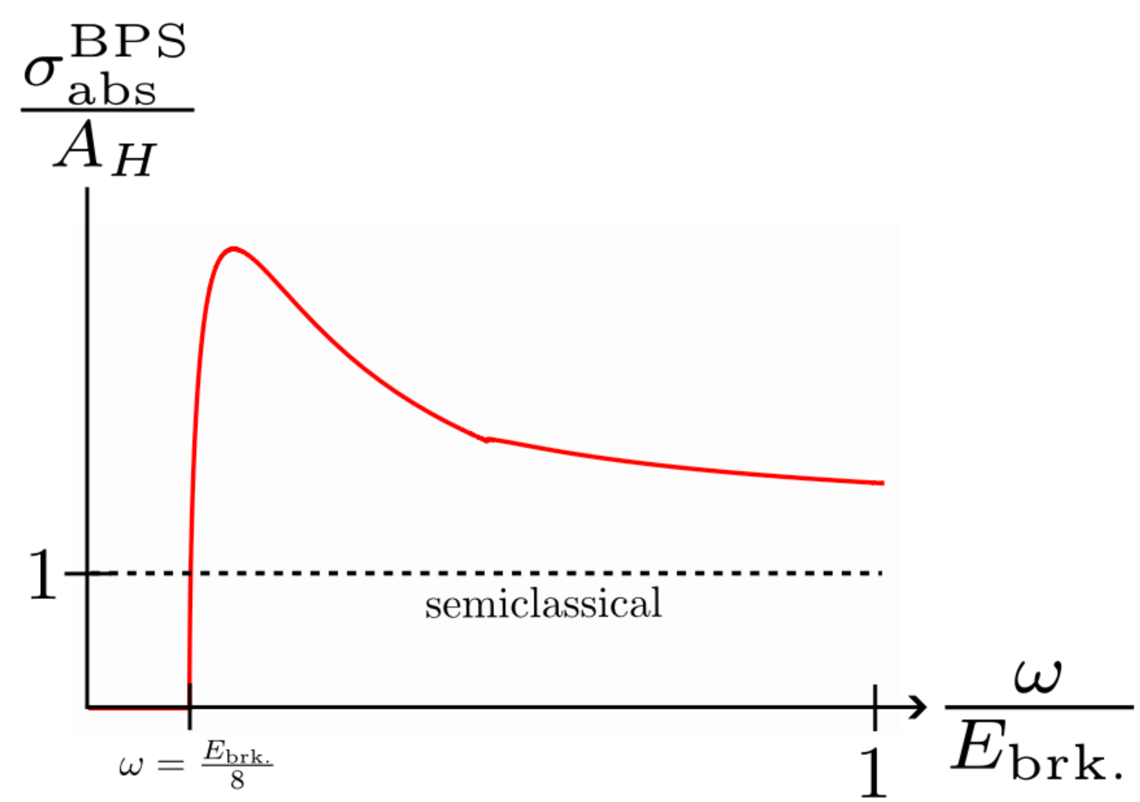


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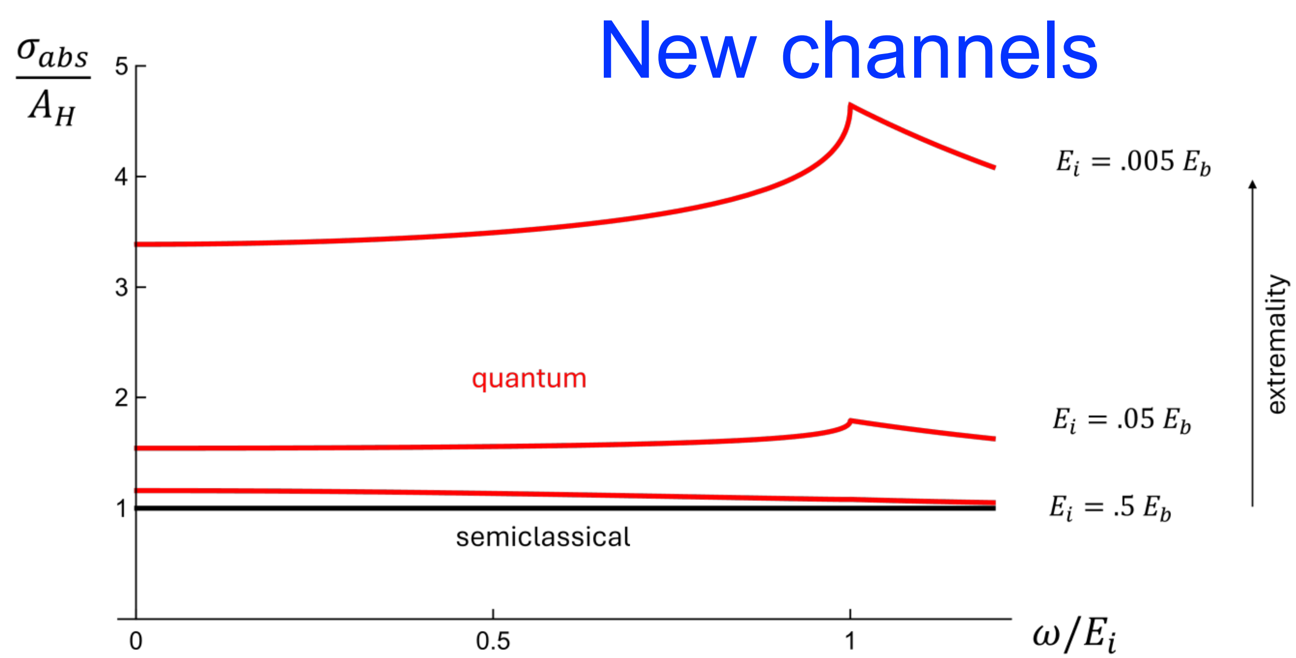


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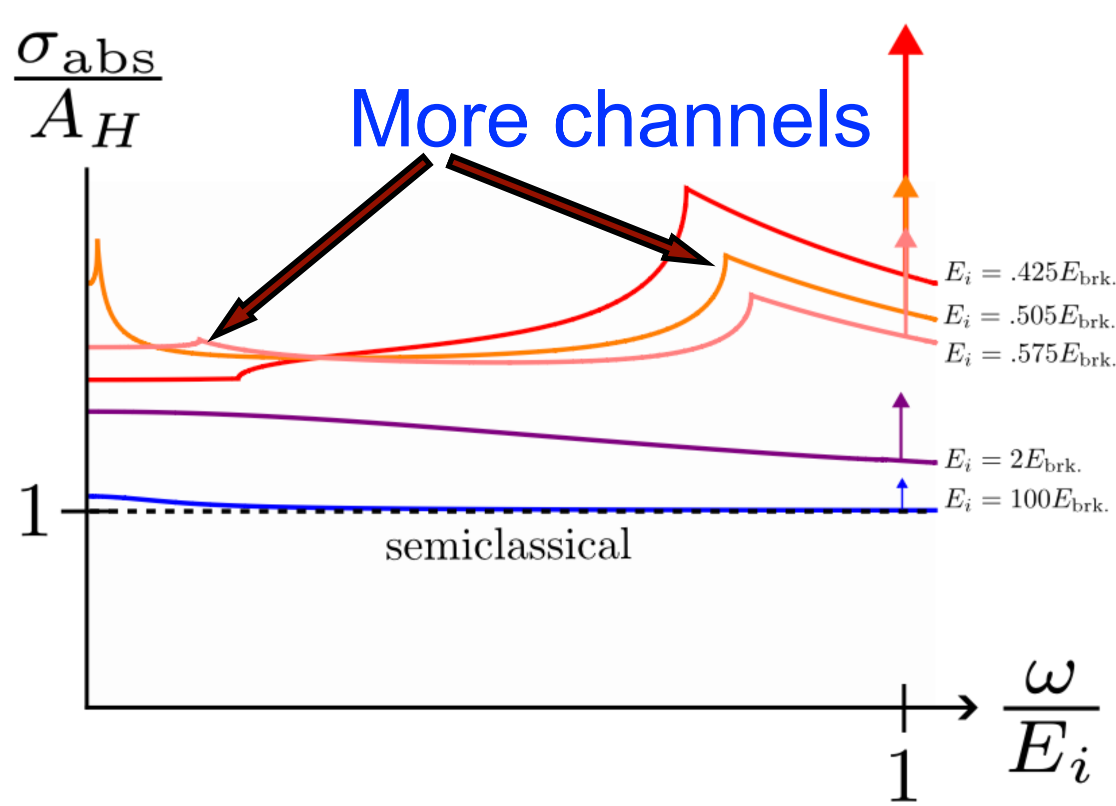


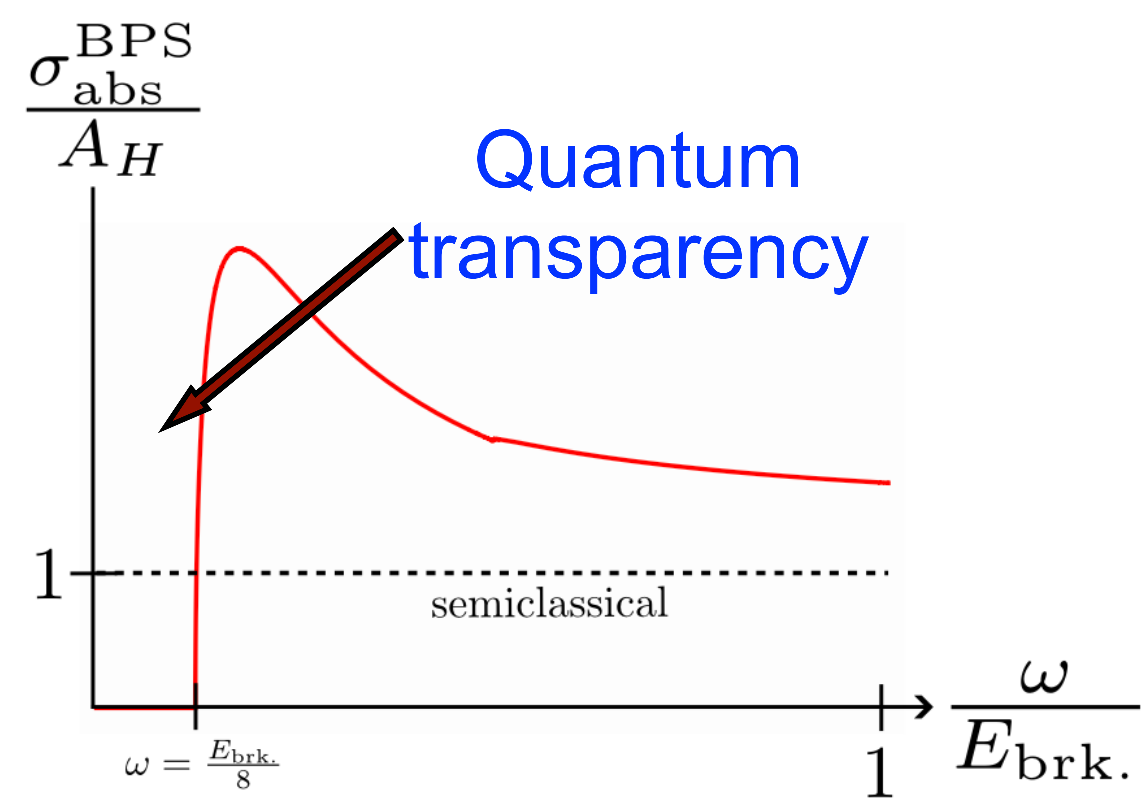


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# 3. More developments

- Extract a signal of *Schwarzian* modes *from* the one-loop *string worldsheet* amplitude (slightly indirectly).

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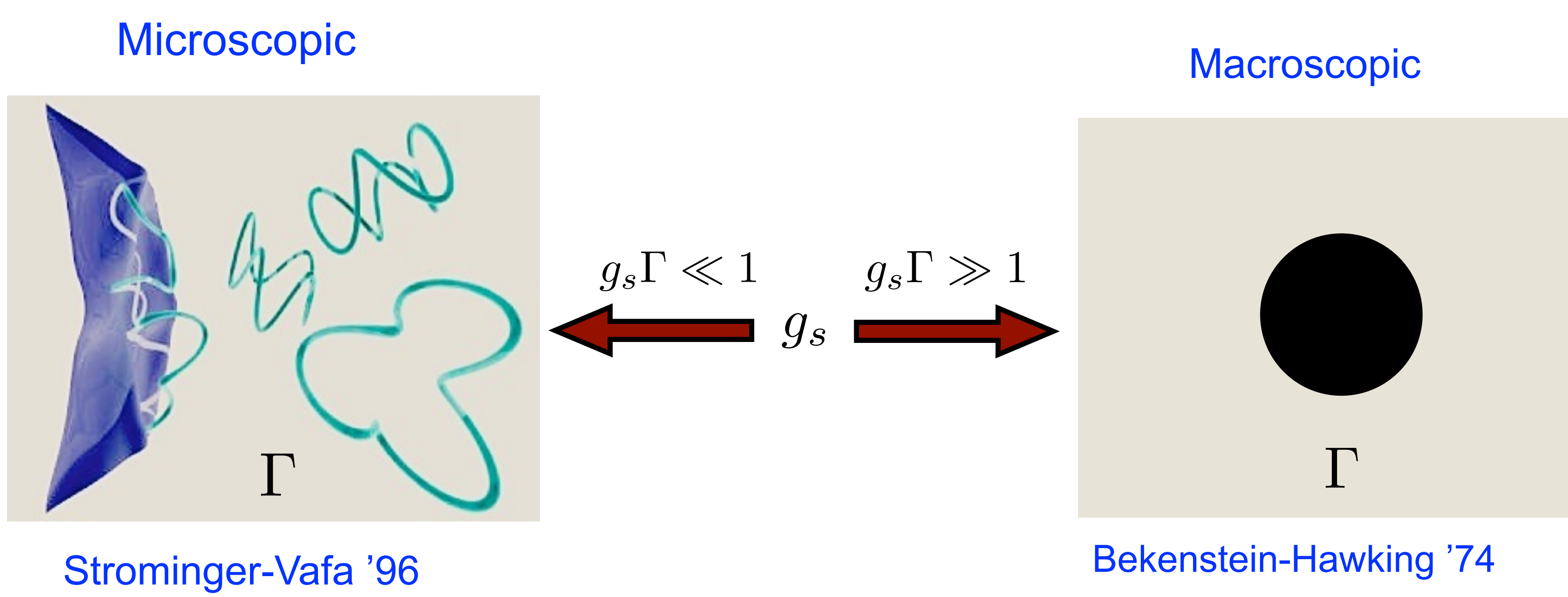
  (Classical) super-radiant instability takes BH away from extremality. Time-scale of decay comparable to Schwarzian time-scale. [Maldacena-S.M., '23 unpublished]

