Holographic confining theories on space-times with constant positive curvature

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Introduction

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- At low energies, they do not propagate as free particles
- They are always found confined inside stable baryons (three quarks)
- This phenomenon is known as confinement
- Made possible by the fact that quantum chromodynamics (QCD) is strongly coupled at low energies

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- Heating up a gas of baryons will break them apart
- Thermal phase transition from a confined baryon gas to a deconfined quark-gluon plasma

Quantum deconfinement transition

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- High energies can be reached by putting the baryon gas in a small box
- Deconfinement transition as the volume of the box is decreased

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- We focus on strongly-coupled large-N QFTs and use holographic duality
- No quarks $\Rightarrow$ confinement in the form of glueball bound states (only gluons)

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- The theory is described by a single dimensionless curvature parameter

$$\mathcal{R} \equiv \frac{d(d-1)}{\alpha^2}\, \lambda^{-2/(d-\Delta)}\,. \tag{2}$$

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[Ghosh–Kiritsis–Nitti–Witkowski '17 and '18]

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  [Ghosh–Kiritsis–Nitti–Witkowski '17 and '18]
- A lot of work on QFTs on curved manifolds in recent years

  [Ghosh, Kiritsis, Nitti, Witkowski, Jokela, Nourry, Préau, Litos, Raymond, Ghodsi, Mashayekhi, Aharony, Urbach, Weiss, Marolf, Rangamani, Van Raamsdonk, Blackman, MacDermott, Karch, O'Bannon, Klebanov, Pufu, Jafferis, Safdi, Freedman, JK, ...]

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- The theory is said to exhibit confinement in flat space if the Wilson loop has an area law
- The Wilson loop cannot be applied as a criterion on the compact sphere directly

Outline

1. **The holographic model**

2. **Classification of gravity solutions**

3. **The phase diagram**

1. The holographic model

Einstein-scalar theory in $d+1$-dimensions

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- The scalar field is dual to the relevant scalar operator $\mathcal{O}$ of dimension $\frac{d}{2} < \Delta < d$
- Look for backreacted domain-wall solutions with spherical S^d-slices

$$ds_{d+1}^2 = du^2 + e^{2A(u)} \alpha^2 ds_{S^d}^2 , \quad \varphi = \varphi(u) , \tag{4}$$

where u is a holographic radial coordinate

Choice of UV behavior of the potential

- Scalar potential is chosen to have a maximum at $\varphi = 0$

$$V(\varphi) = 2\Lambda - \frac{\Delta(d-\Delta)}{2\ell^2}\,\varphi^2 + \mathcal{O}(\varphi^3)\,, \quad \varphi \to 0\,, \tag{5}$$

where $\Lambda = -\frac{d(d-1)}{2\ell^2} < 0$

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- The maximum is reached at $u \to -\infty$ where

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- Asymptotically locally EAdS$_{d+1}$ metric with S^d_α conformal boundary

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- Produces a soft-wall at $u = u_0$ where the slices shrink to zero size $e^{2A(u_0)} = 0$ and space ends

Why exponential asymptotics?

- The dual theory on a flat space $\mathbb{R}^d$ exhibits confinement when

$$b_{\mathrm{c}} < b < \sqrt{d}\, b_{\mathrm{c}} \,, \quad b_{\mathrm{c}} \equiv \frac{1}{\sqrt{2\,(d-1)}} \tag{9}$$

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- Specifically, for domain wall solutions with flat $\mathbb{R}^d$-slices

$$ds^2_{d+1} = dr^2 + e^{2A(r)}\,\alpha^2 ds^2_{\mathbb{R}^d}\,, \quad \varphi = \varphi(r)\,, \tag{10}$$

the holographic Wilson loop exhibits an area law in this case

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- The scalar potential has an uplift only when b is below the Gubser bound $\sqrt{d}\, b_c$

[Gubser '00]

2. Classification of gravity solutions

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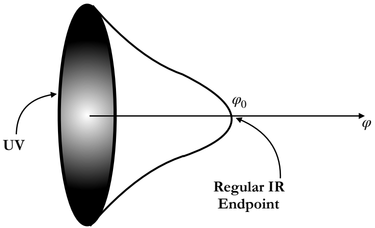
- For type III, the scalar field is finite $\varphi(u_0) = \varphi_0 < \infty$
- For type I and II, the scalar field diverges logarithmically $\varphi(u) \to \infty$ as $u \to u_0$

