Quasi-topological mass generation for 3D linearized gravity

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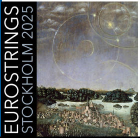

NORDITA, Stockholm, 28th August 2025

A massive history

Motivations

- Universe in accelerated expansion
- LIGO-Virgo $m \leq 10^{-23} \frac{eV}{c^2}$

Solutions? modified gravity?

- Λ as dark energy $\Rightarrow \frac{\Lambda}{M_P^2} \sim 10^{-65}$ ⚠ **but** QFT : $\frac{\Lambda}{M_P^2} \sim 10^{54}$
- $F(R)$ theories? ...
- massive theory $\rightarrow V_{\text{YUK}} \sim \frac{1}{r} e^{-mr}$

Why 3D gravity?

Peculiarities :

- Renormalizable
- Topological
- $\Rightarrow$ no local DoF
- $\Rightarrow$ no propagating massless gravitons
- $\Rightarrow$ no gravitational waves

Good toy model for testing ground of new physics (BH solutions, AdS/CFT, mass terms, ...)

E.Witten, Nucl.Phys.B 311 (1988) 46, *(2+1)-Dimensional Gravity as an Exactly Soluble System*
K.Hinterbichler, Rev.Mod.Phys. 84 (2012) 671-710, *Theoretical Aspects of Massive Gravity*

3D massless linearized gravity: no DoF

The 3D LG action

$$S_{\mathrm{LG}} = \int d^3x \left(\partial_\mu h \partial^\mu h - \partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} - 2\partial_\mu h \partial_\nu h^{\mu\nu} + 2\partial_\rho h_{\mu\nu}\partial^\mu h^{\nu\rho}\right)$$

invariant under

$$\delta_{diff} h_{\mu\nu} = \partial_\mu \lambda_\nu + \partial_\nu \lambda_\mu \ .$$

Customary covariant gauge: harmonic

$$\partial^\nu h_{\mu\nu} - \tfrac{1}{2}\partial_\mu h = 0 \ ,$$

Wave equation from EoM in harmonic gauge

$$\frac{\delta S_{\mathrm{LG}}}{\delta h^{\mu\nu}} = \Box h_{\mu\nu} = 0 \quad \Rightarrow \quad h_{\mu\nu} = C_{\mu\nu} e^{ik_\lambda x^\lambda} \ .$$

On TT-gauge $h^{\mathrm{TT}}_{0\nu} = h^{\mathrm{TT}} = \partial^\mu h^{\mathrm{TT}}_{\mu\nu} = 0$

$$C_{\mu\nu} = 0 \quad \rightarrow \quad \textbf{No propagating DoF}$$

Introducing a mass in 3D GR

Two main massive models :

1) Topologically massive gravity (1982)
S.Deser, R.Jackiw, and S.Templeton, Phys.Rev.Lett. 48, 975-978 (1982), *Three-Dimensional Massive Gauge Theories*
S.Deser, R.Jackiw, and S.Templeton, Ann.Phys. 140, 372-411 (1982), *Topologically Massive Gauge Theories*

2) New massive gravity (2009)
E.A.Bergshoeff, O.Hohm, P.K.Townsend, Phys.Rev.Lett. 102, 201301 (2009), *Massive Gravity in Three Dimensions*

Topologically massive gravity (TMG)

The action

$$S_{\mathrm{TMG}} = \frac{M_P}{2}\int d^3x\left[-\sqrt{-g}\,R - \frac{1}{2\mu_{\mathrm{T}}}\epsilon^{\lambda\mu\nu}\Gamma^{\alpha}_{\lambda\beta}\left(\partial_\mu\Gamma^{\beta}_{\alpha\nu} + \frac{2}{3}\Gamma^{\beta}_{\mu\gamma}\Gamma^{\gamma}_{\nu\alpha}\right)\right],$$

breaks parity, non-topological, but MCS-like topological mass

$$\left(\eta^{\mu\alpha} + \frac{1}{\mu_{\mathrm{T}}}\epsilon^{\mu\lambda\alpha}\partial_\lambda\right)\Box h^{\mathrm{TT}}_{\alpha\beta} = 0 \quad\Rightarrow\quad \left(\Box - \mu_{\mathrm{T}}^2\right)\Box h^{\mathrm{TT}}_{\mu\nu} = 0\,.$$