# Microstates of supersymmetric BHs in string theory



# When “counting” susy microstates in string theory, one really calculates the *index*

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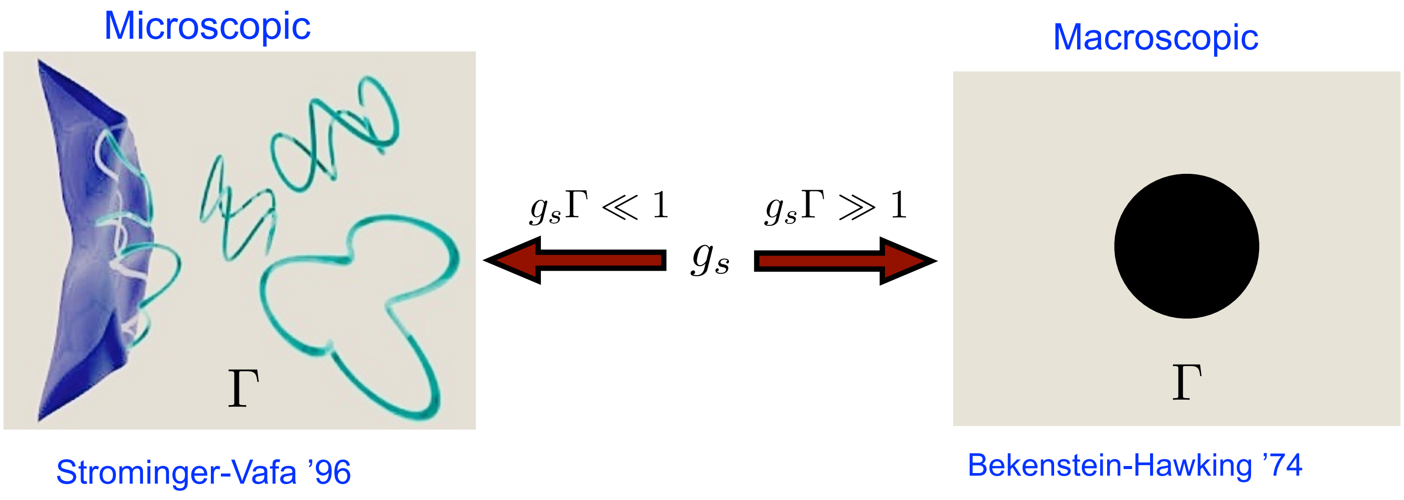

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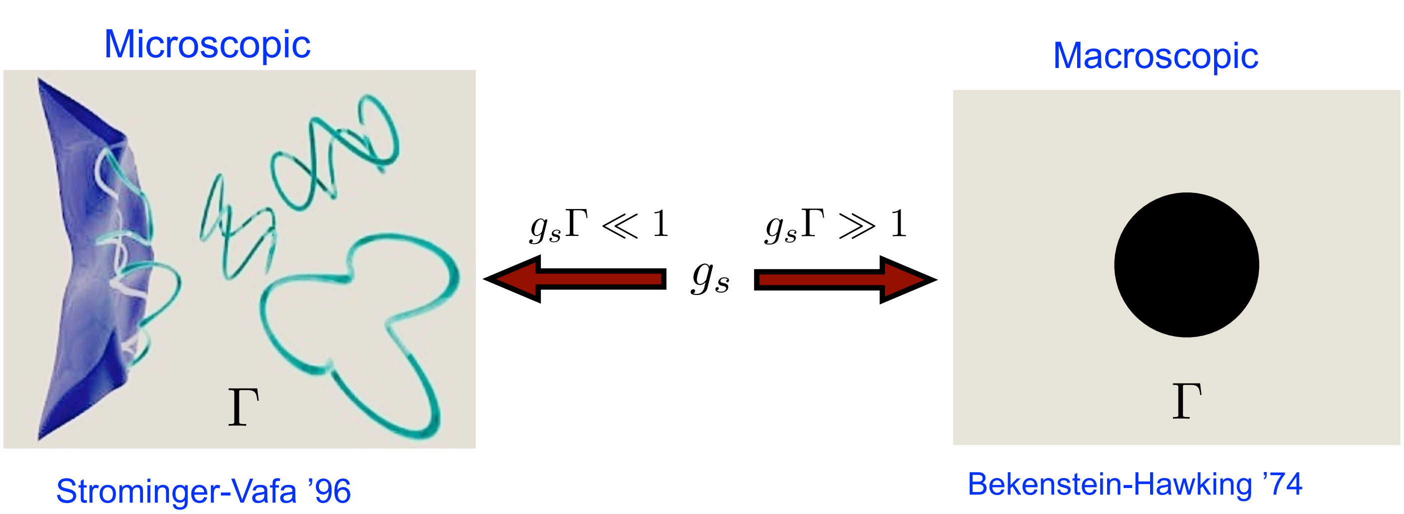

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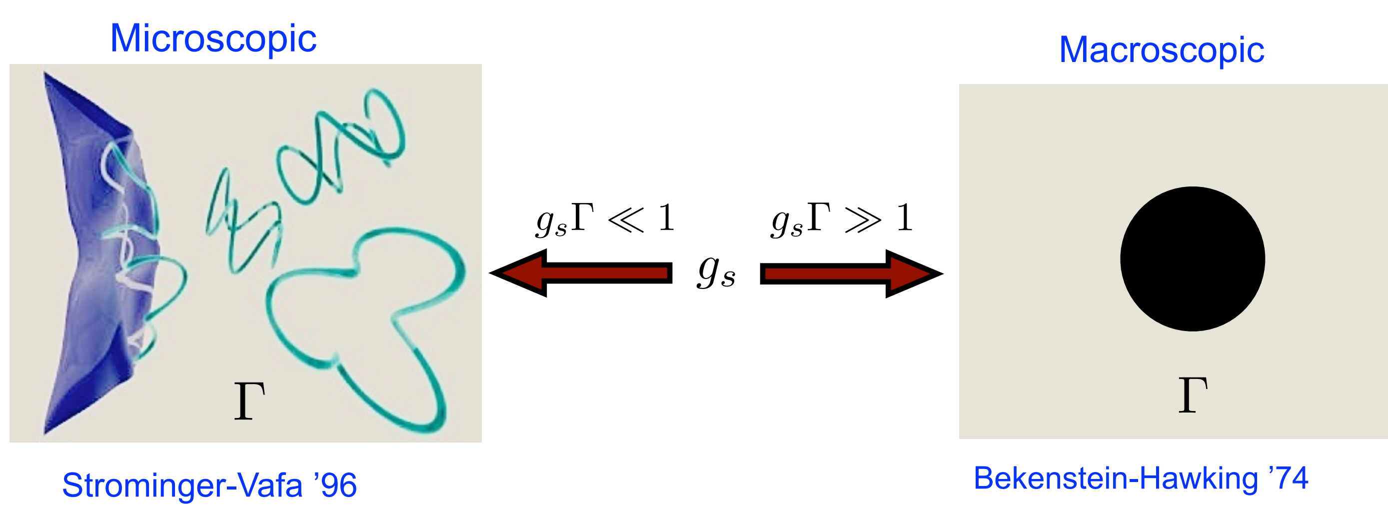

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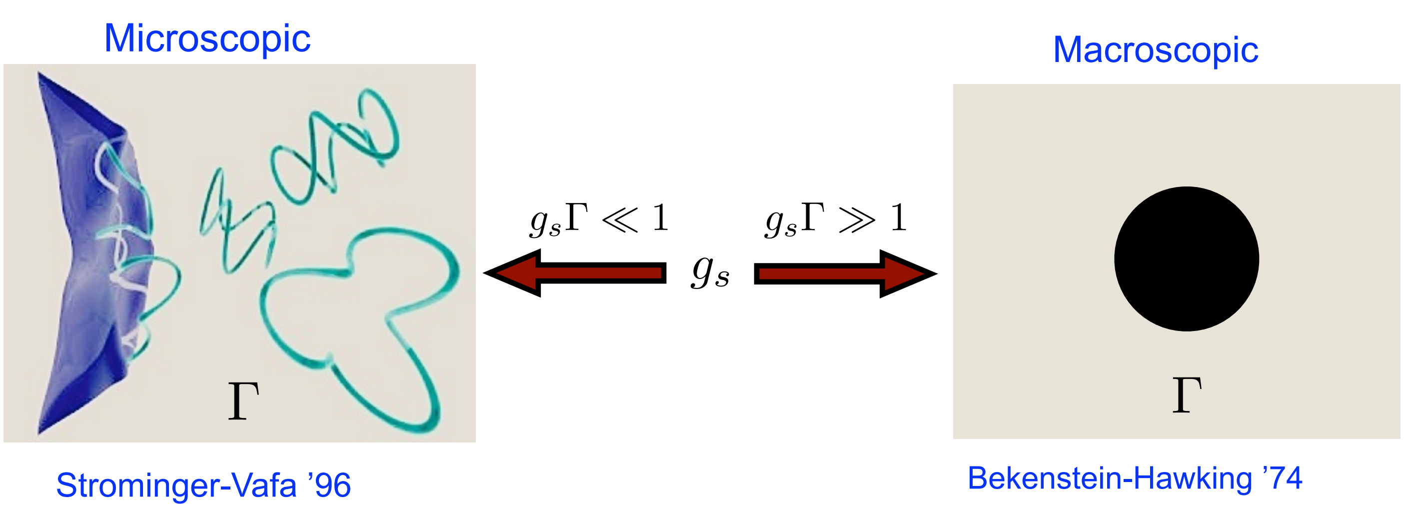
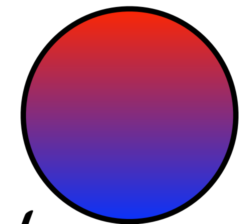


# How can we justify this agreement between microscopic index and grav. entropy?

- For a single BH, we can use 

  (*i*) Quantum decoupling $\mathcal{H}_{\rm BH} \otimes \mathcal{H}_{\rm out}$ [Iliesiu, S.M. Turiaci,'22]

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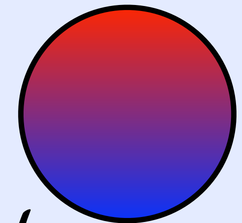

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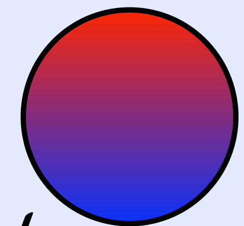

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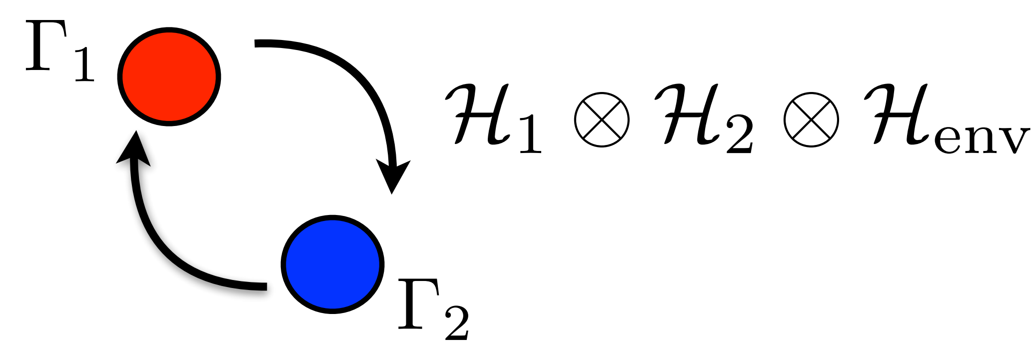

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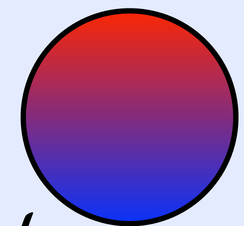

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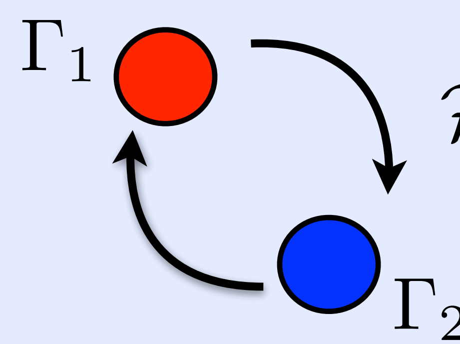


$\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_{\mathrm{env}}$

❖ Cannot decouple both BHs while keeping bnd state!

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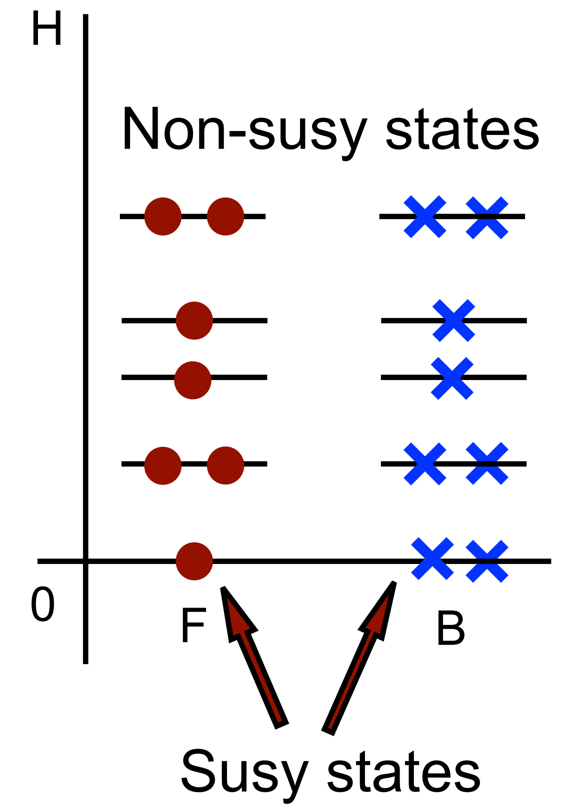

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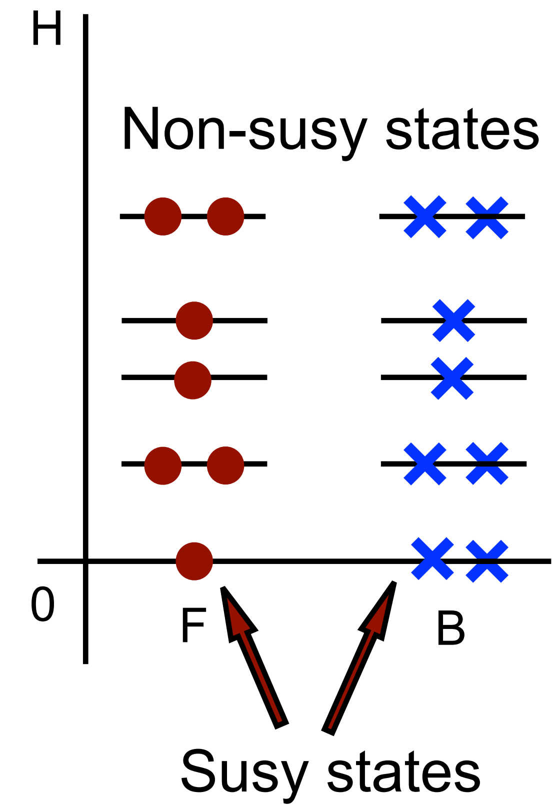

