The CFT Distance Conjecture and Non-Critical Strings

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[2011.10040] with Perlmutter, Rastelli, Vafa
[2410.07309] with Calderon-Infante
[ongoing] with Calderon-Infante, Uranga

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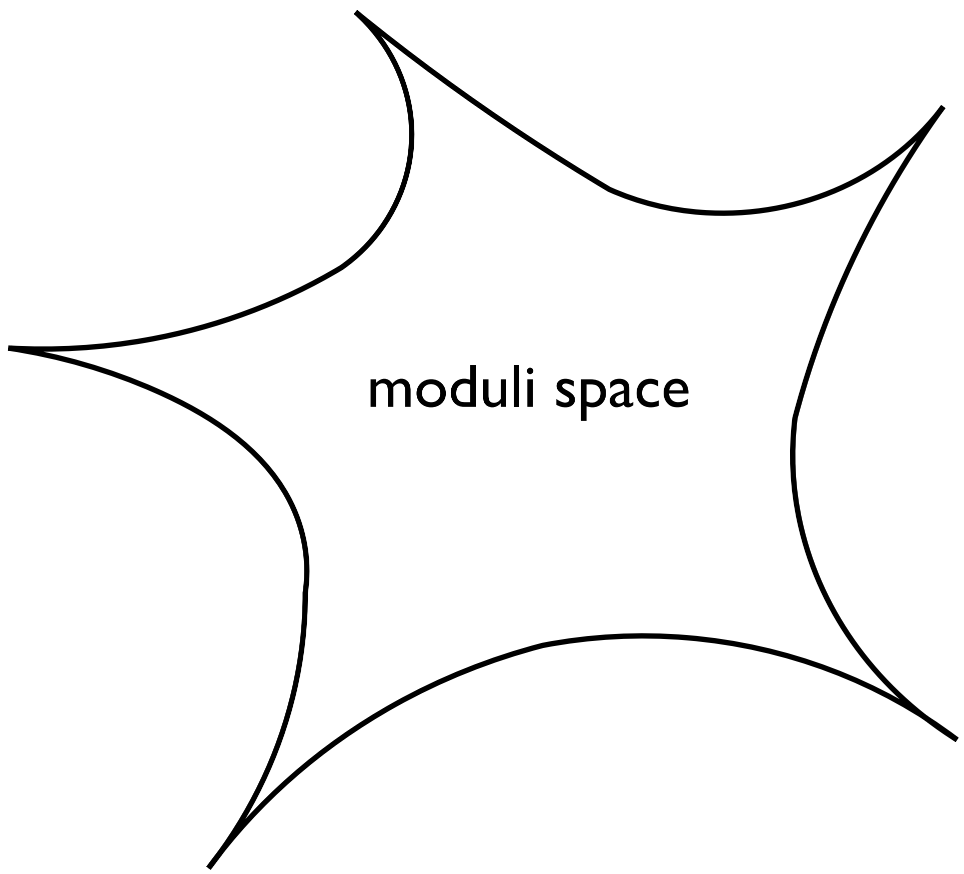
DISTANCE CONJECTURE

Consider an EFT weakly coupled to Einstein gravity with a moduli space parametrized by the vevs of scalar fields ϕ:

$$\mathcal{L} = \frac{M_{\mathrm{pl},d}^{d-2}}{2}\left(R - \frac{1}{2}g_{ij}(\phi)(\partial\phi)^2 + \dots\right)$$



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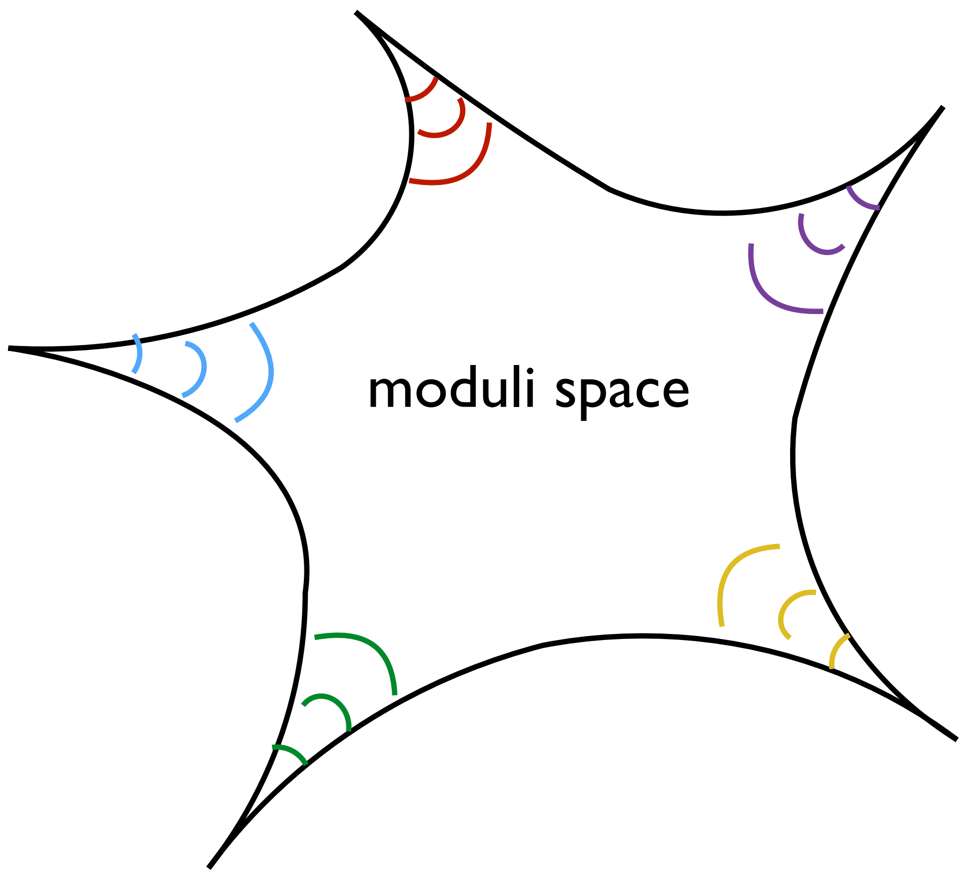