Linearized action

$$S^{lin}_{\mathrm{TMG}} \sim \int d^3x\,\varphi\left(\Box - \mu_{\mathrm{T}}^2\right)\varphi \qquad ; \qquad \varphi \sim h^{\mathrm{T}}_{mn}$$

$\Rightarrow$ one propagating spin-2 DoF with mass μ_{T}.

S.Deser, R.Jackiw, and S.Templeton, Phys.Rev.Lett. 48, 975-978 (1982), *Three-Dimensional Massive Gauge Theories*
S.Deser, R.Jackiw, and S.Templeton, Ann.Phys. 140, 372-411 (1982), *Topologically Massive Gauge Theories*

New massive gravity (NMG)

The action

$$S_{\mathrm{NMG}} = \frac{M_P}{2}\int d^3x\,\sqrt{-g}\,\left[-R + \frac{1}{\mu_{\mathrm{N}}^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2\right)\right],$$

parity-preserving. Mass mechanism FP-like

$$\frac{1}{\mu_{\mathrm{N}}^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2\right) \quad\longleftrightarrow\quad m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right).$$

Linearized EoM

$$\left(\Box - \mu_{\mathrm{N}}^2\right)G^{lin}_{\mu\nu} = 0 \quad ; \quad R^{lin} = 0\,,$$

Two propagating massive DoF $\sim h^{\mathrm{TT}}_{\mu\nu}$, with helicity ± 2 and mass μ_{N}.

E.A.Bergshoeff, O.Hohm, P.K.Townsend, Phys.Rev.Lett. 102, 201301 (2009), *Massive Gravity in Three Dimensions*

Comparisons and issues

TMG v.s. NMG :

	TMG	NMG
Dimensions	3	any
Parity	violated	preserved
Massive DoF	one, spin-2	two, spin-2

TMG & NMG - open questions/aspects :

Higher mass dimensions $\Rightarrow$ $1/\mu_{\mathrm{T}}$ and $1/\mu_{\mathrm{N}}^2$

- "massless" limit : $\lim_{\mu\to\infty}$ $\sim$"$-$EH".
- Physical situation : tiny mass for the graviton ($\mu \to 0$).
- Infinite invariant terms $\to$ effective theory.

Quasi-topological mass generation

Based on :
E.B., E.Lui and N.Maggiore, Phys.Rev.D 112, 044035 (2025), arXiv:2507.20845
Quasi-topological mass generation for 3D linearized gravity

Requirements

1. **Massless limit**
2. **Propagators**
3. **No unphysical ghosts**
4. **Power-counting**
5. **Degrees of Freedom (DoF)**

The quasi-topological massive term

The action

$$S = S_{\mathrm{LG}} + mS_m \ ,$$

with

$$S_m = \int d^3x \, \epsilon^{\mu\nu\rho} h_\mu^{\ \lambda} \partial_\nu h_{\rho\lambda} \ ,$$

invariant under longitudinal diffs

$$\delta_{long} h_{\mu\nu} = \partial_\mu \partial_\nu \Lambda \ .$$

Is it a good massive theory?

1 ✓ massless limit : $S \xrightarrow{m\to 0} S_{\mathrm{LG}}$

4 ✓ power-counting : $[m] = 1$

First attempt: scalar gauge-fixing

Gauge condition: scalar

$$\kappa_0 \partial^\mu \partial^\nu h_{\mu\nu} + \kappa_1 \Box h = 0 \,,$$

with action term

$$S_{gf}^{(long)} = \int d^3x \, b \left(\kappa_0 \partial^\mu \partial^\nu h_{\mu\nu} + \kappa_1 \Box h \right) \,.$$

Full action

$$S_{tot}^{(long)} = S + S_{gf}^{(long)}$$

Propagators? $\rightarrow$ **2** ? **3** ?

