ER for typical EPR

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Eurostrings

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Based on: [2412.08693] [2504.17546] w/ Javier Magán & Brian Swingle

Introduction

- In holography, two ideas appear to be in tension. One is ER=EPR: enough microscopic entanglement generates a connected space (e.g. the black hole interior).

... [van Raamsdonk '13]
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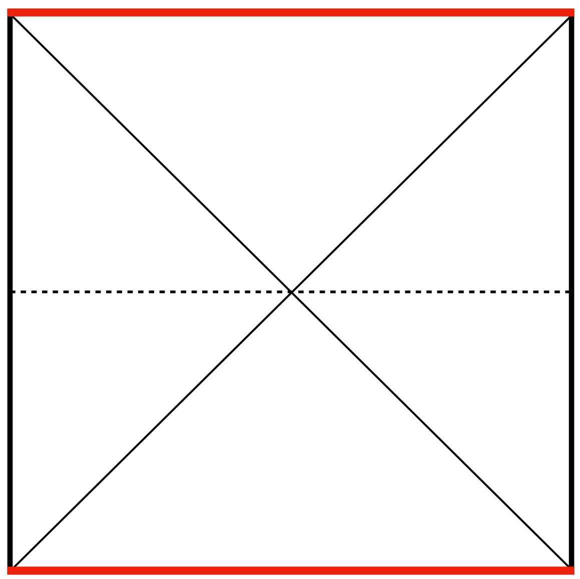

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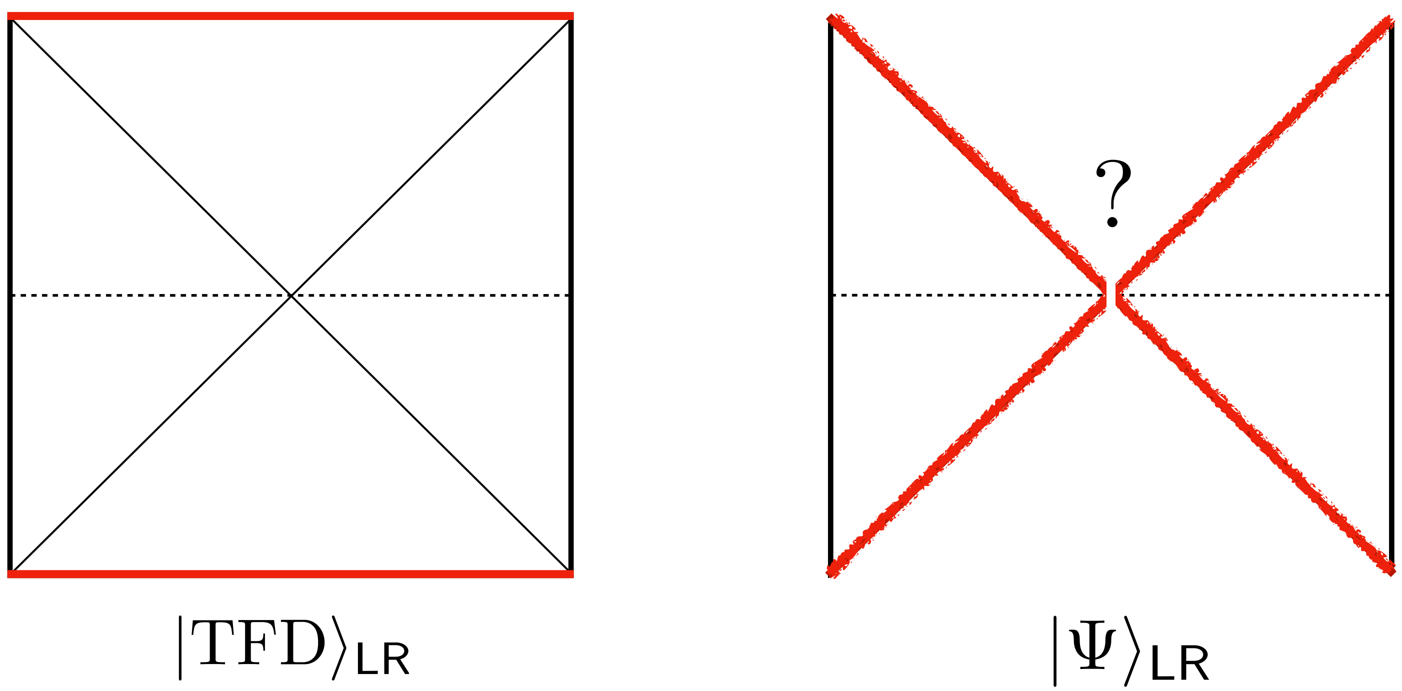

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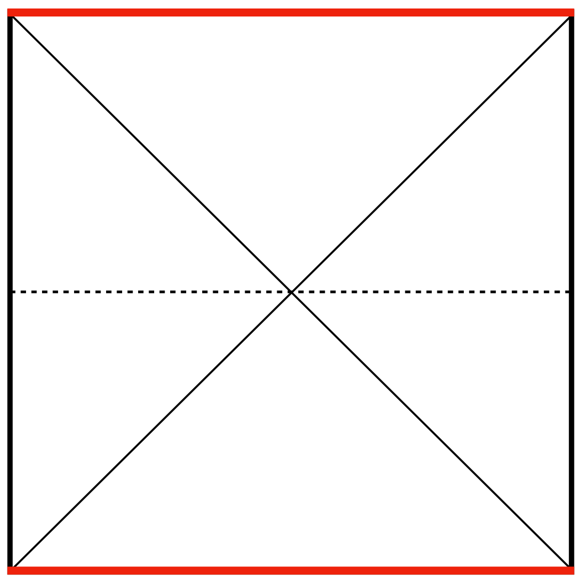

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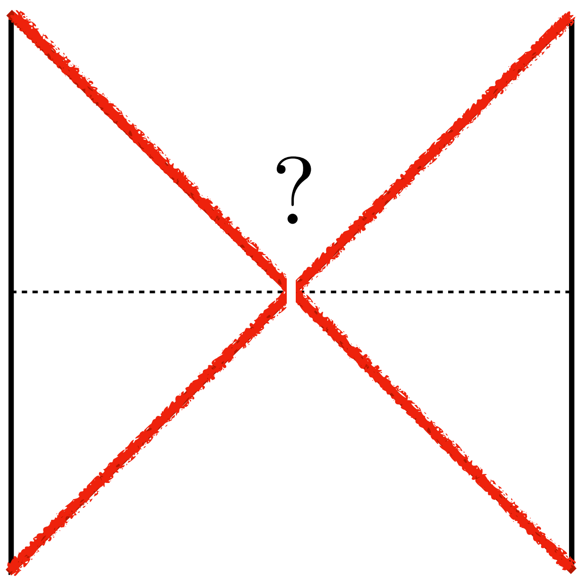

$|\Psi\rangle_{\mathsf{LR}}$

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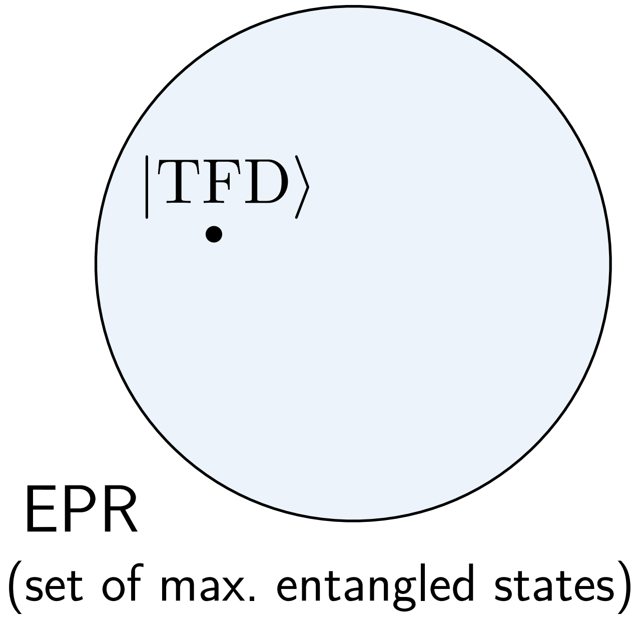
Do generic (typical) EPRs of two black holes have ERs?

Typical EPR

- As an illustrative example, consider a system of N qubits at infinite temperature.



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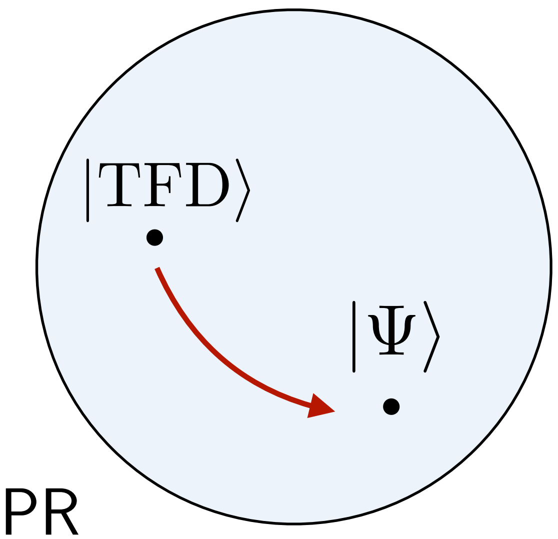

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(set of max. entangled states)

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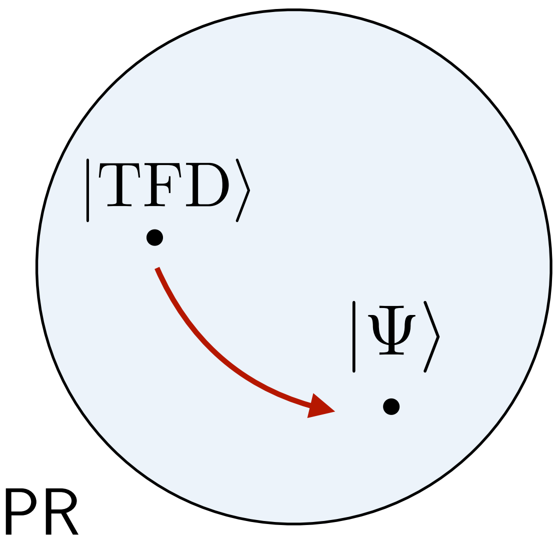

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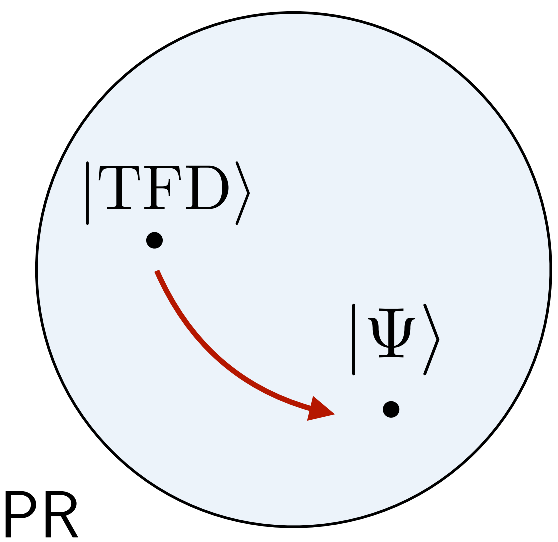

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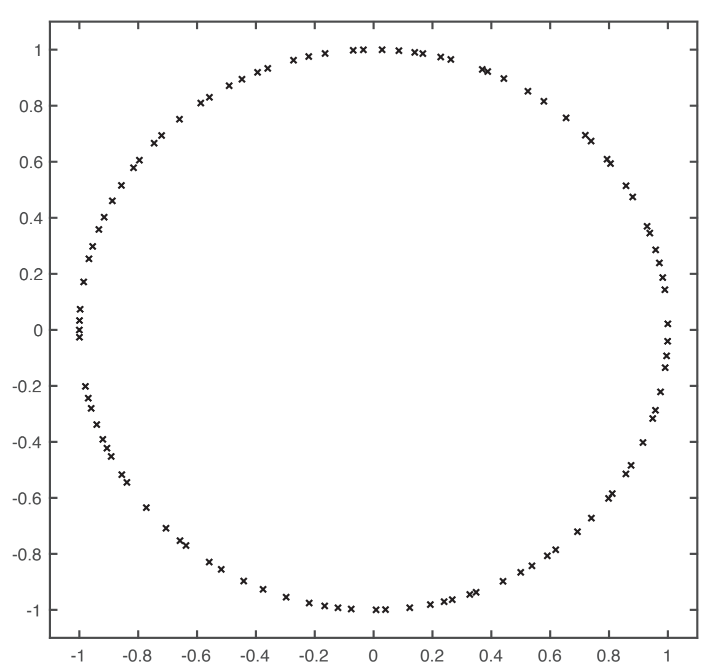

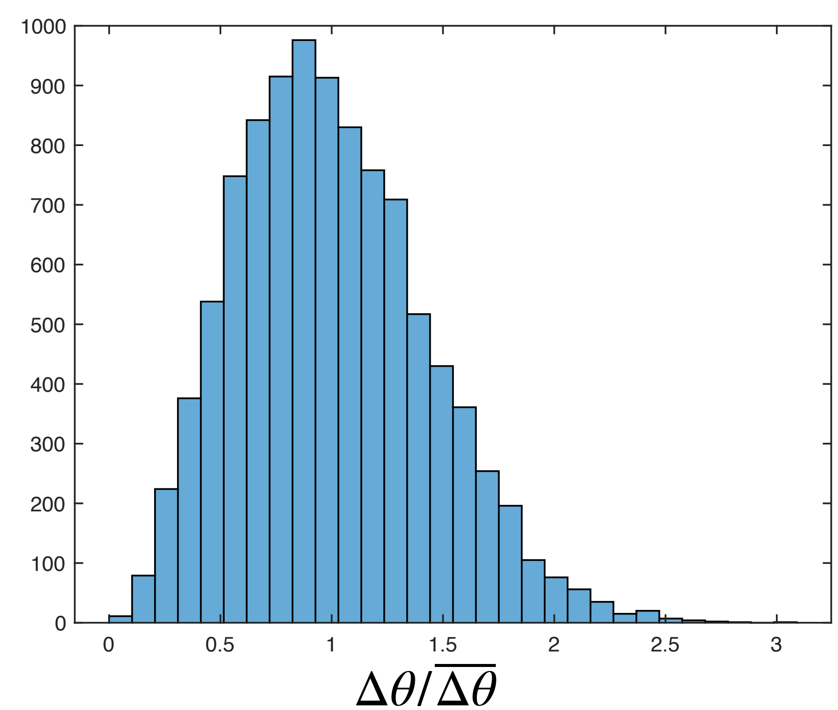

- Typical EPRs have the same entanglement spectrum as the TFD but simple LR correlation functions are $O(2^{-N})$ and very erratically dependent on the specific microstate. This was used in the past to argue against their semiclassicality.

