The generalized Cartan Geometry of α'-corrections

Falk Hassler

Based on work with Martin Cederwall, Achilleas Gitsis, Ondřej Hulík, David Osten, Yuho Sakatani and Luca Scala

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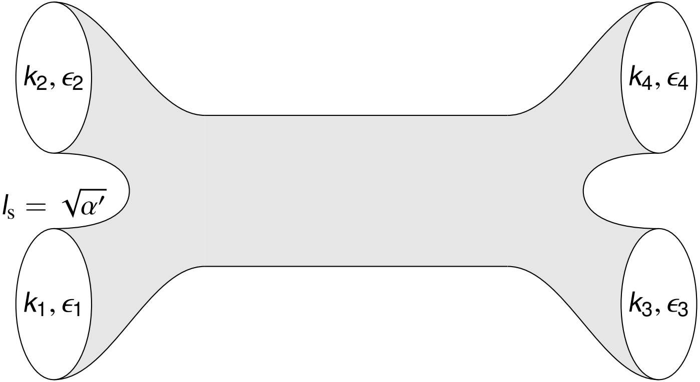
String amplitudes, Effective Field Theory and gravity



massless modes

metric	g_{ij}
gauge potential	B_{ij}
dilaton	ϕ
$\vdots$	

$$\mathcal{A}^{\rm tree}(s,t,u) = g_s^2 \frac{(\alpha')^4}{stu} \frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)\Gamma(1-\alpha' u)}{\Gamma(1+\alpha' s)\Gamma(1+\alpha' t)\Gamma(1+\alpha' u)} \mathcal{R}^4$$

$$= g_s^2 (\alpha')^2 \left[\frac{1}{stu} + 2\zeta(3)(\alpha')^3 + \zeta(5)(\alpha')^5(s^2+t^2+u^2) + \dots \right] \mathcal{R}^4$$

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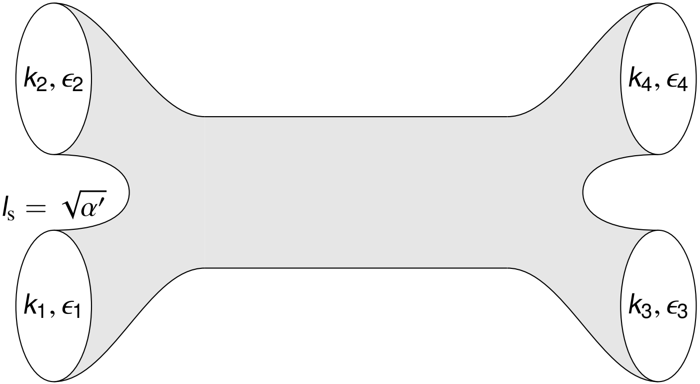

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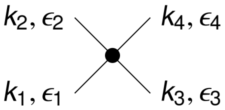

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$R_{ijk}{}^{l}, \qquad H_{ijk} = 3\partial_{[i}B_{jk]}, \qquad \phi, \qquad \nabla_i$ building blocks

$$S = \frac{2\pi}{(4\pi^2\alpha')^4}\int \mathrm{d}x^d\,\sqrt{-g}e^{-2\phi}\left[c_1 R + c_2\nabla_i\phi\nabla^i\phi + c_3 H_{ijk}H^{ijk} + \alpha'(\dots) + \dots\right]$$

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4. fix their coefficients my matching the amplitudes

$$S = \frac{2\pi}{(4\pi^2\alpha')^4}\int \mathrm{d}x^d\,\sqrt{-g}e^{-2\phi}\left[R + 4\nabla_i\phi\nabla^i\phi + \tfrac{1}{12}H_{ijk}H^{ijk} + \alpha'(\dots) + \dots\right]$$

Challenges

- explosion of terms

	coeff.	theories
α'^0	3	all
α'^1	8	bos., het.
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α'^3	872	all

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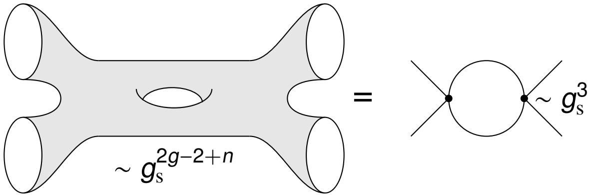

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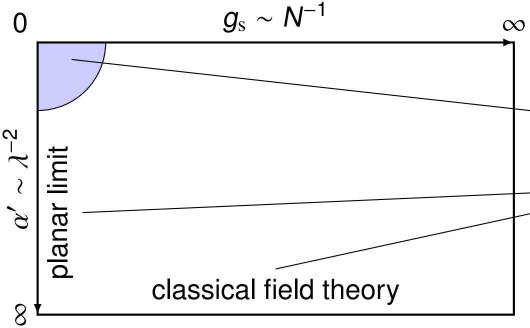

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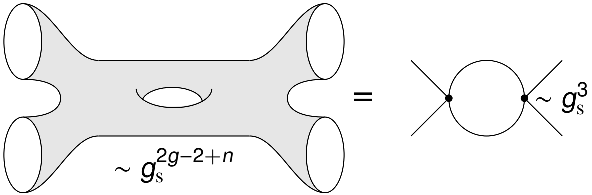

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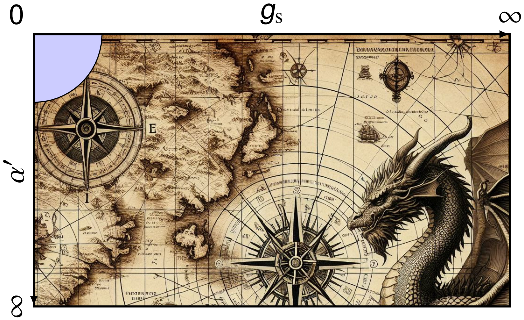

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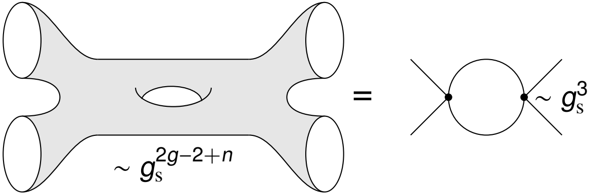

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How to go beyond?