First attempt: no massless limit

Propagator expansion

$$\langle \tilde{h}_{\alpha\beta}(p)\tilde{h}_{\rho\sigma}(-p)\rangle = \sum_{i=0}^{6} c_i(p)A^{(i)}_{\alpha\beta,\rho\sigma}(p) \ ,$$

but

$$c_i \sim \frac{1}{m} + \ldots \quad \text{for some } i$$

No well defined propagator in the massless limit :

Second attempt: vector gauge-fixing

Gauge condition: harmonic

$$\partial^\nu h_{\mu\nu} - \tfrac{1}{2}\partial_\mu h = 0 \quad \rightarrow \quad S_{gf}^{(diff)} = \int d^3x\, b^\mu\left(\partial^\nu h_{\mu\nu} - \tfrac{1}{2}\partial_\mu h\right).$$

Massive term as controlled symmetry breaking :

$$\delta_{diff} S_m|_{gf} = -2\int d^3x \epsilon^{\mu\nu\rho}\lambda_\mu\partial^\alpha\partial_\nu h_{\rho\alpha}|_{gf} = -\int d^3x \epsilon^{\mu\nu\rho}\lambda_\mu\partial_\nu\partial_\rho h = 0$$

Full action

$$S_{tot}^{(diff)} = S + S_{gf}^{(diff)}\,.$$

$$\boxed{\text{Propagators?} \quad \rightarrow \quad \text{2 ?} \quad \text{3 ?}}$$

Well defined propagators!

Propagator expansion

$$\Delta^{(m)}_{\alpha\beta,\rho\sigma} \equiv \langle \tilde{h}_{\alpha\beta}(p)\tilde{h}_{\rho\sigma}(-p)\rangle = \sum_{i=0}^{6} c_i(p)A^{(i)}_{\alpha\beta,\rho\sigma}(p) \ ,$$

with

$$c_i \sim \frac{1}{p^2 + m^2}$$

2 ✓ Well defined propagator : $\Delta^{(m)} \xrightarrow{m\to 0} \Delta^{(\mathrm{LG})}$

massive pole at $p^2 = -m^2$!

3 ✓ No unphysical ghosts/tachyonic poles

Massive spin-2

Given the Barnes-River basis

$$P^{(2)} \equiv \frac{1}{2}\left(\theta_{\mu\alpha}\theta_{\nu\beta} + \theta_{\nu\alpha}\theta_{\mu\beta} - \theta_{\mu\nu}\theta_{\alpha\beta}\right)$$

$$P^{(0,s)} \equiv \frac{1}{2}\theta_{\mu\nu}\theta_{\alpha\beta}\ ;\quad P^{(0,w)} \equiv \omega_{\mu\nu}\omega_{\alpha\beta}\ ;\quad P^{(0,sw)} \equiv \frac{1}{\sqrt{2}}\left(\theta_{\mu\nu}\omega_{\alpha\beta} + \theta_{\alpha\beta}\omega_{\mu\nu}\right)$$

$$\mathcal{E}^{(\theta)} \equiv (\epsilon_{\alpha\lambda\rho}\theta_{\sigma\beta} + \epsilon_{\beta\lambda\rho}\theta_{\sigma\alpha} + \epsilon_{\alpha\lambda\sigma}\theta_{\rho\beta} + \epsilon_{\beta\lambda\sigma}\theta_{\rho\alpha})\partial^{\lambda}\ ,$$

with $P^{(\dots)}$ spin projectors for a symmetric rank-2 tensor field, and

$$\theta_{\mu\nu} \equiv \eta_{\mu\nu} - \omega_{\mu\nu} \quad ; \quad \omega_{\mu\nu} \equiv \frac{1}{\Box}\partial_\mu\partial_\nu\ .$$

The tensor field propagator is

$$\Delta^{(m)} \propto \frac{1}{\Box - m^2}\left(P^{(2)} - \frac{m}{2\Box}\mathcal{E}^{(\theta)}\right) - \frac{1}{\Box}\left(\sqrt{2}P^{(0,sw)} + P^{(0,s)} + 2P^{(0,w)}\right)\ .$$

Massive pole associated to the **spin-2** projector, describing a unitary mode of positive mass m^2 and positive norm.

