# The Statistics of BPS Chaos

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UNIVERSITÀ DEGLI STUDI DI PADOVA

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Work in progress with N. Ceplak, S. Massai and M. Shigemori

*August 26th, 2025*

*Count them* ⟵ Supersymmetric black-hole microstates ⟶ *Describe them from the physics of intersecting branes*

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Supersymmetric black-hole microstates

*Describe them from the physics of intersecting branes*

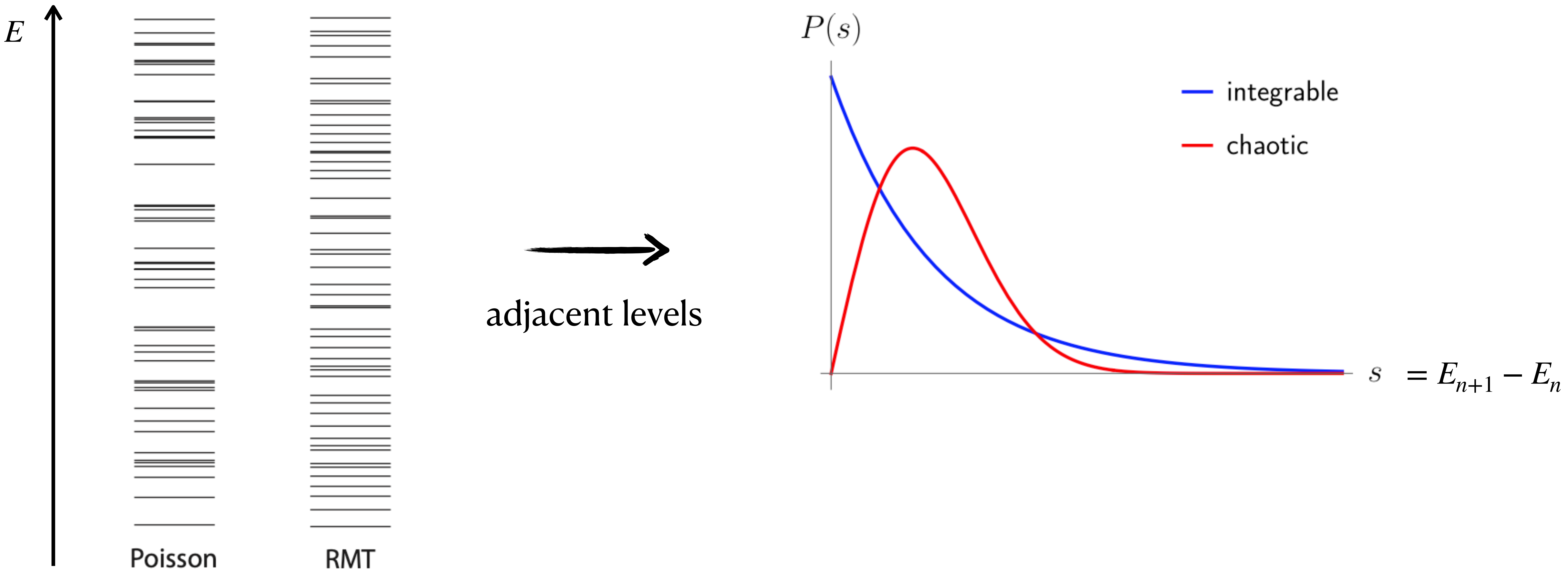
*Collective properties*

***(Late-time) quantum chaos***

→ How does quantum chaos emerge out of supersymmetric BH microstates?

# Late-time chaos and the Thouless time

- Late-time chaos → properties of quantum states taken collectively.
- A prime example of such properties is *eigenvalue repulsion*.



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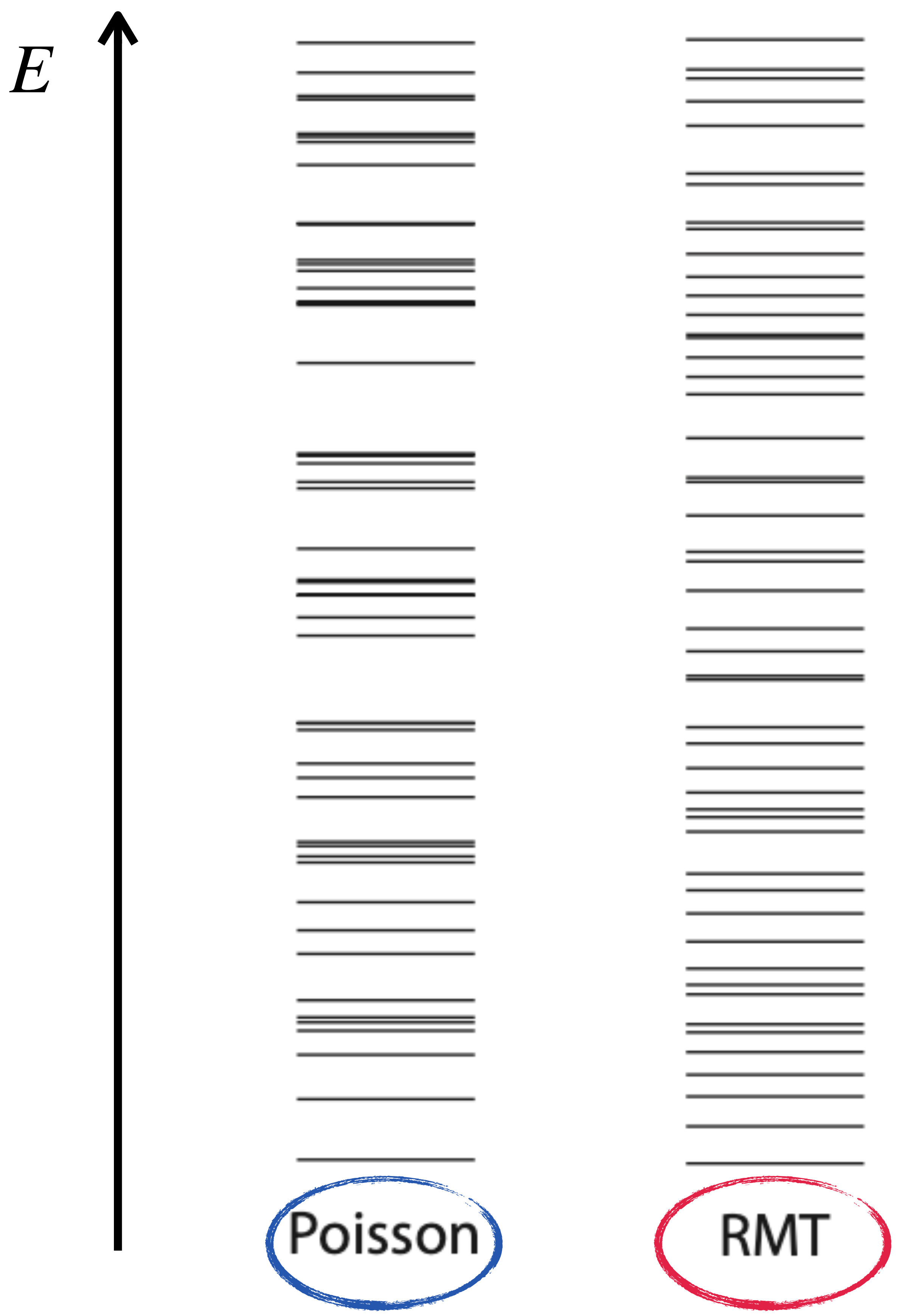