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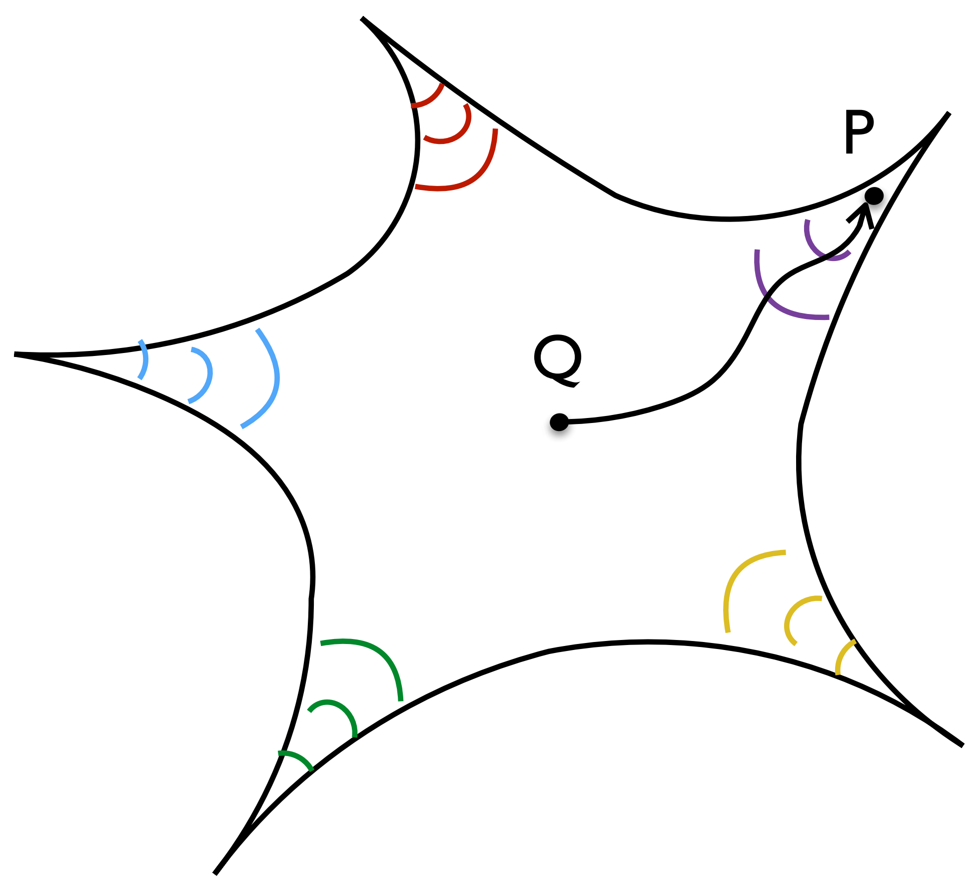
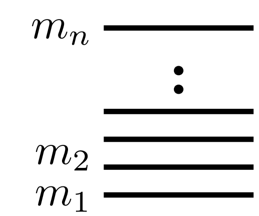


$\exists$ tower of states becoming all light



$$m(P) \sim m(Q)e^{-\alpha\Delta\phi}$$

as $\Delta\phi \to \infty$

Geodesic distance:

$$\Delta\phi = \int_Q^P \sqrt{g_{ij}\frac{d\phi^i}{ds}\frac{d\phi^j}{ds}}ds$$

e.g. string modes, KK states…

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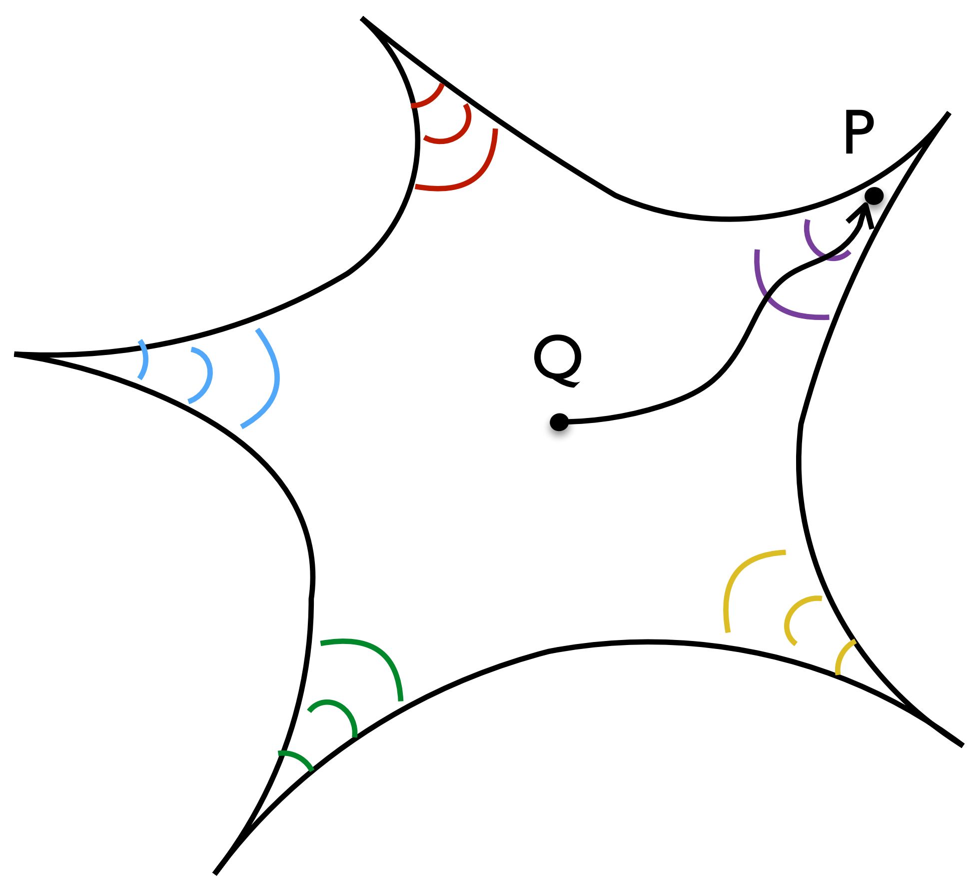

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m_n

$\vdots$

m_2

m_1

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Distance Conjecture: There should be an **infinite tower of states** becoming **exponentially light** (in Planck units) when approaching any **infinite field distance** boundary of the field space [Ooguri-Vafa'06]

