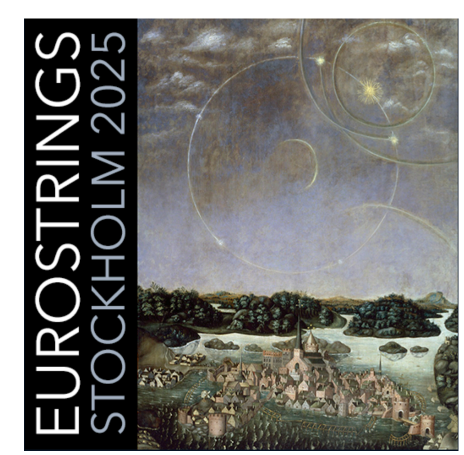
Multi-particle states in CFTs

Agnese Bissi, Uppsala University & ICTP



Based on 2111.06857 and 2412.19788 with
Giulia Fardelli and Andrea Manenti

Set up

In this talk I will discuss correlators in $\mathcal{N} = 4$ Super Yang Mills, with $SU(N)$ gauge group in the regime of large 't Hooft coupling $\lambda = g_{YM}^2 N$ and $N \gg 1$.

Set up

In this talk I will discuss correlators in $\mathcal{N} = 4$ Super Yang Mills, with $SU(N)$ gauge group in the regime of large 't Hooft coupling $\lambda = g_{YM}^2 N$ and $N \gg 1$.

holographic regime

Set up

In this talk I will discuss correlators in $\mathcal{N}=4$ Super Yang Mills, with $SU(N)$ gauge group in the regime of large 't Hooft coupling $\lambda = g_{YM}^2 N$ and $N \gg 1$.

holographic regime

Correlators $\longrightarrow$ Spectrum of the theory

AdS/CFT correspondence

CFT

AdS

AdS/CFT correspondence

CFT

AdS

4 dimensional $\mathcal{N} = 4$
Super Yang Mills with
SU(N) gauge group and
SU(4) R-symmetry

AdS/CFT correspondence

CFT

4 dimensional $\mathcal{N} = 4$ Super Yang Mills with SU(N) gauge group and SU(4) R-symmetry

AdS

type IIB superstring theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$

AdS/CFT correspondence

CFT

4 dimensional $\mathcal{N} = 4$
Super Yang Mills with
SU(N) gauge group and
SU(4) R-symmetry

- rank of the gauge group N
- coupling constant g_{YM}

AdS

type IIB superstring
theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$

AdS/CFT correspondence

CFT

4 dimensional $\mathcal{N} = 4$
Super Yang Mills with
SU(N) gauge group and
SU(4) R-symmetry

- rank of the gauge group N
- coupling constant g_{YM}

AdS

type IIB superstring
theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$

- string length $\sqrt{\alpha'}$
- string coupling g_s

AdS/CFT correspondence

CFT	AdS
4 dimensional $\mathcal{N}=4$ Super Yang Mills with SU(N) gauge group and SU(4) R-symmetry	type IIB superstring theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$
• rank of the gauge group N	• string length $\sqrt{\alpha'}$
• coupling constant g_{YM}	• string coupling g_s

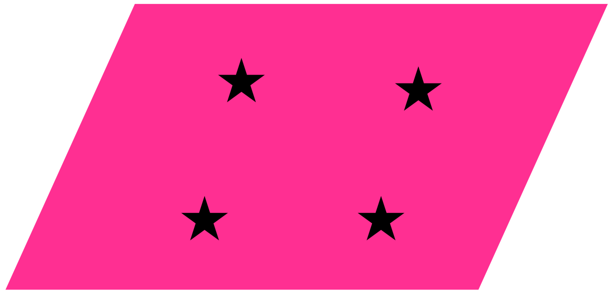
$$\boxed{\begin{gathered} N \sim g_s^{-1} \\ \lambda = g_{YM}^2 N = (\alpha')^{-2} \end{gathered}}$$

Set up

Set up

Weakly coupled regime in the bulk is supergravity and corresponds to large central charge and string length to zero.

Set up

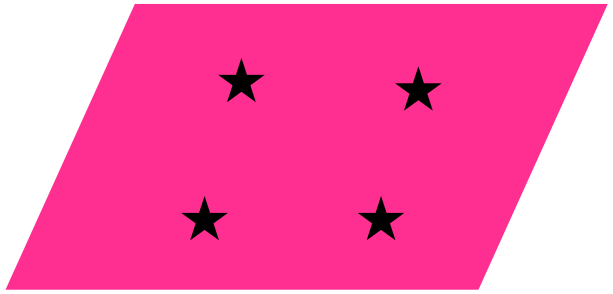


$$\mathbb{R}^{d-1,1} = \partial \mathrm{AdS}_{d+1}$$

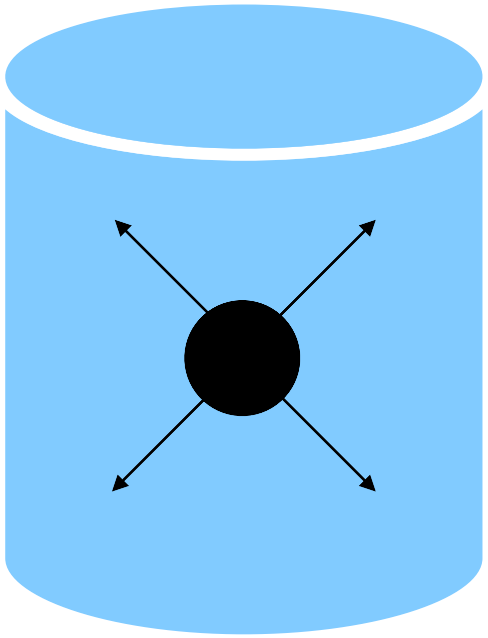
Weakly coupled regime in the bulk is supergravity and corresponds to large central charge and string length to zero.

Four point correlators in a large N expansion

Set up



$\mathbb{R}^{d-1,1} = \partial \mathrm{AdS}_{d+1}$



$\mathrm{AdS}_{d+1} \times \mathrm{S}^q$

Weakly coupled regime in the bulk is supergravity and corresponds to large central charge and string length to zero.

