

A BPS Road to Holography & Matrix Theory

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Foundation

A BPS Road to Holography & Matrix Theory

based on 2410.03591, 2506.19720 with

Chris Blair, Johannes Lahnsteiner, Niels Obers

& work in progress with Dawid Maskalaniec & Utku Zorba

based also on other recent works + upcoming with

Stefano Baiguera, Eric Bergshoeff, Jan de Boer, Paolo Di Vecchia, Stephen Ebert, Joaquim Gomis, Troels Harmark, Kevin Grosvenor, Vendela Kennemyr, Viggo Kraft, Yang Lei, Luca Romano, Evangelos Tsolakidis

see also Troels Harmark's talk tomorrow

Non-Lorentzian Perspective on AdS/CFT

what is non-Lorentzian?

asymptotic behaviors of e.g. $AdS_5 \times S^5$

why is this perspective important?

BPS nature of holography & matrix theory

what can we do with it?

classify holography in string theory

revisit BFSS conjecture for matrix theory

quantum critical membrane
 $S_{\text{susy}} = \frac{1}{2g^2} \int dt d\vec{x} d\theta [\bar{D}^\alpha \gamma^I D_\alpha \gamma^J G_{IJ}(Y) - 2\partial_\alpha Y \partial^\alpha Y]$

beta-functions

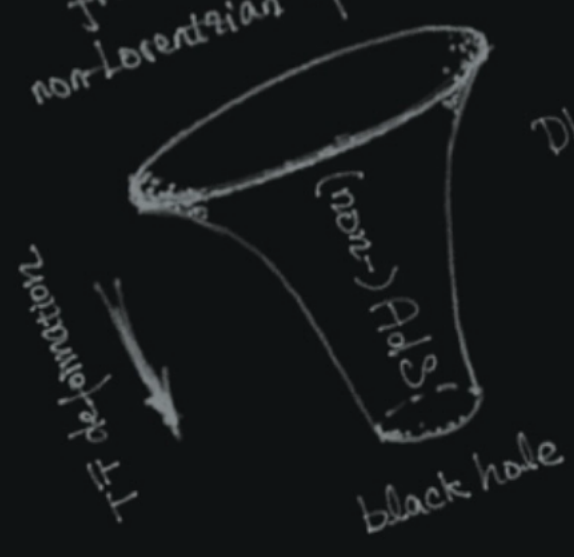
$$\beta_{IJ}^G = \frac{g^2}{4\pi} [-H^{KL} R_{KL(I} G_{J)M} - \frac{1}{2} G_{M(I} \nabla_{J)} (H^{KL} S_{KL}^M)]$$

$$\beta_{IJ}^H = \frac{g^2}{4\pi} [\Sigma_{IJ} + \frac{1}{2} H^{KL} H_{M(I} \Delta_{J)} S_{KL}^M]$$

matrix quantum mechanics for M-theory

$$H_{\text{BFSS}} = R \text{tr} \left(\frac{\pi i \pi_i}{2} + \frac{1}{4} [\gamma^i, \gamma^j]^2 + \theta^\tau \chi_i [\theta, \gamma^i] \right)$$

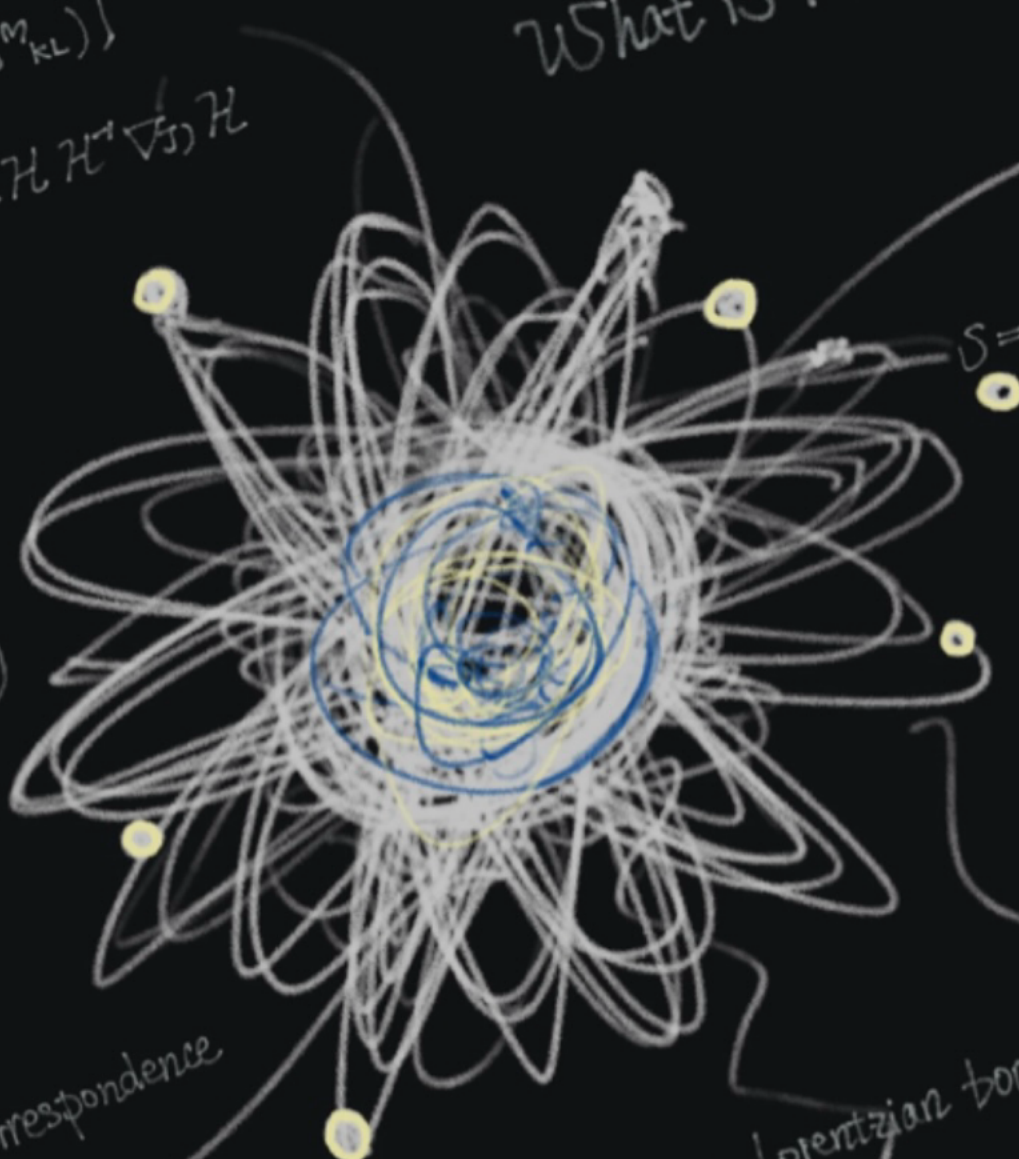
field theory in non-Lorentzian spacetime