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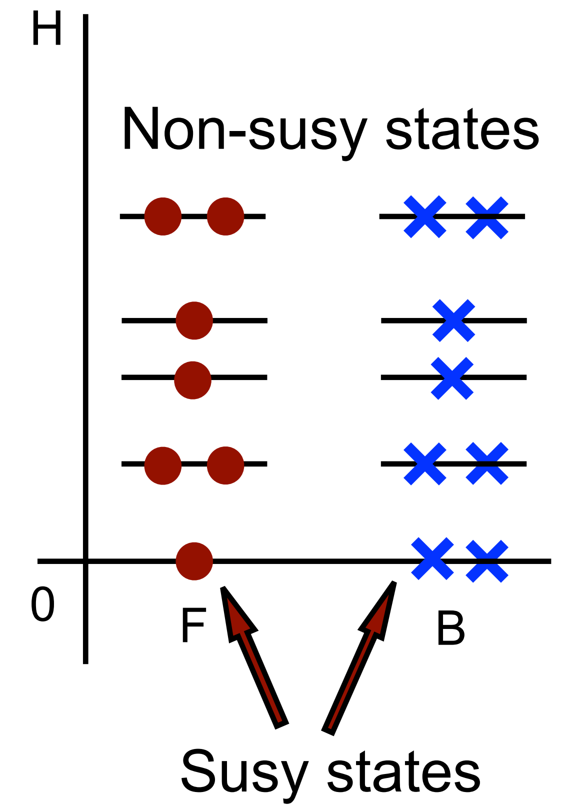


$$Z^{\text{index}}_{\text{grav}} = \int_{\psi_M(t_E+\beta)=\psi_M(t_E)} DG_{MN}\, D\psi_M\; e^{-S_{\text{grav}}(G_{MN},\psi_M)}$$

What are the saddles of the gravitational index?

# There is a tension between supersymmetric BH and the gravitational index

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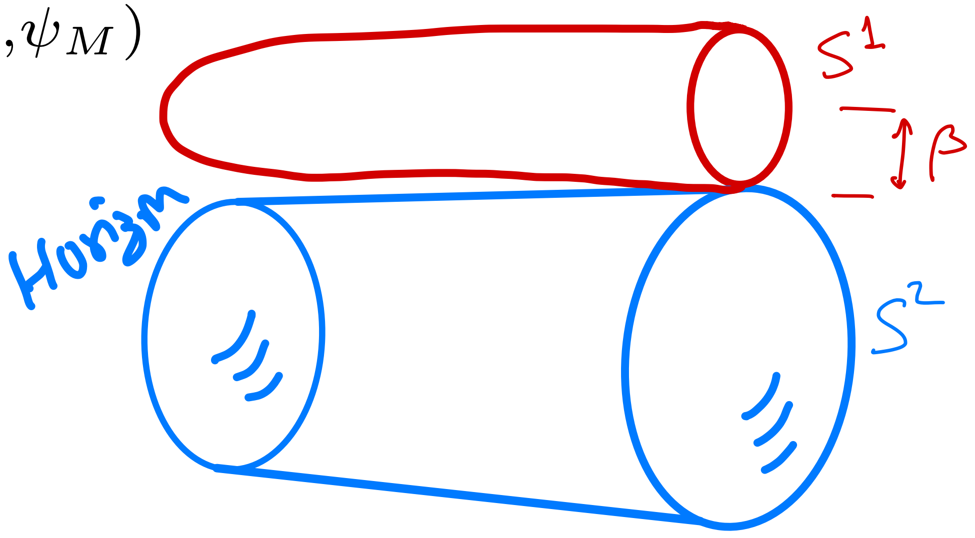

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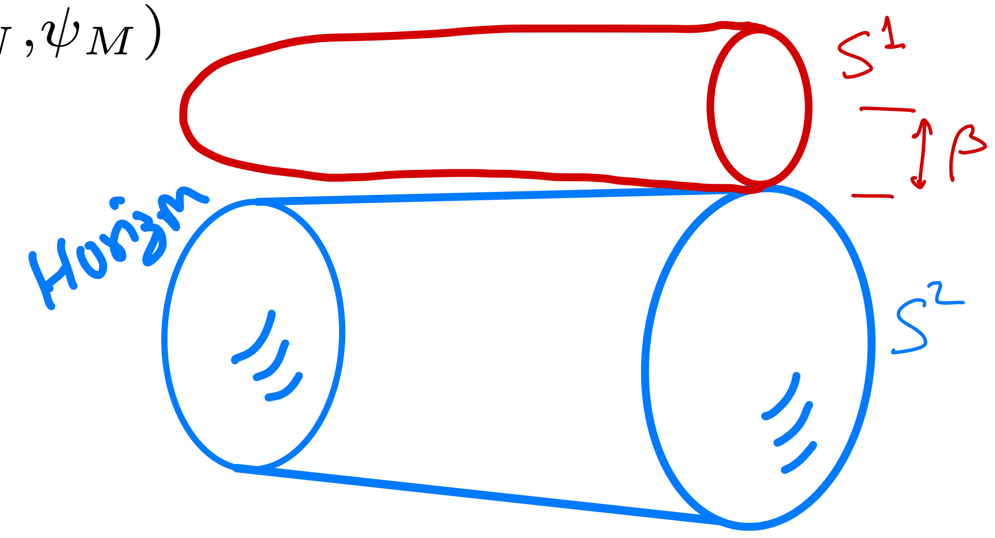


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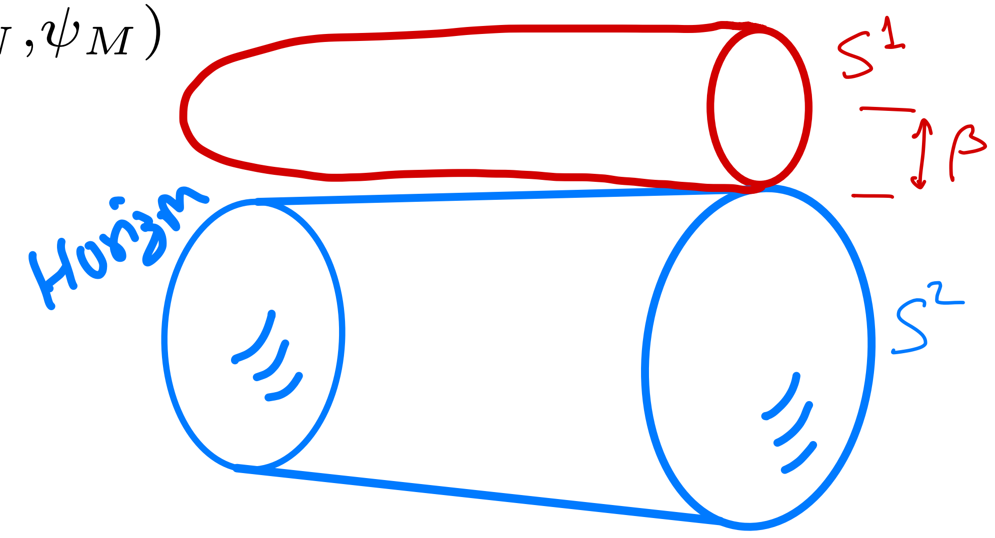


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- Periodic fermions vs smooth spin structure.

# Resolution: true saddles of the gravitational index are Euclidean susy non-extremal BHs

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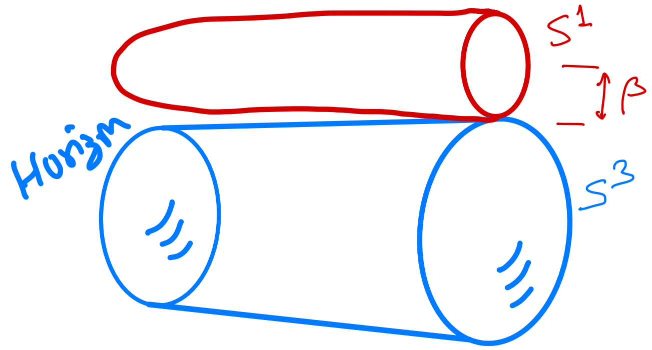


Cabo-Bizet, Cassani, Martelli, S.M. ’18 in AdS5

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  - $(-1)^F = e^{2\pi i R}$ converts problem to finding bosonic (complex) solutions

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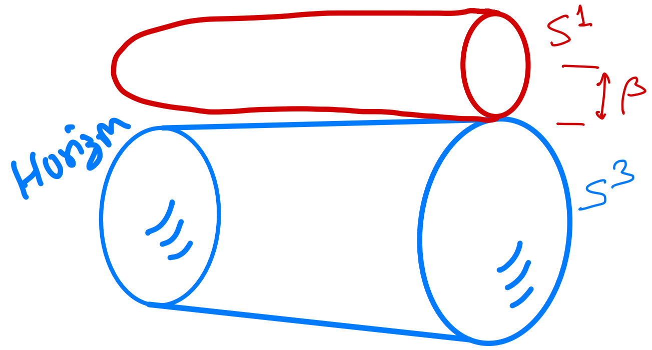


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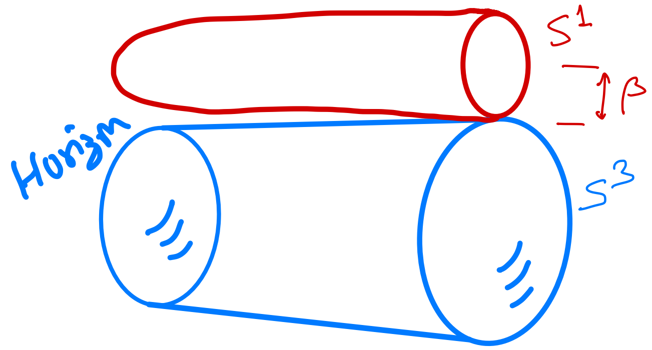


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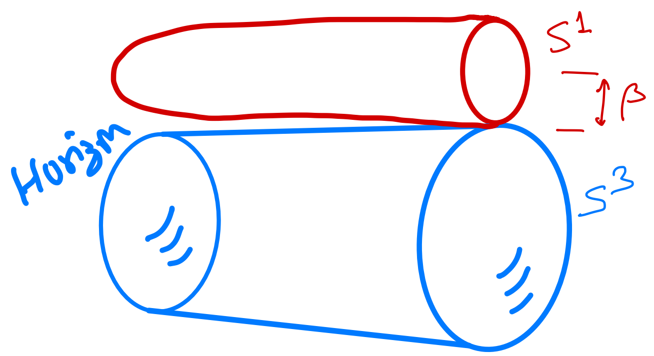


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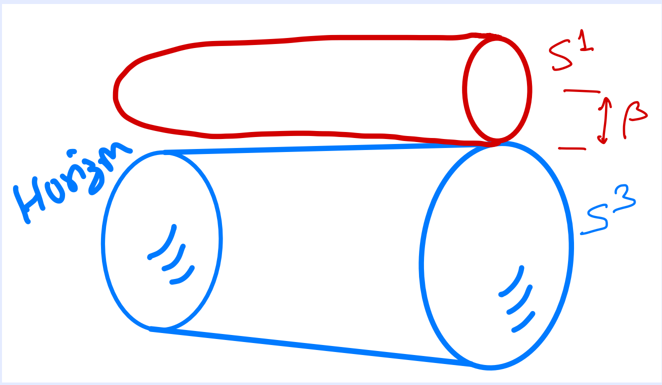


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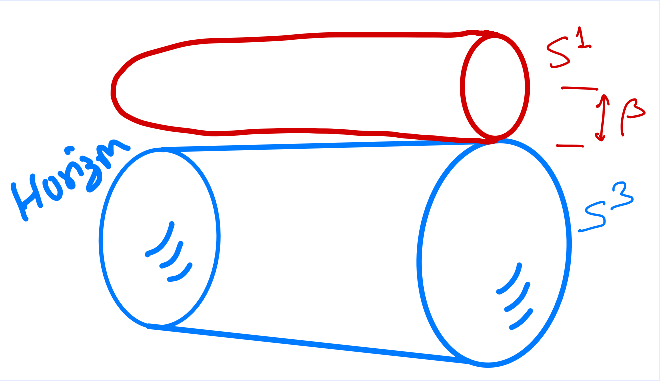


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Generalized to AdS$_d$, flat space

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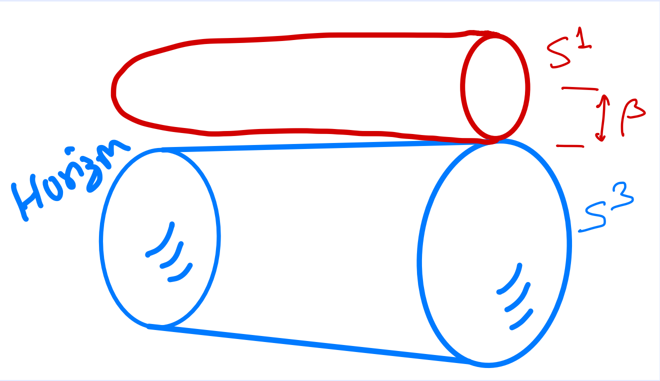


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Generalized to $AdS_d$, flat space

- Note: all these complex saddles allowed by the Konstevich-Segal-Witten criterion [P. Benetti-Genolini, S.M. ’25]

# Simple example: 4d Einstein-Maxwell theory

- Solution determined by two harmonic functions

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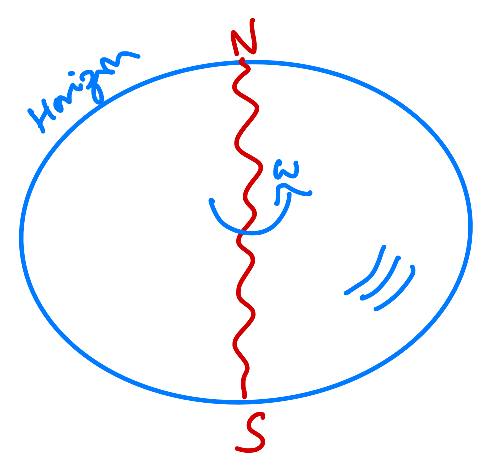
[Hartle-Hawking '72]