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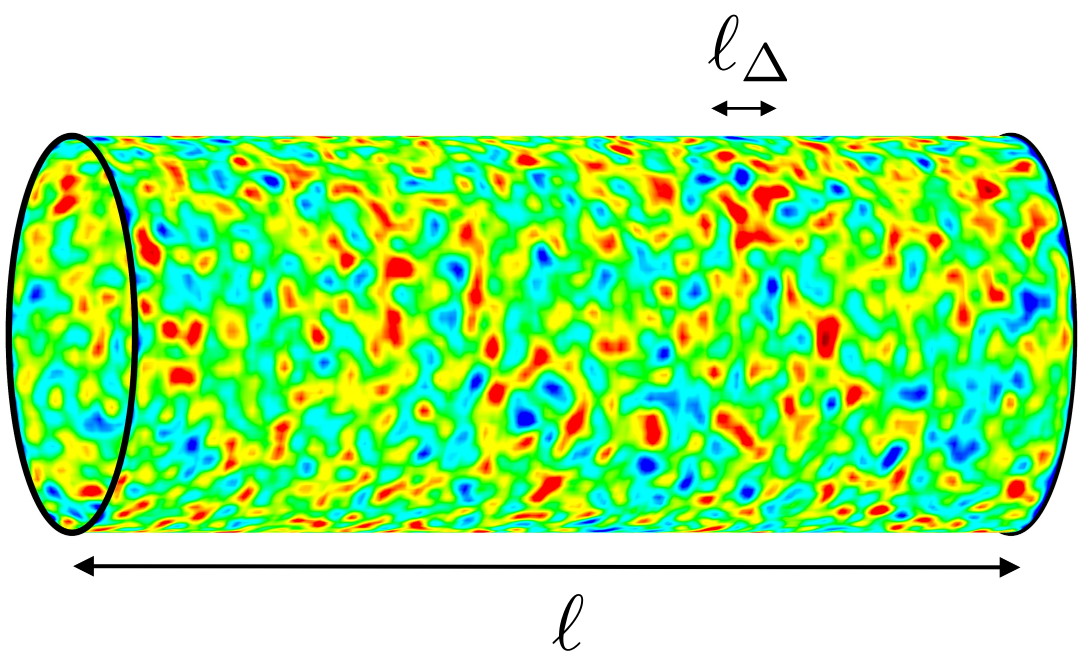
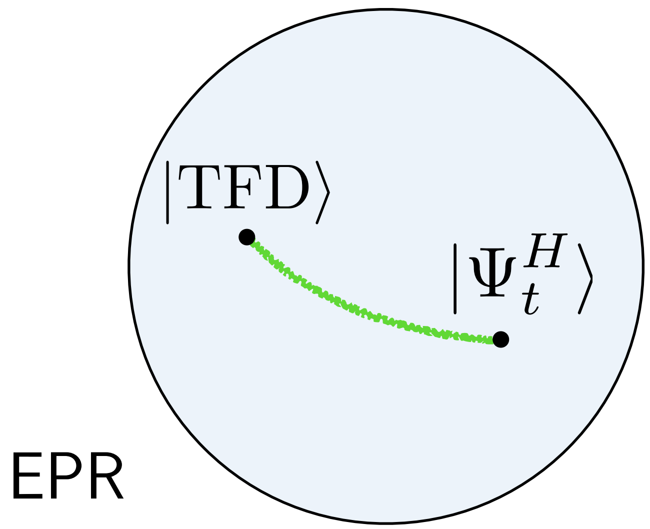
- Outline: (1) Explicitly construct families of ER bridges that incorporate this erraticity semiclassically. The wormholes are long **caterpillars**:



(2) Use the GPI to derive a quantitative correspondence between the length of the wormholes and the amount of microscopic quantum randomness of the states.

Towards typical EPRs

- Starting from the TFD, a natural possibility to construct more generic entangled states is to time-evolve it with the time-independent Hamiltonian H_R.



$$\left|\Psi_t^H\right\rangle = e^{-\mathrm{i}tH_\mathsf{R}}\left|\mathrm{TFD}\right\rangle = \frac{1}{2^{\frac{N}{2}}}\sum_{i=1}^{2^N} e^{-\mathrm{i}tE_i}\left|i^*\right\rangle\left|i\right\rangle$$

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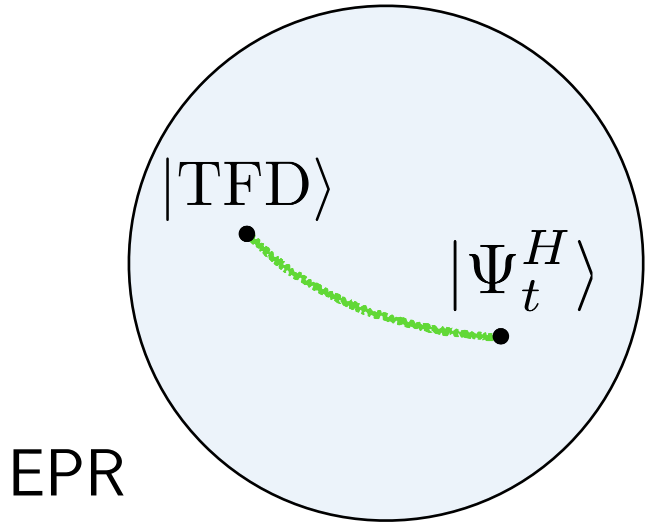

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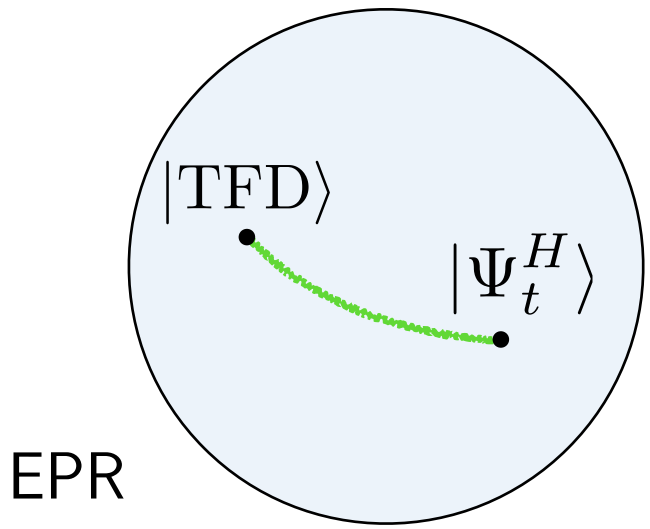

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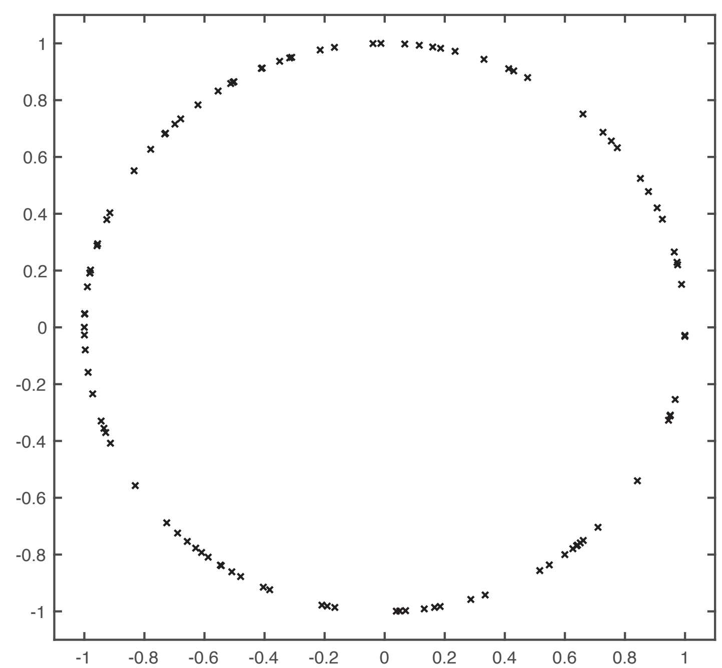
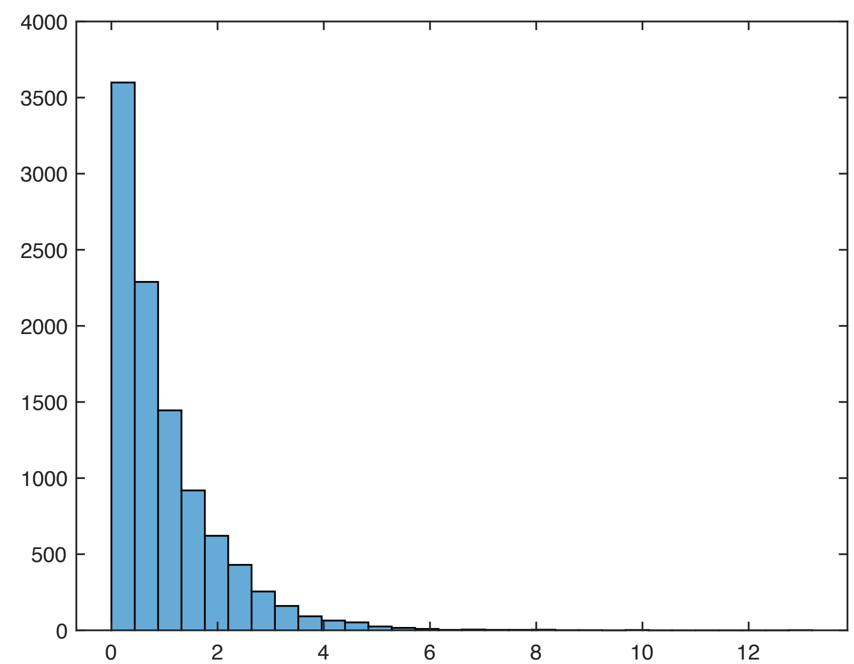
$t \sim (\Delta E)^{-1}$

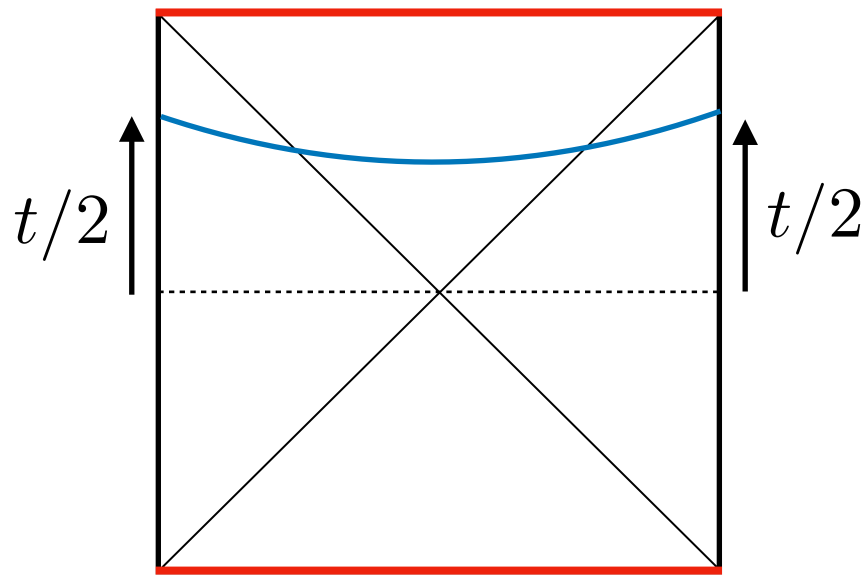
$\theta_i = -tE_i \pmod{2\pi}$





$\Delta\theta/\overline{\Delta\theta}$

- Still, for black holes, the Hamiltonian evolution generates more general examples of ER $=$ EPR. The wormhole stretches under time evolution.



