Universität Hamburg
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RECENT DEVELOPMENTS IN AdS_3/CFT_2 INTEGRABILITY

EUROSTRINGS 2025

DAVIDE POLVARA

BASED ON 2507.12191, 2501.05995 AND 2402.11732

WITH S. FROLOV AND A. SFONDRINI

AdS/CFT CORRESPONDENCE

TYPE IIB SUPERSTRING THEORY ON $AdS_5 \times S^5$

$$\left(T \sim \sqrt{\lambda}\ ,\quad g_s \sim \frac{\lambda}{N}\right)$$

$\longleftrightarrow$

$\mathcal{N}=4$ SYM IN 4d

$$\left(\lambda = g_{YM}^2 N\ ,\quad N\right)$$

STRING TENSION

STRING COUPLING

'T HOOFT COUPLING

RANK OF GAUGE GROUP

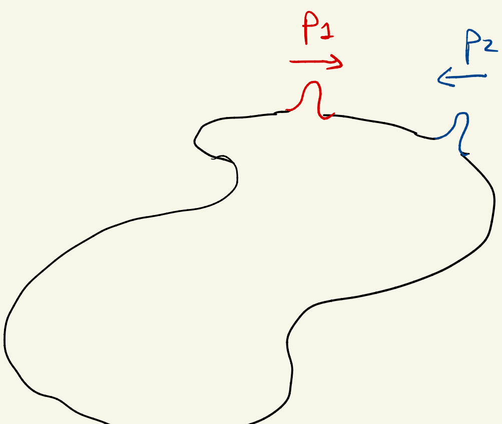
$g_s \to 0 \quad \Longleftrightarrow \quad N \to \infty$ INTEGRABILITY

[MINAHAN, ZAREMBO '02]

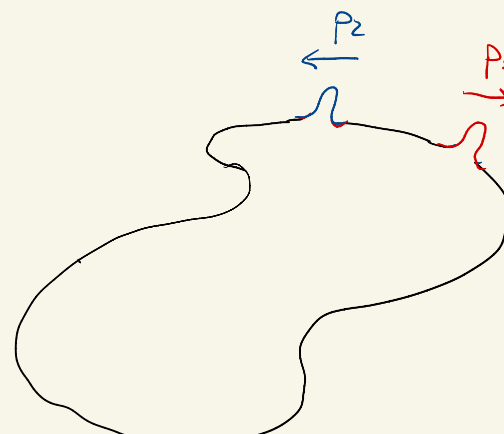
- INTEGRABILITY = LARGE AMOUNT OF CONSERVED CHARGES

$$\Downarrow$$

SCATTERING DIAGONAL AND FACTORIZED
[ZAMOLODCHIKOV & ZAMOLODCHIKOV '78]

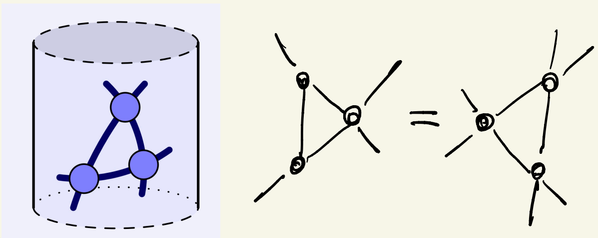


$$= S(p_1, p_2) \times$$



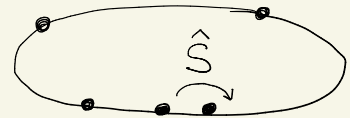
SYSTEMATIC ROUTE TO FIND THE SPECTRUM

LARGE WORLDSHEET



S-MATRIX BOOTSTRAP

LARGE AND PERIODIC

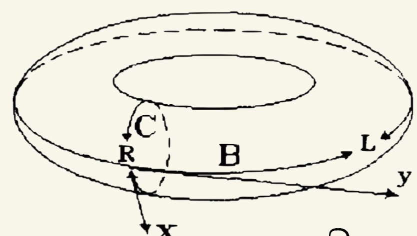


ASYMPTOTIC BETHE ANSATZ

[ARUTYUNOV & FROLOV '09]

[BEISERT ET ALL '10]

FINITE WORLDSHEET

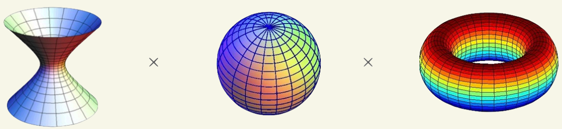


THERMODYNAMIC BETHE ANSATZ (TBA)

QUANTUM SPECTRAL CURVE (QSC) ← . . . ←

Y SYSTEM,
TQ RELATIONS
ETC.

