Black Holes

in Higher Spin AdS_4/CFT_3

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Does Vasiliev higher spin theory make sense?

Holography for higher spin theory black holes

Background

Higher spin AdS_4/CFT_3

Symmetry and simplicity

$$J^s_{\mu_1 \ldots \mu_s} = \sum_{k=0}^{s} a_{sk}\, \partial_{\{\mu_1} \ldots \partial_{\mu_k} \varphi^\dagger \partial_{\mu_{k+1}} \ldots \partial_{\mu_s\}} \varphi \qquad\qquad \partial^\mu J^s_{\mu\mu_1 \ldots \mu_s} = 0$$

- Short distance string theory (on AdS) is higher spin symmetric — Sundborg00, Witten01
- Massless higher spin fields form a classical sub-theory — SezginSundell02
- Pure AdS higher spin theory was known by Vasiliev — FradkinVasiliev87
- The O(N) model is proposed to be the boundary dual — KlebanovPolyakov02
- 3-point interactions match between boundary and AdS — GiombiYin11

Higher spin AdS_4/CFT_3

Phase transitions

- Deconfinement transition is dual to AdS black hole formation — Witten98
- The black hole transition was known to Hawking and Page — HawkingPage84
- The free deconfinement $T_{m,c} \sim 1$ solved by matrix models — Sundborg00 (AharonyMarsano MinwallaPapadodimas VanRamsdonk04)

$$Z(x) = \frac{1}{N!}\int \prod_{i=1}^{N} \frac{d\alpha_i}{2\pi} \prod_{i\neq j} \left| 2\sin\left(\frac{\alpha_i - \alpha_j}{2}\right)\right| \left(\sum_{k=1}^{N} \delta\left(-2\pi k + \sum_i \alpha_i\right)\right)$$
$$\times \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} \left(\zeta_B\left(x^n\right) - (-1)^n \zeta_F\left(x^n\right)\right) \left(-1 + \sum_{i,j} \cos\left(n\left(\alpha_i - \alpha_j\right)\right)\right)\right)$$

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$$U_{m,bd} \sim N^2 T^d \qquad M_{bh} \sim \frac{T^d}{G} \qquad U_{v,bd} \sim N T^d$$

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Higher spin AdS_4/CFT_3

The black hole issue

ShenkerYin11

- “Our main findings are simple to state: in this system there is a large N thermal transition but it occurs at temperatures $T \sim \sqrt{N}$, not $T \sim 1$. In bulk units this corresponds to an energy of order Planck scale, not AdS scale.”
- “This indicates the absence of a thermodynamically stable large AdS-Schwarzschild black hole solution in this theory at $T \sim 1$.”
- “…at temperatures $T \sim 1$ the corrections to the free energy on the sphere are not only nonperturbative in G, but of order $\exp(-N^{3/2})$, too small to be caused by a black hole. This indicates the absence of (uncharged) AdS-Schwarzschild-like black hole solutions in Vasiliev’s theory,…”
- “The bulk interpretation of these results is unclear. The large AdS black hole familar from ordinary AdS/CFT must either be absent in this theory or have subdominant free energy for lower temperatures. The known black hole solution of Didenko-Vasiliev [43] is an extremal charged black hole, whose charges are not yet understood. It is not acandidate for the generic thermal state.”

Higher spin AdS_4/CFT_3

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Higher spin AdS_4/CFT_3

“Vector baryons” ≡ Invariant totally antisymmetric scalar operators in vector model

- “…nontrivial relations among the bilinear invariants first appear at level … ($\sim \frac{4}{3}N^{3/2}$ at large N)…”

Bosonic operators $\mathcal{O}_b = \epsilon^{i_1\cdots i_N}\phi_{i_1}\partial\phi_{i_2}\ldots\partial\partial\ldots\phi_{i_N}$

ShenkerYin11

The **one-vector-baryon** partition function $Z_N(x) = x^{\frac{N}{2}}\prod_{n=0}^{\infty}\left(1+x^n y\right)^{2n+1}\Big|_{y^N}$

- “At temperatures of order 1 we expect leading corrections to the higher spin gas thermal free energy to be of order exp(–Δ/T), nonperturbatively small in N. This gives exp(–$N^{3/2}$/T) for the scalar theory…”