Four point correlators in a large N expansion

Loop amplitudes in AdS

Conformal block decomposition

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{\mathcal{G}(u,v)}{x_{12}^{2\Delta_{\mathcal{O}}}x_{34}^{2\Delta_{\mathcal{O}}}}$$

Conformal block decomposition

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{\mathcal{G}(u,v)}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}}$$

Cross-ratios $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$

Conformal block decomposition

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{\boxed{\mathcal{G}(u,v)}}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}}$$

Cross-ratios

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Conformal block decomposition

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{\boxed{\mathcal{G}(u,v)}}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}}$$

Cross-ratios

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$\mathcal{G}(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}} g_{\Delta,\ell}(u,v)$$

Conformal block decomposition

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{\boxed{\mathcal{G}(u,v)}}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}}$$

Cross-ratios

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$\mathcal{G}(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}} \underline{g_{\Delta,\ell}(u,v)}$$

Conformal blocks

Conformal block decomposition

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{\boxed{\mathcal{G}(u,v)}}{x_{12}^{2\Delta_{\mathcal{O}}}x_{34}^{2\Delta_{\mathcal{O}}}}$$

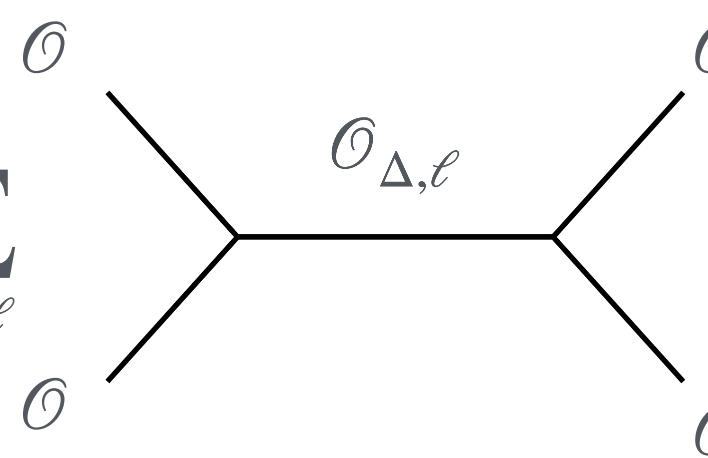
Cross-ratios

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$\mathcal{G}(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}} \underline{g_{\Delta,\ell}(u,v)}$$

Conformal blocks

$$\sum_{\Delta,\ell}$$



Large N expansion

Large N expansion

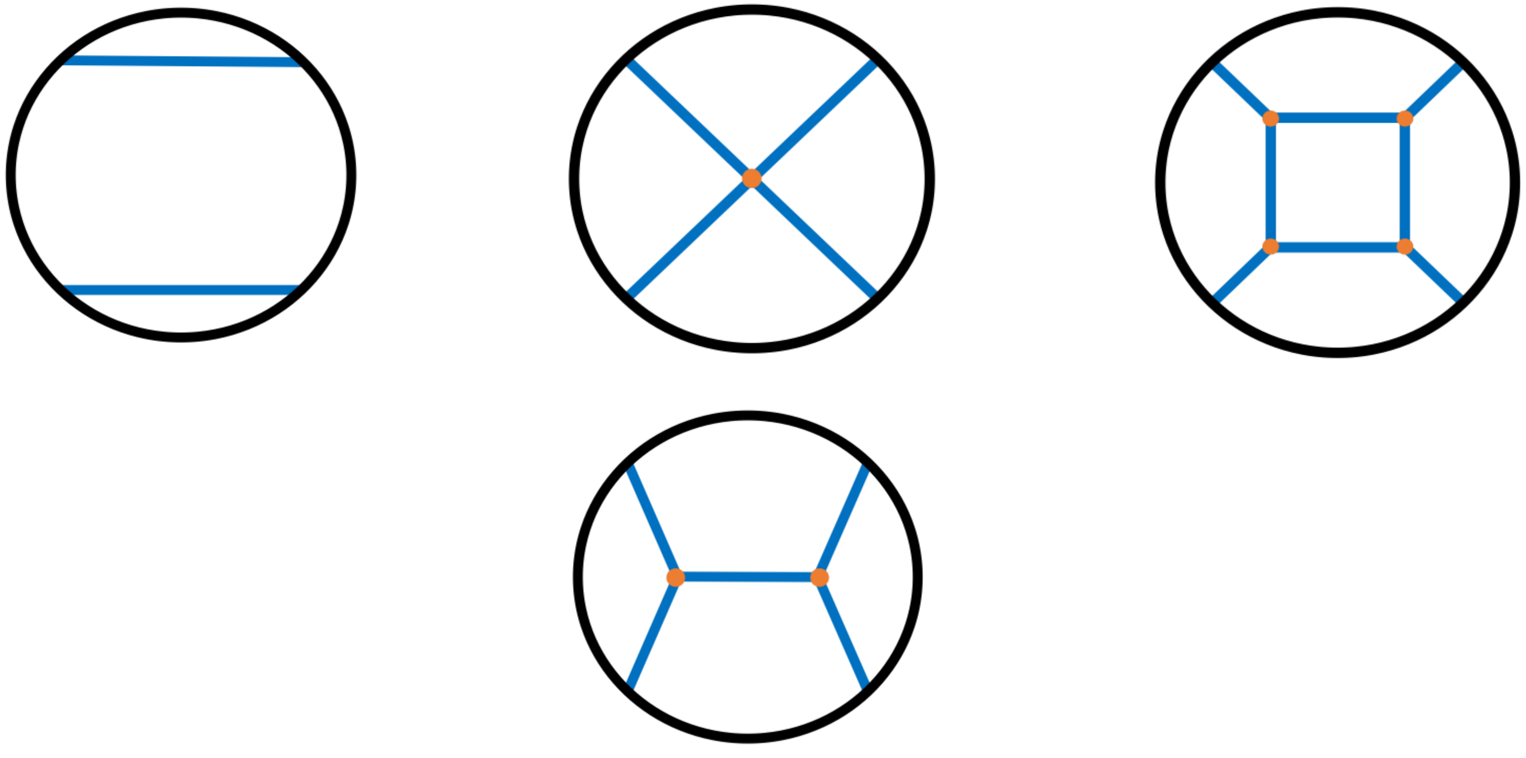
Large N expansion:

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \ldots$$

Large N expansion

Large N expansion:

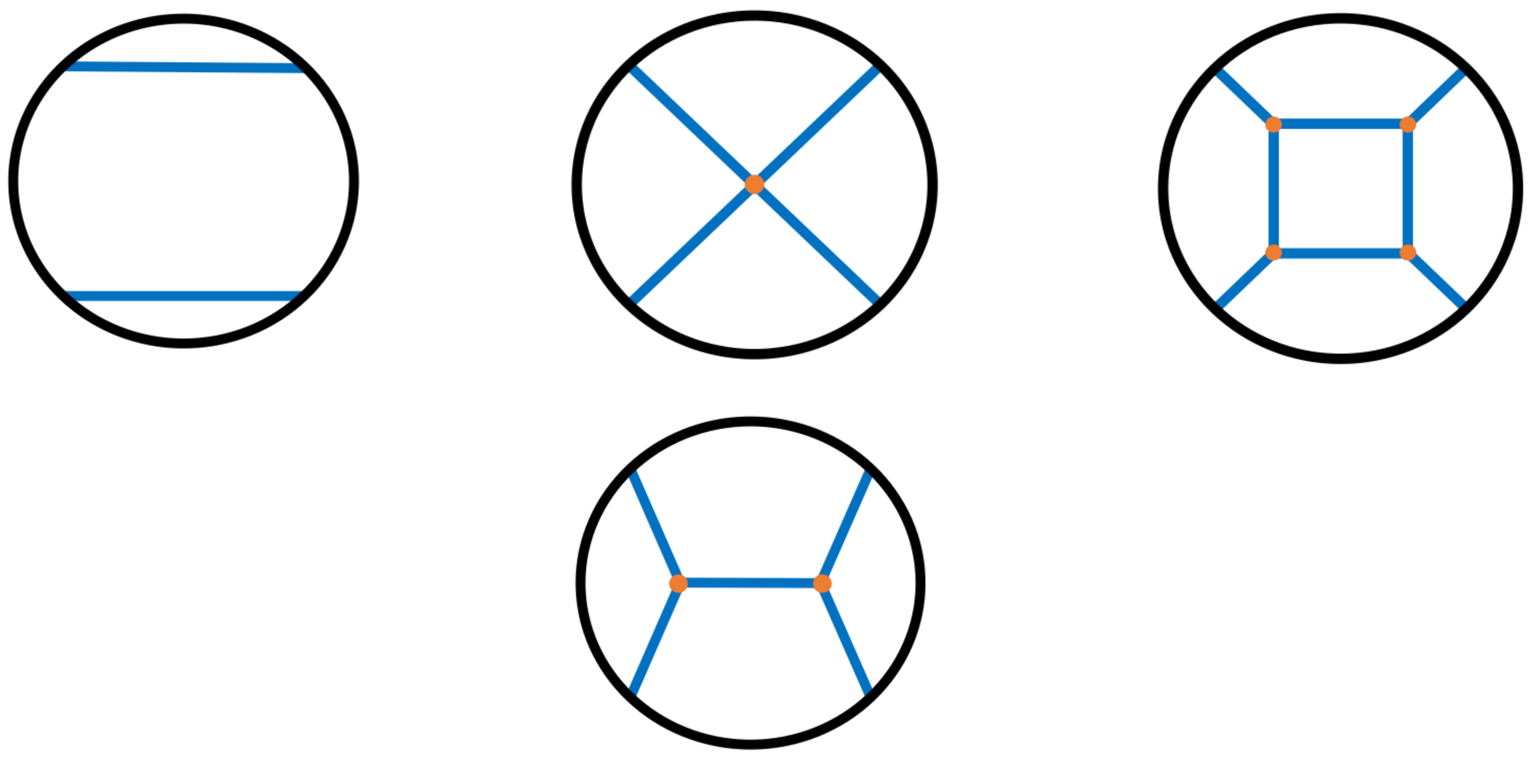
$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \ldots$$



Large N expansion

Large N expansion:

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \ldots$$



Loop expansion!

Spectrum of $\mathcal{N} = 4$ SYM

Spectrum of $\mathcal{N} = 4$ SYM

- The single-trace sector of the spectrum is well understood, for any value of λ (integrability methods)
- The multi-trace sector of the spectrum is much less explored and little is known about the anomalous dimensions of such operators.
- How can we access this piece of information?

Spectrum of $\mathcal{N} = 4$ SYM

- The single-trace sector of the spectrum is well understood, for any value of λ (integrability methods)
- The multi-trace sector of the spectrum is much less explored and little is known about the anomalous dimensions of such operators.
- How can we access this piece of information?

1. Large N expansion of single trace four point functions

Spectrum of $\mathcal{N} = 4$ SYM

- The single-trace sector of the spectrum is well understood, for any value of λ (integrability methods)
- The multi-trace sector of the spectrum is much less explored and little is known about the anomalous dimensions of such operators.
- How can we access this piece of information?

1. Large N expansion of single trace four point functions
2. Large N expansion of four point functions of higher trace op

Spectrum of $\mathcal{N} = 4$ SYM

- The single-trace sector of the spectrum is well understood, for any value of λ (integrability methods)
- The multi-trace sector of the spectrum is much less explored and little is known about the anomalous dimensions of such operators.
- How can we access this piece of information?

1. Large N expansion of single trace four point functions
2. Large N expansion of four point functions of higher trace op
3. Large N expansion of higher point functions of single trace op

see for instance Harris, Kaviraj, Mann, Quintavalle, Schomerus, 2024

Scenario 1.

Study four point functions of single trace operators $\mathcal{O}$ at large N

Scenario 1.

Study four point functions of single trace operators $\mathcal{O}$ at large N

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

Scenario 1.

Study four point functions of single trace operators $\mathcal{O}$ at large N

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

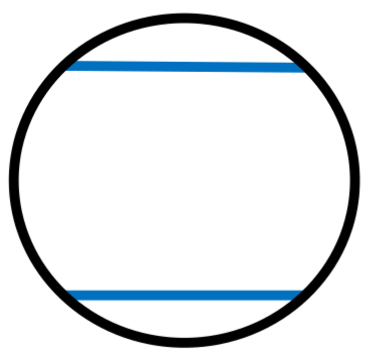
$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \dots$$

Scenario 1.

Study four point functions of single trace operators $\mathcal{O}$ at large N

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \dots$$



DOUBLE
TRACES

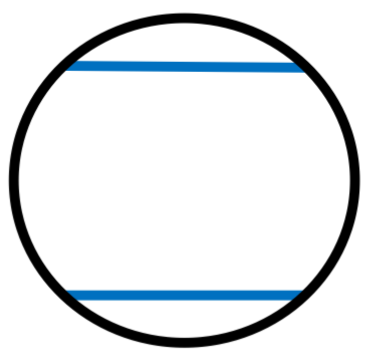
$$\langle \mathcal{O}\mathcal{O}[\mathcal{O}\mathcal{O}]\rangle \sim 1$$

Scenario 1.

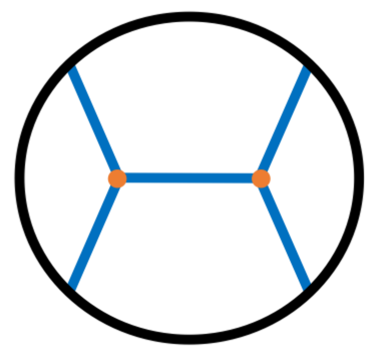
Study four point functions of single trace operators $\mathcal{O}$ at large N