PURPOSE: GENERALISE TO $AdS_3 \times S^3 \times T^4$



FAMILY OF CLASSICALLY INTEGRABLE NON-LINEAR SIGMA MODELS
[A. CAGNAZZO, K. ZAREMBO '12]

WITH 2 FREE PARAMETERS:

1. NSNS FLUX $k \in \mathbb{N}$ (NEAR HORIZON LIMIT OF NS5-F1 SYSTEM)

2. RAMOND-RAMOND FLUX $h \geq 0$ (NEAR HORIZON LIMIT OF D1-D5 BRANES)

WHAT IS KNOWN?

- $h=0$ (PURE NSNS), $k=1,2\ldots$ $\quad WZW_k$

[MALDACENA, OOGURI '00]

$h, k \neq 0$ LARGELY UNEXPLORED REGION

- PURE RAMOND-RAMOND $(k=0)$: INTEGRABILITY

$$\text{TBA} \longleftrightarrow \text{QSC}$$

[FROLOV, SFONDRINI '21]

[EKHAMMAR, VOLIN '21] [CAVAGLIA, GROMOV, STEFAŃSKI, TORRIELLI '21]

[EKHAMMAR, GROMOV, STEFAŃSKI '25]

$h, K \neq 0$: MOST MISTERIOUS BACKGROUND

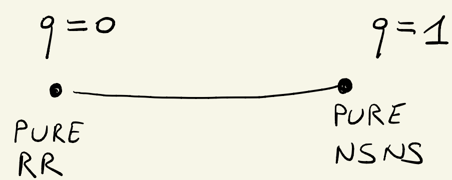
$$S = \int d\tau\, d\sigma \left[-\frac{T}{2}\sqrt{-\gamma}\,\gamma^{ab} G_{\mu\nu}(X) + \frac{K}{2\pi}\,\varepsilon^{ab} B_{\mu\nu}(X) \right] \partial_a X^\mu \partial_b X^\nu$$

$+$ FERMIONS

$\mu, \nu = 0, 1, \ldots, 9 \qquad a, b = \tau, \sigma$

$$h = \sqrt{1 - q^2}\; T \qquad q \in (0, 1)$$

$$\frac{K}{2\pi} = q\, T$$



PLAN OF THE TALK

1. MASSIVE S-MATRICES

2. MASSLESS S-MATRICES

3. CHECKS

4. FUTURE DIRECTIONS

Massive S-matrices

GAUGE FIXING

$$S = \int d\tau\, d\sigma \left[-\frac{T}{2}\sqrt{-\gamma}\;\gamma^{ab} G_{\mu\nu}(X) + \frac{K}{2\pi}\,\varepsilon^{ab} B_{\mu\nu}(X) \right] \partial_a X^\mu \partial_b X^\nu$$

$+$ FERMIONS

3 COORDINATES ON AdS_3: $Z, \bar{Z}, t$

3 COORDINATES ON S^3: $Y, \bar{Y}, \phi$

4 COORDINATES ON T^4: X_J $\quad J = 5, 6, 7, 8$

$$ds^2 = -\left(\frac{4+|Z|^2}{4-|Z|^2}\right)^2 dt^2 + \left(\frac{4}{4-|Z|^2}\right)^2 dZ\, d\bar{Z}$$

$$+\left(\frac{4-|Y|^2}{4+|Y|^2}\right)^2 d\phi^2 + \left(\frac{4}{4+|Y|^2}\right)^2 dY\, d\bar{Y} \quad + \sum_{J=5}^{8} dX_J\, dX_J$$

GAUGE FIXING

$$S = \int d\tau\, d\sigma \left[-\frac{T}{2}\sqrt{-\gamma}\, \gamma^{ab} G_{\mu\nu}(X) + \frac{K}{2\pi}\, \varepsilon^{ab} B_{\mu\nu}(X) \right] \partial_a X^\mu \partial_b X^\nu$$

