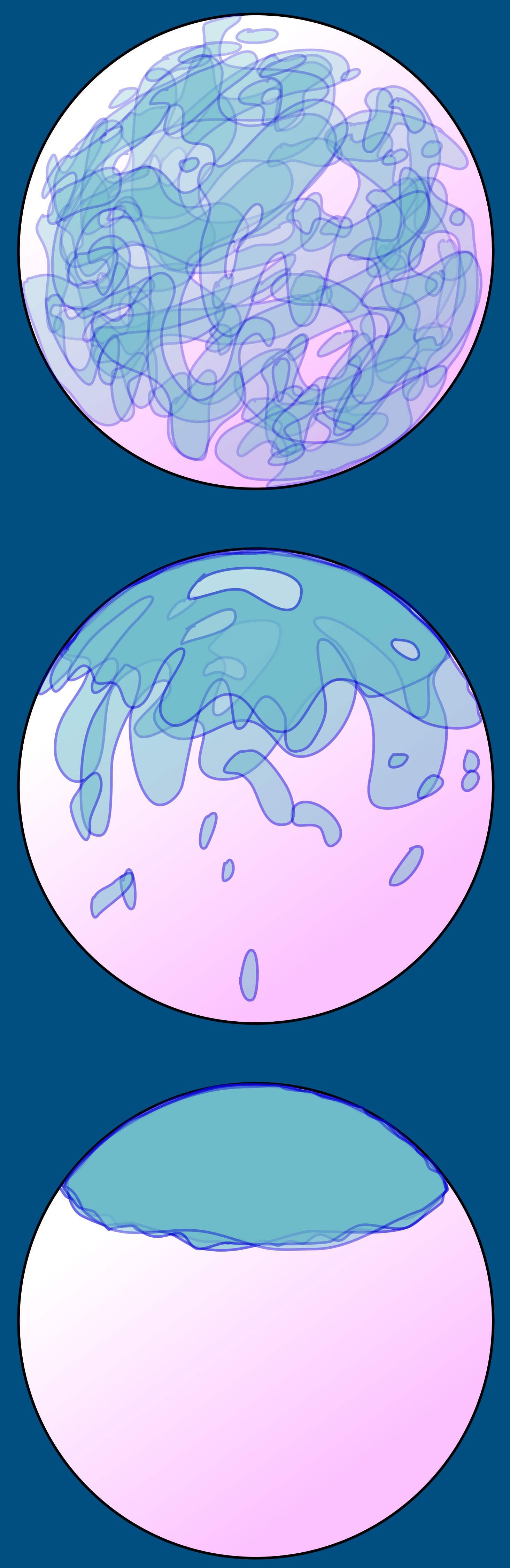
Jackson R. Fliss

UNIVERSITY OF CAMBRIDGE

Minimal areas from entangled matrices

2408.05274

w/ Alex Frenkel, Sean Hartnoll, and Ronak Soni



Eurostrings 2025

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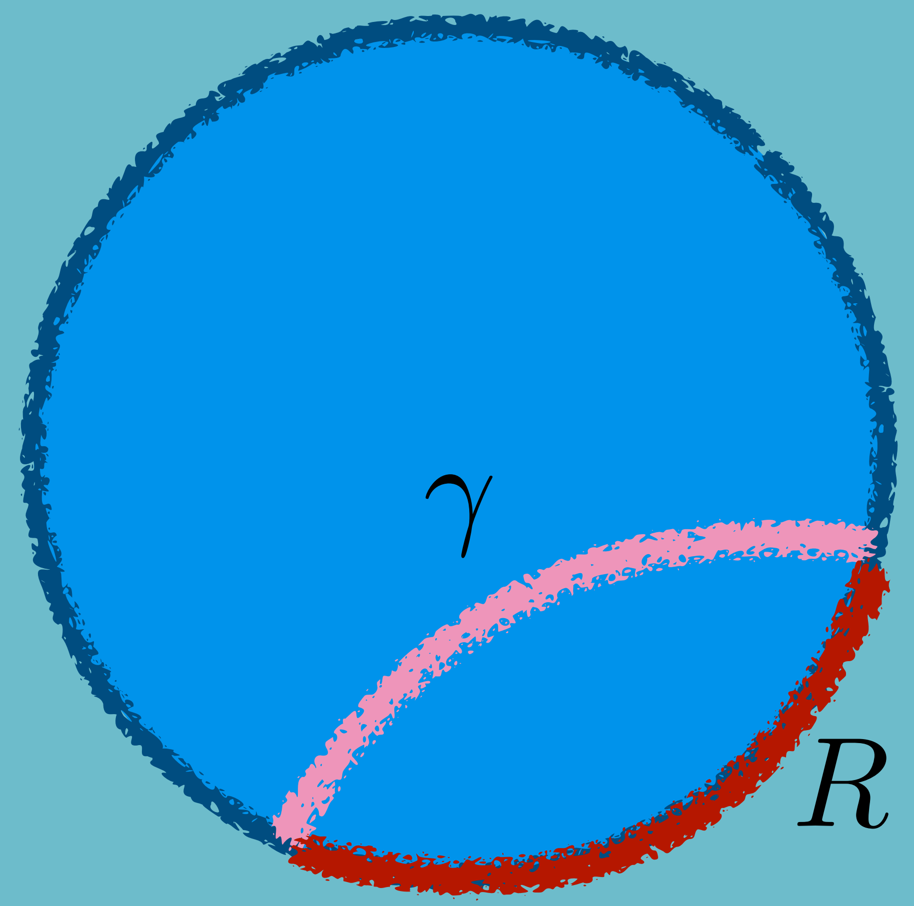

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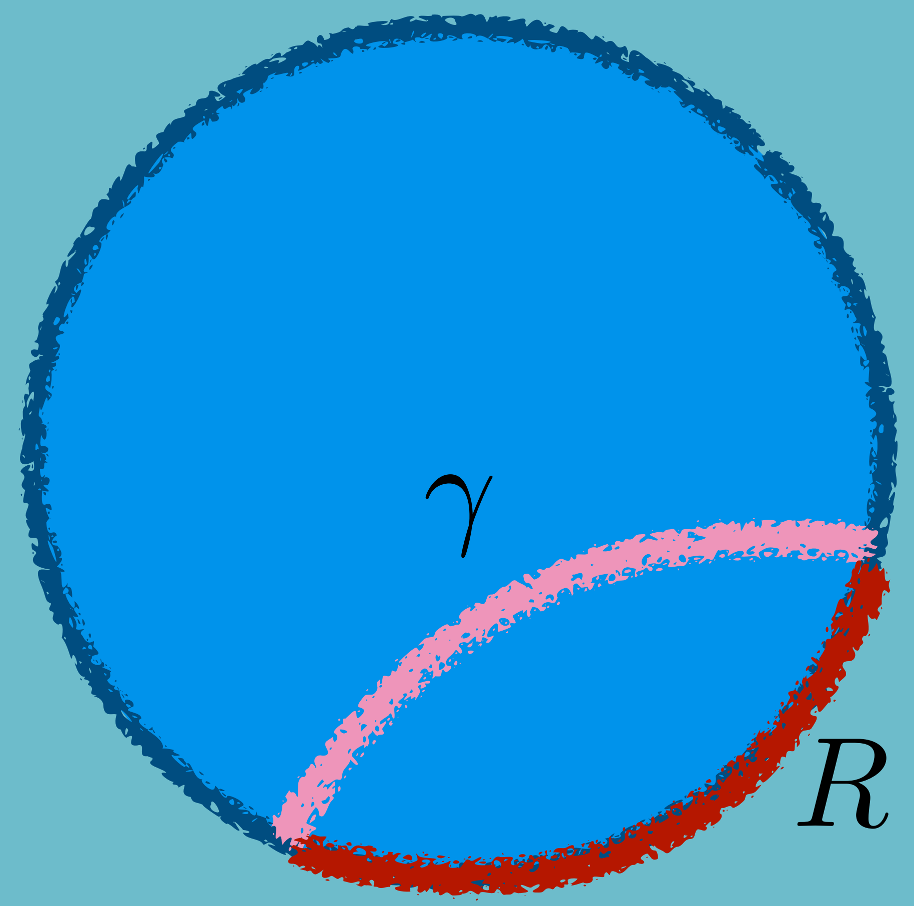