Idea: new symmetries

1. select relevant degrees of freed
2. identify their symmetries*)
3. ...

additional symmetries

- fermionic
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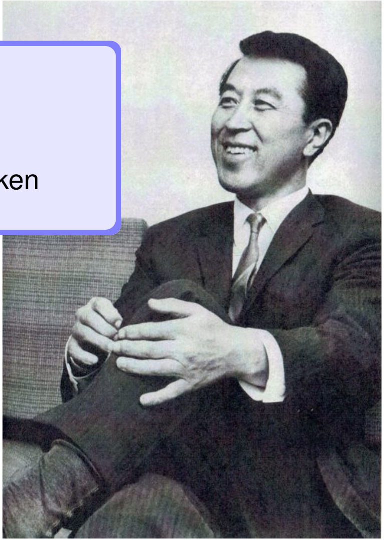

Yoichiro Nambu, 1965

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Coset construction (non-linear realization)

- G = the full symmetry group
- H = residual symmetries after spontaneous breaking
- introduce Mauer-Cartan form $\Omega = g^{-1}\mathrm{d}g$ for $g \in G/H$
- expand it in broken generators t_a as $\Omega = \Omega^a t_a + \dots$

Lagrangian

$$L = L(\Omega^a, D_a)$$

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  +

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Degrees of freedom in linearized gravity. Part I: the complex

- frame field $$e_i^a = \delta_i^a + \varepsilon_i^a$$
- symmetries $$\delta\varepsilon^a = \mathrm{d}\xi^a + \Lambda^a{}_b \mathrm{d}x^b$$

useful relations

$$g_{ij} = \eta_{ab} e_i^a e_j^b$$

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$$\mathrm{d}^2 = 0$$

$$\tau^2 = 0$$

$$\mathrm{d} \circ \tau + \tau \circ \mathrm{d} = 0$$

Degrees of freedom in linearized gravity. Part II: cohomology [see i.e. Vasiliev 05]

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- compute cohomology for the exact sequence

$$
0 \longrightarrow \Lambda_{ab} \xrightarrow{\tau} \varepsilon_{ai} \longrightarrow 0
$$

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Degrees of freedom in linearized gravity. Part II: cohomology [see i.e. Vasiliev 05]

$$
\begin{tikzcd}
 & \yng(2) & & & \\
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$$
\begin{tikzcd}
\overset{\square}{\xi^a} \arrow[r, "\mathrm{d}"] & \overset{\square\square}{\varepsilon^a} \arrow[r] & T^a \arrow[r] & \mathrm{BI}(T^a) \arrow[r] & \cdots \\
\Lambda^a{}_b \arrow[r] \arrow[ur, "\tau"] & \omega^a{}_b \arrow[r] \arrow[ur] & \overset{\square\square}{R^a{}_b} \arrow[r] \arrow[ur] & \overset{\square}{\mathrm{BI}(R^a{}_b)} \arrow[r] \arrow[ur] & \cdots \\
 & C^a{}_b \arrow[r] \arrow[ur] & \mathrm{BI}(C^a{}_b) \arrow[r] \arrow[ur] & \cdots &
\end{tikzcd}
$$

- compute cohomology for all diagonal exact sequences
- only with Weyl tensor cohomology classes of ε^a and $R^a{}_b$ match

Degrees of freedom in linearized gravity. Part II: cohomology [see i.e. Vasiliev 05]

$$
\begin{tikzcd}
\overset{\square}{\xi^a} \arrow[r, "\mathrm{d}"] & \overset{\square\square}{\varepsilon^a} \arrow[r] & \overset{\overset{0}{\shortparallel}}{T^a} \arrow[r] \arrow[dl, bend left] & \mathrm{BI}(T^a) \arrow[r] & \cdots \\
\Lambda^a{}_b \arrow[r] \arrow[ur, "\tau"] & \omega^a{}_b \arrow[r] \arrow[ur] & \overset{\square\square}{R^a{}_b} \arrow[r] \arrow[ur] & \overset{\square}{\mathrm{BI}(R^a{}_b)} \arrow[r] \arrow[ur] & \cdots \\
 & C^a{}_b \arrow[r] \arrow[ur] & \mathrm{BI}(C^a{}_b) \arrow[r] \arrow[ur] & \cdots &
\end{tikzcd}
$$

- compute cohomology for all diagonal exact sequences
- only with Weyl tensor cohomology classes of ε^a and $R^a{}_b$ match
- after imposing torsion constraint ($T^a = 0$), we get

$$\text{gauge} \longrightarrow \text{dof} \longrightarrow \text{eom} \longrightarrow \text{Bianchi}$$

The background independent version and Cartan geometry

- idea: compress complex into a chain

$$
\begin{tikzcd}
\xi^a \arrow[r, "\mathrm{d}"] & \varepsilon^a \arrow[r] & T^a \arrow[r] & \mathrm{BI}(T^a) \arrow[r] & \cdots \\
\Lambda^a{}_b \arrow[r] \arrow[ur, "\tau"] & \omega^a{}_b \arrow[r] \arrow[ur] & R^a{}_b \arrow[r] \arrow[ur] & \mathrm{BI}(R^a{}_b) \arrow[r] \arrow[ur] & \cdots \\
 & C^a{}_b \arrow[r] \arrow[ur] & \mathrm{BI}(C^a{}_b) \arrow[r] \arrow[ur] & \cdots &
\end{tikzcd}
$$

The background independent version and Cartan geometry

- idea: compress complex into a chain

$$\hat{\xi} = \xi^a P_a + \Lambda^{ab} M_{ab}$$

Poincaré algebra

P_a	translations
M_{ab}	Lorentz transformations

$$\begin{tikzcd}
\hat{\xi} & & & & \\
\xi^a \arrow[u] \arrow[r, "\mathrm{d}"] & \varepsilon^a \arrow[r] & T^a \arrow[r] & \mathrm{BI}(T^a) \arrow[r] & \cdots \\
\Lambda^a{}_b \arrow[r] \arrow[ur, "\tau"] & \omega^a{}_b \arrow[r] \arrow[ur] & R^a{}_b \arrow[r] \arrow[ur] & \mathrm{BI}(R^a{}_b) \arrow[r] \arrow[ur] & \cdots \\
& C^a{}_b \arrow[r] \arrow[ur] & \mathrm{BI}(C^a{}_b) \arrow[r] \arrow[ur] & \cdots &
\end{tikzcd}$$

The background independent version and Cartan geometry

- idea: compress complex into a chain

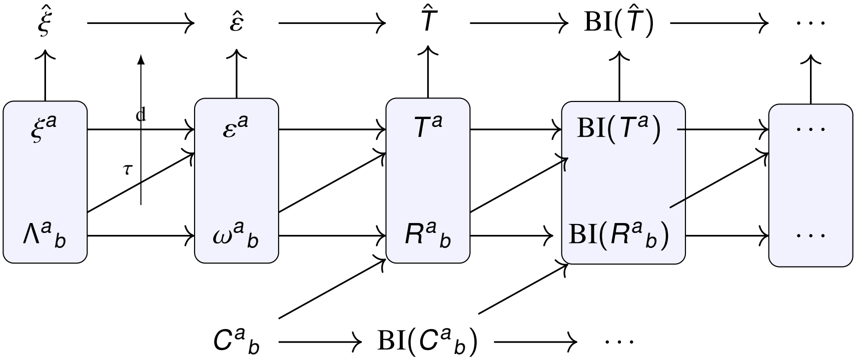
$$\hat{\xi} = \xi^a P_a + \Lambda^{ab} M_{ab}\,, \qquad \hat{\varepsilon} = \varepsilon^a P_a + \omega^{ab} M_{ab}$$

$$\begin{tikzcd}
\hat{\xi} \arrow[r] & \hat{\varepsilon} & & & \\
\xi^a \arrow[u] \arrow[r, "\mathrm{d}"] & \varepsilon^a \arrow[u] \arrow[r] & T^a \arrow[r] & \mathrm{BI}(T^a) \arrow[r] & \cdots \\
\Lambda^a{}_b \arrow[r] \arrow[ur, "\tau"] & \omega^a{}_b \arrow[r] \arrow[ur] & R^a{}_b \arrow[r] \arrow[ur] & \mathrm{BI}(R^a{}_b) \arrow[r] \arrow[ur] & \cdots \\
& C^a{}_b \arrow[r] \arrow[ur] & \mathrm{BI}(C^a{}_b) \arrow[r] \arrow[ur] & \cdots &
\end{tikzcd}$$

