Towards holography for the IKKT matrix model

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Based on work with H. Samtleben [2503.08771] [25xx.xxxxx]

Eurostrings 2025, Nordita
28th of August 2025

Funded by
DFG Deutsche Forschungsgemeinschaft
German Research Foundation

ENS DE LYON

(Euclidean) IKKT matrix model

$$S_{\mathrm{IKKT}} = -\mathrm{Tr}\left(\frac{1}{4}[X_a\,,X_b]\,[X^a\,,X^b] + \frac{1}{2}\bar{\Psi}\,\Gamma^a\,[\Psi\,,X_a]\right)$$

[Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

- Theory of $N \times N$ matrices in **0 dimension**, with 16 supersymmetries and **SO(10) invariance**.

- Originally proposed as a **non-perturbative definition IIB superstring theory**.

  ⟶ How does spacetime and gravity emerge?

- « Worldpoint theory » for the **D(-1) brane**.

  ⟶ Appears on the field theory side of the **holographic dualities** (for the extremal case p= -1):

strings/supergravity on the near-horizon geometry of Dp-branes ⟷ (p+1)-dimensional super Yang-Mills theories with 16 supercharges

$\sim AdS_{p+2} \times S^{8-p}$

[Itzhaki, Maldacena, Sonnenschein, Yankielowicz '98]

[Boonstra, Skenderis, Townsend '98]

Despite intriguing and highly non-trivial results in the IKKT model, this **duality has so far remained largely unexplored…**

(Euclidean) IKKT matrix model

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Result:

Lowest KK fluctuations around S^9 described by a one-dimensional « supergravity » theory ⟷ Lowest BPS multiplet of gauge invariant operators in the IKKT matrix model

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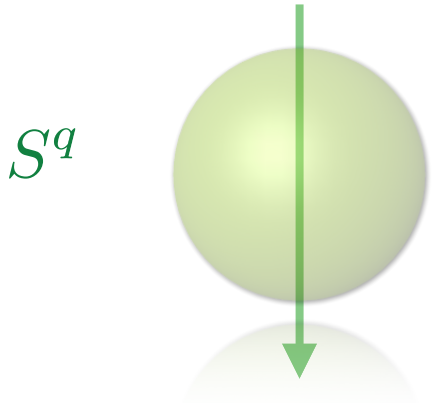
1. Lowest KK fluctuations around S^9 described by a one-dimensional « supergravity » theory ⟷ **2.** Lowest BPS multiplet of gauge invariant operators in the IKKT matrix model

Sphere truncations of gravity

D: Gravity theory



D-q: Gravity theory with $SO(q+1)$ Yang-Mills and scalar potential

« Pauli truncation » [1953]