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Higher spin AdS_4/CFT_3

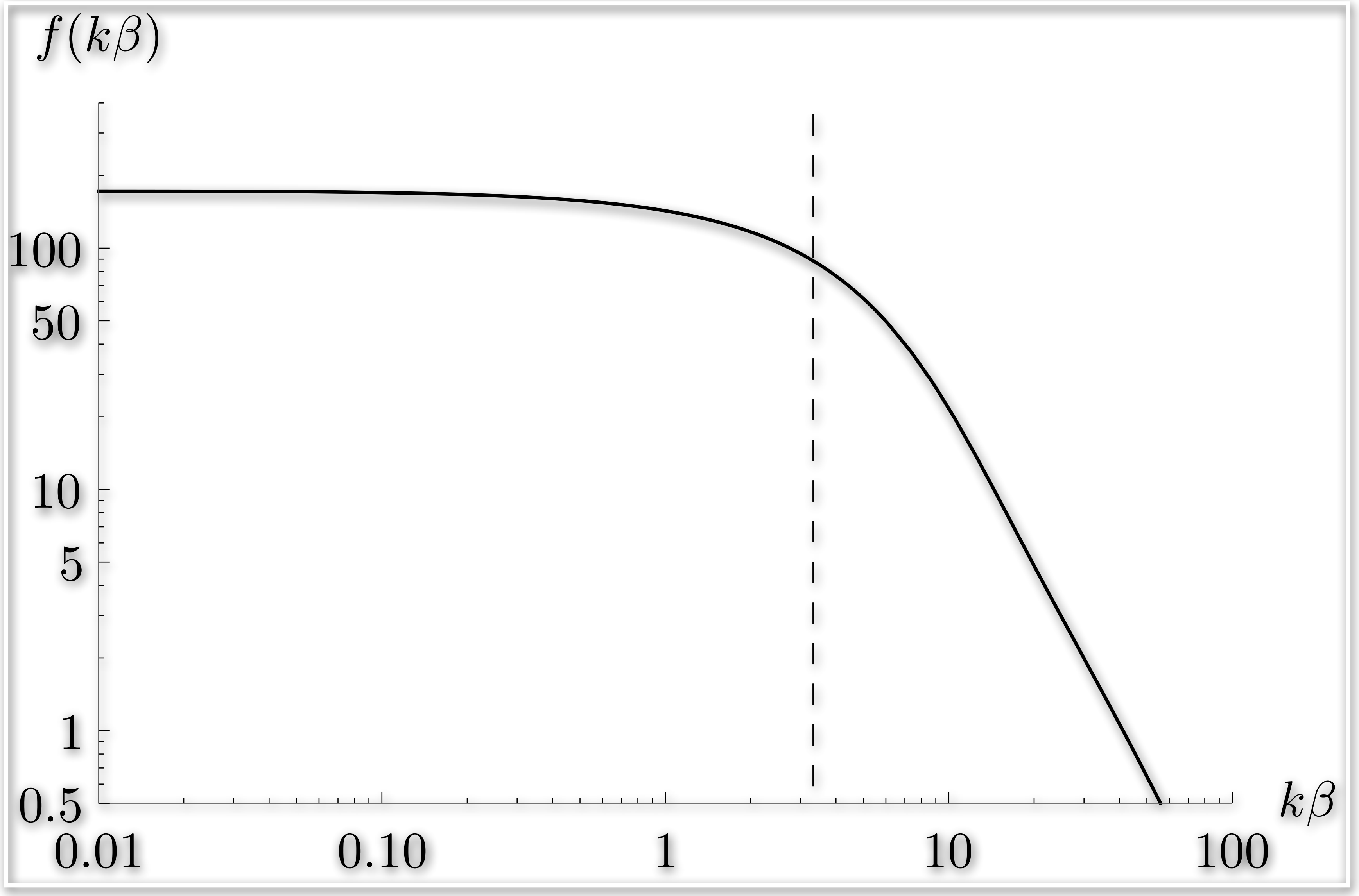
Uncanny similarities to black holes

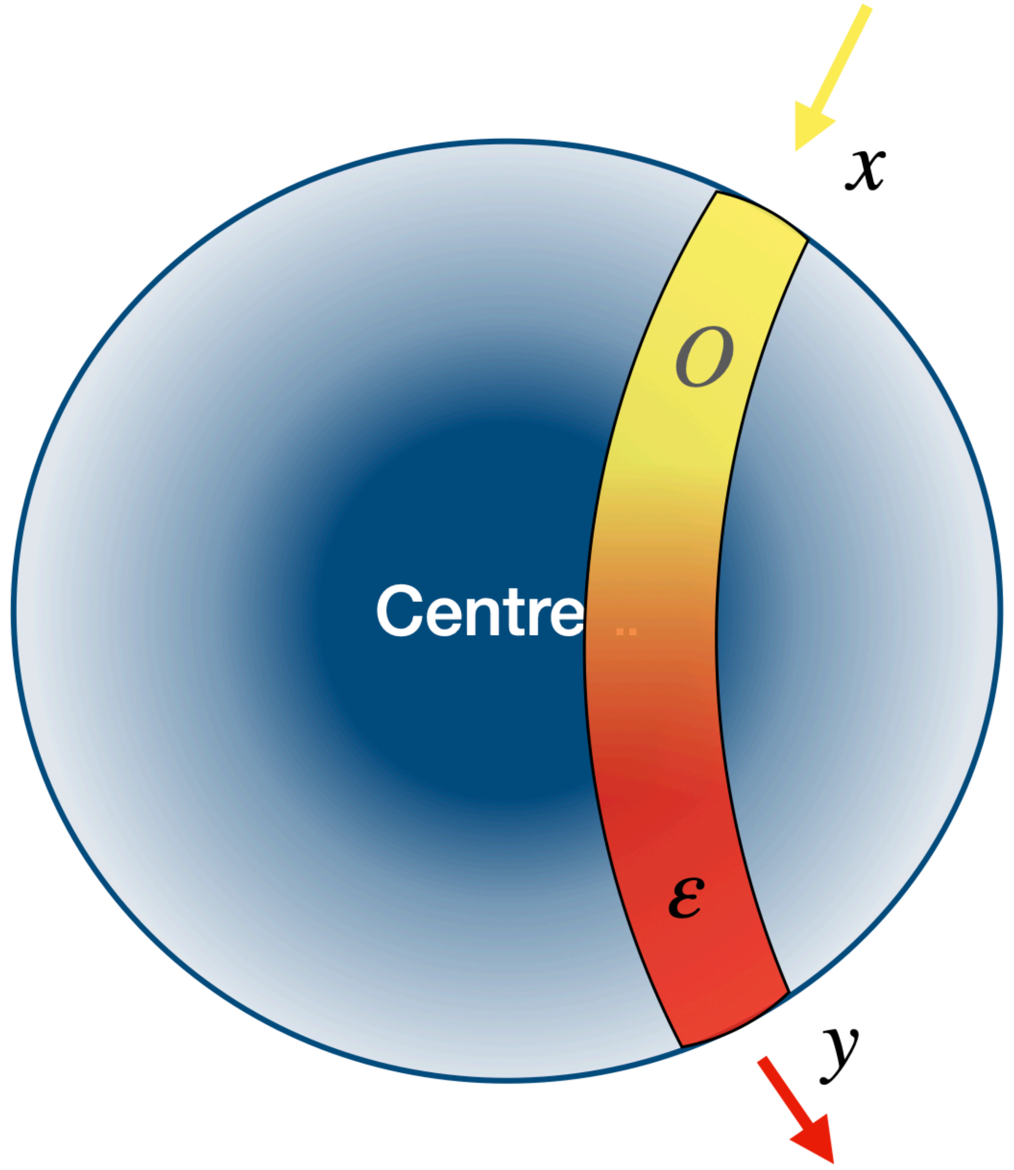
Jevicki et al

SundborgThorlacius et al

SundborgEngelsöy

$$G_{O\varepsilon}(x,y) = \langle O(x)\varepsilon(y)\rangle$$





- Thermalisation – decay of correlators on thermal time scale
- Evanescent modes – modes inside the black hole photon sphere
- Tidal excitation – scalars converting to gravitons close to black holes

The high temperature phase of higher spin theory
is different from that of string theory/gravity

The high temperature phase of higher spin theory
is similar to that of string theory/gravity

Take aways

- There are interesting and numerous superheavy "vector baryon" states in the boundary theory.
- Just below the deconfinement transition heavy microstates become significant.
- A "new" bulk/boundary dictionary makes higher spin AdS_4/CFT_3 similar to standard holography (though the cosmological constant scales as boundary $\sqrt{N}$).

Super heavy states

Condensation nuclei for black holes?