The background independent version and Cartan geometry

- idea: compress complex into a chain

$$\hat{\xi} = \xi^a P_a + \Lambda^{ab} M_{ab}\,, \qquad \hat{\varepsilon} = \varepsilon^a P_a + \omega^{ab} M_{ab}\,, \qquad \ldots$$



The background independent version and Cartan geometry

- idea: compress complex into a chain

$$\hat{\xi} = \xi^a P_a + \Lambda^{ab} M_{ab}\,, \qquad \hat{\varepsilon} = \varepsilon^a P_a + \omega^{ab} M_{ab}\,, \qquad \ldots$$

- and use exterior derivative

$$\hat{\mathrm{d}} = \mathrm{d} + \hat{e}\wedge \qquad \text{with}$$

$$\hat{e} = e_i^a P_a \mathrm{d}x^i + \omega^{ab} M_{ab} \qquad \underline{\text{Cartan connection}}$$

$$\hat{\xi} \xrightarrow{\hat{\mathrm{d}}} \hat{\varepsilon} \longrightarrow \hat{T} \longrightarrow \mathrm{BI}(\hat{T}) \longrightarrow \cdots$$

The background independent version and Cartan geometry

- idea: compress complex into a chain

$$\hat{\xi} = \xi^a P_a + \Lambda^{ab} M_{ab}\,, \qquad \hat{\varepsilon} = \varepsilon^a P_a + \omega^{ab} M_{ab}\,, \qquad \ldots$$

- and use exterior derivative

$\hat{\mathrm{d}} = \mathrm{d} + \hat{e}\wedge$ with

$\hat{e} = e_i^a P_a \mathrm{d}x^i + \omega^{ab} M_{ab}$ Cartan connection

- substituting $\hat{\varepsilon} \to \hat{e}$ in the chain

$$\hat{\xi} \xrightarrow{\hat{\mathrm{d}}} \hat{e} \longrightarrow \hat{T} \longrightarrow \mathrm{BI}(\hat{T}) \longrightarrow \cdots$$

The background independent version and Cartan geometry

- idea: compress complex into a chain

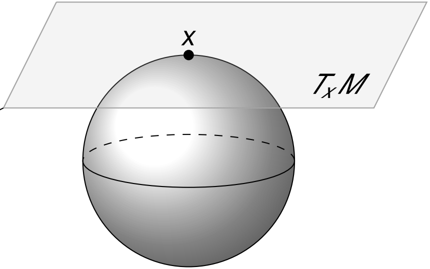
$$\hat{\xi} = \xi^a P_a + \Lambda^{ab} M_{ab}\,, \qquad \hat{\varepsilon} = \varepsilon^a P_a + \omega^{ab} M_{ab}\,, \qquad \ldots$$

- and use exterior derivative

$$\hat{\mathrm{d}} = \mathrm{d} + \hat{e}\wedge \qquad \text{with}$$

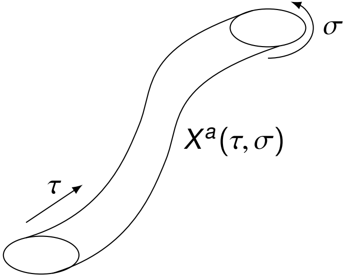
$$\hat{e} = e_i^a P_a \mathrm{d}x^i + \omega^{ab} M_{ab} \qquad \underline{\text{Cartan connection}}$$

- substituting $\hat{\varepsilon} \to \hat{e}$ in the chain
- only input is <u>model space</u> G/H



$$\hat{\xi} \xrightarrow{\hat{\mathrm{d}}} \hat{e} \longrightarrow \hat{T} \longrightarrow \mathrm{BI}(\hat{T}) \longrightarrow \cdots$$

Where is the Poincaré$_{\infty}$?



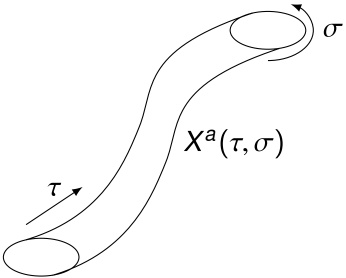
minimal coupling

$$-\int B\,, \qquad B = \tfrac{1}{2}B_{ab}\mathrm{d}x^a \wedge \mathrm{d}x^b$$

three-form field strength

$$H = \mathrm{d}B$$

Where is the Poincaré$_\infty$?



minimal coupling

$$-\int B\,,\qquad B=\tfrac{1}{2}B_{ab}\mathrm{d}x^a\wedge \mathrm{d}x^b$$

three-form field strength

$$H=\mathrm{d}B$$

- gauge transformation

$$\delta B=\mathrm{d}\varphi+L_\xi B$$

diffeomorphism

- gauge transformation for the gauge transformation

$$\delta\varphi=\mathrm{d}\chi$$

Where is the Poincaré$_\infty$?

$$\begin{array}{ccc} \bullet & & \square + \overline{\square} \\ \chi & \xrightarrow{D} & \xi_A \end{array}$$

- gauge transformation

$$\delta B = \mathrm{d}\varphi + L_\xi B\,, \qquad \text{with combined parameter} \qquad \xi^A = \begin{pmatrix} \xi^a & \varphi_a \end{pmatrix}$$

- gauge transformation for the gauge transformation

$$\delta\varphi = \mathrm{d}\chi$$

Where is the Poincaré$_\infty$?

$$
\begin{tikzcd}
\bullet & \square + \overline{\square} & & & \\
\chi \arrow[r, "D"] & \xi_A \arrow[r] & \varepsilon_{AB} \arrow[r] & T_{ABC} \arrow[r] & \cdots \\
 & \Lambda_{AB} \arrow[r] \arrow[ur, "\tau"] & \omega_{ABC} \arrow[r] \arrow[ur] & R_{ABCD} \arrow[r] \arrow[ur] & \cdots
\end{tikzcd}
$$

- gauge transformation

$$\delta B = \mathrm{d}\varphi + L_\xi B\,, \qquad \text{with combined parameter} \qquad \xi^A = \begin{pmatrix} \xi^a & \varphi_a \end{pmatrix}$$

- gauge transformation for the gauge transformation

$$\delta\varphi = \mathrm{d}\chi$$

Where is the Poincaré$_{\infty}$?

$$
\begin{tikzcd}
\bullet & \square + \overline{\square} & & & \\
\chi \arrow[r, "D"] & \xi_A \arrow[r] & \varepsilon_{AB} \arrow[r] & T_{ABC} \arrow[r] & \cdots \\
 & \Lambda_{AB} \arrow[r] \arrow[ur, "\tau"] & \omega_{ABC} \arrow[r] \arrow[ur] & R_{ABCD} \arrow[r] \arrow[ur] & \cdots
\end{tikzcd}
$$

