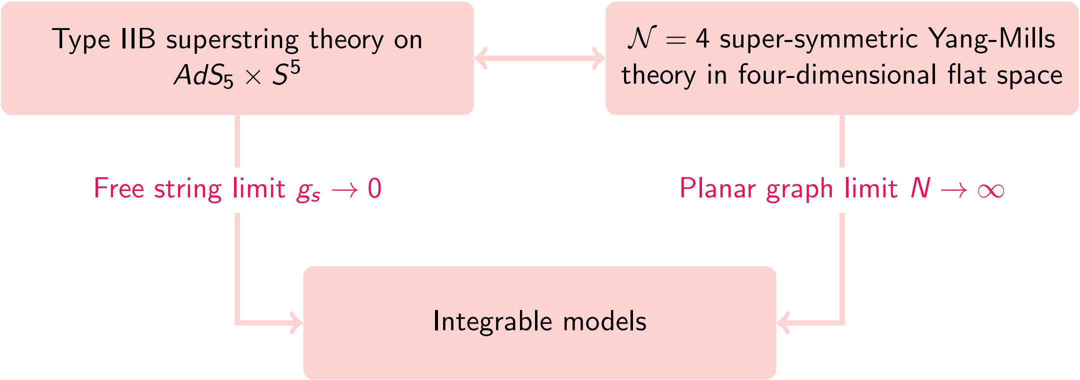
# Integrable deformations of $AdS_3$ strings

Fiona Seibold

EPFL

Type IIB superstring theory on $AdS_5 \times S^5$ $\longleftrightarrow$ $\mathcal{N} = 4$ super-symmetric Yang-Mills theory in four-dimensional flat space


Type IIB superstring theory on $AdS_5 \times S^5$
$\mathcal{N} = 4$ super-symmetric Yang-Mills theory in four-dimensional flat space
Free string limit $g_s \to 0$
Planar graph limit $N \to \infty$
Integrable models

Type IIB superstring theory on $AdS_5 \times S^5$ $\longleftrightarrow$ $\mathcal{N} = 4$ super-symmetric Yang-Mills theory in four-dimensional flat space

Free string limit $g_s \to 0$

Planar graph limit $N \to \infty$

Integrable models

- Are there other superstring theories for which the integrability program can be applied?

- More integrable theories from **lower-dimensional** AdS/CFT setups

$AdS_5 \times \mathcal{M}_5$

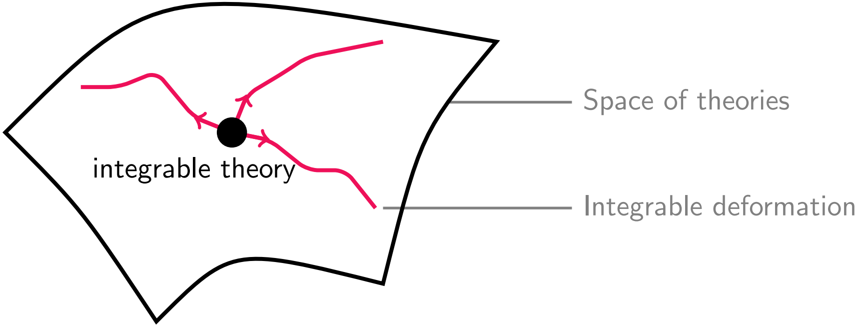
$AdS_4 \times \mathcal{M}_6$

$AdS_3 \times \mathcal{M}_7$

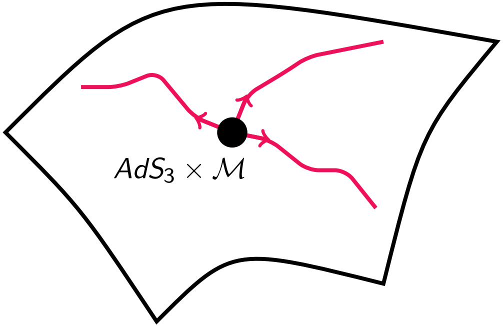
$AdS_2 \times \mathcal{M}_8$

Less (super)symmetries

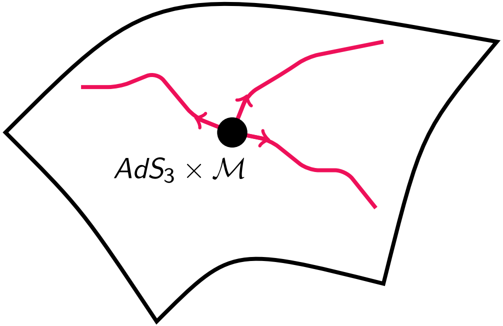
- More integrable theories from **integrable deformations**



- In this talk: integrable supersymmetric deformations of $AdS_3$ strings



- In this talk: integrable supersymmetric deformations of $AdS_3$ strings




- Mainly based on work with **B. Hoare, A. Retore, A. Sfondrini, A. Tseytlin**

- There are three local type IIB $AdS_3$ string vacua preserving 16 real supercharges

$$AdS_3 \times S^3 \times T^4 \ , \qquad AdS_3 \times S^3 \times K3 \ , \qquad AdS_3 \times S^3 \times S^3 \times S^1 \ .$$

[Cvetic, Giveon, Kutasov, Larsen, Rahmfeld, Boonstra, Petters, Skenderis, Strominger, ...]