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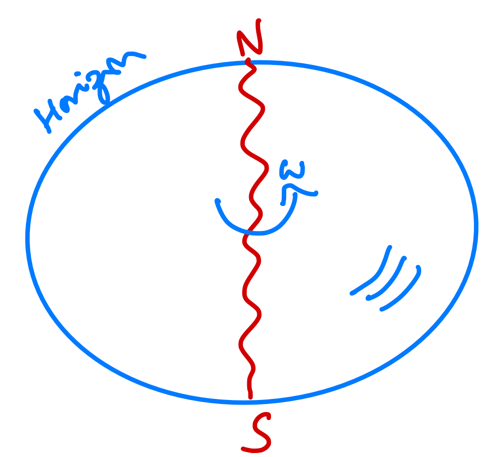


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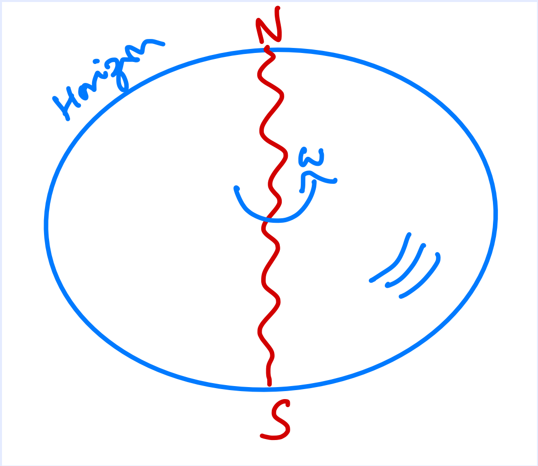


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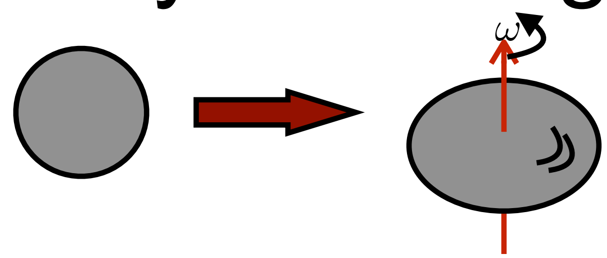


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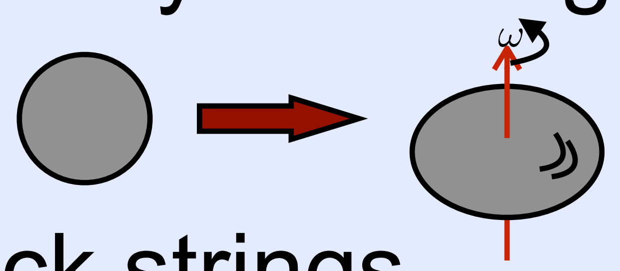


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- 5d AF sugra: BHs and black strings

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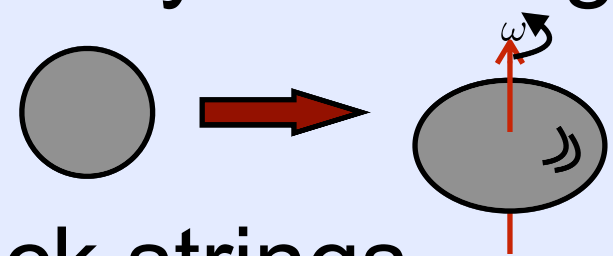


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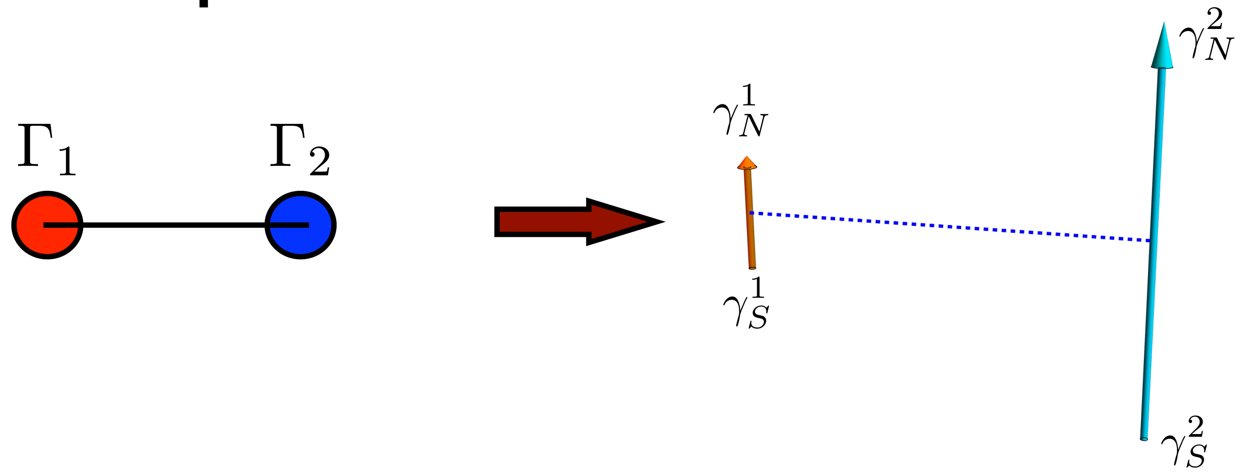
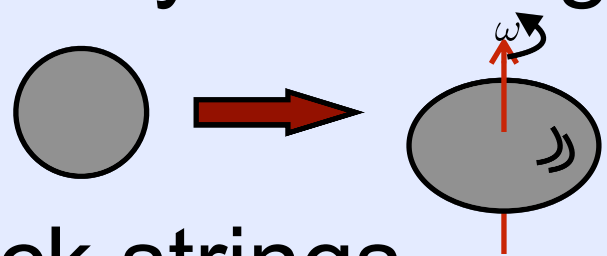
- 4d AF space: multi-BH bound states, wall-crossing




[Boruch, Iliesiu, S.M., Turiaci, '25]

# A systematic application of this idea leads to new solutions for the *gravitational index*

- 4d asymptotic flat supergravity: Rotating IWP solutions. New attractor mechanism




[Boruch, Iliesiu, S.M., Turiaci, '23]

[Chen, S.M., Turiaci, '24]

- 5d AF sugra: BHs and black strings

[Cassani, Ruipérez, Turetta, '24; Adhikari, Dharanipragada, Goswami, Virmani '24]

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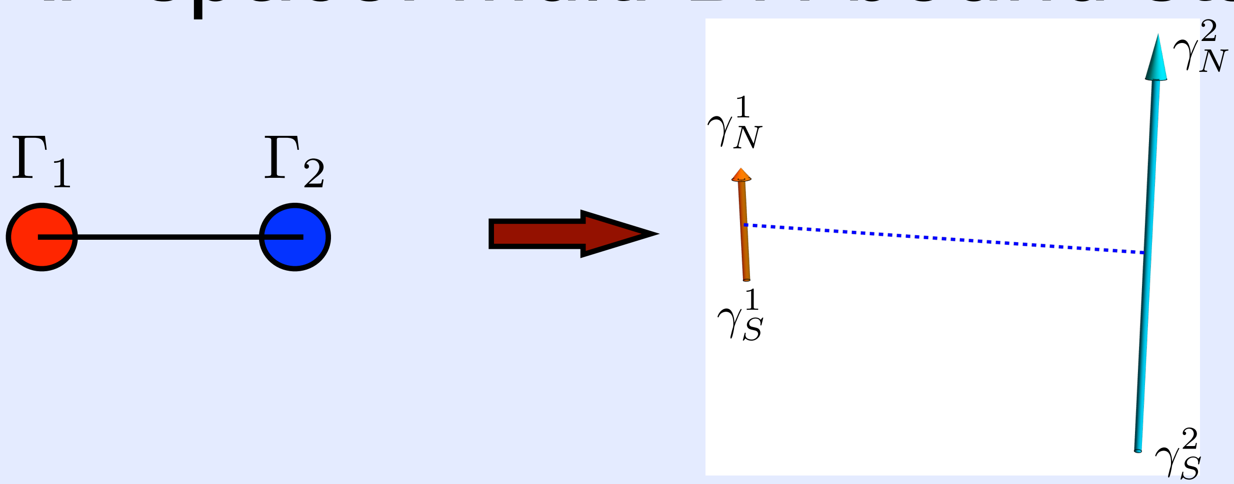


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- 5d AF space: BH bnd states, Black Rings [Cassani, Ruipérez, Turetta, '25] [Boruch, Emparan, Iliesiu, S.M., Turiaci, To appear.]

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Thank you! Questions?

# New attractors for the 4d index

[J.Boruch, L.V.Iliesiu, S.M., G.J.Turiaci, ’23]

# IWP saddles can be generalized to the full Type II string theory/CY3

- Multiple gauge fields in string theory. For every gauge field, split the harmonic function source into N/S

$$H(\mathbf{x}) = h + \frac{\gamma_N}{|\mathbf{x} - \mathbf{x}_N|} + \frac{\gamma_S}{|\mathbf{x} - \mathbf{x}_S|} \qquad \gamma_N + \gamma_S = \Gamma$$

*Fixed charges (monopole)*

*New parameters (dipole)* $\gamma_N - \gamma_S$ $|\mathbf{x}_N - \mathbf{x}_S|$

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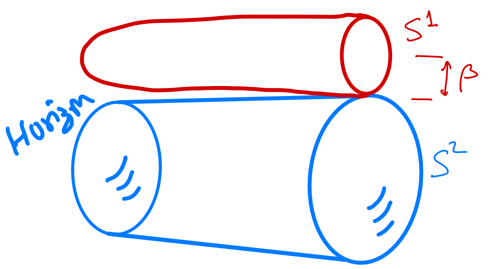
- Susy + smoothness fixes all parameters in terms of monopole charges and temperature

[J.Boruch, L.V.Iliesiu, S.M., G.J.Turiaci, ’23]

# Free energy is determined by new attractor mechanism [J.Boruch, L.V.Iliesiu, S.M., G.J.Turiaci, ’23]



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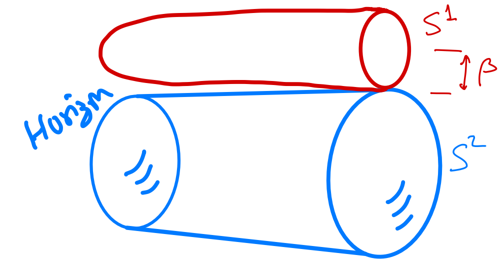
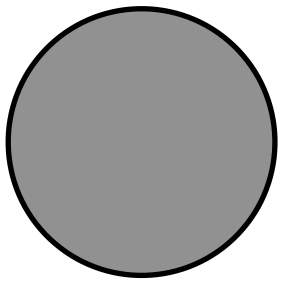



Extremal attractors

[Ferrara, Kallosh, Strominger ’95]



- Scalars constant on horizon

$$Y^I_{\rm ext} - (Y^I_{\rm ext})^* = iP^I$$

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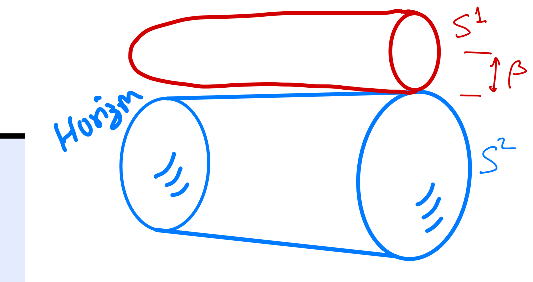


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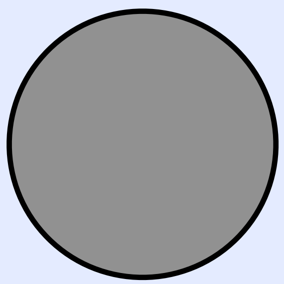

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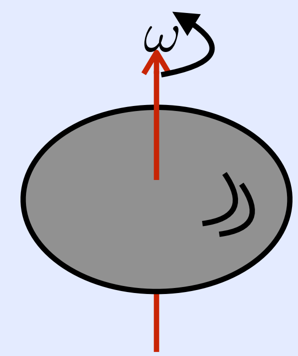


- Scalars depend on angle, moduli, $\beta$

NP $\quad Y^I = Y^I_{\text{ext}} \qquad \overline{Y^I} = 0$

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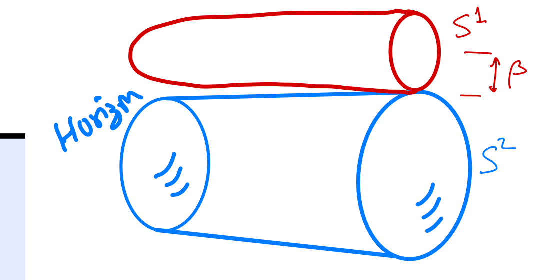


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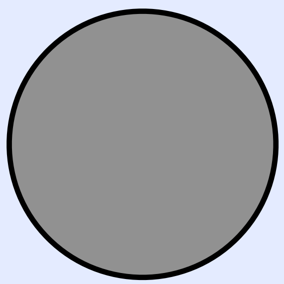

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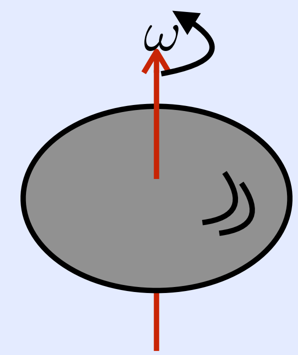