DLCQ/DLCQ^m correspondence



What is M-theory?



matrix p-brane theory

$$S = \frac{i}{2} \int d^2\sigma (-\partial_\sigma X^A \partial_\sigma X_A + \partial_\sigma X^A \partial_\tau X_A + \lambda_A \partial_\sigma X^A)$$

nonrelativistic string theory

$$S = \frac{i}{2} \int d\tau (a_\alpha X^A \partial_\tau X^A + \lambda \bar{\psi} \psi + \bar{\lambda} \partial_\tau \bar{\psi})$$

supergravity from invariant theory

$$24 S_4^{(3)} x^4 + 24 S_3^{(3)} x^3 y + 12 S_2^{(3)} x^2 y^2 + 4 S_1^{(3)} x y^3 + S_0^{(3)} y^4$$

$$2 S_2^{(5)} x^2 + 2 S_1^{(5)} x y + S_0^{(5)} y^2$$

$$I_N = \sum_{m=0}^{N-1} (-1)^m S_m S_{N-m-1}$$

non-Lorentzian bootstrap?

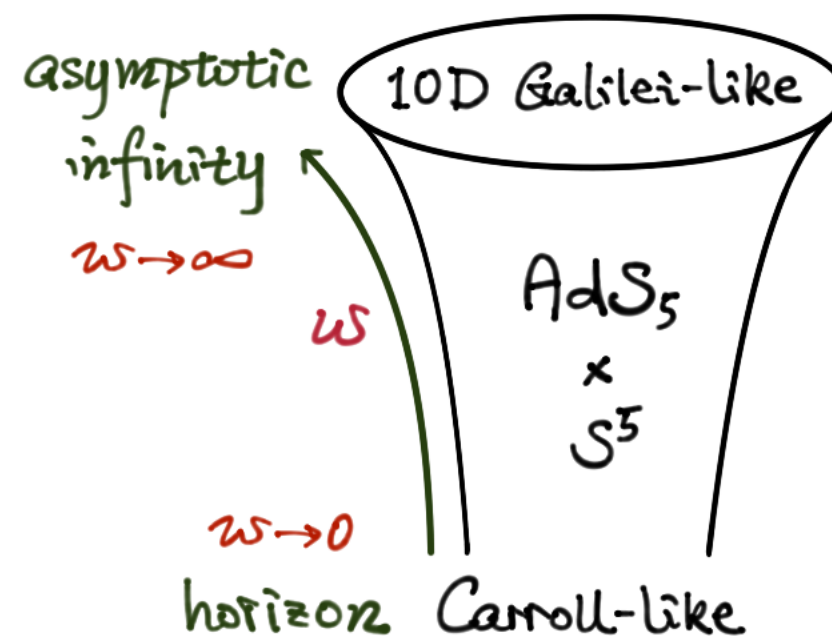
Holography

Asymptotics of $AdS_5 \times S^5$

* classic example: AdS_5/CFT_4

$$ds^2 = \omega (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{1}{\omega} (dr^2 + r^2 d\Omega_5^2)$$

$$C_4 = \frac{\omega^2}{g_s} dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \quad \omega = \frac{r^2}{l^2}$$



why non-Lorentzian? no 10D metric description

manifest Galilei-like / Carroll-like boost symmetry?

Probe D3-Brane

$$S_{D3} = - \frac{1}{g_s \alpha'^2} \int d^4 \sigma \sqrt{-\det \left(\omega \eta_{\alpha\beta} + \frac{1}{\omega} \partial_\alpha X^i \partial_\beta X^i + F_{\alpha\beta} \right)} + \frac{\omega^2}{g_s \alpha'^2} \int d^4 \sigma$$

$i=4, \dots, 9$

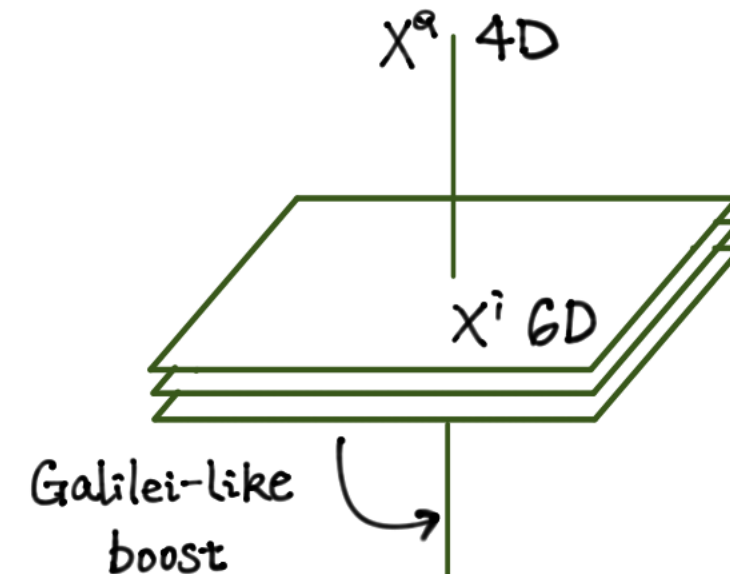
$$\xrightarrow[\text{BPS}]{\omega \rightarrow \infty} - \frac{1}{g_s \alpha'^2} \int d^4 X \left(\frac{1}{2} \partial_a X^i \partial^a X^i + \frac{1}{4} F_{ab} F^{ab} \right)$$