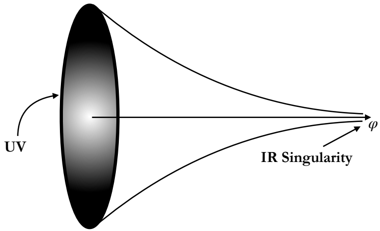
[JK–Kiritsis–Nitti '25]

Geometry	$\varphi(u_0)$
type III	finite
type II	∞
type I	∞

Types of gravity solutions



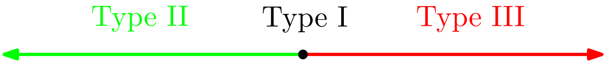
Type III



Type I-II

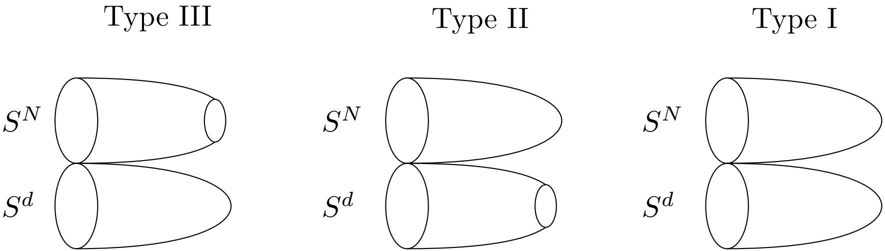
The space of solutions

Geometry	$\varphi(u_0)$	Integration constant
type III	finite	yes
type II	∞	yes
type I	∞	no, unique



Uplift to pure Einstein gravity

- The solutions uplift to $S^d \times S^N$ domain wall metrics in $d + N + 1$ dimensions



3. The phase diagram

The free energy

- Quantity of interest is the renormalized Euclidean path integral

$$Z(\mathcal{R}) \equiv \int [d\Phi]\, e^{-I_{\mathrm{QFT}}} \tag{13}$$

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- For fixed $\mathcal{R}$, the solution with the smallest on-shell action gives $F(\mathcal{R})$

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Holographic deconfinement transition

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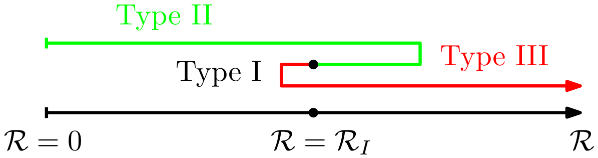

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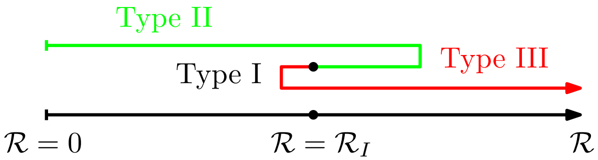

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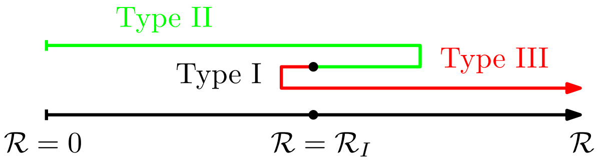

- Thus there must be a transition from II to III solutions in the vicinity of $\mathcal{R} = \mathcal{R}_{\mathrm{I}}$
- This is the holographic deconfinement transition as $\mathcal{R}$ increases

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- Type III solution corresponds to the deconfined phase
- This is also reflected in the spectrum of fluctuations (spectrum of glueball bound states)

[Ghodsi–Kiritsis–Mashayekhi–Nitti '25]

The order of the transition

$$V(\varphi) \sim V_\infty \, e^{2b\varphi} \, , \quad \varphi \to \infty \tag{15}$$

- There is a special value b_{E} (the Efimov bound) for the exponent b defined by

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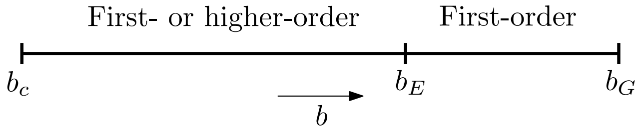
- It controls the order of the deconfinement transition



[JK–Kiritsis–Nitti ’25]

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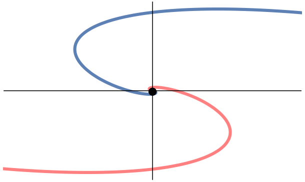

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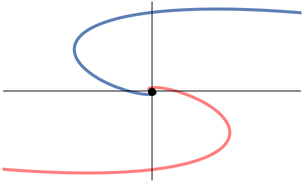