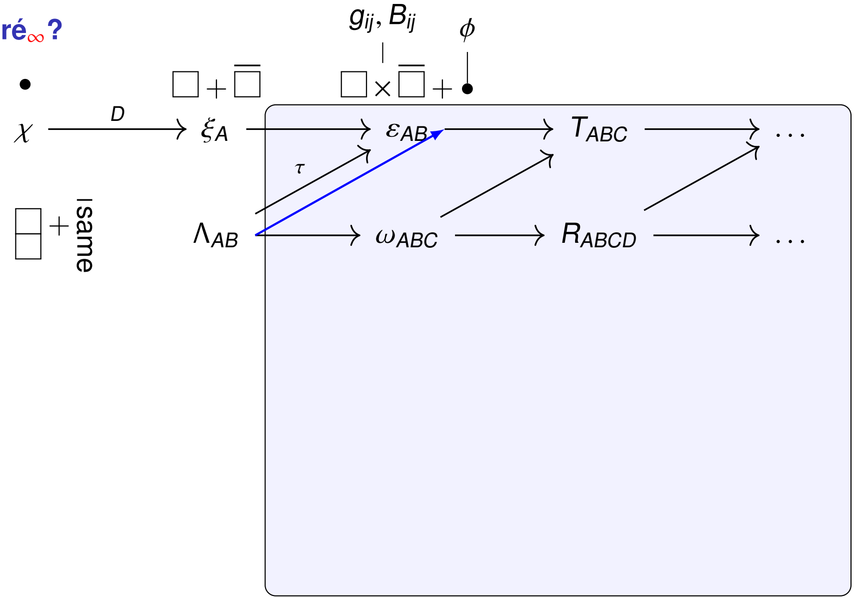
fixed by <u>generalized</u> Cartan geometry

Where is the Poincaré$_{\infty}$?

$$
\begin{tikzcd}
\bullet & \square + \overline{\square} & \square \times \overline{\square} + \bullet & & \\
\chi \arrow[r, "D"] & \xi_A \arrow[r] & \varepsilon_{AB} \arrow[r] & T_{ABC} \arrow[r] & \cdots \\
& \Lambda_{AB} \arrow[r] \arrow[ur, "\tau"] & \omega_{ABC} \arrow[r] \arrow[ur] & R_{ABCD} \arrow[r] \arrow[ur] & \cdots
\end{tikzcd}
$$

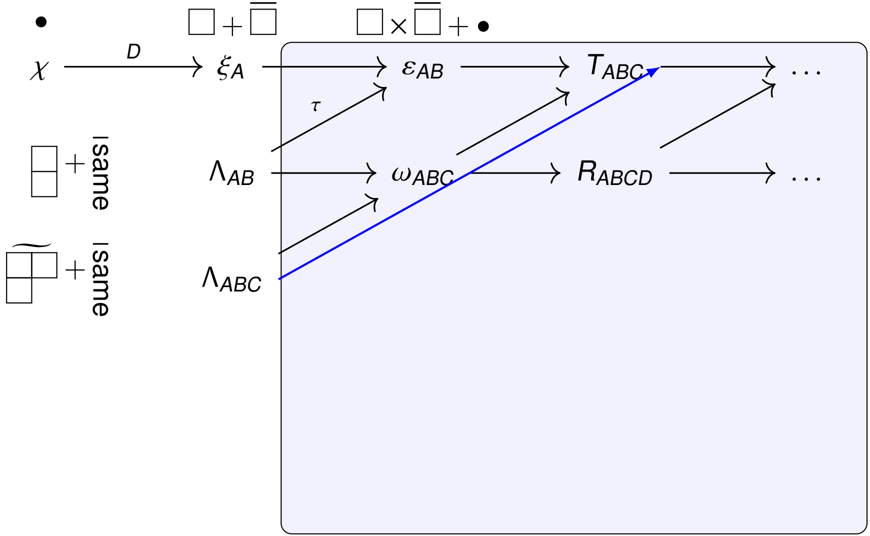
fixed by generalized Cartan geometry

Where is the Poincaré$_\infty$?



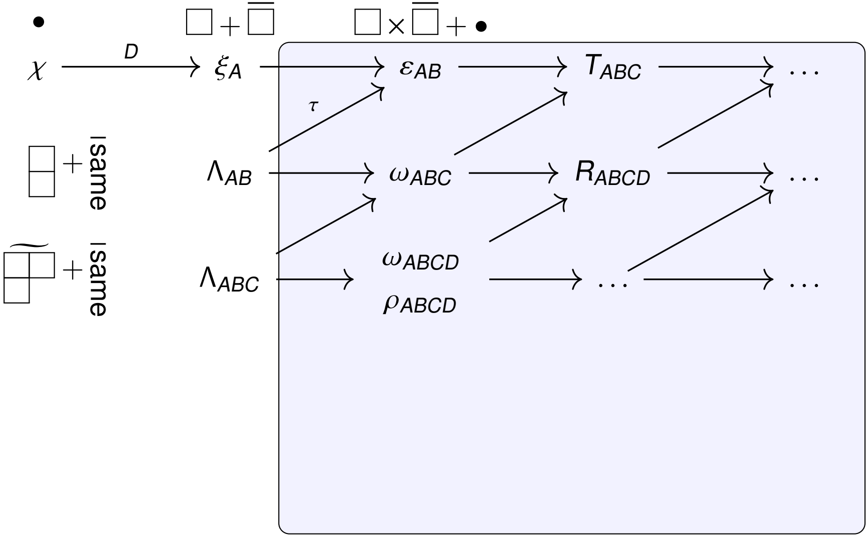
fixed by <u>generalized</u> Cartan geometry

Where is the Poincaré$_\infty$?



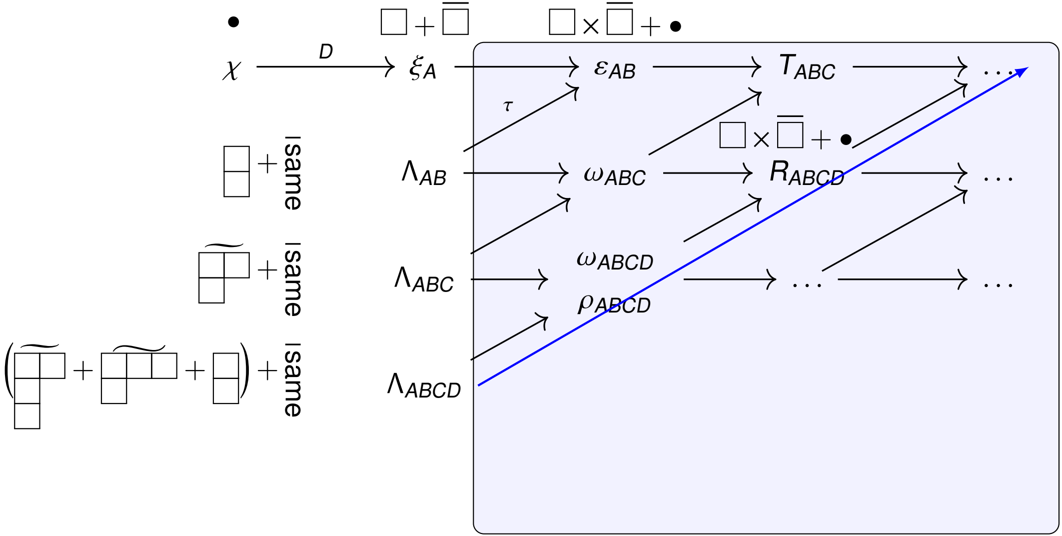
fixed by <u>generalized</u> Cartan geometry

Where is the Poincaré$_\infty$?