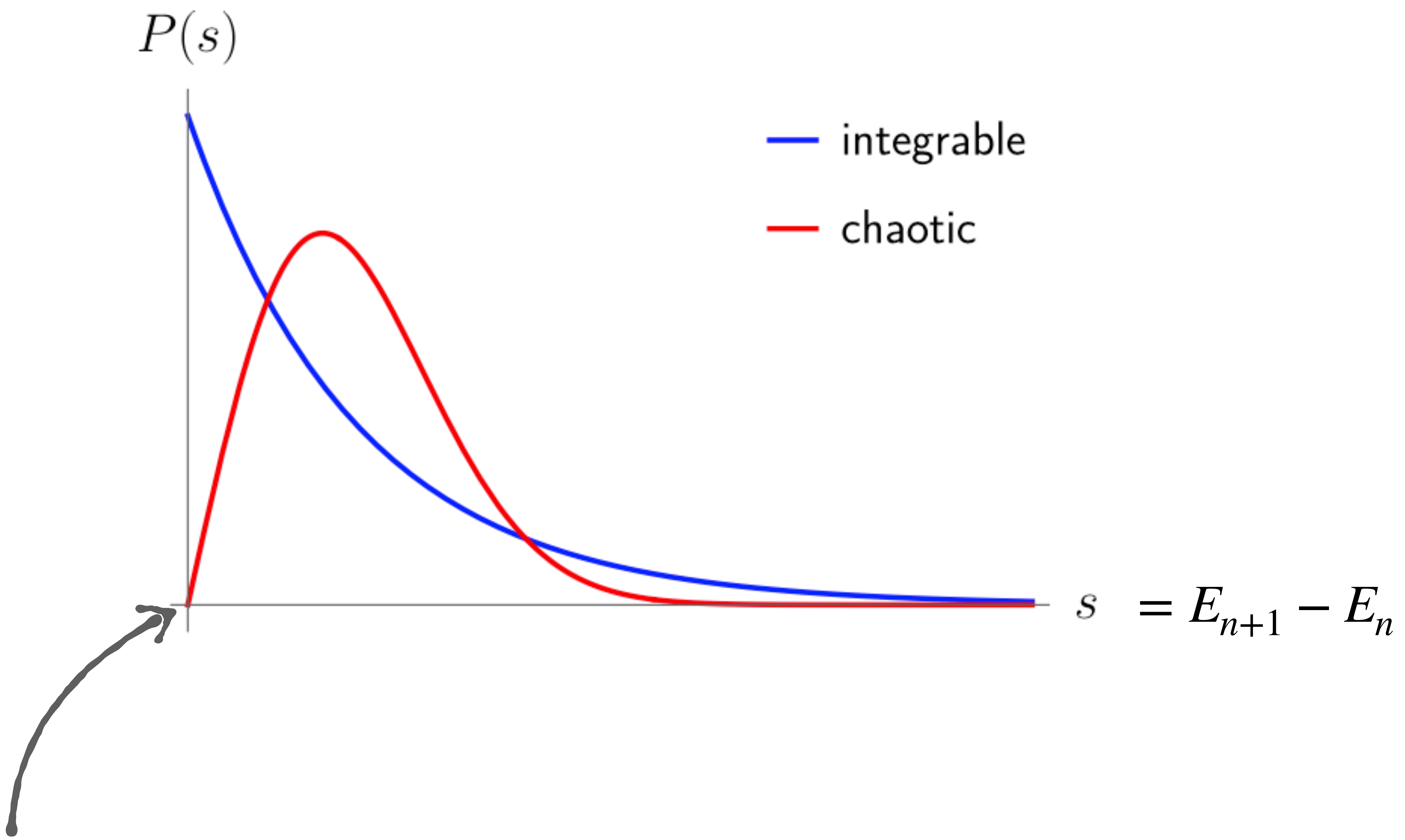


Zero probability for degenerate levels

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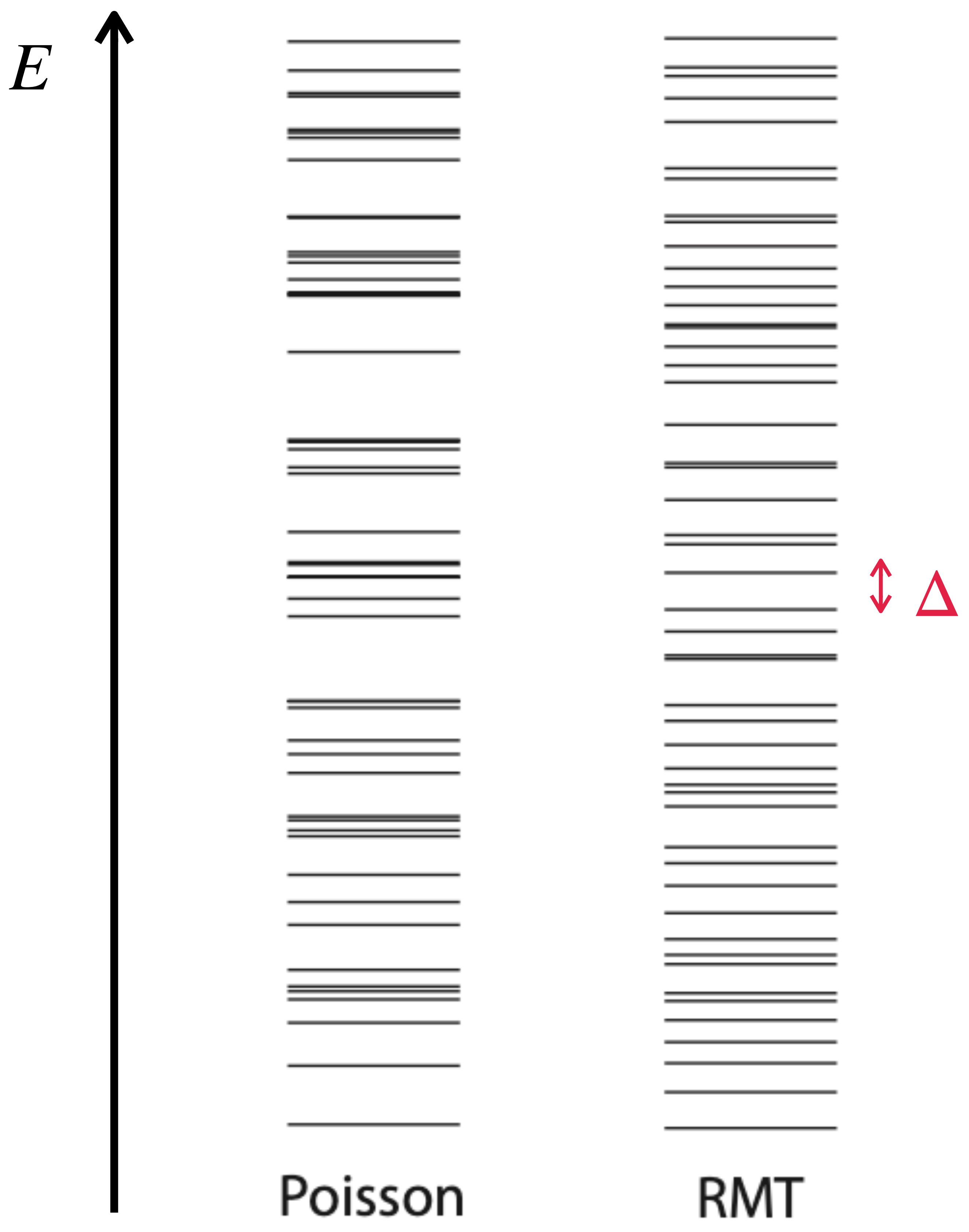


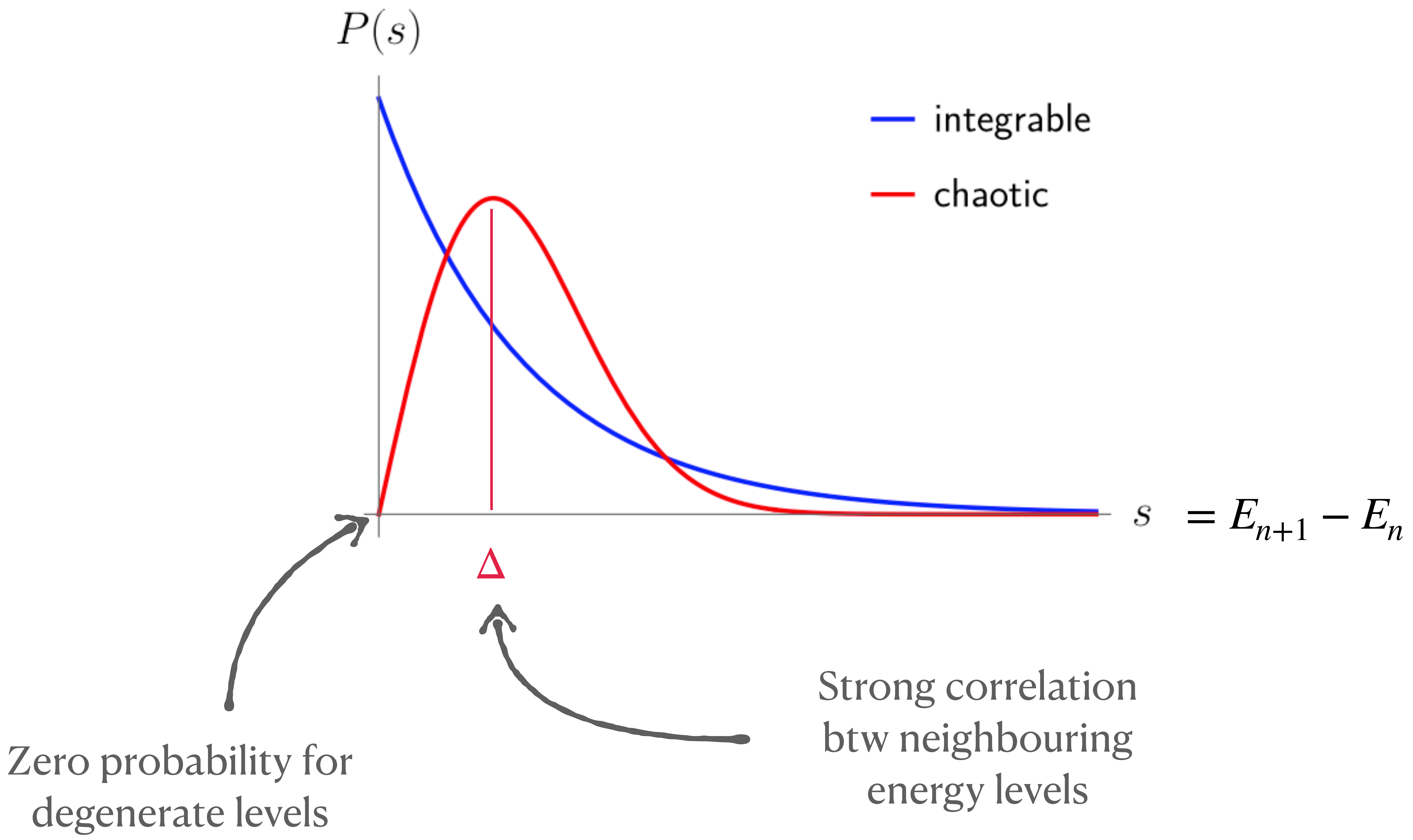

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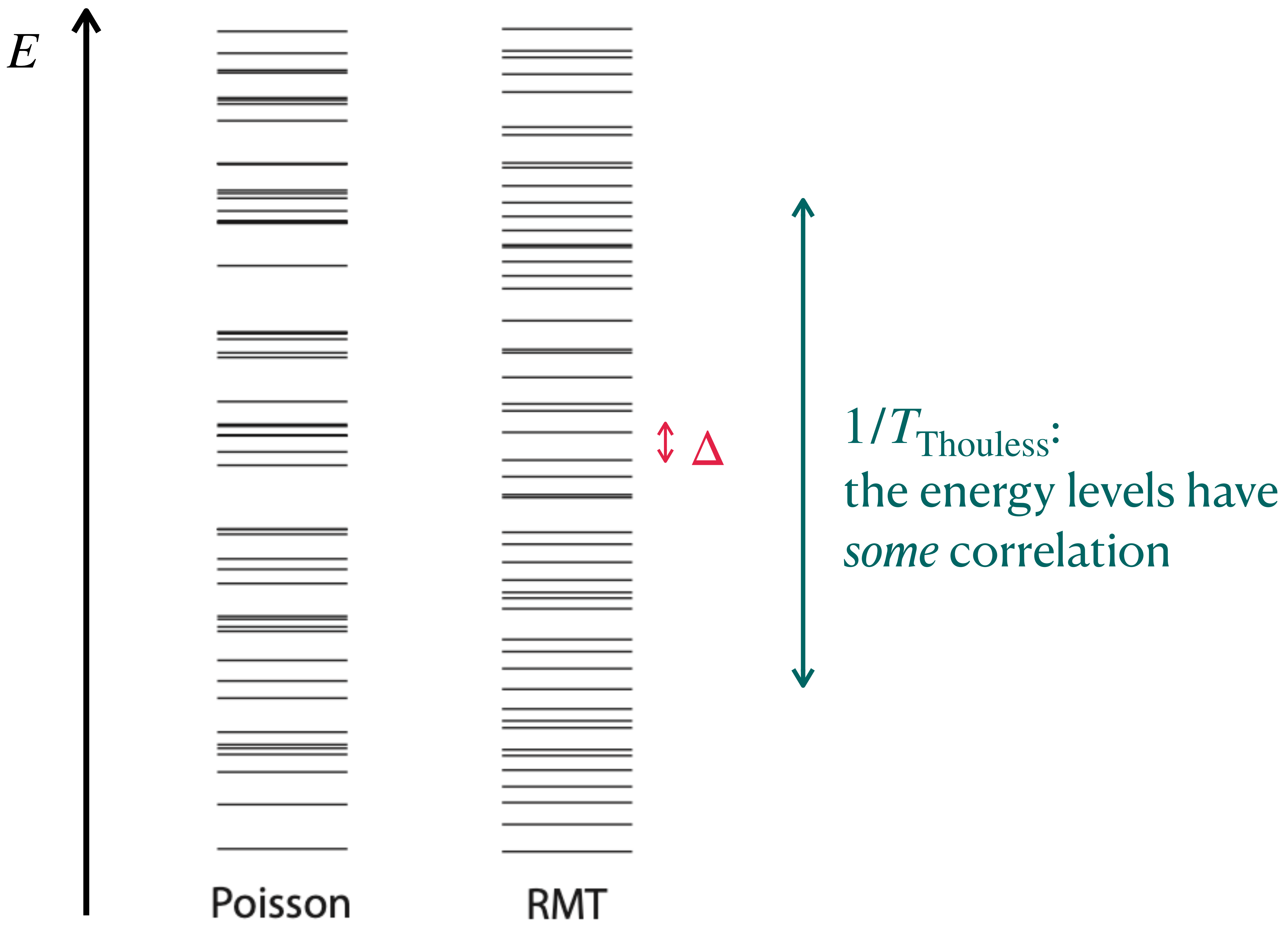


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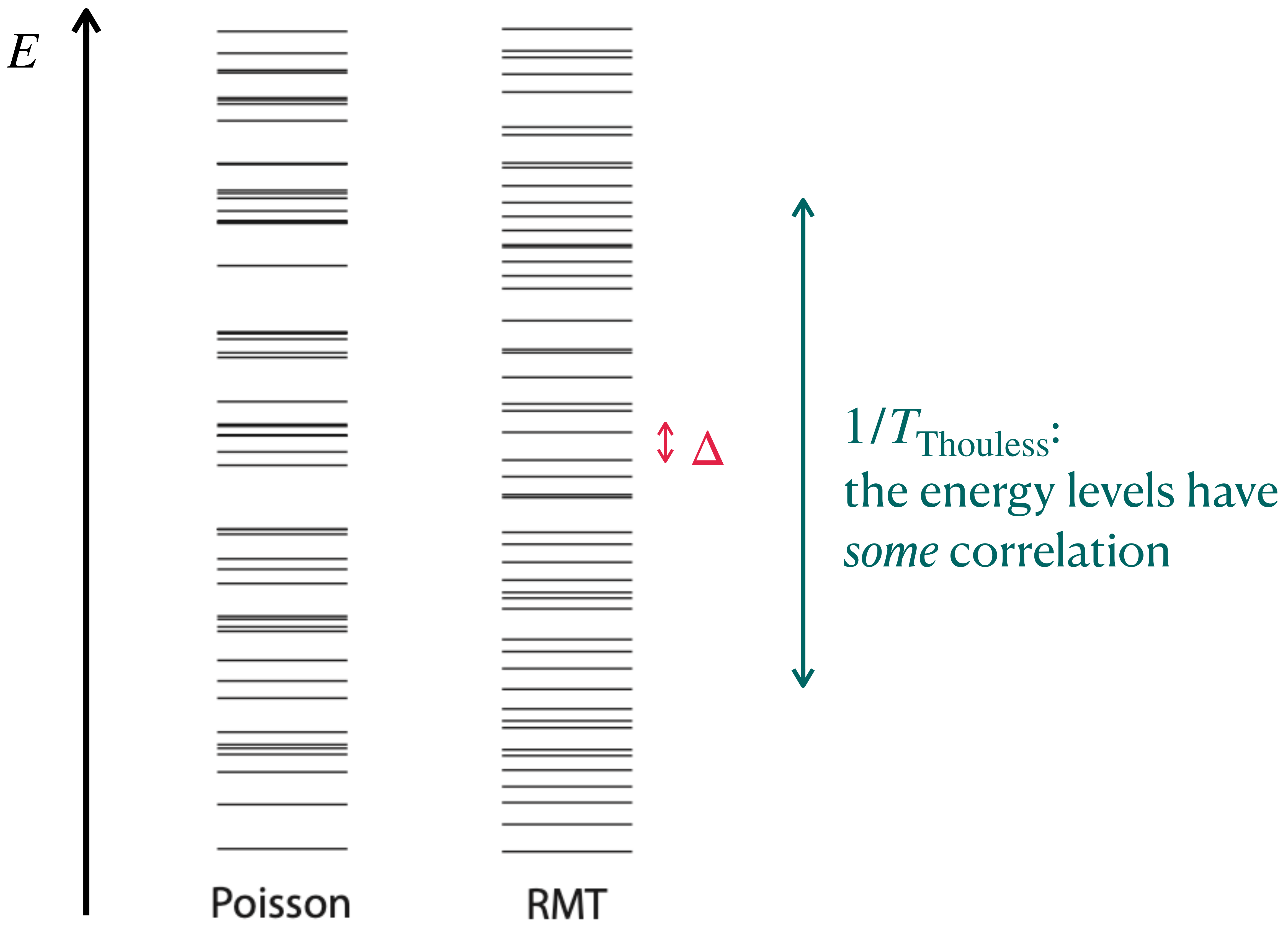