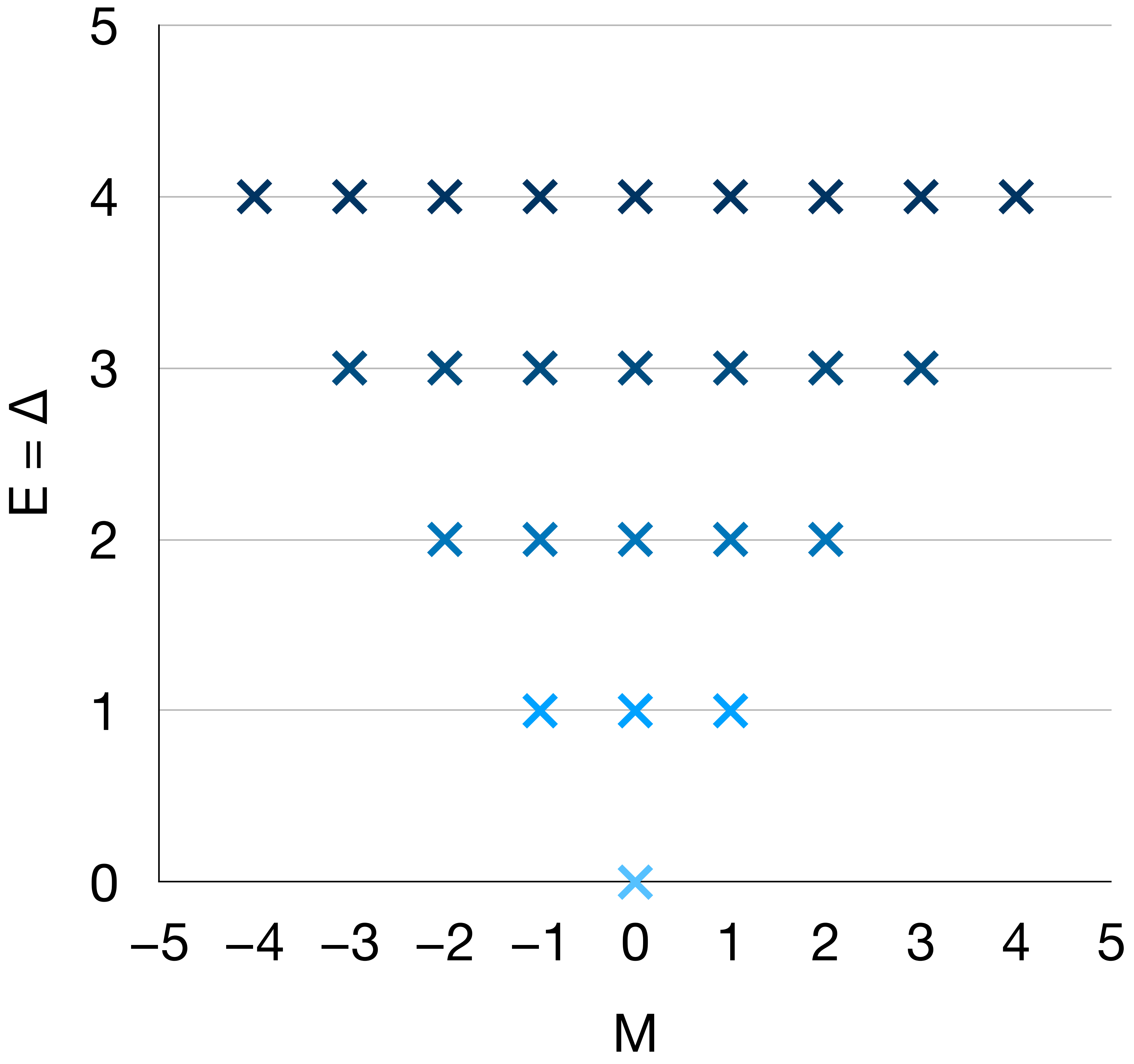
Super heavy states

The Fermi gas picture

Due to the ϵ tensor the constituent fields φ in the SO(N) model baryon operator $B = \epsilon^{n_1 \dots n_N} \partial^{k_1} \varphi_{n_1} \partial^{k_2} \varphi_{n_2} \dots \partial^{k_N} \varphi_{n_N}$ obeys an **exclusion principle**.

- Effectively a gas of N free “fermions”.
- The spacetime vectors given by the derivatives ∂_μ are 3-vectors for the boundary of AdS_4.
- For the **groundstate**, we fill all the lowest energy levels, with as few derivatives as possible, respecting Pauli.
- $\Delta_0 \sim \dfrac{2}{3} N^{3/2}$