- There are three local type IIB $AdS_3$ string vacua preserving 16 real supercharges

$$AdS_3 \times S^3 \times T^4 \ , \qquad AdS_3 \times S^3 \times K3 \ , \qquad AdS_3 \times S^3 \times S^3 \times S^1 \ .$$

[Cvetic, Giveon, Kutasov, Larsen, Rahmfeld, Boonstra, Petters, Skenderis, Strominger, ...]

- (1) and (2) have superisometry $\mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R$

- There are three local type IIB $AdS_3$ string vacua preserving 16 real supercharges

$$AdS_3 \times S^3 \times T^4 \,, \qquad AdS_3 \times S^3 \times K3 \,, \qquad AdS_3 \times S^3 \times S^3 \times S^1 \,.$$

[Cvetic, Giveon, Kutasov, Larsen, Rahmfeld, Boonstra, Petters, Skenderis, Strominger, ...]

- (1) and (2) have superisometry $\mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R$
- (3) has superisometry $\mathfrak{d}(2,1;\alpha)_L \oplus \mathfrak{d}(2,1;\alpha)_R$

- There are three local type IIB $AdS_3$ string vacua preserving 16 real supercharges

$$AdS_3 \times S^3 \times T^4 \;, \qquad AdS_3 \times S^3 \times K3 \;, \qquad AdS_3 \times S^3 \times S^3 \times S^1 \;.$$

[Cvetic, Giveon, Kutasov, Larsen, Rahmfeld, Boonstra, Petters, Skenderis, Strominger, ...]

- (1) and (2) have superisometry $\mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R$
- (3) has superisometry $\mathfrak{d}(2,1;\alpha)_L \oplus \mathfrak{d}(2,1;\alpha)_R$
- Parameter $\alpha$ controls ratio between sphere radii, with (3) $\to$ (1) in limit $\alpha \to 0, 1$

- There are three local type IIB $AdS_3$ string vacua preserving 16 real supercharges

$$AdS_3 \times S^3 \times T^4 \,, \qquad AdS_3 \times S^3 \times K3 \,, \qquad AdS_3 \times S^3 \times S^3 \times S^1 \,.$$

[Cvetic, Giveon, Kutasov, Larsen, Rahmfeld, Boonstra, Petters, Skenderis, Strominger, ...]

- (1) and (2) have superisometry $\mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R$
- (3) has superisometry $\mathfrak{d}(2,1;\alpha)_L \oplus \mathfrak{d}(2,1;\alpha)_R$
- Parameter $\alpha$ controls ratio between sphere radii, with (3) $\to$ (1) in limit $\alpha \to 0, 1$
- Can be supported by pure NSNS flux, pure RR flux, or a mixture of both

- There are three local type IIB $AdS_3$ string vacua preserving 16 real supercharges

$$AdS_3 \times S^3 \times T^4 \,, \qquad AdS_3 \times S^3 \times K3 \,, \qquad AdS_3 \times S^3 \times S^3 \times S^1 \,.$$

[Cvetic, Giveon, Kutasov, Larsen, Rahmfeld, Boonstra, Petters, Skenderis, Strominger, ...]

- (1) and (2) have superisometry $\mathfrak{psu}(1,1|2)_L \oplus \mathfrak{psu}(1,1|2)_R$
- (3) has superisometry $\mathfrak{d}(2,1;\alpha)_L \oplus \mathfrak{d}(2,1;\alpha)_R$
- Parameter $\alpha$ controls ratio between sphere radii, with (3) $\to$ (1) in limit $\alpha \to 0, 1$
- Can be supported by pure NSNS flux, pure RR flux, or a mixture of both
- All are **integrable** (free strings) [Babichenko, Stefanski, Zarembo '10] [Cagnazzo, Zarembo '12]

1. Integrable $AdS_3$ strings
2. TsT transformations
3. Trigonometric and Elliptic deformations
4. From classical to quantum integrability: the worldsheet S-matrix
5. Conclusions and Outlook

- Perhaps the simplest integrable deformation is the “TsT” transformation

[Lunin Maldacena ’05] [Frolov ’05] [Alday Arutyunov Frolov ’05] [...]

- Perhaps the simplest integrable deformation is the "TsT" transformation

  [Lunin Maldacena '05] [Frolov '05] [Alday Arutyunov Frolov '05] [...]