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- Characterises how strongly chaotic a quantum system is
  - $T_{\text{Thouless}} \sim \mathcal{O}(1)$ : $\exists$ correlation btw *all* energy levels in the microcanonical window. → « strong chaos »
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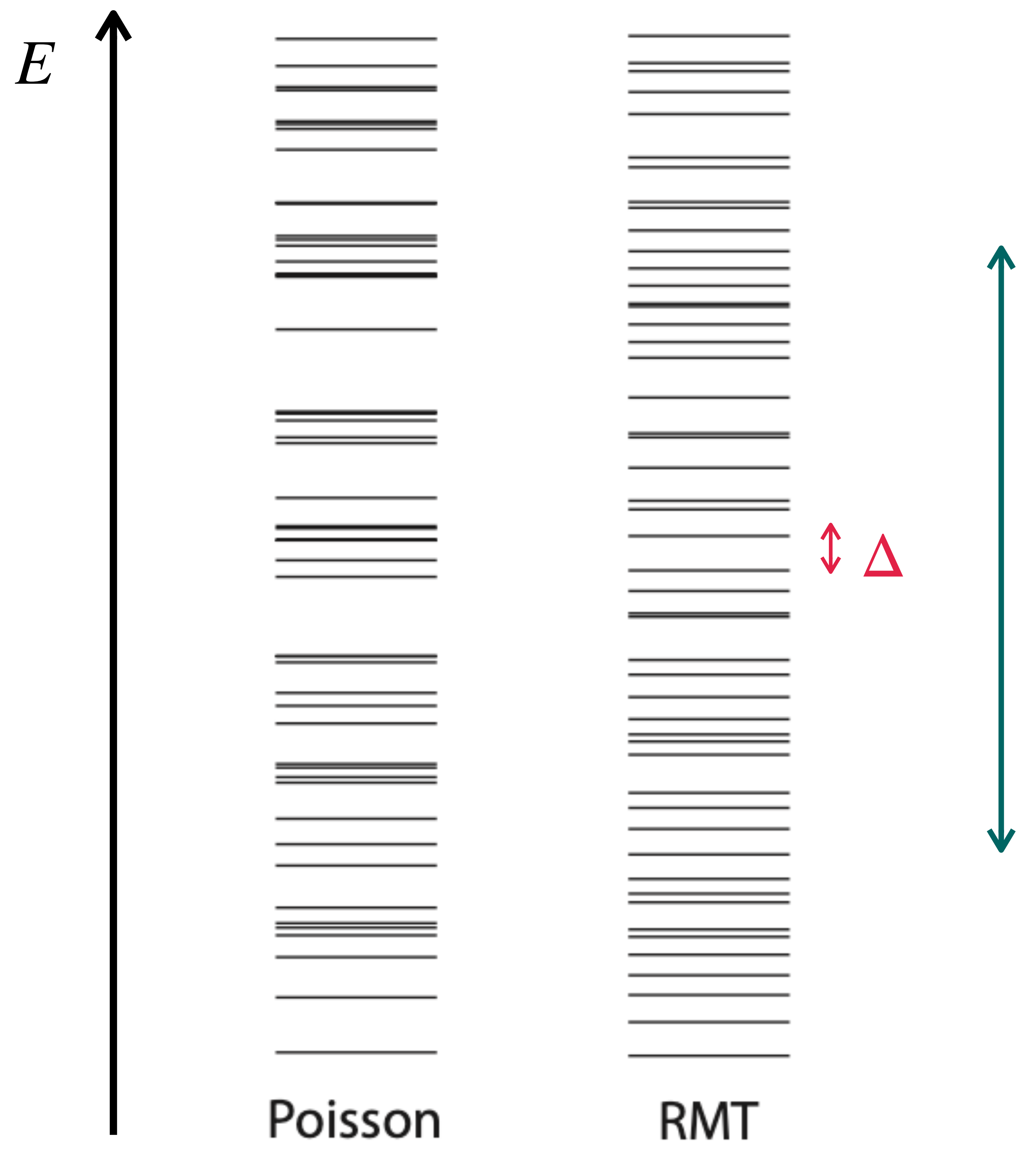


$1/T_{\text{Thouless}}$:
the energy levels have
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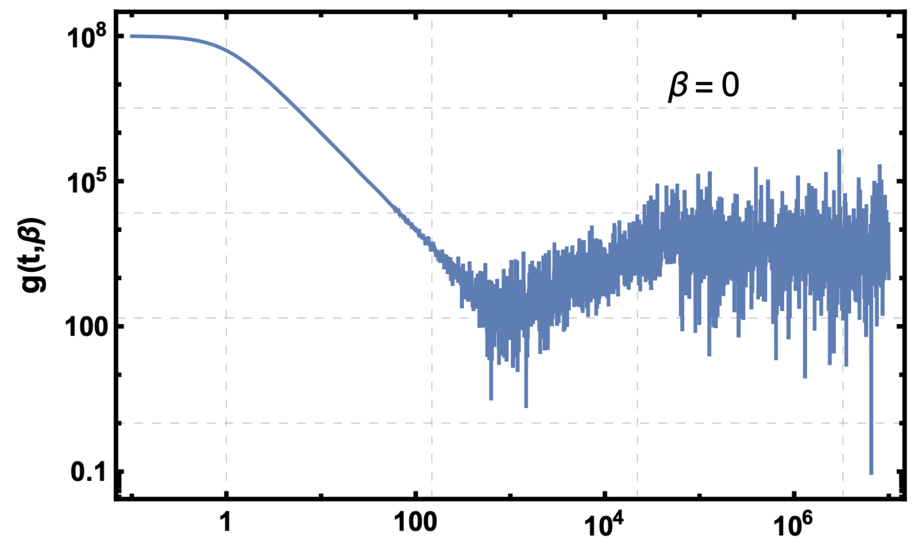


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- In practice: corresponds to the start of the ramp of the *spectral form factor*

# Black holes and BPS chaos

- Supersymmetric BHs are also expected to have properties of chaos.

- **Chaos for BPS systems**: Replace hamiltonian by BPS projection of any « simple » operator $\hat{\mathcal{O}} = P_{BPS}\mathcal{O}P_{BPS}$, and "time"-evolve the system with it.

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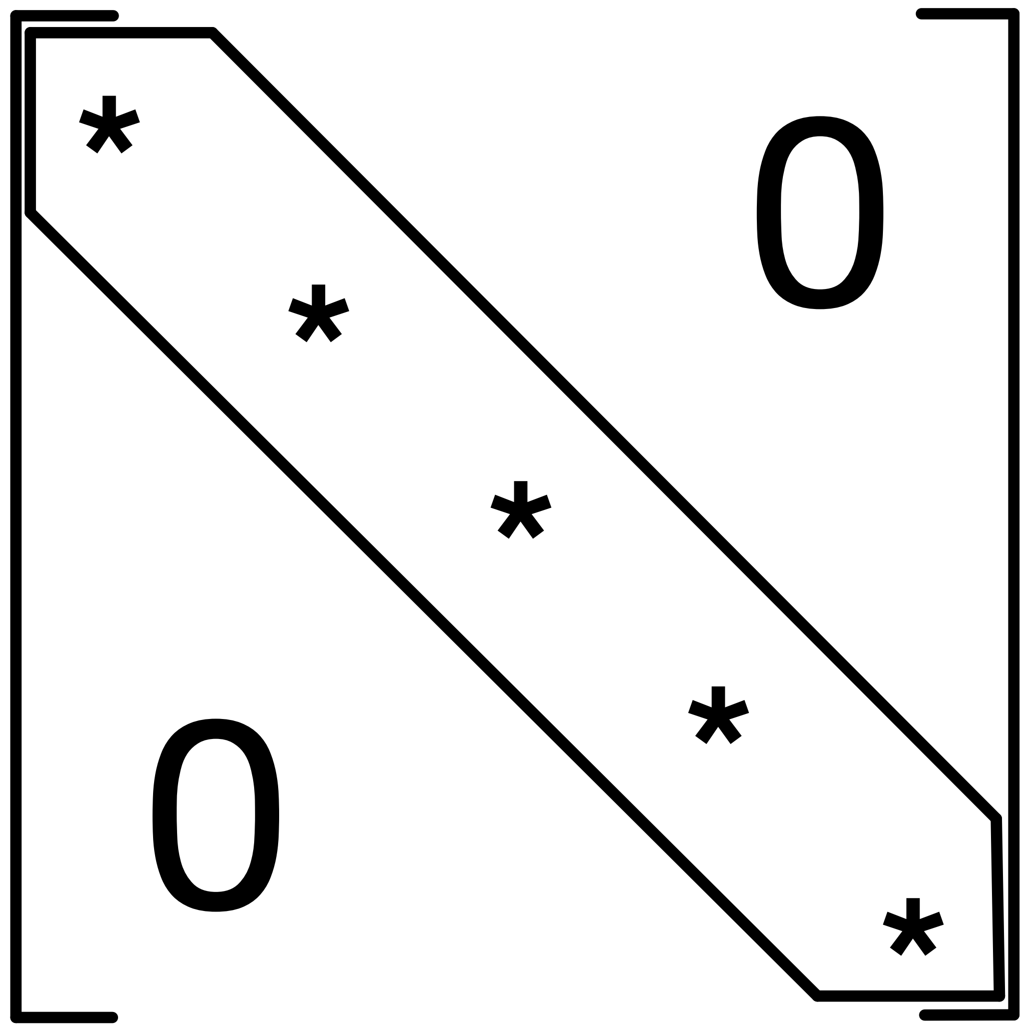


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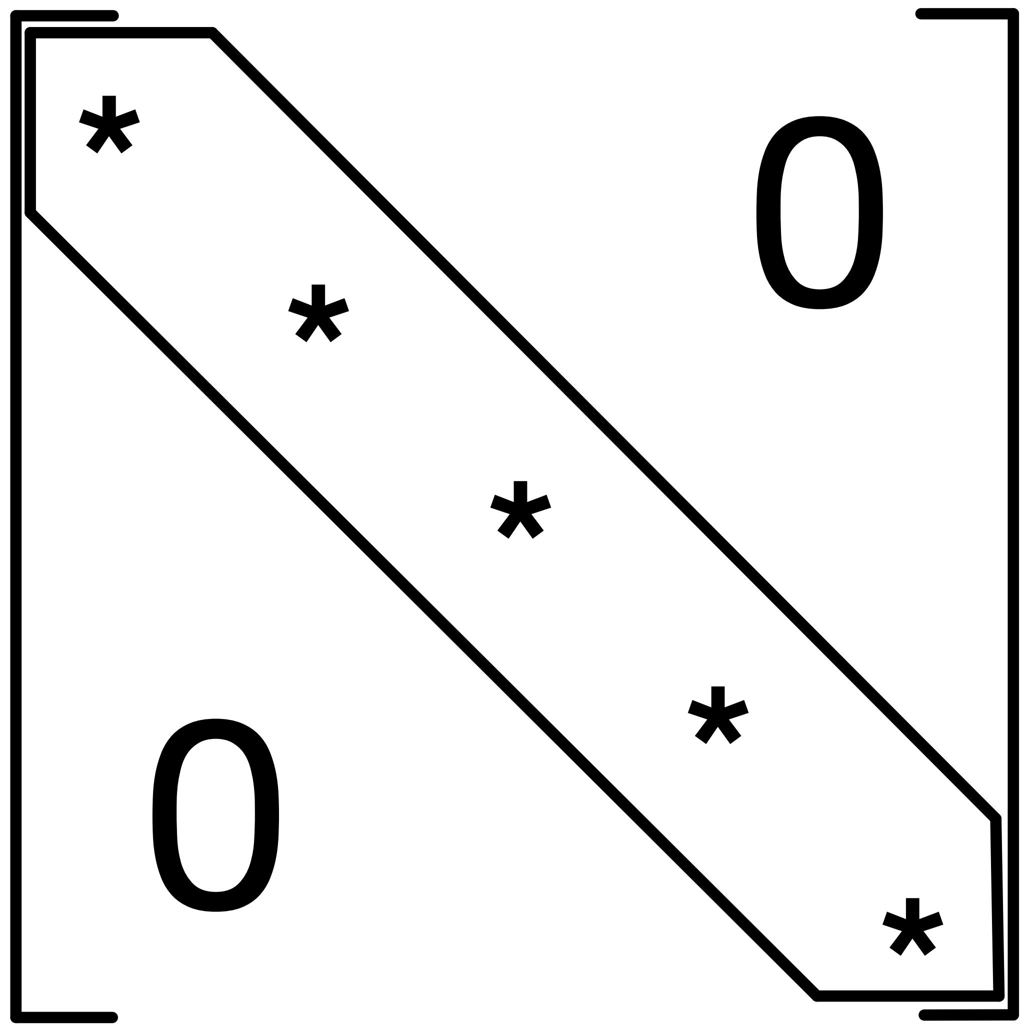
  $\rightarrow$ interaction with the nearest neighbours

$$T_{\text{Thouless}} \sim T_{\text{diffusion}} \sim \left(\frac{\text{size of the matrix}}{\text{size of the band}}\right)^2$$