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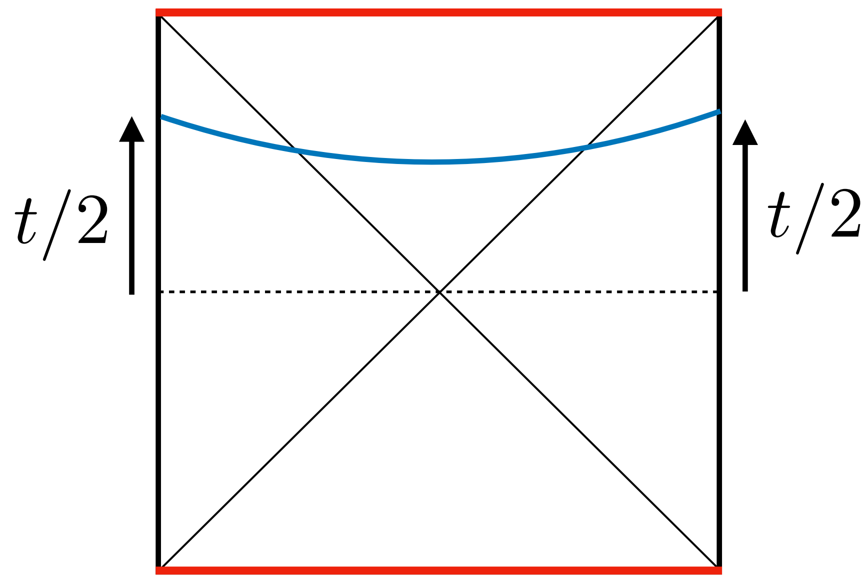

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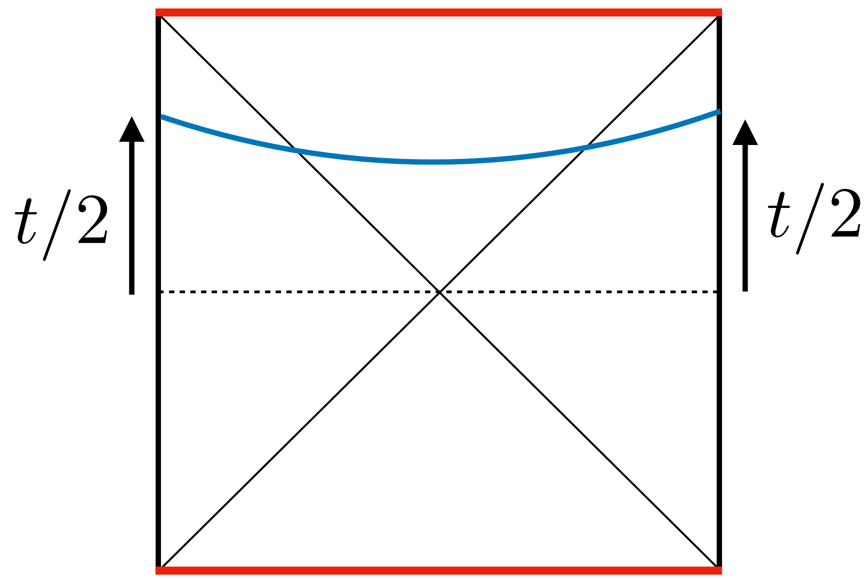

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- At Heisenberg times $t = O(\beta e^S)$, the state becomes maximally complex and yet atypical. Non-perturbative effects are expected to be important at this timescale.

[Susskind '20] [Stanford, Yang '22] ...
[Iliesiu, Mezei, Sarosi '21]

Brownian time evolution

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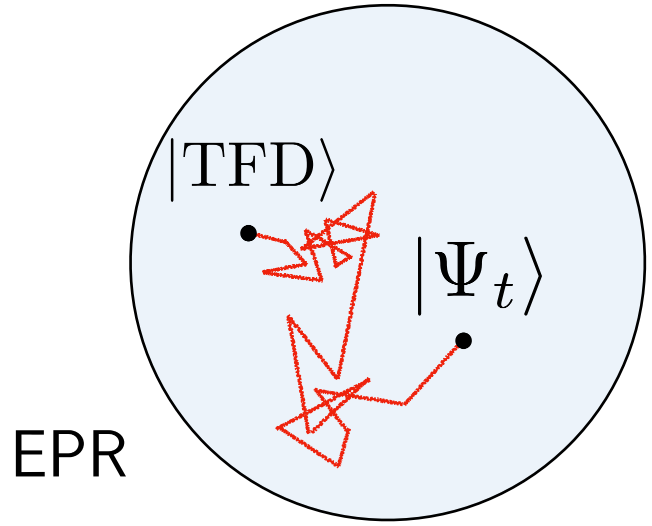
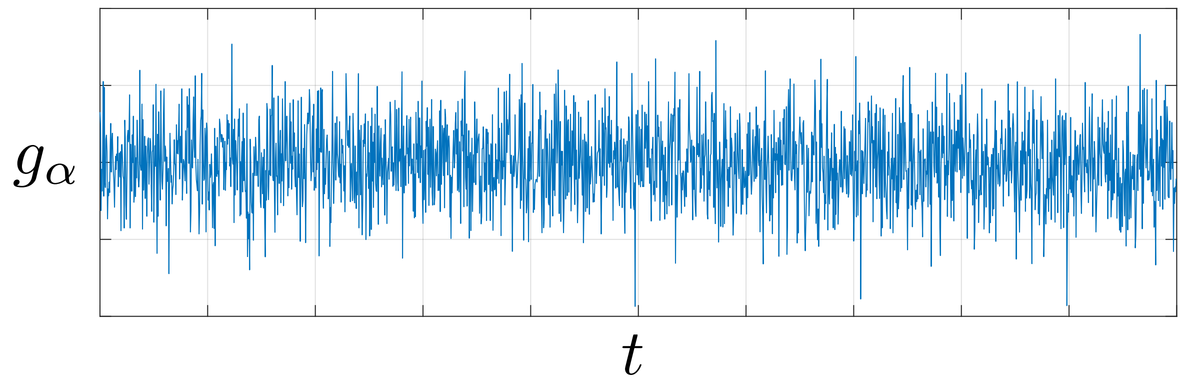
[Lashkari et al. '11]

$$|\Psi_t\rangle = U(t)\,|\mathrm{TFD}\rangle \qquad H(t) = \sum_{\alpha=1}^{K} g_\alpha(t)\mathcal{O}_\alpha$$





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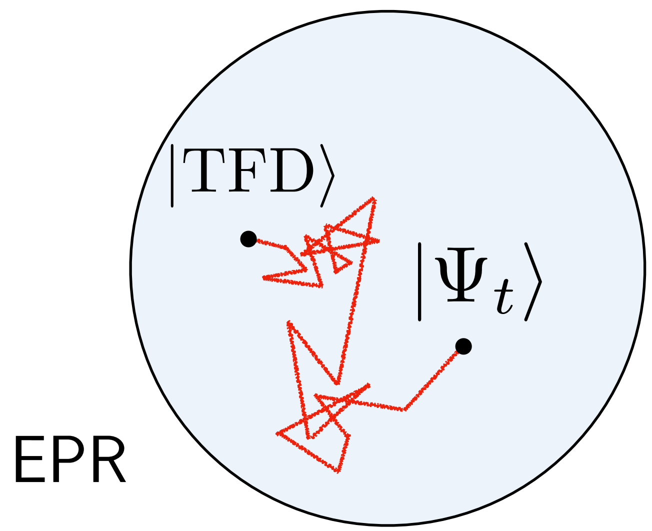

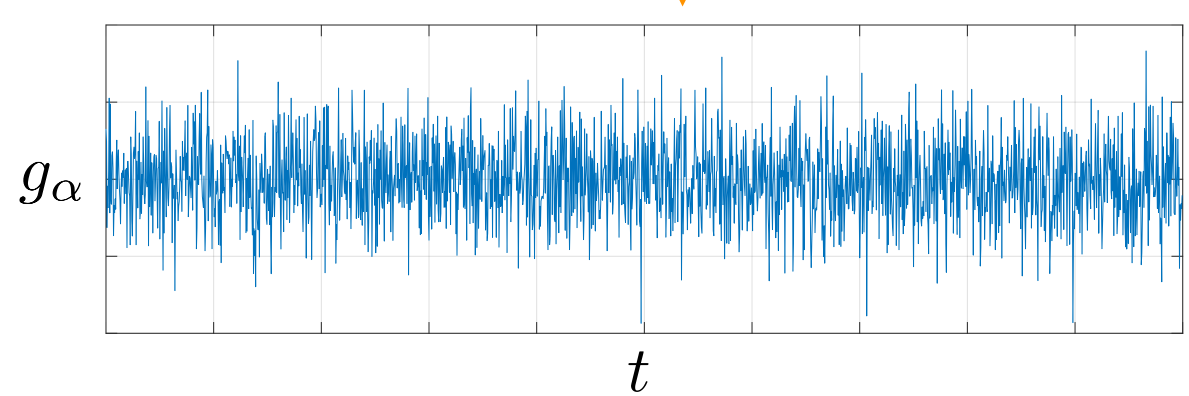

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- The dynamics is the continuous version of a random quantum circuit: at each time the system evolves via an infinitesimal "random gate" $U(t,\delta t) = e^{-\mathrm{i}\delta t\, H(t)}$.

- Brownian dynamics can be formally solved exactly on average. As an example, consider the spectral form factor of $U(t)$.

$$\mathsf{SFF}(t) = \overline{|\mathrm{Tr}\, U(t)|^2} =$$

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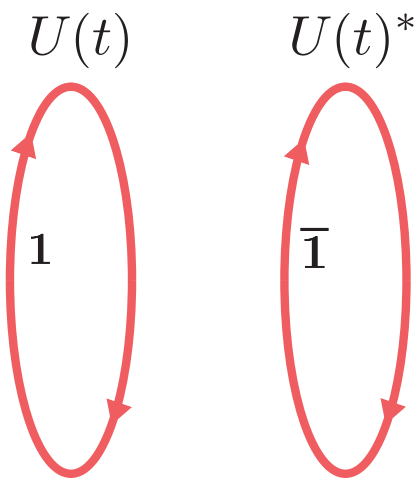

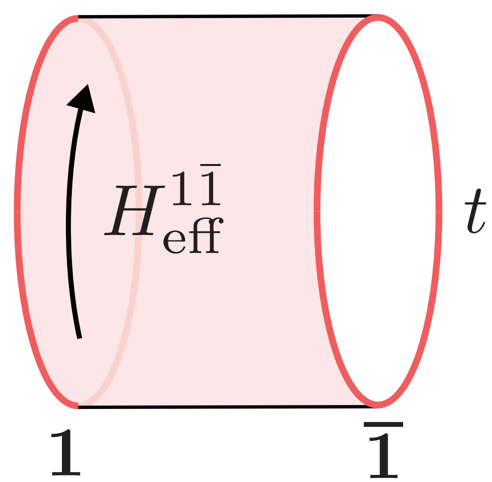

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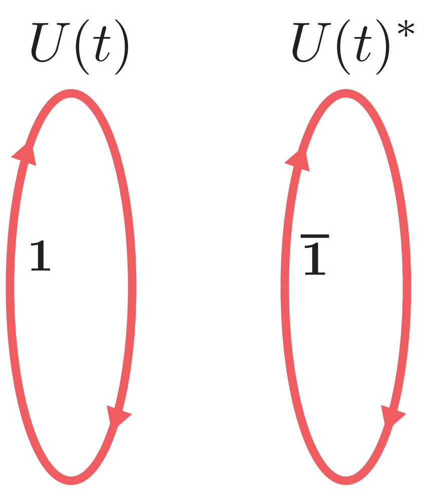