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \dots$$



DOUBLE TRACES



DOUBLE TRACES

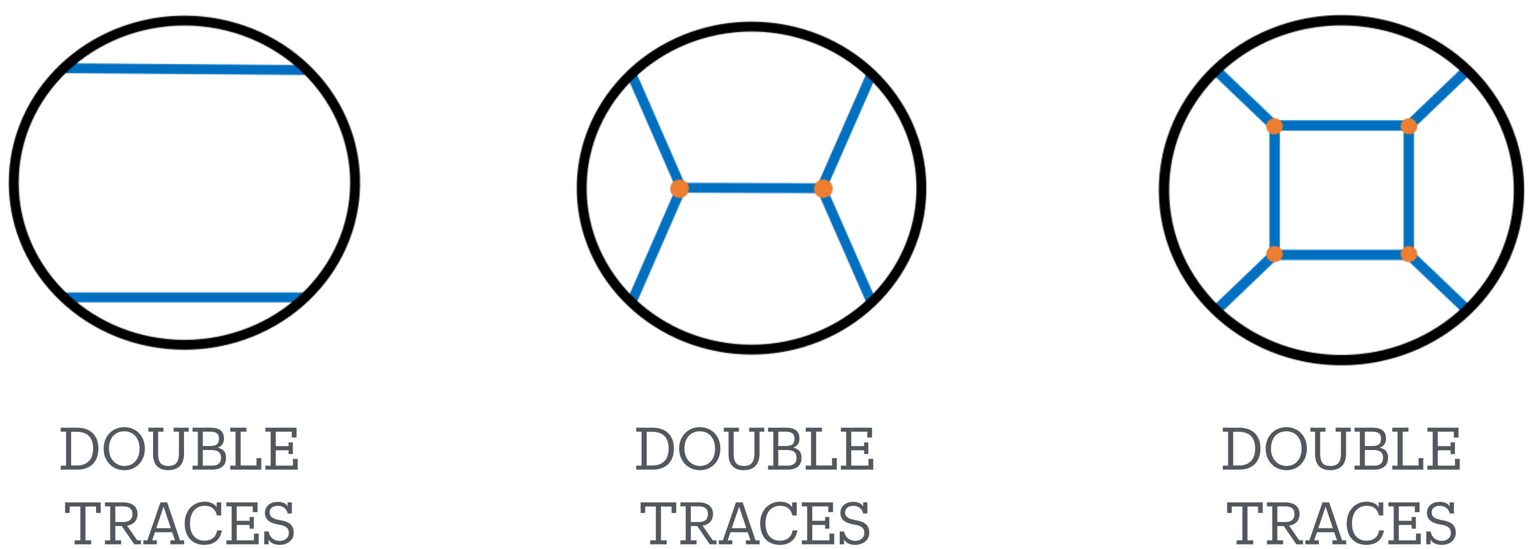
$$\langle \mathcal{O}\mathcal{O}[\mathcal{O}\mathcal{O}]\rangle \sim 1$$

Scenario 1.

Study four point functions of single trace operators $\mathcal{O}$ at large N

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \dots$$



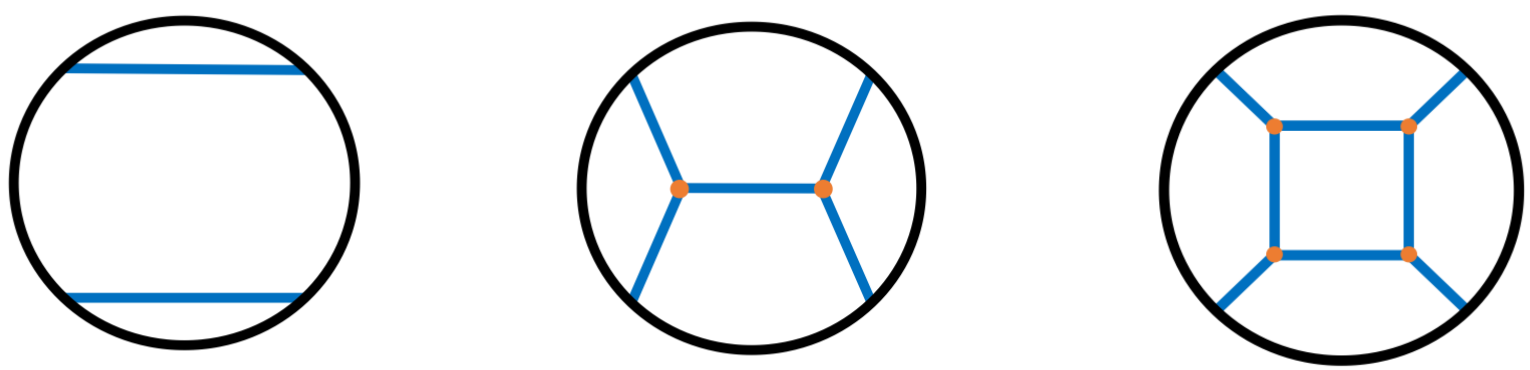
$$\langle \mathcal{O}\mathcal{O}[\mathcal{O}\mathcal{O}]\rangle \sim 1 \qquad \langle \mathcal{O}\mathcal{O}[\mathcal{O}_1\mathcal{O}_2\dots\mathcal{O}_m]\rangle \sim N^{-m}$$

Scenario 1.

Study four point functions of single trace operators $\mathcal{O}$ at large N

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \dots$$



DOUBLE TRACES | DOUBLE TRACES | DOUBLE TRACES

DOUBLE TRACES

TRIPLE TRACES

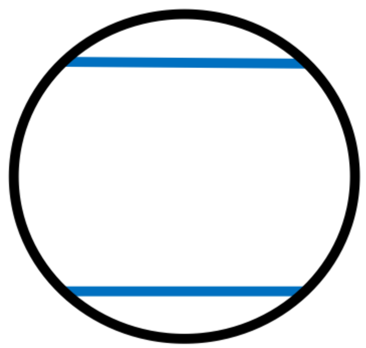
$$\langle \mathcal{O}\mathcal{O}[\mathcal{O}\mathcal{O}]\rangle \sim 1 \qquad \langle \mathcal{O}\mathcal{O}[\mathcal{O}_1\mathcal{O}_2\dots\mathcal{O}_m]\rangle \sim N^{-m}$$

Scenario 1.

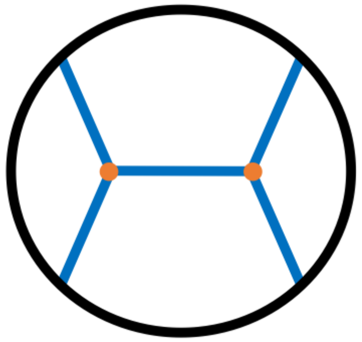
Study four point functions of single trace operators $\mathcal{O}$ at large N

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

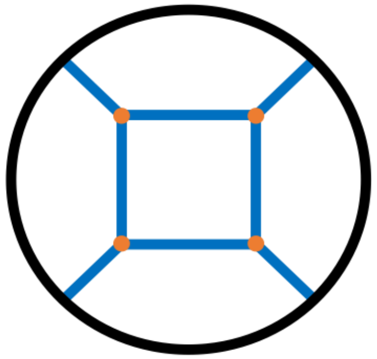
$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \dots$$



DOUBLE TRACES



DOUBLE TRACES



DOUBLE TRACES

DOUBLE TRACES

TRIPLE TRACES

$$\langle \mathcal{O}\mathcal{O}[\mathcal{O}\mathcal{O}]\rangle \sim 1 \qquad \langle \mathcal{O}\mathcal{O}[\mathcal{O}_1\mathcal{O}_2\dots\mathcal{O}_m]\rangle \sim N^{-m}$$

$[\mathcal{O}_1\mathcal{O}_2\dots\mathcal{O}_m]$ are m-trace operators, with $\langle[\mathcal{O}_1\mathcal{O}_2\dots\mathcal{O}_m][\mathcal{O}_1\mathcal{O}_2\dots\mathcal{O}_m]\rangle = 1$

Scenario 2.

Study four point functions of higher trace operators

$$\mathcal{O}_{DT} \sim \left[\mathcal{O}\mathcal{O}\right].$$

Scenario 2.

Study four point functions of higher trace operators

$$\mathcal{O}_{DT} \sim \left[\mathcal{O}\mathcal{O}\right].$$

One example is

$$\langle \mathcal{O}_{DT}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

Scenario 2.

Study four point functions of higher trace operators

$$\mathcal{O}_{DT} \sim \left[\mathcal{O}\mathcal{O}\right].$$

One example is

$$\langle \mathcal{O}_{DT}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

In this case, triple-trace operators appear already at leading order in the large N expansion.