[Erdös, Knowles '13]

$\hat{\mathcal{O}} =$ 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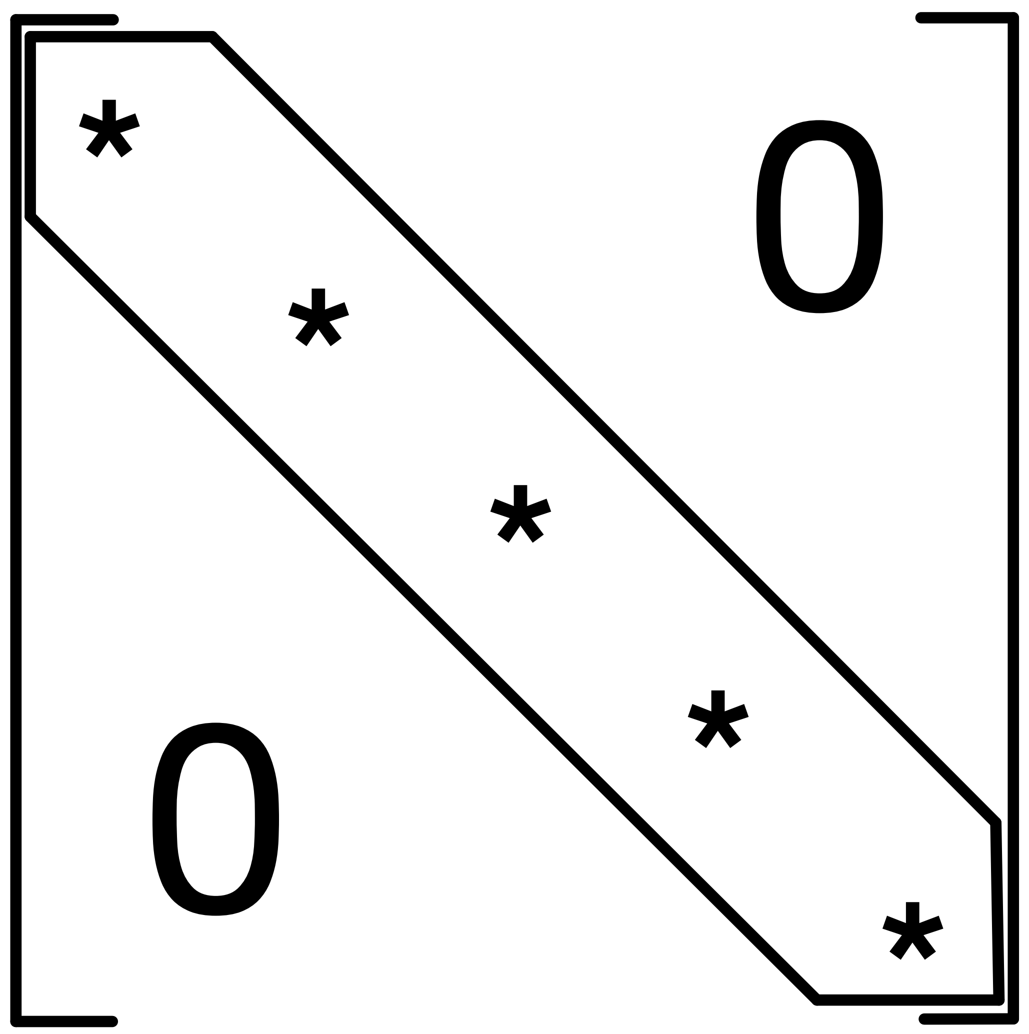
$\hat{\mathcal{O}} =$ 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**Can one estimate $T_{\text{Thouless}}$ for the Strominger-Vafa-like black-hole microstates (D1-D5-P)?**

# The D1-D5-P system

| | $t$ | $\mathbb{R}^4$ | $S^1_y$ | $T^4$ |
|---|---|---|---|---|
| D5 | $-$ | $\cdot$ | $-$ | $-$ |
| D1 | $-$ | $\cdot$ | $-$ | $\sim$ |
| P | $-$ | $\cdot$ | $\rightarrow$ | $\sim$ |

- The brane system backreacts into a black-hole solution, at $g_s N \gg 1$.

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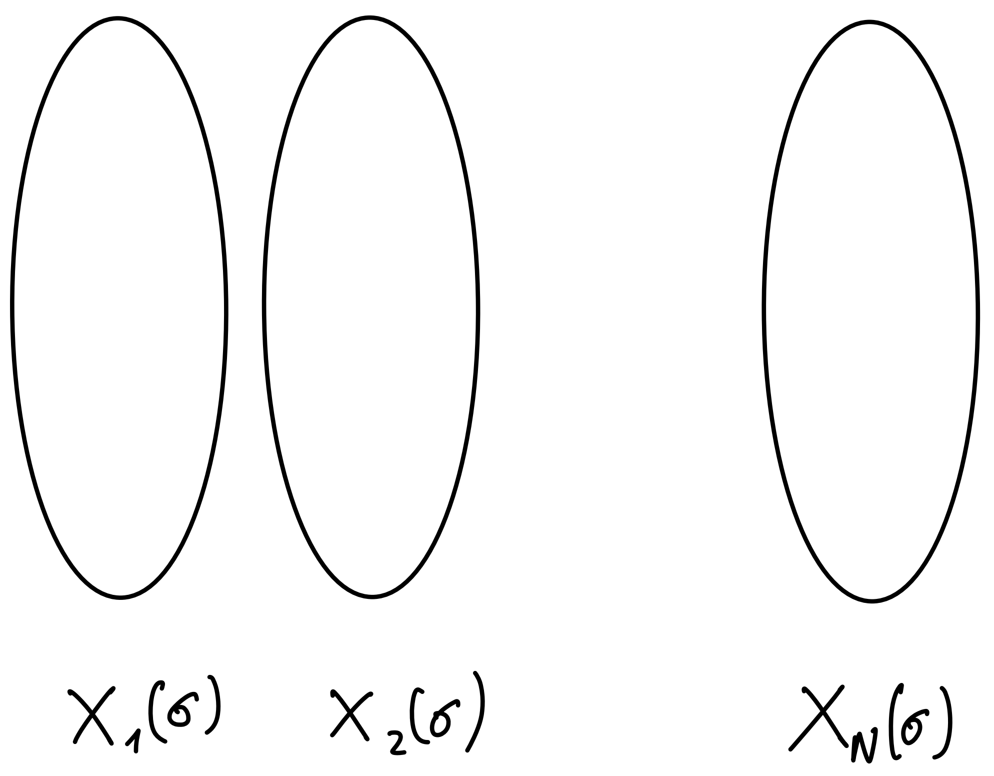
  $\rightarrow$ low energy limit at finite $g_s$ is a 2d CFT

Deformation of the D1-D5 CFT at the free orbifold point ($g_s = 0$).

# The D1-D5 CFT at the free orbifold point

- 2d sigma-model on $(T^4)^N/S_N$ :

  $N = N_1 N_5$ copies of fields on $S^1$; specify boundary conditions to glue the copies together




$$X_i(\sigma = 2\pi) = X_{g(i)}(\sigma = 0)$$
$$(g \in S_N)$$

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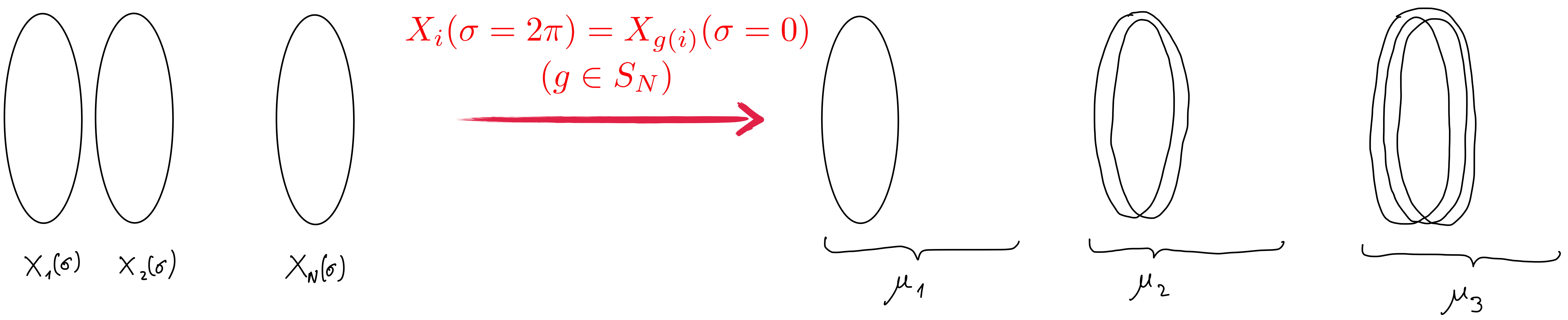

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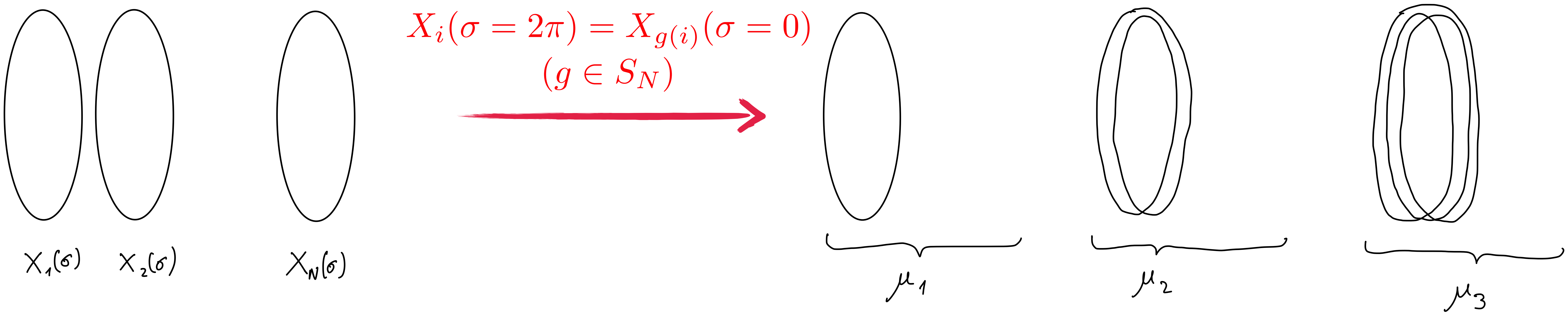

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- **Two-charge states**:
  1. Specify how the $N$ copies combine into strands.