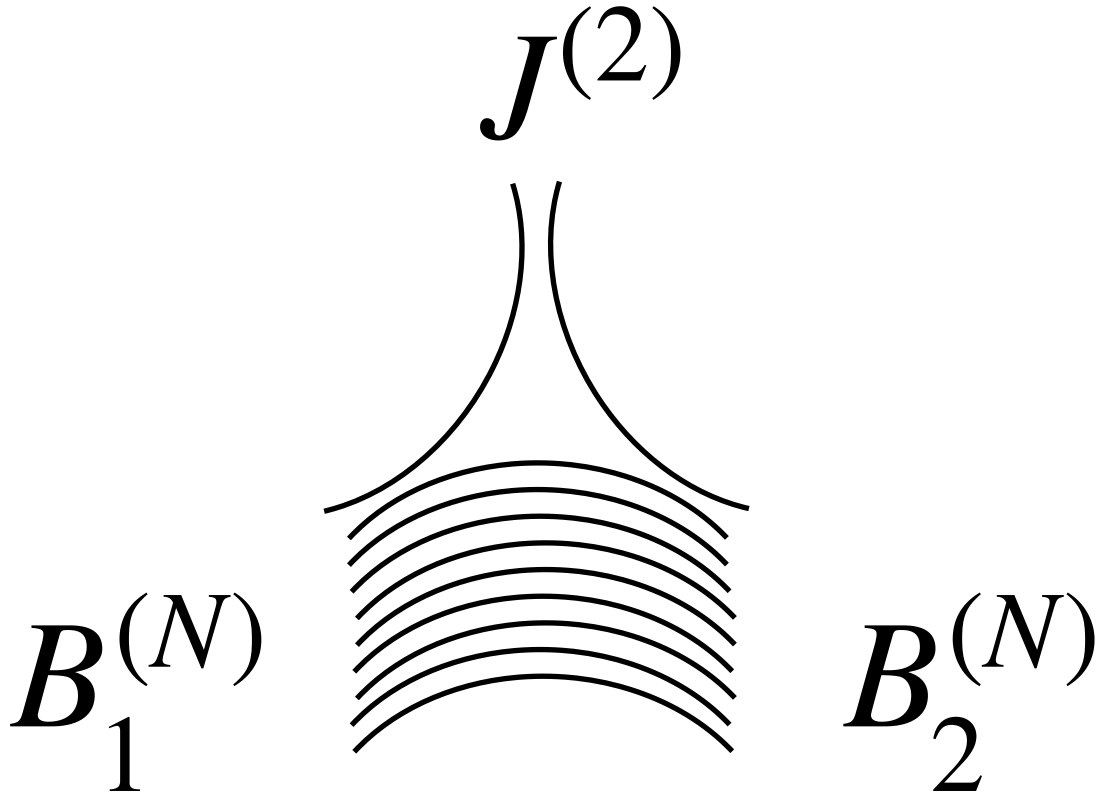
For N a squared integer:
Scalar baryon groundstate
— Filled shell



Bulk duals of super heavy states

Higher spin charges and (generalised) Didenko-Vasiliev black holes

- For higher spin currents J and vector baryons B_i the 3-point correlators $\left\langle J(x_0)B_1(x_1)B(x_2)\right\rangle \neq 0$ in general.
- Vector baryons carry higher spin charges.
- Just as Didenko-Vasiliev black holes.



$$\left\langle J(x_0)B_1(x_1)B(x_2)\right\rangle \neq 0$$

Statistical mechanics of vector baryons

The grand canonical ensemble

- For fermions the grand canonical ensemble is convenient.
- The chemical potential potential μ is chosen to get $N = \langle N \rangle = \partial_\mu \log \mathcal{Z}$.
- Study high temperatures, $\beta \ll 1$.

$$Z_N(x) = x^{\frac{N}{2}} \prod_{n=0}^{\infty} \left(1 + x^n y\right)^{2n+1} \Bigg|_{y^N}$$

$$x = e^{-\beta}, \quad y = e^{\beta\mu}$$

$$\mathcal{Z}(x, y) \approx \exp\left(-\sum_{m=1}^{\infty} \frac{(-1)^m}{m} \frac{y^m x^{m/2}(x^m + 1)}{(1 - x^m)^2} \right)$$