Propagating massive DoF: $h_{\mu\nu}$ decomposition

Irreducible reps. of the rotation group

$$h_{00} = \phi \qquad ; \qquad h_{0i} = u_i + \partial_i \omega$$

$$h_{ij} = \underbrace{\boxed{(\eta_{ij} - \hat{\partial}_i \hat{\partial}_j)\varphi}}_{\text{Transverse}} + \underbrace{\boxed{\hat{\partial}_i \hat{\partial}_j \chi}}_{\text{Longit.}} + \underbrace{\boxed{\partial_i \xi_j + \partial_j \xi_i}}_{\text{Solenoidal}} \ ,$$

where

$$\hat{\partial}_i \equiv \frac{\partial_i}{\sqrt{\nabla^2}} \quad ; \quad u_i = \epsilon_{0ij} \partial^j u \quad ; \quad \xi_i = \epsilon_{0ij} \partial^j \xi \ ,$$

so that

$$\partial^i h^{\mathrm{T}}_{ij} = 0 \quad ; \quad \epsilon^{0ij} \partial_i \partial^k h^{\mathrm{L}}_{jk} = 0 \quad ; \quad \partial^i \partial^j h^{\mathrm{S}}_{ij} = 0 \ ,$$

and

$$\partial^i u_i = 0 \quad ; \quad \partial^i \xi_i = 0 \ .$$

S.Deser, R.Jackiw, and S.Templeton, Ann.Phys. 140, 372-411 (1982), *Topologically Massive Gauge Theories*

Propagating massive DoF

The action becomes

$$S \sim \int d^3x \left[\phi\nabla^2(\varphi - mu) + \omega\nabla^2\left(2\partial_0\varphi - m\nabla^2\xi - m\partial_0 u\right) + \right.$$
$$\left. +\chi\left(\partial_0^2\varphi - m\partial_0\nabla^2\xi\right) + \left(\nabla^2 u - \nabla^2\partial_0\xi\right)^2 - m\varphi\nabla^2(u - \partial_0\xi)\right] .$$

Constraints from ϕ, ω and χ give

$$\varphi = mu \quad ; \quad \nabla^2\xi = \partial_0 u \,,$$

thus

$$S \sim \int d^3x \left[u\left(\Box - m^2\right)\Box u\right] \quad \Rightarrow \quad \left(\Box - m^2\right)\Box u = 0 \,.$$

5 ✓ **One massive propagating DoF : $\Box u$**

Physical interpretation

Decomposition of the Ricci tensor

$$R_{ij} \equiv R^{\mathrm{T}}_{ij} + R^{\mathrm{L}}_{ij} + R^{\mathrm{S}}_{ij} = (\eta_{ij} - \hat{\partial}_i\hat{\partial}_j)\mathcal{R}^{\mathrm{T}} + \hat{\partial}_i\hat{\partial}_j\mathcal{R}^{\mathrm{L}} + \partial_i\mathcal{R}^{\mathrm{S}}_j + \partial_j\mathcal{R}^{\mathrm{S}}_i \ .$$

Linearized Ricci tensor in the harmonic gauge

$$R_{\mu\nu} \propto \Box h_{\mu\nu} \ .$$

Projecting, on-shell

$$\left(\eta^{ij} - \hat{\partial}^i\hat{\partial}^j\right)R_{ij} \quad \Rightarrow \quad \mathcal{R}^{\mathrm{T}} \sim \Box u \ .$$

Thus

$$\left(\Box - m^2\right)R^{\mathrm{T}}_{ij} = 0 \ .$$

S.Deser, R.Jackiw, and S.Templeton, Phys.Rev.Lett. 48, 975-978 (1982), *Three-Dimensional Massive Gauge Theories*
S.Deser, R.Jackiw, and S.Templeton, Ann.Phys. 140, 372-411 (1982), *Topologically Massive Gauge Theories*