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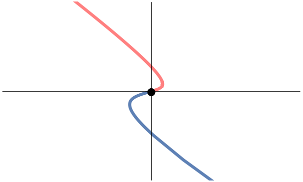

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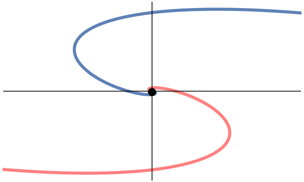

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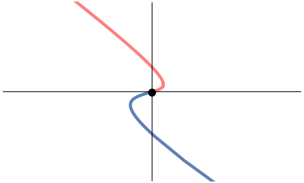

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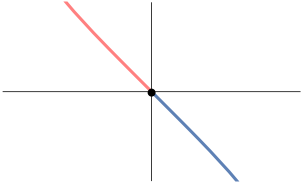

$b < b_{\mathrm{E}}$ 2nd possibility

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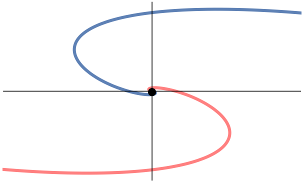

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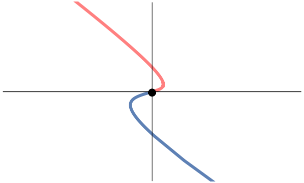

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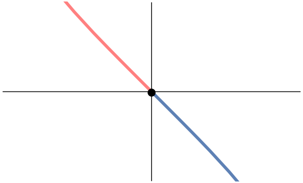
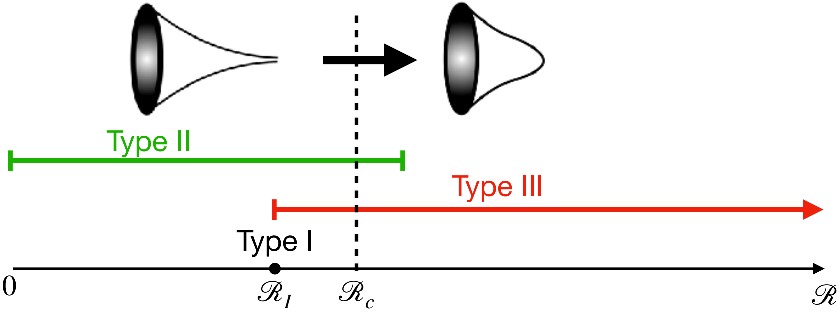


$b < b_{\mathrm{E}}$ 2nd possibility

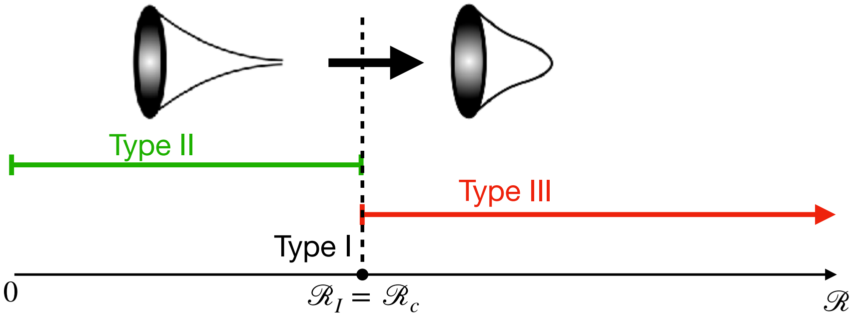
- Second- or higher-order
- Not infinite-order

Holographic deconfinement transition



first-order transition

Holographic deconfinement transition



second- or higher-order transition

Concrete numerical example

Choice of the potential for numerics

- For numerics, we consider the potential (for $0 < \varphi < \infty$)

$$V(\varphi) = -\frac{d(d-1)}{\ell^2} - \left(\frac{\Delta(\Delta - d)}{2\ell^2} + 4V_\infty\, b^2\right)\varphi^2 + 4V_\infty \sinh^2(b\varphi)\,. \tag{17}$$

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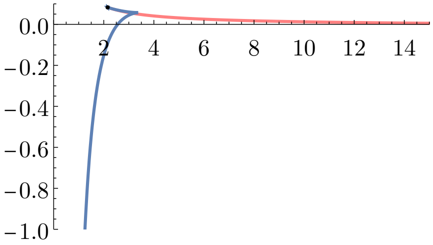
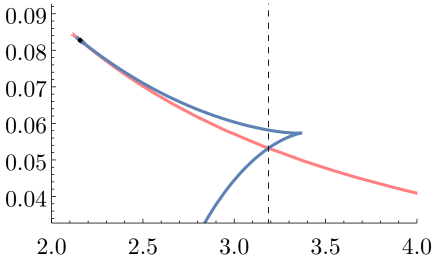
- Solve equations of motion numerically
- Compute on-shell actions and the free energy numerically (for $d = 4$)