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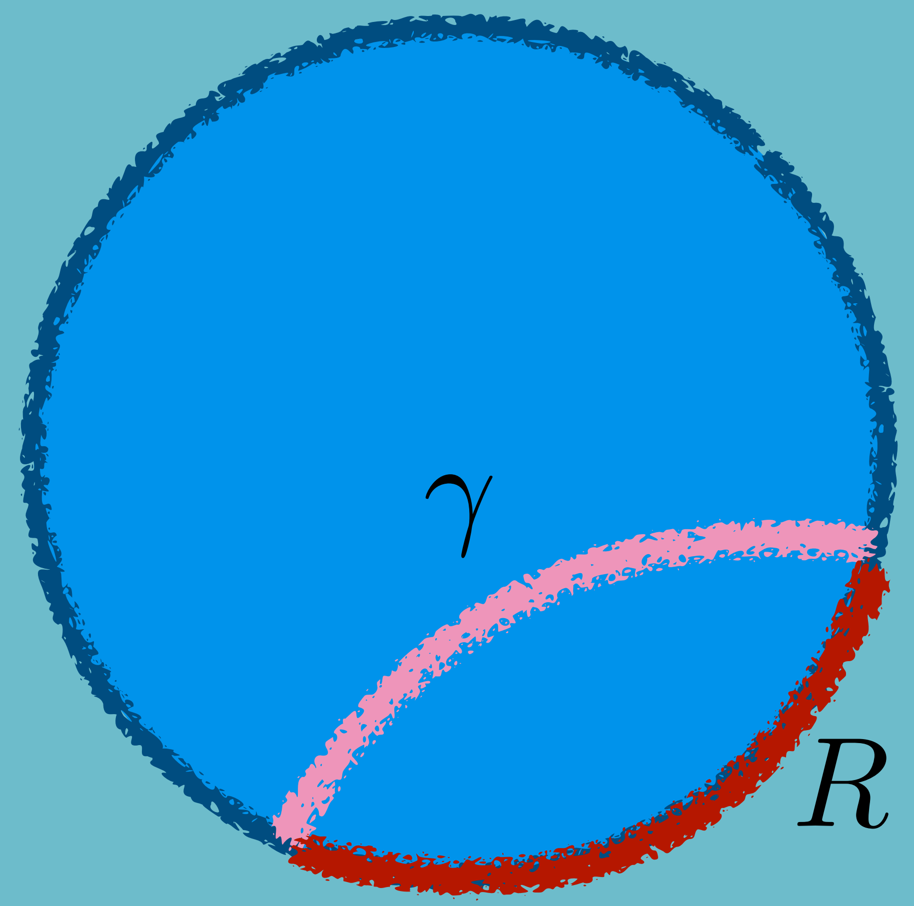

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Look "under the hood" of this mechanism…

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Consequence of diffeomorphism invariance, subject to some gauge-invariant condition

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This talk: how these features arise in matrix quantum mechanics

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(c=1 matrix model)

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Not (necessarily) quantum gravity or holography.
Regardless, interesting, tractable, models displaying similar key features

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- Non-commutative sphere of radius $\dfrac{\nu N}{2}$

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- At large *N* acts on membrane theory as ***volume preserving diffeomorphisms.***
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The goal: VPD-invariant entanglement entropies

The result:

$$S_{EE} = \min_{\substack{\Sigma, \\ |\Sigma| \text{ fixed}}} \frac{|\partial\Sigma|}{g_{\text{eff}}} \log\left(g_{\text{eff}} N \frac{|\Sigma|}{|\partial\Sigma|}\right)$$

The result:

Area as state counting

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Saddle-point in average over VPDs that change the subsystem

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Subsystems and gauge (in)variance

- Initial partition: matrix elements in a *MxM* block in a fixed basis

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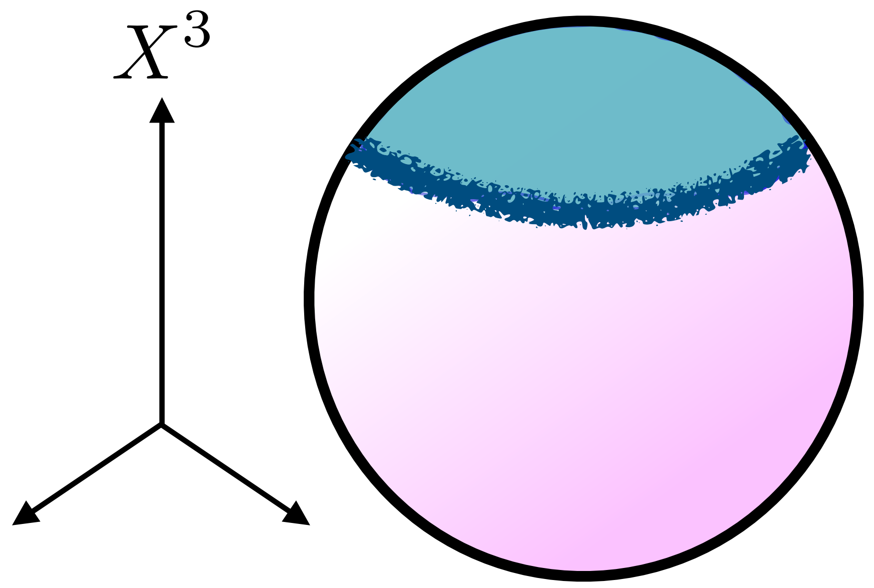

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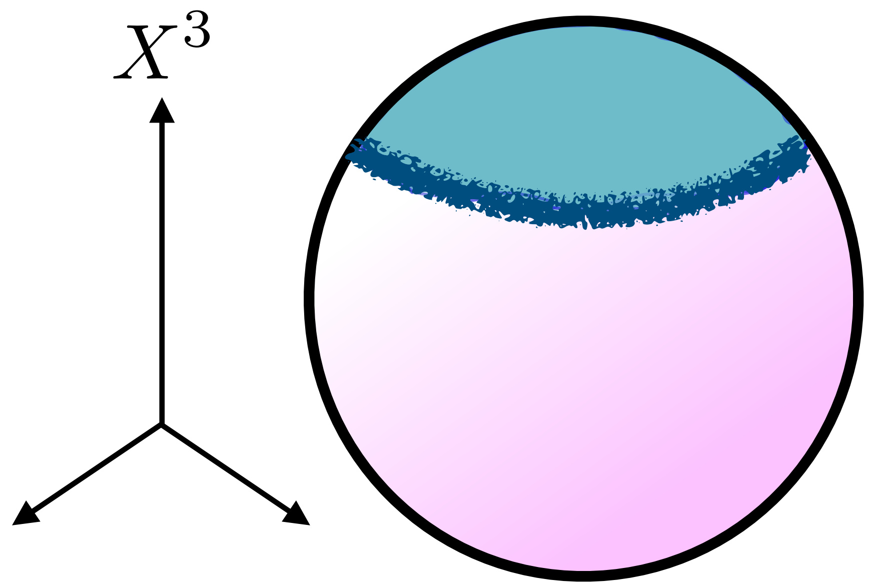