static gauge
 $X^{0,\dots,3} = \sigma^{0,\dots,3}$

non-abelianization $\leadsto \mathcal{N}=4$ SYM

Galilei-like boost $\begin{cases} \delta X^a = 0 \\ \delta X^i = v^i_a X^a \end{cases}$

$$\delta S_{D3} = - \frac{2}{g_s \alpha'^2} \int d^4 X v^i_a \partial^a X^i$$



$\mathcal{N}=4$ SYM sees 10D Galilei-like geometry [Blair, Lahnsteiner, Obers, ZY '24] [Guijosa '25]

Relation to Matrix Theory

[de Wit, Hope, Nicolai '88]

[Banks, Fischler, Shenker, Susskind '96]

D-particle case: $\mathcal{N}=4$ SYM on $T^3 \xrightarrow{\text{T-dual}} \text{BFSS matrix theory}$

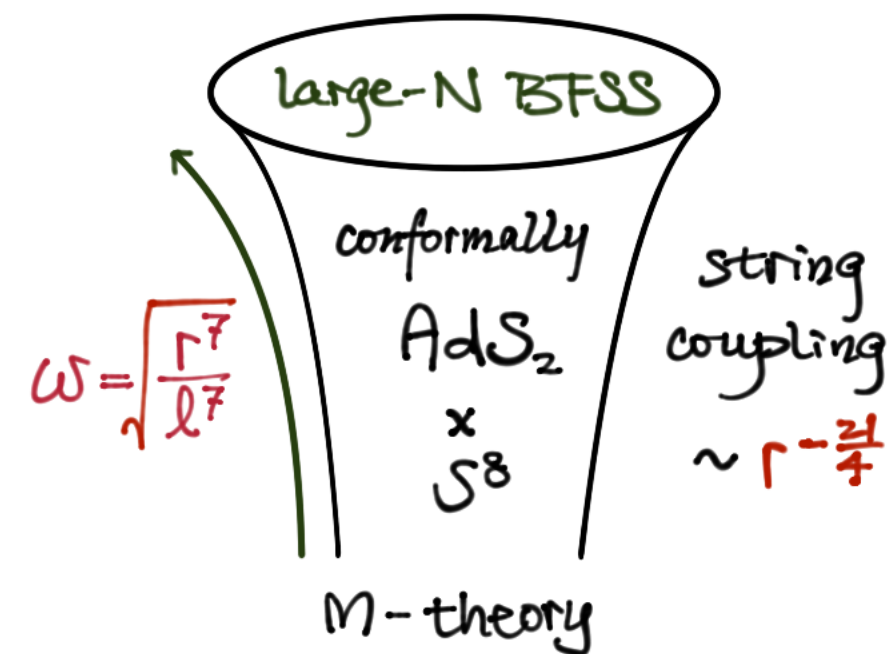


Newton-like instantaneous force

$$ds^2 = -\omega dt^2 + \frac{1}{\omega} (dr^2 + r^2 d\Omega_8^2)$$

$$C_1 = \frac{\omega^2}{g_s} dt$$

$$e^{\Phi} = \omega^{-\frac{3}{2}} g_s$$



[Itzhaki, Maldacena, Sonnenschein, Yankielowicz '98]

Probe D0-Brane

assume ω is constant

$$ds^2 = -\omega dt^2 + \frac{1}{\omega}(dr^2 + r^2 d\Omega_8^2)$$

$$C_1 = \frac{\omega^2}{g_s} dt$$

$$e^{\Phi} = \omega^{-\frac{3}{2}} g_s$$

limit of type IIA superstring

$\omega \rightarrow \infty$: infinite speed of light c

Matrix 0-brane Theory (MOT)

[Blair, Lahnsteiner, Obers, ZY '23]

[Gomis, ZY '23]

$$S_{D0} = -mc \int d\tau \sqrt{c^2 \dot{X}_0^2 - \dot{X}_i^2} + \frac{1}{\sqrt{\alpha'}} \int d\tau \frac{c^2}{g_s} \dot{X}^0$$

$$m = \frac{1}{g_s \sqrt{\alpha'}}$$

$$= -\frac{1}{\sqrt{\alpha'}} \int d\tau \frac{1}{c^{-\frac{3}{2}} g_s} \sqrt{c \dot{X}_0^2 - \frac{1}{c} \dot{X}_i^2} + \frac{1}{\sqrt{\alpha'}} \int d\tau \frac{c^2}{g_s} \dot{X}^0$$

static gauge
 $X^0 = \tau$

$$\xrightarrow[\text{BPS}]{c \rightarrow \infty} \frac{1}{g_s \sqrt{\alpha'}} \int d\tau \frac{1}{2} \dot{X}_i^2 \xrightarrow{\text{non-abelian}} \text{BFSS matrix theory}$$

[de Wit, Hope, Nicolai '88]

[Banks, Fischler, Shenker, Susskind '96]