+ FERMIONS

3 COORDINATES ON AdS_3: $Z, \bar{Z}, \cancel{t}$

3 COORDINATES ON S^3: $Y, \bar{Y}, \cancel{\phi}$

LIGHT-CONE GAUGE FIXING

4 COORDINATES ON T^4: X_J $\quad J = 5, 6, 7, 8$

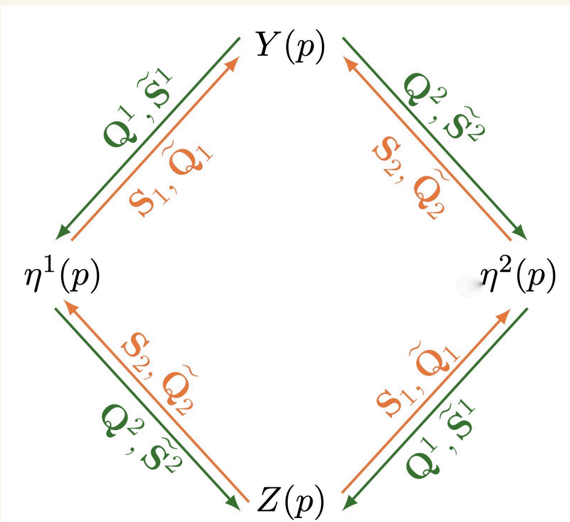
$$ds^2 = -\left(\frac{4+|Z|^2}{4-|Z|^2}\right)^2 dt^2 + \left(\frac{4}{4-|Z|^2}\right)^2 dZ\, d\bar{Z}$$
$$+ \left(\frac{4-|Y|^2}{4+|Y|^2}\right)^2 d\phi^2 + \left(\frac{4}{4+|Y|^2}\right)^2 dY\, d\bar{Y} + \sum_{J=5}^{8} dX_J\, dX_J$$

MASSIVE PARTICLES

SHORT REPRESENTATIONS OF $psu(1|1)^{\oplus 4}_{c.c}$

[BORSATO, OHLSSON SAX, SFONDRINI, STEFANSKI '13]
[HOARE, STEPANCHUK, TSEYTLIN '13]

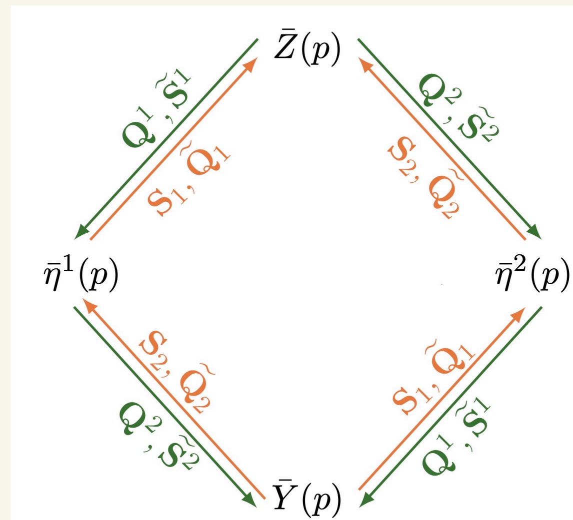
$$E_L(p) = \sqrt{\Big(1 + \underbrace{\frac{k}{2\pi}}_{\text{NSNS}}\, p\Big)^2 + 4\, \underbrace{h^2}_{\text{RR}} \sin^2\!\Big(\frac{p}{2}\Big)} \qquad E_R(p) = \sqrt{\Big(-1 + \frac{k}{2\pi}\, p\Big)^2 + 4\, h^2 \sin^2\!\Big(\frac{p}{2}\Big)}$$



LEFT

$k \in \mathbb{N}$

$h \geq 0$



RIGHT

- WORLDSHEET S-MATRIX KNOWN (UP TO THE DRESSING FACTORS)

$$\hat{\mathbb{S}}_{ij}^{kl}(p_1,p_2) = \Sigma(p_1,p_2)\, S_{ij}^{kl}(p_1,p_2)$$ [LLOYD, OHLSSON SAX, SFONDRINI, STEFANSKI '14]

Σ: FIXED BY UNITARITY, CROSSING FUSION ETC.