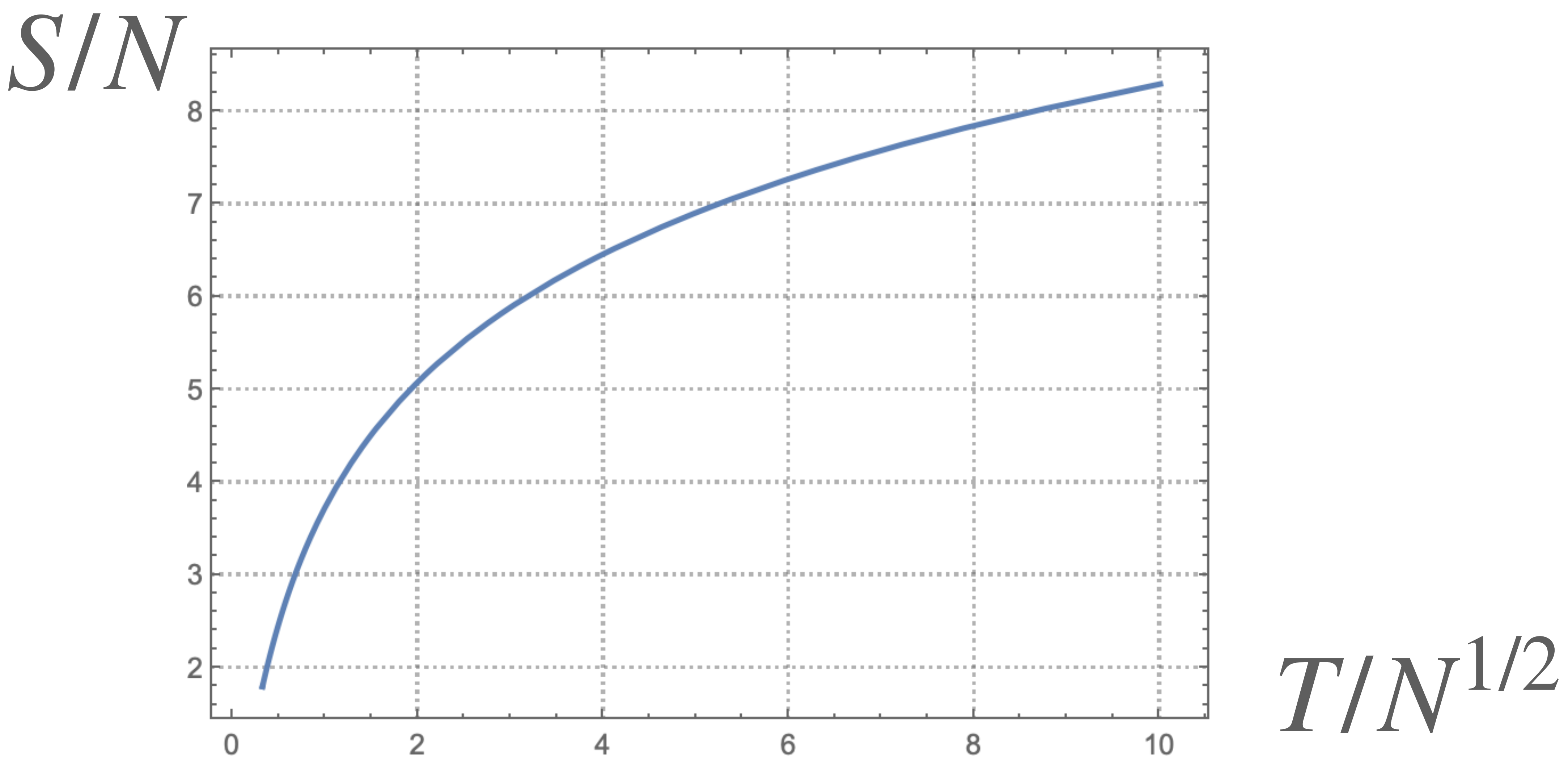
$$N \approx -\frac{2}{\beta^2} \mathrm{Li}_2(-e^{\beta\mu}) \qquad \log \mathcal{Z} \approx -\frac{2}{\beta^2} \mathrm{Li}_3(-e^{\beta\mu})$$