[JK–Kiritsis–Nitti '25]

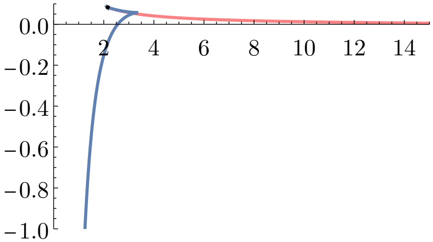
Free energy above the Efimov bound

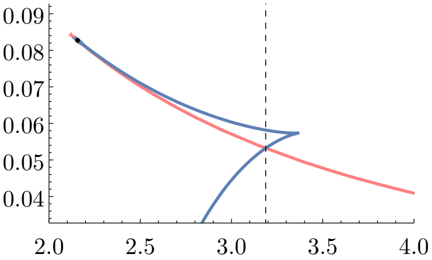




- Free energy $F = F(\mathcal{R})$ above the Efimov bound $b_{\mathrm{E}} < b < b_{\mathrm{G}}$

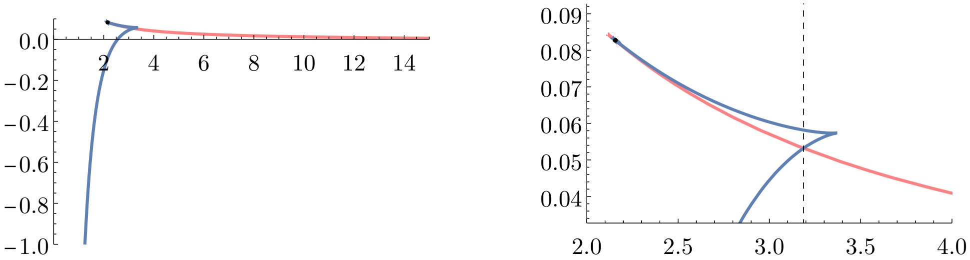
Free energy above the Efimov bound





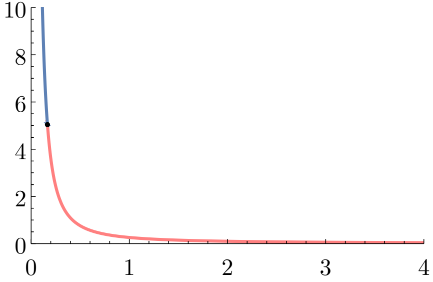
- Free energy $F = F(\mathcal{R})$ above the Efimov bound $b_{\mathrm{E}} < b < b_{\mathrm{G}}$
- First-order transition

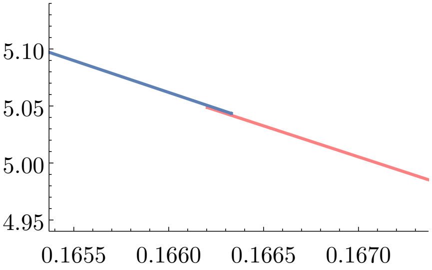
Free energy above the Efimov bound



- Free energy $F = F(\mathcal{R})$ above the Efimov bound $b_{\mathrm{E}} < b < b_{\mathrm{G}}$
- First-order transition
- Type I solution is thermodynamically unstable

Free energy below the Efimov bound





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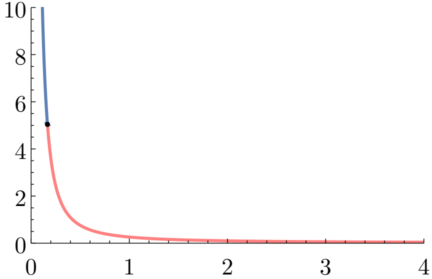
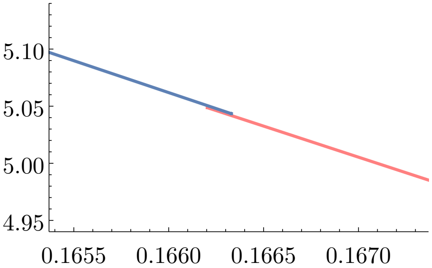
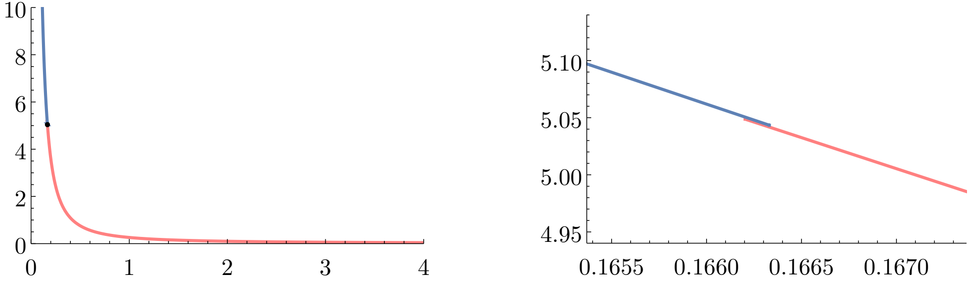
Free energy below the Efimov bound





