Higgsing 5d SCFTs from Geometry, Branes & Quivers

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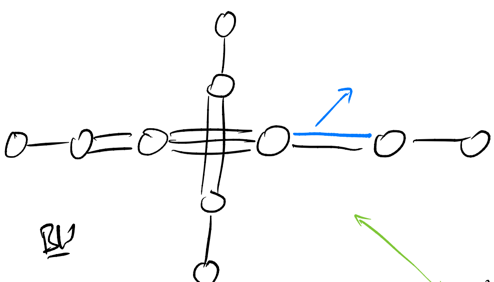
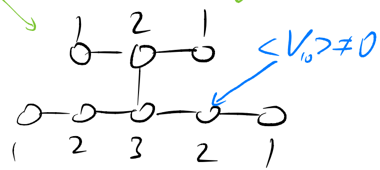


BW

$$\left\{ \begin{aligned} xy &= z^2 \\ uv &= z^3 + Xz \end{aligned} \right\}$$

CY3



MQ

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- based on several works with:

A. Bourget, A. Hanany, M. Sperling, Z. Zhong, ...

(brane webs, magnetic quivers, Higgsing / Hasse diagrams)

- and in particular upcoming work with:

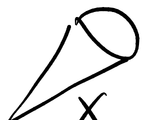
M. De Marco, M. Del Zotto, A. Sangiovanni

(geometry, deformations, connection to above)

5d $\mathcal{N}=1$ SCFTs in String Theory

two common constructions:

1) M-theory on local CY3

$\mathbb{R}^{1,4} \times$  X

Coulomb Branch ~ Resolutions of X

Higgs Branch ~ Deformations of X

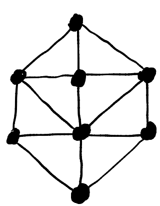
"geometric engineering"

2) Worldvolume theory on 5-brane web in IIB (w/ 7-branes)

Coulomb Branch ~ breathing of web

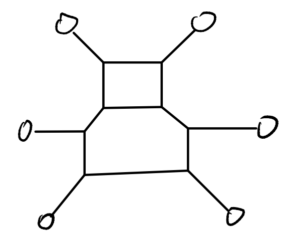
Higgs Branch ~ decomposition of web

if toric:

toric diagram 

dual

$\longleftrightarrow$

 brane web

- CB of 5d SCFT is easy to describe both in the geometric engineering and the brane web construction.

  not our focus today!

- HB of 5d SCFT is not (or let's say barely) understood in geometric engineering approach.
  However it is by now well understood in the brane web, via magnetic quivers!

  → Goal: better our understanding of Higgs branch in geom. eng. using branes & quivers

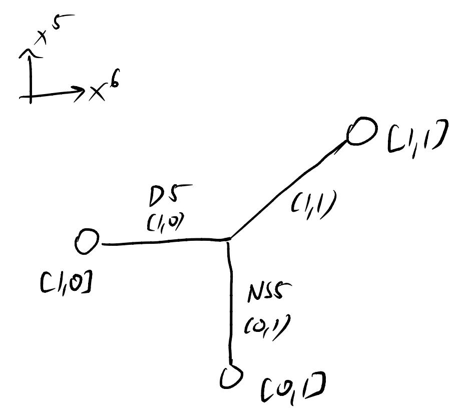
Brane Webs (5d generalisation of 3d Hanany-Witten)

IIB	0	1	2	3	4	5	6	7	8	9
(1,0)5 = D5	x	x	x	x	x		x			
(0,1)5 = NS5	x	x	x	x	x	x				
(p,q)5	x	x	x	x	x	angle α				
[p,q]7	x	x	x	x	x			x	x	x

$\underbrace{0,1,2,3,4}_{so(1,4)}$ $\quad$ $\underbrace{7,8,9}_{so(3)_R}$

$\tan\alpha = \frac{q}{p}$ for $\tau = i$

cartoon:

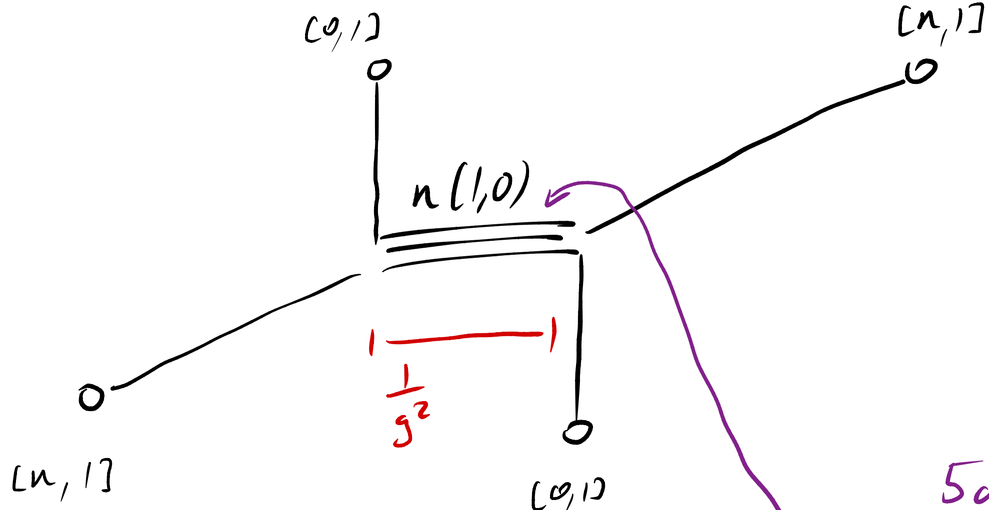


monodromy cut

$$M_{[p,q]} = \begin{pmatrix} 1-pq & p^2 \\ -q^2 & 1+pq \end{pmatrix}$$

brane webs ~ tropical curve (+ 7 branes)

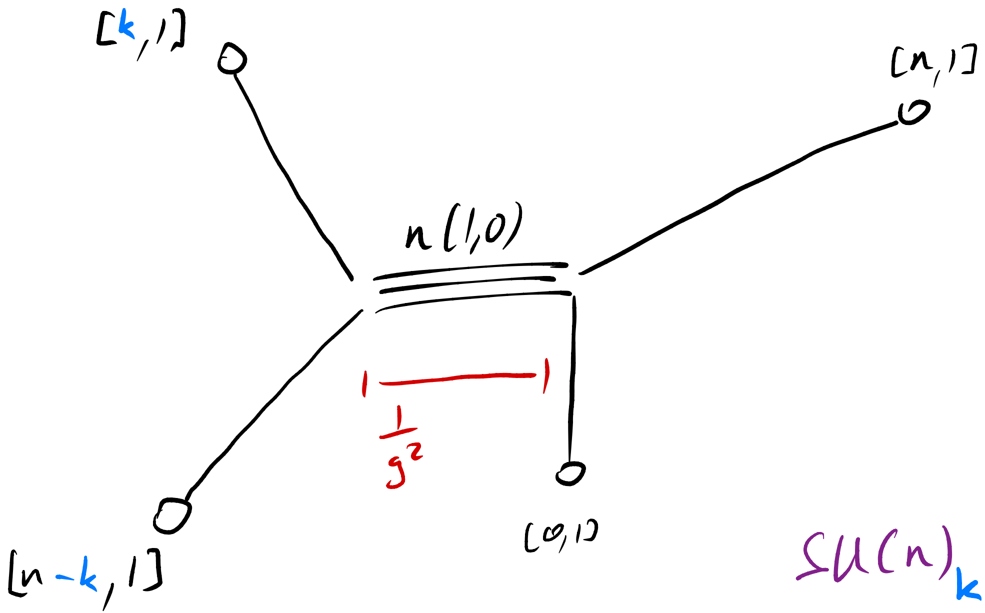
Let's construct some 5d $\mathcal{N}=1$ theories!