S: FIXED BY $[Q,S]=0$

- MORE SPECIFICALLY:

$$(\mathbb{S}_{LL})_{ij}^{kl} = \Sigma_{LL}(p_1,p_2)\left(S_{LL}(p_1,p_2)\right)_{ij}^{kl} \qquad (\mathbb{S}_{RR})_{ij}^{kl} = \Sigma_{RR}(p_1,p_2)\left(S_{RR}(p_1,p_2)\right)_{ij}^{kl}$$

$$(\mathbb{S}_{LR})_{ij}^{kl} = \Sigma_{LR}(p_1,p_2)\left(S_{LR}(p_1,p_2)\right)_{ij}^{kl} \qquad (\mathbb{S}_{RL})_{ij}^{kl} = \Sigma_{RL}(p_1,p_2)\left(S_{RL}(p_1,p_2)\right)_{ij}^{kl}$$

$$a, b \in \{L, R\}$$

$$\Sigma_{ab}(p_1, p_2) = \Sigma^{\mathrm{ODD}}_{ab}(p_1, p_2) \quad \Sigma^{\mathrm{EVEN}}_{ab}(p_1, p_2)$$

EXPRESSED IN TERMS OF GENERALISED RELATIVISTIC RAPIDITIES.

GENERALIZATION OF BES DRESSING FACTOR

[OHLSSON SAX, RIABCHENKO, STEFANSKI '23]

[S. FROLOV, DP, A. SFONDRINI '24]

[S. FROLOV, DP, A. SFONDRINI '25] × 2

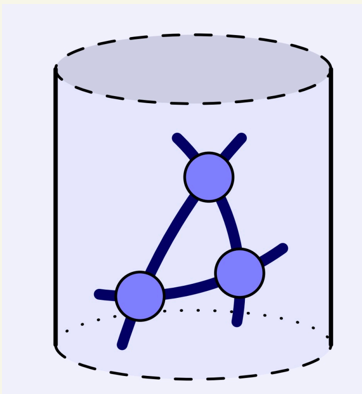
STRING REGION $\longleftrightarrow$ MIRROR REGION

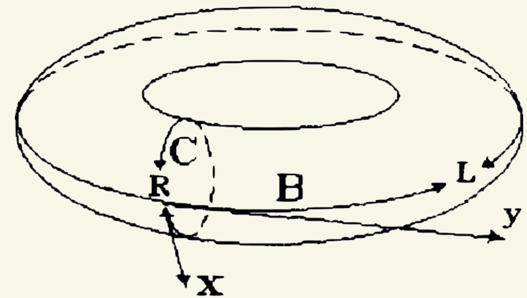
DOUBLE WICK ROTATION
OF TIME & SPACE

$$(\tau, \sigma) \longleftrightarrow (i\tilde{\sigma}, i\tilde{\tau})$$

[ZAMOLODCHIKOV '89]

[ARUTYUNOV, FROLOV '09]

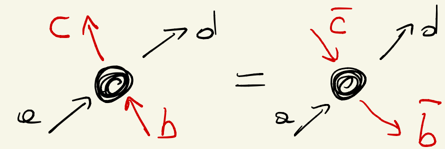




$$(E, p) \longleftrightarrow (i\tilde{p}, i\tilde{E})$$

$$E^2 = \left(m + \frac{\kappa}{2\pi} p\right)^2 + 4h^2 \sin^2\left(\frac{p}{2}\right) \longleftrightarrow -\tilde{p}^2 = \left(m + \frac{i\kappa}{2\pi}\tilde{E}\right)^2 - 4h^2 \operatorname{sh}^2\left(\frac{\tilde{E}}{2}\right)$$

$$S(p_1,p_2) \longleftrightarrow S(\tilde{p}_1,\tilde{p}_2)$$

Properties	String	Mirror	
1. Braiding U.	✓	✓	$S_{YY}(p_1,p_2)\,S_{YY}(p_2,p_1)=1$
2. Crossing	✓	✓	
3. LR sym.	✓	✓	$S_{YY}(p_1,p_2) \xleftrightarrow{K\leftrightarrow -K} S_{\bar{Y}\bar{Y}}(p_1,p_2)$
4. Unitarity	✓	✗	$\lvert S_{YY}(p_1,p_2)\rvert^2=1 \quad p_1,p_2\in\mathbb{R}$
5. Parity	✗	✓	$S_{YY}(p_1,p_2)=S_{YY}(-p_2,-p_1)$
6. CP	✓	✗	$S_{YY}(p_1,p_2)=S_{\bar{Y}\bar{Y}}(-p_2,-p_1)$

BOUND STATES

BY FUSING THE S-MATRICES OF FUNDAMENTAL PARTICLES WE OBTAIN THE BOUND STATES S-MATRICES

"MASS": ±1, ±2, ..