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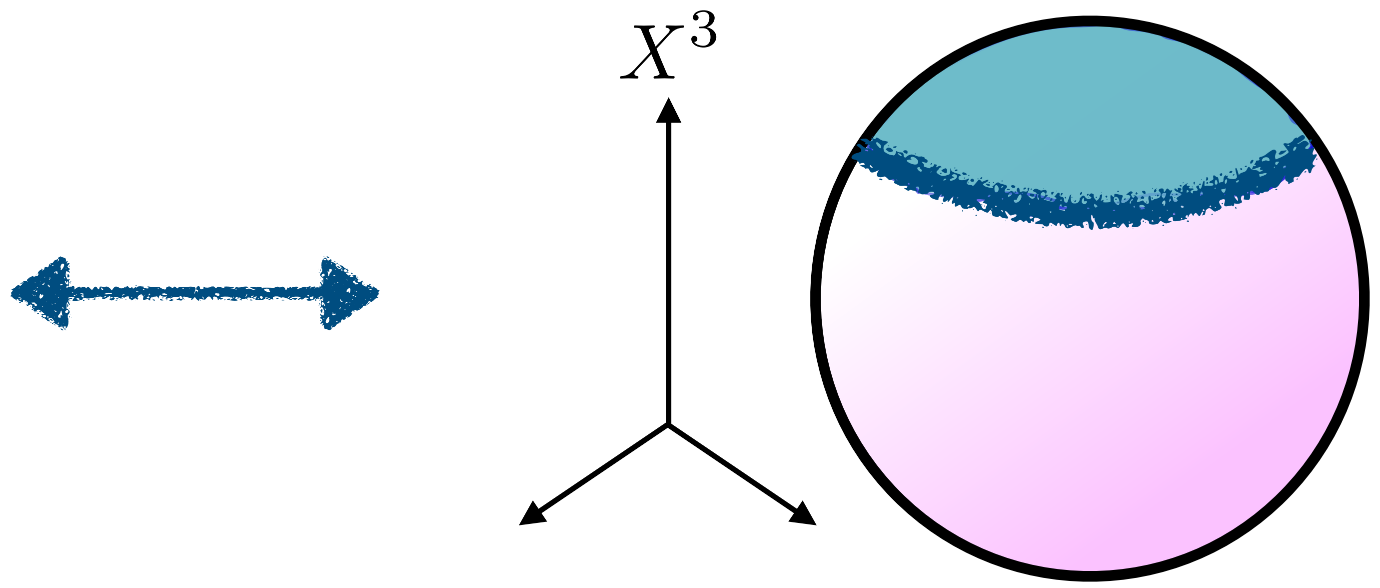

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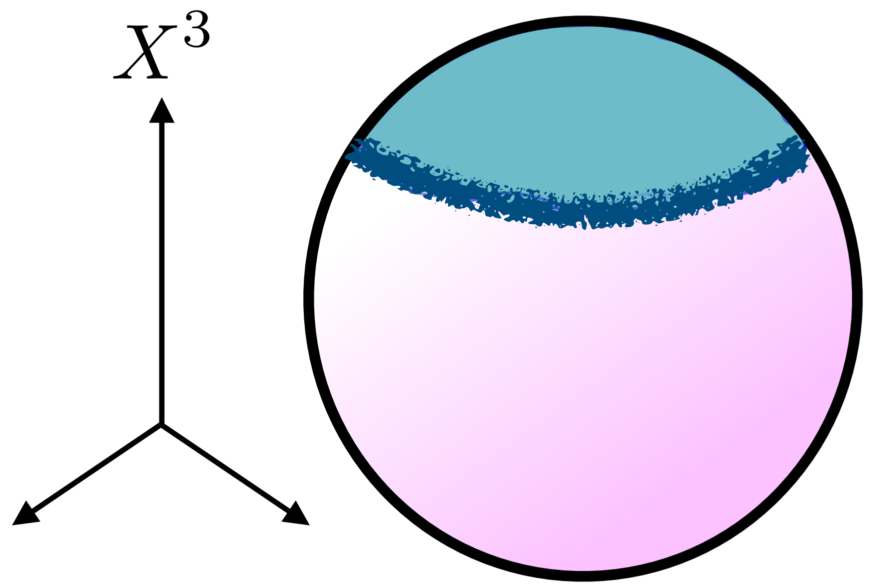

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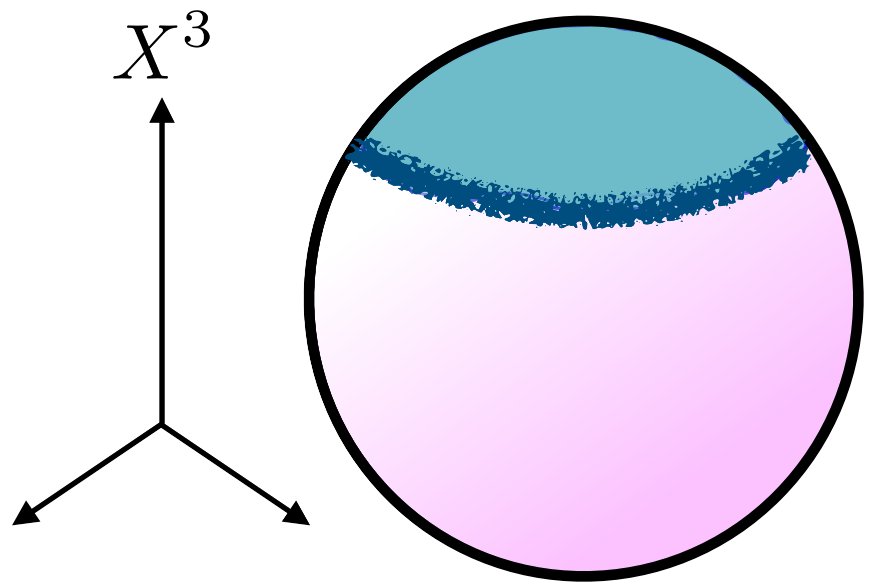

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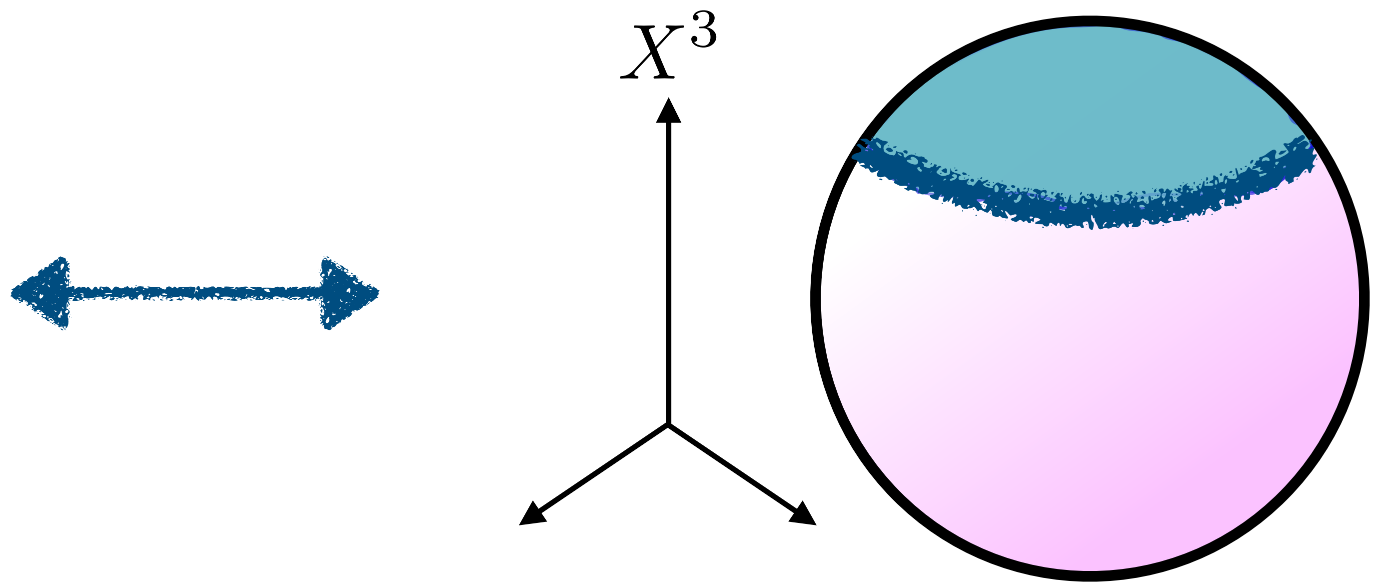