BRS symmetry

BRS field transformation

$$
\begin{aligned}
s h_{\mu\nu} &= \partial_\mu c_\nu + \partial_\nu c_\mu & s c_\mu &= 0 \\
s \bar{c}_\mu &= b_\mu & s b_\mu &= 0\,,
\end{aligned}
$$

and

$$s^2 = 0 \quad ; \quad s S_{\text{LG}} = 0 \quad ; \quad s(S_{\text{LG}} + S_{gf}) = 0\,,$$

with

$$
\begin{aligned}
S_{gf} &= s \int d^3x\, \bar{c}^\mu \left(\partial^\nu h_{\mu\nu} - \tfrac{1}{2}\partial_\mu h\right) \\
&= \int d^3x\, \left[b^\mu \left(\partial^\nu h_{\mu\nu} - \tfrac{1}{2}\partial_\mu h\right) - \bar{c}^\mu \Box c_\mu\right]\,,
\end{aligned}
$$

broken by S_m.

Emerging BRS-like massive symmetry

New, modified, BRS operator

$$
\begin{aligned}
s_m h_{\mu\nu} &= \partial_\mu c_\nu + \partial_\nu c_\mu \qquad & s_m c_\mu &= 0 \\
s_m \bar{c}_\mu &= b_\mu \qquad & s_m b_\mu &= 2m\epsilon_{\mu\nu\rho}\partial^\nu c^\rho \ ,
\end{aligned}
$$

and now

$$
s_m(S_{\rm LG} + S_{gf} + mS_m) = 0 \ .
$$

New algebraic structure $\{s_m, \delta\}$

$$
s_m^2 = \delta \quad ; \quad [s_m \, , \, \delta] = \delta^2 = 0 \ ,
$$

with

$$
\delta\bar{c}_\mu = 2m \, \epsilon_{\mu\nu\rho}\partial^\nu c^\rho \quad ; \quad \delta\left\{h_{\mu\nu} \, , \, c_\mu \, , \, b_\mu\right\} = 0 \ .
$$

$\boxed{\textbf{$s_m, \delta$ uniquely identify the theory !}}$

Final Remarks

Review of results

The 3D massive LG presented here has

1. ✓ **Massless limit**
2. ✓ **Propagators**
3. ✓ **No unphysical ghosts**
4. ✓ **Power-counting**
5. ✓ **Degrees of Freedom (DoF)**
6. ! **Uniquely identified by the BRS-like symmetry s_m** !

Future prospects

- Nonlinear theory?
- 4D extension?
- Extended symmetry

Thanks for your attention!

4D massive gravity: Fierz-Pauli

Linearised massive action

$$S = S_{\text{LG}} + S_{\text{FP}}\ ,$$

with

$$S_{\text{LG}} = \int d^4x\ \left(\partial_\mu h \partial^\mu h - \partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} - 2\partial_\mu h \partial_\nu h^{\mu\nu} + 2\partial_\rho h_{\mu\nu}\partial^\mu h^{\nu\rho}\right)$$

$$S_{\text{FP}} = \int d^4x \left(m_1^2 h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2\right)\ ,$$

and FP tuning

$$m_1^2 + m_2^2 = 0\ .$$

Issues: breaks diffs invariance, vDVZ discontinuity, massless limit.
H.vanDam, M.J.G.Veltman, Nucl.Phys.B 22, 397-411 (1970), *massive and massless Yang-Mills and gravitational fields*
V.I.Zakharov, JETP Lett. 12, 312 (1970) *Linearized gravitation theory and the graviton mass*

Can be recovered with proper gauge-fixing
G.Gambuti,N.Maggiore, Phys.Lett.B 807, 135530 (2020), *A note on harmonic gauge(s) in massive gravity*,
and Eur.Phys.J.C 81, no.2, 171 (2021), *Fierz-Pauli theory reloaded: from a theory of a symmetric tensor field to linearized massive gravity*