     → Distribute the winding number $N$ into strands of different lengths

  $$N = \sum_{n \geq 1} n\mu_n$$

  2. Choose a polarisation (16 of them) on each strand.

# Organising the microstates

- Fix $N$. Regroup the microstates according to the number of strands. [Chen, Lin, Shenker '24]

Single strand of length $N$

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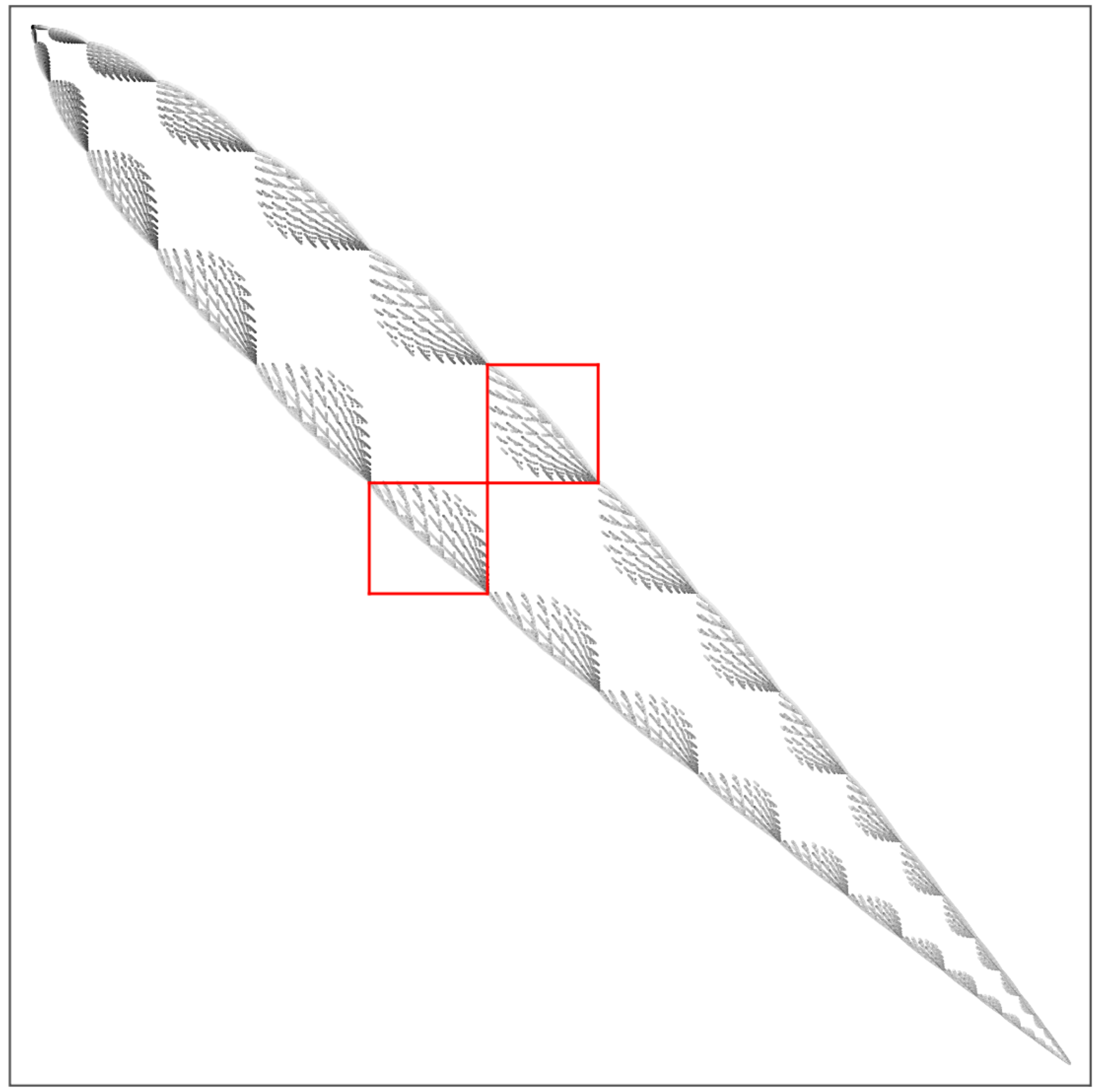
$$\hat{\Sigma}_2 =$$



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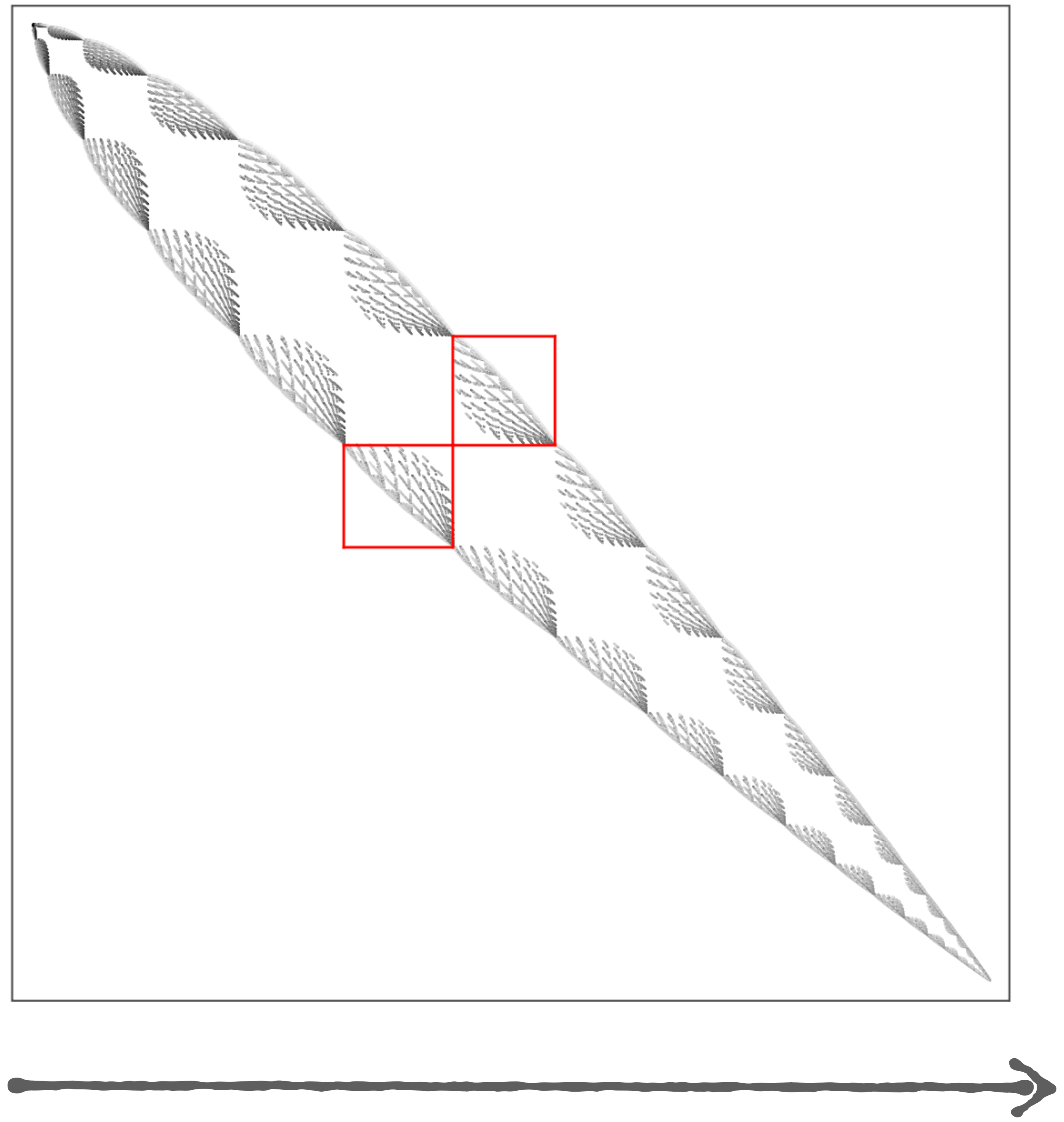


- Each group only interacts with its nearest neighbour

  $\rightarrow$ banded matrix

$$\text{size of the band} = \frac{\text{size of the biggest group}}{\text{total number of microstates}} \sim \frac{1}{\sqrt{N}}$$

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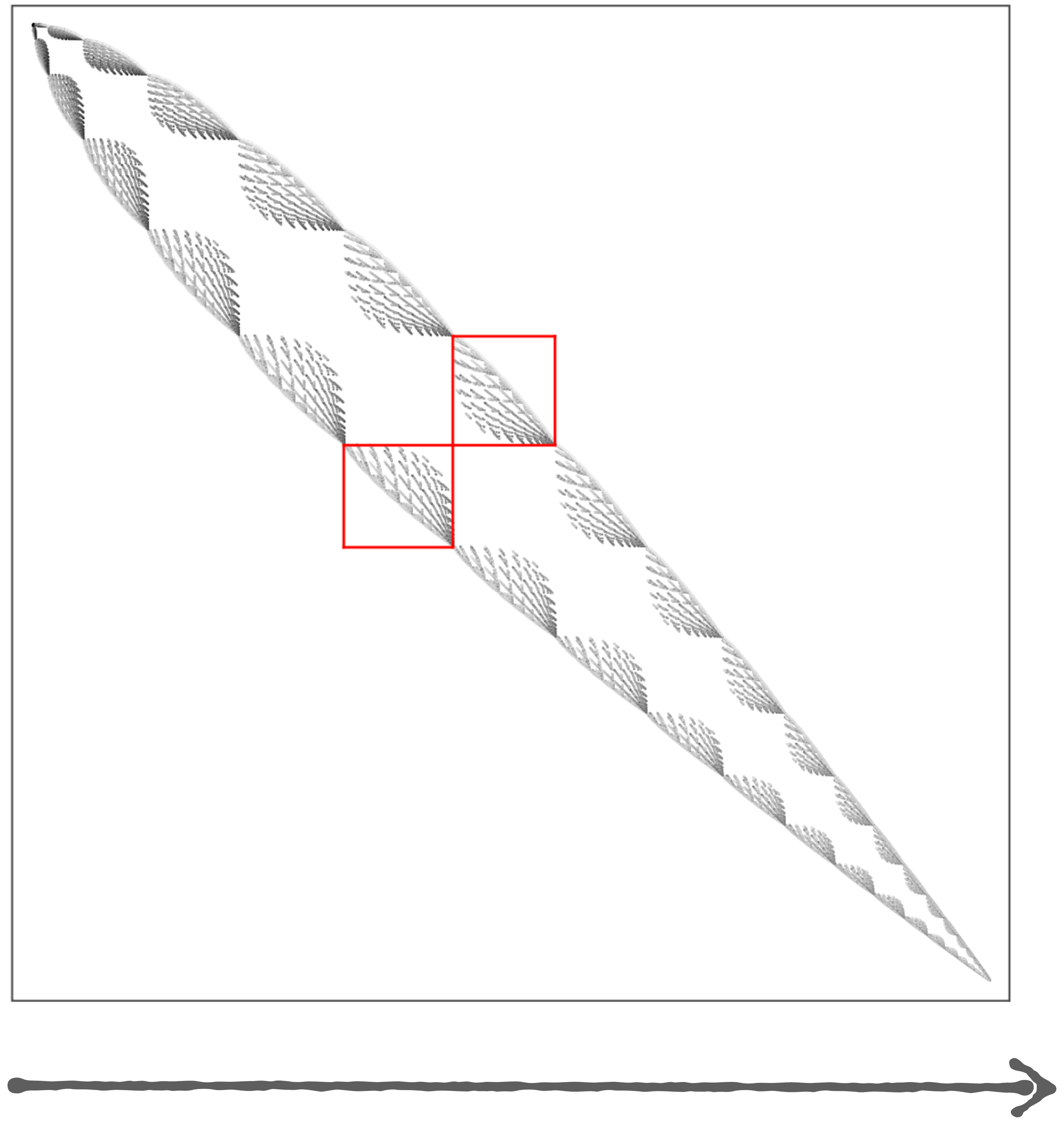

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- In line with: $\langle N_{\text{strands}} \rangle = \sharp \sqrt{N} \ln N$

  (most microstates are in the middle of [1,$N$])

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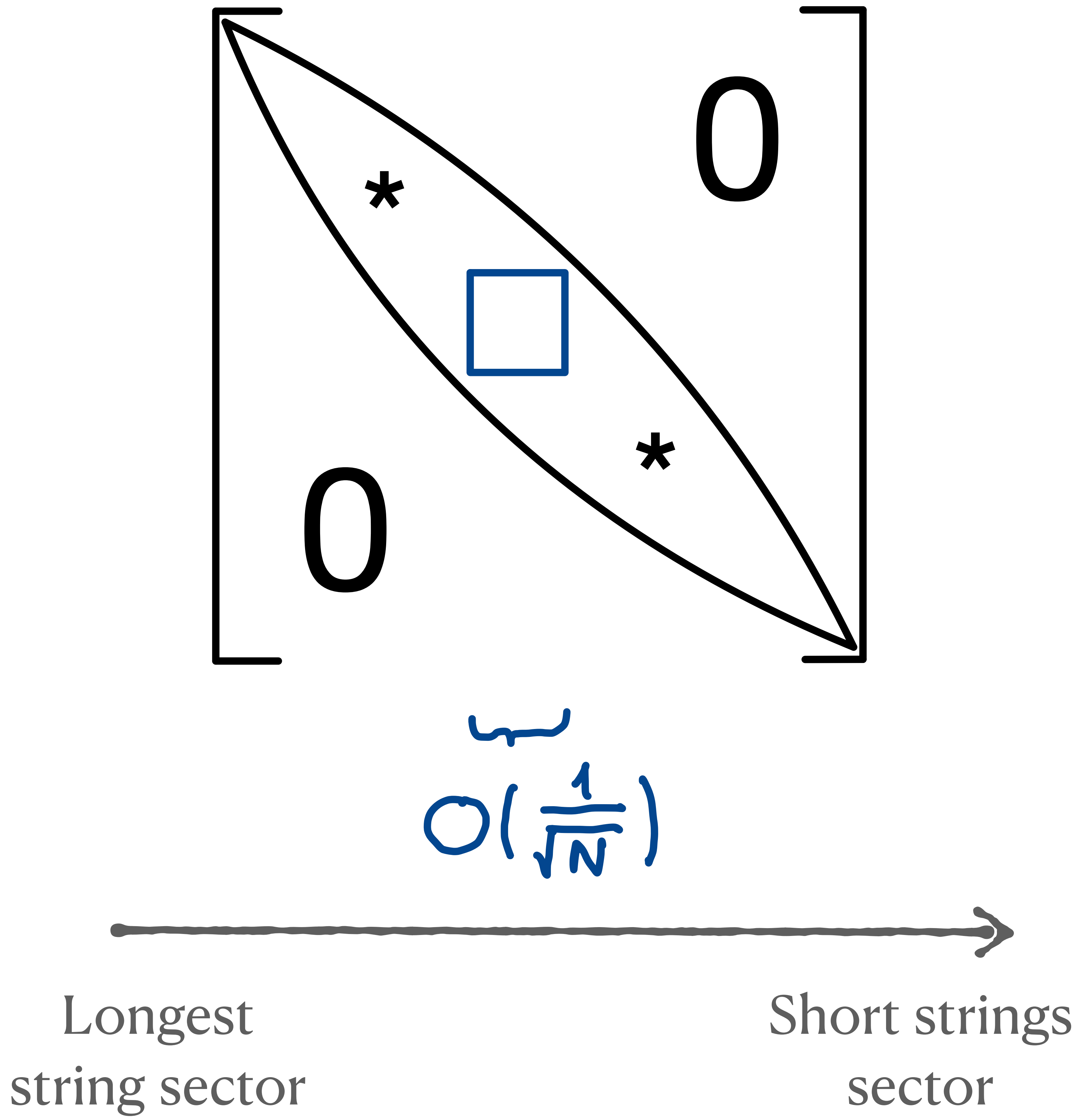