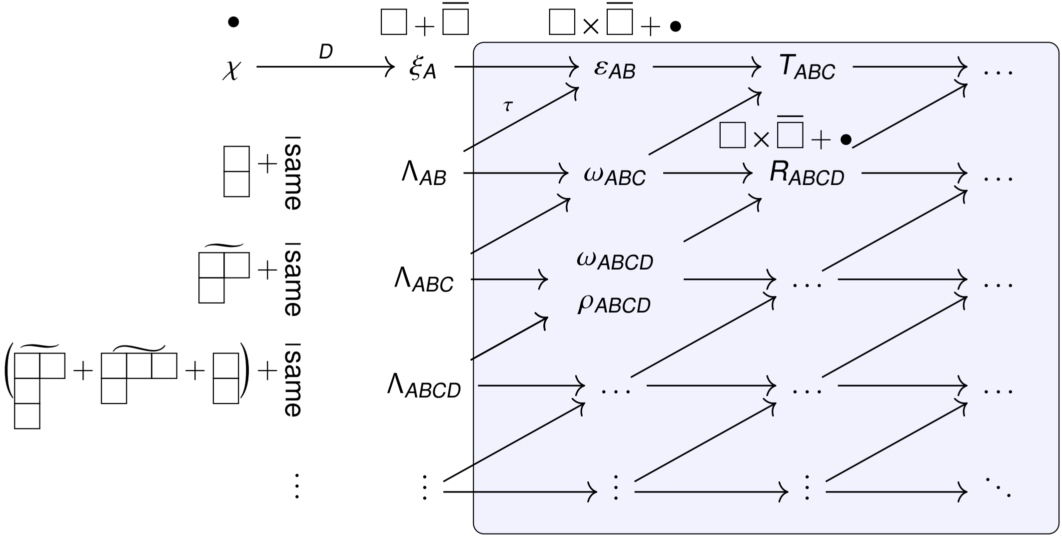
fixed by generalized Cartan geometry

Where is the Poincaré$_\infty$?



fixed by <u>generalized</u> Cartan geometry

Where is the Poincaré$_\infty$?



fixed by generalized Cartan geometry

Relevant features of generalized Cartan geometry [Poláček, Siegel 13; Butter, FH, Pope, Zhang 23; FH, Hulik, Osten 24]

$$\begin{tikzcd}
\hat{\chi} \arrow[rr] & & \hat{\xi} \arrow[dl, dash] \arrow[dr, dash] \arrow[rr, "\hat{D}"] & & \hat{E} \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] \arrow[r] & \hat{T} \arrow[r] & \mathrm{BI}(\hat{T}) \arrow[r] & \cdots \\
& \xi & & \Lambda \quad E & \omega & \rho & &
\end{tikzcd}$$

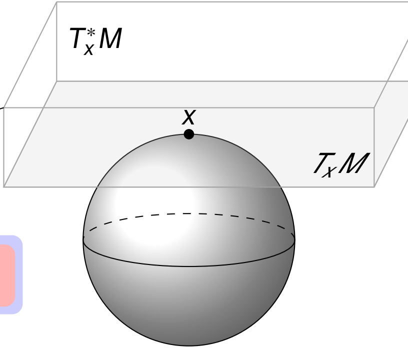
- new connection ρ with corresponding curvature

Relevant features of generalized Cartan geometry [Poláček, Siegel 13; Butter, FH, Pope, Zhang 23; FH, Hulik, Osten 24]

$$
\begin{tikzcd}
\hat{\chi} \arrow[rr] & & \hat{\xi} \arrow[rrr, "\hat{D}"] \arrow[dl, dash] \arrow[dr, dash] & & & \hat{E} \arrow[rr] \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & & \hat{T} \arrow[r] & \mathrm{BI}(\hat{T}) \arrow[r] & \cdots \\
& \xi & & \Lambda & E & \omega & \rho & & &
\end{tikzcd}
$$

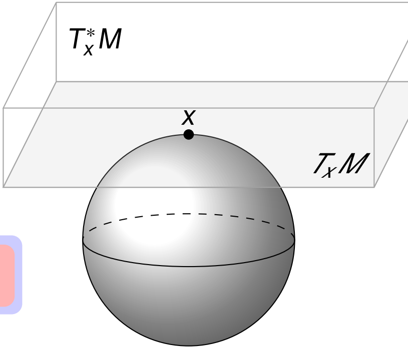
- new connection ρ with corresponding curvature
- model space is double coset $\widetilde{H} \backslash G / H$ generated by

$$\underbrace{\ldots \quad \tilde{t}^{ABC} \quad \tilde{t}^{AB}}_{\widetilde{H}} \quad P_A \quad \underbrace{t_{AB} \quad t_{ABC} \quad \ldots}_{H}$$



Relevant features of generalized Cartan geometry [Poláček, Siegel 13; Butter, FH, Pope, Zhang 23; FH, Hulik, Osten 24]

$$\begin{tikzcd}
\hat{\chi} \arrow[r] & \hat{\xi} \arrow[r, "\hat{D}"] \arrow[dl, no head] \arrow[d, no head] & \hat{E} \arrow[r] \arrow[d, no head] \arrow[dr, no head] \arrow[drr, no head] & \hat{T} \arrow[r] & \mathrm{BI}(\hat{T}) \arrow[r] & \cdots \\
\xi & \Lambda & E & \omega & \rho &
\end{tikzcd}$$

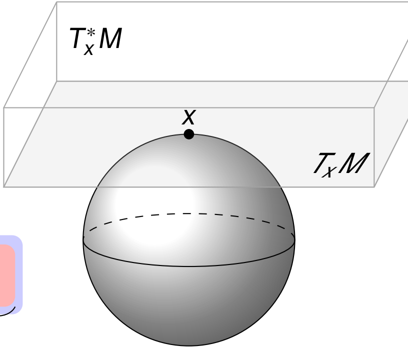


- new connection ρ with corresponding curvature
- model space is double coset $\widetilde{H} \backslash G / H$ generated by

$$\underbrace{\dots \quad \tilde{t}^{ABC} \quad \tilde{t}^{AB} \quad P_A}_{\text{fixed by } \tau^2 = 0 \text{ \& cohomology}} \quad t_{AB} \quad t_{ABC} \quad \dots$$