Half-BPS operators in $\mathcal{N} = 4$ SYM

Half-BPS operators in $\mathcal{N} = 4$ SYM

$$\Delta_{\mathcal{O}_p} = p$$

$$\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_q \rangle = f(N)$$

$[0,p,0]$ of $SU(4)_R$

Half-BPS operators in $\mathcal{N} = 4$ SYM

$$\Delta_{\mathcal{O}_p} = p$$

$$m^2 = \Delta_p(\Delta_p - 4)$$

$$\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_q \rangle = f(N)$$

$[0,p,0]$ of $SU(4)_R$

S^5 angular momentum is p

Half-BPS operators in $\mathcal{N} = 4$ SYM

$\Delta_{\mathcal{O}_p} = p$

$m^2 = \Delta_p(\Delta_p - 4)$

$\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_q \rangle = f(N)$

$[0,p,0]$ of $SU(4)_R$

S^5 angular momentum is p

Scalar operators s_p

Half-BPS operators in $\mathcal{N} = 4$ SYM

$$\Delta_{\mathcal{O}_p} = p$$

$$\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_q \rangle = f(N)$$

$[0,p,0]$ of $SU(4)_R$

$$m^2 = \Delta_p(\Delta_p - 4)$$

S^5 angular momentum is p

Scalar operators s_p

p=2	$\mathcal{O}_2 \sim \mathrm{Tr}\,\phi^2$
p=3	$\mathcal{O}_3 \sim \mathrm{Tr}\,\phi^3$
p=4	$\mathcal{O}_4 \sim \mathrm{Tr}\,\phi^4$

Half-BPS operators in $\mathcal{N} = 4$ SYM

$$\Delta_{\mathcal{O}_p} = p$$

$$m^2 = \Delta_p(\Delta_p - 4)$$

$$\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_q \rangle = f(N)$$

$[0,p,0]$ of $SU(4)_R$

S^5 angular momentum is p

Scalar operators s_p

p=2	$\mathcal{O}_2 \sim \mathrm{Tr}\,\phi^2$
p=3	$\mathcal{O}_3 \sim \mathrm{Tr}\,\phi^3$
p=4	$\mathcal{O}_4 \sim \mathrm{Tr}\,\phi^4$

Graviton

Kaluza Klein modes

Half-BPS operators in $\mathcal{N} = 4$ SYM

$$\Delta_{\mathcal{O}_p} = p$$

$$m^2 = \Delta_p(\Delta_p - 4)$$

$$\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_q \rangle = f(N)$$

$[0,p,0]$ of $SU(4)_R$

S^5 angular momentum is p

Scalar operators s_p

p=2	$\mathcal{O}_2 \sim \mathrm{Tr}\,\phi^2$
p=3	$\mathcal{O}_3 \sim \mathrm{Tr}\,\phi^3$
p=4	$\mathcal{O}_4 \sim \mathrm{Tr}\,\phi^4$

Graviton

Kaluza Klein modes

$\mathcal{O}_4 \sim \mathrm{Tr}\,\phi^2 \mathrm{Tr}\,\phi^2$

Bound 2-particle state

Quarter-BPS operators in $\mathcal{N} = 4$ SYM

Quarter-BPS operators in $\mathcal{N} = 4$ SYM

Quarter-BPS multi trace operators

$$\boxed{\begin{gathered}[q, p, q] \text{ of } SU(4)_R \\ \Delta = 2q + p\end{gathered}}$$

Quarter-BPS operators in $\mathcal{N} = 4$ SYM

Quarter-BPS multi trace operators

$[q, p, q]$ of $SU(4)_R$

$\Delta = 2q + p$

Bound states

Quarter-BPS operators in $\mathcal{N} = 4$ SYM

Quarter-BPS multi trace operators

$[q, p, q]$ of $SU(4)_R$

$$\Delta = 2q + p$$

Bound states

An example:

Quarter-BPS operators in $\mathcal{N} = 4$ SYM

Quarter-BPS multi trace operators

$[q, p, q]$ of $SU(4)_R$

$\Delta = 2q + p$

Bound states

An example:

$$\mathcal{O}_{02} \sim \mathrm{Tr}\phi^i\phi^j\mathrm{Tr}\phi^k\phi^l P_{ijkl}$$

Quarter-BPS operators in $\mathcal{N} = 4$ SYM

Quarter-BPS multi trace operators

$[q, p, q]$ of $SU(4)_R$

$\Delta = 2q + p$

Bound states

An example:

$$\mathcal{O}_{02} \sim \mathrm{Tr}\phi^i\phi^j\mathrm{Tr}\phi^k\phi^l P_{ijkl}$$

$$\Delta = 4 \qquad [2,0,2]_R$$

This talk

This talk

Consider four point correlators made of single and double trace operators, half or quarter BPS.

This talk

Consider four point correlators made of single and double trace operators, half or quarter BPS.

By disentangling the protected part of the correlators, infer information about the anomalous dimension of double and higher trace operators.

Four point functions of $\frac{1}{2}$ BPS operators

$$\langle \mathcal{O}_{\Delta_1}\mathcal{O}_{\Delta_2}\mathcal{O}_{\Delta_3}\mathcal{O}_{\Delta_4}\rangle = K(\Delta_i, x_i, y_i)\ \ \mathcal{G}_{\Delta_i}(u, v, \sigma, \tau)$$

Four point functions of $\frac{1}{2}$ BPS operators

$$\langle \mathcal{O}_{\Delta_1}\mathcal{O}_{\Delta_2}\mathcal{O}_{\Delta_3}\mathcal{O}_{\Delta_4}\rangle = \boxed{K(\Delta_i, x_i, y_i)}\ \mathcal{G}_{\Delta_i}(u, v, \sigma, \tau)$$

Kinematical term

Four point functions of $\frac{1}{2}$ BPS operators

$$\langle \mathcal{O}_{\Delta_1}\mathcal{O}_{\Delta_2}\mathcal{O}_{\Delta_3}\mathcal{O}_{\Delta_4}\rangle = \boxed{K(\Delta_i, x_i, y_i)}\ \mathcal{G}_{\Delta_i}(u, v, \sigma, \tau)$$

Kinematical term

x_i Space time coordinates

y_i R-symmetry coordinates

Four point functions of $\frac{1}{2}$ BPS operators

$$\langle \mathcal{O}_{\Delta_1}\mathcal{O}_{\Delta_2}\mathcal{O}_{\Delta_3}\mathcal{O}_{\Delta_4}\rangle = \boxed{K(\Delta_i, x_i, y_i)}\ \mathcal{G}_{\Delta_i}(u, v, \underline{\sigma, \tau})$$

Kinematical term harmonic cross ratios

x_i Space time coordinates

y_i R-symmetry coordinates

Four point functions of $\frac{1}{2}$ BPS operators

$$\langle \mathcal{O}_{\Delta_1}\mathcal{O}_{\Delta_2}\mathcal{O}_{\Delta_3}\mathcal{O}_{\Delta_4}\rangle = \boxed{K(\Delta_i, x_i, y_i)}\ \mathcal{G}_{\Delta_i}(u, v, \underline{\sigma, \tau})$$