Probe String

fundamental string in M0T

$$S = -\frac{T}{2} \int d^2\sigma \partial_\alpha X^\mu \partial^\alpha X_\mu$$

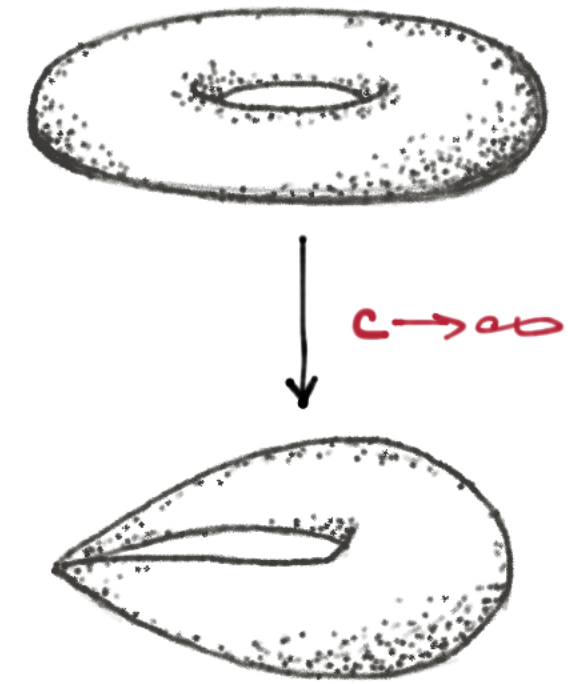
$$\xrightarrow{c \rightarrow \infty} -\frac{T}{2} \int d^2\sigma \left(\lambda \partial_\sigma X^0 + \partial_\tau X^0 \partial_\tau X^0 + \partial_\sigma X^i \partial_\sigma X^i \right)$$

[Gomis, ZY '23] fundamental string in M0T

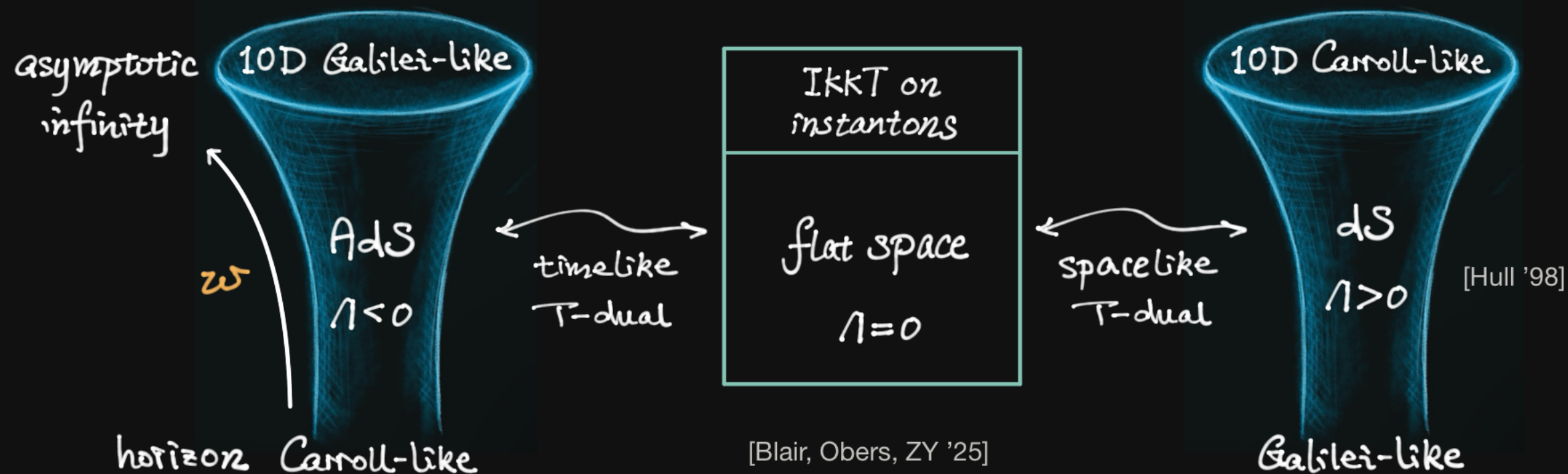
T-dualize X^0 : $S = -\frac{T}{2} \int d^2\sigma \partial_\sigma X^\mu \partial_\sigma X_\mu$

tensionless string

[Isberg, Lindström, Sundborg, Theodoridis '93]



Holography through Looking-Glass

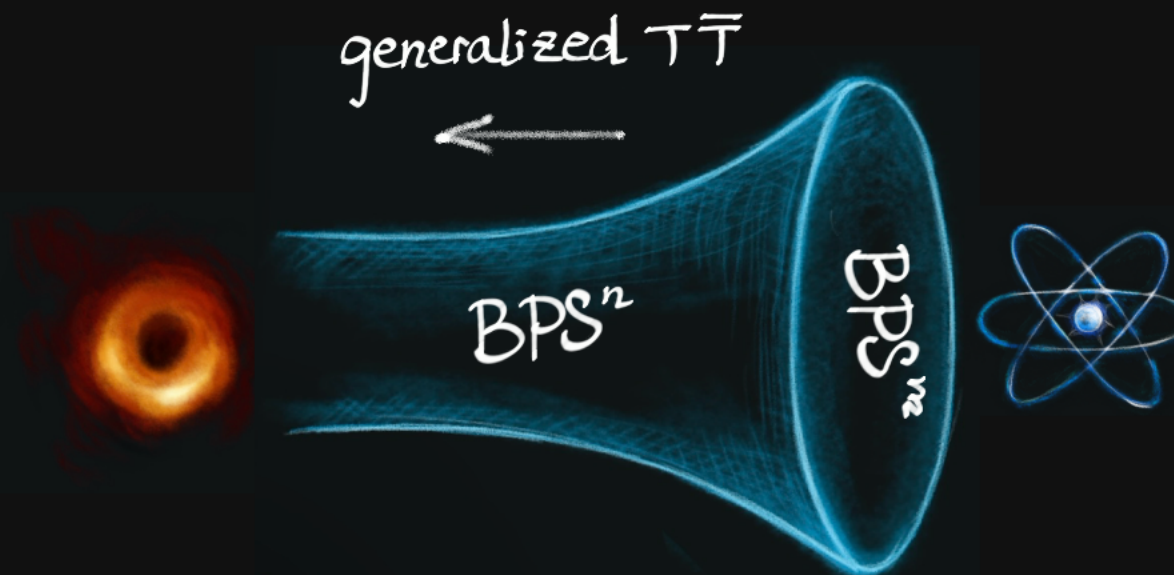


consecutive BPS limits

[De Boer, Dijkgraaf, Harmark, Obers]
[Blair, Lahnsteiner, Obers, ZY '24]



$m \rightarrow n$
near-horizon
limits



non-Lorentzian holography from top-down?