Relevant features of generalized Cartan geometry [Poláček, Siegel 13; Butter, FH, Pope, Zhang 23; FH, Hulik, Osten 24]

$$\begin{tikzcd}
\hat\chi \arrow[r] & \hat\xi \arrow[r, "\hat D"] \arrow[dl, no head] \arrow[d, no head] & \hat E \arrow[r] \arrow[d, no head] \arrow[dr, no head] \arrow[drr, no head] & \hat T \arrow[r] & \mathrm{BI}(\hat T) \arrow[r] & \cdots \\
\xi & \Lambda & E & \omega & \rho &
\end{tikzcd}$$



- new connection ρ with corresponding curvature
- model space is double coset $\widetilde{H}\backslash G/H$ generated by

$$\underbrace{\cdots \quad \tilde t^{ABC} \quad \tilde t^{AB} \quad P_A}_{\text{fixed by } \tau^2 = 0 \text{ \& cohomology}} \quad \underbrace{t_{AB} \quad t_{ABC} \quad \cdots}$$

- specified by a symmetric, invariant bilinear form κ on Lie(H)

A tower of corrections

$$
\begin{tikzcd}
\chi \arrow[r, "D"] & \xi_A \arrow[r] & \varepsilon_{AB} \arrow[r] & T_{ABC} \arrow[r] & \cdots \\
& \Lambda_{AB} \arrow[r] \arrow[ur, "\tau"] & \omega_{ABC} \arrow[r] \arrow[ur] & R_{ABCD} \arrow[r] \arrow[ur] & \cdots \\
& \Lambda_{ABC} \arrow[r] \arrow[ur] & \begin{matrix} \omega_{ABCD} \\ \rho_{ABCD} \end{matrix} \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \\
& \Lambda_{ABCD} \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \\
& \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \ddots
\end{tikzcd}
$$

A tower of corrections

$$
\begin{tikzcd}
\chi \arrow[r, "D"] & \xi_A \arrow[r] & \varepsilon_{AB} \arrow[r] & T_{ABC} \arrow[r] & \cdots \\
& \Lambda_{AB} \arrow[r] \arrow[ur, "\tau"] & \omega_{ABC} \arrow[r] \arrow[ur] & R_{ABCD} \arrow[r] \arrow[ur] & \cdots \\
& \Lambda_{ABC} \arrow[r] \arrow[ur] & \begin{array}{c} \omega_{ABCD} \\ \rho_{ABCD} \end{array} \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \\
& \Lambda_{ABCD} \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \\
& \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \ddots
\end{tikzcd}
$$

$$\Lambda^{i-2} = \Lambda^{A_1\ldots A_i} t_{A_1\ldots A_i}$$

$$\omega_A^{i-2} = \omega_A^{B_1\ldots B_i} t_{B_1\ldots B_i}$$

$$\vdots$$

$$\rho^{i-2\, j-2} = \tfrac{1}{2}\rho^{A_1\ldots A_i B_1\ldots B_j} t_{A_1\ldots A_i} \wedge t_{B_1\ldots B_j}$$

A tower of corrections

$$
\begin{tikzcd}
\chi \arrow[r, "D"] & \xi_A \arrow[r] & \varepsilon_{AB} \arrow[r] & T_{ABC} \arrow[r] & \cdots \\
& t_0 \ni \Lambda^0 \arrow[r] \arrow[ur, "\tau"] & \omega^0_A \arrow[r] \arrow[ur] & R^0_{AB} \arrow[r] \arrow[ur] & \cdots \\
& t_1 \ni \Lambda^1 \arrow[r] \arrow[ur] & \begin{matrix}\omega^1_A \\ \rho^{00}\end{matrix} \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \\
& t_2 \ni \Lambda^2 \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \\
& \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \ddots
\end{tikzcd}
$$

$$\Lambda^{i-2} = \Lambda^{A_1\ldots A_i} t_{A_1\ldots A_i}$$

$$\omega_A^{i-2} = \omega_A^{B_1\ldots B_i} t_{B_1\ldots B_i}$$

$$\vdots$$

$$\rho^{i-2\,j-2} = \tfrac{1}{2}\rho^{A_1\ldots A_i B_1\ldots B_j} t_{A_1\ldots A_i} \wedge t_{B_1\ldots B_j}$$

A tower of corrections

$$
\begin{tikzcd}
\chi \arrow[r, "D"] & \xi_A \arrow[r] & \overset{\square\times\overline{\square}+\bullet}{\varepsilon_{AB}} \arrow[r, red] & \overset{\substack{0\\ \shortparallel}}{T_{ABC}} \arrow[r] \arrow[dl, red, "\square\times\overline{\square}+\bullet"] & \cdots \\
 & t_0 \ni \Lambda^0 \arrow[r] \arrow[ur, "\tau"] & \omega^0_A \arrow[r] & R^0_{AB} \arrow[r] \arrow[ur] & \cdots \\
 & t_1 \ni \Lambda^1 \arrow[r] \arrow[ur] & \begin{matrix}\omega^1_A\\ \rho^{00}\end{matrix} \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \\
 & t_2 \ni \Lambda^2 \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \\
 & \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \ddots
\end{tikzcd}
$$

A tower of corrections

$$\begin{tikzcd}
\chi \arrow[r, "D"] & \xi_A \arrow[r] & \overset{\square\times\overline{\square}+\bullet}{\varepsilon_{AB}} \arrow[r, red] & \overset{0}{\overset{\shortparallel}{T_{ABC}}} \arrow[r] \arrow[dl, red] & \cdots \\
& t_0 \ni \Lambda^0 \arrow[r] \arrow[ur, "\tau"] & \omega^0_A \arrow[r, red] & \overset{\square\times\overline{\square}+\bullet}{R^0_{AB}} \arrow[r] \arrow[ur] \arrow[dl, red] & \cdots \\
& t_1 \ni \Lambda^1 \arrow[r] \arrow[ur] & \begin{matrix}\omega^1_A\\ \rho^{00}\end{matrix} \arrow[r] & \cdots \arrow[r] \arrow[ur] & \cdots \\
& t_2 \ni \Lambda^2 \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \arrow[r] \arrow[ur] & \cdots \\
& \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \ddots
\end{tikzcd}$$

A tower of corrections

$$
\begin{tikzcd}
\chi \arrow[r, "D"] & \xi_A \arrow[r] & \overset{\square \times \overline{\square} + \bullet}{\varepsilon_{AB}} \arrow[r] & \overset{\substack{0 \\ \shortparallel}}{T_{ABC}} \arrow[r] \arrow[dl] & \cdots & \\
& t_0 \ni \Lambda^0 \arrow[r] \arrow[ur, "\tau"] & \omega^0_A \arrow[r] & \overset{\square \times \overline{\square} + \bullet}{R^0_{AB}} \arrow[r] \arrow[dl] \arrow[ur] & \cdots & {} \arrow[ddd, "\sqrt{\alpha'}"'] \\
& t_1 \ni \Lambda^1 \arrow[r] \arrow[ur] & \begin{matrix} \omega^1_A \\ \rho^{00} \end{matrix} \arrow[r] & \cdots \arrow[r] \arrow[dl] \arrow[ur] & \cdots & \\
& t_2 \ni \Lambda^2 \arrow[r] \arrow[ur] & \cdots \arrow[r] & \cdots \arrow[r] \arrow[dl] \arrow[ur] & \cdots & \\
& \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] & \vdots \arrow[r] \arrow[ur] & \ddots & {}
\end{tikzcd}
$$