- Can be applied whenever the theory has two $U(1)$ isometries
  - T-duality in $\varphi_1 \to \tilde{\varphi}_1$ (type IIB $\to$ type IIA)
  - Shift $\varphi_2 \to \varphi_2 + \beta\tilde{\varphi}_1$
  - T-duality in $\tilde{\varphi}_1 \to \varphi_1$ (type IIA $\to$ type IIB)

- Perhaps the simplest integrable deformation is the "TsT" transformation

  [Lunin Maldacena '05] [Frolov '05] [Alday Arutyunov Frolov '05] [...]

- Can be applied whenever the theory has two $U(1)$ isometries
  - T-duality in $\varphi_1 \to \tilde{\varphi}_1$ (type IIB $\to$ type IIA)
  - Shift $\varphi_2 \to \varphi_2 + \beta\tilde{\varphi}_1$
  - T-duality in $\tilde{\varphi}_1 \to \varphi_1$ (type IIA $\to$ type IIB)

- Maps string backgrounds to string backgrounds (Buscher rules)

- Perhaps the simplest integrable deformation is the "TsT" transformation

  [Lunin Maldacena '05] [Frolov '05] [Alday Arutyunov Frolov '05] [...]

- Can be applied whenever the theory has two $U(1)$ isometries
  - T-duality in $\varphi_1 \to \tilde{\varphi}_1$ (type IIB $\to$ type IIA)
  - Shift $\varphi_2 \to \varphi_2 + \beta\tilde{\varphi}_1$
  - T-duality in $\tilde{\varphi}_1 \to \varphi_1$ (type IIA $\to$ type IIB)
- Maps string backgrounds to string backgrounds (Buscher rules)
- Often has an interpretation in holography (especially if TsT in compact space)

Lunin-Maldacena background (TsT in $S^5$) $\Leftrightarrow$ Leigh-Strasser deformation ($\mathcal{N} = 1$)

# TsT transformations: application to $AdS_3 \times S^3 \times S^3 \times S^1$



- TsT transformations of the pure RR $AdS_3 \times S^3 \times S^3 \times S^1$

$$AdS_3 \times S^3_1 \times S^3_2 \times S^1$$

- TsT transformations of the pure RR $AdS_3 \times S^3 \times S^3 \times S^1$

$$
\begin{array}{ccccccc}
AdS_3 & \times & S^3_1 & \times & S^3_2 & \times & S^1 \\
U(1)_L \times U(1)_R & \times & U(1)_{1,L} \times U(1)_{1,R} & \times & U(1)_{2,L} \times U(1)_{2,R} & \times & U(1)
\end{array}
$$

## TsT transformations: application to $AdS_3 \times S^3 \times S^3 \times S^1$



- TsT transformations of the pure RR $AdS_3 \times S^3 \times S^3 \times S^1$

$$
\begin{array}{ccccccc}
AdS_3 & \times & S^3_1 & \times & S^3_2 & \times & S^1 \\
U(1)_L \times U(1)_R & \times & U(1)_{1,L} \times U(1)_{1,R} & \times & U(1)_{2,L} \times U(1)_{2,R} & \times & U(1)
\end{array}
$$

# TsT transformations: application to $AdS_3 \times S^3 \times S^3 \times S^1$



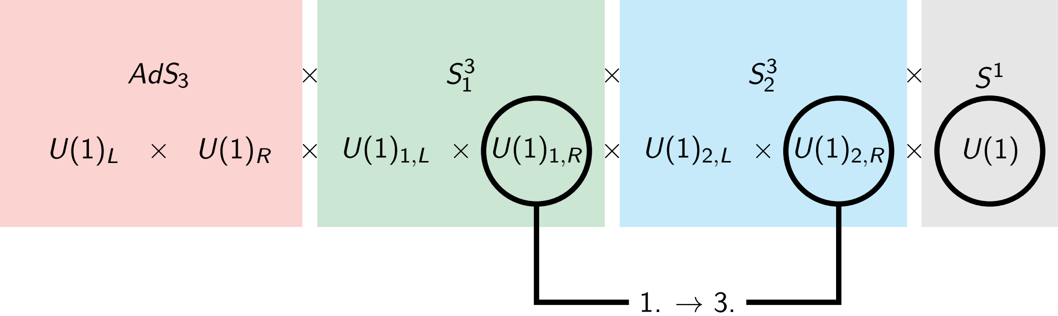
- TsT transformations of the pure RR $AdS_3 \times S^3 \times S^3 \times S^1$

$$AdS_3 \times S^3_1 \times S^3_2 \times S^1$$

$$U(1)_L \times U(1)_R \times U(1)_{1,L} \times U(1)_{1,R} \times U(1)_{2,L} \times U(1)_{2,R} \times U(1)$$

1. $\rightarrow$ 2.