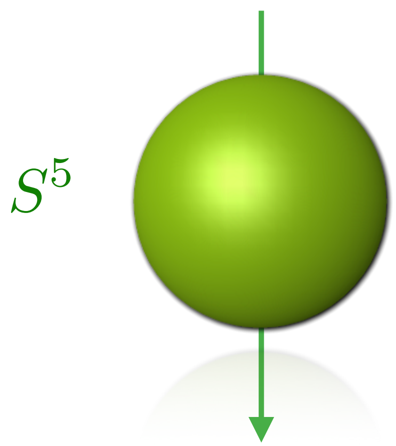
Does not work for **pure gravity**…

must start from specific matter-coupled gravity theories

$\longrightarrow$ (maximal) supergravities

Sphere truncations of supergravities

D=10 IIB supergravity



D=5 SO(6) gauged supergravity

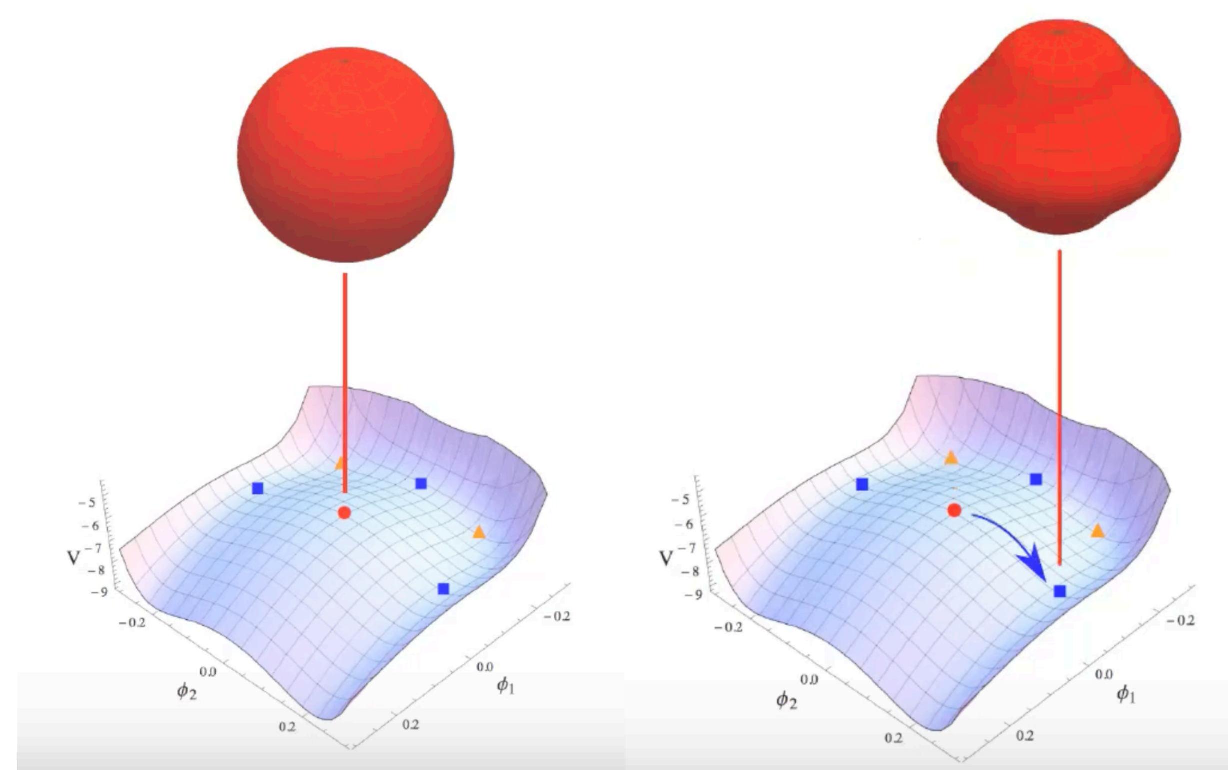
[Baguet, Hohm, Samtleben '15]

$AdS_5 \times S^5$ ⟷ D3-brane

Conformal brane

Consistent sphere truncations of gravity theories are hard to construct, but:

- Allow to uplift all solutions of the lower-dimensional theory.



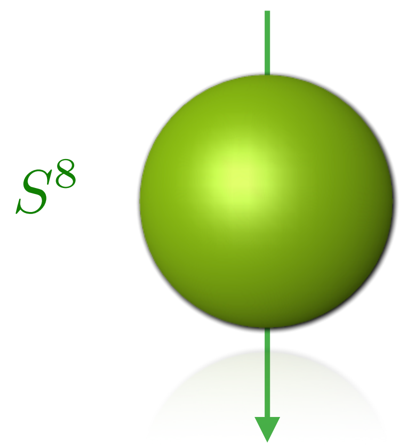
- Powerful tools for precision holography

⟶ using holographic renormalization technique

[Kanitscheider, Skenderis, Taylor '08]

Sphere truncations of supergravities

D=10 IIA supergravity



D=2 SO(9) gauged supergravity

[Bossard, FC, Kleinschmidt, Inverso ‘23]

$AdS_5 \times S^5 \longleftrightarrow$

$e^{\Phi}(AdS_2 \times S^8) \longleftrightarrow$ D0-brane

(Non)-conformal branes

Consistent sphere truncations of gravity theories are hard to construct, but:

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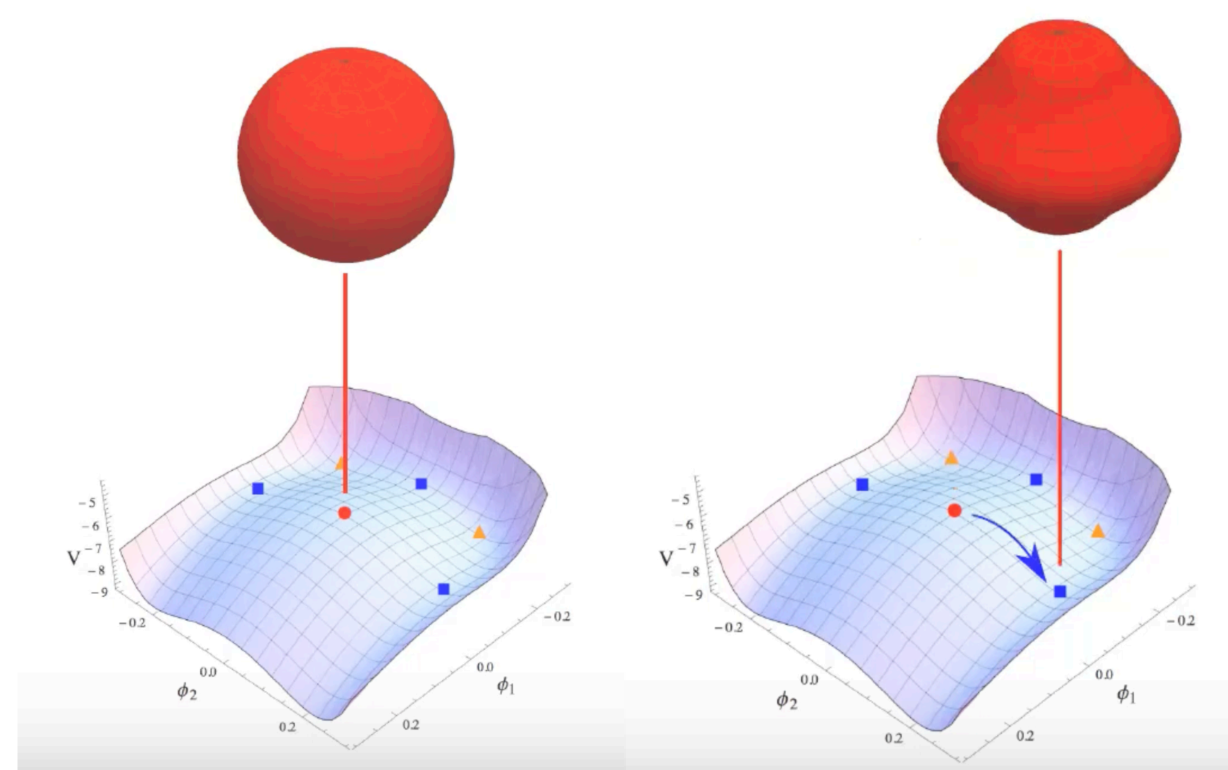

- Powerful tools for precision holography

  $\longrightarrow$ 2pt/3pt functions in BFSS

[Ortiz, Samtleben, Tsimpis ’14] [Bobev, Mera Alvarez, Paul ’25]

Kaluza-Klein truncations around all Dp brane solutions have been constructed except for the

$$ds^2_{10} = dt^2 + t^2\,\Omega_9^2$$

$$e^{\Phi} = g_s\Big(1 + \frac{Q}{t^8}\Big) = \chi^{-1}$$

[Gibbons, Green, Perry '96]

[Ooguri, Skenderis '98]

Holographic coordinate t

Sphere truncation to one dimension

[FC, Samtleben '25]

D=10: (Euclidean) gravity + dilaton + axion

$$\mathcal{L}_{10} = E\Big(R - \frac{1}{2}(\partial\Phi)^2 + e^{2\Phi}(\partial\chi)^2\Big)$$

S^9

$$ds^2_{10} = e^{9\phi}\,\Delta\,\mathrm{e}^2\,dt^2 + \mathrm{g}^{-2}\,e^{\phi}\,T^{-1}_{ij}\,D\mu^i\,D\mu^j$$

$$e^{\Phi} = e^{-4\phi}\,\Delta^{-1}$$

$$\chi = -\frac{1}{2\,\mathrm{eg}}\Big(T^{-1}_{ij}\,D_t T_{kj}\,\mu^i\mu^k + \dot\phi\Big)$$

D=1: + scalars

$$\mathcal{L}_1 = 10\,\mathrm{e}^{-1}\,\dot\phi^2 + \frac{1}{4}\mathrm{e}^{-1}\,D_t T^{-1}_{ij}\,D_t\,T_{ij} - \frac{1}{2}\,\mathrm{e}\,\mathrm{g}^2\,e^{8\phi}\,\big(2\,T_{ij}T_{ij} - (T_{ii})^2\big)$$

$$i\,,j = 1\,,\ldots 10$$

No curvature tensors in one dimension

→ Variation with respect to Einbein and gauge fields lead to first order constraints.