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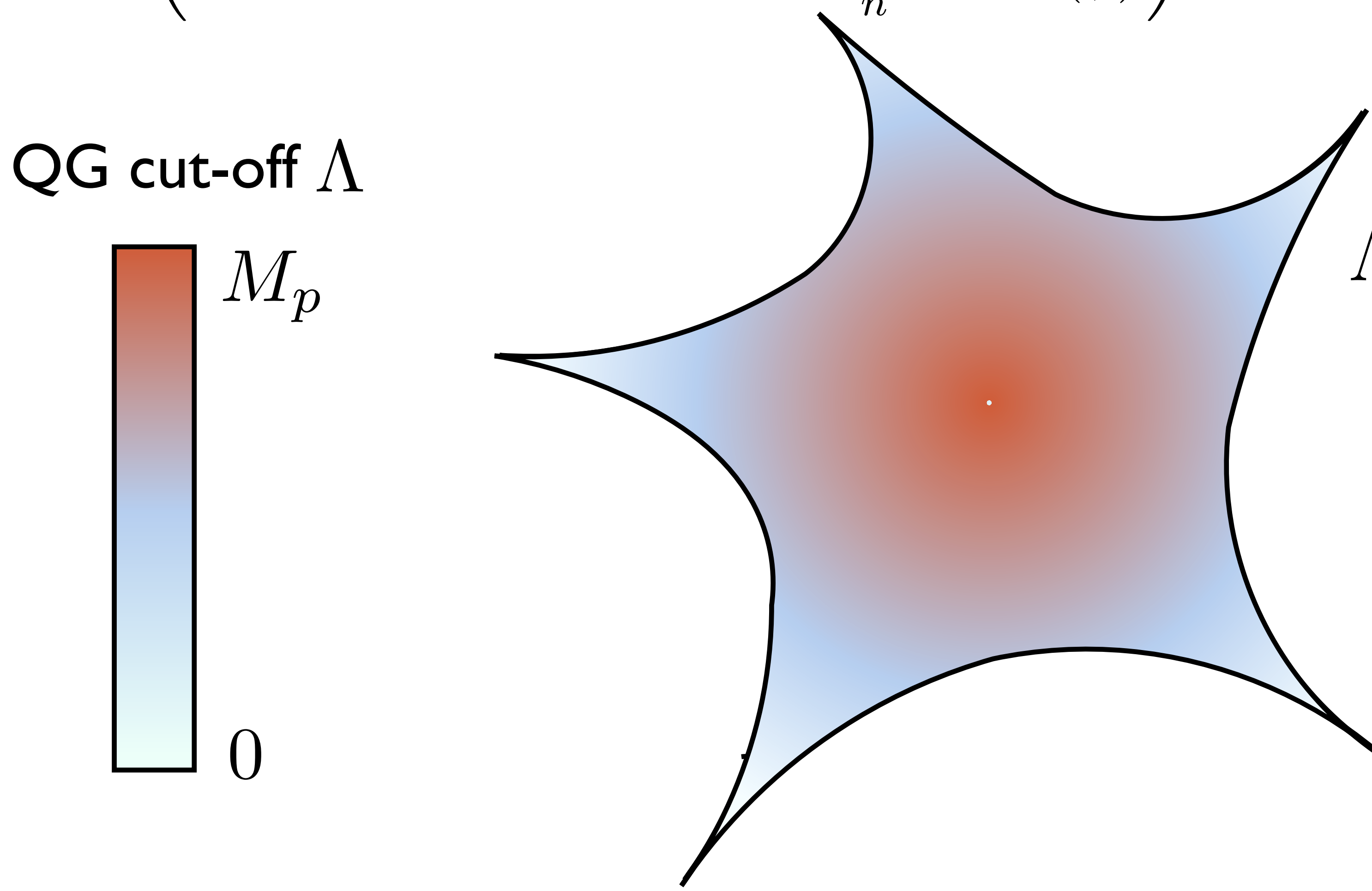
$$\mathcal{L} = \frac{M_{\mathrm{pl},d}^{d-2}}{2}\left(R - \frac{1}{2}g_{ij}(\phi)(\partial\phi)^2 + \cdots + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda^{n-2}(\phi)}\right)$$



$$\Lambda \sim M_p \exp(-\lambda\Delta\phi) \ll M_p$$

as $\quad \phi \to \infty$

It gives information about when and how Einstein gravity breaks down, and a quantum gravity description takes over

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$$m \sim M_p e^{-\alpha \Delta\phi}$$

$$\frac{1}{\sqrt{D-2}} \leq \alpha \leq \sqrt{\frac{D-1}{D-2}}$$

critical string tower

1-dim KK tower

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Can we prove its universality without relying on particular examples?

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In AdS backgrounds, it gets translated to a CFT statement that we can aim to prove (or disprove) universally

Is it a quantum gravity constraint or a string lamppost effect?

CFT DISTANCE CONJECTURE

Consider $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$

Bulk moduli space $\longleftrightarrow$ Conformal manifold (space of exactly marginal couplings)

field metric $\longleftrightarrow$ Zamolodchikov metric $|x-y|^{2d}\langle O_i(x)O_j(y)\rangle = g_{ij}(t^i)$

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Consider a local unitary CFT with a conformal manifold:

$\exists$ **infinite tower of primary operators** saturating the unitarity bound at **every infinite distance limit** measured by Zamolodchikov metric, such that

$$\gamma_J \sim e^{-\alpha d(\tau,\tau')} \quad \text{as} \quad d(\tau,\tau') \to \infty$$

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CFT Distance conjecture:

For d>2: [Perlmutter,Rastelli,Vafa,IV'21] [Baume,Calderon-Infante'21]
For d=2 [Kontsevich-Soibelman'00, Archarya-Douglas'06]

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More universal than expected!

It works in all known examples (even non-holographic CFTs)

no large N, no large gap
(non-Einstein gravity)

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This talk: Beginning of a classification of infinite distance limits in 4d conformal manifolds

→ leads to tensionless limits of different (non-)critical string backgrounds

↓

classification of free strings

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The only known examples of d>2 non-compact conformal manifolds occur in 4d SCFTs, so let's focus on d=4

(see though [Bobev et al'23] for d=3 candidates)

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Are they always associated to the critical string or more exotic objects?

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$$g_{YM} \to 0 \quad \Rightarrow \quad \mathrm{CFT}_{\mathrm{free}} \times \mathrm{CFT}'$$

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$$d(\tau, \tau') = \int \sqrt{g_{\tau\bar{\tau}}} = \beta \log(\mathrm{Im}\tau) \to \infty$$

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$$\gamma_J \sim f(J) g_{YM}^2 \sim f(J) \exp\left(-\frac{d(\tau,\tau')}{\beta}\right) \to 0$$

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$$\beta^2 = 24 \, \mathrm{dim} G$$

gauge group getting free

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If there is a weakly coupled AdS dual (large N), it implies: [Perlmutter,Rastelli,Vafa,IV'21]

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gauge group getting free

Lower bound for the exponential rate α !

EXAMPLE

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Weak coupling limit maps to the tensionless limit of the Type IIB string

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Comment: This result is valid for $\lambda = g_{YM}^2 N \to 0$ but it can be connected to the supergravity result (valid for $\lambda = g_{YM}^2 N \to \infty$) using integrability results

$$\alpha = \frac{1}{\sqrt{D-2}} = \frac{1}{\sqrt{3}}$$

[Calderon-Infante,IV'24]

Spin-off result: This can be used to provide an argument for the absence of AdS scale separation

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What about other 4d SCFTs?

What are the possible values of the exponential rate?

What does the HS tower correspond to?

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To start a classification, we first restrict our attention to 4d SCFTs with large N and a simple factor of the gauge group

only one gauge coupling

CLASSIFICATION OF LIMITS IN 4d SCFTs

Full classification of 4d (Lagrangian) SCFTs with large N and simple factor for the gauge group [Bhardwaj, Tachikawa'13] [Razamat, Sabag, Zafrir'20]

1)

SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=4$ $SU(N)$	3	0	0	0	0	0	0	$\frac{N^2-1}{4}$	$\frac{N^2-1}{4}$
$\mathcal{N}=2$ $SU(N)$	1	2	2	0	0	4	4	$\frac{3N^2+3N-2}{12}$	$\frac{6N^2+3N-5}{24}$
$\mathcal{N}=2$ $SU(N)$	1	1	1	1	1	0	0	$\frac{3N^2-2}{12}$	$\frac{6N^2-5}{24}$
$\mathcal{N}=1$ $SU(N)$	2	1	1	0	0	2	2	$\frac{6N^2+3N-5}{24}$	$\frac{12N^2+3N-11}{48}$
$\mathcal{N}=4$ $USp(2N)$	–	0	–	3	–	0	–	$\frac{N(2N+1)}{4}$	$\frac{N(2N+1)}{4}$
$\mathcal{N}=2$ $USp(2N)$	–	2	–	1	–	8	–	$\frac{6N^2+9N-1}{12}$	$\frac{12N^2+12N-1}{24}$
$\mathcal{N}=1$ $USp(2N)$	–	3	–	0	–	12	–	$\frac{4N^2+8N-1}{8}$	$\frac{8N^2+10N-1}{16}$
$\mathcal{N}=4$ $SO(N)$	–	3	–	0	–	0	–	$\frac{N(N-1)}{8}$	$\frac{N(N-1)}{8}$