Kinematical term harmonic cross ratios

x_i Space time coordinates

y_i R-symmetry coordinates

$\mathcal{G}_{\Delta_i}(u, v, \sigma, \tau)$ contains information of primaries transforming under

$$[0,\Delta_1,0] \times [0,\Delta_2,0] \cap [0,\Delta_3,0] \times [0,\Delta_4,0]$$

Four point functions of $\frac{1}{2}$ BPS operators

Superconformal Ward Identities:

Four point functions of $\frac{1}{2}$ BPS operators

Superconformal Ward Identities:

disentangle the contribution of protected operators in the

$[0,\Delta_1,0] \times [0,\Delta_2,0] \cap [0,\Delta_3,0] \times [0,\Delta_4,0]$

Four point functions of $\frac{1}{2}$ BPS operators

Superconformal Ward Identities: disentangle the contribution of protected operators in the

$$[0,\Delta_1,0] \times [0,\Delta_2,0] \cap [0,\Delta_3,0] \times [0,\Delta_4,0]$$

E.g. $$[0,2,0] \times [0,2,0] = [0,0,0] + [0,2,0] + [0,4,0] + [1,2,1] + [2,0,2] + [1,0,1]$$

Four point functions of $\frac{1}{2}$ BPS operators

Superconformal Ward Identities:

disentangle the contribution of protected operators in the

$$[0,\Delta_1,0] \times [0,\Delta_2,0] \cap [0,\Delta_3,0] \times [0,\Delta_4,0]$$

E.g. $$[0,2,0] \times [0,2,0] = [0,0,0] + [0,2,0] + [0,4,0] + [1,2,1] + [2,0,2] + [1,0,1]$$

$\frac{1}{2}$ BPS operators

Four point functions of $\frac{1}{2}$ BPS operators

Superconformal Ward Identities:

disentangle the contribution of protected operators in the

$[0,\Delta_1,0] \times [0,\Delta_2,0] \cap [0,\Delta_3,0] \times [0,\Delta_4,0]$

E.g. $[0,2,0] \times [0,2,0] = [0,0,0] + [0,2,0] + [0,4,0] + [1,2,1] + [2,0,2] + [1,0,1]$

$\frac{1}{2}$ BPS operators

$$\mathcal{G}_{\Delta_i}(u,v,\sigma,\tau) = \kappa + \mathcal{G}^f(u,v,\sigma,\tau) + R(u,v,\sigma,\tau)\mathcal{H}(u,v,\sigma,\tau)$$

Four point functions of $\frac{1}{2}$ BPS operators

Superconformal Ward Identities:

disentangle the contribution of protected operators in the

$[0,\Delta_1,0] \times [0,\Delta_2,0] \cap [0,\Delta_3,0] \times [0,\Delta_4,0]$

E.g. $[0,2,0] \times [0,2,0] = [0,0,0] + [0,2,0] + [0,4,0] + [1,2,1] + [2,0,2] + [1,0,1]$

$\frac{1}{2}$ BPS operators

$$\mathcal{G}_{\Delta_i}(u,v,\sigma,\tau) = \kappa + \boxed{\mathcal{G}^f(u,v,\sigma,\tau)} + R(u,v,\sigma,\tau)\mathcal{H}(u,v,\sigma,\tau)$$

protected (do not depend on λ)

Four point functions of $\frac{1}{2}$ BPS operators

Superconformal Ward Identities:

disentangle the contribution of protected operators in the

$$[0,\Delta_1,0] \times [0,\Delta_2,0] \cap [0,\Delta_3,0] \times [0,\Delta_4,0]$$

E.g. $$[0,2,0] \times [0,2,0] = [0,0,0] + [0,2,0] + [0,4,0] + [1,2,1] + [2,0,2] + [1,0,1]$$

$\frac{1}{2}$ BPS operators

$$\mathcal{G}_{\Delta_i}(u,v,\sigma,\tau) = \kappa + \boxed{\mathcal{G}^f(u,v,\sigma,\tau)} + R(u,v,\sigma,\tau)\boxed{\mathcal{H}(u,v,\sigma,\tau)}$$

protected (do not depend on λ) long multiplet

Four point functions of $\dfrac{1}{2}$ BPS operators

$$\mathcal{H}(u,v,\sigma,\tau) = \sum_{\Delta,\ell,R} a^R_{\Delta,\ell} Y^R(\sigma,\tau) g_{\Delta+4,\ell}(u,v)$$

Four point functions of $\frac{1}{2}$ BPS operators

$$\mathcal{H}(u,v,\sigma,\tau) = \sum_{\Delta,\ell,R} a^R_{\Delta,\ell} Y^R(\sigma,\tau) \underline{g_{\Delta+4,\ell}(u,v)}$$

superconformal blocks

Four point functions of $\frac{1}{2}$ BPS operators

$$\mathcal{H}(u, v, \sigma, \tau) = \sum_{\Delta,\ell,R} a^R_{\Delta,\ell} Y^R(\sigma,\tau) \underline{g_{\Delta+4,\ell}(u,v)}$$

superconformal blocks

R=

$[0,\Delta_1 - 2,0] \times [0,\Delta_2 - 2,0] \cap [0,\Delta_3 - 2,0] \times [0,\Delta_4 - 2,0]$

Four point functions of $\frac{1}{2}$ BPS operators

$$\mathcal{H}(u,v,\sigma,\tau) = \sum_{\Delta,\ell,R} a^R_{\Delta,\ell} Y^R(\sigma,\tau) \underline{g_{\Delta+4,\ell}(u,v)}$$

superconformal blocks

R=

$$[0,\Delta_1 - 2,0] \times [0,\Delta_2 - 2,0] \cap [0,\Delta_3 - 2,0] \times [0,\Delta_4 - 2,0]$$

Strategy:

Four point functions of $\frac{1}{2}$ BPS operators

$$\mathcal{H}(u, v, \sigma, \tau) = \sum_{\Delta,\ell,R} a^R_{\Delta,\ell} Y^R(\sigma,\tau) \underline{g_{\Delta+4,\ell}(u,v)}$$

superconformal blocks

$R=$
$$[0,\Delta_1 - 2,0] \times [0,\Delta_2 - 2,0] \cap [0,\Delta_3 - 2,0] \times [0,\Delta_4 - 2,0]$$

Strategy:
- compute the protected part of the correlator
- use crossing symmetry to see which consequences this part has on the $\mathcal{H}(u, v, \sigma, \tau)$

Inversion Formula

It is possible to write a relation that invert the OPE allowing us to reconstruct the correlator by knowing only its singularities as $v \to 0$ or $\bar{z} \to 1$

Caron Huot 2017

Simmons-Duffin Stanford Witten 2017

Inversion Formula

It is possible to write a relation that invert the OPE allowing us to reconstruct the correlator by knowing only its singularities as $v \to 0$ or $\bar{z} \to 1$

How?