Sphere truncation to one dimension

[FC, Samtleben '25]

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$$i\,,j = 1\,,\ldots 10$$

$$\det(T_{ij}) = 1$$

SO(10) invariant solution: $T_{ij} = \delta_{ij}$ $\quad \mathrm{e}^{-1} = 2\mathrm{g}e^{5\phi}$ $\quad e^{\phi} = t + c$ $\xrightarrow[\text{D=10}]{\text{Uplifts to}}$ (Near-horizon of) D(-1) instanton

IKKT is supersymmetric, so let's consider fermions

Maximal SUSY 1D implies…

[FC, Samtleben '25]

Bosons

e	Einbein		
ϕ	dilaton		
$A_t^{[ij]}$	$SO(10)_g$ gauge fields		
$T_{(ij)}$	$\dfrac{SL(10)}{SO(10)}$	scalars	$\mathbf{54}$
$a_{[ijk]}$	scalars		$\mathbf{120}$

SO(10) Fermions *(32 comp. spinors)*

ψ	gravitini	
λ	« spin-1/2 »	
χ_a	« spin-1/2 », Γ^a-traceless	$\mathbf{144} \oplus \overline{\mathbf{144}}$

$$\{\Gamma^a , \Gamma^b\} = 2\,\delta^{ab}\,\mathbf{1}_{32}$$

$$a\,, b = 1\,, \dots 10$$

Holomorphic fermions

[Nicolai '78]

$$\mathcal{L}_1 = 10\,\mathrm{e}^{-1}\,\dot{\phi}^2 + \frac{1}{4}\mathrm{e}^{-1}\,D_t\,T^{-1}_{ij}\,D_t\,T_{ij} - \frac{1}{12}\mathrm{e}^{-1}\,e^{-2\phi}\,T^{-1}_{ij}T^{-1}_{kl}T^{-1}_{mn}\,D_t a_{ikm}\,D_t a_{jln} - \frac{\mathrm{e}}{2}\mathbf{V}[T\,,\phi\,,a]$$
$$+\mathcal{L}_{\text{top.}}[a] + 20\,\bar{\lambda}\,\mathcal{D}_t\lambda + 2\,\bar{\chi}^a\,\mathcal{D}_t\chi_a + \mathcal{L}_{\text{Noether}} + \mathcal{L}_{\text{Yukawa}}$$

Full scalar potential

$$\mathbf{V}[T\,,\phi\,,a] = \mathrm{g}^2\,e^{8\phi}\left(2\,T_{ij}T_{ij} - (T_{ii})^2 + \frac{1}{2}e^{30\phi}\left(a_{ijk}\,a_{lmn}\,T_{il}T^{-1}_{jm}T^{-1}_{kn} - 2\,a_{ijk}\,a_{ijl}\,T^{-1}_{kl}\right)\right) + \ldots + \mathcal{O}(a^{10})$$

Killing spinors at $a_{ijk} = 0$

SUSY variations

$$\delta_\epsilon \psi = \mathcal{D}_t\, \epsilon - \mathrm{g}\, \frac{\mathrm{e}}{4} e^{4\phi}\, T_{ii}\, \Gamma_*\, \epsilon$$

$$\delta_\epsilon \lambda = \frac{1}{2} \mathrm{e}^{-1} \dot{\phi}\, \Gamma_*\, \epsilon + \frac{1}{10}\, \mathrm{g}\, e^{4\phi}\, T_{ii}\, \epsilon$$

$$\delta_\epsilon \chi_a = (\mathcal{D}T \ldots)_{ab}\, \Gamma^b \epsilon + (T \ldots)_{ab}\, \Gamma^b \Gamma_* \epsilon$$

Chirality matrix $\Gamma_* = \begin{pmatrix} \mathbf{1}_{16} & 0 \\ 0 & -\mathbf{1}_{16} \end{pmatrix}$