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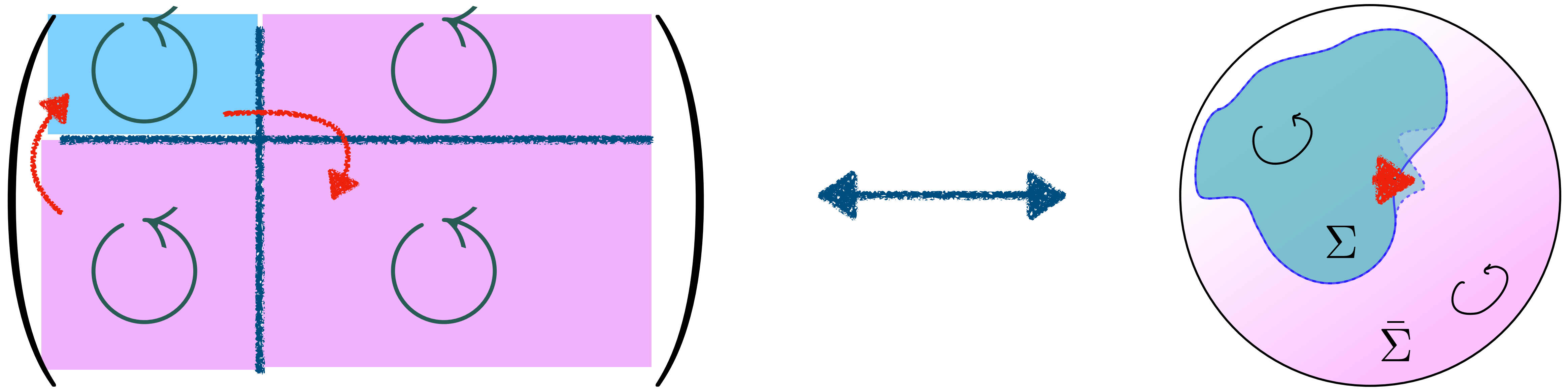

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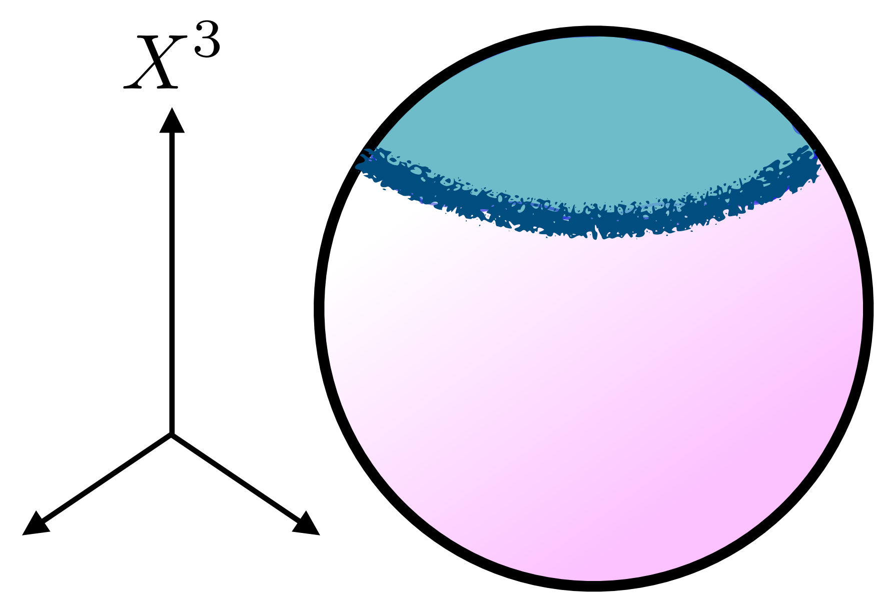

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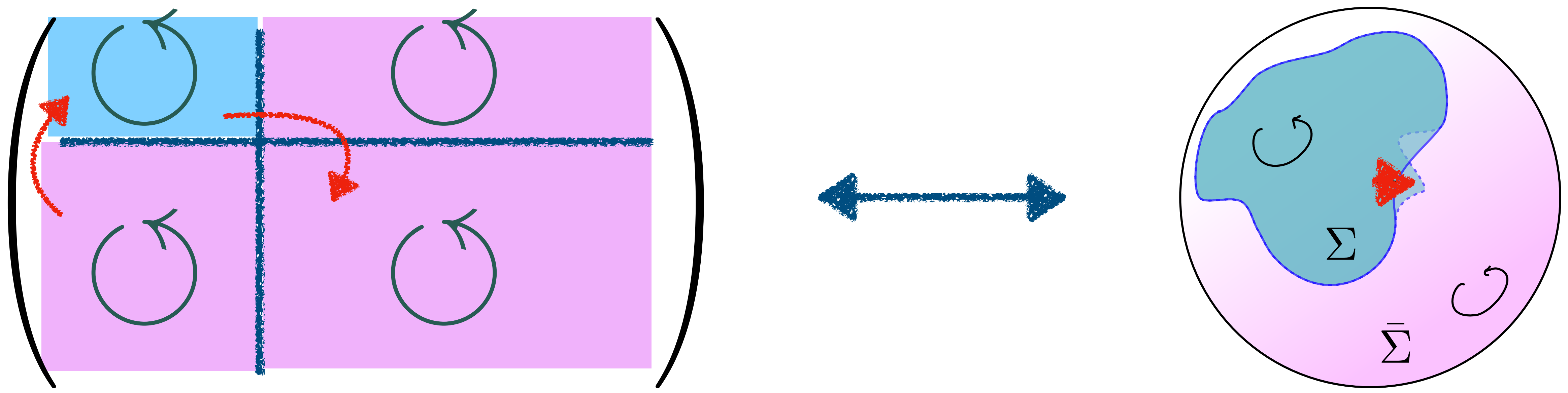
$$U(M) \times U(N-M)$$



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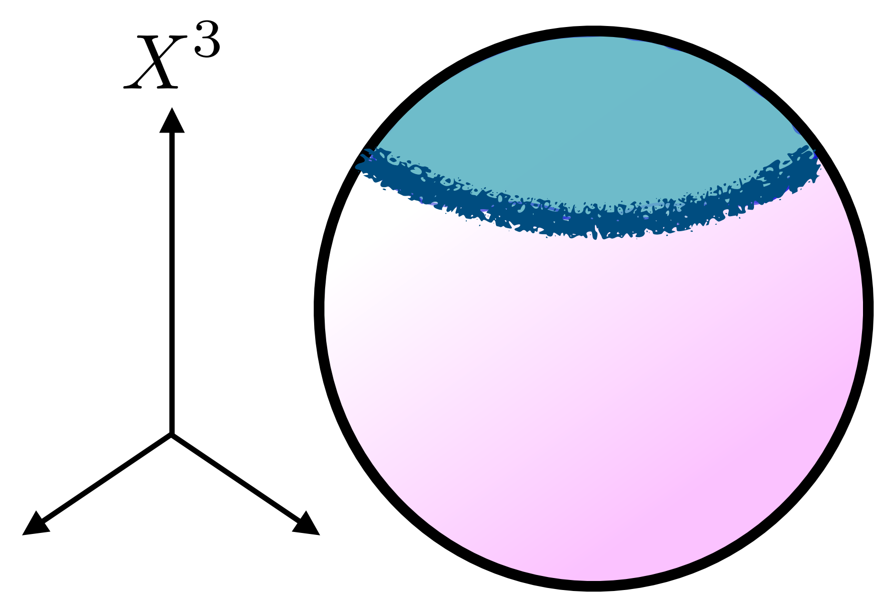