- TsT transformations of the pure RR $AdS_3 \times S^3 \times S^3 \times S^1$



- Start with pure RR, TsT in the right Cartan $U(1)_R$ of $S^3$ and $U(1)$ of $S^1$ [FS, Sfondrini '25]

- Start with pure RR, TsT in the right Cartan $U(1)_R$ of $S^3$ and $U(1)$ of $S^1$ [FS, Sfondrini '25]
- Gives warped geometry

$$ds^2 = R^2 ds^2(AdS_3) + R_1^2 ds^2(S^3) + R_2^2 ds^2(S^3_\Delta) + dx^2 \ ,$$

$$ds^2(S^3_\Delta) = \frac{1}{4}(\sin^2\theta d\xi_L^2 + d\theta^2 + \Delta A^2) \ , \qquad A = \left(d\xi_R - \cos\theta d\xi_L\right)$$

$$\beta = \frac{2}{R_2}\sqrt{\frac{1-\Delta}{\Delta}} \ , \qquad 0 \leq \Delta \leq 1 \ .$$

- Start with pure RR, TsT in the right Cartan $U(1)_R$ of $S^3$ and $U(1)$ of $S^1$ [FS, Sfondrini '25]
- Gives warped geometry

$$ds^2 = R^2 ds^2(AdS_3) + R_1^2 ds^2(S^3) + R_2^2 ds^2(S^3_\Delta) + dx^2 \ ,$$

$$ds^2(S^3_\Delta) = \frac{1}{4}(\sin^2\theta d\xi_L^2 + d\theta^2 + \Delta A^2) \ , \qquad A = \left(d\xi_R - \cos\theta d\xi_L\right)$$

$$\beta = \frac{2}{R_2}\sqrt{\frac{1-\Delta}{\Delta}} \ , \qquad 0 \le \Delta \le 1 \ .$$

- Dilaton & Fluxes

$$\Phi = \text{const} \ , \quad H_3 = \frac{1}{2}R_2\sqrt{1-\Delta}\, dA \wedge dx \ , \quad F_3 = 2(R^2\Omega(AdS_3) + R_1^2\Omega(S^3) + R_2^2\Omega(S^3)) \ ,$$

$$F_5 = R_2\sqrt{1-\Delta}(1+\star)(R^2\Omega(AdS_3) + R_1^2\Omega(S^3)) \wedge A \wedge dx$$

- Strong deformation limit [FS, Sfondrini '25]

$$\xi_R \to \frac{2}{R_2}\frac{\xi_R}{\sqrt{\Delta}}\,, \qquad \Delta \to 0\,.$$

- Strong deformation limit [FS, Sfondrini '25]

$$\xi_R \to \frac{2}{R_2}\frac{\xi_R}{\sqrt{\Delta}} \,, \qquad \Delta \to 0 \,.$$

- Flat direction appears, dilaton & fluxes remain finite

$$\mathrm{d}s^2 = R^2 ds^2(AdS_3) + R_1^2 ds^2(S^3) + \frac{R_2^2}{4} ds^2(S^2) + d\xi_R^2 + dx^2 \,.$$

$$\Phi = \text{const} \,, \quad H_3 = \frac{R_2}{2}\Omega(S^2) \wedge dx \,, \quad F_3 = \frac{R_2}{2}\Omega(S^2) \wedge d\xi_R$$
$$F_5 = (1+\star)(R^2\Omega(AdS_3) + R_1^2\Omega(S^3)) \wedge d\xi_R \wedge dx \,.$$

- Strong deformation limit [FS, Sfondrini '25]

$$\xi_R \to \frac{2}{R_2}\frac{\xi_R}{\sqrt{\Delta}} \,, \qquad \Delta \to 0 \,.$$

- Flat direction appears, dilaton & fluxes remain finite

$$\mathrm{d}s^2 = R^2 ds^2(AdS_3) + R_1^2 ds^2(S^3) + \frac{R_2^2}{4} ds^2(S^2) + d\xi_R^2 + dx^2 \,.$$

$$\Phi = \text{const} \,, \quad H_3 = \frac{R_2}{2}\Omega(S^2) \wedge dx \,, \quad F_3 = \frac{R_2}{2}\Omega(S^2) \wedge d\xi_R$$
$$F_5 = (1+\star)(R^2\Omega(AdS_3) + R_1^2\Omega(S^3)) \wedge d\xi_R \wedge dx \,.$$

- Gives the $AdS_3 \times S^3 \times S^2 \times T^2$ background, which is classically integrable [Wulff '14]

# Interpolating integrable backgrounds



$AdS_3 \times S^3 \times S^3 \times S^1$

$\boxed{AdS_3 \times S^3 \times S^2 \times T^2}$

$$AdS_3 \times S^3 \times S^3 \times S^1$$

$$AdS_3 \times S^3 \times S^2 \times T^2 \qquad\qquad AdS_3 \times S^2 \times S^2 \times T^3$$