Killing spinor equations:

$$\boxed{\delta_\epsilon(\text{fermions}) \overset{!}{=} 0}$$

Only two possibilities: non-supersymmetric configurations or

1/2-supersymmetric configurations:

$$\delta_\epsilon \lambda = \ldots \left(\mathbf{1}_{32} \pm \Gamma_*\right) \epsilon \overset{!}{=} 0$$

$$\delta_\epsilon \chi_a = (\ldots)_{ab}\, \Gamma^b \left(\mathbf{1}_{32} \pm \Gamma_*\right) \epsilon \overset{!}{=} 0$$

$$\longrightarrow \quad \epsilon_- = \begin{pmatrix} \varepsilon(t) \\ 0 \end{pmatrix}$$

Requires the bosonic fields to satisfy 1st order equations that

- imply the field equations
- can be solved exactly

General 1/2-SUSY solutions:

$$e^{-\phi}\, T_{ij} = \begin{pmatrix} e^{\phi_1} & & \\ & \ddots & \\ & & e^{\phi_{10}} \end{pmatrix} \quad \text{with} \quad e^{-\phi_i} = t + c_i$$

break SO(10)

Dual to states in the `Coulomb branch'

Smeared D instantons

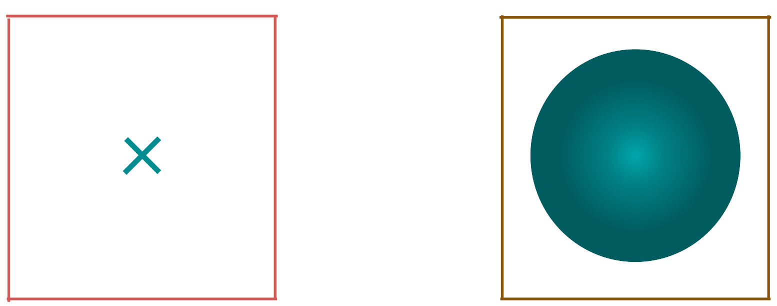
[analogous to smeared D3 branes
Freedman, Gubser, Pilch, Warner '00]

Solutions preserving $SO(p) \times SO(10-p)$

$$e^{-\phi} T_{ij} = \begin{pmatrix} \frac{1}{t+c_1} & & & & & \\ & \ddots & & & & \\ & & \frac{1}{t+c_1} & & & \\ & & & \frac{1}{t+c_2} & & \\ & & & & \ddots & \\ & & & & & \frac{1}{t+c_2} \end{pmatrix} \xrightarrow[\text{D=10}]{\text{Uplifts to}} \begin{aligned} ds^2_{10} &= |\vec{dx}|^2 \\ e^{\Phi(\vec{x})} &\sim \int_{\mathbb{R}^{10}} \vec{dy}\, \frac{\sigma_p(\vec{x}\,,\vec{y}\,,c_1\,,c_2)}{|\vec{x}-\vec{y}|^8} \end{aligned}$$

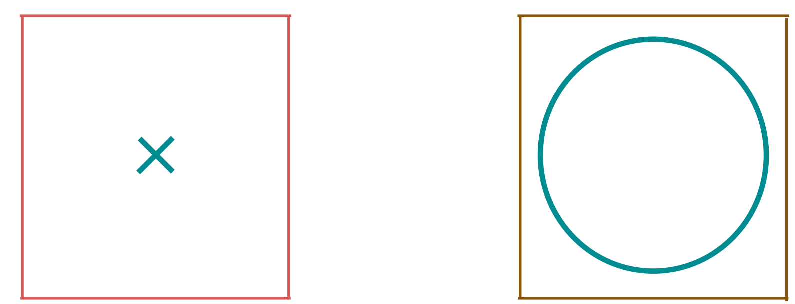
brane
distribution

<u>p>2</u>



$\mathbb{R}^p$ $\mathbb{R}^{10-p}$

<u>p=2</u>



$\mathbb{R}^2$ $\mathbb{R}^8$

- By also breaking SO(10-p): ball/sphere distributions deformed to ellipsoids.
- Further uplift to **pp-waves solutions** of pure Lorentzian **gravity in D=12** [Tseytlin '97]

$$\longrightarrow \quad ds^2_{12} = dx_+ dx_- + e^{\Phi}\, dx_- dx_- + ds^2_{10}$$