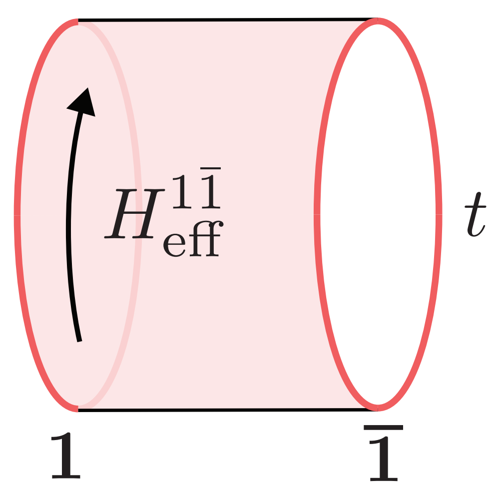

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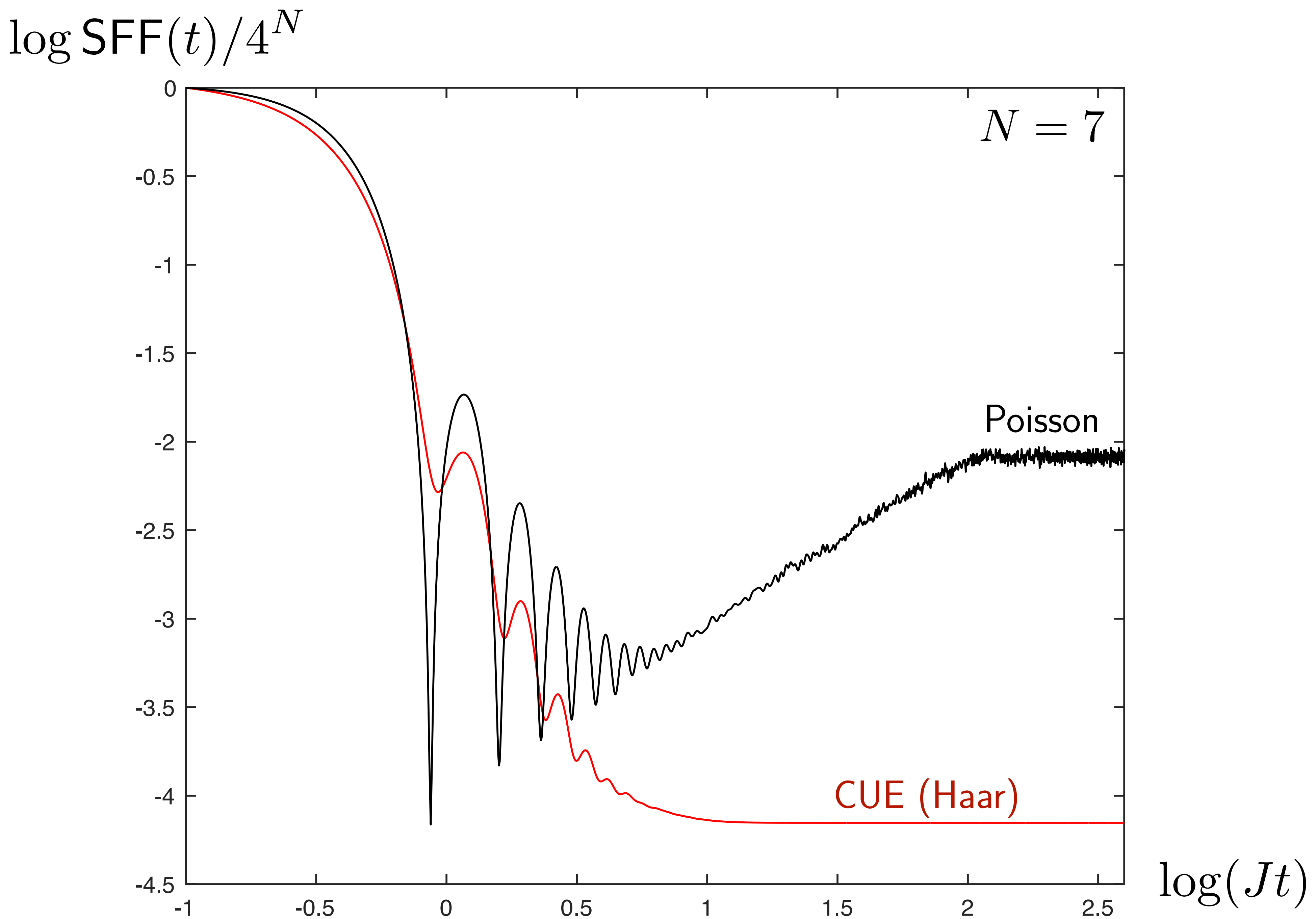
$$H_{\text{eff}}^{1\bar{1}} \, |\mathbf{1}\rangle = 0$$

- The effective Hamiltonian is gapped and its unique GS is the infinite temperature TFD. At late times this produces:

$$\mathsf{SFF}(t) \approx 1 + N_* \, e^{-2tE_{\text{gap}}}$$

- The late time SFF signals that the Brownian evolution dynamically forms a generic unitary (an thus a generic EPR) which incorporates the eigenphase repulsion of Haar.



Finite temperature

- AdS/CFT systems that describe black holes. We construct $|\Psi_t\rangle$ with the Brownian time-evolution applied to a finite temperature TFD.

[Shenker, Stanford '13]
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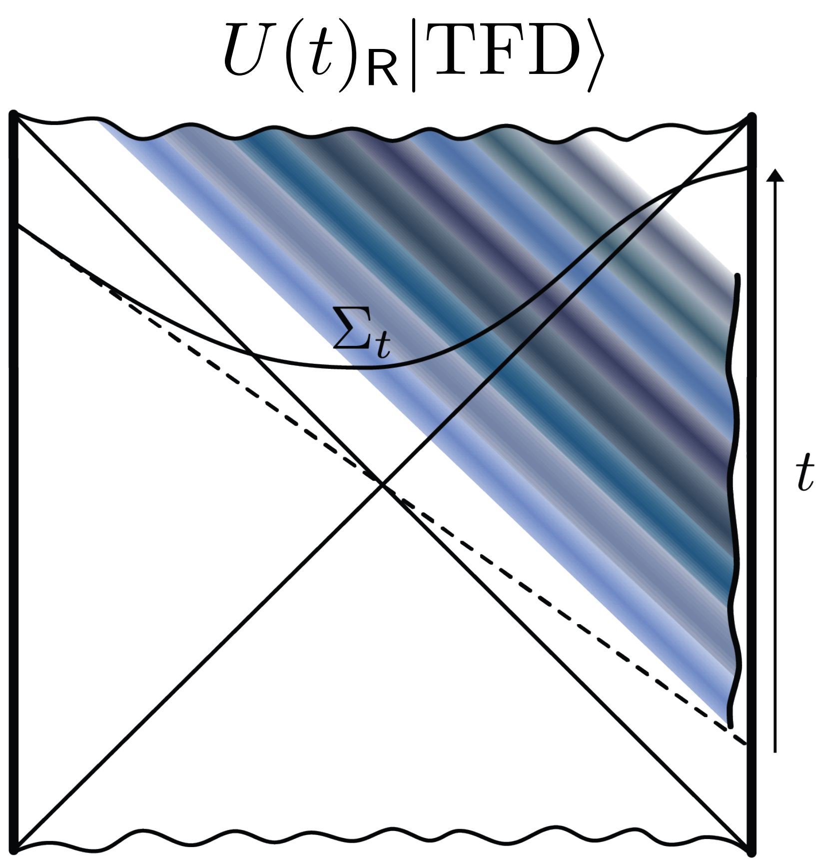

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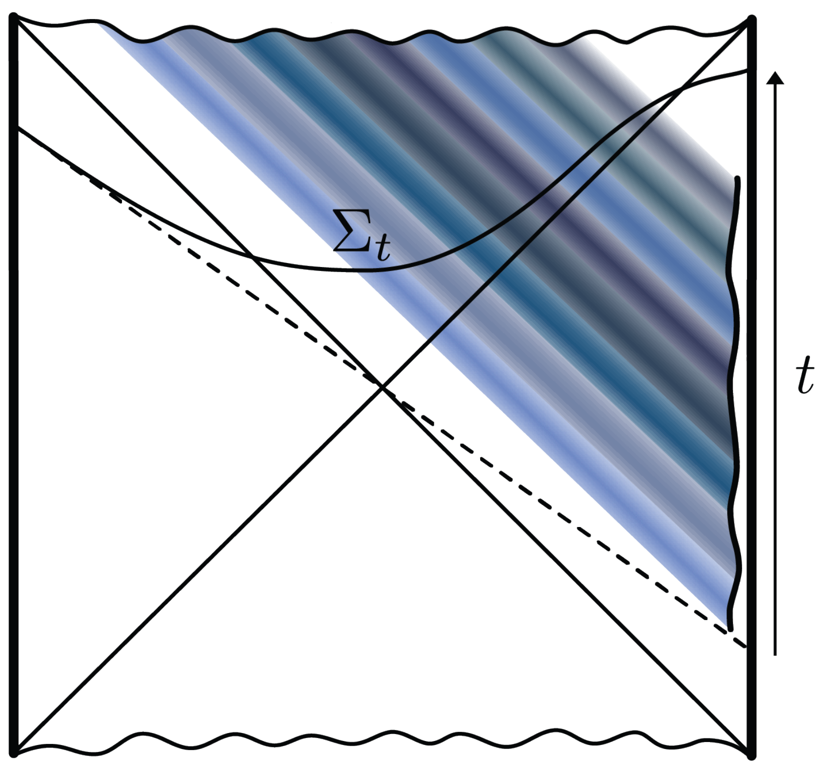

$$Jt_{\max} \sim \frac{M_0}{K\delta E} \sim \frac{S}{K}$$

- Out-of-equilibrium at $O(S/K)$ times.

- Incorporate Euclidean evolution to gradually cool the state down. This injects the perturbations from the Euclidean past and directly creates them in the BH interior.

$$U(\Gamma_t) = e^{-\delta\beta H_0} U_I(n, \delta t) \cdots e^{-\delta\beta H_0} U_I(2, \delta t) e^{-\delta\beta H_0} U_I(1, \delta t)$$

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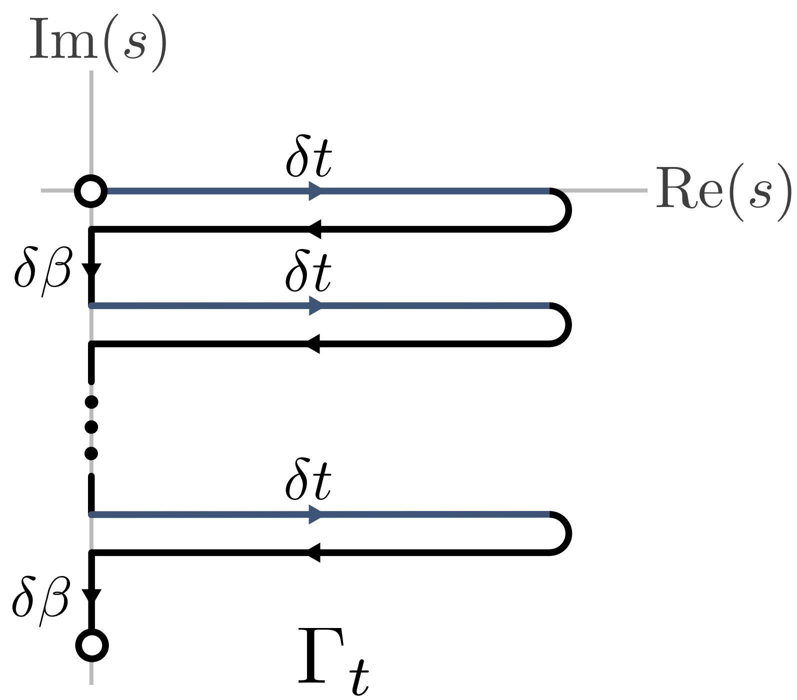

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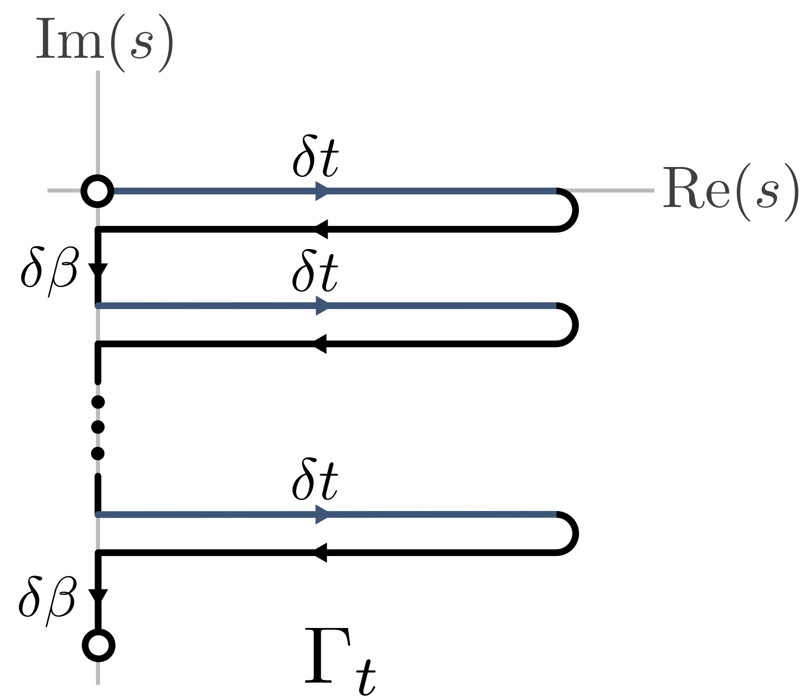

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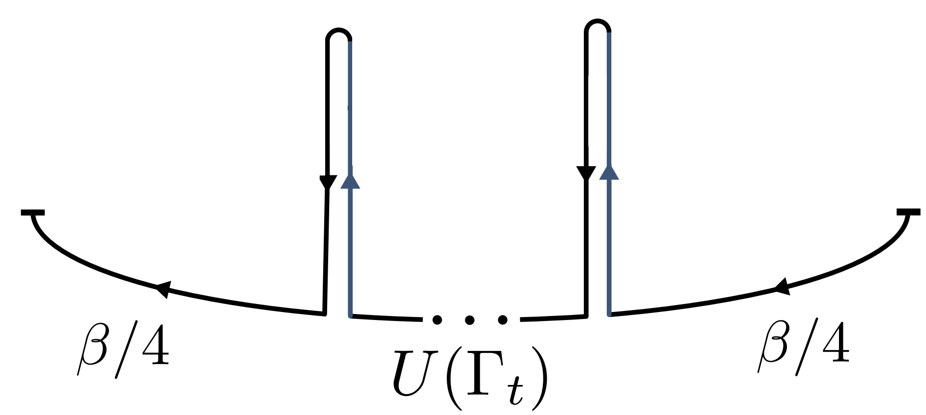
- The CFT path integral prepares an equilibrium version of the time-evolved TFD.

