EXPLORING BLACK HOLE HILBERT SPACES VIA THE GRAVITY PATH INTEGRAL

arXiv: 2410.00091
arXiv: 2412.06884

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Black hole entropy

The Question

- Bekenstein and Hawking: black holes are thermodynamic objects with an entropy

$$S_{\mathrm{BH}} = \frac{\mathcal{A}}{4 G_N}$$

- If this entropy counts microstates, then

$$\dim\left(\mathcal{H}_{\mathrm{BH}}\right) = e^{S_{\mathrm{BH}}}$$

- **Exponentially large number of black hole microstates?**

Paths to Resolution

- String theory approach [Strominger and Vafa 1996]
- "Bag of gold" geometries [Wheeler 1964]

Black hole microstates

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A Breakthrough

Can we give a universal microscopic explanation for the entropy of black holes?

Microscopic Origin of the Entropy of Black Holes in General Relativity

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Family of States

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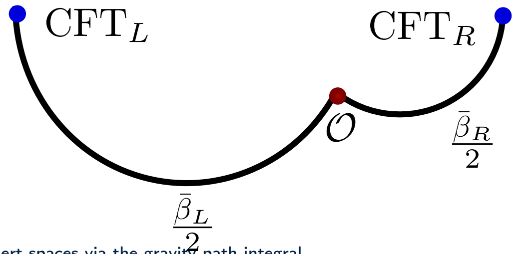
Family of States in Holographic CFT

Consider states on the tensor product of two identical copies of a d-dimensional CFT defined on $\mathbb{S}^{d-1} \times \mathbb{R}$

$$|\Psi\rangle = \sum_{n,m} \Psi_{mn} |E_m\rangle_L \otimes |E_n\rangle_R$$

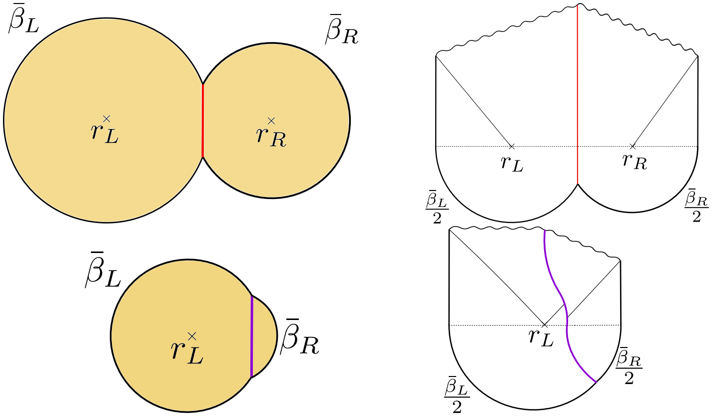
$\Psi_{mn} = \frac{e^{-\frac{1}{2}\bar{\beta}_L E_m - \frac{1}{2}\bar{\beta}_R E_n}}{\sqrt{Z_\Psi}} \langle m|\mathcal{O}|n\rangle$ and $Z_\Psi = \mathrm{Tr}\left[\mathcal{O}^\dagger e^{-\bar{\beta}_L H} \mathcal{O} e^{-\bar{\beta}_R H}\right]$



Family of States

Family of States in Gravity

Operators $\mathcal{O}$ in CFT $\leftrightarrow$ Shell of matter in gravity



Microcanonical Projection

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The Projection Operator

- Assume $\mathcal{H} = \bigoplus_E \mathcal{H}^E$
- Define the microcanonical projector $\Pi_E : \bigoplus_{E'} \mathcal{H}^{E'} \to \mathcal{H}^E$
- Black hole microstates

$$|\Psi^E\rangle = \frac{1}{\sqrt{Z_\Psi}} \sum_{|E_m\rangle_L \otimes |E_n\rangle_R \in \mathcal{H}^E} \mathcal{O}_{mn} |E_m\rangle_L \otimes |E_n\rangle_R$$

- In practice, use inverse Laplace transform

$$f(E) \propto \int d\beta e^{\beta E} F(\beta)$$

Counting states

Strategy

- Introduce states $|\Psi_k^E\rangle$, where $\mathcal{O}^{(k)}$ are a product of n scalar operators $\mathcal{O}_{\Delta_k}$ that create shells with mass $m_k = k m_{\Delta_1}$

$$\mathcal{H}^E_{\mathrm{bulk}}(\Omega) \equiv \mathrm{Span}\{|\Psi_k^E\rangle, \quad k = 1, \ldots, \Omega\}$$

- Define the Gram matrix $G_{ij} = \langle \Psi_i^E | \Psi_j^E \rangle$

$$\dim\left(\mathcal{H}^E_{\mathrm{bulk}}(\Omega)\right) = \mathrm{rank}\,(G) = \Omega - \dim\left(\mathrm{Ker}\,(G)\right)$$

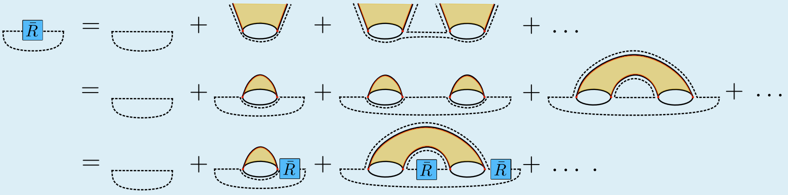
- Find $\mathrm{Ker}\,(G)$ using the resolvent matrix $R_{ij}(\lambda) = \left(\frac{1}{\lambda \mathbb{1} - G}\right)_{ij}$

$$R_{ij}(\lambda) = \frac{1}{\lambda}\delta_{ij} + \sum_{n=1}^{\infty} \frac{1}{\lambda^{n+1}} (G^n)_{ij}$$