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$$U(M) \times U(N-M)$$

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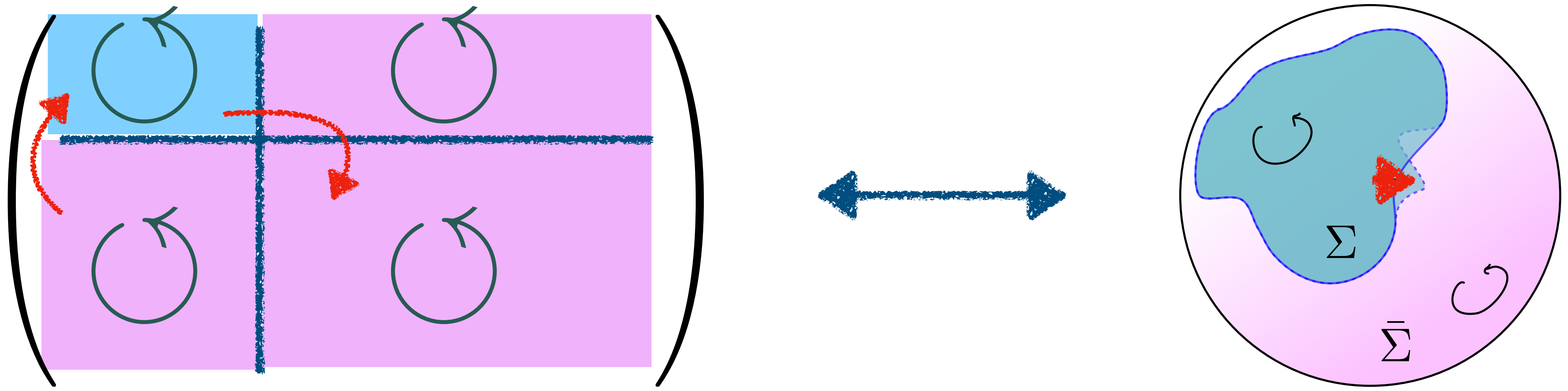

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- ***Gauge-invariant question:***

what is the entropy of *any* *MxM* matrix block?	$\longleftrightarrow$	**what is the entropy of *any* region of fixed volume?**

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- Start with a gauge-fixed state $|\psi_{\rm gf}\rangle \in \mathcal{H}_{\rm ext}$ and average over gauge orbit

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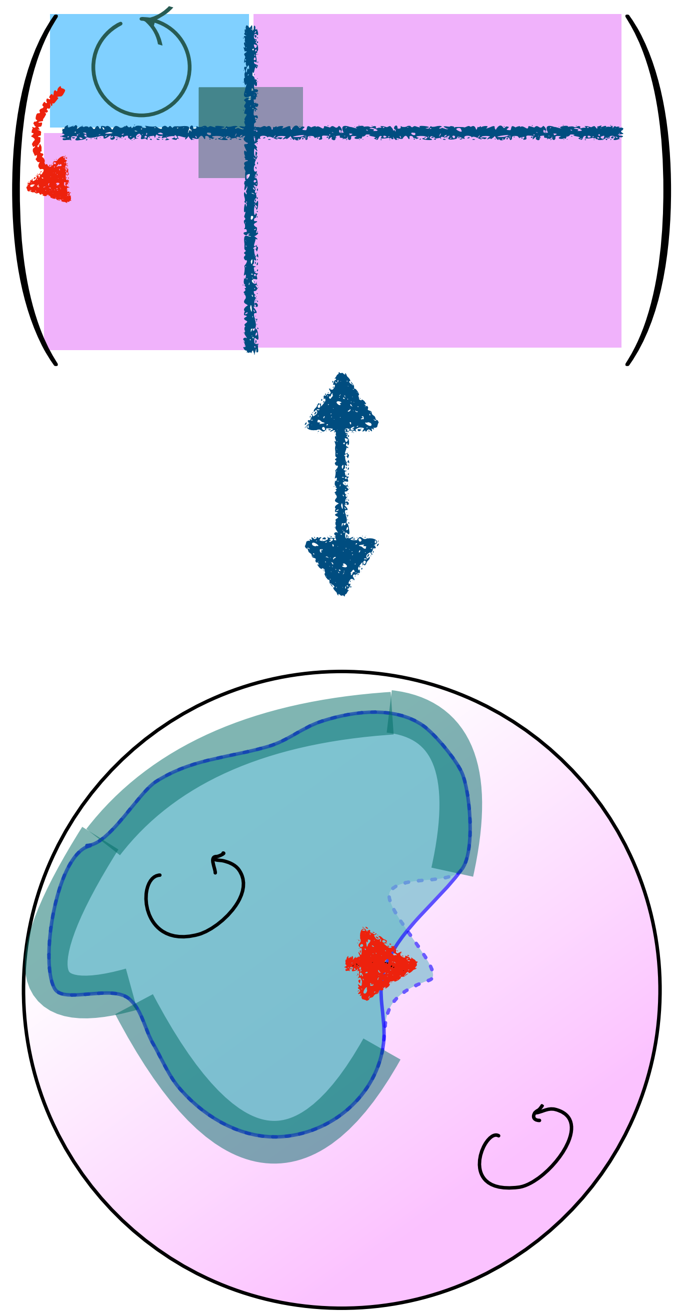

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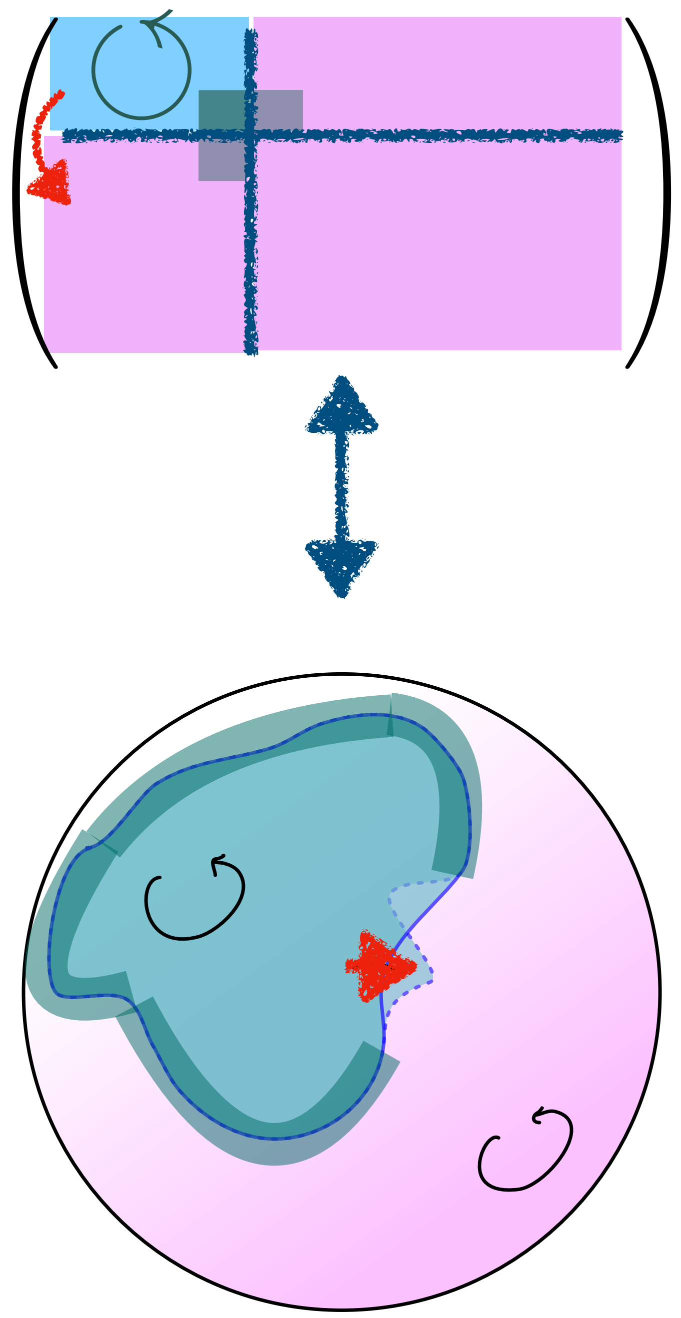

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    - Maximally entangled edge modes in irreps μ_Σ of *U(M)*.
- F : no analogue in local gauge theory
    - $V \in F$ leads to different subsystems being traced out.