[Lambert, Smith '24] [Blair, Lahnsteiner, Obers, ZY '24 '25] [Harmark, Lahnsteiner, Obers '25] ...
also a different proposal in [Fontanella, Nieto García '24]

Intermezzo: M-Theory Uplift

DLCQ M-Theory

Discrete Light Cone Quantization

$$ds_{11}^2 = e^{\frac{4}{3}\Phi} (R_s^{-1} dy + C_1)^2 + e^{-\frac{2}{3}\Phi} ds_{10}^2$$

compactify
 $R_s = g_s l_s$  y

$\omega \rightarrow \infty$
 infinite boost

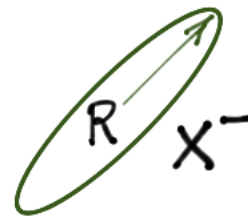
$$ds_{10}^2 = -2\omega dt^2 + \frac{1}{\omega} (dr^2 + r^2 d\Omega_8^2)$$

$$C_1 = \frac{\omega^2}{g_s} dt$$

$$e^{\Phi} = \omega^{-\frac{3}{2}} g_s$$

$\omega \rightarrow \infty$

MOT



null compactification

DLCQ M-Theory

$$ds_{11}^2 = 2 dX^- dX^+ + dx^i dx^i$$

$$(y, t, x^i) = l_p \left(X^-, \frac{X^+}{l_s}, \frac{x^i}{l_s} \right)$$

Kaluza-Klein modes in X^- : BFSS matrix theory

quantum critical membrane

$$S_{\text{susy}} = \frac{1}{2g^2} \int dt d\vec{x} d\theta [\bar{D}^\alpha \gamma^I D_\alpha \gamma^J G_{IJ}(\gamma) - 2\partial_\alpha \gamma^\alpha \partial_\alpha \gamma^\alpha]$$

beta-functions

$$\beta_{IJ}^G = \frac{g^2}{4\pi} [-H^{KL} R_{KL(I} G_{J)M} - \frac{1}{2} G_{M(I} \nabla_{J)} (H^{KL} S_{KL}^M)]$$

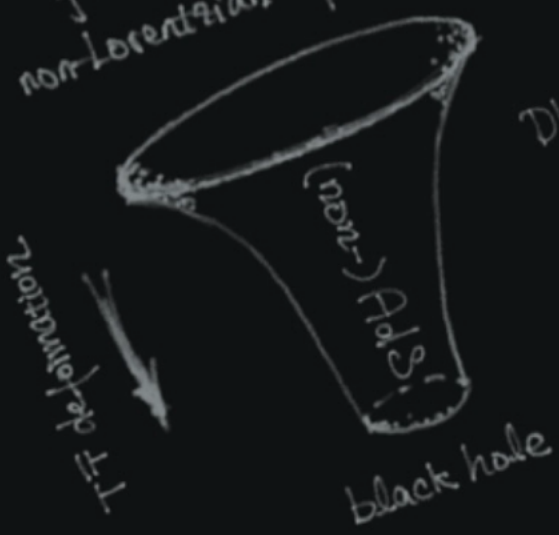
$$-\frac{g^2}{8\pi} \text{tr} [\mathcal{H}^{-1} (\mathcal{H} + \tilde{\mathcal{H}})^{-1} \nabla_{(I} \mathcal{H} \mathcal{H}^{\dagger} \nabla_{J)} \mathcal{H}]$$

$$\beta_{IJ}^H = \frac{g^2}{4\pi} [\Sigma_{IJ} + \frac{1}{2} H^{KL} H_{M(I} \Delta_{J)} S_{KL}^M]$$

matrix quantum mechanics for M-theory

$$H_{\text{BFSS}} = R \text{tr} \left(\frac{\pi i \pi_i}{2} + \frac{1}{4} [\gamma^i, \gamma^j]^2 + \theta^\tau \chi_i [\theta, \gamma^i] \right)$$