Counting States

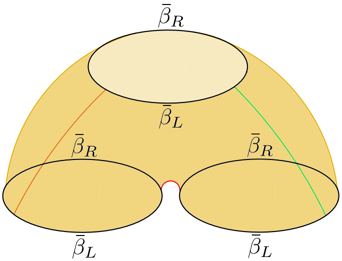
$$R(\lambda) = \frac{\Omega}{\lambda} + \frac{1}{\lambda}\sum_{n=1}^{\infty}\sum_{i=1}^{\Omega}\frac{1}{\lambda^n}(G^n)_{ii}$$

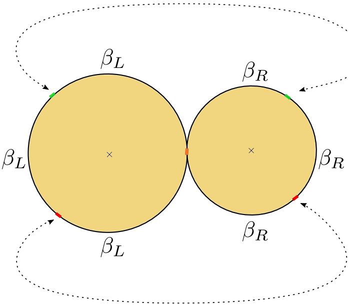
Semiclassical Approximation for Trace of the Resolvent



$$\overline{R}(\lambda) = \frac{\Omega}{\lambda} + \frac{1}{\lambda}\sum_{n=1}^{\infty}\sum_{i=1}^{\Omega}\frac{\overline{R}^n}{\Omega^n}\overline{(G^n)_{ii}}\Big|_{\mathrm{conn.}}$$

Counting States





$$\overline{\langle\Psi_{k_1}|\Psi_{k_2}\rangle\langle\Psi_{k_2}|\Psi_{k_3}\rangle\dots\langle\Psi_{k_n}|\Psi_{k_1}\rangle}\Big|_{\text{conn.}} = \frac{Z(n\beta_L)Z(n\beta_R)e^{-nI_{\text{univ.}}}}{\left(Z(\beta_L)Z(\beta_R)e^{-I_{\text{univ.}}}\right)^n}$$

$$\overline{\langle\Psi^E_{k_1}|\Psi^E_{k_2}\rangle\langle\Psi^E_{k_2}|\Psi^E_{k_3}\rangle\dots\langle\Psi^E_{k_n}|\Psi^E_{k_1}\rangle}\Big|_{\text{conn.}} = e^{(1-n)(S_{BH}(E_L)+S_{BH}(E_R))}$$

Counting States

Solving the Schwinger-Dyson Equation

$$\lambda \overline{R} = \Omega + \frac{\overline{R} e^{S_L+S_R}}{e^{S_L+S_R} - \overline{R}}$$

Solution:

$$\overline{R_+} = \frac{\Omega - e^{S_L+S_R}}{\lambda} \Theta\left(\Omega - e^{S_L+S_R}\right) + R_0 + O(\lambda)$$

$$\Rightarrow \overline{\dim\left(\mathcal{H}^E_{\text{bulk}}(\Omega)\right)} = \begin{cases} \Omega, & \Omega < e^{S_L+S_R} \\ e^{S_L+S_R}, & \Omega \geq e^{S_L+S_R} \end{cases}$$

Factorization of Microcanonical Bulk Traces

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Microcanonical Bulk Trace

$$\mathrm{Tr}_{\mathcal{H}^E_{\mathrm{bulk}}}\left(f(H_L)g(H_R)\right) = f(E_L)g(E_R)\mathrm{dim}(\mathcal{H}^E_{\mathrm{bulk}})$$

- coarse-grained

$$\overline{\dim\left(\mathcal{H}^E_{\mathrm{bulk}}(\Omega)\right)} = \begin{cases} \Omega, & \Omega < e^{S_L+S_R} \\ e^{S_L+S_R}, & \Omega \geq e^{S_L+S_R} \end{cases}$$

- fine-grained (*)

$$\overline{\left(\mathrm{Tr}_{\mathcal{H}^E_{\mathrm{bulk}}}\left(f(H_L)g(H_R)\right) - \overline{\mathrm{Tr}_{\mathcal{H}^E_{\mathrm{bulk}}}\left(f(H_L)g(H_R)\right)}\right)^2} = 0$$

Factorization of Microcanonical Bulk Traces

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Fine-grained Factorization

- Boruch et al. 2024 **How the Hilbert space of two-sided black holes factorises** [arXiv:2406.04396]
- Balasubramanian et al. 2025 **Factorization of the Hilbert space of eternal black holes in general relativity** [arXiv:2410.00091]
- Balasubramanian and Yildirim 2025a **A Nonperturbative Toolkit for Quantum Gravity** [arXiv:2504.16986]
- Balasubramanian and Yildirim 2025b **The Nonperturbative Hilbert Space of Quantum Gravity With One Boundary** [arXiv:2506.04319]

Further Investigation and Outlook

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Further Investigation

- Climent et al. 2024 **Universal construction of black hole microstates** [arXiv:2401.08775]
- Balasubramanian et al. 2024 **Counting microstates of out-of-equilibrium black hole fluctuations** [arXiv:2412.06884]
- Abdalla et al. 2025 **The gravitational path integral from an observer's point of view** [arXiv:2501.02632]
- ...

Outlook

- Hilbert space of black holes formed from collapse
- Quantum corrections
- de Sitter microstates
- ...

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