WORLD SHEET MOMENTUM

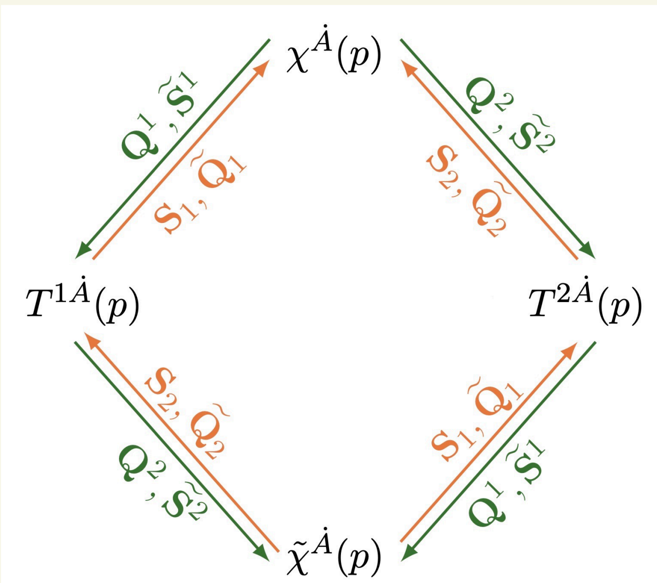
$$E_{w.s} = \sqrt{\left(m + \frac{K}{2\pi} p\right)^2 + 4 h^2 \sin^2 \frac{p}{2}}$$

$$S_{YY}^{m_1 m_2}(p_1, p_2) = \prod_{J_1=1}^{m_1} \prod_{J_2=1}^{m_2} S_{YY}(p_{J_1}, p_{J_2})$$

Massless S-matrices

MASSLESS PARTICLES

$$E(p) = \sqrt{\frac{k^2}{4\pi^2} p^2 + 4 h^2 \sin^2 \frac{p}{2}}$$



THERE ARE TWO SHORT REPRESENTATIONS

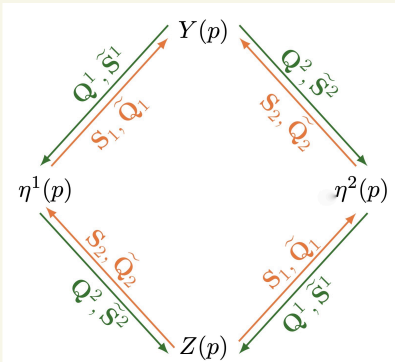
$\dot{A} = 1, 2$ SU(2) INDEX

$$S = \Sigma^{\infty} \left(S_{su(2)}\right)_{\dot{A}\dot{B}}^{\dot{C}\dot{D}} \, S_{psu(1|1)_{c.e}}^{\oplus 2}$$

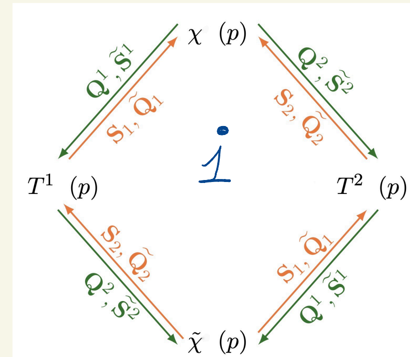
MASSLESS DRESSING FACTOR

Massless S-matrices as the $m \to 0$ limit of the massive ones

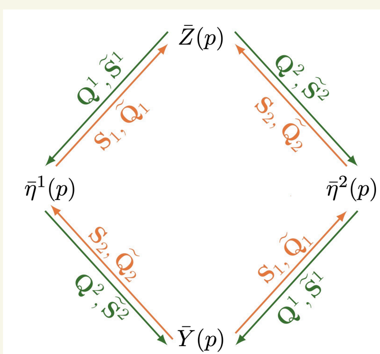
$$E_L(p) = \sqrt{\left(m + \frac{k}{2\pi}p\right)^2 + 4h^2\sin^2\left(\frac{p}{2}\right)}$$