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$$-I_{\text{on-shell}} = \pi(q_I Y^I + 2iF)\big|_N + \pi(q_I \overline{Y}^I - 2i\overline{F})\big|_S$$

$$= -\beta M_{\rm BPS} + S^{\rm extremal}_{\rm BH}(\Gamma)$$

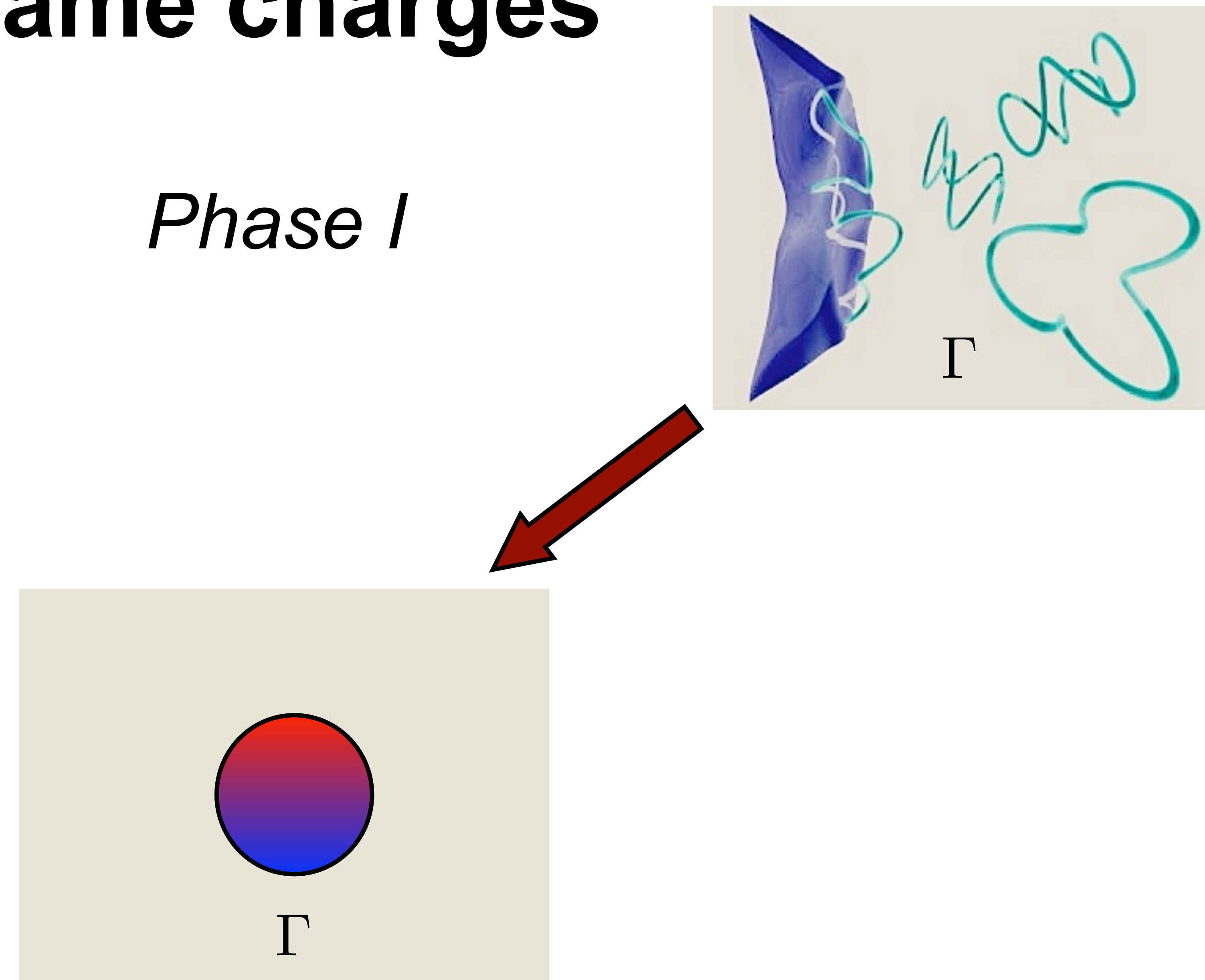
[cf Ooguri-Strominger-Vafa ’04]

# BH bound states and wall-crossing

[J.Boruch, L.V.Iliesiu, S. M., G.J.Turiaci, ’25]

# Multiple grav. configurations can exist with same charges

*Phase I*




$S(\Gamma)$

[Bates-Denef ’00; Denef-Moore ’07]

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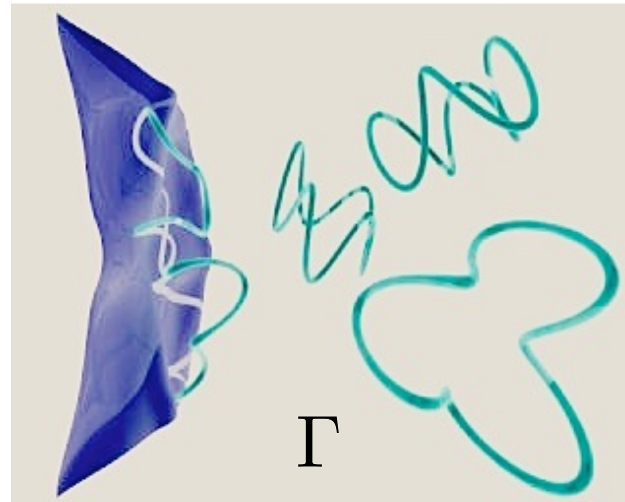


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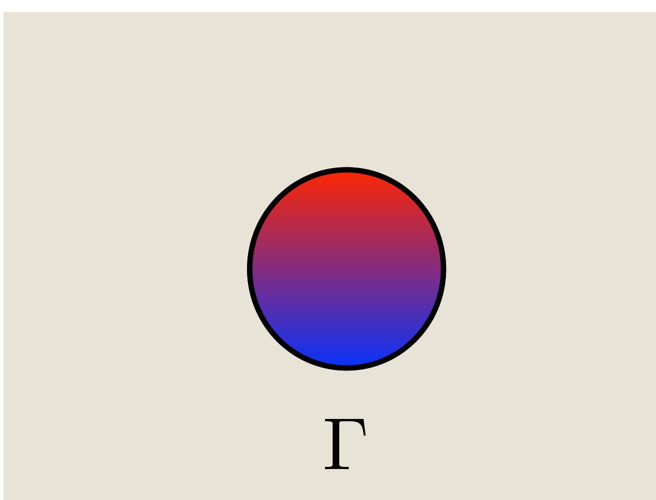


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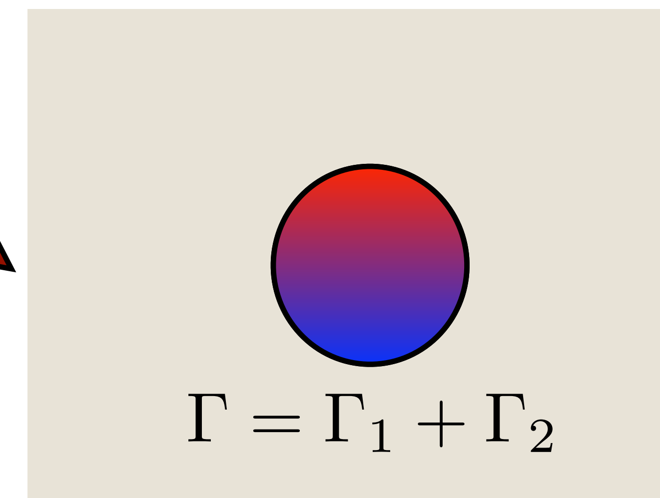


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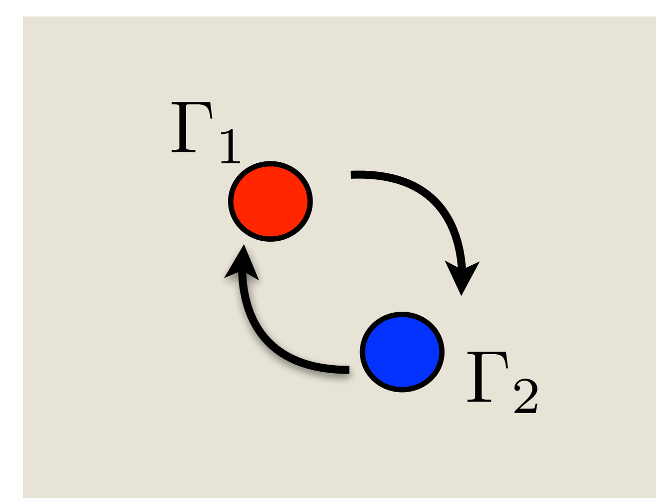


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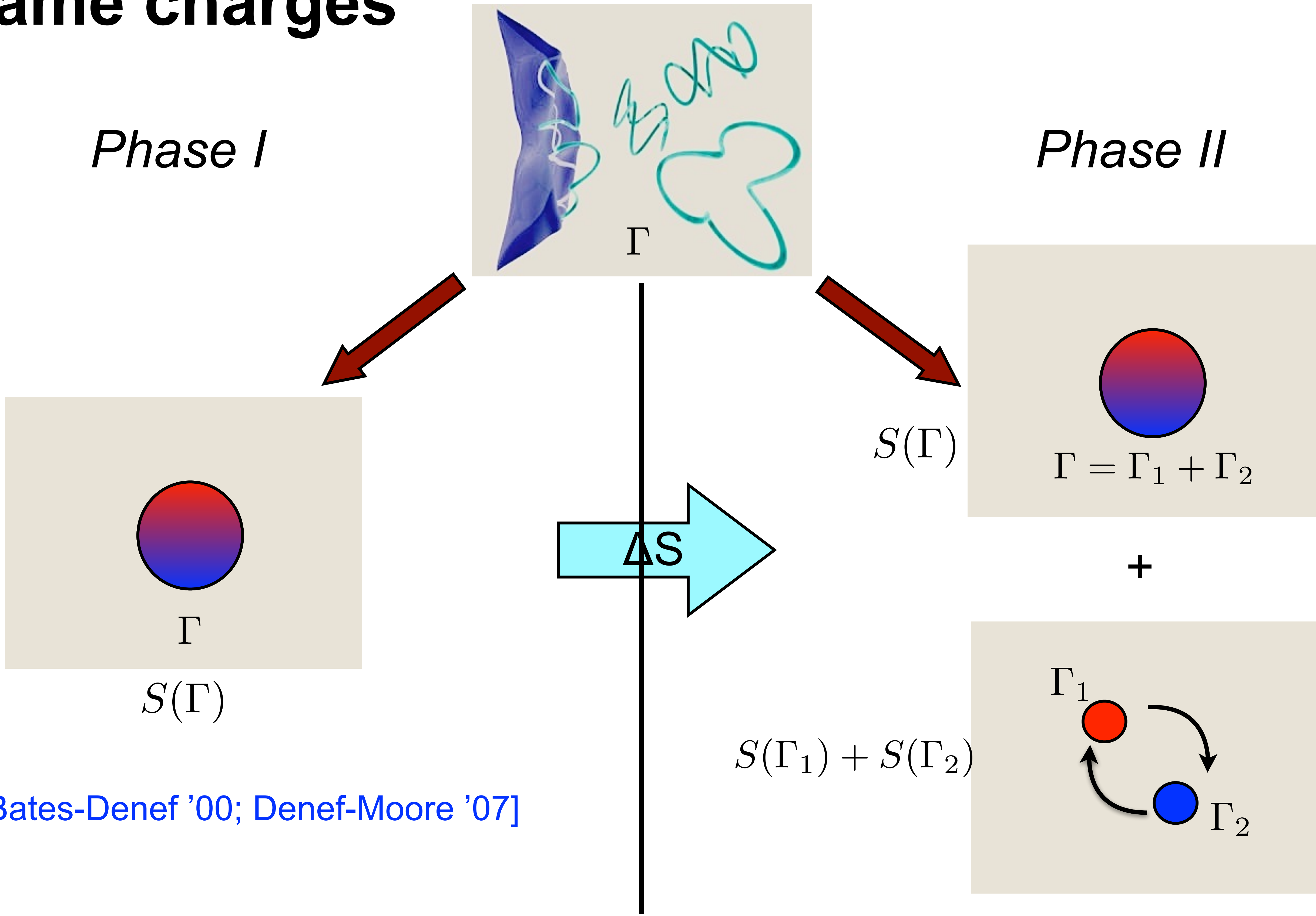
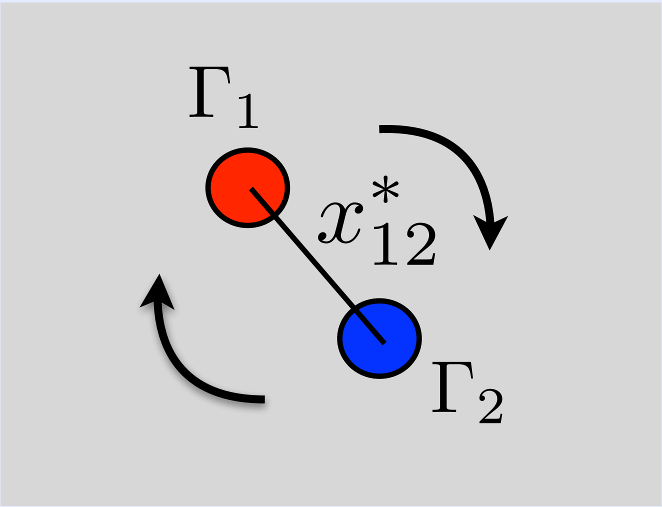



[Bates-Denef '00; Denef-Moore '07]

# 2-BH bound states in N=2 sugra are rigid, and exist only in regions of moduli space




Distance fixed in terms of charges $\Gamma_{1,2}$ and asymptotic scalar moduli $h$ .

$$x^*_{12} = -\frac{\langle \Gamma_1, \Gamma_2 \rangle}{\langle \Gamma_1, h \rangle}$$

Bates-Denef '00

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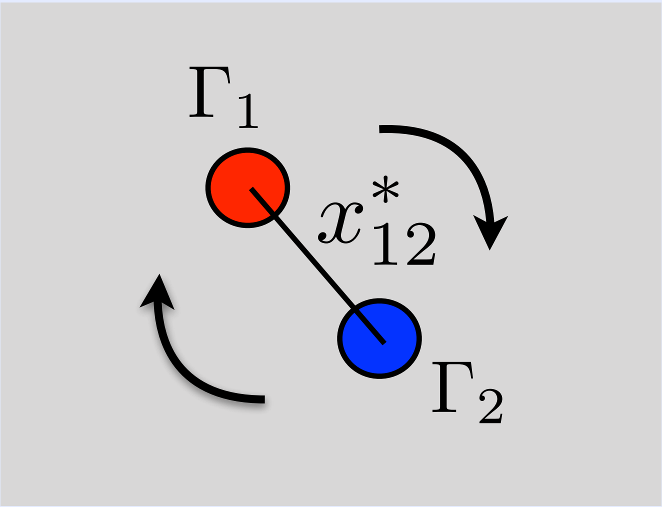


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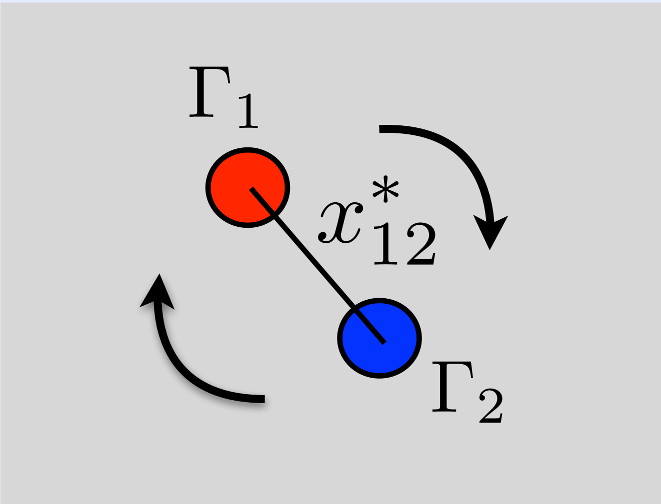