Back to the matrix model

$$S_{\rm IKKT} = -{\rm Tr}\Big(\frac{1}{4}[X_a\,,X_b]\,[X^a\,,X^b] + \frac{1}{2}\bar\Psi\,\Gamma^a\,[\Psi\,,X_a]\Big)$$

16 supercharges $\Gamma_*\epsilon = \epsilon$

algebra: $[\delta_{\epsilon_1}\,,\delta_{\epsilon_2}] = \delta_\theta^{SU(N)}$

X_a and Ψ are $SU(N)$ matrices, transforming in the $\mathbf{10}$ and $\mathbf{16_s}$ of $SO(10)$.

- Single trace operators: $\mathcal{O} = {\rm Tr}\Big[X\,X\ldots\Psi\ldots X\ldots\Psi\ldots\Big]$ $\longrightarrow$ arrange in supermultiplets

Tower of BPS multiplets: $\sum_{n=2}^{\infty}\mathcal{B}_n$ which combine SO(10) rep.

$$\mathcal{B}_n = [n,0000]_n \oplus [n-1,0001]_{n+\frac12} \oplus [n-2,0100]_{n+1}\oplus [n-3,1010]_{n+\frac32} \oplus [n-3,0020]_{n+2} \oplus [n-4,2000]_{n+2} \oplus [n-4,1010]_{n+\frac52} \oplus [n-4,0100]_{n+3} \oplus [n-4,0001]_{n+\frac72} \oplus [n-4,0000]_{n+4}$$

$$\mathcal{O}_{a_1\ldots a_n} = {\rm Tr}\big[X_{((a_1}X_{a_2}\ldots X_{a_n))}\big]$$

[Morales, Samtleben '05]

- 1D supergravity fields $\longleftrightarrow$ lowest BPS multiplet of operators

$\#_{\rm bos.} = \#_{\rm ferm.} + 1$

$$\mathcal{B}_2 = \boxed{\mathbf{54}_{+2} \oplus \mathbf{144}_{+\frac52} \oplus \mathbf{120}_{+3}} \ominus \mathbf{45}_{+4} \ominus \mathbf{16}_{+\frac92}$$

Lowest BPS multiplet

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$\mathcal{B}_2$ multiplet of operators

1D supergravity fluctuations

54	$\mathcal{O}_{ab} = \mathrm{Tr}[X_a X_b] - \frac{1}{10}\,\delta_{ab}\,\mathrm{Tr}[X^c X_c]$	T_{ab}
144	$\mathcal{O}^a = \mathrm{Tr}[X^a\,\Psi] - \frac{1}{9}\,\mathrm{Tr}[X_b\,\Gamma^{ab}\Psi]$	χ^a
120	$\mathcal{O}_{abc} = \mathrm{Tr}\,[X_a[X_b, X_c]] - \frac{1}{8}\,\mathrm{Tr}\,\left[\bar{\Psi}\Gamma_{abc}\Psi\right]$	a_{abc}

Dictionary purely based on kinematics

Lowest BPS multiplet

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16 supercharges $\quad \Gamma_*\epsilon = \epsilon$

algebra: $[\delta_{\epsilon_1}\,, \delta_{\epsilon_2}] = \delta_\theta^{SU(N)}$

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$\mathcal{B}_2$ multiplet of operators

SUSY variations

	Operator	SUSY variation
54	$\mathcal{O}_{ab} = \mathrm{Tr}[X_a X_b] - \frac{1}{10}\,\delta_{ab}\,\mathrm{Tr}[X^c X_c]$	$\delta_\epsilon \mathcal{O}_{ab} = \frac{9}{5}\bar{\epsilon}\,\Gamma^{[a}\,\mathcal{O}^{b]}$
144	$\mathcal{O}^a = \mathrm{Tr}[X^a\,\Psi] - \frac{1}{9}\,\mathrm{Tr}[X_b\,\Gamma^{ab}\Psi]$	$\delta_\epsilon \mathcal{O}_a = \frac{1}{18}\left(7\,\Gamma^{bc}\,\epsilon\,\mathcal{O}_{abc} - \Gamma_{abcd}\,\epsilon\,\mathcal{O}^{bcd}\right)$
120	$\mathcal{O}_{abc} = \mathrm{Tr}\,[X_a[X_b, X_c]] - \frac{1}{8}\,\mathrm{Tr}\left[\bar{\Psi}\Gamma_{abc}\Psi\right]$	$\delta_\epsilon \mathcal{O}_{abc} = 0$

Nilpotent structure of SUSY variations $\longrightarrow$ Deformations by VEV of $\mathcal{O}_{ab}$ preserve 16 supercharges

Future directions

- Computation of **holographic correlators** in SO(10) supergravity using holographic renormalization.