$|\Psi_t\rangle \propto$



Semiclassical duals: caterpillars

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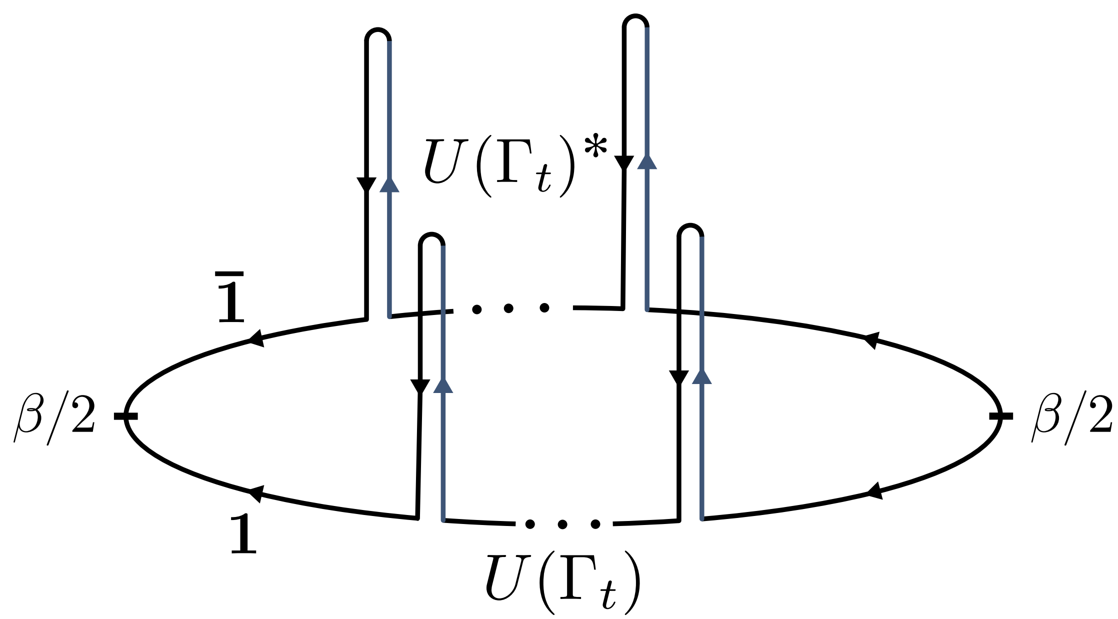

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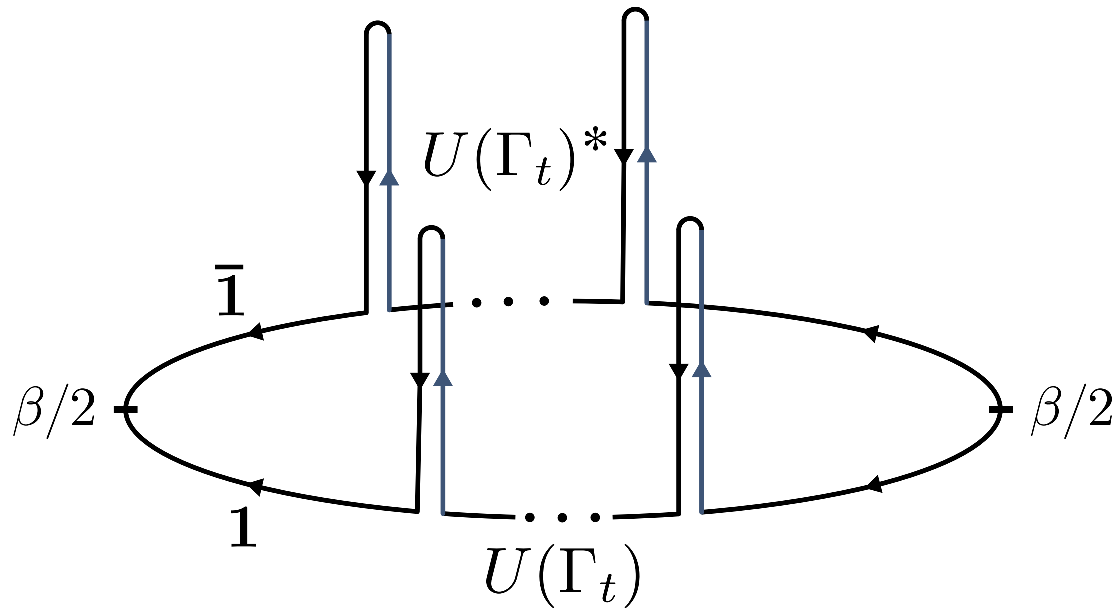

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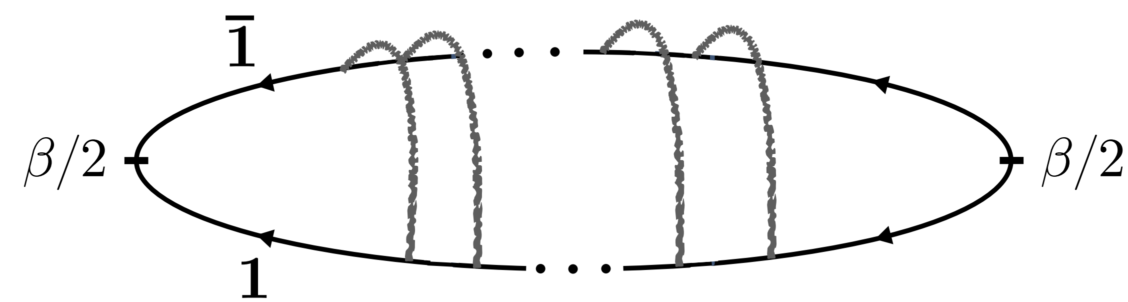

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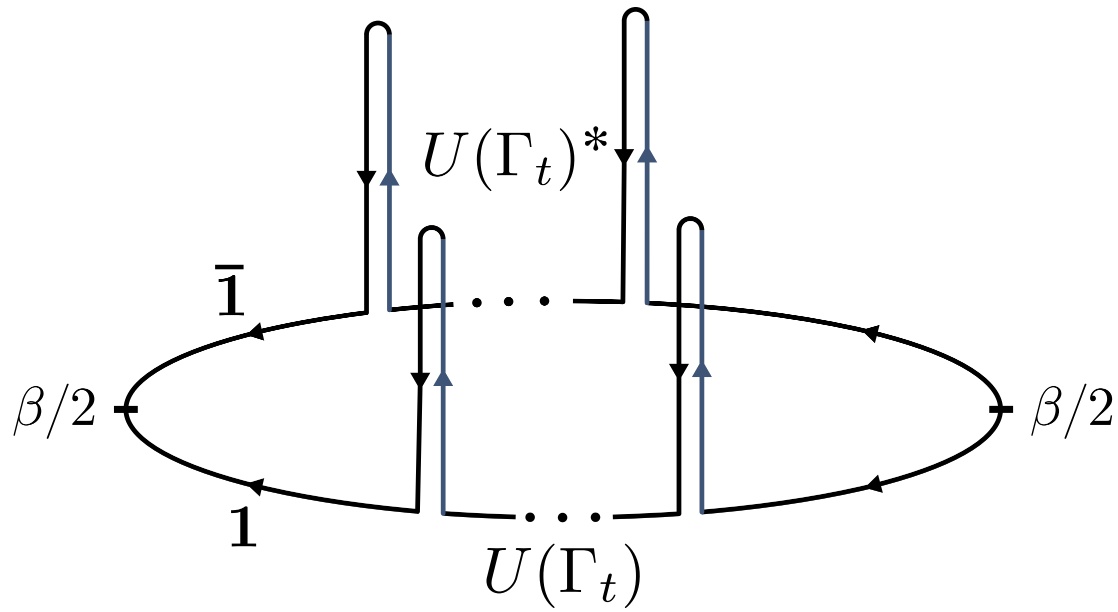

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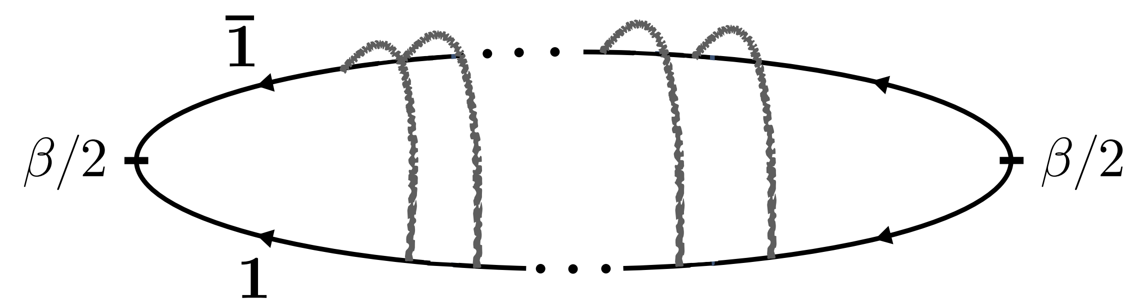


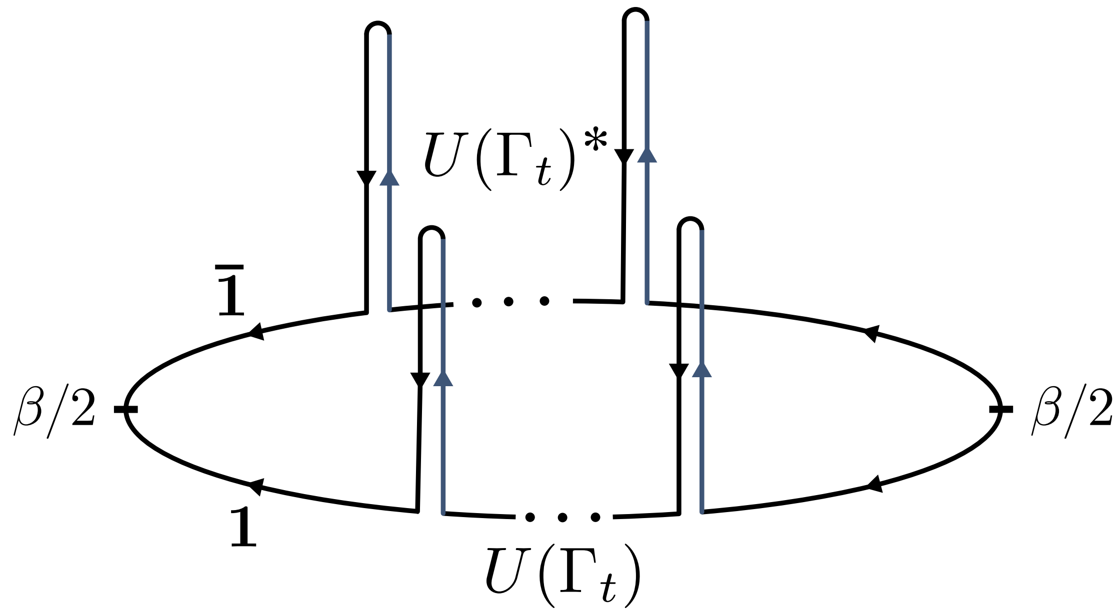
- The effective Hamiltonian is a "double-trace" or Maldacena-Qi Hamiltonian which has appeared in holography in the context of eternal traversable wormholes.

$$H_{\text{eff}} = H_0^1 + H_0^{\bar{1}} + \frac{J}{2}\sum_\alpha \left(O_\alpha^1 - O_\alpha^{\bar{1}\,*} \right)^2$$