Statistical mechanics of vector baryons

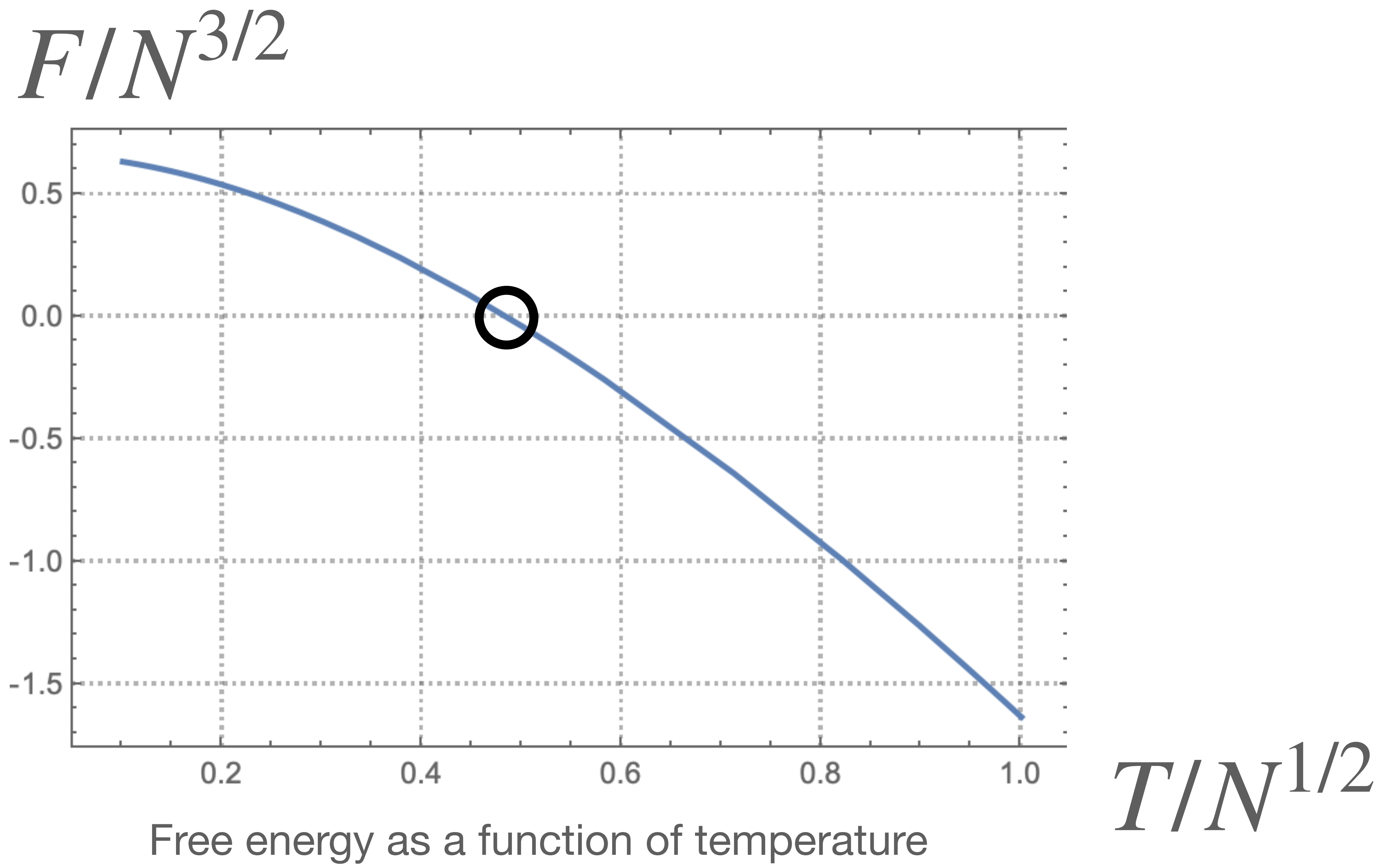
State functions

- The thermodynamic state functions scale
- Entropy as N
- Free and internal energy as $N^{3/2,}$ as groundstate energy