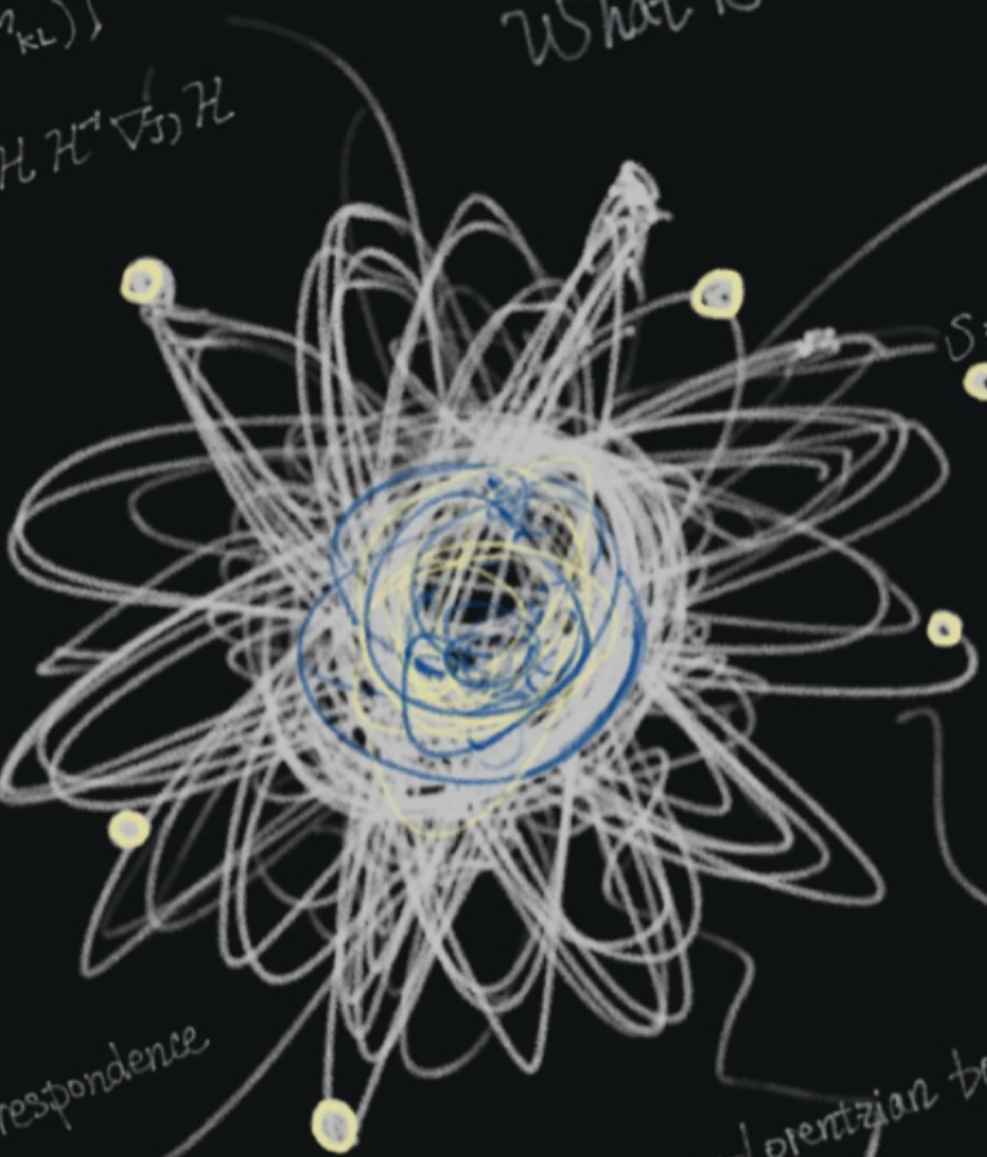
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$$S = \frac{i}{2} \int d\sigma (\partial_\sigma X^A \partial_\sigma X^A + \lambda \bar{\partial} X + \bar{\lambda} \partial X)$$

supergravity from invariant theory

$$24 S_4^{(3)} x^4 + 24 S_3^{(3)} x^3 y + 12 S_2^{(3)} x^2 y^2 + 4 S_1^{(3)} x y^3 + S_0^{(3)} y^4$$

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$$I_N = \sum_{m=0}^{N-1} (-1)^m S_m S_{N-m-1}$$

non-Lorentzian bootstrap?

Matrix Theory

BFSS Matrix Theory Conjecture

* bound-state of N D0-particles in M0J

$$S_{\text{BFSS}} = \frac{1}{R} \int d\tau \operatorname{tr} \left\{ \frac{1}{2} \dot{X}^i{}^2 + \frac{1}{4} [X^i, X^j]^2 + \text{fermionic sector} \right\}$$

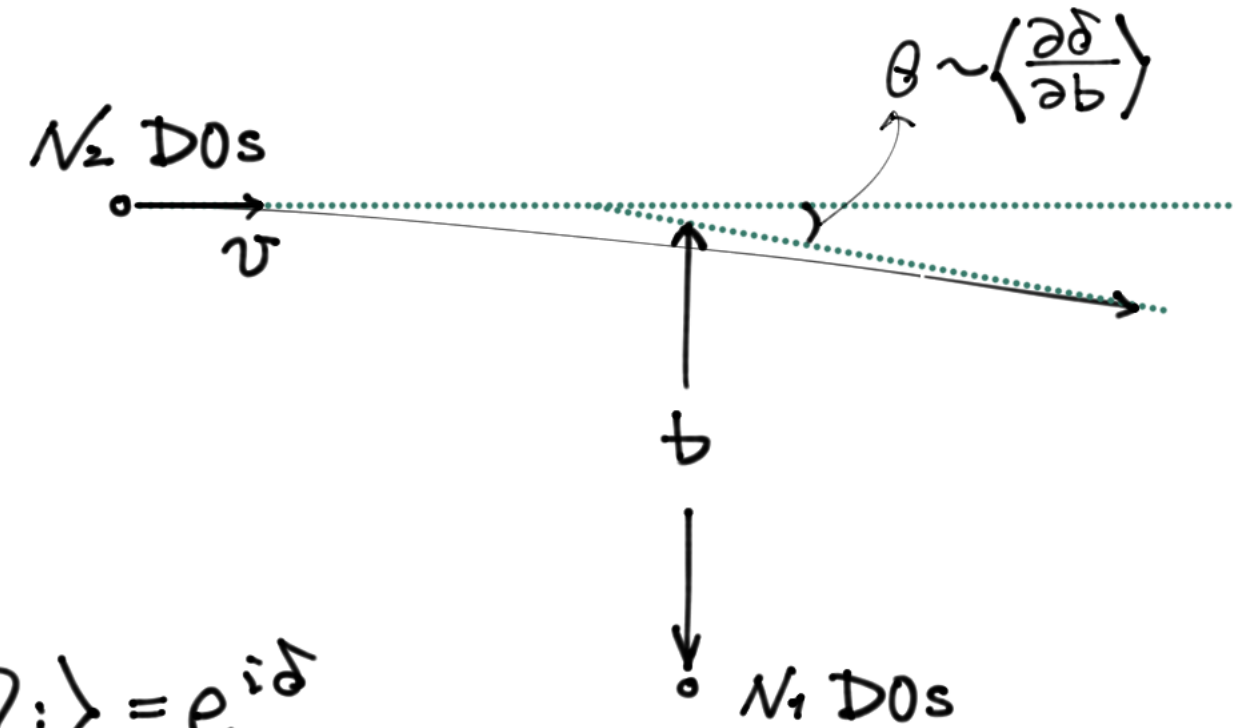
* supergraviton with $P_- = \frac{N}{R}$ in DLCQ M-theory

$$ds_{11}^2 = 2 dX^- dX^+ + dX^i dX^i$$


$N \rightarrow \infty$: P_- becomes continuous $\rightarrow$ decompactification

BFSS matrix theory $\xrightarrow{N \rightarrow \infty}$ M-theory in flat spacetime