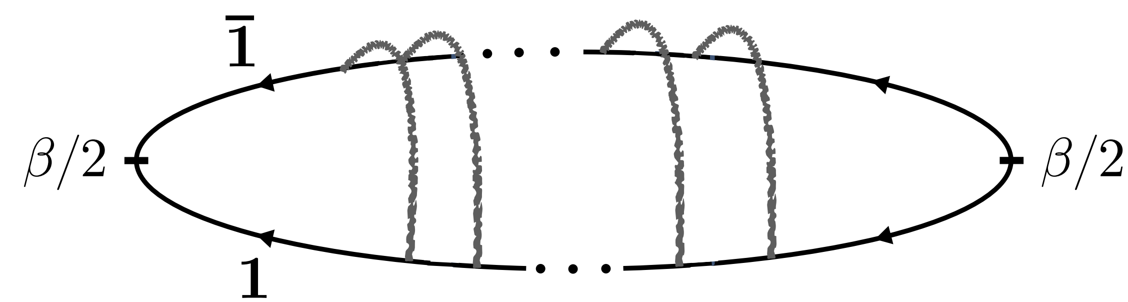
[Maldacena, Qi '18]
[Cotrell et al. '18]

Semiclassical duals: caterpillars

- Use GPI to prepare the semiclassical dual to $|\Psi_t\rangle$ computing its norm. The geometry of individual instances is too complicated → take average over couplings.



$$\overline{Z_{\Psi_t}} = \langle \mathrm{TFD} | \, e^{-tH_{\mathrm{eff}}} \, | \mathrm{TFD} \rangle$$



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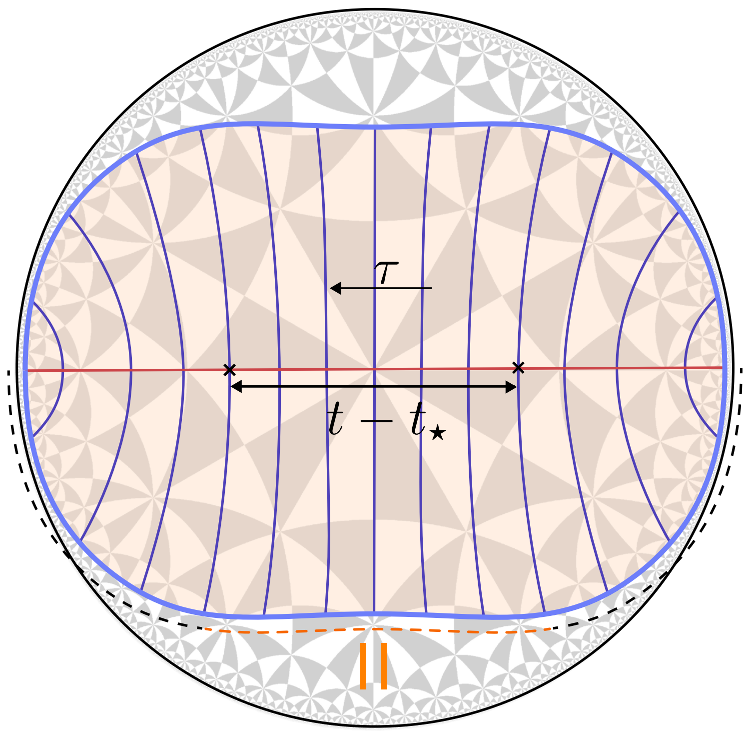
- We want to evaluate the average norm using the GPI.

$$\overline{Z_{\Psi_t}} = e^{-I} Z_{1-\mathrm{loop}}$$

- In SYK, at low temperatures, we can solve for the disk saddle point, using the Liouville particle formulation of JT gravity: [Maldacena, Qi '18]



$$I = \frac{N}{2}\int \mathrm{d}u \left(\frac{1}{2}\dot{L}^2 + 2e^{-L} - \eta e^{-\Delta L}\right)$$

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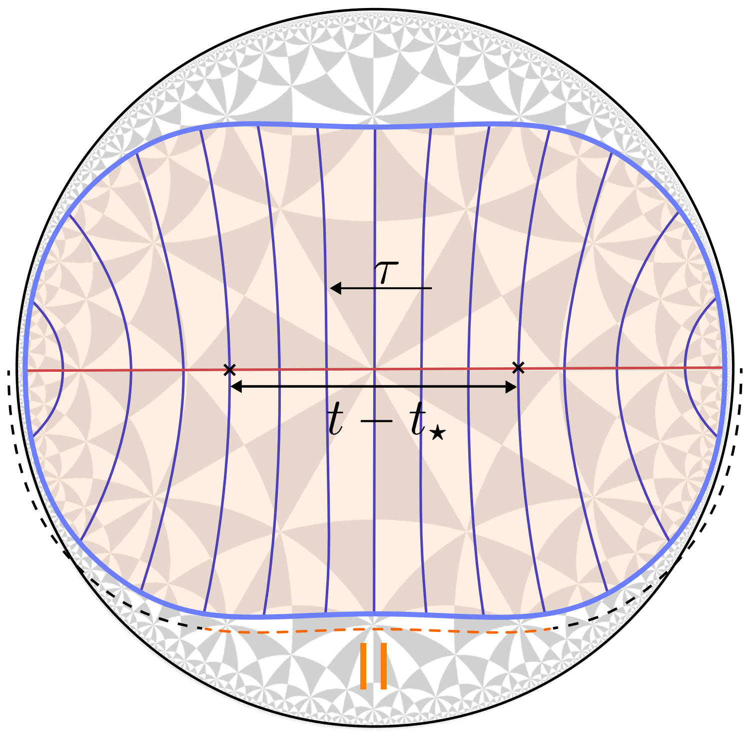

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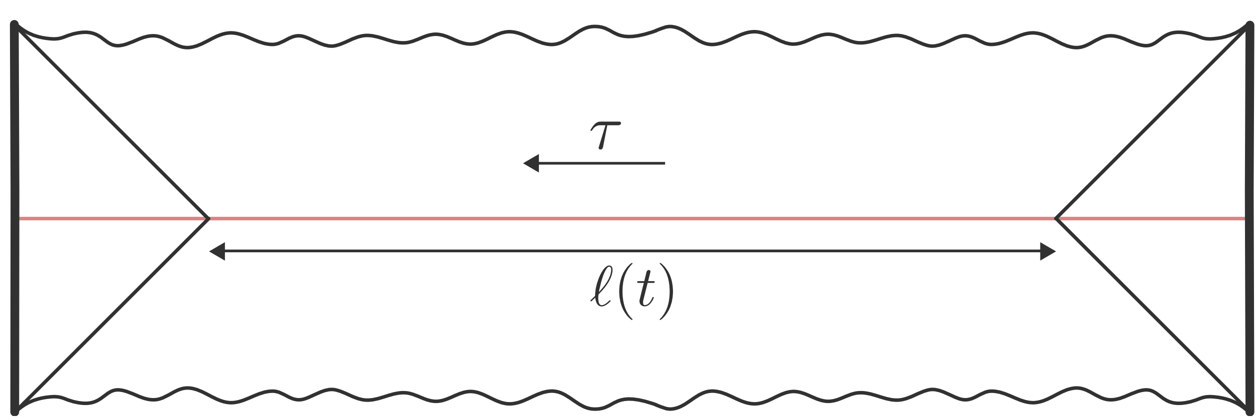
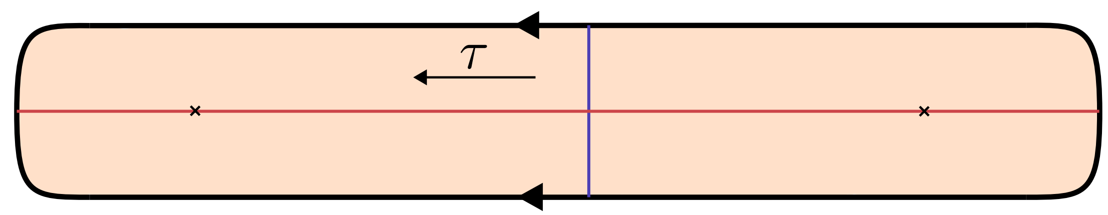
- The consequence is that the length of the wormhole (red slice) grows linearly with t.



$$\ell(t) \sim t - t_\star$$

- In higher dimensions we can argue for the same result for $O(c)$ single-trace matter operators using general properties of the effective Hamiltonian: it has a unique ground state that looks like a finite-temperature TFD and it is gapped.

[Cotrell et al. '18]



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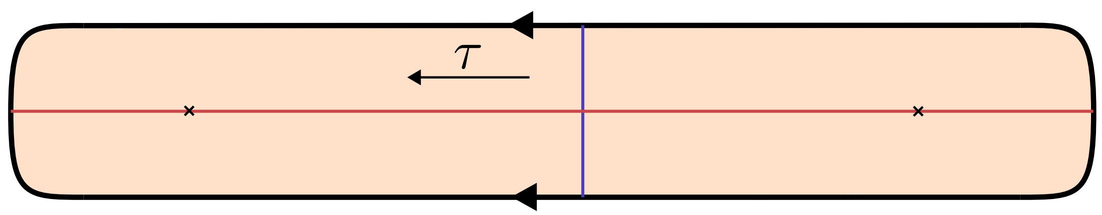
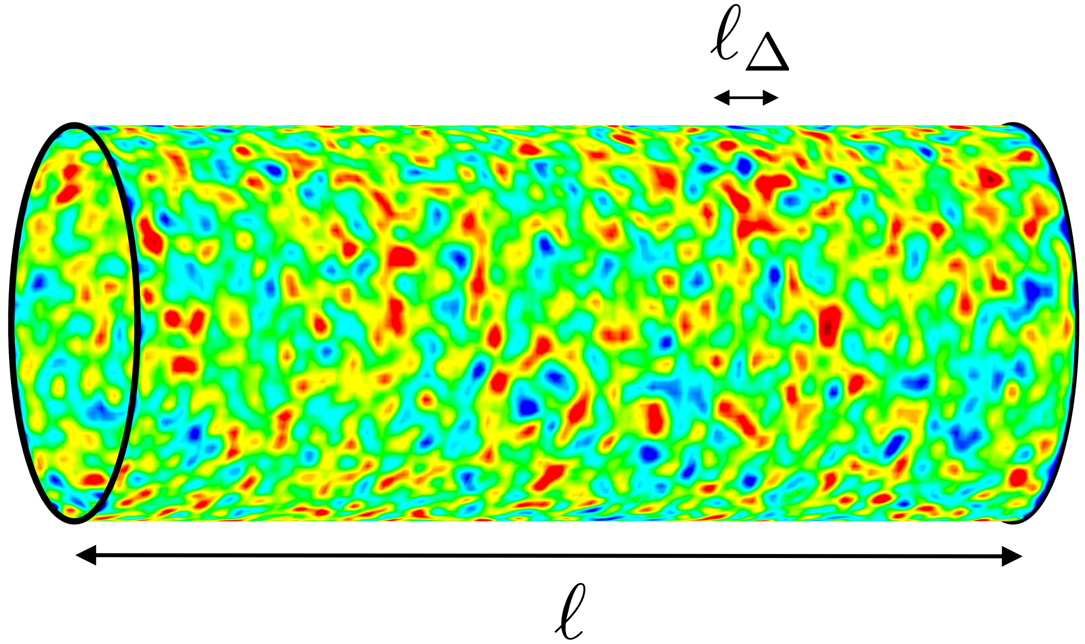


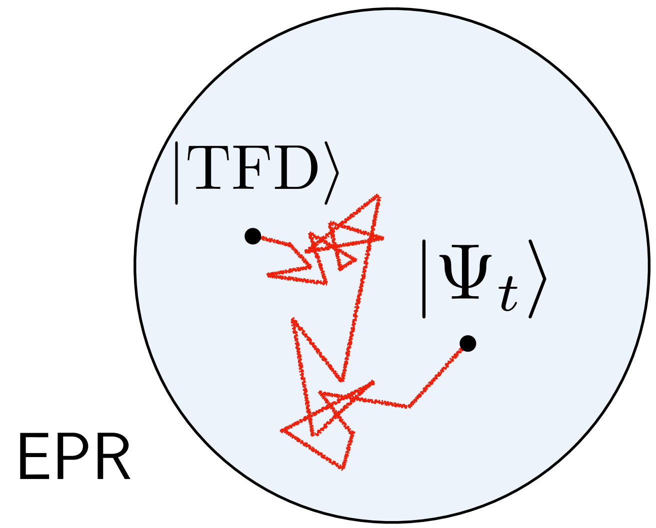
- The slope of the linear growth can be related to the energy gap E_{gap} of the effective Hamiltonian using the lightest excitations on the wormhole as a "clock":



$$\frac{\ell}{\ell_\Delta} = E_{\mathrm{gap}}(t - t_\star)$$

Randomness growth in gravity

- Universally, the Brownian dynamics produces a dense exploration of Hilbert space and the ensemble of states becomes more and more random in time.