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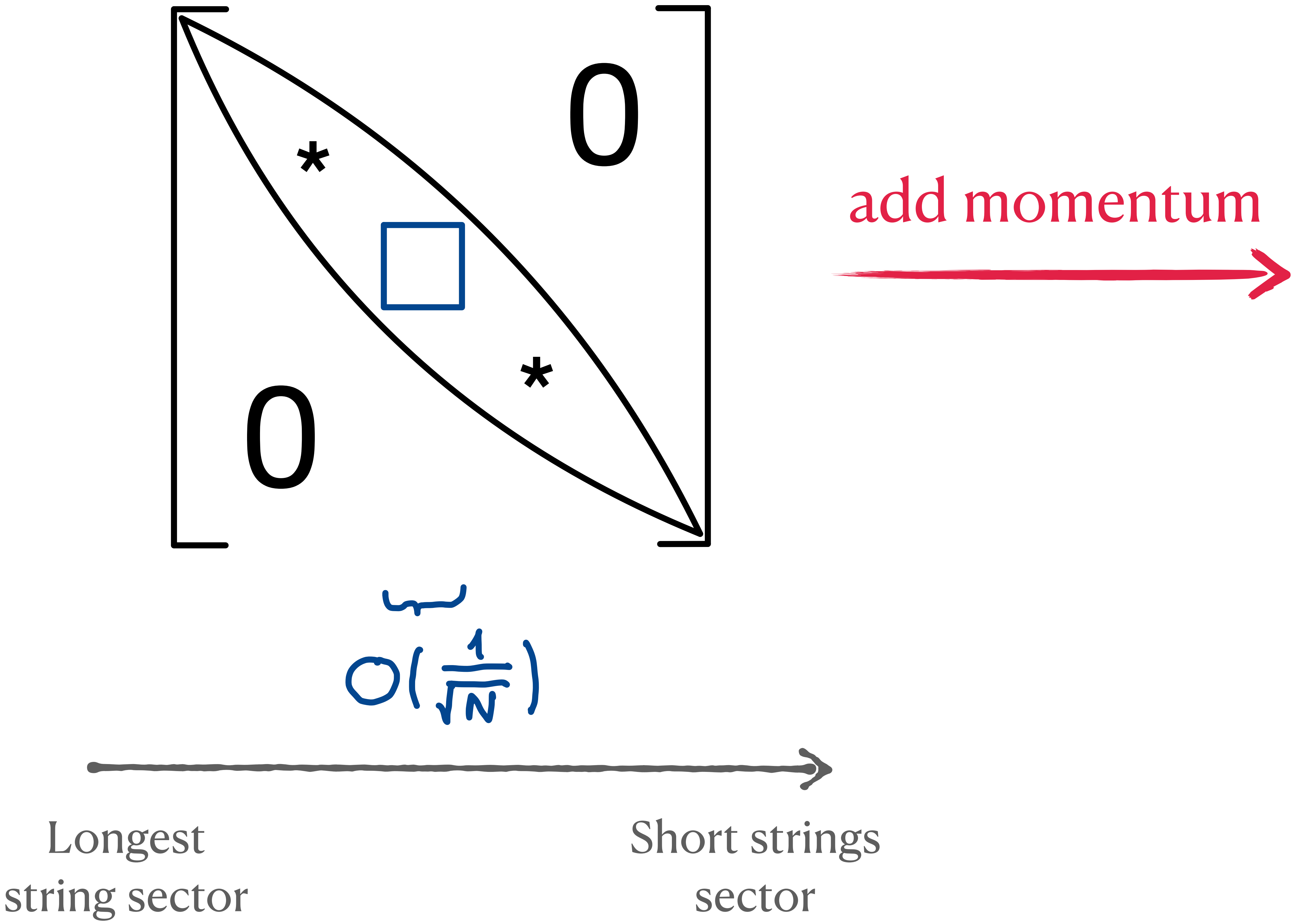

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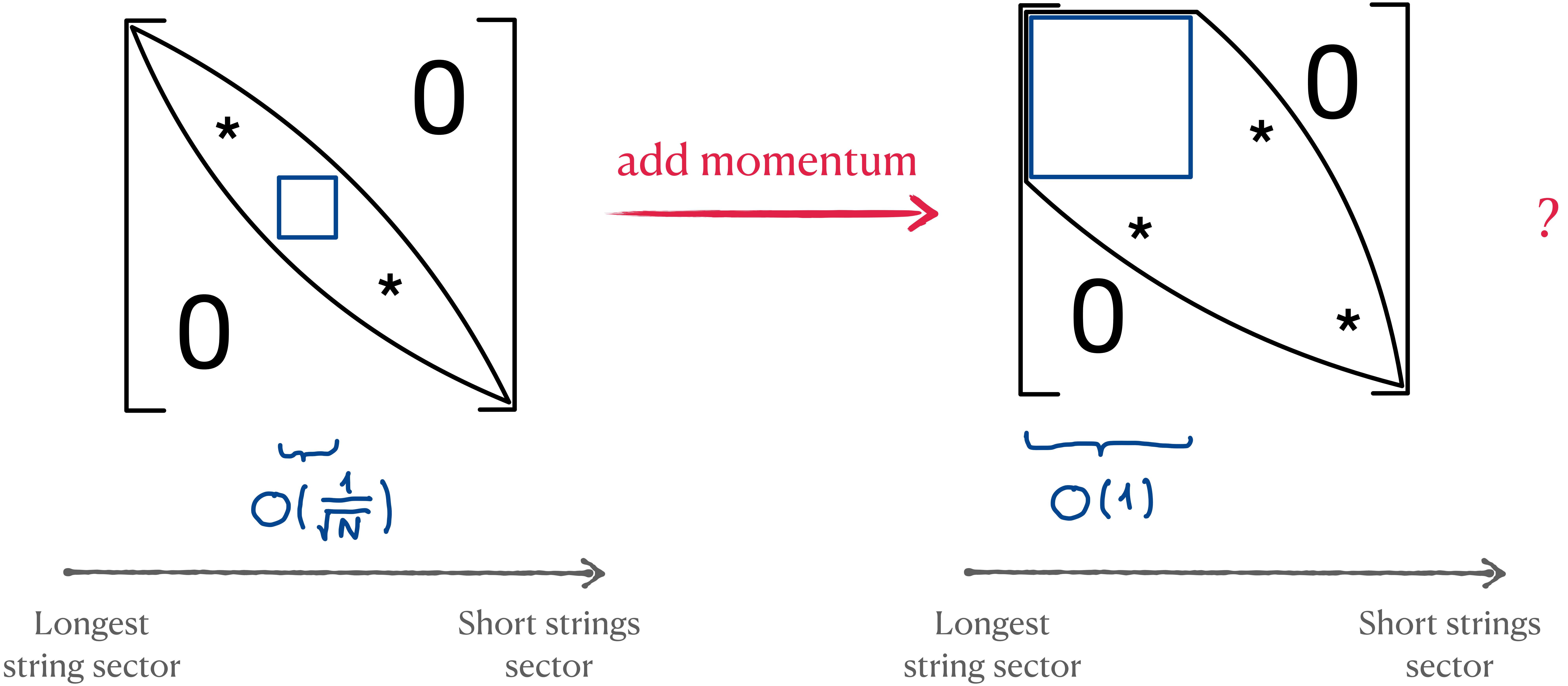

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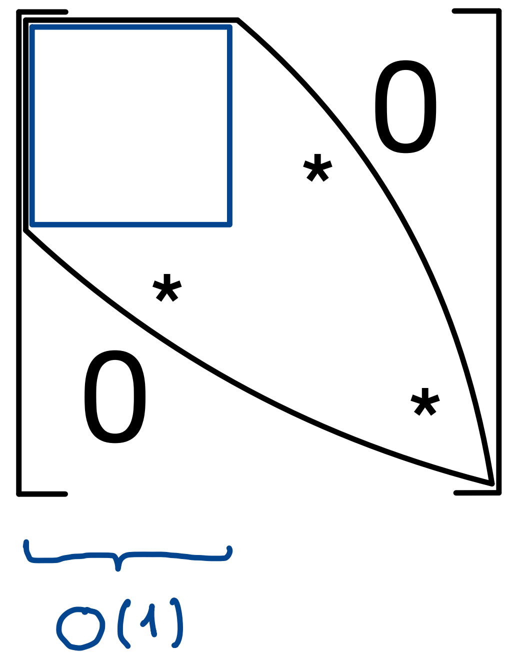
Possible to show that
$\langle N_{\text{strands}} \rangle \approx \mathcal{O}(1)$ ?



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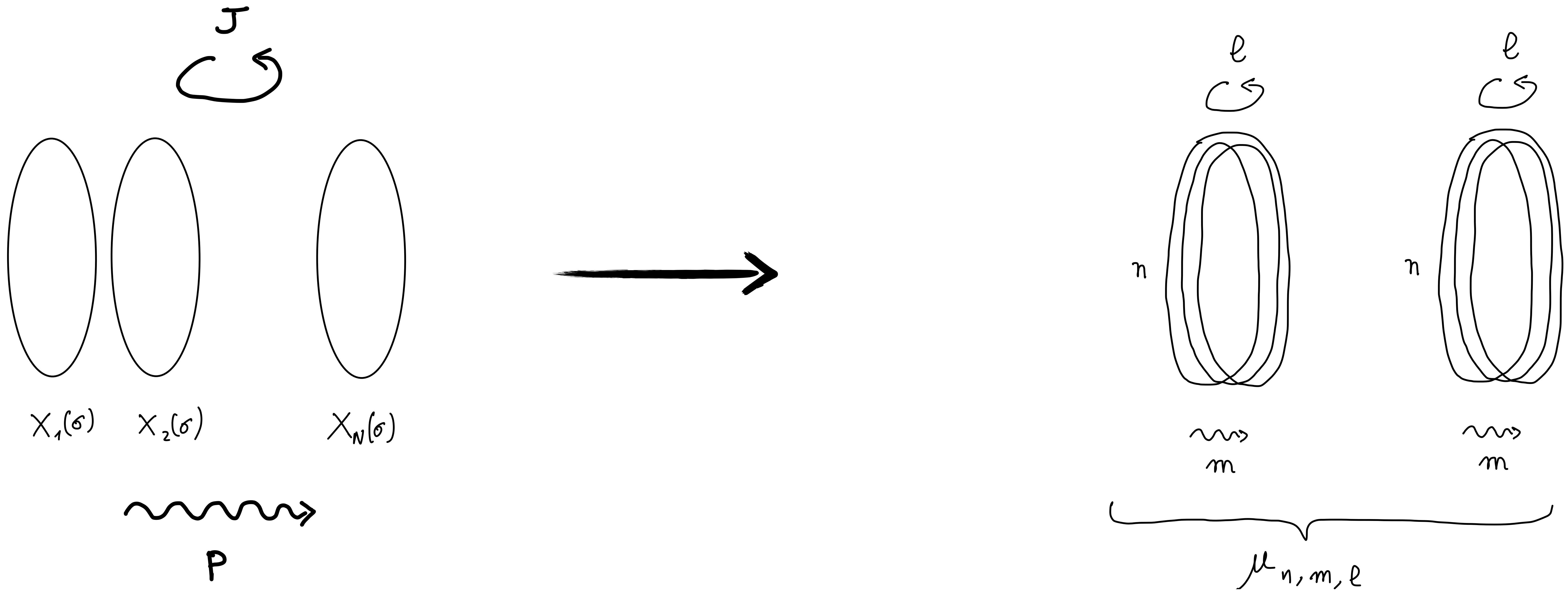