Test: Eikonal Scattering



* perturbative BFSS matrix theory

amplitude $\mathcal{A} = \lim_{t \rightarrow \infty} \langle \Omega_f | e^{-i H_{\text{BFSS}} t} | \Omega_i \rangle = e^{i\delta}$

$$\delta = \int dt V(b^2 + v^2 t^2) \quad V(v) = \frac{15}{16} N_1 N_2 \frac{v^4}{17} + \dots$$

effective coupling $g_{\text{eff}} \sim \frac{N R l_s^2}{b^3} \quad R = g_s l_s$

valid for finite N & small R : 11D SUGRA invalid

however, SUGRA seems to work for $2 \rightarrow 2$ scattering...

Supergraviton Scattering

* perturbative 11D supergravity

Aichelburg-Sexl metric $\omega^2 = \frac{r^7}{l^7}$

$$ds_{11}^2 = 2dX^+dX^- + dx^i dx^i + \frac{1}{\omega^2} (dX^-)^2$$

spacelike

dim. reduction in X^- to $\mathbb{R}^4$

$$ds^2 = -2\omega dt^2 + \frac{1}{\omega} (dr^2 + r^2 d\Omega_8^2)$$

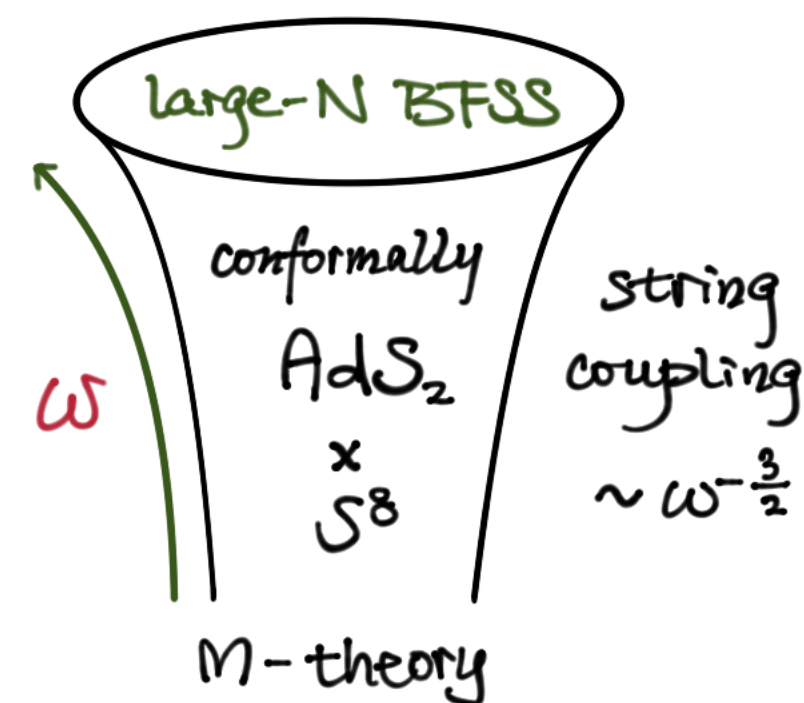
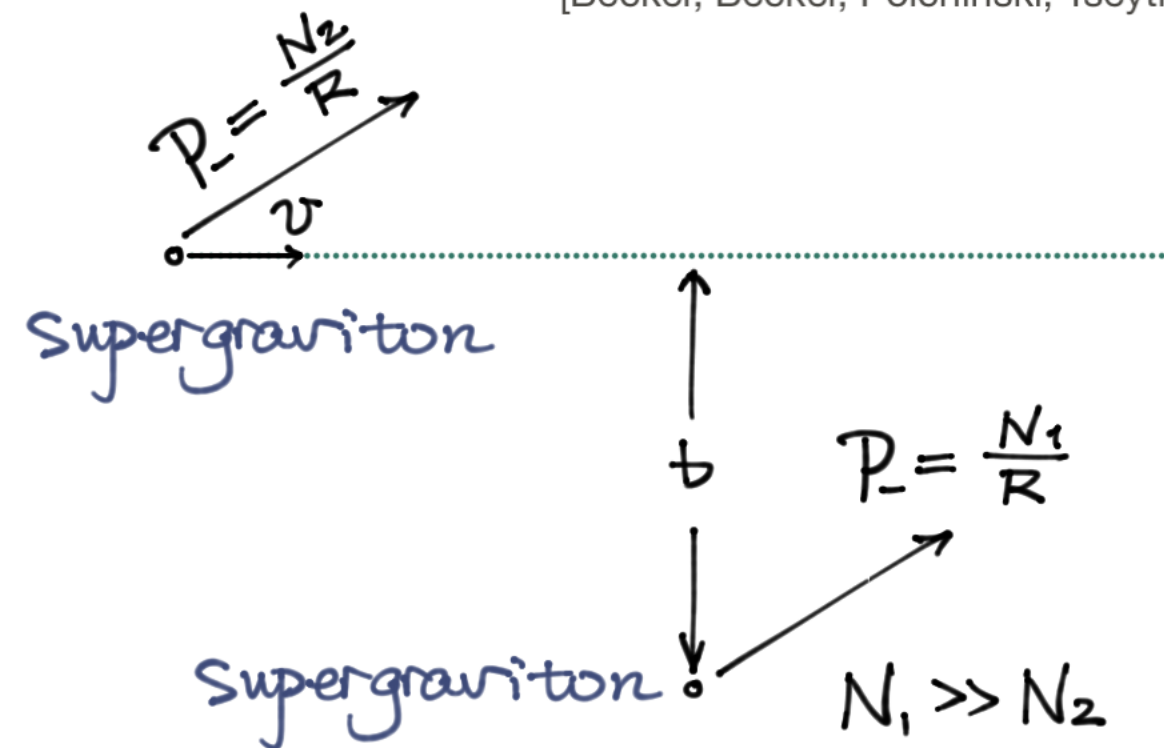
$$C_1 = \frac{2\omega^2}{g_s} dt$$