Area from the $U(M)$ charge

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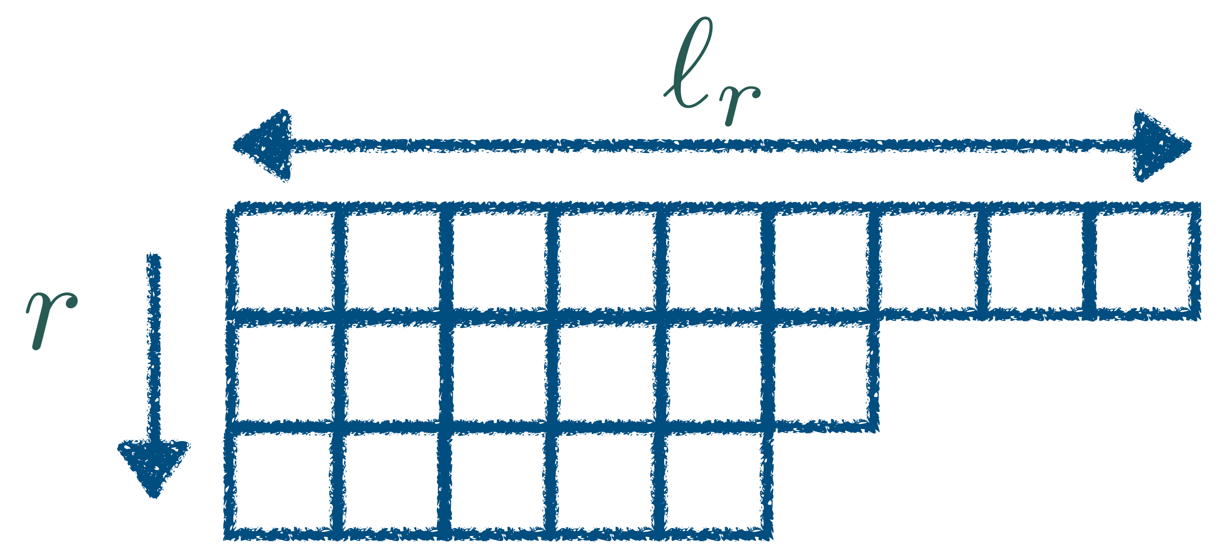

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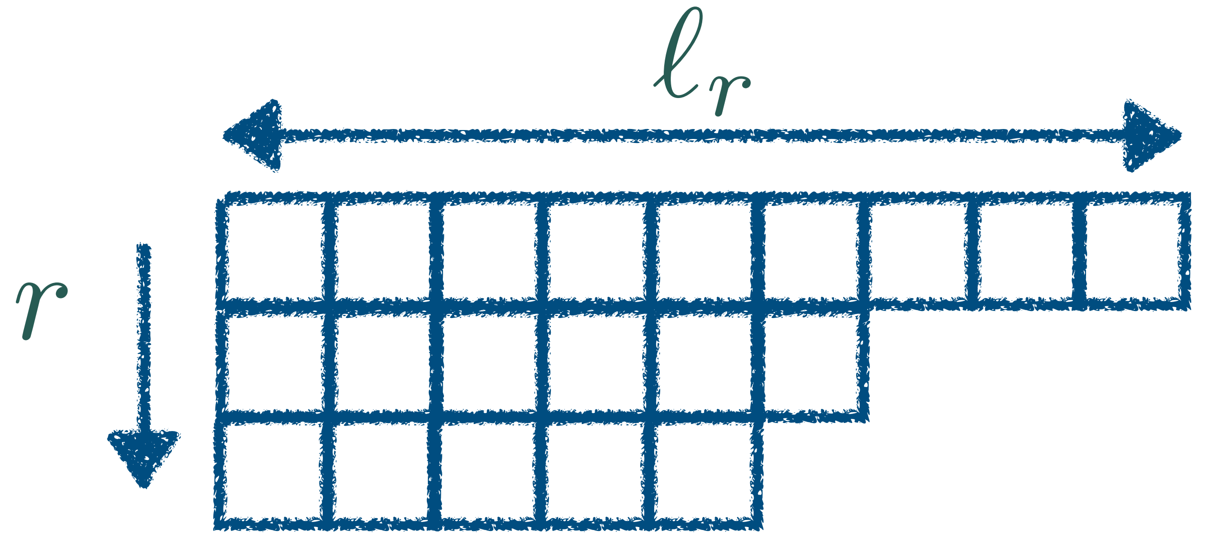

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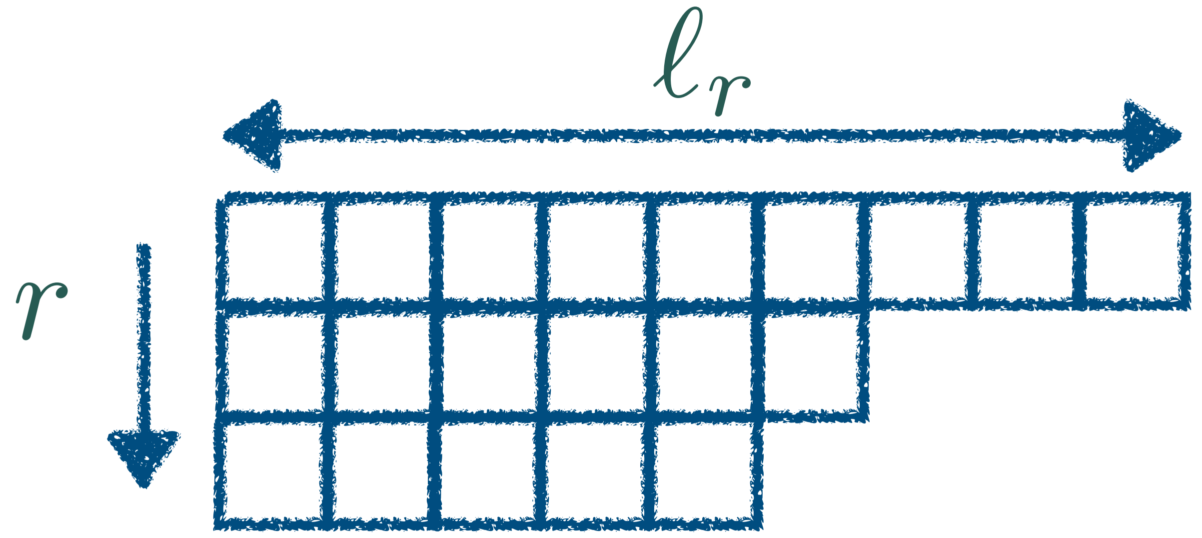

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[Frenkel; '23]

$$\log \mathsf{d}_{\mu^\star} \approx \sum_r \ell_r \log \frac{M}{\ell_r} = \sum_r |\partial\Sigma_{(r)}| \log\left(\frac{N|\Sigma|}{|\partial\Sigma_{(r)}|}\right)$$

what is the entropy of *any MxM* matrix block?

