Better Alone

On the String Theory of a Single NS5-brane

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In collaboration with Emil Martinec — arXiv: 2506.22300

Decoupling Limit

	Bulk	Boundary
N **D3**	$AdS_5 \times S^5$ String Theory	4d $\mathcal{N} = 4$ SYM
n_5 **NS5**	**A**symptotically **L**inear **D**ilaton String Theory	6d **L**ittle **S**tring **T**heory

[Giveon, Kutasov '99]

Decoupling the NS5 branes

$$ds^2 = dx^2_{1,5} + \left(1 + \frac{n_5\,\alpha'}{r^2}\right)\left(dr^2 + r^2\,d\Omega^2_3\right)$$

$$e^{2\Phi} = g_s^2\left(1 + \frac{n_5\,\alpha'}{r^2}\right)$$

Decouple the dynamics on the NS5 brane from the bulk by $g_s \to 0 \quad r \to 0$

$$ds^2 = dx^2_{5,1} + n_5\,\alpha'\left(d\rho^2 + d\Omega^2_3\right)$$

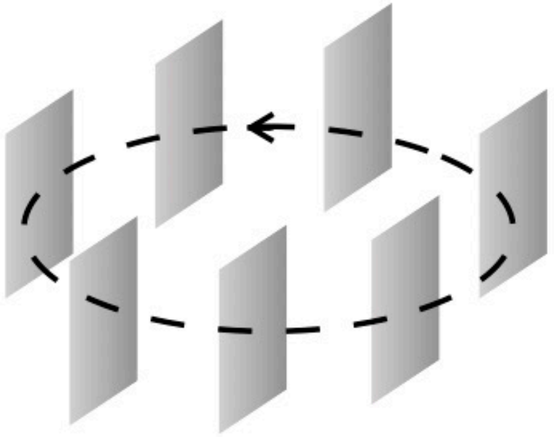
$$\Phi = -\,\rho$$

Worldsheet CFT

$$\mathbb{R}^{5,1} \times \mathbb{R}^{(1)}_{\rho} \times SU(2)^{(1)}_{n_5}$$

[Callan, Harvey, Strominger '91]

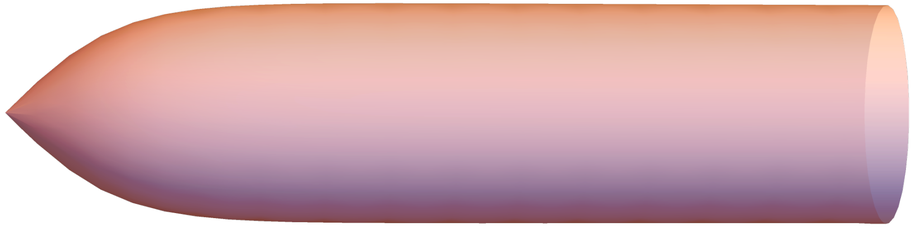
Separating the NS5 branes



$$g_s \to 0 \quad R \to 0 \quad \text{with} \quad \alpha' \text{ and } \frac{R}{g_s} \text{ fixed}$$

$$ds^2 = dx_{1,5}^2 + n_5\,\alpha'\left(d\rho^2 + \tanh^2\rho\, d\theta^2 + d\sigma^2 + \tan^2\sigma\, d\phi^2\right)$$

$$\Phi = \Phi_0 - \log\cosh\rho$$



$$R_\theta = \sqrt{n_5}\,\ell_s$$

[Giveon, Kutasov ‘99]

Worldsheet CFT

The resulting worldsheet CFT is

$$\left(\frac{SL(2,\mathbb{R})^{(1)}_{n_5}}{U(1)} \times \frac{SU(2)^{(1)}_{n_5}}{U(1)} \right) \Big/ \mathbb{Z}_{n_5} \quad \times \quad \mathbb{R}^{5,1}$$

[Giveon, Kutasov ‘99]

An equivalent formulation is the null gauged WZW model

$$\frac{SL(2,\mathbb{R})^{(1)}_{n_5} \times SU(2)^{(1)}_{n_5}}{U(1) \times U(1)} \quad \times \quad \mathbb{R}^{5,1}$$

[Israel, Kounnas, Pakman, Troost ’04; Martinec, Massai ’17; Martinec, Massai, Turton ’18-’25; AD, Martinec ’24]

Little String Theory

- **NOT** a local Quantum Field Theory

- **Strongly Coupled**: No small parameter in which to organize a perturbative expansion of the boundary theory. String tension and brane separation are the only scales left.

- **Non-Abelian**: multiple NS5 branes

The Virtue of Solitude

For a single NS5 brane, $n_5 = 1$, one expects Little String Theory:

- **No longer strongly-coupled**: the size of the linear dilaton throat is comparable to the string length and the string cannot probe the strongly coupled region.