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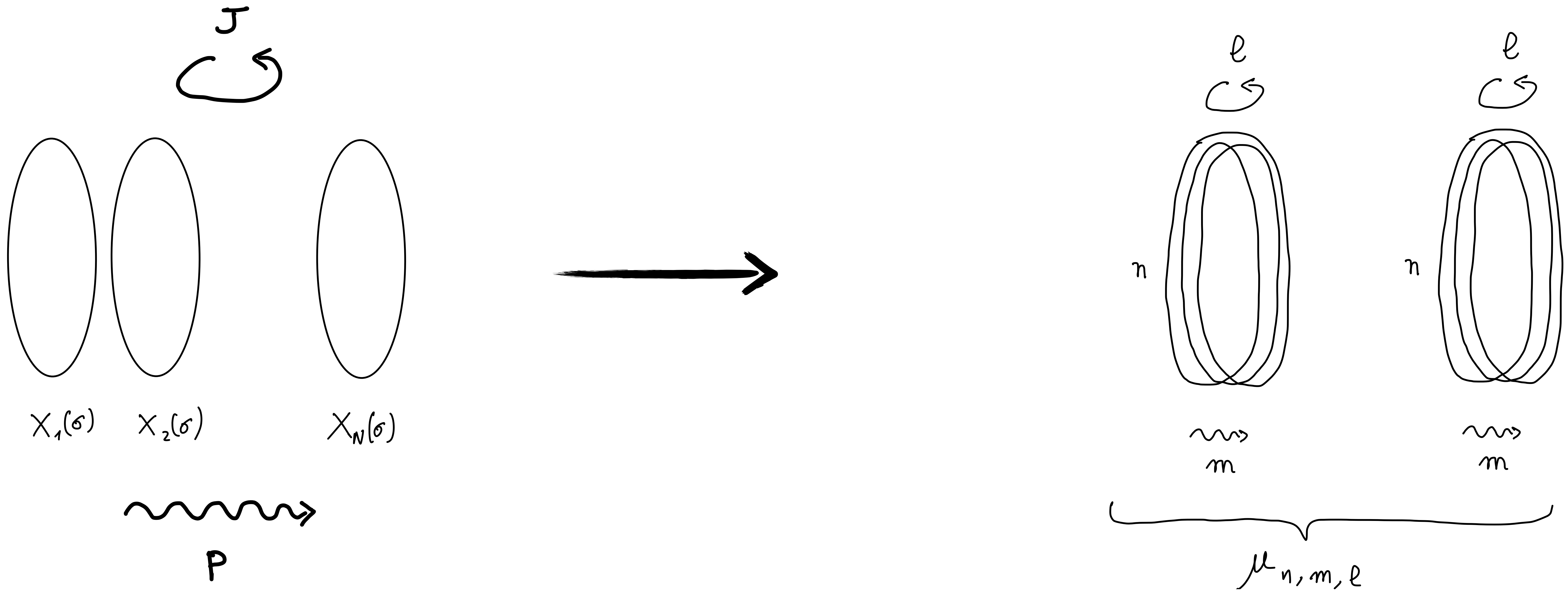

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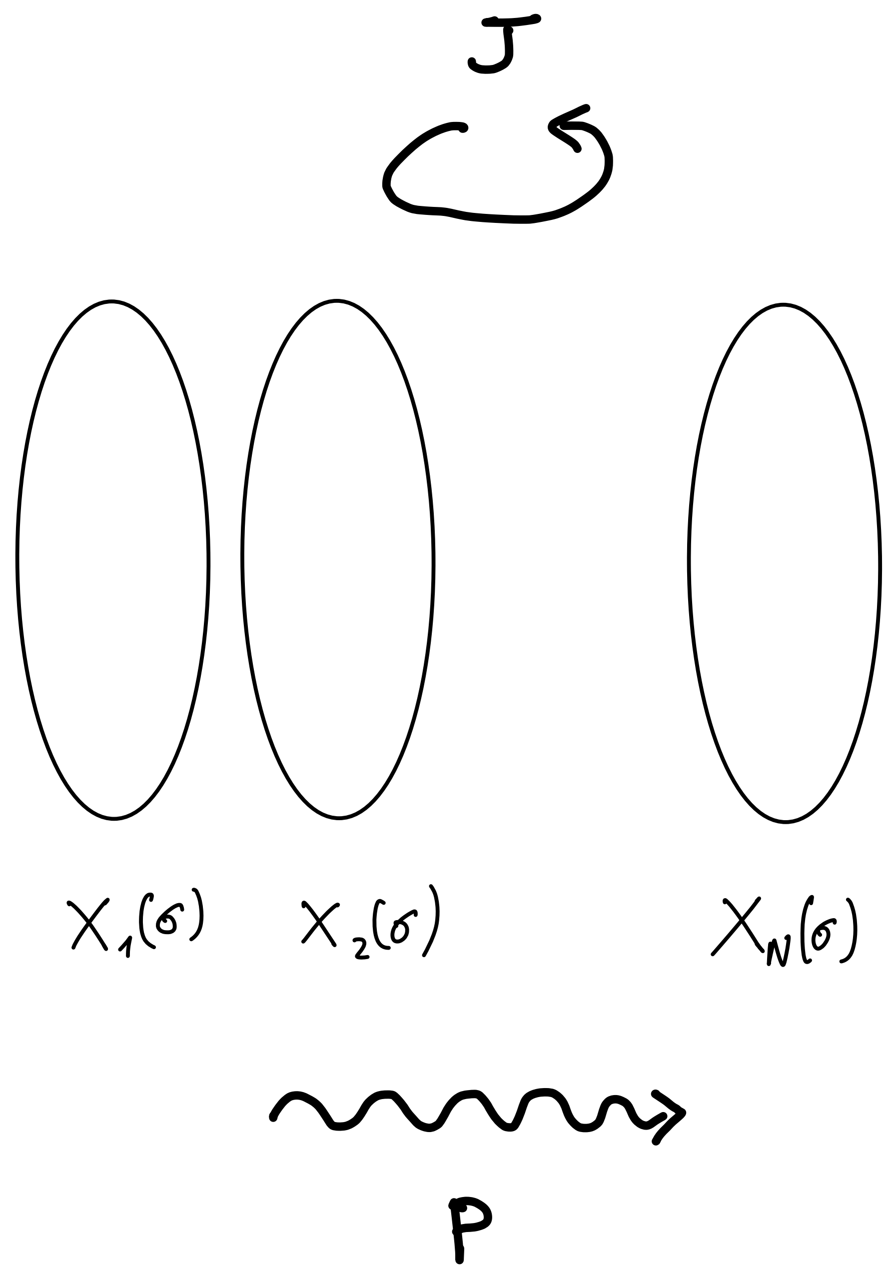


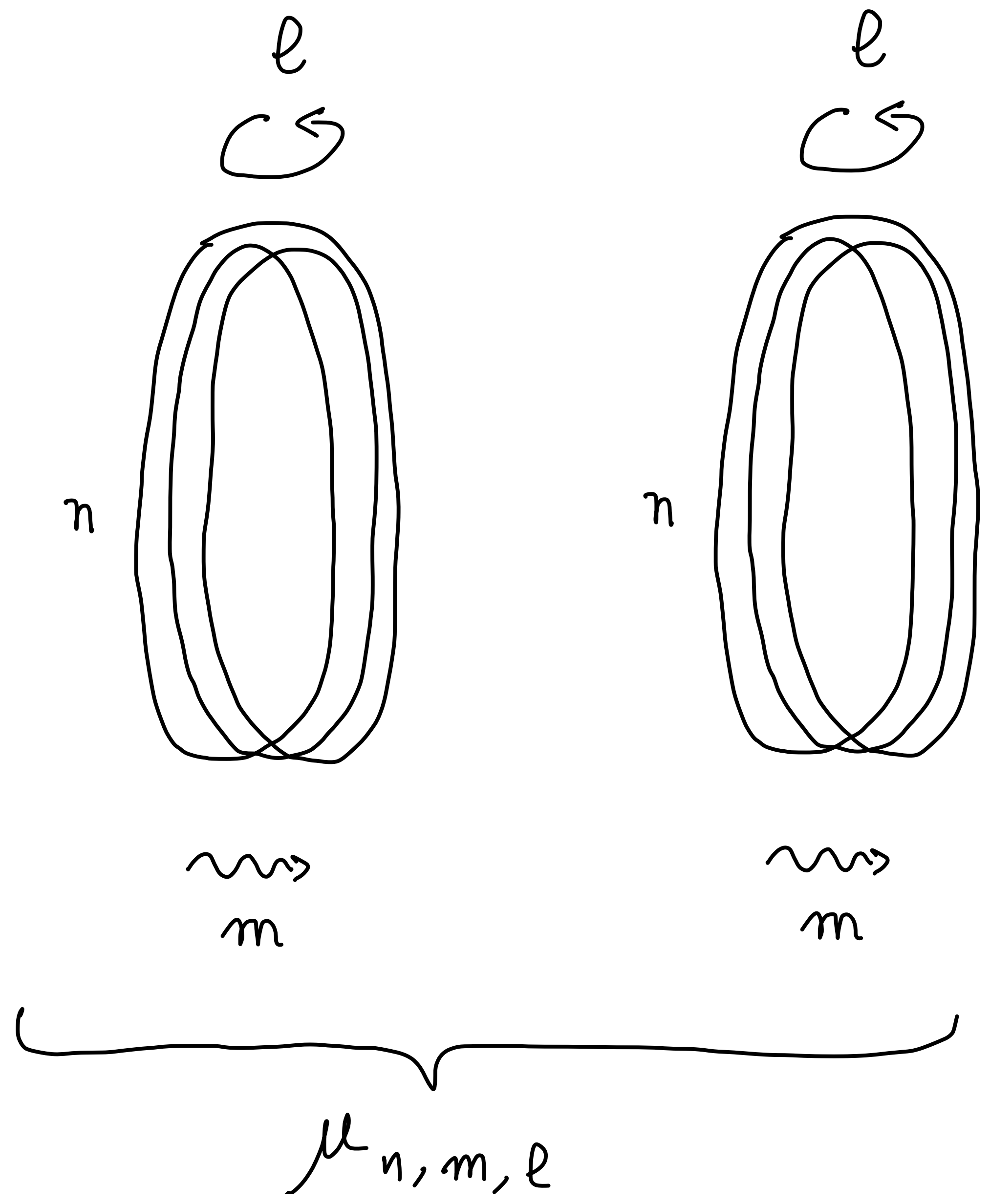

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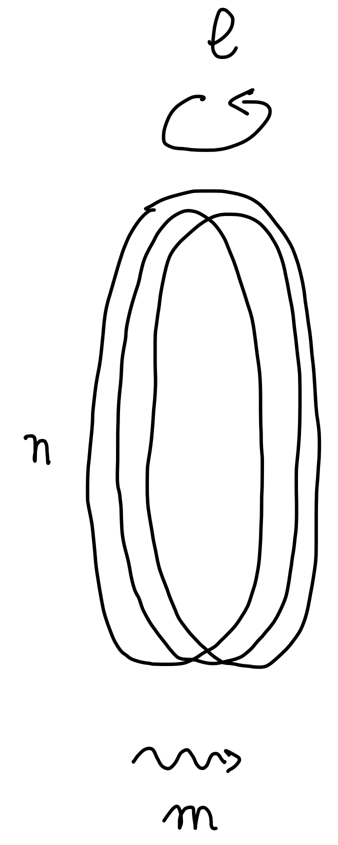
- How many microstates? Counting problem in microcanonical ensemble.
  - *Single strand*: degeneracy of $c(n, m, l)$
  - $\mu_{n,m,l}$ *of them*: $\begin{pmatrix} c(n,m,l) + \mu_{n,m,l} - 1 \\ \mu_{n,m,l} \end{pmatrix}$ (bosons), or $\begin{pmatrix} c(n,m,l) \\ \mu_{n,m,l} \end{pmatrix}$ (fermions).