2)

SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=2$ $SU(N)$	1	0	0	0	0	$2N$	$2N$	$\frac{2N^2-1}{6}$	$\frac{7N^2-5}{24}$
$\mathcal{N}=1$ $SU(N)$	0	0	1	1	0	$2N-4$	$2N+4$	$\frac{8N^2-3}{24}$	$\frac{14N^2-9}{48}$
$\mathcal{N}=2$ $USp(2N)$	–	0	–	1	–	$4N+4$	–	$\frac{N(4N+3)}{6}$	$\frac{N(14N+9)}{24}$
$\mathcal{N}=2$ $SO(N)$	–	1	–	0	–	$2N-4$	–	$\frac{N(2N-3)}{12}$	$\frac{N(7N-9)}{48}$
$\mathcal{N}=1$ $SO(N)$	–	0	–	1	–	$2N-8$	–	$\frac{4N^2-9N-1}{24}$	$\frac{7N^2-12N-1}{48}$

3)

SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=2$ $SU(N)$	1	1	1	0	0	$N+2$	$N+2$	$\frac{7N^2+3N-4}{24}$	$\frac{13N^2+3N-10}{48}$
$\mathcal{N}=2$ $SU(N)$	1	0	0	1	1	$N-2$	$N-2$	$\frac{7N^2-3N-4}{24}$	$\frac{13N^2-3N-10}{48}$
$\mathcal{N}=1$ $SU(N)$	2	0	0	0	0	N	N	$\frac{7N^2-5}{24}$	$\frac{13N^2-11}{48}$
$\mathcal{N}=1$ $SU(N)$	1	0	1	1	0	$N-4$	$N+4$	$\frac{7N^2-4}{24}$	$\frac{13N^2-10}{48}$
$\mathcal{N}=1$ $USp(2N)$	–	1	–	1	–	$2N+6$	–	$\frac{14N^2+15N-1}{24}$	$\frac{26N^2+21N-1}{48}$
$\mathcal{N}=1$ $USp(2N)$	–	2	–	0	–	$2N+10$	–	$\frac{14N^2+21N-2}{24}$	$\frac{26N^2+27N-2}{48}$
$\mathcal{N}=1$ $SO(N)$	–	0	–	2	–	$N-10$	–	$\frac{7N^2-21N-4}{48}$	$\frac{13N^2-27N-4}{96}$
$\mathcal{N}=1$ $SO(N)$	–	1	–	1	–	$N-6$	–	$\frac{7N^2-15N-2}{48}$	$\frac{13N^2-21N-2}{96}$

CLASSIFICATION OF LIMITS IN 4d SCFTs

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1)

SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=4$ $SU(N)$	3	0	0	0	0	0	0	$\frac{N^2-1}{4}$	$\frac{N^2-1}{4}$
$\mathcal{N}=2$ $SU(N)$	1	2	2	0	0	4	4	$\frac{3N^2+3N-2}{12}$	$\frac{6N^2+3N-5}{24}$
$\mathcal{N}=2$ $SU(N)$	1	1	1	1	1	0	0	$\frac{3N^2-2}{12}$	$\frac{6N^2-5}{24}$
$\mathcal{N}=1$ $SU(N)$	2	1	1	0	0	2	2	$\frac{6N^2+3N-5}{24}$	$\frac{12N^2+3N-11}{48}$
$\mathcal{N}=4$ $USp(2N)$	–	0	–	3	–	0	–	$\frac{N(2N+1)}{4}$	$\frac{N(2N+1)}{4}$
$\mathcal{N}=2$ $USp(2N)$	–	2	–	1	–	8	–	$\frac{6N^2+9N-1}{12}$	$\frac{12N^2+12N-1}{24}$
$\mathcal{N}=1$ $USp(2N)$	–	3	–	0	–	12	–	$\frac{4N^2+8N-1}{8}$	$\frac{8N^2+10N-1}{16}$
$\mathcal{N}=4$ $SO(N)$	–	3	–	0	–	0	–	$\frac{N(N-1)}{8}$	$\frac{N(N-1)}{8}$

$$\alpha = \frac{1}{\sqrt{2}}$$

2)

SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=2$ $SU(N)$	1	0	0	0	0	$2N$	$2N$	$\frac{2N^2-1}{6}$	$\frac{7N^2-5}{24}$
$\mathcal{N}=1$ $SU(N)$	0	0	1	1	0	$2N-4$	$2N+4$	$\frac{8N^2-3}{24}$	$\frac{14N^2-9}{48}$
$\mathcal{N}=2$ $USp(2N)$	–	0	–	1	–	$4N+4$	–	$\frac{N(4N+3)}{6}$	$\frac{N(14N+9)}{24}$
$\mathcal{N}=2$ $SO(N)$	–	1	–	0	–	$2N-4$	–	$\frac{N(2N-3)}{12}$	$\frac{N(7N-9)}{48}$
$\mathcal{N}=1$ $SO(N)$	–	0	–	1	–	$2N-8$	–	$\frac{4N^2-9N-1}{24}$	$\frac{7N^2-12N-1}{48}$

$$\alpha = \sqrt{\frac{2}{3}}$$

3)

SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=2$ $SU(N)$	1	1	1	0	0	$N+2$	$N+2$	$\frac{7N^2+3N-4}{24}$	$\frac{13N^2+3N-10}{48}$
$\mathcal{N}=2$ $SU(N)$	1	0	0	1	1	$N-2$	$N-2$	$\frac{7N^2-3N-4}{24}$	$\frac{13N^2-3N-10}{48}$
$\mathcal{N}=1$ $SU(N)$	2	0	0	0	0	N	N	$\frac{7N^2-5}{24}$	$\frac{13N^2-11}{48}$
$\mathcal{N}=1$ $SU(N)$	1	0	1	1	0	$N-4$	$N+4$	$\frac{7N^2-4}{24}$	$\frac{13N^2-10}{48}$
$\mathcal{N}=1$ $USp(2N)$	–	1	–	1	–	$2N+6$	–	$\frac{14N^2+15N-1}{24}$	$\frac{26N^2+21N-1}{48}$
$\mathcal{N}=1$ $USp(2N)$	–	2	–	0	–	$2N+10$	–	$\frac{14N^2+21N-2}{24}$	$\frac{26N^2+27N-2}{48}$
$\mathcal{N}=1$ $SO(N)$	–	0	–	2	–	$N-10$	–	$\frac{7N^2-21N-4}{48}$	$\frac{13N^2-27N-4}{96}$
$\mathcal{N}=1$ $SO(N)$	–	1	–	1	–	$N-6$	–	$\frac{7N^2-15N-2}{48}$	$\frac{13N^2-21N-2}{96}$

$$\alpha = \sqrt{\frac{7}{12}}$$

THREE TYPES OF WEAK COUPLING LIMITS

Considering all large N 4d SCFTs with simple gauge factor:

We get only three values for the exponential rate of the HS tower!