5d $\mathcal{N}=1$

$SU(n)_0$ pure gauge theory

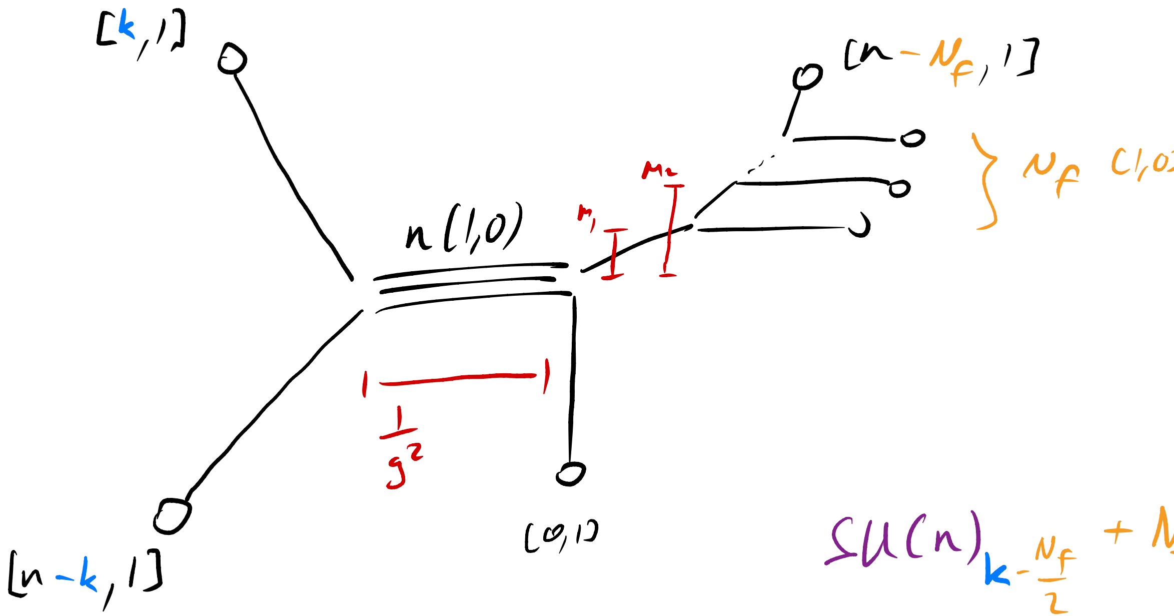
Let's construct some 5d $\mathcal{N}=1$ theories!



$SU(n)_k$ pure gauge theory

CS level

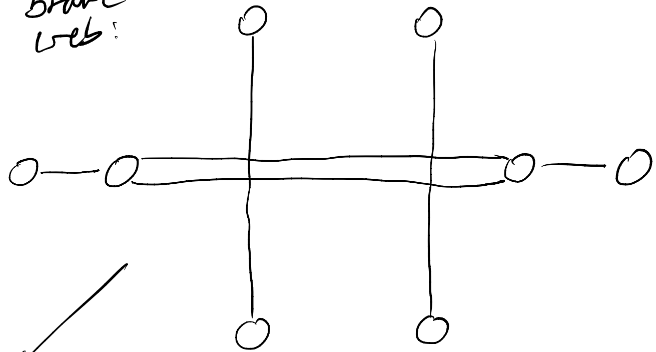
Let's construct some 5d $\mathcal{N}=1$ theories!

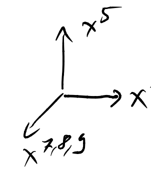


$SU(n)_{k-\frac{N_f}{2}} + N_f$ fund. hypers

SU(2) + 4 F, finite coupling

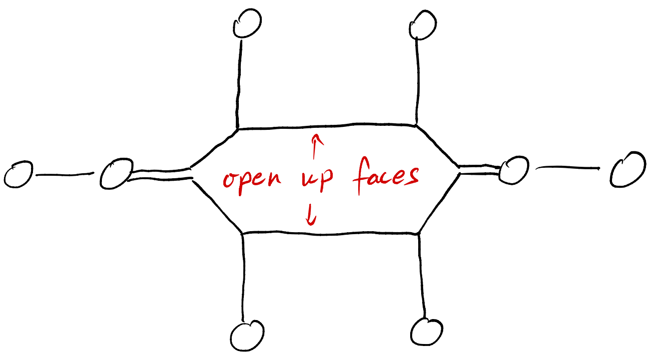
brane web:

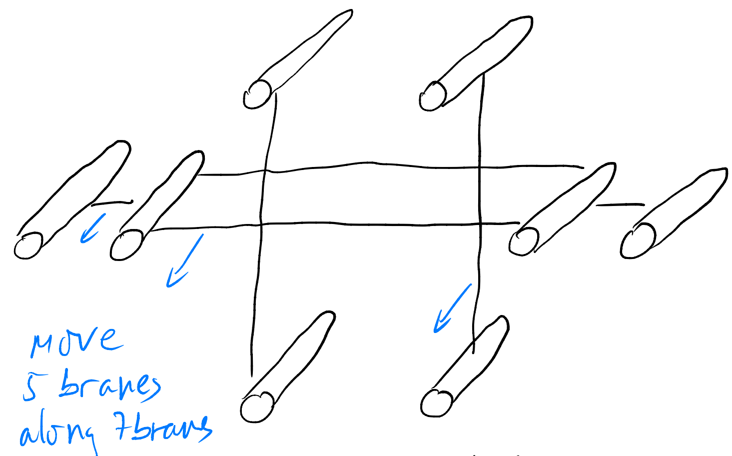




Coulom branch

Higgs branch



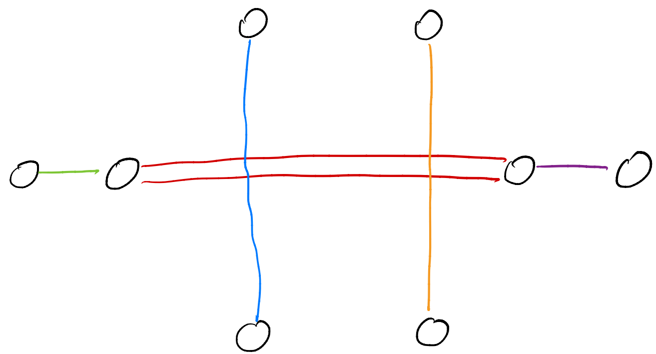


$\Rightarrow$ 1 Coulom branch modulus

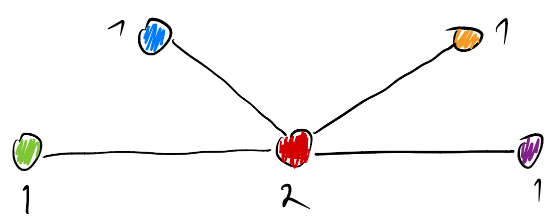
$\Rightarrow$ 5 Higgs branch moduli

Will suppress $x^{7,8,9}$ directions, and use colours instead!

Higgs branch!



From this we obtain a $\boxed{\text{magnetic quiver}}$ [Cabrera, Hanany, Yagi; ...]



Have $\mathcal{H}\left(\begin{array}{c}\boxed{4}\\ | \\ \circ\ SU(2)\end{array}\right) = \mathcal{C}^{3d}\left(\begin{array}{ccccc} & \overset{1}{\circ} & & \overset{1}{\circ} & \\ & & \diagdown \ \diagup & & \\ \underset{1}{\circ} & - & \underset{2}{\circ} & - & \underset{1}{\circ}\end{array}\right)$

classical Higgs branch, independent of spacetime dimension

3d Coulomb branch

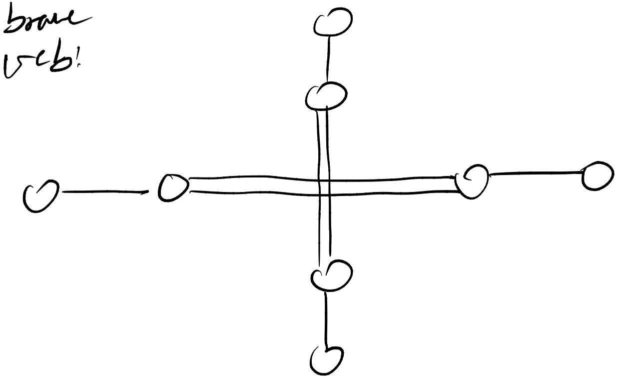
reminiscent of 3d Mirror Symmetry

now what about SCFT UV fixed point of $SU(2)+4F$, the Seiberg E_5 theory?