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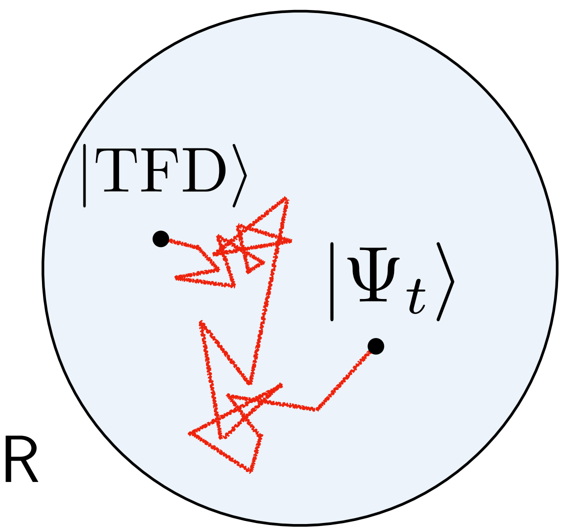
EPR



$\boxed{k=1}$

$$\overline{\langle \Psi_t | \mathcal{A} | \Psi_t \rangle} \;=\; \overline{\langle \Psi_{\mathsf{EPR}} | \, \mathcal{A} \, | \Psi_{\mathsf{EPR}} \rangle} \quad \forall \; \mathcal{A}$$

$\boxed{k=2}$

$$\overline{\langle \Psi_t | \mathcal{A} | \Psi_t \rangle^2} \;, \quad \overline{S_2(\rho_{\mathsf{L}})} \quad \dots$$

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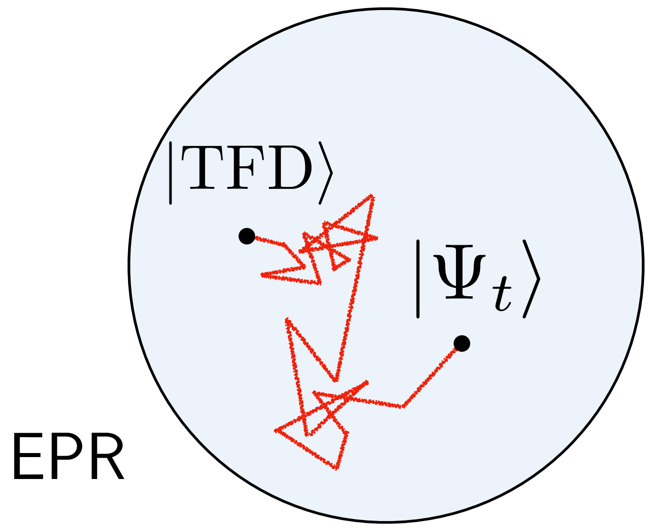

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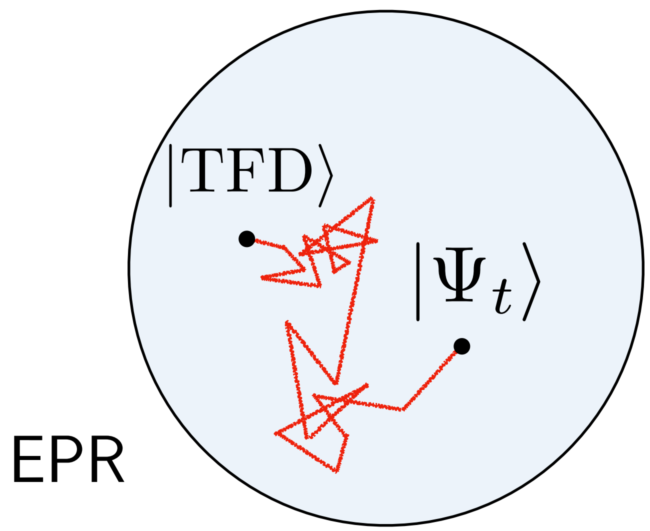

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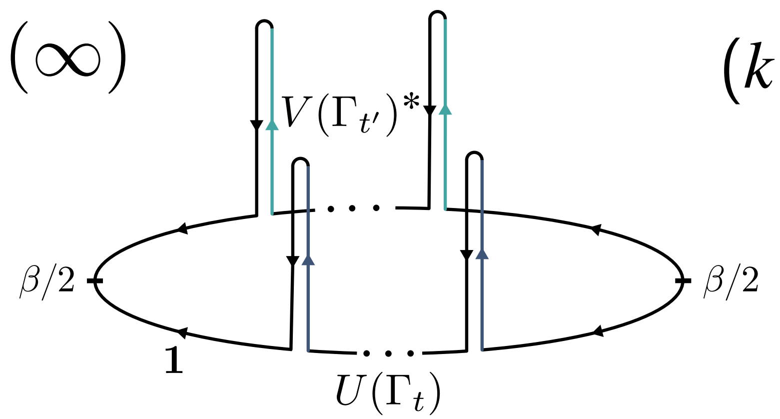
- For black holes, how does randomness grow with time t?

- The distance to k-design is encoded in the statistics of inner products, whose moments are known as the **frame potentials** $F_k(t)$ of the ensemble of states.

$$\mathrm{distance}(k) = F_k(t) - F_k(\infty)$$

$$F_k(t) = \overline{|\langle \Phi_t | \Psi_t \rangle|^{2k}}$$



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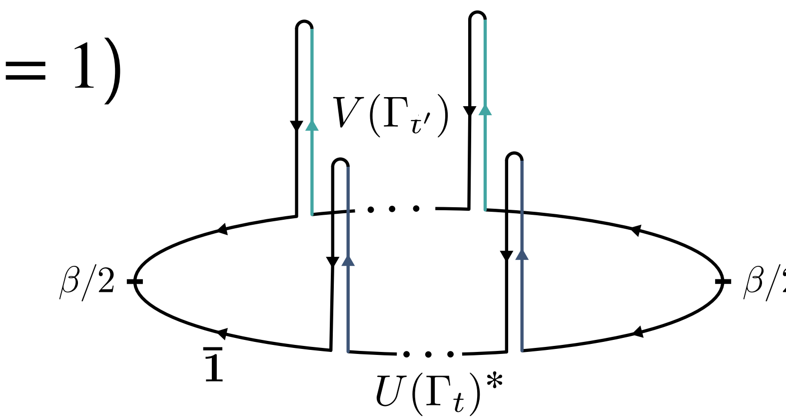

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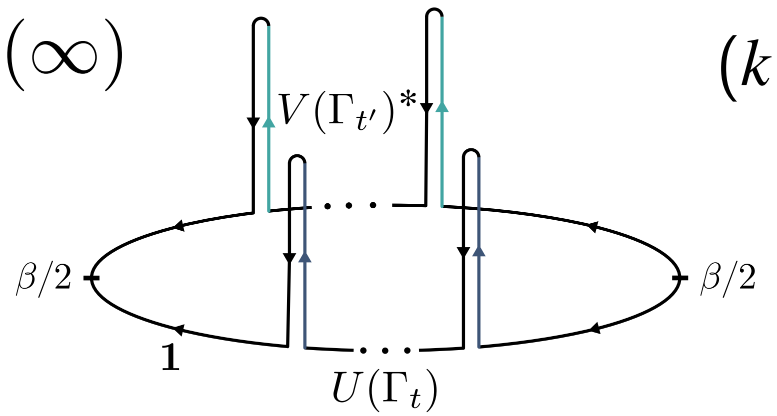

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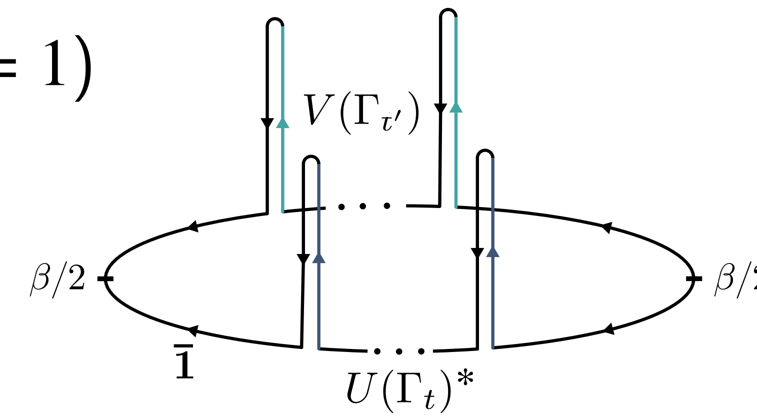

- The FP is a thermal partition function for a $2k$-replica effective Hamiltonian at inverse temperature $2t$. At late times its low-energy sector dominates, which breaks replica symmetry and it is captured by k copies of the 2-replica physics of H_{eff}.

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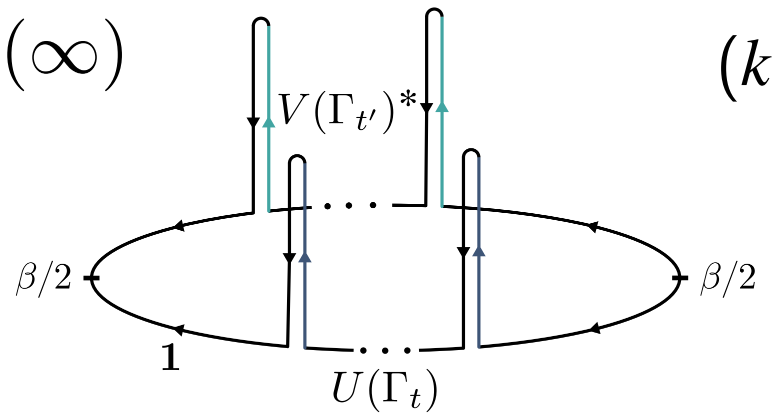

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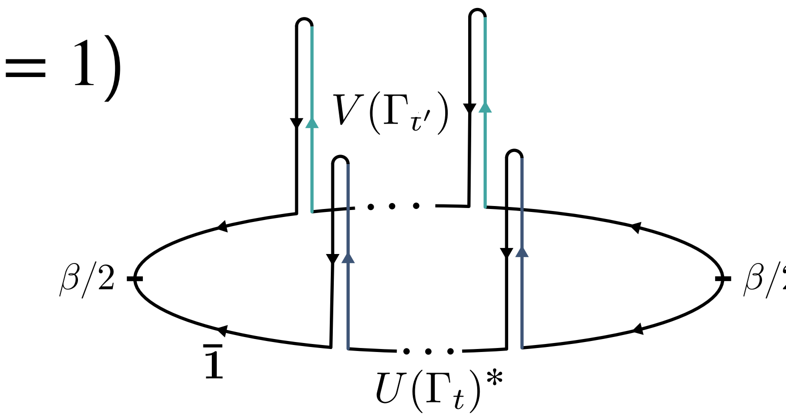

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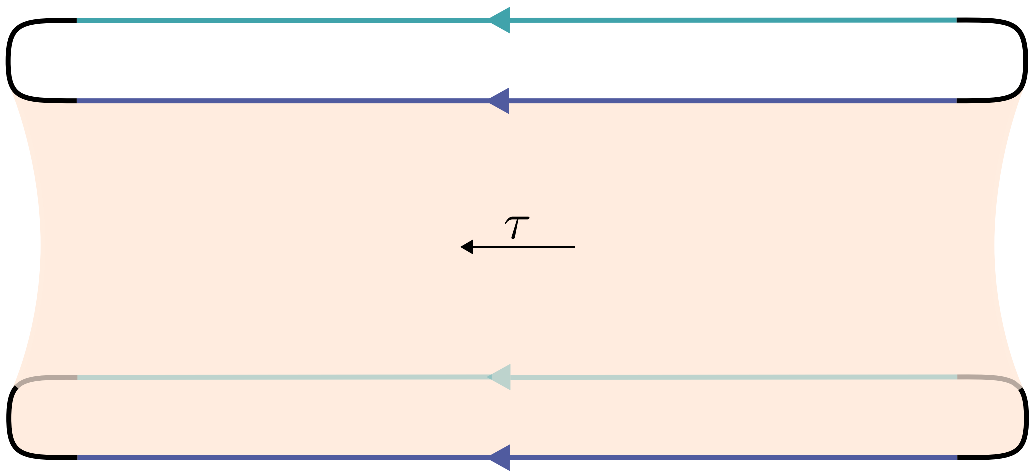
[Magán, Swingle, MS '24 '25]



$$F_k(t) \sim F_k(\infty)\left(1 + N_* e^{-2tE_{\text{gap}}}\right)$$

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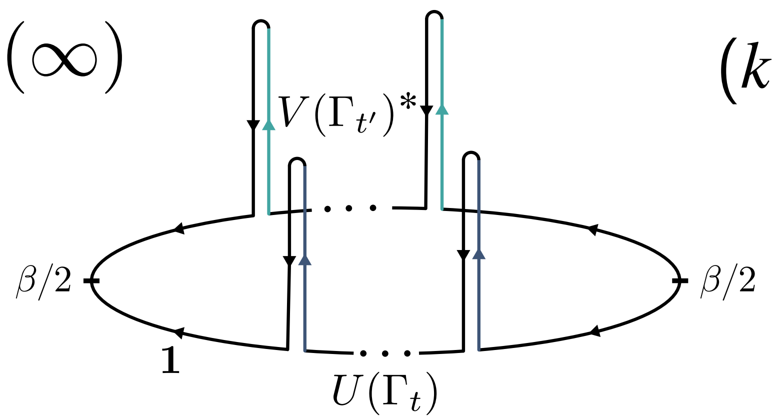