# The counting problem

- Partition function

$$Z(p,q,y) = \prod_{n,m,l} \sum_{\mu_{n,m,l}\geq 0} (p^n q^m y^l)^{\mu_{n,m,l}} \binom{c(n,m,l)+\mu_{n,m,l}-1}{\mu_{n,m,l}} \qquad \text{(bosons)}$$

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BH degeneracy: distribute the charges while accounting for the symmetrisation

$$d(\vec{N}) = \sum_{\mu_1\vec{n_1}+\ldots+\mu_k\vec{n_k}=\vec{N}} \prod_{i=1}^{k} \binom{c(\vec{n_i})+\mu_i-1}{\mu_i}$$

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# The counting problem

- Partition function

$$Z(p,q,y) = \prod_{n,m,l} \sum_{\mu_{n,m,l}\geq 0} (p^n q^m y^l)^{\mu_{n,m,l}} \binom{c(n,m,l)+\mu_{n,m,l}-1}{\mu_{n,m,l}} \qquad \text{(bosons)}$$

Compare

$$N = \sum_{n,m,l} n\,\mu_{n,m,l}$$

$$P = \sum_{n,m,l} m\,\mu_{n,m,l}$$

$$J = \sum_{n,m,l} l\,\mu_{n,m,l}$$

$$Z(p,q,y) = \sum_{N,P,J} d(N,P,J)\, p^N q^P y^J$$

BH degeneracy: distribute the charges while accounting for the symmetrisation

$$d(\vec{N}) = \sum_{\mu_1\vec{n_1}+\ldots+\mu_k\vec{n_k}=\vec{N}} \prod_{i=1}^{k} \binom{c(\vec{n_i})+\mu_i-1}{\mu_i}$$

Track down the number of strands

$$d(\vec{N}, s) = \sum_{\substack{\mu_1\vec{n_1}+\ldots+\mu_k\vec{n_k}=\vec{N} \\ \mu_1+\ldots+\mu_k=s}} \prod_{i=1}^{k} \binom{c(\vec{n_i})+\mu_i-1}{\mu_i}$$

# In the grand-canonical ensemble

- $(p, q, y)$ as fugacities: $p = e^{-\beta_N}$, $q = e^{-\beta_P}$, $y = e^{-\beta_J}$
- System thermalises and

$$\langle \mu_{n,m,l} \rangle = \underbrace{f_{\text{stat}}}_{\text{occupation factor}}(n, m, l)\, \underbrace{\rho}_{\text{density of states}}(n, m, l) = \frac{c(n, m, l)}{e^{\beta_N n + \beta_P m + \beta_J l} \mp 1}$$

- Unknown: temperatures.

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- Mean number of strands

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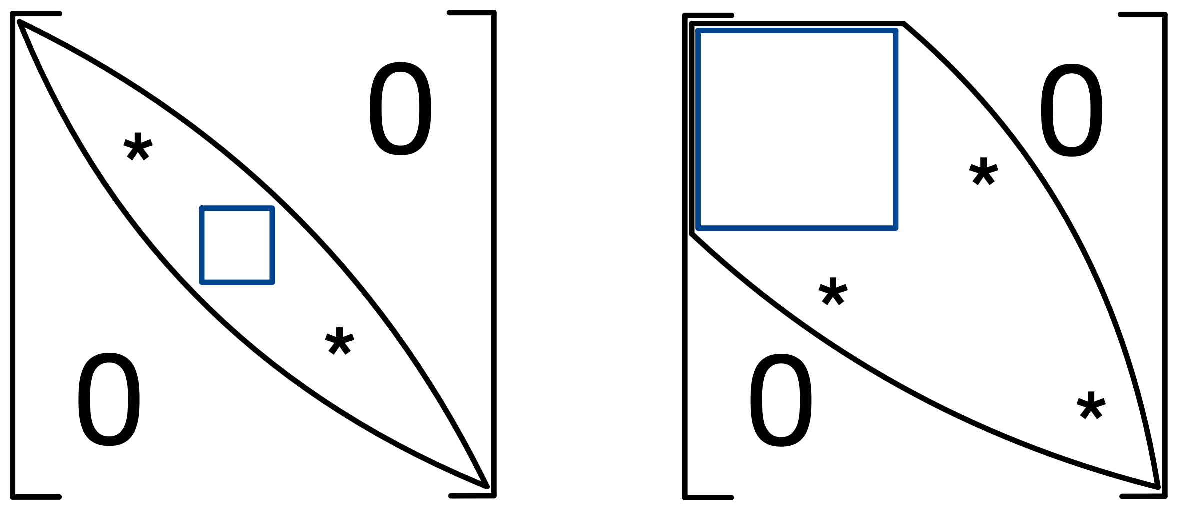

# Partition function for the index

- Play the same game but with the index as the partition function:

[See Samir's talk this morning]

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[Dijkgraaf, (Moore), Verlinde, Verlinde '96]
[Castro, Murthy '08]

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  $\rightarrow$ Attractor equations for the axio-dilaton!

[Cardoso, de Wit, Käppeli, Mohaupt '04]

Thermalisation of microstates in grand-canonical ensemble with *index* as partition function $\longrightarrow$ Attractor equations

# Partition function for the degeneracy

Thermalisation of microstates in grand-canonical ensemble
with *degeneracy* as partition function

→

Statistics of BPS microstates into groups

↓

Potential of microstates to exhibit strong chaos

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WIP

Statistics of BPS microstates into groups

$\downarrow$

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- *Caveat:* The analysis is at the free orbifold point; the strength of chaos can differ at $g_s > 0$.

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WIP

- *Caveat:* The analysis is at the free orbifold point; the strength of chaos can differ at $g_s > 0$.

How does quantum chaos emerge out of supersymmetric BH microstates?

→ Does strong chaos only emerge as a gravitational effect?

→ If yes, how far away from strong chaos is the free orbifold point?

# Conclusion

Strong vs. weak chaos for D1-D5-P

Strong vs. weak chaos
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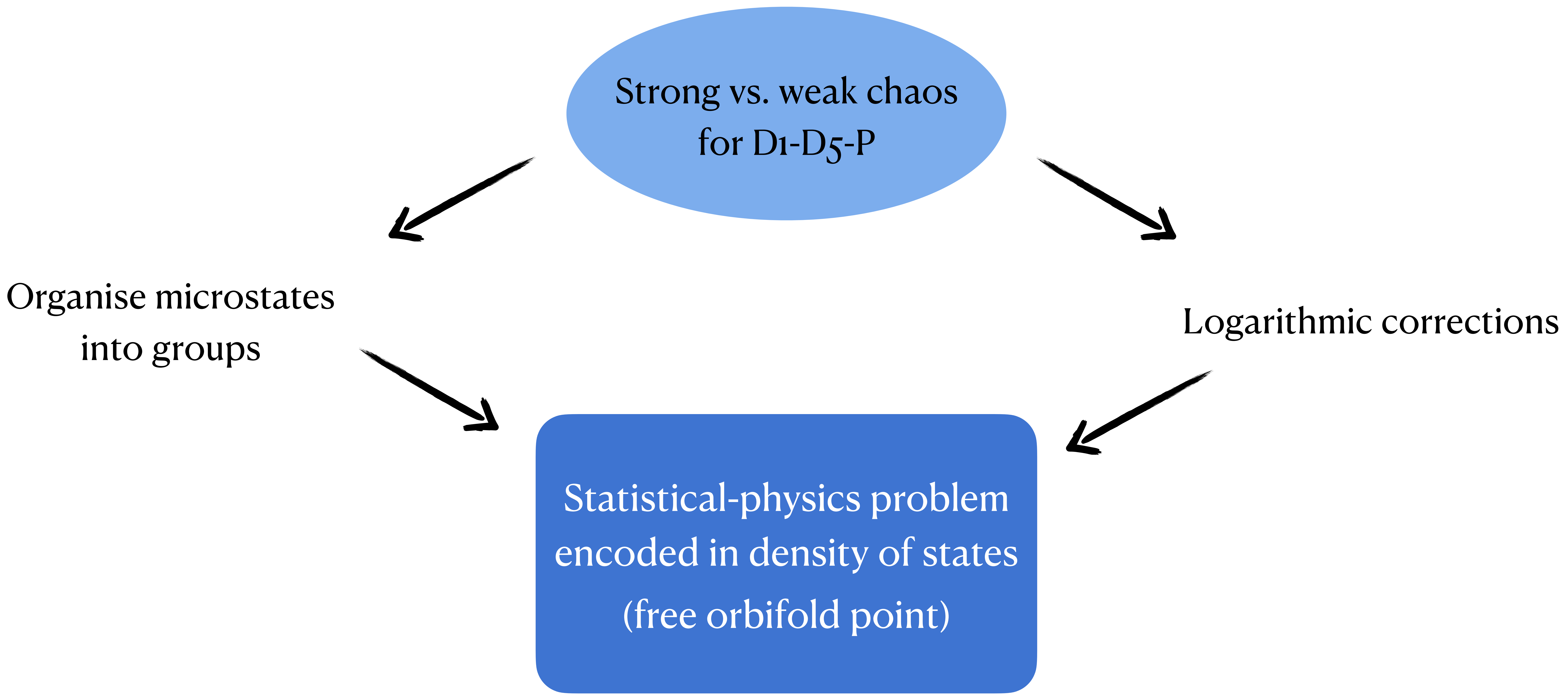
Organise microstates
into groups

Logarithmic corrections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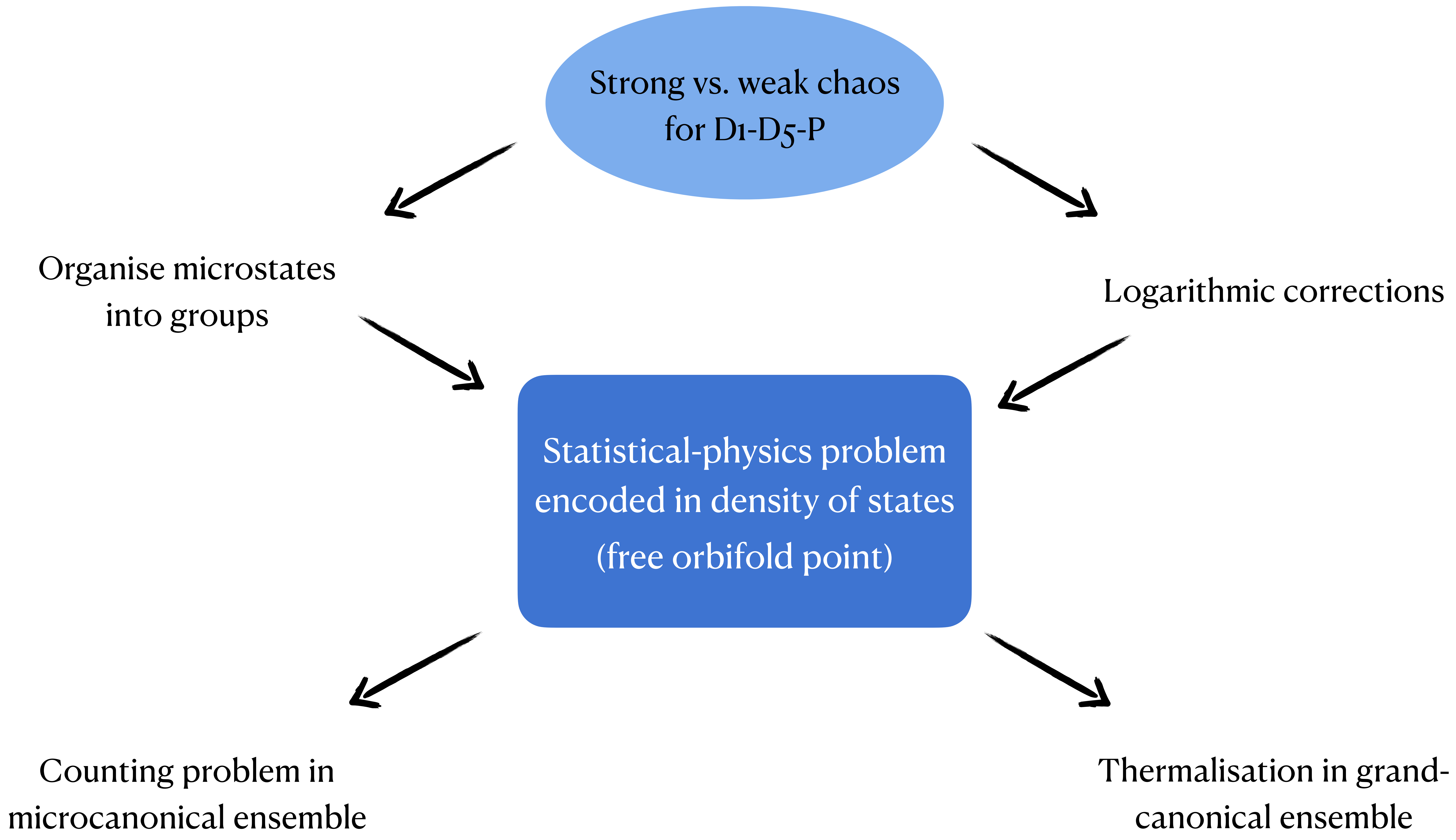

Strong vs. weak chaos for D1-D5-P
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Counting problem in microcanonical ensemble
Thermalisation in grand-canonical ensemble

# Outlook

The Statistics of BPS Chaos

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→ statistics about the states that become non-BPS when one tunes $g_s \neq 0$

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→ formulate fortuity with a symmetry between $(N, P)$?

[See also Siyul's poster]

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The Statistics of BPS Chaos

(speculation)

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[See also Siyul's poster]

***Swampland***

→ what if the principle behind the Distance Conjecture was strong chaos?

[See also Irene's talk on Thursday]

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*Thank you!*