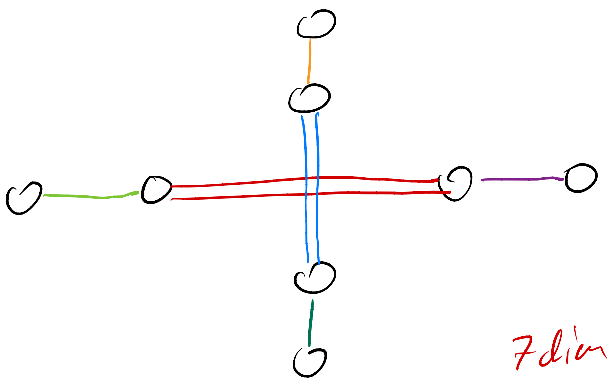
$SU(2) + 4F$

∞ coupling

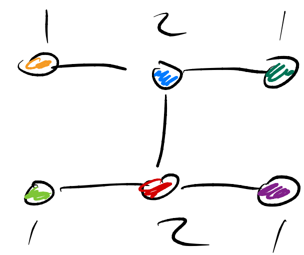
brane web!



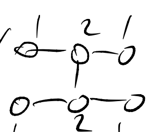
HB!



MQ!



7 dim Higgs branch!

$\mathcal{C}^{3d}\Bigg($  $\Bigg) = e_5 = \mathcal{H}(E_5 \text{ SCFT}, SU(2)+4F\,\infty)$ ✓

min nilp. orbit closure

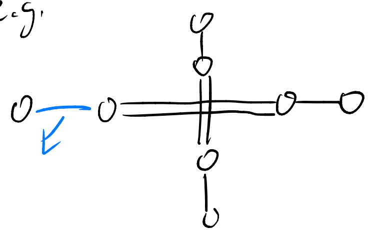
Now, what is a Higgsing (giving a VEV to a field) in this picture?

Don't have direct Field Theory description!

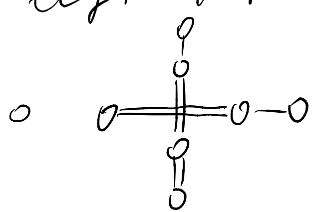
<u>In brane web:</u>

Can send a single brane away

e.g.



left with

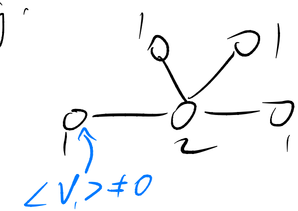


describes free hypers!

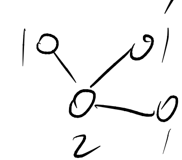
<u>In magnetic quiver:</u>

Can give VEV to a monopole op.

e.g.



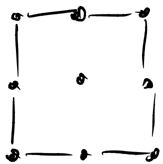
breaks theory to



free twisted hypers!

Now what about geometric engineering?

toric diagram:



describes a complete intersection

$$\left\{\begin{array}{l} xy = z^2 \\ uv = z^2 \end{array}\right\}$$

local CY3 singularity

Using [Collinucci, Valandro] can obtain deformations

$$\left\{\begin{array}{l} xy = z^2 + \alpha_5 u + \alpha_6 v + \alpha_7 z \\ uv = z^2 + \alpha_1 x + \alpha_2 y + \alpha_3 z + \alpha_4 \end{array}\right\}$$