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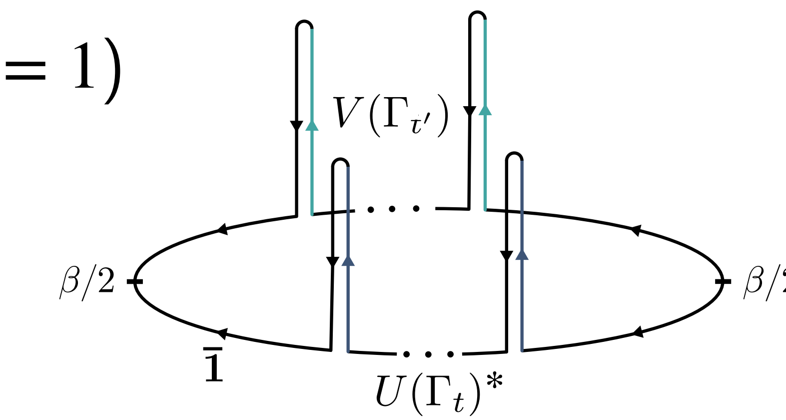

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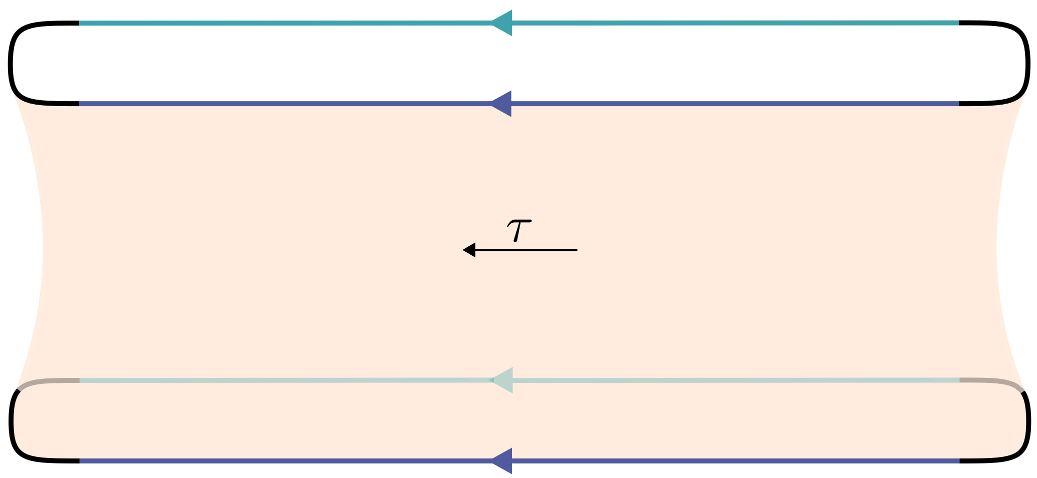

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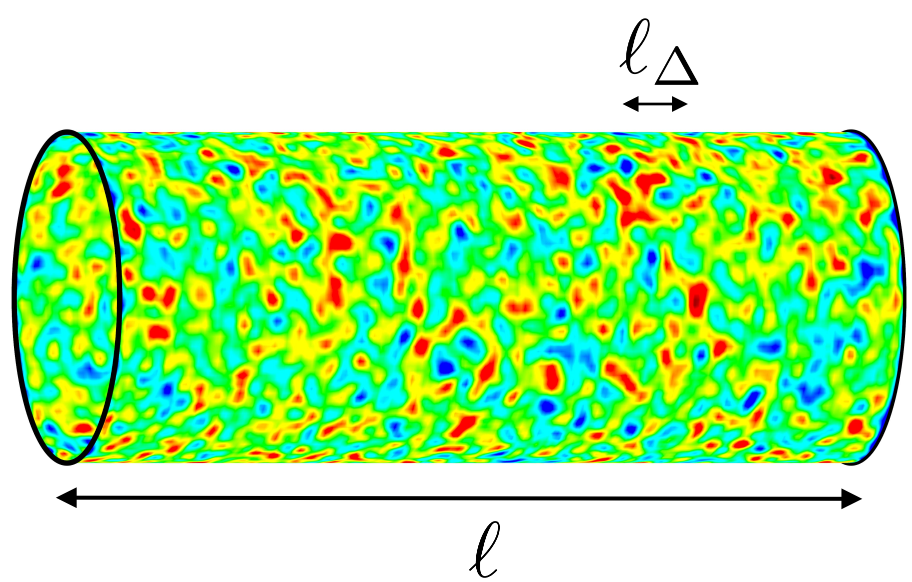
$$\boxed{k \sim E_{\text{gap}} t}$$

- Randomness grows linearly with the same slope as the average length of the wormhole.

Wormhole length = randomness

- Combining both linear growths we arrive to our main result:

> The ensemble of caterpillars of length ℓ and matter correlation scale ℓ_Δ forms an ε-approximate quantum state k-design of the black holes for
> $$k \sim \frac{\ell - \ell_\varepsilon}{\ell_\Delta} \qquad (\ell_\varepsilon = \ell_\Delta \log \varepsilon^{-1})$$



- This constitutes a "complexity" = geometry relation in holography, derived from the GPI.

Discussion

- Ensembles of caterpillars provide a window into the generic structure of the Hilbert space of black holes in any theory of gravity with low-energy matter.

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- Some open questions:

  - Firewalls? What is the ratio of caterpillars with firewalls and without firewalls?
    [Stanford, Yang '22]

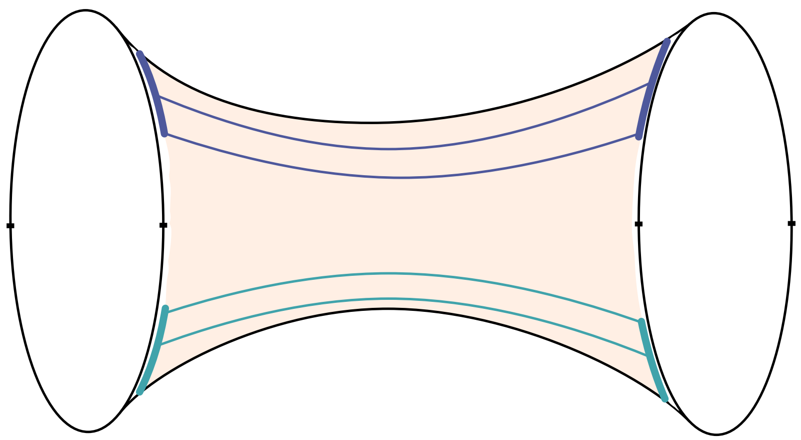
  - Plateau signal of EPR correlators?

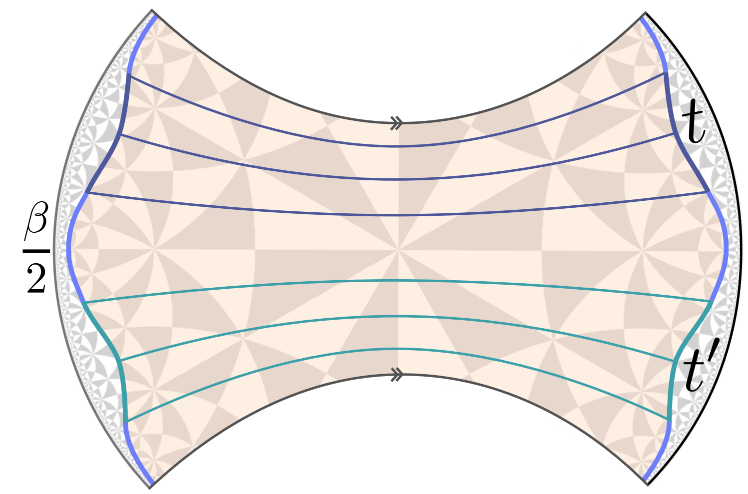
  - Formation of k-designs of black holes in other ways, e.g., performing measurements?
    [Maldacena, Kourkoulou '17] [de Boer et al. '19]

  - Matter that forms the caterpillar falls into the singularity. Is there any relation between the genericity of the microstates and the genericity of the singularity?

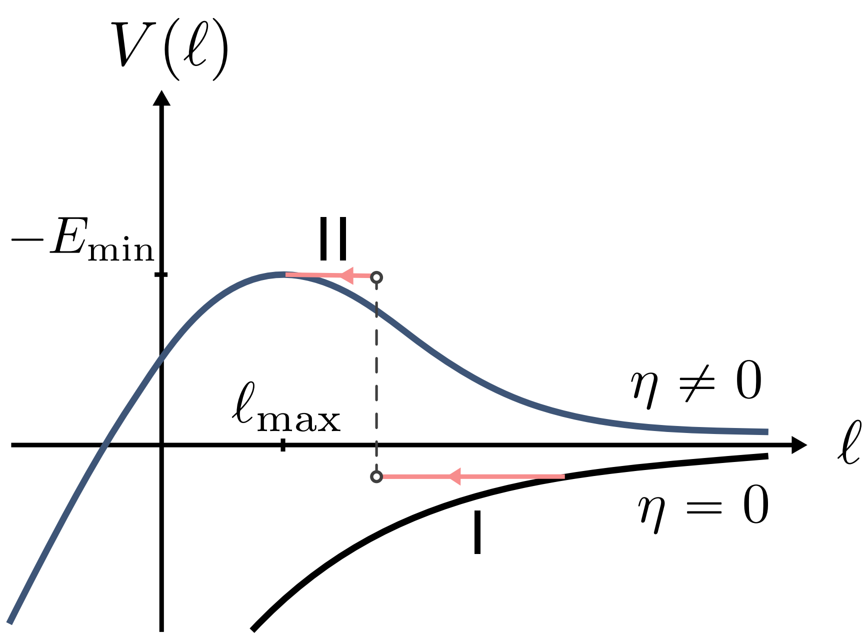
Thanks!

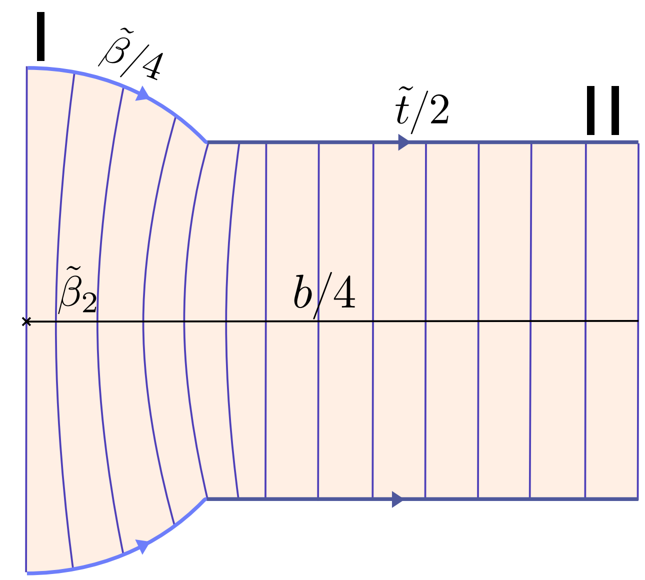
- In JT gravity the relevant two-boundary wormhole for the linear growth of randomness is a stabilized double trumpet.





- It can be constructed as a classical solution of the Euclidean geodesic length degree of freedom. It is stabilized by the interaction term coming from the bulk fields.

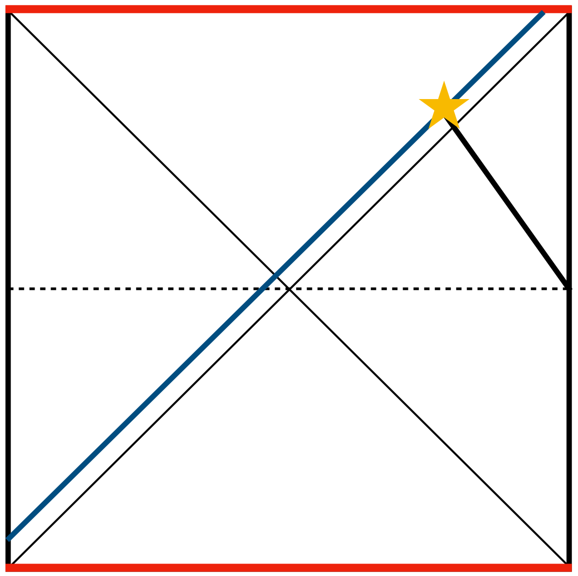




Firewalls in typical states?

- Firewalls can simply correspond to matter shocks in a highly boosted frame.

[Shenker, Stanford '13][Susskind '15]
[Stanford, Yang '22] ...



- It seems from the average the geometry that only past-evolved caterpillars have firewalls. Are states with firewalls indistinguishable from states without firewalls?