Caron Huot 2017

Simmons-Duffin Stanford Witten 2017

Inversion Formula

It is possible to write a relation that invert the OPE allowing us to reconstruct the correlator by knowing only its singularities as $v \to 0$ or $\bar{z} \to 1$

How?

$$c_{\Delta,\ell} \sim \int_0^1 dz d\bar{z}\ \mu(z,\bar{z})\ \text{dDisc}[\mathcal{G}(z,\bar{z})]$$

Caron Huot 2017

Simmons-Duffin Stanford Witten 2017

Inversion Formula

It is possible to write a relation that invert the OPE allowing us to reconstruct the correlator by knowing only its singularities as $v \to 0$ or $\bar{z} \to 1$

How?

$$c_{\Delta,\ell} \sim \int_0^1 dz d\bar{z}\ \mu(z,\bar{z})\ \mathrm{dDisc}[\mathcal{G}(z,\bar{z})]$$

kernel

double discontinuity

Caron Huot 2017

Simmons-Duffin Stanford Witten 2017

Inversion Formula

It is possible to write a relation that invert the OPE allowing us to reconstruct the correlator by knowing only its singularities as $v \to 0$ or $\bar{z} \to 1$

How?

$$c_{\Delta,\ell} \sim \int_0^1 dz d\bar{z}\ \mu(z,\bar{z})\ \text{dDisc}[\mathcal{G}(z,\bar{z})]$$

kernel

double discontinuity

$$\text{dDisc}[\mathcal{G}(z,\bar{z})] = \mathcal{G}_{eucl}(z,\bar{z}) - \frac{1}{2}\mathcal{G}^{\circlearrowleft}(z,\bar{z}) - \frac{1}{2}\mathcal{G}^{\circlearrowright}(z,\bar{z})$$

analytic continuation around $\bar{z} \to 1$

Caron Huot 2017

Simmons-Duffin Stanford Witten 2017

Inversion Formula

It is possible to write a relation that invert the OPE allowing us to reconstruct the correlator by knowing only its singularities as $v \to 0$ or $\bar{z} \to 1$

How?

$$c_{\Delta,\ell} \sim \int_0^1 dz d\bar{z}\ \underbrace{\mu(z,\bar{z})}_{\textbf{kernel}}\ \underbrace{\mathrm{dDisc}[\mathcal{G}(z,\bar{z})]}_{\textbf{double discontinuity}}$$

Caron Huot 2017

Simmons-Duffin Stanford Witten 2017

Inversion Formula

It is possible to write a relation that invert the OPE allowing us to reconstruct the correlator by knowing only its singularities as $v \to 0$ or $\bar{z} \to 1$

How?

$$c_{\Delta,\ell} \sim \int_0^1 dz d\bar{z}\ \underbrace{\mu(z,\bar{z})}_{\textbf{kernel}}\ \underbrace{\mathrm{dDisc}[\mathcal{G}(z,\bar{z})]}_{\textbf{double discontinuity}}$$

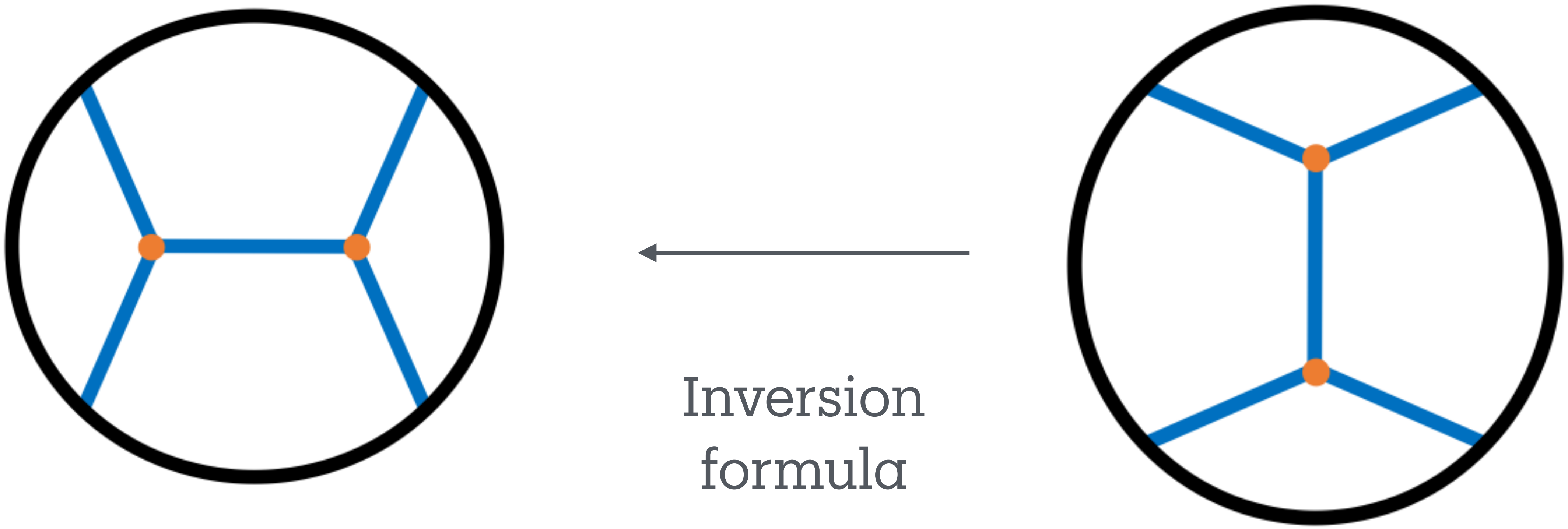
$$c_{\Delta,\ell} \xrightarrow[\Delta \to \Delta_k]{} \frac{a_{\Delta_k,\ell}}{\Delta - \Delta_k}$$

has poles at the dimension of the exchanged operator with residue the square of the three point function

Caron Huot 2017

Simmons-Duffin Stanford Witten 2017

Strategy



Non- Protected operators

Protected operators

Double trace $\frac{1}{2}$ BPS

Consider operators with $\Delta = 4$, transforming under $[0,4,0]_R$

$$\mathcal{O}_4^{SP} = \left(\frac{4(N^2+1)}{(N^2-1)(N^2-4)(N^2-9)} \right)^{1/2} \left(t_4 - \frac{2N^2-3}{N(N^2+1)} t_2^2 \right)$$

$$\mathcal{O}_4^{DT} = \left(\frac{2}{N^4-1} \right)^{1/2} t_2^2 \qquad\qquad t_p = \mathrm{Tr}(y \cdot \phi)^p$$

Double trace $\dfrac{1}{2}$ BPS

Consider operators with $\Delta = 4$, transforming under $[0,4,0]_R$

$$\mathcal{O}_4^{SP} = \left(\frac{4(N^2+1)}{(N^2-1)(N^2-4)(N^2-9)}\right)^{1/2}\left(t_4 - \frac{2N^2-3}{N(N^2+1)}t_2^2\right)$$

$$\mathcal{O}_4^{DT} = \left(\frac{2}{N^4-1}\right)^{1/2} t_2^2 \qquad\qquad t_p = \mathrm{Tr}(y\cdot\phi)^p$$