$\alpha_i \to$ def params

7 dim Higgs branch

can add constant term in either equation, no difference

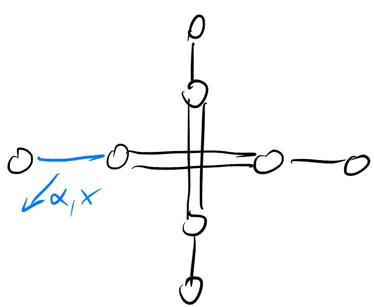
generic $\alpha_i \to$ get smooth CY3

Now only turn on single deformation:

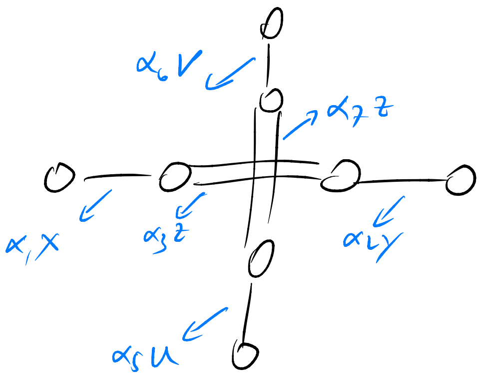
$$\begin{cases} xy = z^2 \\ uv = z^2 + \alpha_1 x \end{cases} \qquad \alpha_1 \neq 0$$

The resulting singularity is of compound Du Val (cDV) type, engineering free hypers [Collinucci, De Marco, Sangiovanni, Valandro].

We can in fact match this deformation exactly with the brane move and monopole VEV of before!



We can match all single brane move with a specific deformation:



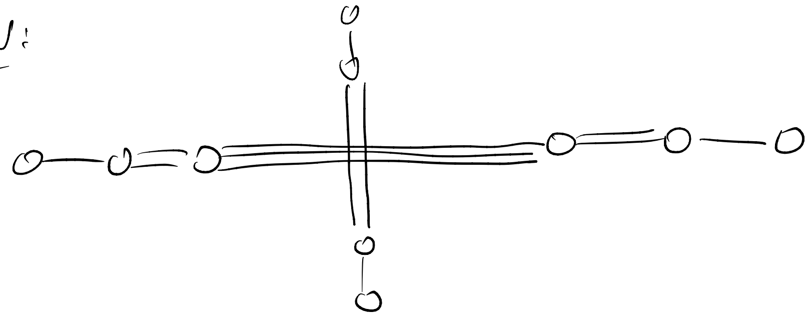
of course there is some arbitrariness due to the symmetry!

More interesting example:

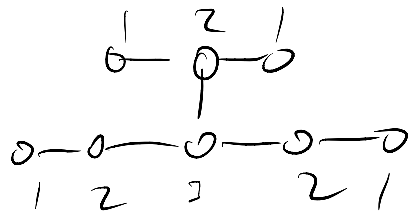
$\widehat{SU(3)+6F}$:

use this notation for ∞ coupling

BW:



MQ:



CY3:

$$\left\{ \begin{array}{l} xy = z^2 \\ uv = z^3 \end{array} \right\}$$

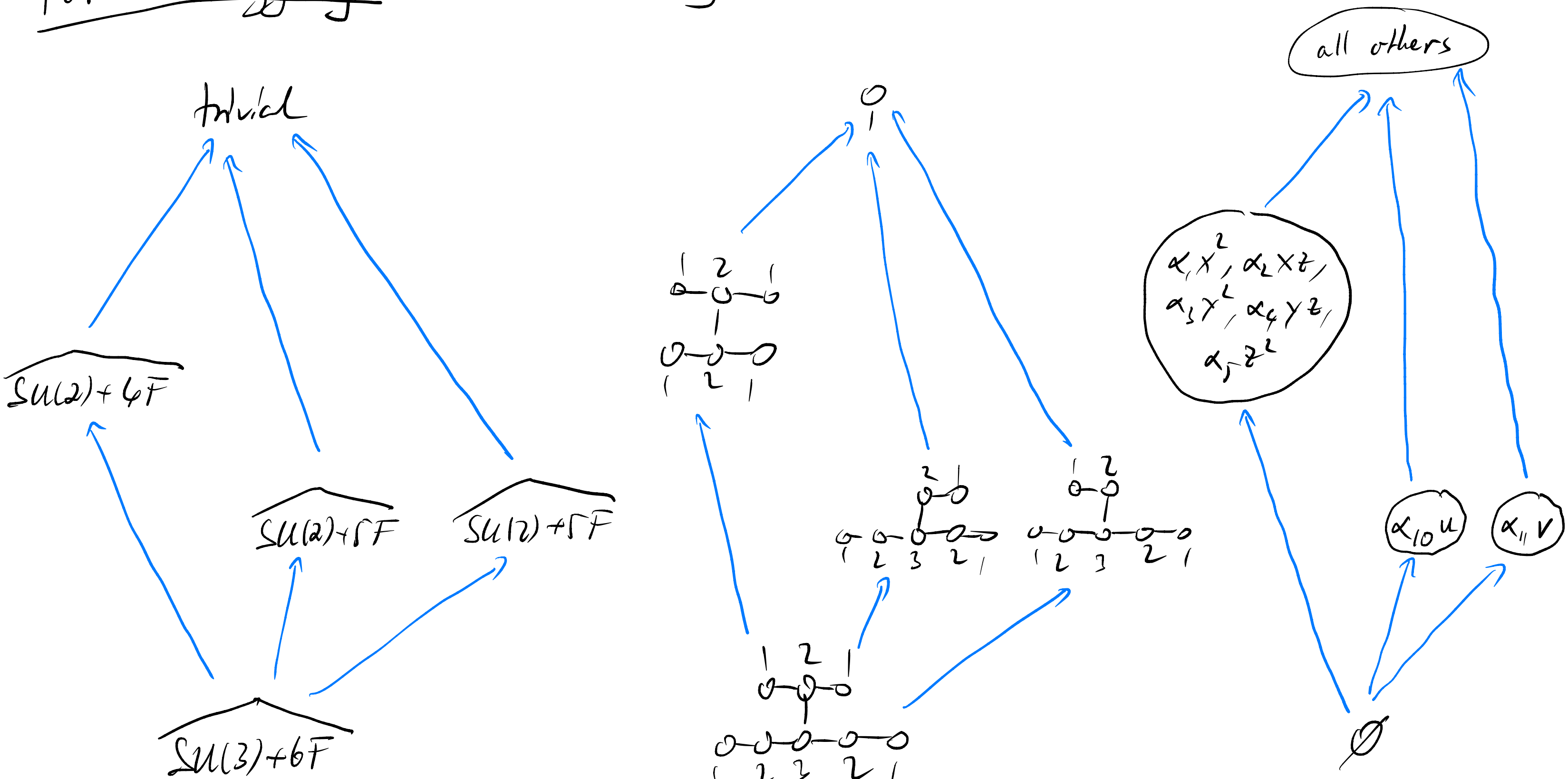
Here the defs are:

$$\left\{\begin{array}{l} xy = z^2 + \alpha_{10} u + \alpha_{11} v + \alpha_{12} z \\ uv = z^3 + \alpha_1 x^2 + \alpha_2 xz + \alpha_3 y^2 + \alpha_4 yz + \alpha_5 z^2 \\ \qquad + \alpha_6 x + \alpha_7 y + \alpha_8 z + \alpha_9 \end{array}\right\}$$

Getting complicated, but the deformation group nicely according to which partial Higgsing they trigger!

partial Higgsings are captured in a Hasse diagram!

Partial Higgsings: corresponding: MQs , deformations



The nodes in the Hasse diagram correspond to symplectic leaves in Higgs branch.

→ Are able to probe the stratification of the Higgs branch via deformations of the CY3 equations

This matches (where applicable) the work of [Bourget, Collinucci, Schäfer-Nameki] and [Alexeev, Argüz, Bousseau].

Outlook:

→ Obtain full Higgs branch from geometry?

→ Understand non toric brane webs better by understanding the corresponding deformed geometries!

Thank You!