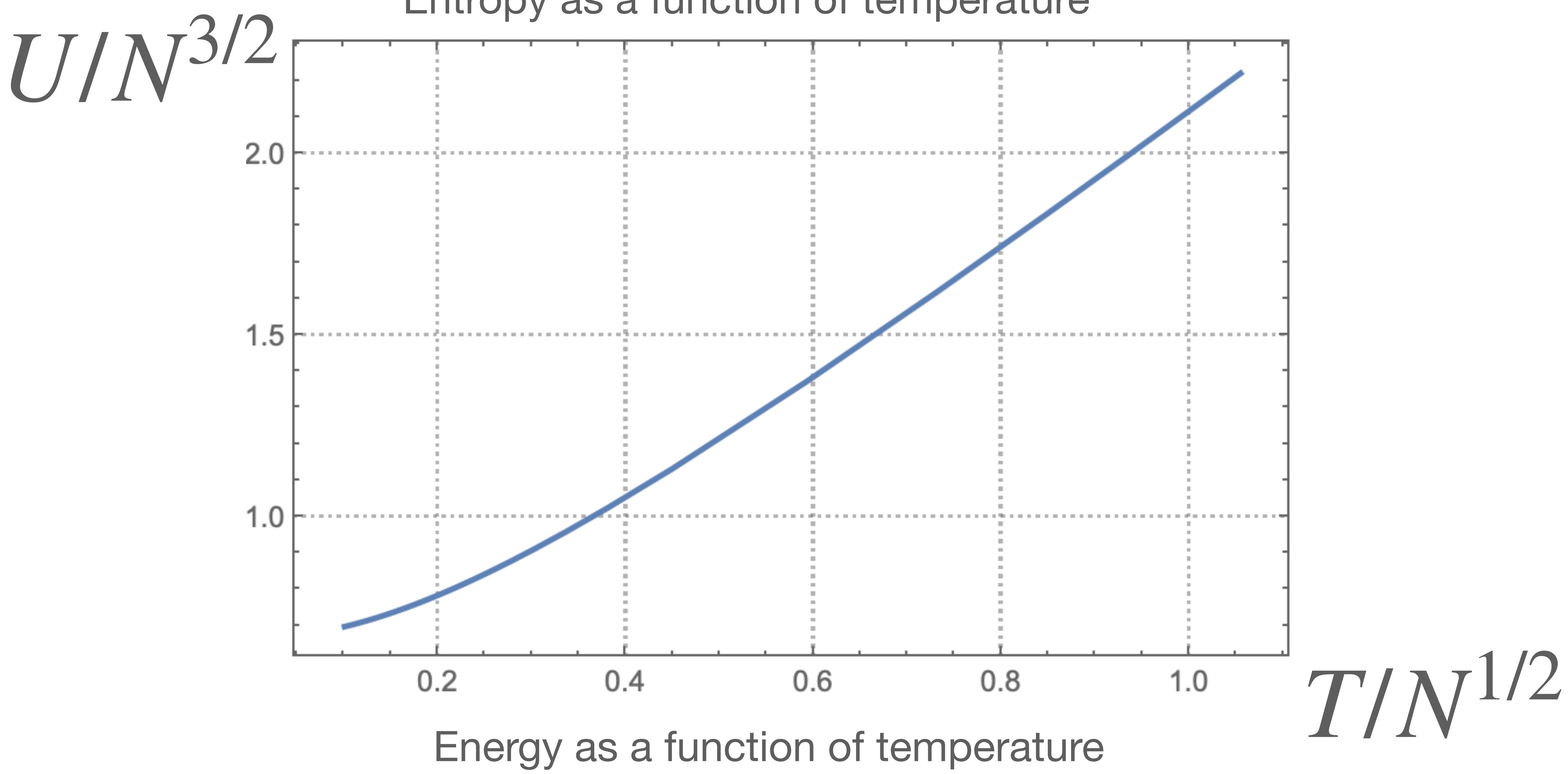
- F < 0 for T > $T_0 \approx 0.48 \sqrt{N}$



Entropy as a function of temperature



Free energy as a function of temperature



Energy as a function of temperature

Statistical mechanics of vector baryons

Vector baryons as condensation nuclei?

Confined,
no vector
baryon

Confined,
+ vector
baryon

Deconfined,
+ vector
baryon

$T_0 \quad T_c$

Blackbody
radiation,

no
condensation
nucleus or
“star”

Blackbody
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+condensation
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Blackbody
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nucleus or
“star”
+black hole

Units for higher spin holography

Measuring G and Λ

Units for higher spin holography

Assuming that the largest black holes are AdS-Schwarschild

- Try a new perspective on higher spin black holes
- Since some pieces are missing in the vector model/higher spin theory duality, start by an assumption and check consequences.

- Assumption: **The largest higher spin black holes are AdS-Schwarzschild**
- This is a kind of correspondence limit, which has a chance of being classical
- Large higher spin black hole $\equiv$ dual to high T equilibrium of vector model

Units for higher spin holography

Eliminate geometry in the high T limit of AdS-Schwarzschild

- Assumption: **The largest higher spin black holes are AdS-Schwarzschild**

$$\beta = \frac{4\pi l^2 r_+}{l^2 + 3r_+^2} \rightarrow \frac{4\pi l^2}{3r_+}$$

$$M = \frac{1}{2G} r_+ \left(1 + \frac{r_+^2}{l^2}\right) \rightarrow \frac{r_+^3}{2Gl^2}$$

Leading order: $\frac{2GM}{l} \sim \frac{l^3}{\beta^3}$

Correction term: $\frac{2G\delta M}{l} \sim -\frac{l}{\beta}$

Units for higher spin holography

Find AdS scale and Newton’s constant

- Assumption: **The largest higher spin black holes are AdS-Schwarzschild**

Determine the AdS length scale by cancelling G: $$-\frac{l^2}{\beta^2} \sim \frac{M}{\delta M}$$

Then find Newton's constant G: $$G \sim -\frac{l^2}{\beta \delta M} \sim \frac{\beta M}{\delta M^2}$$

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Boundary: $M = U \sim N \frac{\hbar}{\beta_0} \frac{\beta_0^3}{\beta^3}$

Boundary: $\delta M = \delta U \sim -N^2 \frac{\hbar}{\beta}$

Non-trivial large N calculation

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