Orthogonal: $\langle \mathcal{O}_4^{SP}\mathcal{O}_4^{DT}\rangle = 0$

Double trace $\frac{1}{2}$ BPS

Consider operators with $\Delta = 4$, transforming under $[0,4,0]_R$

$$\mathcal{O}_4^{SP} = \left(\frac{4(N^2+1)}{(N^2-1)(N^2-4)(N^2-9)} \right)^{1/2} \left(t_4 - \frac{2N^2-3}{N(N^2+1)} t_2^2 \right)$$

$$\mathcal{O}_4^{DT} = \left(\frac{2}{N^4-1} \right)^{1/2} t_2^2 \qquad\qquad t_p = \mathrm{Tr}(y \cdot \phi)^p$$

Orthogonal: $\langle \mathcal{O}_4^{SP} \mathcal{O}_4^{DT} \rangle = 0$

And: $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_4^{SP} \rangle = 0 \qquad \langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_4^{DT} \rangle \neq 0$

Double trace $\frac{1}{2}$ BPS

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_4^{DT} \mathcal{O}_4^{DT} \rangle$$

- Used supersymmetry and Ward identities to disentangle the contribution of protected vs non protected operators

Double trace $\frac{1}{2}$ BPS

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_4^{DT} \mathcal{O}_4^{DT} \rangle$$

- Used supersymmetry and Ward identities to disentangle the contribution of protected vs non protected operators

- The non protected part is

$$\mathcal{H}^{DT}_{2244} = 2 + \frac{2}{v^2} + \frac{4}{N^2-1}\left(1 + \frac{1}{v^2} + \frac{6}{v} - 2u^2 \bar{D}_{2422}(u,v)\right)$$

Results

Results

- Intermediate operators are $SU(4)_R$ singlets

Results

- Intermediate operators are $SU(4)_R$ singlets
- Compute the anomalous dimensions and correction to the OPE coefficients of double traces and triple traces appearing in the OPE

Results

- Intermediate operators are $SU(4)_R$ singlets
- Compute the anomalous dimensions and correction to the OPE coefficients of double traces and triple traces appearing in the OPE
- At twist $\tau = \Delta - \ell$=6 there are triple trace operators $\sim \mathrm{Tr}\phi^2\mathrm{Tr}\phi^2\mathrm{Tr}\phi^2$

Results

- Intermediate operators are $SU(4)_R$ singlets
- Compute the anomalous dimensions and correction to the OPE coefficients of double traces and triple traces appearing in the OPE
- At twist $\tau = \Delta - \ell$=6 there are triple trace operators $\sim \mathrm{Tr}\phi^2\mathrm{Tr}\phi^2\mathrm{Tr}\phi^2$
- Results are lengthy but explicitly written in the paper

Results

- Intermediate operators are $SU(4)_R$ singlets
- Compute the anomalous dimensions and correction to the OPE coefficients of double traces and triple traces appearing in the OPE
- At twist $\tau = \Delta - \ell$=6 there are triple trace operators $\sim \mathrm{Tr}\phi^2\mathrm{Tr}\phi^2\mathrm{Tr}\phi^2$
- Results are lengthy but explicitly written in the paper
- There are issues with mixing: there is more than one state with the same quantum number.

Results

- Intermediate operators are $SU(4)_R$ singlets
- Compute the anomalous dimensions and correction to the OPE coefficients of double traces and triple traces appearing in the OPE
- At twist $\tau = \Delta - \ell$=6 there are triple trace operators $\sim \mathrm{Tr}\phi^2\mathrm{Tr}\phi^2\mathrm{Tr}\phi^2$
- Results are lengthy but explicitly written in the paper
- There are issues with mixing: there is more than one state with the same quantum number.
- Our results agree with supergravity based ones as in Aprile, Giusto, Russo.

Double trace $\frac{1}{4}$ BPS

Double trace $\frac{1}{4}$ BPS

We studied several correlators involving half and quarter BPS operators and used the same strategy as before

Double trace $\dfrac{1}{4}$ BPS

We studied several correlators involving half and quarter BPS operators and used the same strategy as before

$\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_{02} \rangle$ protected

Double trace $\dfrac{1}{4}$ BPS

We studied several correlators involving half and quarter BPS operators and used the same strategy as before

$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_{02} \rangle$ protected

$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_{02} \mathcal{O}_{02} \rangle$

$\langle \mathcal{O}_{02} \mathcal{O}_{02} \mathcal{O}_{02} \mathcal{O}_{02} \rangle$

reconstructed it up to N^{-2} from the the protected part

Double trace $\frac{1}{4}$ BPS

Difficulties:

Double trace $\dfrac{1}{4}$ BPS

Difficulties:

- Proliferation of R-symmetry structures, for instance $\mathcal{O}_2 \times \mathcal{O}_{02}$ has 10 R-symmetry structures and $\mathcal{O}_{02} \times \mathcal{O}_{02}$ has 42.

Double trace $\frac{1}{4}$ BPS

Difficulties:

- Proliferation of R-symmetry structures, for instance $\mathcal{O}_2 \times \mathcal{O}_{02}$ has 10 R-symmetry structures and $\mathcal{O}_{02} \times \mathcal{O}_{02}$ has 42.

- The solution to the Superconformal Ward identities is not known.

Double trace $\frac{1}{4}$ BPS

Difficulties:

- Proliferation of R-symmetry structures, for instance $\mathcal{O}_2 \times \mathcal{O}_{02}$ has 10 R-symmetry structures and $\mathcal{O}_{02} \times \mathcal{O}_{02}$ has 42.

- The solution to the Superconformal Ward identities is not known.

- The form of the Superconformal block is not known.

some progress in Hansen, Heslop, Puerta Ramisa

Double trace $\dfrac{1}{4}$ BPS

Difficulties:

- Proliferation of R-symmetry structures, for instance $\mathcal{O}_2 \times \mathcal{O}_{02}$ has 10 R-symmetry structures and $\mathcal{O}_{02} \times \mathcal{O}_{02}$ has 42.

- The solution to the Superconformal Ward identities is not known.

- The form of the Superconformal block is not known.

some progress in Hansen, Heslop, Puerta Ramisa

- We computed the anomalous dimension and OPE coefficient of classes of operators (conformal primaries) up to twist 6.

Conclusions

Conclusions

- Study four point functions of protected operators to get information about the dimension of higher trace operators.

Conclusions

- Study four point functions of protected operators to get information about the dimension of higher trace operators.

- For double trace operators $\tau \to 2\Delta_{ext} + 2n + \gamma$ for large spin, with $\gamma \to \dfrac{A}{\ell^{\tau_{min}}}$. What happens for higher trace operators?

Conclusions

- Study four point functions of protected operators to get information about the dimension of higher trace operators.

- For double trace operators $\tau \to 2\Delta_{ext} + 2n + \gamma$ for large spin, with $\gamma \to \dfrac{A}{\ell^{\tau_{min}}}$. What happens for higher trace operators?

- Need to understand the superconformal blocks for quarter-BPS operators.

Conclusions

- Study four point functions of protected operators to get information about the dimension of higher trace operators.

- For double trace operators $\tau \to 2\Delta_{ext} + 2n + \gamma$ for large spin, with $\gamma \to \dfrac{A}{\ell^{\tau_{min}}}$. What happens for higher trace operators?

- Need to understand the superconformal blocks for quarter-BPS operators.

- Systematics of triple and quadruple traces.