Incorporating *F*

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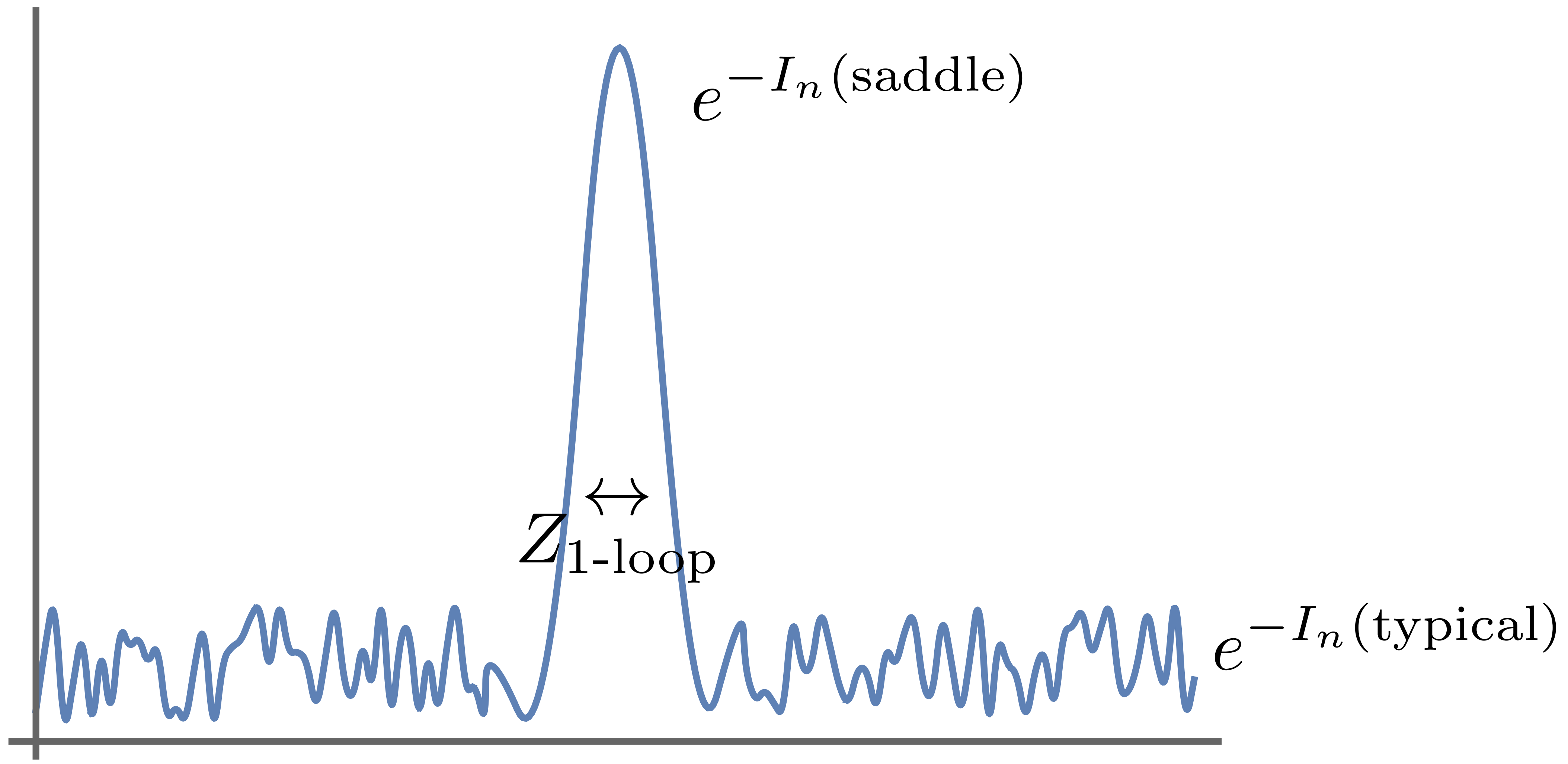

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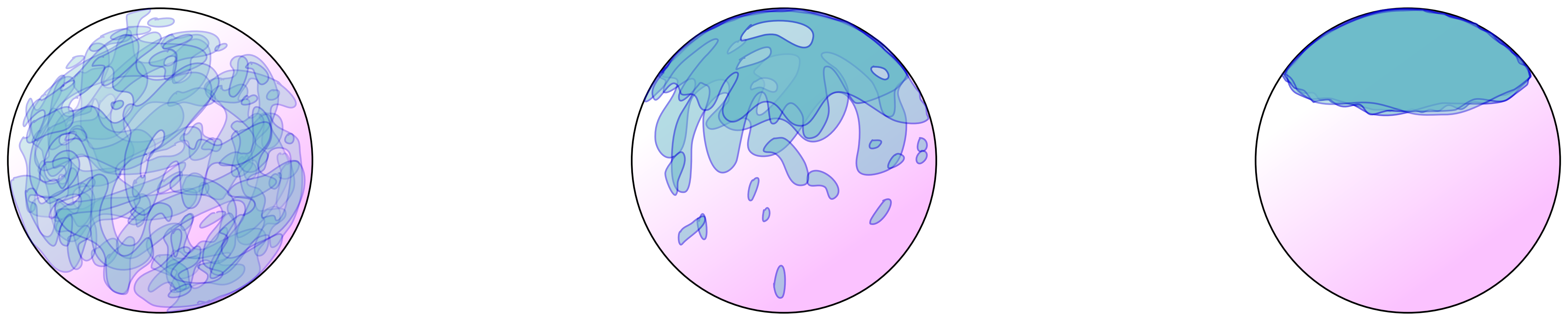

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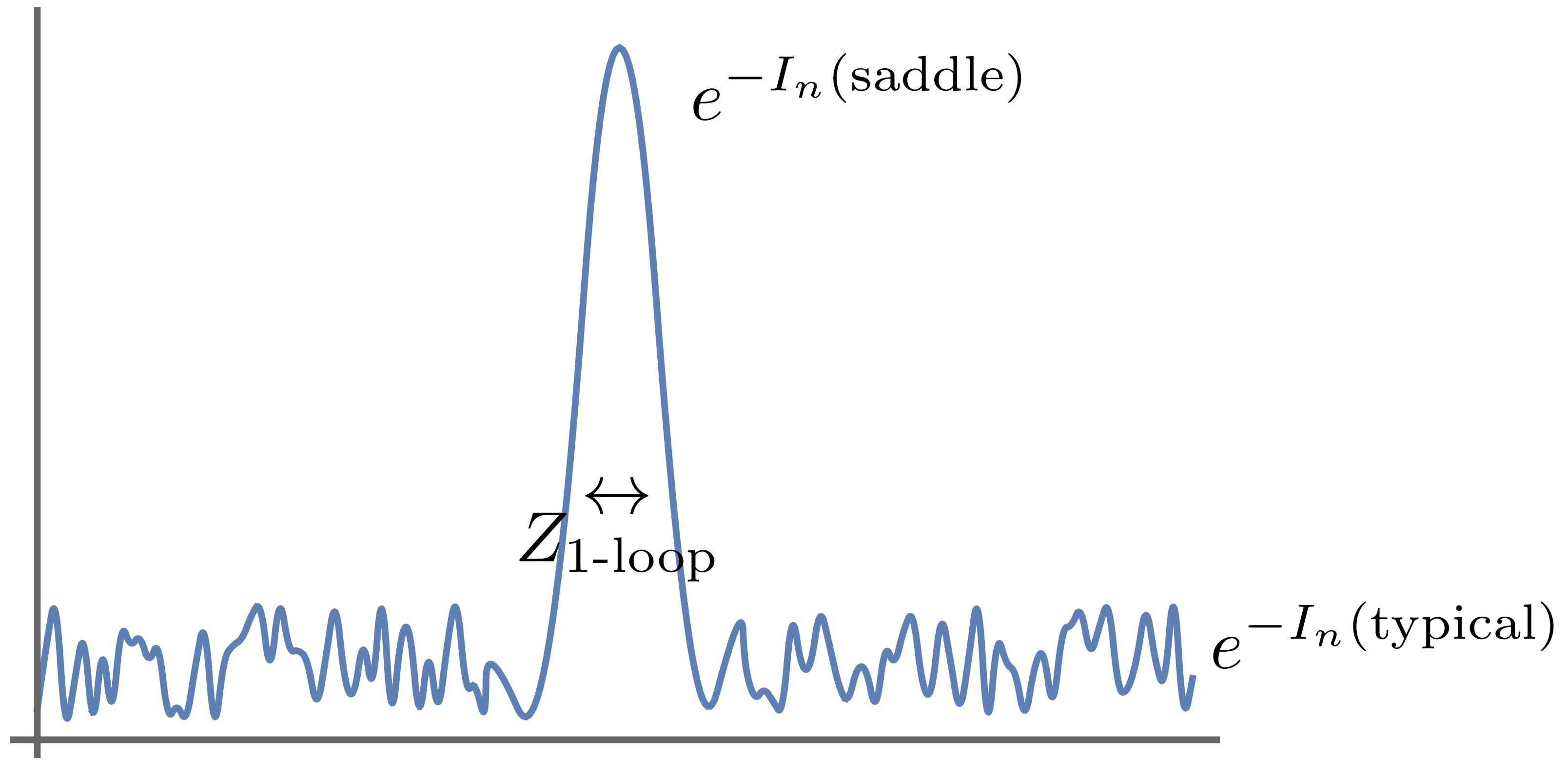
- ***Towards an RT formula outside of holography:*** *gauge fixing to more abstract features than a boundary. QRFs.*