A tower of corrections

$$R^0_{AB} = \mathrm{d}\omega^0_A + \underbrace{\sum_{i,j}[\omega^i_A, \omega^j_B]|_{t_0} + \dots}_{\text{non-linear terms from gen. Cartan}}$$

$$\begin{tikzcd}
\chi \arrow[r, "D"] & \xi_A \arrow[r] & \varepsilon_{AB} \arrow[r] & T_{ABC} \arrow[r] & \dots \\
t_0 \ni \Lambda^0 \arrow[r] \arrow[ur, "\tau"] & \omega^0_A \arrow[r] \arrow[ur] & R^0_{AB} \arrow[r] \arrow[ur] & \dots \arrow[ddd, "\sqrt{\alpha'}"] \\
t_1 \ni \Lambda^1 \arrow[r] \arrow[ur] & \begin{matrix}\omega^1_A \\ \rho^{00}\end{matrix} \arrow[r] \arrow[ur] & \dots \arrow[r] \arrow[ur] & \dots \\
t_2 \ni \Lambda^2 \arrow[r] \arrow[ur] & \dots \arrow[r] \arrow[ur] & \dots \arrow[r] \arrow[ur] & \dots \\
\vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \ddots
\end{tikzcd}$$

A tower of corrections

$$R^0_{AB} = \mathrm{d}\omega^0_A + \color{blue}{\sum_{i,j} [\omega^i_A, \omega^j_B]|_{t_0}} + \dots$$

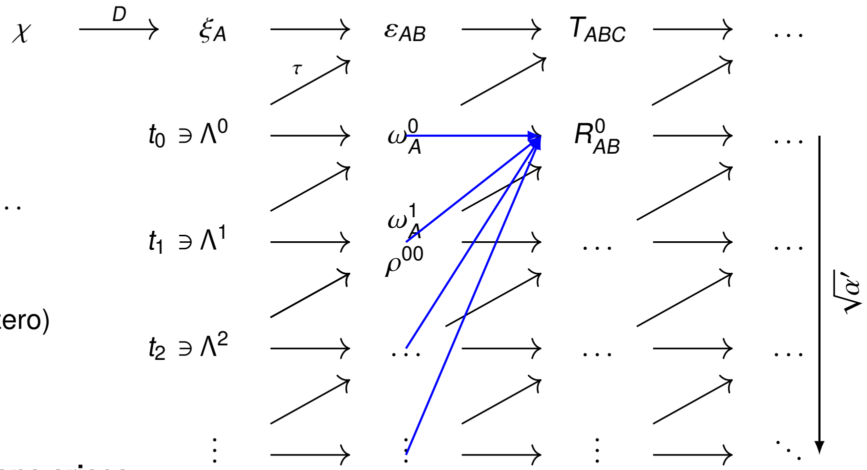
H is graded (κ vanishes)

$[t_i, t_j] \subset t_{i+j}$

no α'-corrections

$$\begin{tikzcd}
\chi \arrow[r, "D"] & \xi_A \arrow[r] & \varepsilon_{AB} \arrow[r] & T_{ABC} \arrow[r] & \dots & \\
 & t_0 \ni \Lambda^0 \arrow[r] \arrow[ur, "\tau"] & \omega^0_A \arrow[r] \arrow[ur] & R^0_{AB} \arrow[r] \arrow[ur] & \dots & {} \arrow[dddd, "\sqrt{\alpha'}"] \\
 & t_1 \ni \Lambda^1 \arrow[r] \arrow[ur] & \begin{matrix}\omega^1_A \\ \rho^{00}\end{matrix} \arrow[r] \arrow[ur] & \dots \arrow[r] \arrow[ur] & \dots & \\
 & t_2 \ni \Lambda^2 \arrow[r] \arrow[ur] & \dots \arrow[r] \arrow[ur] & \dots \arrow[r] \arrow[ur] & \dots & \\
 & \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \vdots \arrow[r] \arrow[ur] & \ddots & {}
\end{tikzcd}$$

A tower of corrections



$$R^0_{AB} = \mathrm{d}\omega^0_A + \sum_{i,j} [\omega^i_A, \omega^j_B]|_{t_0} + \dots$$

H is a filtration (κ non-zero)

$$[t_i, t_j] \subset \bigcup_{k \leq i+j} t_k$$

a tower of α'-corrections arises

A skyline and an evaded no-go theorem

- admissible κ's are parameterized by $\kappa = \kappa(\underline{a, b}, \underline{c, d}, e, f, \ldots)$

very likely (WIP)

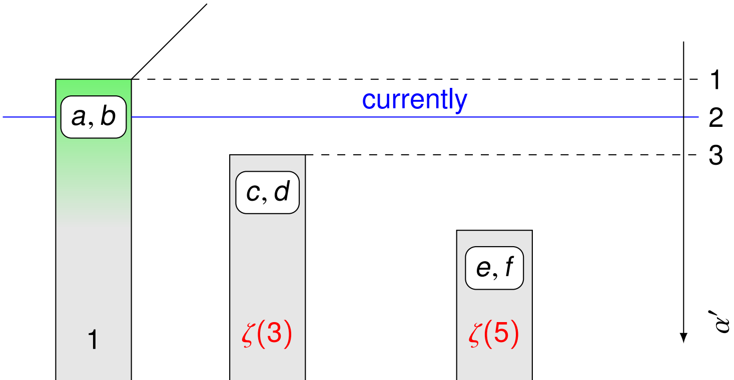
have to be there [Achilleas Gitsis, FH 24]

A skyline and an evaded no-go theorem

- admissible κ's are parameterized by $\kappa = \kappa(\underline{a, b}, \underline{c, d}, e, f, \ldots)$
- each parameter creates a tower of α'-corrections

known from generalized Bergshoeff-de Roo identification

[Bergshoeff, de Roo 89; Marques, Nunez 2015; Baron, Lescano, Marques 2018; Baron, Marques 2022]

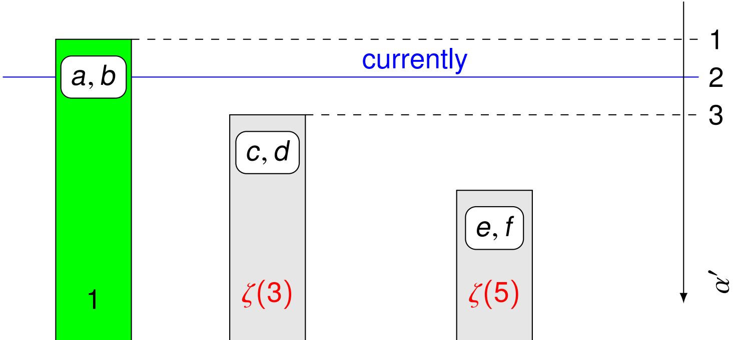


	theories
α'^0	all
α'^1	bos., het.
α'^2	bos., het.
α'^3	all

A skyline and an evaded no-go theorem

- admissible κ's are parameterized by $\kappa = \kappa(\underline{a, b}, \underline{c, d}, e, f, \ldots)$
- each parameter creates a tower of α'-corrections
- no-go for α'^3-tower from deformed O(d)×O(d) symmetry [Hsia, Kamal, Wulff 24]