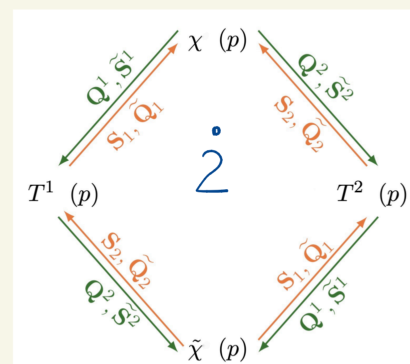
$$\xrightarrow{m \to 0^+}$$



$$E_R(p) = \sqrt{\left(m - \frac{k}{2\pi}p\right)^2 + 4h^2\sin^2\left(\frac{p}{2}\right)}$$



$$\xrightarrow{m \to 0^+}$$



CHECKS

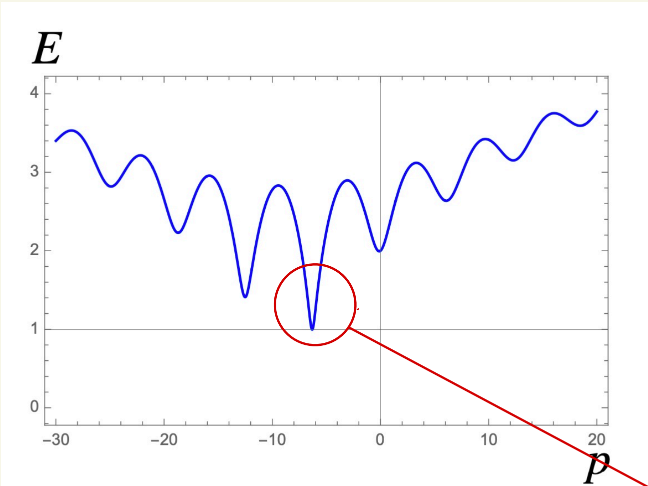
1. WE HAVE ANALYTIC EXPRESSIONS FOR MIXED-FLUX S MATRICES SATISFYING ALL SYMMETRY REQUIREMENTS.

2. AGREEMENT WITH PERTURBATIVE RESULTS IN NEAR BMN LIMIT AT TREE LEVEL AND 1-LOOP CUT CONSTRUCTABLE PART [BIANCHI, HOARE '14] [SUNDIN, WULFF '14]

3. Agreement in the relativistic limit.

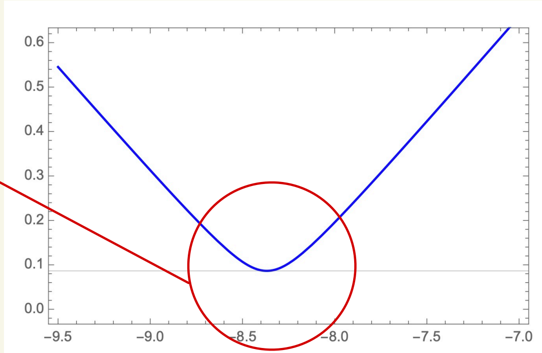
[Frolov, DP, Sfondrini '23]

[Fontanella, Ohlsson Sax, Stefański, Torrielli '19]



$$p = -\frac{2\pi}{k} m + h\, q$$

$$h \ll 1$$



$J = 1, 2, \dots, k-1$

$$E = \sqrt{\left(m + \frac{k}{2\pi} p\right)^2 + 4h^2 \sin^2 \frac{p}{2}} \longrightarrow E = \sin\left(\frac{\pi}{k} J\right) \operatorname{ch}\theta$$

FUTURE DIRECTIONS

1. TAKE k FINITE AND $h \to 0$. DOES THE SPECTRUM AGREE WITH THE WZW MODEL?

2. TAKE $k=1$ AND SMALL h. CAN WE REPRODUCE THE RESULTS OF SYMMETRIC PRODUCT ORBIFOLD THEORY $\mathrm{Sym}_N(T^4)$?

[GIRIBET, HULL, KLEBAN, PORRATI, RABINOVICI '18]
[EBERHARDT, GABERDIEL, GOPAKUMAR '18] [GABERDIEL, GOPAKUMAR, NAIRZ '23]
[FABRI, SFONDRINI, SKRZYPEK '25] ...

3. Find the spectrum numerically at all values of h and k.

4. TBA $\rightleftarrows$ QSC

5. More general integrable backgrounds

Mixed flux with $T^4 \longrightarrow S^3 \times S^1$

THANK you