  → Comparaison with IKKT will be subtle. Normalization?

- 1D supergravity with $SO(p,q)$ gaugings? KK **truncation on hyperboloid** $H_{p,q}$? [Hull, Warner '88]

- KK truncation of $\mathrm{IIB}^{\star}$ supergravity to 1D? [Blair, Obers, Yan '25]

  → Tool to study holography for the Lorentzian IKKT model? [Anagnostopoulos, Asano, Kawai, Nishimura, Tsuchiya…]

- Detailed study of the vacua landscapes of 1D gauged supergravities. Non-vanishing a_{ijk} :

  → solutions dual to mass deformations ?

  Polarized IKKT [Bonelli '02] [Hartnoll, Liu '24] [Komatsu, and al '24]

  Spherical branes [Bobev, Bomans, Gautason, Minahan, Nedelin '18 '19 '24]

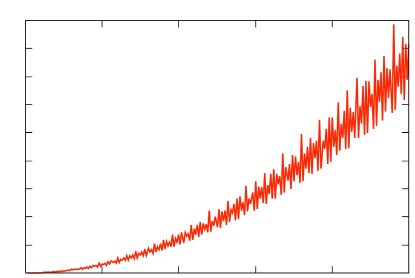
- 1D « supergravity » seems like an appropriate framework for supersymmetric localization…

Reproduce parts of:

(speculative)

$$Z_{\mathrm{IKKT}}(N) = \frac{(2\pi)^{(10N+11)(N-1)/2}}{N^{5/2}\Pi_{k=1}^{N-1}k!}\,\sigma_2(N)$$

[Krauth, Nicolai, Staudacher '98] [Moore, Nekrasov, Shatashvili '98]



Summary

- We presented sphere truncations of axion-dilaton (Euclidean) gravity to one dimension.

  ⟶ For IIB supergravity, it captures the lowest KK fluctuations (in the 'gravity sector') around the D(-1) instanton background.

- We constructed the maximally supersymmetric completion (32 supercharges) of the resulting one-dimensional model.

  ⟶ Derived a general class of 1/2-supersymmetric solutions: uplifts describe D(-1) instantons smeared in a flat ten-dimensional space.

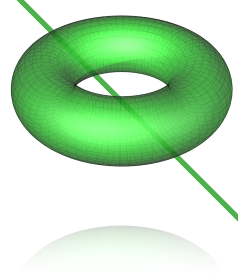
- We proposed the holographic dictionary between the lowest BPS multiplet of single trace operators in the IKKT model and the one-dimensional supergravity multiplet.

Thank you for your attention

Tack för din uppmärksamhet

Pure gravity on the torus

D: Pure gravity



T^q

Einstein-Hilbert action

$$G_{MN}(x,\cancel{y}) = \begin{pmatrix} g_{\mu\nu} & \rho^{2/q} A_\mu{}^p\, T_{pn} \\ \rho^{2/q} A_\nu{}^p\, T_{mp} & \rho^{2/q} T_{mn} \end{pmatrix}$$

D-q: Theory with rigid symmetry $\mathcal{G}$

$$S_d = \frac{1}{\kappa_d^2} \int d^d x\, e\, \rho \Big[R^{(d)} - \frac{1}{4} \rho^{2/q}\, T_{mn}\, F^m_{\mu\nu}\, F^{\mu\nu\, n} + \frac{q-1}{q} \rho^{-2}\, \partial_\mu \rho\, \partial^\mu \rho + \frac{1}{4} \partial_\mu T^{-1}_{mn}\, \partial^\mu T_{mn} \Big]$$