- **Abelian**: we only have one brane

The Goal

- Define the **worldsheet** string theory of a **single NS5 brane**
- Identify its **boundary dual**
- Solve an **exact** incarnation of Little String Theory **holography**

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Define the **worldsheet** string theory of a **single NS5 brane**

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Hard to Be Alone

Naive Guess

$$\frac{SL(2,\mathbb{R})^{(1)}_{n_5} \times SU(2)^{(1)}_{n_5}}{U(1) \times U(1)} \quad \times \quad \mathbb{R}^{5,1}$$

Hard to Be Alone

Naive Guess

$$\frac{SL(2,\mathbb{R})^{(1)}_{n_5=1} \times SU(2)^{(1)}_{n_5=1}}{U(1) \times U(1)} \quad \times \quad \mathbb{R}^{5,1}$$

Problem

$$SU(2)^{(1)}_{n_5=1} = SU(2)_{n_5=-1} \oplus 3 \text{ free fermions} \quad \Longrightarrow \quad \textbf{Not unitary!}$$

A similar puzzle for tensionless $AdS_3 \times S^3 \times \mathbb{T}^4$

At $n_5 \geq 2$

$$SL(2,\mathbb{R})^{(1)}_{n_5} \times SU(2)^{(1)}_{n_5} \quad \longleftrightarrow \quad PSU(1,1\,|\,2)_{n_5}$$

Field content

- Bosonic $SL(2,\mathbb{R})_{n_5+2} \times SU(2)_{n_5-2}$
- 6 free fermions

- Bosonic $SL(2,\mathbb{R})_{n_5+2} \times SU(2)_{n_5-2}$
- 4 bc systems of dim (1,0)

[Berkovits, Vafa, Witten '99]

A similar puzzle for tensionless $AdS_3 \times S^3 \times \mathbb{T}^4$

At $n_5 = 1$ it is then natural to consider the worldsheet theory

$$PSU(1,1\,|\,2)_1 \times \mathbb{T}^4$$

[Eberhardt, Gaberdiel, Gopakumar '18]

$PSU(1,1\,|\,2)_1$ admits a free field realization in terms of

- 1 $\beta\gamma$ system with dim (1,0)
- 2 bc systems with dim (1,0)

[Beem, Ferrari '23; AD, Knighton, Naderi '23]

Like a Bad Scrabble Rack

Refined Guess

$$\frac{PSU(1,1\,|\,2)_1}{U(1)\times U(1)} \quad \times \quad \mathbb{R}^{5,1}$$

The current we are going to gauge cannot be written in terms of the free fields generating $PSU(1,1\,|\,2)_1$

Our alphabet is too small

$n_5 \geq 2$ and $n_5 = 1$ comparison

$PSU(1,1\,|\,2)_{n_5}$ is generated by

$n_5 \geq 2$

- Bosonic $SL(2,\mathbb{R})_{n_5+2} \times SU(2)_{n_5-2}$
- 4 bc systems of dim (1,0)

$n_5 = 1$

- 1 $\beta\gamma$ system with dim (1,0)
- 2 bc systems with dim (1,0)

Our strategy

- We insist on **extending the map** between hybrid and RNS and find a free field realization for bosonic $SL(2,\mathbb{R})_3 \times SU(2)_{-1}$ in terms of 3 $\beta\gamma$ systems

- We obtain a new free field realization of $PSU(1,1\,|\,2)_1$ in terms of 3 $\beta\gamma$ **systems** and **4** bc **systems**

- **Not Unitary** and **Not equivalent** to the tensionless string

- **Extra constraint** relating the 4 bc systems restores unitarity and equivalence with the tensionless string

A larger alphabet

We overcome the obstacle in three steps:

1. **Alternative free field realization** of $PSU(1,1\,|\,2)_1$ and construct a map to RNS variables

2. **Additional constraint** restores unitarity

3. **Complete equivalence** with the original formulation of the tensionless string

The Single Brane String Theory Unlocked

We provide a **worldsheet formulation** of the single NS5 brane string theory in terms of a **free field worldsheet CFT** subject to **three** mutually compatible **BRST constraints**:

- Familiar String BRST operator
- U(1) gauging BRST operator
- Unitarity BRST operator unique to $n_5 = 1$

Future directions

- **Dual 5+1d abelian gauge theory**: how does the holographic map work in detail?
- **Solve the BRST cohomology**: what is the spacetime symmetry algebra?
- **Correlation functions**: Localizations? Free theory?
- **String theory dual of $T\overline{T}$ deformation**: a variant of the gauging embedding describes the string dual of the $T\overline{T}$-deformed symmetric product orbifold