# Interpolating integrable backgrounds



$$
\begin{tikzcd}
 & AdS_3 \times S^3 \times S^3 \times S^1 \arrow[dl, leftrightarrow, bend right] \arrow[dr, leftrightarrow, bend left] \arrow[d, dashed, "\alpha \to 0, 1"] & \\
AdS_3 \times S^3 \times S^2 \times T^2 & AdS_3 \times S^3 \times T^4 \arrow[d, leftrightarrow] & AdS_3 \times S^2 \times S^2 \times T^3 \\
 & AdS_3 \times S^2 \times T^5 &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & AdS_3 \times S^3 \times S^3 \times S^1 \arrow[dl, leftrightarrow, bend right] \arrow[dr, leftrightarrow, bend left] \arrow[dd, dashed, "\alpha \to 0,1"] & \\
AdS_3 \times S^3 \times S^2 \times T^2 & & AdS_3 \times S^2 \times S^2 \times T^3 \\
 & AdS_3 \times S^3 \times T^4 \arrow[d, leftrightarrow] & \\
 & AdS_3 \times S^2 \times T^5 &
\end{tikzcd}
$$

- Properties of the interpolating backgrounds: [Orlando, Reffert, Uruchurtu '10] [FS, Sfondrini '25]
  - preserve the $AdS_3$ factor
  - preserve classical integrability
  - preserve 8 supersymmetries, one copy of $\mathfrak{d}(2,1;\alpha)$ or $\mathfrak{psu}(1,1|2)$

- Can also deform $AdS_3$: time-like, space-like and null warping

## Comments on TsT transformations



- Can also deform $AdS_3$: time-like, space-like and null warping
- Integrable supersymmetric interpolation to backgrounds with $AdS_2$ geometry

# Comments on TsT transformations

- Can also deform $AdS_3$: time-like, space-like and null warping
- Integrable supersymmetric interpolation to backgrounds with $AdS_2$ geometry
- TsT transformations are special cases of **homogeneous Yang-Baxter** deformations

[Klimcik '02 '08] [Delduc Magro Vicedo '13 '14] [Kawaguchi, Matsumoto, Yoshida '14] [van Tongeren '15]

- Can also deform $AdS_3$: time-like, space-like and null warping
- Integrable supersymmetric interpolation to backgrounds with $AdS_2$ geometry
- TsT transformations are special cases of **homogeneous Yang-Baxter** deformations

  [Klimcik '02 '08] [Delduc Magro Vicedo '13 '14] [Kawaguchi, Matsumoto, Yoshida '14] [van Tongeren '15]
- Equivalent to strings in undeformed geometry but with **twisted b.c.**

  [Alday, Arutyunov, Frolov '05] [van Tongeren '15] [Borsato, Driezen, Miramontes '22]

## Plan

- Strings propagating in a warped $S^3_\beta$ geometry and **no B-field** [Cherednik '81]

$$ds^2 = e_1^2 + e_2^2 + \beta e_3^2 \ , \qquad e_k = \mathrm{Tr}[g^{-1} dg J_k] \ , \qquad g \in SU(2) \ .$$

- Strings propagating in a warped $S^3_\beta$ geometry and **no B-field** [Cherednik '81]

$$ds^2 = e_1^2 + e_2^2 + \beta e_3^2 \ , \qquad e_k = \mathrm{Tr}[g^{-1} dg J_k] \ , \qquad g \in SU(2) \ .$$

- Broken $\mathfrak{su}(2)_R$ symmetry gets promoted to a Drinfel'd-Jimbo **quantum group**

$$[J_3, J_\pm] = 2J_\pm \ , \qquad [J_+, J_-] = \frac{q^{J_3} - q^{-J_3}}{q - q^{-1}} \ .$$

- Strings propagating in a warped $S^3_\beta$ geometry and **no B-field** [Cherednik '81]

$$ds^2 = e_1^2 + e_2^2 + \beta e_3^2 \ , \qquad e_k = \mathrm{Tr}[g^{-1} dg J_k] \ , \qquad g \in SU(2) \ .$$

- Broken $\mathfrak{su}(2)_R$ symmetry gets promoted to a Drinfel'd-Jimbo **quantum group**

$$[J_3, J_\pm] = 2J_\pm \ , \qquad [J_+, J_-] = \frac{q^{J_3} - q^{-J_3}}{q - q^{-1}} \ .$$

- Deforms the geometry locally ($\neq$ twisting)

- Quantum deformations can be generalised to cosets and **supercosets**

[Klimcik '02 '08] [Delduc, Magro, Vicedo '13 '14] [Hoare '14] [FS '19]

- Quantum deformations can be generalised to cosets and **supercosets**

  [Klimcik '02 '08] [Delduc, Magro, Vicedo '13 '14] [Hoare '14] [FS '19]

- Deformation of the $AdS_3 \times S^3 \times T^4$ and $AdS_3 \times S^3 \times S^3 \times S^1$ superstring

- Quantum deformations can be generalised to cosets and **supercosets**

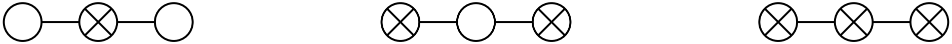
  [Klimcik '02 '08] [Delduc, Magro, Vicedo '13 '14] [Hoare '14] [FS '19]