$$e^{\Phi} = \omega^{-\frac{3}{2}} g_s$$

$$\omega^2 = \frac{r^7}{l^7}$$

back to holography: $g_s \ll 1$, $g_{\text{eff}} \gg 1$

mismatches for $3 \rightarrow 3$ amplitudes... [Helling, Plefka, Serone, Waldron '99]



Non-Lorentzian Supergravity

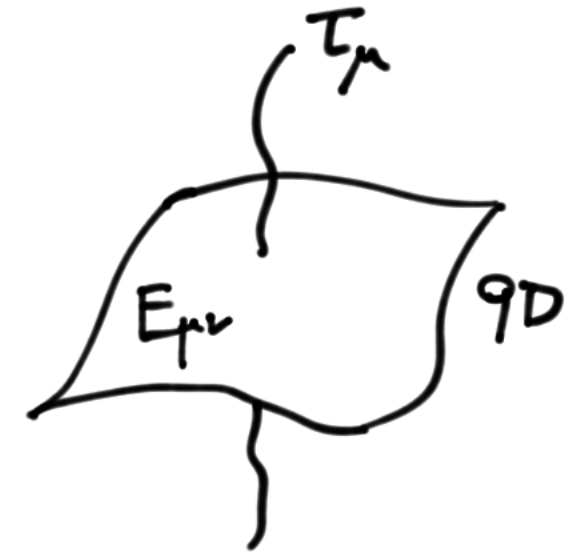
* what is perturbative/finite- N BFSS matrix theory?

11D SUGRA in DLCQ $\xrightarrow{\text{null reduction}}$ singular, but in an understandable way!

non-Lorentzian M0T SUGRA?

M0T limit
 $\omega \rightarrow \infty$
of IIA SUGRA

$$\begin{aligned} ds^2 &= -\omega (\tau_\mu dx^\mu)^2 + \frac{1}{\omega} E_{\mu\nu} dx^\mu dx^\nu \\ C_1 &= \omega^2 e^{-\Phi} \tau_\mu dx^\mu + \underbrace{V}_{\text{Newton potential}} \\ e^\Phi &= \omega^{-\frac{3}{2}} e^\varphi \end{aligned}$$



* finite- N BFSS: D0-particle worldline formalism for
10D non-Lorentzian quantum gravity

U-dual to matrix string: 2nd quantized non-rel. strings

[Motl '97]

[Dijkgraaf, Verlinde, Verlinde '97]

perturbative, unitary, UV-complete

[Klebanov, Maldacena '00]

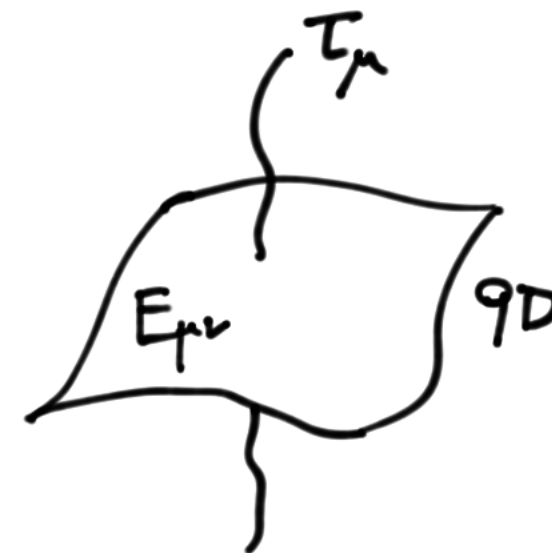
[Gomis, Ooguri '00]

[Danielsson, Guijosa, Kruczenski '00]

Effective Potential from Poisson Equation

IIA SUGRA coupled to D0-particle

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[e^{-2\Phi} (R + 4\partial_\mu \Phi \partial^\mu \Phi) - |dG|^2 \right] \\ - \frac{1}{l_s} \int ds e^{-\Phi} \sqrt{-\dot{x}^\mu \dot{x}^\nu G_{\mu\nu}} + \frac{1}{l_s} \int C_1$$



Poisson Eqn. $\lim_{\omega \rightarrow \infty} \frac{1}{\omega} \delta_\omega S = 0$

$$\Rightarrow 2e^\Phi \tau^\mu E^{\rho\sigma} \nabla_\rho (dV)_{\mu\sigma} + \dots = -\frac{\kappa_{10}^2}{2l_s} \int d\tau \frac{\delta^{(10)}(x - X(\tau))}{\det(\tau_\mu E_\mu^i)} e^\Phi \frac{(E_{\mu\nu} \dot{x}^\mu \dot{x}^\nu)^2}{(\tau_\rho \dot{x}^\rho)^3}$$

$$\Rightarrow V(\tau) = \frac{15}{16} \frac{v^4}{r^7} \rightarrow \text{lowest quantum correction in BFSS}$$

* this is only a proof of concept

* higher-order terms: perturbation theory of D0-branes in M0T sugra?

Summary

- * finite- N BFSS matrix theory: non-Lorentzian SUSRA
- * Large- N limit: AdS/CFT
- * unification of BPS decoupling limits in string/M-theory
- * implications for holography: flat space, de Sitter, non-Lorentzian...
- * undo BPS decoupling limit: $T\bar{T}$ deformation for D-branes
-

This is a new arena with many more to explore!

Thank You!