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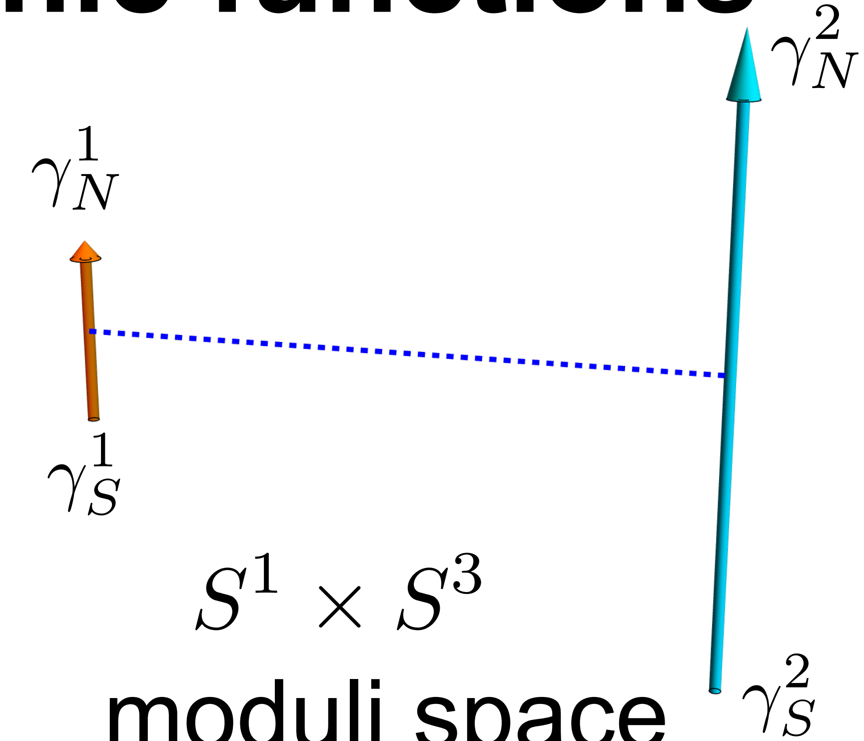
- For fixed charges and scalar moduli, moduli space of 2-centered bnd state is $S^2$ (global symmetries).

# Index saddles for BH bound states can be constructed using harmonic functions


$S^2$ moduli space




$S^1 \times S^3$
moduli space

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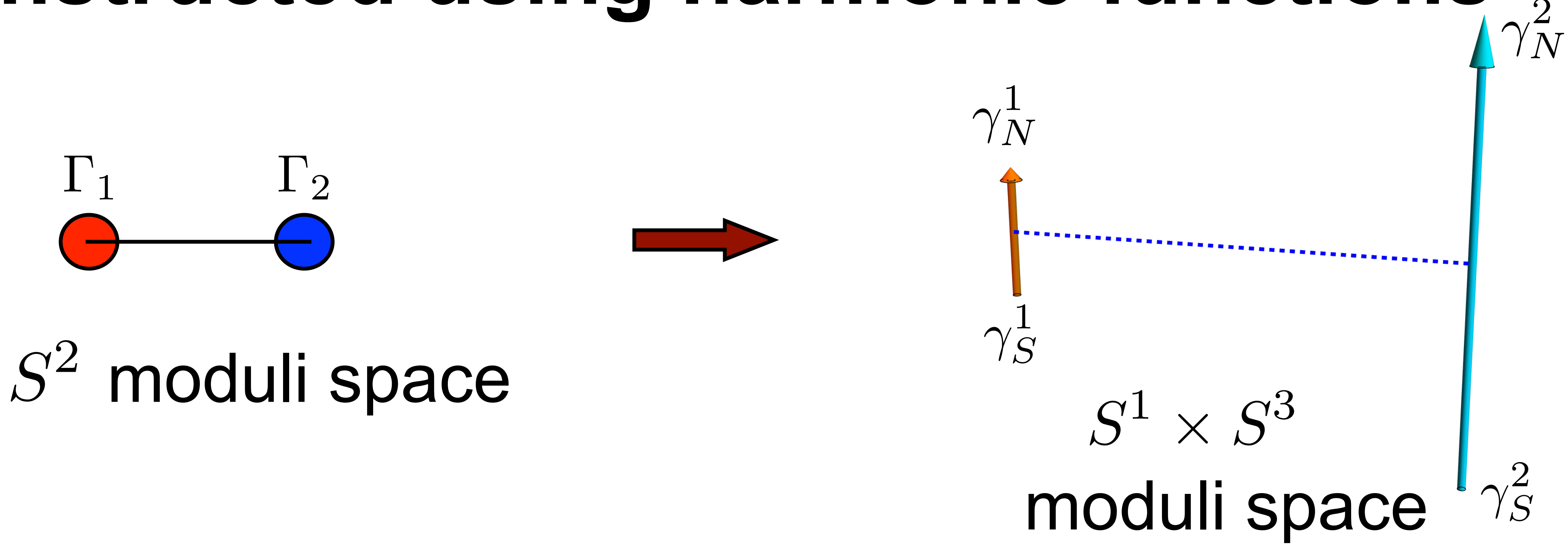


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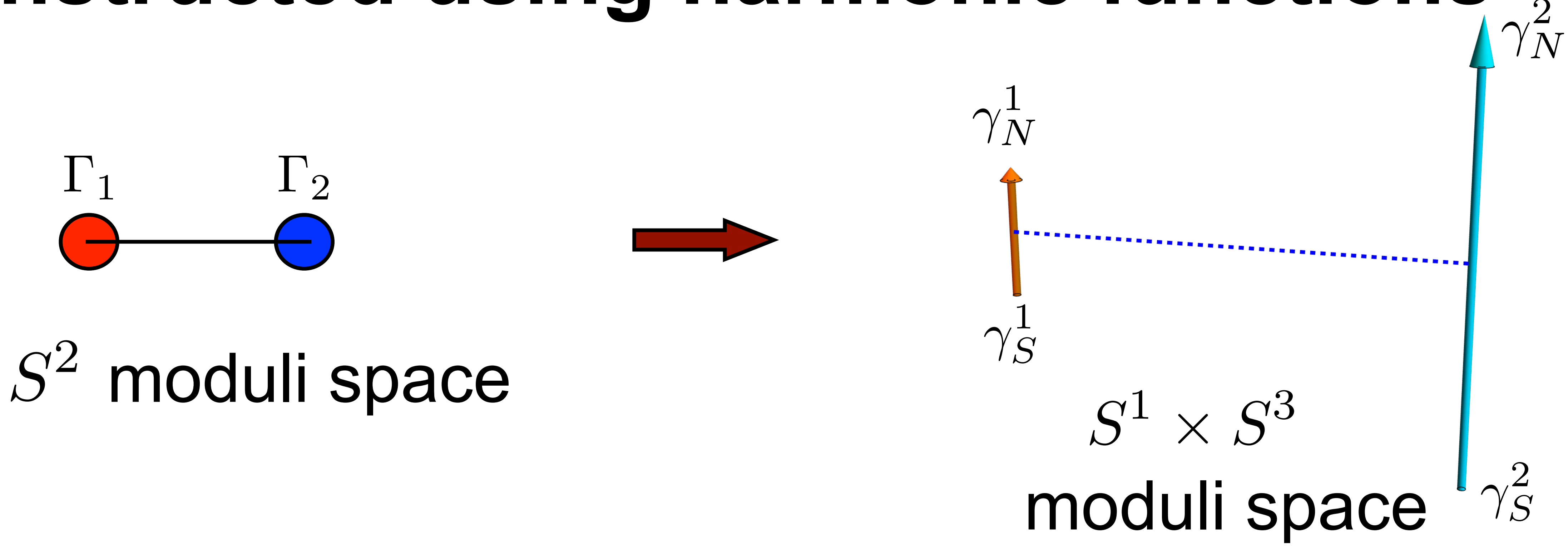


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- Remarkably,

$$-I_{\text{on-shell}} = -\beta|Z(\Gamma, h_\infty)| + S^{\text{ext}}(\Gamma_1) + S^{\text{ext}}(\Gamma_2)$$

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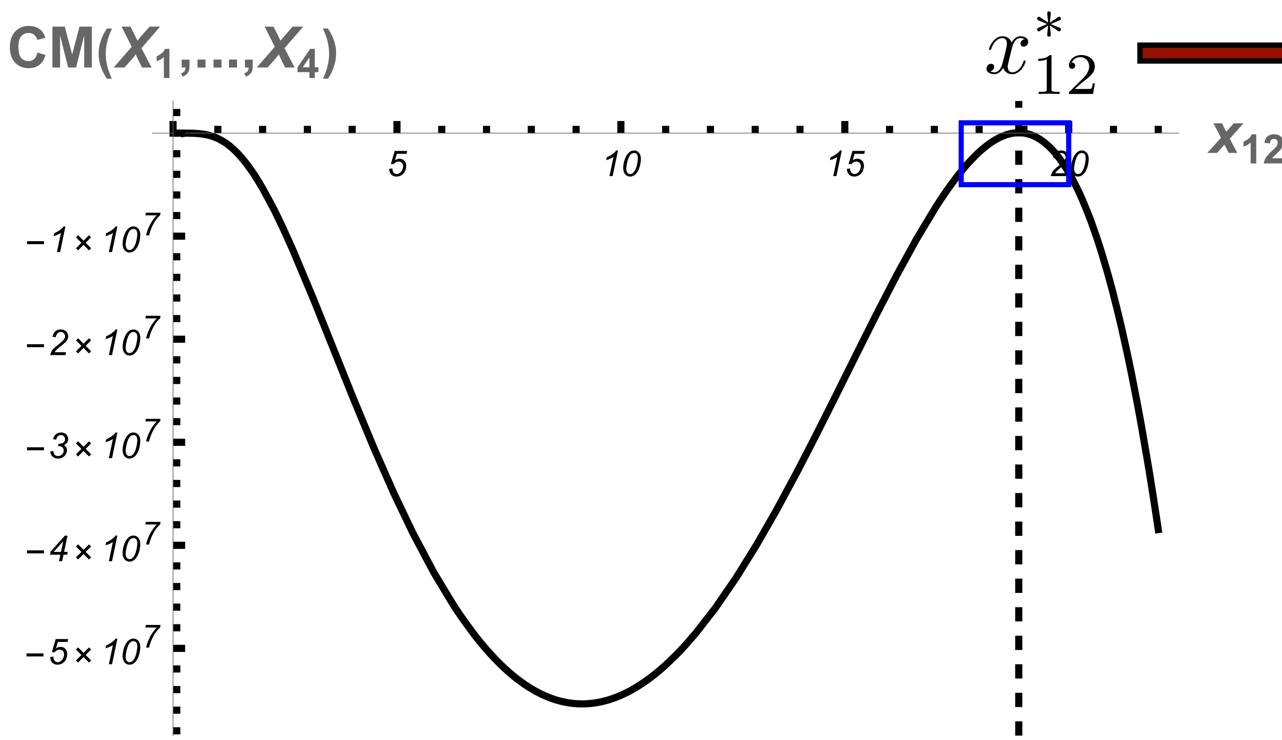
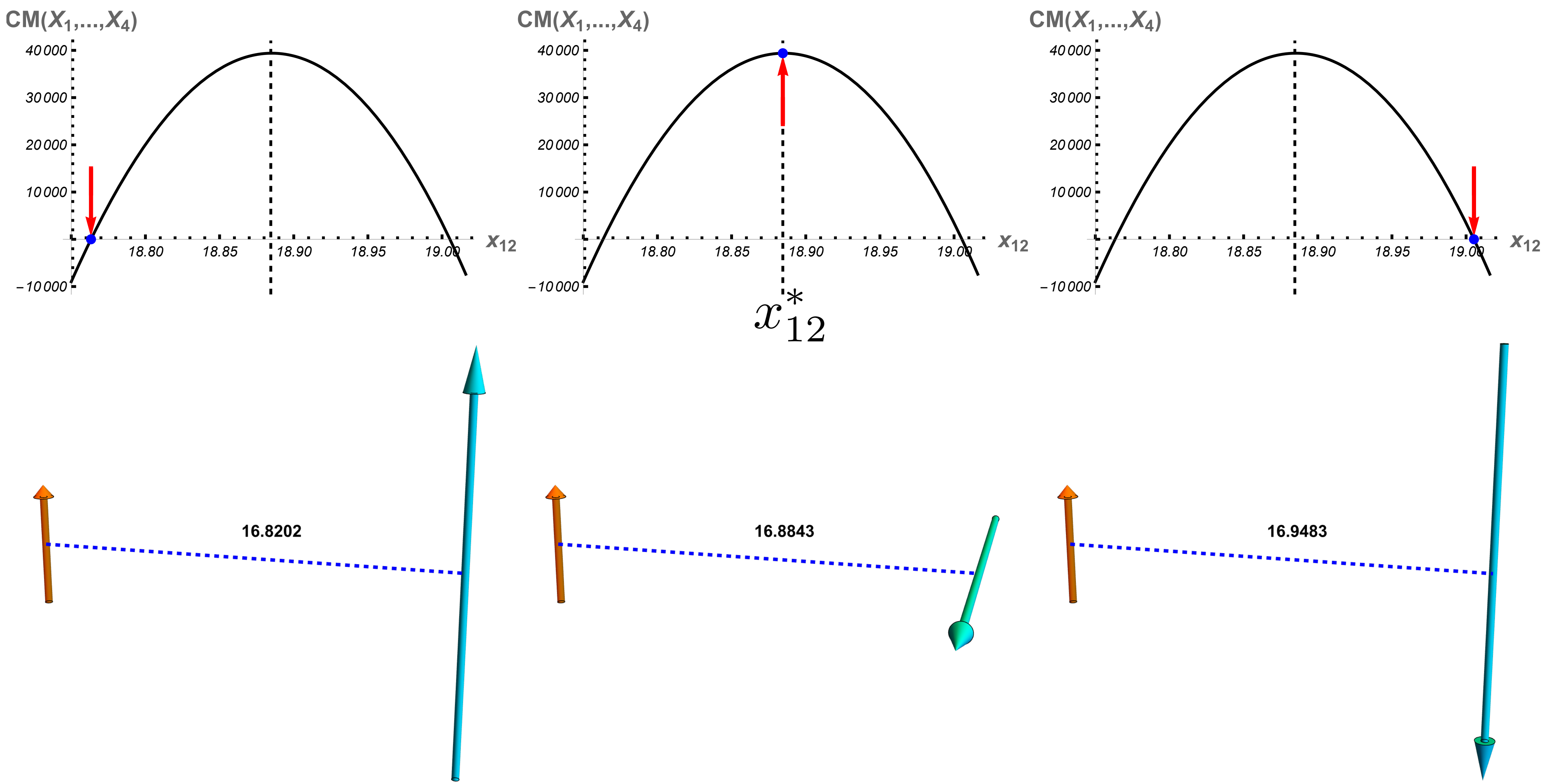