- Deformation of the $AdS_3 \times S^3 \times T^4$ and $AdS_3 \times S^3 \times S^3 \times S^1$ superstring
- Preserves 8 SUSY if deform only $\mathfrak{psu}(1,1|2)_R$ or $\mathfrak{d}(2,1;\alpha)_R$ copy

- Quantum deformations can be generalised to cosets and **supercosets**

  [Klimcik '02 '08] [Delduc, Magro, Vicedo '13 '14] [Hoare '14] [FS '19]

- Deformation of the $AdS_3 \times S^3 \times T^4$ and $AdS_3 \times S^3 \times S^3 \times S^1$ superstring
- Preserves 8 SUSY if deform only $\mathfrak{psu}(1,1|2)_R$ or $\mathfrak{d}(2,1;\alpha)_R$ copy
- Inequivalent deformations captured by different Dynkin diagrams



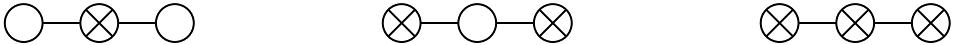
## Trigonometric deformation



- Quantum deformations can be generalised to cosets and **supercosets**

  [Klimcik '02 '08] [Delduc, Magro, Vicedo '13 '14] [Hoare '14] [FS '19]

- Deformation of the $AdS_3 \times S^3 \times T^4$ and $AdS_3 \times S^3 \times S^3 \times S^1$ superstring
- Preserves 8 SUSY if deform only $\mathfrak{psu}(1,1|2)_R$ or $\mathfrak{d}(2,1;\alpha)_R$ copy
- Inequivalent deformations captured by different Dynkin diagrams



- Only the fully fermionic Dynkin diagram leads to a supergravity background

  [Borsato Wulff '16] [Hoare, FS '18] [FS '19]

- Elliptic deformation introduces an additional deformation parameter [Cherednik '81]

$$ds^2 = \beta_1 e_1^2 + \beta_2 e_2^2 + \beta_3 e_3^2$$

# Elliptic deformation of $S^3$

- Elliptic deformation introduces an additional deformation parameter [Cherednik '81]

$$ds^2 = \beta_1 e_1^2 + \beta_2 e_2^2 + \beta_3 e_3^2$$

- Generalised to $AdS_3$ but **not yet** to (super)coset models

[Lacroix, Wallberg '24] [Hoare, Retore, FS '24]

- Elliptic deformation introduces an additional deformation parameter [Cherednik '81]

$$ds^2 = \beta_1 e_1^2 + \beta_2 e_2^2 + \beta_3 e_3^2$$

- Generalised to $AdS_3$ but **not yet** to (super)coset models

  [Lacroix, Wallberg '24] [Hoare, Retore, FS '24]

- Embedding of the elliptic $AdS_3 \times S^3 \times T^4$ geometry into supergravity

  [Hoare Retore FS '24] [Hoare FS '25]

- Ansatz for the RR fluxes

$$\Phi = 0 \;, \qquad H_3 = 0 \;, \qquad F_1 = 0 \;, \qquad F_3 = F_3^{(4)} \;, \qquad F_5 = \textstyle\sum_{i=1}^{3} F_3^{(i)} \wedge J_2^{(i)} \;.$$

$$F_3^{(i)} = \mathrm{x}_i^{(1)} f_3^{(1)} + \mathrm{x}_i^{(2)} f_3^{(2)} + \mathrm{x}_i^{(3)} f_3^{(3)} + \mathrm{x}_i^{(4)} f_3^{(4)} \;, \qquad j = 1, 2, 3, 4 \;.$$

- Basis of self-dual and orthogonal three-forms

$$f_3^{(1)} = 2(-e^0 \wedge e^2 \wedge e^4 + e^1 \wedge e^3 \wedge e^5) \;, \qquad f_3^{(2)} = 2(-e^1 \wedge e^2 \wedge e^3 - e^0 \wedge e^4 \wedge e^5) \;,$$
$$f_3^{(3)} = 2(-e^0 \wedge e^1 \wedge e^5 + e^2 \wedge e^3 \wedge e^4) \;, \qquad f_3^{(4)} = 2(e^0 \wedge e^1 \wedge e^2 + e^3 \wedge e^4 \wedge e^5) \;.$$

- Basis of self-dual and orthogonal two-forms on torus

$$J_2^{(1)} = e^6 \wedge e^7 - e^8 \wedge e^9 \;, \qquad J_2^{(2)} = e^6 \wedge e^8 + e^7 \wedge e^9 \;, \qquad J_2^{(3)} = e^6 \wedge e^9 - e^7 \wedge e^8 \;,$$

- Supergravity imposes constraints on norms and scalar products of $\mathbf{x}^{(j)}$

($\rightarrow$ 10 parameters in the RR fluxes)