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For overall weak coupling limits: $\alpha = \sqrt{\frac{2c}{\dim G}} = \frac{1}{\sqrt{4a/c - 2}}$

CFT central charges

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→ what about these ones?

For overall weak coupling limits: $\alpha = \sqrt{\frac{2c}{\mathrm{dim}G}} = \frac{1}{\sqrt{4a/c-2}}$

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Swampland insight: the exact value of α gives us information about the microscopic (gravitational) nature of the tower

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This suggests that the three values correspond to three different towers of oscillator string modes

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Swampland insight: the exact value of α gives us information about the microscopic (gravitational) nature of the tower

This suggests that the three values correspond to three different towers of oscillator string modes

(i.e., three different types of tensionless string limits)

Let's check this and investigate which free strings we get!

HAGEDORN BEHAVIOUR

We compute the thermal partition function on the 3-sphere of all the previous gauge theories in the free limit

$$Z = \sum_{states} e^{-E_i/T}$$

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We obtain that they all exhibit a Hagedorn-like density of states with a Haggedorn temperature that depends only on α

$$\rho(E) \sim e^{E/T_H} \;, \quad \boxed{T_H = T_H(a/c) = T_H(\alpha)}$$

[Calderon-Infante,IV'24]

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[Calderon-Infante,IV'24]

This Hagedorn-density of states suggests indeed that the HS tower originates from three different string-like objects in the bulk

But which ones?

TENSIONLESS STRINGS

1) $\alpha = \frac{1}{\sqrt{2}}$ for $\frac{a}{c} = 1$

2) $\alpha = \sqrt{\frac{7}{12}}$ for $\frac{a}{c} = \frac{13}{14}$

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→ non-Einstein theories (no large gap)

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Let's construct an interpolating model starting from a holographic theory:

Holographic $\mathrm{SCFT}_1 = G_1 \times G_2$ $\xrightarrow{g_1 \to 0}$ Non-holographic $\mathrm{SCFT}_2 = G_2$

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difficult to analyse in the bulk…

Let's construct an interpolating model starting from a holographic theory:

$$\underset{\text{SCFT}_1 = G_1 \times G_2}{\text{Holographic}} \xrightarrow{g_1 \to 0} \underset{\text{SCFT}_2 = G_2}{\text{Non-holographic}}$$

Use this interpolating model to infer the bulk interpretation of the weak coupling limit $g_2 \to 0$ in SCFT_2

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1)

SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=4\ SU(N)$	3	0	0	0	0	0	0	$\frac{N^2-1}{4}$	$\frac{N^2-1}{4}$
$\mathcal{N}=2\ SU(N)$	1	2	2	0	0	4	4	$\frac{3N^2+3N-2}{12}$	$\frac{6N^2+3N-5}{24}$
$\mathcal{N}=2\ SU(N)$	1	1	1	1	1	0	0	$\frac{3N^2-2}{12}$	$\frac{6N^2-5}{24}$
$\mathcal{N}=1\ SU(N)$	2	1	1	0	0	2	2	$\frac{6N^2+3N-5}{24}$	$\frac{12N^2+3N-11}{48}$
$\mathcal{N}=4\ USp(2N)$	–	0	–	3	–	0	–	$\frac{N(2N+1)}{4}$	$\frac{N(2N+1)}{4}$
$\mathcal{N}=2\ USp(2N)$	–	2	–	1	–	8	–	$\frac{6N^2+9N-1}{12}$	$\frac{12N^2+12N-1}{24}$
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$\mathcal{N}=4\ SO(N)$	–	3	–	0	–	0	–	$\frac{N(N-1)}{8}$	$\frac{N(N-1)}{8}$

$$\alpha = \frac{1}{\sqrt{2}}$$

2)

SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=2\ SU(N)$	1	0	0	0	0	$2N$	$2N$	$\frac{2N^2-1}{6}$	$\frac{7N^2-5}{24}$
$\mathcal{N}=1\ SU(N)$	0	0	1	1	0	$2N-4$	$2N+4$	$\frac{8N^2-3}{24}$	$\frac{14N^2-9}{48}$
$\mathcal{N}=2\ USp(2N)$	–	0	–	1	–	$4N+4$	–	$\frac{N(4N+3)}{6}$	$\frac{N(14N+9)}{24}$
$\mathcal{N}=2\ SO(N)$	–	1	–	0	–	$2N-4$	–	$\frac{N(2N-3)}{12}$	$\frac{N(7N-9)}{48}$
$\mathcal{N}=1\ SO(N)$	–	0	–	1	–	$2N-8$	–	$\frac{4N^2-9N-1}{24}$	$\frac{7N^2-12N-1}{48}$

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$\mathcal{N}=2\ SU(N)$	1	1	1	0	0	$N+2$	$N+2$	$\frac{7N^2+3N-4}{24}$	$\frac{13N^2+3N-10}{48}$
$\mathcal{N}=2\ SU(N)$	1	0	0	1	1	$N-2$	$N-2$	$\frac{7N^2-3N-4}{24}$	$\frac{13N^2-3N-10}{48}$
$\mathcal{N}=1\ SU(N)$	2	0	0	0	0	N	N	$\frac{7N^2-5}{24}$	$\frac{13N^2-11}{48}$
$\mathcal{N}=1\ SU(N)$	1	0	1	1	0	$N-4$	$N+4$	$\frac{7N^2-4}{24}$	$\frac{13N^2-10}{48}$
$\mathcal{N}=1\ USp(2N)$	–	1	–	1	–	$2N+6$	–	$\frac{14N^2+15N-1}{24}$	$\frac{26N^2+21N-1}{48}$
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$\mathcal{N}=1\ SO(N)$	–	1	–	1	–	$N-6$	–	$\frac{7N^2-15N-2}{48}$	$\frac{13N^2-21N-2}{96}$

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$\mathcal{N}=2\ SU(N)$	1	2	2	0	0	4	4	$\frac{3N^2+3N-2}{12}$	$\frac{6N^2+3N-5}{24}$
$\mathcal{N}=2\ SU(N)$	1	1	1	1	1	0	0	$\frac{3N^2-2}{12}$	$\frac{6N^2-5}{24}$
$\mathcal{N}=1\ SU(N)$	2	1	1	0	0	2	2	$\frac{6N^2+3N-5}{24}$	$\frac{12N^2+3N-11}{48}$
$\mathcal{N}=4\ USp(2N)$	–	0	–	3	–	0	–	$\frac{N(2N+1)}{4}$	$\frac{N(2N+1)}{4}$
$\mathcal{N}=2\ USp(2N)$	–	2	–	1	–	8	–	$\frac{6N^2+9N-1}{12}$	$\frac{12N^2+12N-1}{24}$
$\mathcal{N}=1\ USp(2N)$	–	3	–	0	–	12	–	$\frac{4N^2+8N-1}{8}$	$\frac{8N^2+10N-1}{16}$
$\mathcal{N}=4\ SO(N)$	–	3	–	0	–	0	–	$\frac{N(N-1)}{8}$	$\frac{N(N-1)}{8}$

$$\alpha = \frac{1}{\sqrt{2}}$$

example! [Gadde,Pomoni,Rastelli'20]