Thank you for your attention.

Extra slides

Fine print on the saddle point



$$\mathrm{Tr}\rho_{\text{in}}^n \approx Z_{\text{1-loop}} e^{-I_n(\text{saddle})} + e^{-I_n(\text{typical})}$$

- *Establishing the existence + dominance of a saddle point.*
  - the value of $I_n(\text{saddle})$ itself and that it is a minimum
  - $Z_{\text{1-loop}}$: the order of fluctuations about the saddle point
  - $I_n(\text{typical})$: value of $\log \mathrm{d}_{\mu_V^\star}$ for a typical configuration

Haar averaged integrals

Can be checked, e.g. in the fuzzy sphere state

more serious fine print…

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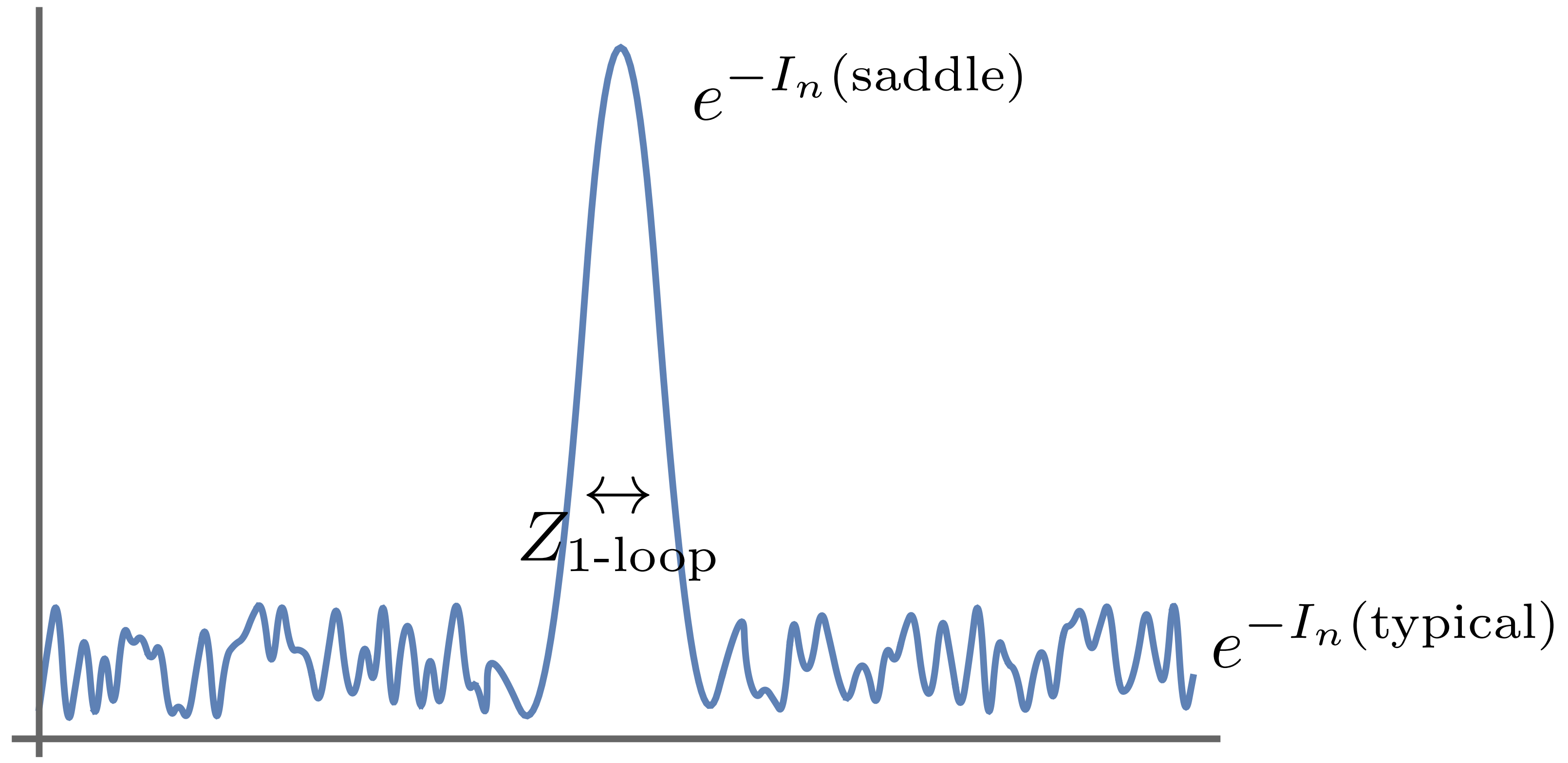

- *U(N)* is much larger than smooth continuum VPDs.
  - *F* contains many elements that act non-geometrically (send Σ to fractal and/or fragmented Planck-sized regions)
  - *These non-geometric maps proliferate and wash out the saddle-point*

We find it necessary to "coarse-grain" the integral over F to elements that act geometrically on Σ

Does not seem special to matrix character of the problem… may be generic

On coarse-grained diffeos

Return to the factorization map

$$|\psi\rangle = \int_F dV \int_{U(M)} dU \int_{U(N-M)} d\tilde{U}\, \hat{\pi}(U\tilde{U}V)|\psi_{\rm gf}\rangle$$

- Integrals over *U(M)*, *U(N-M)* $\longrightarrow$ reduced state at any *V* is *indistinguishable* against VPDs preserving a subregion.

- Integral over *F* $\longrightarrow$ different reduced states that are *distinguishable* for different *V*'s.

- The reduced state of a low-energy observer in any sector should not be orthogonal to one related by a Planck-sized diffeo $\longrightarrow$ remove from integral.

In practicality: integrate over a quotient of F

  - Still large enough to account for geometric VPDs as $N \to \infty$, but suppressed enough that the geometric area dominates as a saddle.

E.g. in the fuzzy sphere state:

- A “low-tech” solution…

$$F = U(N')/(U(M') \otimes U(N' - M')) \qquad N' = N/p \quad M' = M/p$$

- By comparing $I_n(\text{saddle})$ to $Z_{\text{1-loop}}$ and to $I_n(\text{typical})$ we find a range of *p* such that the minimal area dominates.

$$N^{3/4} \ll p \ll N$$