No problem, we don't need this symmetry!



	theories
α'^0	all
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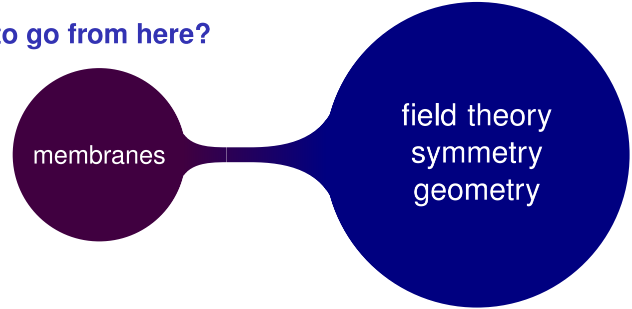
α'^2	bos., het.
α'^3	all

Where to go from here?

field theory
symmetry
geometry

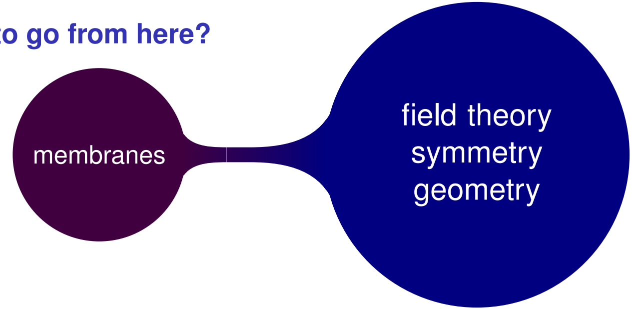
Where to go from here?



generalized Cartan geometry:

$$\begin{tikzcd}
{\color{red}\hat{\chi}} \arrow[rr] & & \hat{\xi} \arrow[rrr, "\hat{D}"] \arrow[dl, dash] \arrow[dr, dash] & & & \hat{E} \arrow[rr] \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & & \hat{T} \arrow[r] & \cdots \\
& \xi & & \Lambda & E & \omega & {\color{red}\rho} & &
\end{tikzcd}$$

Where to go from here?

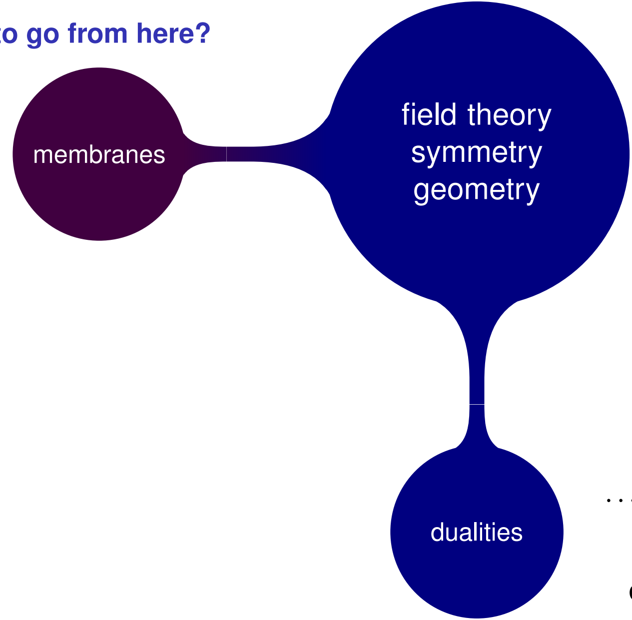


exceptional Cartan geometry: [FH, Yuho Sakatani 23]

tensor hierarchy

$$
\begin{tikzcd}
\cdots \arrow[r] & \hat{\chi} \arrow[rr] & & \hat{\xi} \arrow[dl, dash] \arrow[dr, dash] \arrow[rrr, "\hat{D}"] & & & \hat{E} \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] \arrow[drr, dash] \arrow[rrr] & & & \hat{T} \arrow[r] & \cdots \\
& & \xi & & \Lambda & E & \omega & \rho & \cdots & &
\end{tikzcd}
$$

Where to go from here?

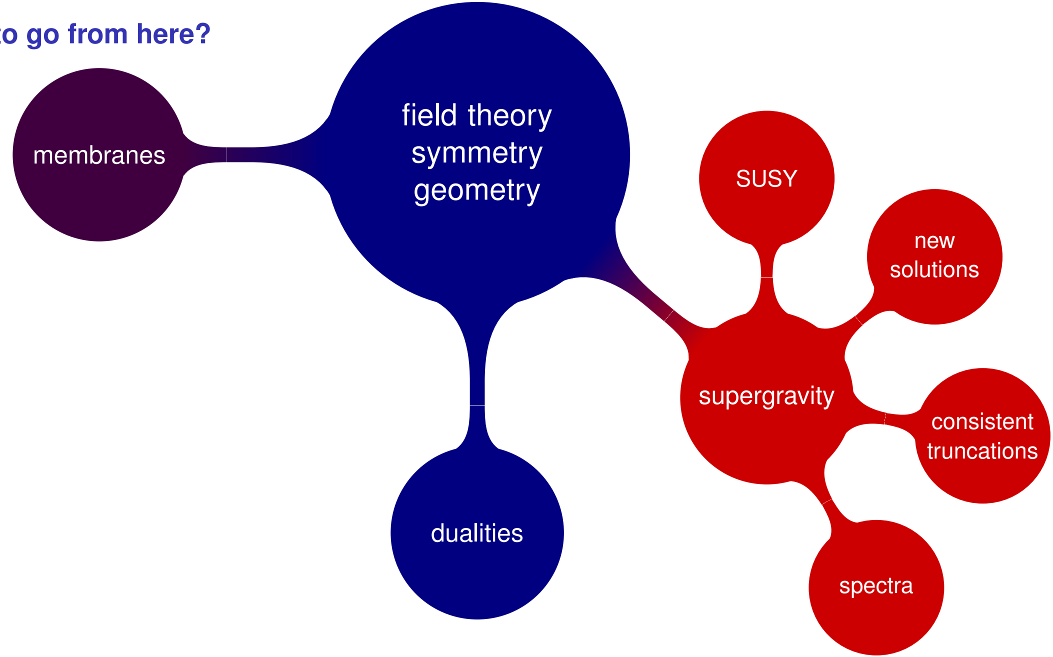


constant

$$\cdots \longrightarrow \hat{E} \xrightarrow{\hat{D}} \hat{T} \longrightarrow \cdots$$

different realizations [FH 17; ...]

Where to go from here?



Where to go from here?