2)

SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=2\ SU(N)$	1	0	0	0	0	$2N$	$2N$	$\frac{2N^2-1}{6}$	$\frac{7N^2-5}{24}$
$\mathcal{N}=1\ SU(N)$	0	0	1	1	0	$2N-4$	$2N+4$	$\frac{8N^2-3}{24}$	$\frac{14N^2-9}{48}$
$\mathcal{N}=2\ USp(2N)$	–	0	–	1	–	$4N+4$	–	$\frac{N(4N+3)}{6}$	$\frac{N(14N+9)}{24}$
$\mathcal{N}=2\ SO(N)$	–	1	–	0	–	$2N-4$	–	$\frac{N(2N-3)}{12}$	$\frac{N(7N-9)}{48}$
$\mathcal{N}=1\ SO(N)$	–	0	–	1	–	$2N-8$	–	$\frac{4N^2-9N-1}{24}$	$\frac{7N^2-12N-1}{48}$

$$\alpha = \sqrt{\frac{2}{3}}$$

3)

SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=2\ SU(N)$	1	1	1	0	0	$N+2$	$N+2$	$\frac{7N^2+3N-4}{24}$	$\frac{13N^2+3N-10}{48}$
$\mathcal{N}=2\ SU(N)$	1	0	0	1	1	$N-2$	$N-2$	$\frac{7N^2-3N-4}{24}$	$\frac{13N^2-3N-10}{48}$
$\mathcal{N}=1\ SU(N)$	2	0	0	0	0	N	N	$\frac{7N^2-5}{24}$	$\frac{13N^2-11}{48}$
$\mathcal{N}=1\ SU(N)$	1	0	1	1	0	$N-4$	$N+4$	$\frac{7N^2-4}{24}$	$\frac{13N^2-10}{48}$
$\mathcal{N}=1\ USp(2N)$	–	1	–	1	–	$2N+6$	–	$\frac{14N^2+15N-1}{24}$	$\frac{26N^2+21N-1}{48}$
$\mathcal{N}=1\ USp(2N)$	–	2	–	0	–	$2N+10$	–	$\frac{14N^2+21N-2}{24}$	$\frac{26N^2+27N-2}{48}$
$\mathcal{N}=1\ SO(N)$	–	0	–	2	–	$N-10$	–	$\frac{7N^2-21N-4}{48}$	$\frac{13N^2-27N-4}{96}$
$\mathcal{N}=1\ SO(N)$	–	1	–	1	–	$N-6$	–	$\frac{7N^2-15N-2}{48}$	$\frac{13N^2-21N-2}{96}$

$$\alpha = \sqrt{\frac{7}{12}}$$

INTERPOLATING MODEL FOR N=2 SQCD

[Gadde,Pomoni,Rastelli'20] [Dei,Pietro,Ferrero,Rastelli'24]

CFT	$\mathcal{N}=2 \;\; \text{SQCD}$ $SU(N) \text{ with } N_f = 2N$
Brane picture	
Gravity dual	

INTERPOLATING MODEL FOR N=2 SQCD

[Gadde,Pomoni,Rastelli'20] [Dei,Pietro,Ferrero,Rastelli'24]

CFT	$\mathcal{N}=2$ $SU(N)\times SU(N)$	$\xrightarrow{g_1\to 0}$	$\mathcal{N}=2$ SQCD $SU(N)$ with $N_f=2N$
Brane picture			
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Gravity dual	$\mathrm{AdS}_5\times S^5/\mathbb{Z}_2$		

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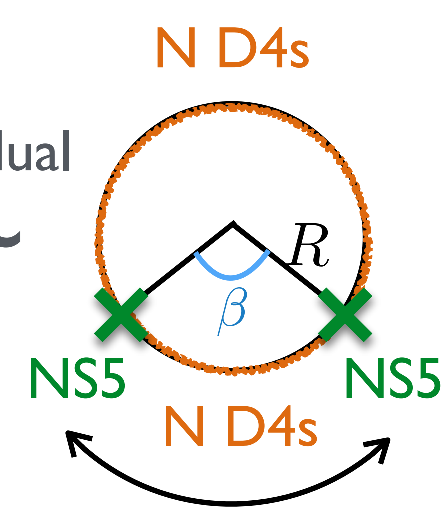
Brane picture	D3-branes on A1 singularity $\mathbb{R}^2\times\mathbb{R}^4/\mathbb{Z}_2$	T-dual ~	 $\tau_0 = 2\pi\beta R$
Gravity dual	$\mathrm{AdS}_5\times S^5/\mathbb{Z}_2$		

INTERPOLATING MODEL FOR N=2 SQCD

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$$\mathcal{N}=2$$
$$SU(N)\times SU(N)$$

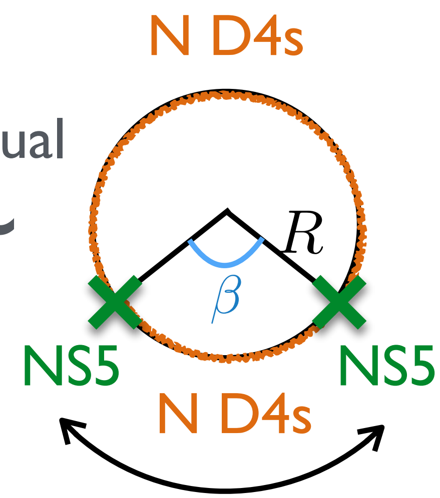
$$\xrightarrow{g_1\to 0}$$

$$\mathcal{N}=2\ \ \text{SQCD}$$
$$SU(N)\ \text{with}\ N_f=2N$$

Brane picture

D3-branes on
A1 singularity
$$\mathbb{R}^2\times\mathbb{R}^4/\mathbb{Z}_2$$

T-dual
~



$$\tau_0 = 2\pi\beta R$$

double-scaling
limit of LST

$$\tau_0\to 0$$
$$g_s^A\to 0$$
$$\frac{\tau_0}{g_s^A}\sim\frac{1}{g_2^2}\ \text{fixed}$$

D3/D5-branes on
non-critical string
background

$$\mathbb{R}^{5,1}\times\left(\frac{SL(2,\mathbb{R})_2}{U(1)}\right)/\mathbb{Z}_2$$

Gravity dual

$$\mathrm{AdS}_5\times S^5/\mathbb{Z}_2$$

INTERPOLATING MODEL FOR N=2 SQCD

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CFT

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Brane picture

D3-branes on A1 singularity

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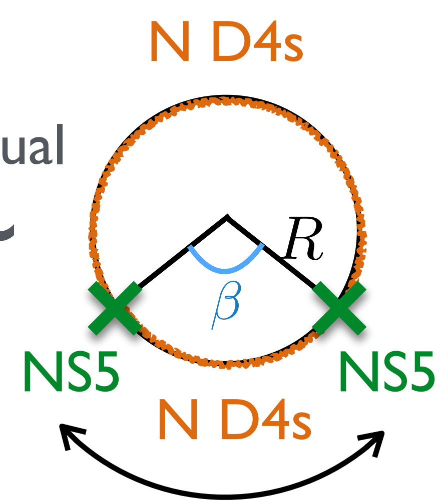
T-dual