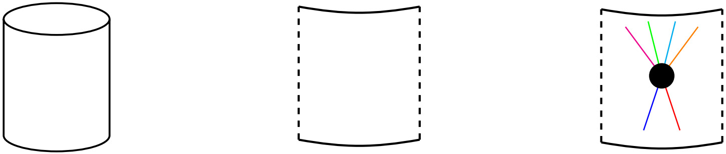
- Supergravity imposes constraints on norms and scalar products of $\mathbf{x}^{(j)}$
(→ 10 parameters in the RR fluxes)
- Background preserves 8 SUSY $\Leftrightarrow$ vectors $\mathbf{x}^{(j)}$ lie in a two-plane
(→ 8 parameters in the RR fluxes)

- Supergravity imposes constraints on norms and scalar products of $\mathbf{x}^{(j)}$ ($\to$ 10 parameters in the RR fluxes)
- Background preserves 8 SUSY $\Leftrightarrow$ vectors $\mathbf{x}^{(j)}$ lie in a two-plane ($\to$ 8 parameters in the RR fluxes)
- Killing vectors and spinors generate a $\mathfrak{psu}(1,1|2)$ superalgebra

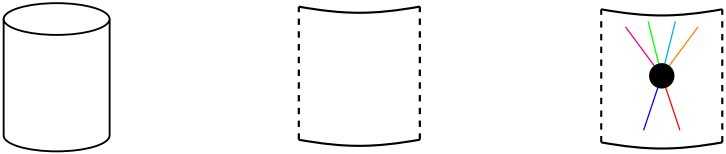
- Supergravity imposes constraints on norms and scalar products of $\mathbf{x}^{(j)}$
(→ 10 parameters in the RR fluxes)
- Background preserves 8 SUSY ⇔ vectors $\mathbf{x}^{(j)}$ lie in a two-plane
(→ 8 parameters in the RR fluxes)
- Killing vectors and spinors generate a $\mathfrak{psu}(1,1|2)$ superalgebra
- Bosonic model is integrable. **What about the superstring?**

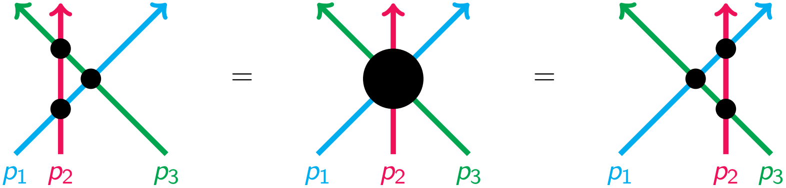
# Plan

String sigma model → Gauge-fixed model → Massive 2D QFT → Scattering matrix



String sigma model → Gauge-fixed model → Massive 2D QFT → Scattering matrix





- Light-cone coordinates

$$X^+ = \frac{1}{2}(T + \Phi) \ , \qquad X^- = T - \Phi \ .$$

- Uniform light-cone gauge ($\leftrightarrow$ static gauge in the T-dual frame)

$$X^+ = \tau \ , \qquad P_- = 1 \ .$$

- Light-cone Hamiltonian

$$H = -\int d\sigma \, P_+(\mathcal{X}, P_{\mathcal{X}}, \acute{\mathcal{X}})$$

- For $AdS_3 \times S^3 \times T^4$: non-relativistic theory for

$$4 \text{ massive B.} + 4 \text{ massive F.} + 4 \text{ massless B.} + 4 \text{ massless F.}$$

- For $AdS_3 \times S^3 \times T^4$: non-relativistic theory for

$$4 \text{ massive B.} + 4 \text{ massive F.} + 4 \text{ massless B.} + 4 \text{ massless F.}$$

- S-matrix bootstrapped using symmetries:

$$\Delta_{21}(X)\,\mathbb{S} = \mathbb{S}\,\Delta_{12}(X)\ , \qquad \forall X \in \mathcal{A}$$

where $\mathcal{A}$ is the symmetry surviving the light-cone gauge-fixing and $\Delta(X)$ is the coproduct

[Borsato, Hoare, Ohlsson-Sax, Stefanski, Sfondrini, Torrielli, Tseytlin, Wulff ...]

- For $AdS_3 \times S^3 \times T^4$: non-relativistic theory for

$$4 \text{ massive B.} + 4 \text{ massive F.} + 4 \text{ massless B.} + 4 \text{ massless F.}$$

- S-matrix bootstrapped using symmetries:

$$\Delta_{21}(X)\,\mathbb{S} = \mathbb{S}\,\Delta_{12}(X)\ , \qquad \forall X \in \mathcal{A}$$

where $\mathcal{A}$ is the symmetry surviving the light-cone gauge-fixing and $\Delta(X)$ is the coproduct

[Borsato, Hoare, Ohlsson-Sax, Stefanski, Sfondrini, Torrielli, Tseytlin, Wulff ...]