- Free energy $F = F(\mathcal{R})$ below the Efimov bound $b_{\mathrm{c}} < b < b_{\mathrm{E}}$
- Second-order transition according to the numerics

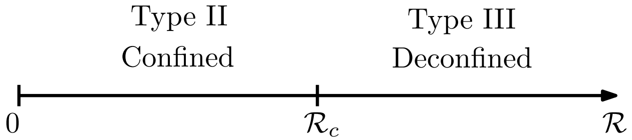
Free energy below the Efimov bound



- Free energy $F = F(\mathcal{R})$ below the Efimov bound $b_c < b < b_E$
- Second-order transition according to the numerics
- Transition through the type I solution

Conclusions

- We demonstrated that a (holographic) deconfinement transition can be induced by curvature



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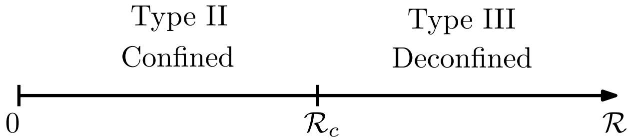
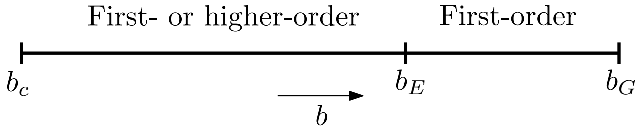


- The order of the transition depends on the exponent b of the potential $V \sim e^{2b\varphi}$



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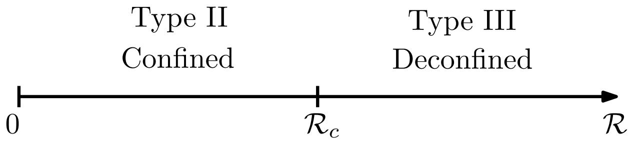

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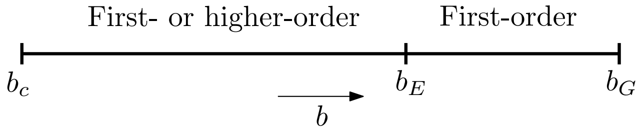

Thank you

Confinement-deconfinement transition

- Without quarks in $N = 3$ QCD, the transition is first-order
- With one massive quark (real world), the transition is a crossover

Numerical values

- $d = 4$:

$$b_{\mathrm{c}} \approx 0.408\,, \quad b_{\mathrm{E}} \approx 0.516\,, \quad b_{\mathrm{G}} \approx 0.816 \tag{18}$$

- $\alpha = \ell = 1$, $V_\infty = -1$ and $\Delta = 5/2$ (larger than $d/2 = 2$ and smaller than $d = 4$)
- We considered $b = 0.47$ and $b = 0.65$

Gubser bound

- Type II solution only exists below the Gubser bound
- For flat slices, there are no regular solutions when $b > b_{\mathrm{G}}$ (since no type II)
- No solution reaching $\varphi = \infty$ exists when $b > b_{\mathrm{G}}$, but it bounces back
- Certain IR boundary terms in the on-shell action vanish below the Gubser bound

Dimensionless vev

- It gives

$$\mathcal{C} \equiv \frac{\Delta_-}{d} \langle \mathcal{O} \rangle \, |\varphi_-|^{-\Delta_+/\Delta_-} \, . \tag{19}$$

- It is given by the derivative of the free energy

$$F'(\mathcal{R}) = \mathcal{N} \left(\frac{2\,\mathcal{C}(\mathcal{R})}{\mathcal{R}^3} - \frac{1}{96} \frac{1}{\mathcal{R}} \right) , \tag{20}$$

Below the confinement bound

- Exponents $b < b_c$ obtained by dimensional reduction on a torus