~



$$\tau_0 = 2\pi\beta R$$

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Gravity dual

$$\text{AdS}_5 \times S^5/\mathbb{Z}_2$$

$\longrightarrow$

Backreacted sub-critical background with 7 geometric dimensions including a factor $AdS_5 \times S^1$

INTERPOLATING MODELS

[Calderon-Infante,UrangaIV'ongoing]

We construct interpolating models for all the previous gauge theories

2)

SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=2\ SU(N)$	1	0	0	0	0	$2N$	$2N$	$\frac{2N^2-1}{6}$	$\frac{7N^2-5}{24}$
$\mathcal{N}=1\ SU(N)$	0	0	1	1	0	$2N-4$	$2N+4$	$\frac{8N^2-3}{24}$	$\frac{14N^2-9}{48}$
$\mathcal{N}=2\ USp(2N)$	–	0	–	1	–	$4N+4$	–	$\frac{N(4N+3)}{6}$	$\frac{N(14N+9)}{24}$
$\mathcal{N}=2\ SO(N)$	–	1	–	0	–	$2N-4$	–	$\frac{N(2N-3)}{12}$	$\frac{N(7N-9)}{48}$
$\mathcal{N}=1\ SO(N)$	–	0	–	1	–	$2N-8$	–	$\frac{4N^2-9N-1}{24}$	$\frac{7N^2-12N-1}{48}$

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SUSY & Group	n_{Ad}	n_A	$n_{\bar{A}}$	n_S	$n_{\bar{S}}$	n_F	$n_{\bar{F}}$	c	a
$\mathcal{N}=2\ SU(N)$	1	1	1	0	0	$N+2$	$N+2$	$\frac{7N^2+3N-4}{24}$	$\frac{13N^2+3N-10}{48}$
$\mathcal{N}=2\ SU(N)$	1	0	0	1	1	$N-2$	$N-2$	$\frac{7N^2-3N-4}{24}$	$\frac{13N^2-3N-10}{48}$
$\mathcal{N}=1\ SU(N)$	2	0	0	0	0	N	N	$\frac{7N^2-5}{24}$	$\frac{13N^2-11}{48}$
$\mathcal{N}=1\ SU(N)$	1	0	1	1	0	$N-4$	$N+4$	$\frac{7N^2-4}{24}$	$\frac{13N^2-10}{48}$
$\mathcal{N}=1\ USp(2N)$	–	1	–	1	–	$2N+6$	–	$\frac{14N^2+15N-1}{24}$	$\frac{26N^2+21N-1}{48}$
$\mathcal{N}=1\ USp(2N)$	–	2	–	0	–	$2N+10$	–	$\frac{14N^2+21N-2}{24}$	$\frac{26N^2+27N-2}{48}$
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$\mathcal{N}=2$ $USp(2N)$	–	0	–	1	–	$4N+4$	–	$\frac{N(4N+3)}{6}$	$\frac{N(14N+9)}{24}$
$\mathcal{N}=2$ $SO(N)$	–	1	–	0	–	$2N-4$	–	$\frac{N(2N-3)}{12}$	$\frac{N(7N-9)}{48}$
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$\mathcal{N}=1$ $SU(N)$	2	0	0	0	0	N	N	$\frac{7N^2-5}{24}$	$\frac{13N^2-11}{48}$
$\mathcal{N}=1$ $SU(N)$	1	0	1	1	0	$N-4$	$N+4$	$\frac{7N^2-4}{24}$	$\frac{13N^2-10}{48}$
$\mathcal{N}=1$ $USp(2N)$	–	1	–	1	–	$2N+6$	–	$\frac{14N^2+15N-1}{24}$	$\frac{26N^2+21N-1}{48}$
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$\mathcal{N}=1\ \ SU(N)$	2	0	0	0	0	N	N	$\frac{7N^2-5}{24}$	$\frac{13N^2-11}{48}$
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$\mathcal{N}=1\ SU(N)$	0	0	1	1	0	$2N-4$	$2N+4$	$\frac{8N^2-3}{24}$	$\frac{14N^2-9}{48}$
$\mathcal{N}=2\ USp(2N)$	–	0	–	1	–	$4N+4$	–	$\frac{N(4N+3)}{6}$	$\frac{N(14N+9)}{24}$
$\mathcal{N}=2\ SO(N)$	–	1	–	0	–	$2N-4$	–	$\frac{N(2N-3)}{12}$	$\frac{N(7N-9)}{48}$
$\mathcal{N}=1\ SO(N)$	–	0	–	1	–	$2N-8$	–	$\frac{4N^2-9N-1}{24}$	$\frac{7N^2-12N-1}{48}$

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Weak coupling limit with $\alpha = \sqrt{2/3}$ = tensionless limit of the non-critical string $\mathbb{R}^{5,1} \times \left(\frac{SL(2,\mathbb{R})_2}{U(1)}\right)/\mathbb{Z}_2$

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$\mathcal{N}=2\ SU(N)$	1	0	0	1	1	$N-2$	$N-2$	$\frac{7N^2-3N-4}{24}$	$\frac{13N^2-3N-10}{48}$
$\mathcal{N}=1\ SU(N)$	2	0	0	0	0	N	N	$\frac{7N^2-5}{24}$	$\frac{13N^2-11}{48}$
$\mathcal{N}=1\ SU(N)$	1	0	1	1	0	$N-4$	$N+4$	$\frac{7N^2-4}{24}$	$\frac{13N^2-10}{48}$
$\mathcal{N}=1\ USp(2N)$	–	1	–	1	–	$2N+6$	–	$\frac{14N^2+15N-1}{24}$	$\frac{26N^2+21N-1}{48}$
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Approaching 3 NS5's + different content of $D4, D6, O4^{\pm}, O6^{\pm}$

Weak coupling limit with $\alpha = \sqrt{7/12}$ = tensionless limit of the string $\mathbb{R}^{5,1} \times \left(\frac{SL(2,\mathbb{R})_3}{U(1)} \times \frac{SU(2)_3}{U(1)}\right)/\mathbb{Z}_3$

CONCLUSIONS

- The Distance Conjecture implies a universal property for the variation of scaling dimensions as moving in the conformal manifold of CFTs, which holds even for non-Einstein theories

- We start a classification of infinite distance limits (weak coupling limits) in the conformal manifold of 4d SCFTs, which correspond to tensionless (non-)critical string limits in the bulk dual

- For 4d SCFTs with large N and a simple gauge factor, there are only three types of weak coupling limits.