d-dimensional gravity

$U(1)^q$ Maxwell

$F^m_{\mu\nu} = 2\,\partial_{[\mu} A^m_{\nu]}$

dilaton

$SL(q)/SO(q)$ sigma model

Rigid symmetry $\mathcal{G} = R^+ \times SL(q)$

Inherited from higher-dimensional diffeomorphisms

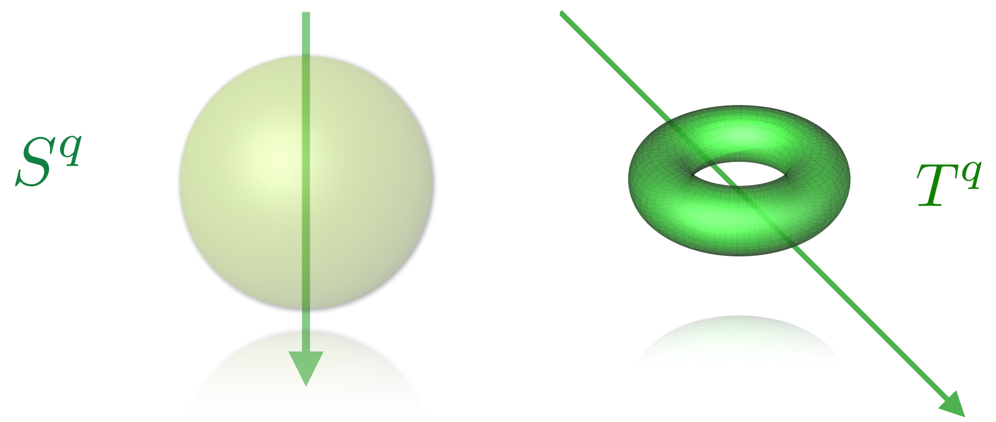
$$T \longrightarrow \Lambda\, T\, \Lambda^T$$

$$A_\mu \longrightarrow \lambda^{-1/q} \Lambda^{-1} A_\mu$$

$$\rho \longrightarrow \lambda^{1/q} \rho$$

Sphere truncations of gravity

D: Gravity theory



Necessary condition:

$$SO(q+1) \subset \mathcal{G}$$

[Cvetic, Gibbons, Lü, Pope '03]

D-q: Gravity theory with $SO(q+1)$ YM and scalar potential

Theory with rigid symmetry $\mathcal{G}$

Doesn't work for pure gravity: $\mathcal{G} = GL(q) \not\supset SO(q+1)$

Requires symmetry enhancement of $\mathcal{G}$

→ must start from matter-coupled gravity (~ supergravity)

Two families

- Gravity + p-form — Conformal branes
- Gravity + p-form + dilaton — Non-conformal branes

Sphere truncations to two dimensions

- 2nd family: Gravity + p-form + dilaton

D=11: Gravity

$\downarrow S^1$

D=10: Gravity + Maxwell + dilaton

$\downarrow S^2$

D=8: Gravity + $SO(3)$ YM + scalars

D=11: Gravity

$\downarrow S^1$

D=10: Gravity + Maxwell + dilaton

$\downarrow S^8$

D=2: Gravity + $SO(9)$ YM + scalars

Something tricky in this case

Affine enhancement $\mathcal{G} = \widehat{SL(9)} \supset SO(9)$

[FC, Samtleben '23]

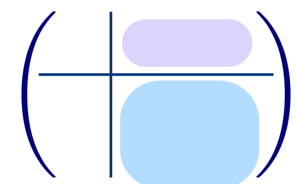
[Bossard, FC, Kleinschmidt, Inverso '23]

Two paths to D=2 on the torus

D=q+3: Pure gravity

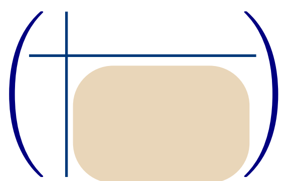
T^{q+1}

T^q



D=3: $R^+ \times SL(q)$ symmetry

Dualisation of the KK vectors
into scalars $\Longrightarrow SL(q+1)_E$ symmetry



[Ehlers 56']

S^1

D=2: $R^+ \times SL(q+1)$ symmetry $\Longleftrightarrow$ $R^+ \times SL(q+1)_E$ symmetry

Realised on scalars dual to each other

Two (on-shell) equivalent versions of the D=2 theory

[Geroch '71]

$$\mathcal{G} = \text{"} SL(q+1) \times SL(q+1)_E \sim \widehat{SL(q+1)} \text{"}$$

Affine extension