- Satisfies the Yang-Baxter equation

- Different choices of light-cone coordinates correspond to $T\bar{T}, J\bar{T}$ etc. deformations

[Frolov '19] [van Tongeren, Sfondrini '19] [Borsato, Driezen, Hoare, Retore, FS '23]

- Different choices of light-cone coordinates correspond to $T\bar{T}, J\bar{T}$ etc. deformations

[Frolov '19] [van Tongeren, Sfondrini '19] [Borsato, Driezen, Hoare, Retore, FS '23]

- Homogeneous YB (ex: TsT transformations) correspond to a **Drinfel'd twist**

$$\mathbb{S}_{YB} = \mathcal{F}^{-1}\,\mathbb{S}\mathcal{F}, \qquad (\mathcal{F}^{-1}\otimes 1)(\Delta\otimes 1)\mathcal{F}^{-1} = (1\otimes\mathcal{F}^{-1})(1\otimes\Delta)\mathcal{F}^{-1}\,.$$

[van Tongeren '15]

## S-matrix in deformed theories



- Different choices of light-cone coordinates correspond to $T\bar{T}, J\bar{T}$ etc. deformations

[Frolov '19] [van Tongeren, Sfondrini '19] [Borsato, Driezen, Hoare, Retore, FS '23]

- Homogeneous YB (ex: TsT transformations) correspond to a **Drinfel'd twist**

$$\mathbb{S}_{YB} = \mathcal{F}^{-1}\,\mathbb{S}\mathcal{F}, \qquad (\mathcal{F}^{-1}\otimes 1)(\Delta\otimes 1)\mathcal{F}^{-1} = (1\otimes\mathcal{F}^{-1})(1\otimes\Delta)\mathcal{F}^{-1}\,.$$

[van Tongeren '15]

- For trigonometric deformation can exploit the hidden Drinfel'd-Jimbo **quantum group**

[Beisert, Koroteev '08] [Hoare 14] [FS, van Tongeren, Zimmermann '21]

- For elliptic deformation: symmetries and exact S-matrix **not yet known**

- For elliptic deformation: symmetries and exact S-matrix **not yet known**
- Focus on perturbative tree-level S-matrix in the massive sector

$$H = H_2 + H_3 + H_4 + \ldots \ , \qquad \mathbb{S} = 1 + i\mathbb{T} + \ldots$$

- For elliptic deformation: symmetries and exact S-matrix **not yet known**
- Focus on perturbative tree-level S-matrix in the massive sector

$$H = H_2 + H_3 + H_4 + \dots \ , \qquad \mathbb{S} = 1 + i\mathbb{T} + \dots$$

- Satisfies the classical Yang-Baxter equation ($\rightarrow$ **integrability**) [Hoare FS '25]

# Towards an exact S-matrix for the elliptic deformation 

- For elliptic deformation: symmetries and exact S-matrix **not yet known**
- Focus on perturbative tree-level S-matrix in the massive sector

$$H = H_2 + H_3 + H_4 + \dots \,, \qquad \mathbb{S} = 1 + i\mathbb{T} + \dots$$

- Satisfies the classical Yang-Baxter equation ($\rightarrow$ **integrability**) [Hoare FS '25]
- Completely new form, reminescent of an 8-vertex model

- For elliptic deformation: symmetries and exact S-matrix **not yet known**
- Focus on perturbative tree-level S-matrix in the massive sector

$$H = H_2 + H_3 + H_4 + \dots \,, \qquad \mathbb{S} = 1 + i\mathbb{T} + \dots$$

- Satisfies the classical Yang-Baxter equation ($\rightarrow$ **integrability**) [Hoare FS '25]
- Completely new form, reminescent of an 8-vertex model
- In a limit goes to the tree-level S-matrix on $AdS_2 \times S^2$ [Abbott, Murugan, Sundin, Wulff '13]

# Conclusions

- Very rich space of integrable deformations of $AdS_3$ strings
- Some preserve 8 supersymmetries
- Some interpolate between known integrable backgrounds
- Constructed a new elliptic deformation of the $AdS_3 \times S^3 \times T^4$ superstring
- Tree-level S-matrix compatible with integrability, new interesting features

- Deformed action, symmetries and exact S-matrix in the elliptic deformation?
  [Costello, Delduc, Klimcik, Magro, Vicedo, Witten, Yamazaki, . . . ]
- Relation to exact S-matrices constructed by bootstrapping the YBE?
  [de Leeuw, Pribytok, Retore, Ryan . . . ]
- Relation to other integrable interpolating S-matrices? [Torrielli '25]
- Bethe Ansatz, TBA and QSC for deformed $AdS_3$?
  [Arutyunov, Klabbers, van Tongeren, FS, Sfondrini . . . ]
- Integrability in other families of susy backgrounds with warped $AdS_3$ and $S^3$ geometry?
  [Donos, Gauntlett, Sparks '09] [Maurelli, Noris, Oyarzo, Samtleben, Trigiante '25]
- Lift the deformation to the brane setup? [Lozano, Nunez, Ramirez, Skenderis, . . . ]
- Interpretation in holography?

Thank you !