[Extremal 2-BH bound state]

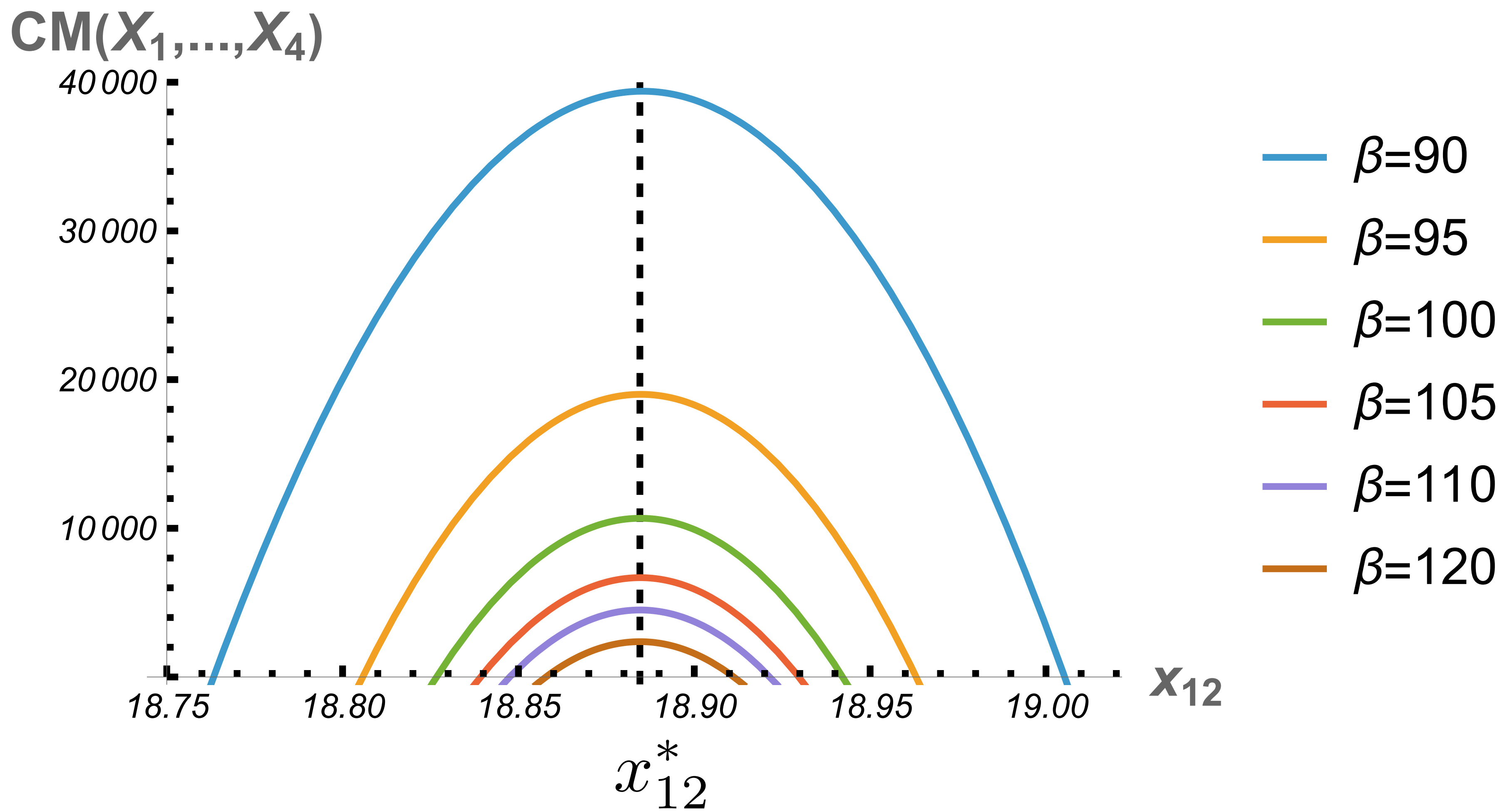
$\beta = 90$

# The Cayley-Menger condition cuts out a small region close to the extremal distance




$\beta = 90$

# Decreasing temperature narrows the 2-BH moduli space to the extremal distance

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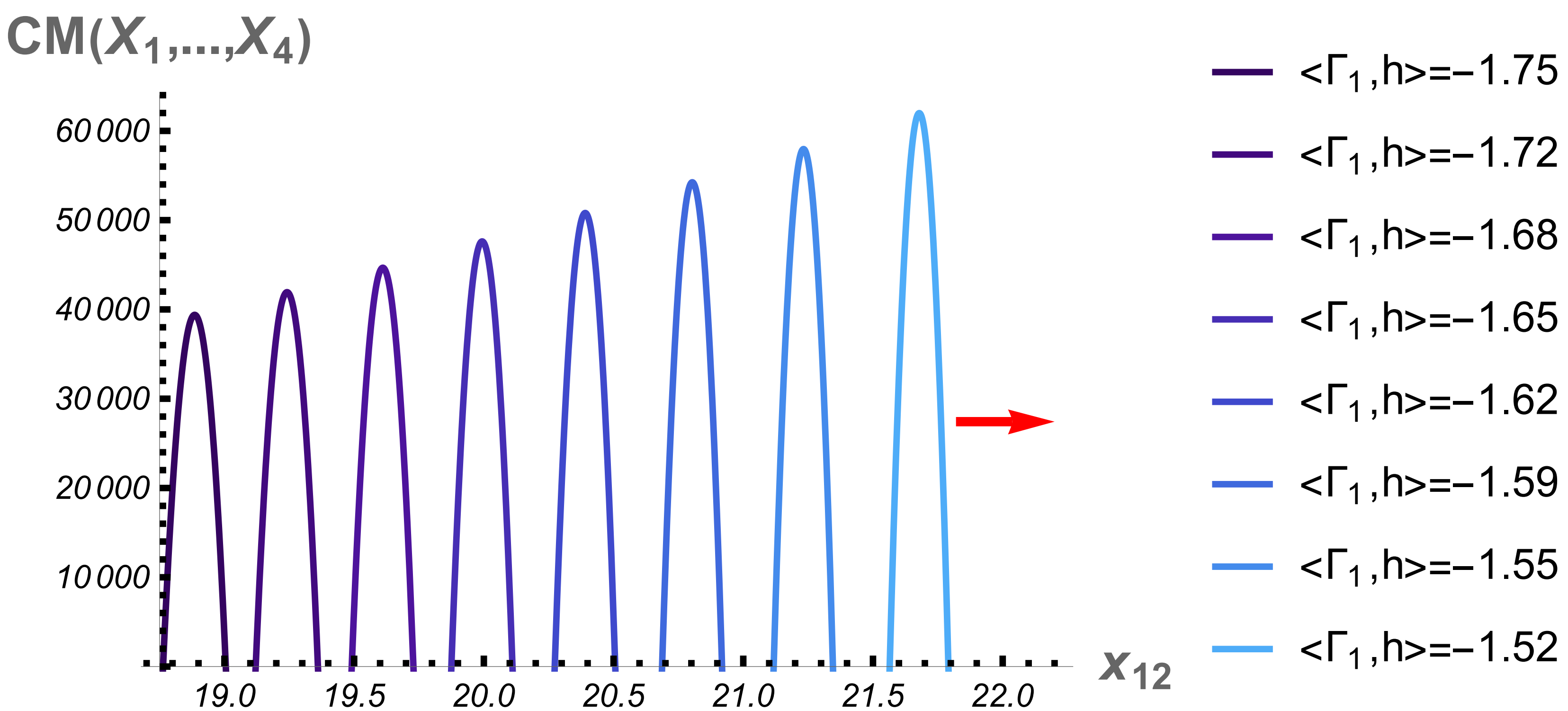
# The index moduli space varies with scalar moduli “carrying” the extremal distance



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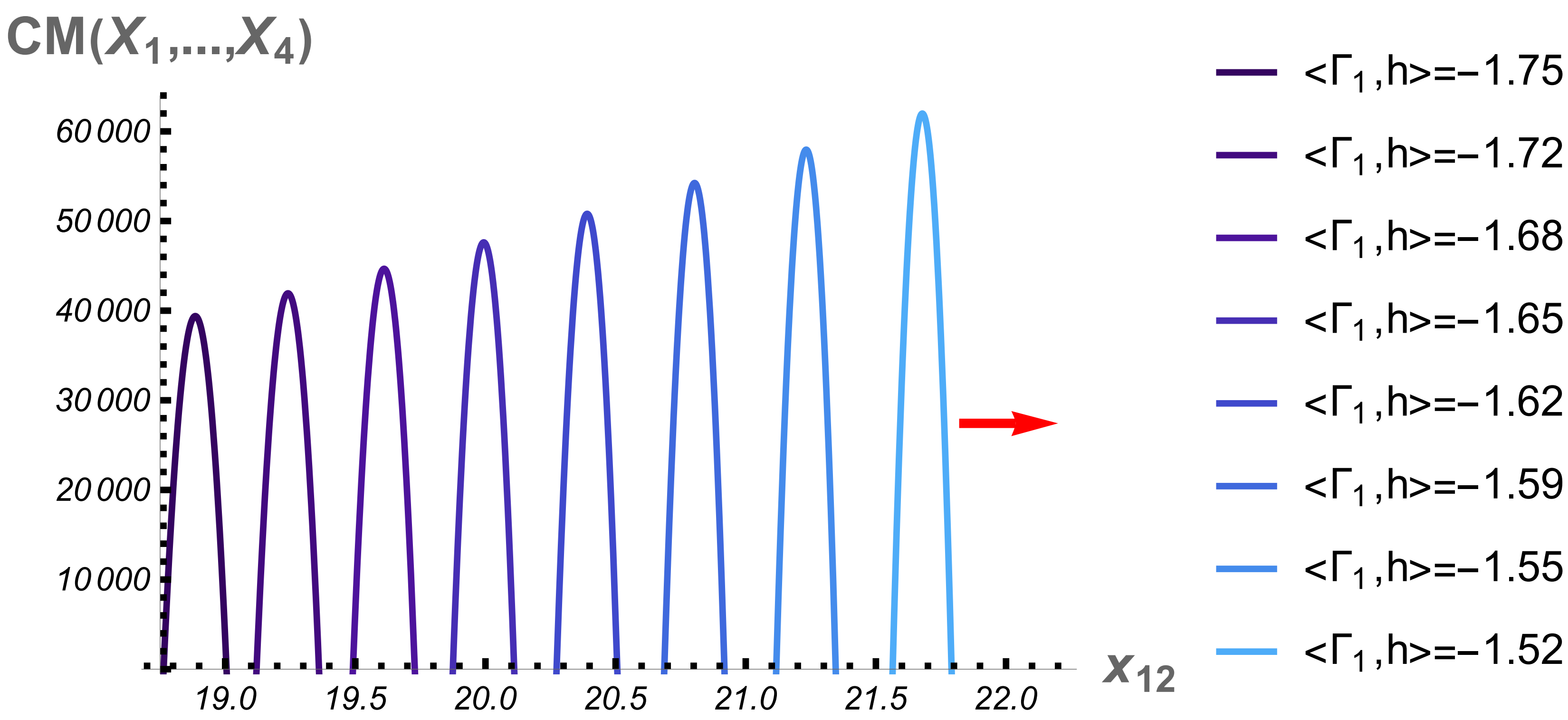


- As one approaches the wall of marginal stability in asymptotic moduli space, index moduli space $\to \infty$
- Whole moduli space disappears exactly upon crossing the wall!

# Index saddles for 5d black strings

[J.Boruch, R. Emparan, L.V.Iliesiu, S.M. ’25]

# Solns of Type IIA string theory can be lifted to solns of M-theory on circle

$$ds^2_{5d} = (2V(x))^{2/3}(d\psi + A^0)^2 + (2V(x))^{-1/3} ds^2_{4d}$$

V= Volume(CY3) in string units

[Gaiotto, Strominger, Yin '05] [Castro, Davis, Kraus, Larsen '07]

[Behrndt, Cardoso, Mahapatra' 05; de Wit, Katamadas, '09]

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$$ds_{5d}^2 = (2V(x))^{2/3}(d\psi + A^0)^2 + (2V(x))^{-1/3} ds_{4d}^2$$

$$\psi \sim \psi + 4\pi$$

M-theory circle

$$\frac{R_M}{\ell_5} = 4\pi(2V_\infty)^{1/3}$$

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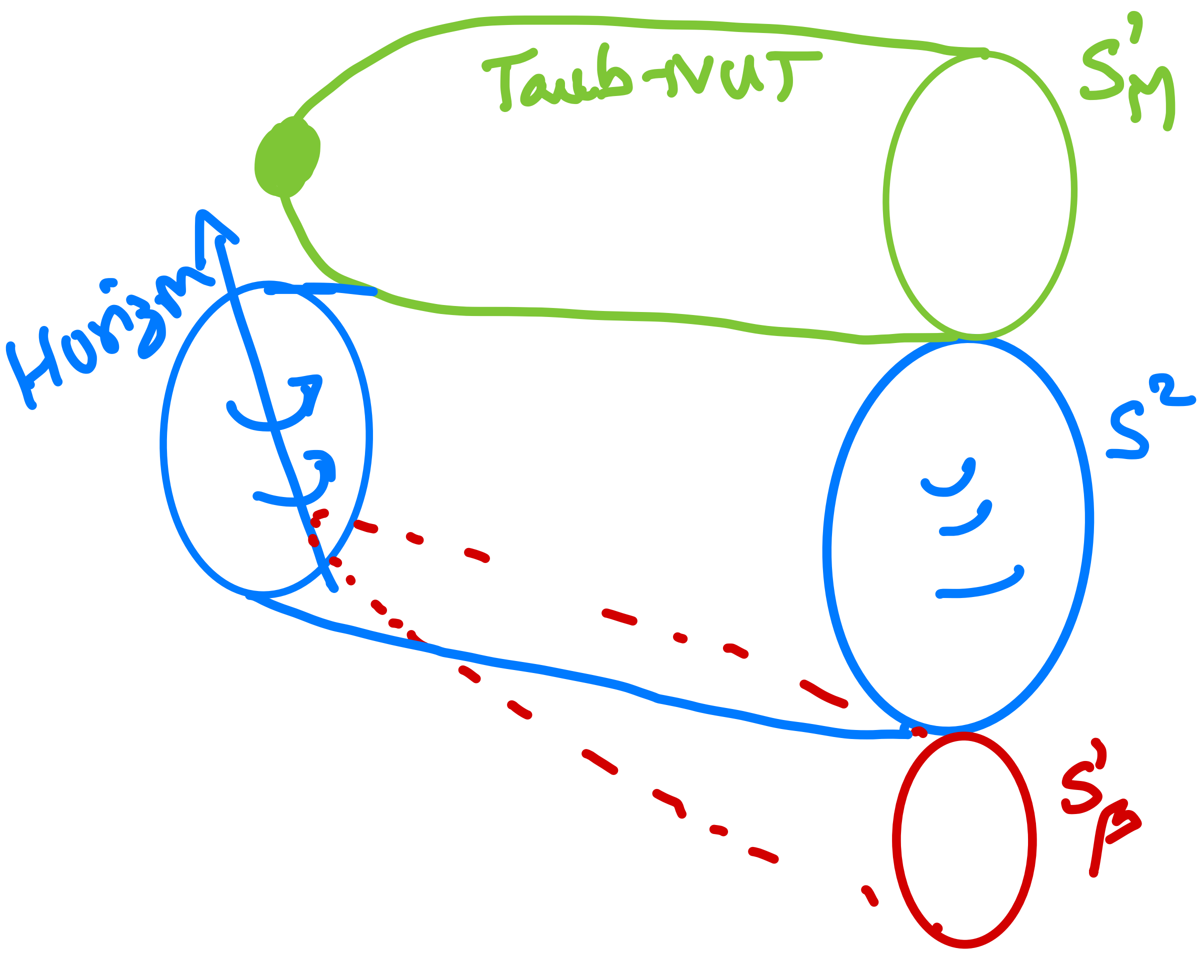


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5d Black hole index

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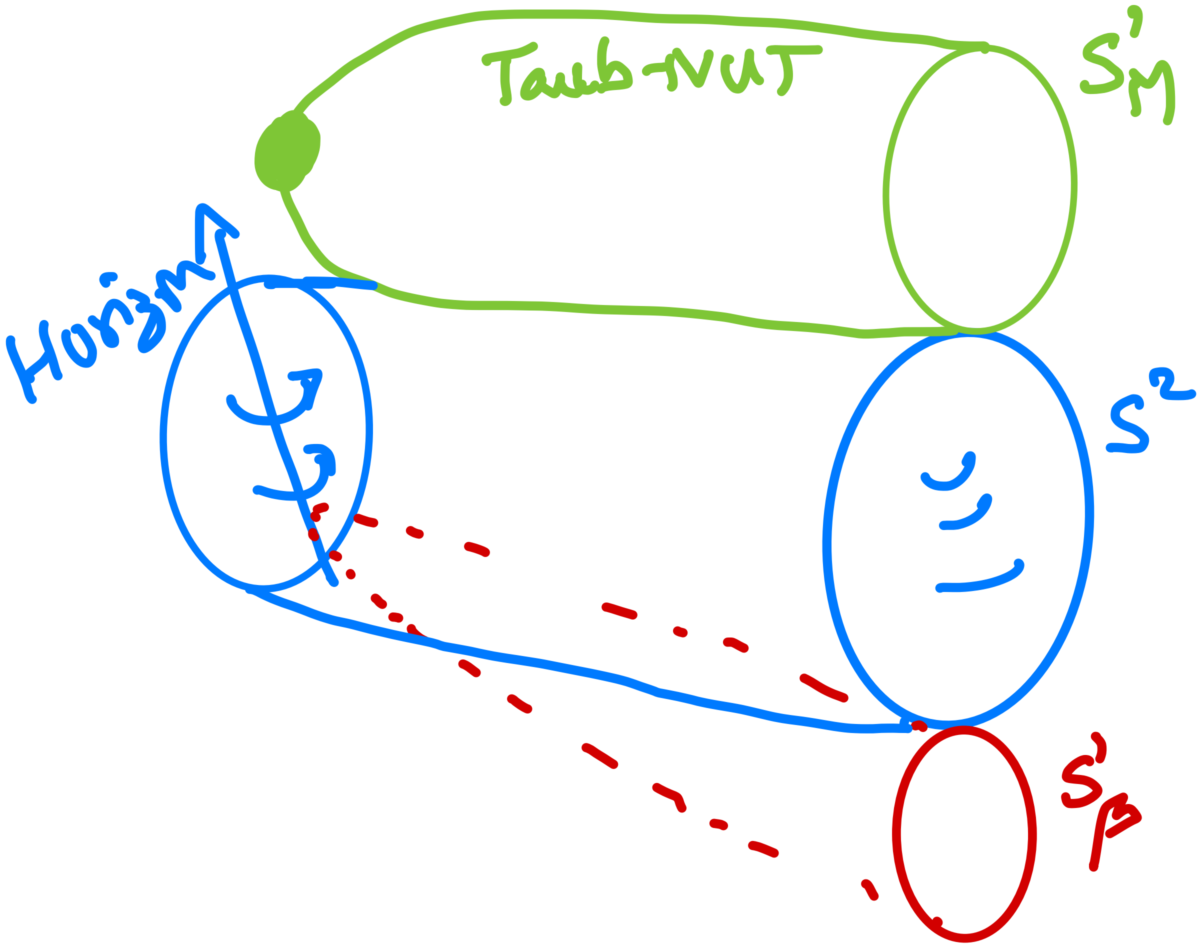
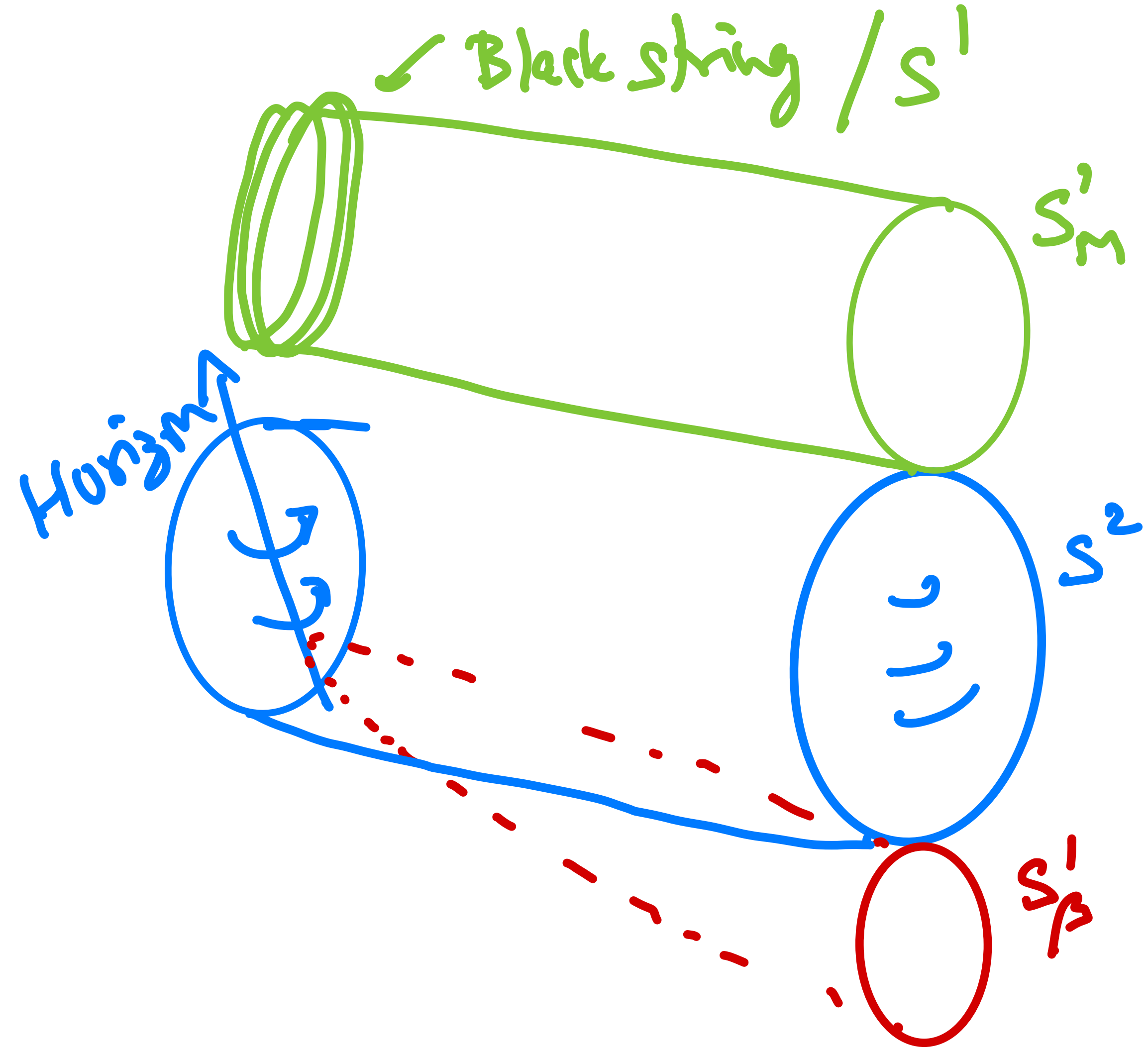



Magnetic charge

$$P^0 = 1$$

$$P^0 = 0$$

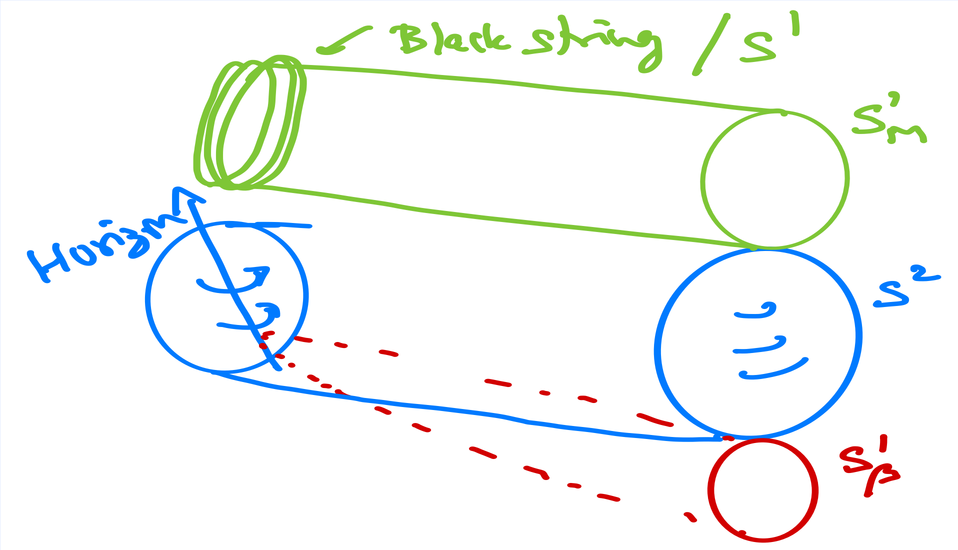
5d Black hole index

5d Black string index

# The $T \to 0$ limit leads to the extremal black string (preserving susy throughout)

- Finite temperature saddles for index of wrapped black string in $\mathbb{R}^3 \times S^1_M \times S^1_{\beta_{5d}}$

$$\frac{\beta_{5d}}{\ell_5} = \frac{\beta_{4d}}{\ell_4 \left(2V_\infty\right)^{1/6}}$$



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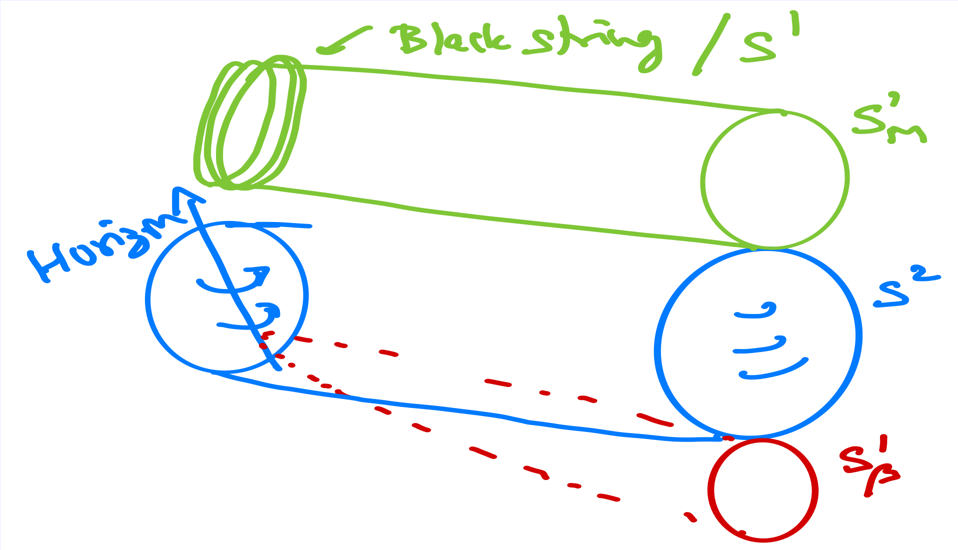


$$\frac{\beta_{5d}}{\ell_5} = \frac{\beta_{4d}}{\ell_4\,(2V_\infty)^{1/6}}$$

- $\beta_{5d}/\ell_5 \to \infty \qquad R_M/\ell_5 = \text{fixed}$

$\Longrightarrow$ extremal (wrapped) Black String in 5d AF space

Near-horizon $\mathrm{AdS}_3 \times S^2$ can be decoupled as usual

# We can also decouple the supersymmetric black string at finite temperature

- Take $P^0 = 0 \qquad \beta_{5d}/\ell_5 \to \infty \qquad \beta_{5d}/R_M = \mathrm{fixed}$
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- Decouple $\Rightarrow$ Gravitational saddle dual to $\mathrm{SCFT}_2$ elliptic genus [Maldacena-Strominger-Witten'98]
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$\Rightarrow$ independent of $\overline{\tau}$

$\Rightarrow$ equal to Cardy formula!

# 3. Can we extract a signal of Schwarzian modes from the string worldsheet?

Yes, although slightly indirectly:

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Superstring generalizations to $\mathrm{AdS}_3 \times S^3 \times \begin{matrix} T^4 \\ S^3 \times S^1 \end{matrix}$

[S.M., Rangamani '24, '25]

$T \to 0$

- Rich gravitational phenomena: multi-BH bound states

  - independent of

[Bates, Denef, '00; Denef, Moore '07]

What are saddle points? Susy BH?