  We show that the partition function at the free limit exhibits a Hagedorn-like density of states and construct interpolating models to identify the associated tensionless string

Thank you!

back-up slides

Critical string limit

If the bulk dual is Einstein gravity (so that a=c) → the higher spin tower always comes from the critical string

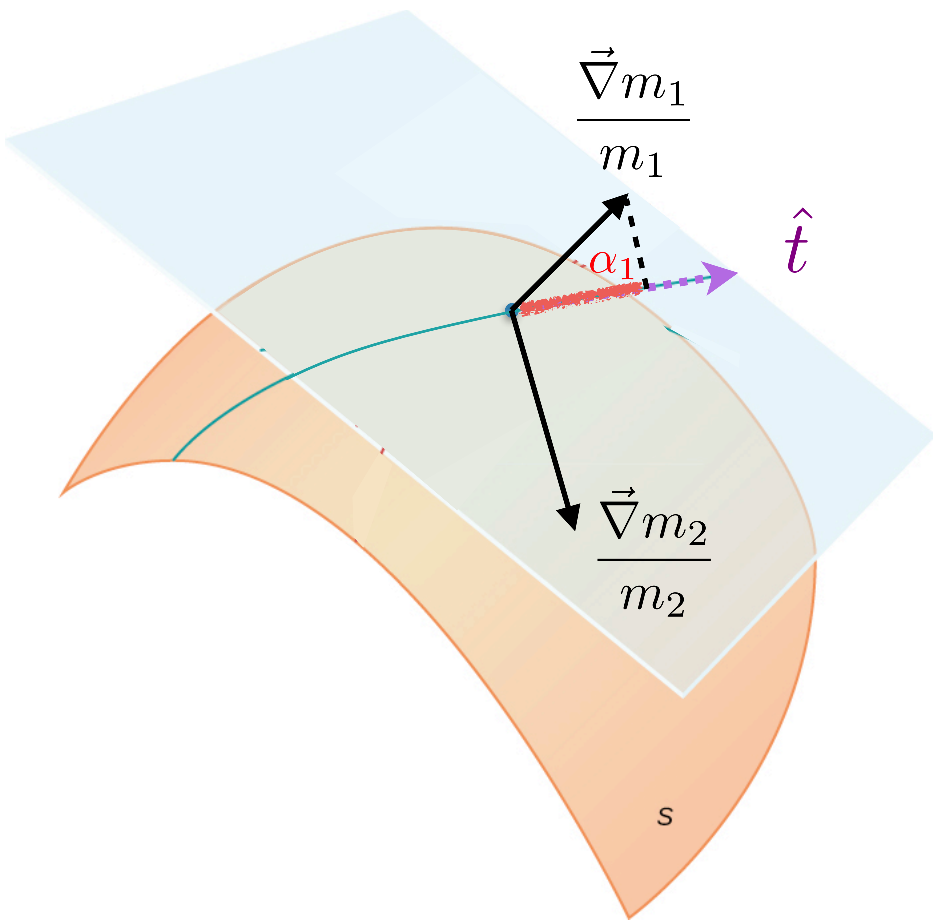
Exponential rate: $\alpha = \frac{1}{\sqrt{2}}$

How is it compatible with

$$\left|\frac{\vec{\nabla} m}{m}\right| = \frac{1}{\sqrt{d-2}} = \frac{1}{\sqrt{3}} \quad ?$$

- Quantization of the string changes in AdS
- $\alpha = \frac{\vec{\nabla} m}{m} \cdot \hat{t}$ is the projection on the conformal manifold

but $m(R_{S^5}, g_s) \longleftrightarrow m(N, g_{YM})$



Convex Hull in N=4 SYM

Let us focus on N=4 SYM / $AdS_5 \times S^5$ [Calderon-Infante,IV' ongoing]

- Field theory pert. regime: $\lambda \ll 1$

$$\gamma \sim N g_{YM}^2$$

$$m \sim \frac{\sqrt{\gamma}}{N^{2/3}} \longrightarrow \frac{\vec{\nabla} m_{HS}}{m_{HS}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{30}}\right)$$

$$\left|\frac{\vec{\nabla} m_{\text{HS}}}{m_{\text{HS}}}\right| = \sqrt{\frac{8}{15}}$$

- Supergravity regime: $\lambda \gg 1$

$$T_s = M_{p,5}^2\, g_s^{1/2} R^{-5/4}$$

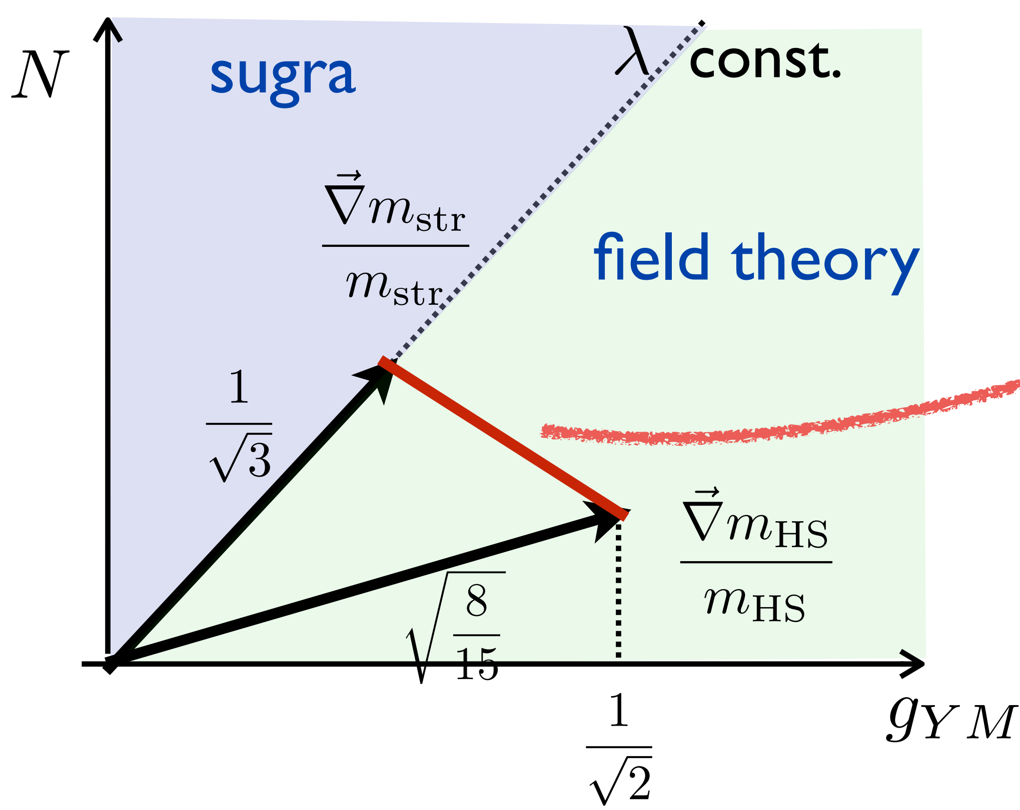
$$m \sim \sqrt{T_s} \longrightarrow \frac{\vec{\nabla} m_{str}}{m_{str}} = \left(\frac{1}{2\sqrt{2}}, \frac{\sqrt{30}}{12}\right)$$

$$\left|\frac{\vec{\nabla} m_{\text{str}}}{m_{\text{str}}}\right| = \frac{1}{\sqrt{3}}$$

$$R = M_{p,5}^{-1} N^{2/3}$$

$$g_s = g_{YM}^2$$

$$\lambda = g_{YM}^2 N$$



Results from integrability as λ changes

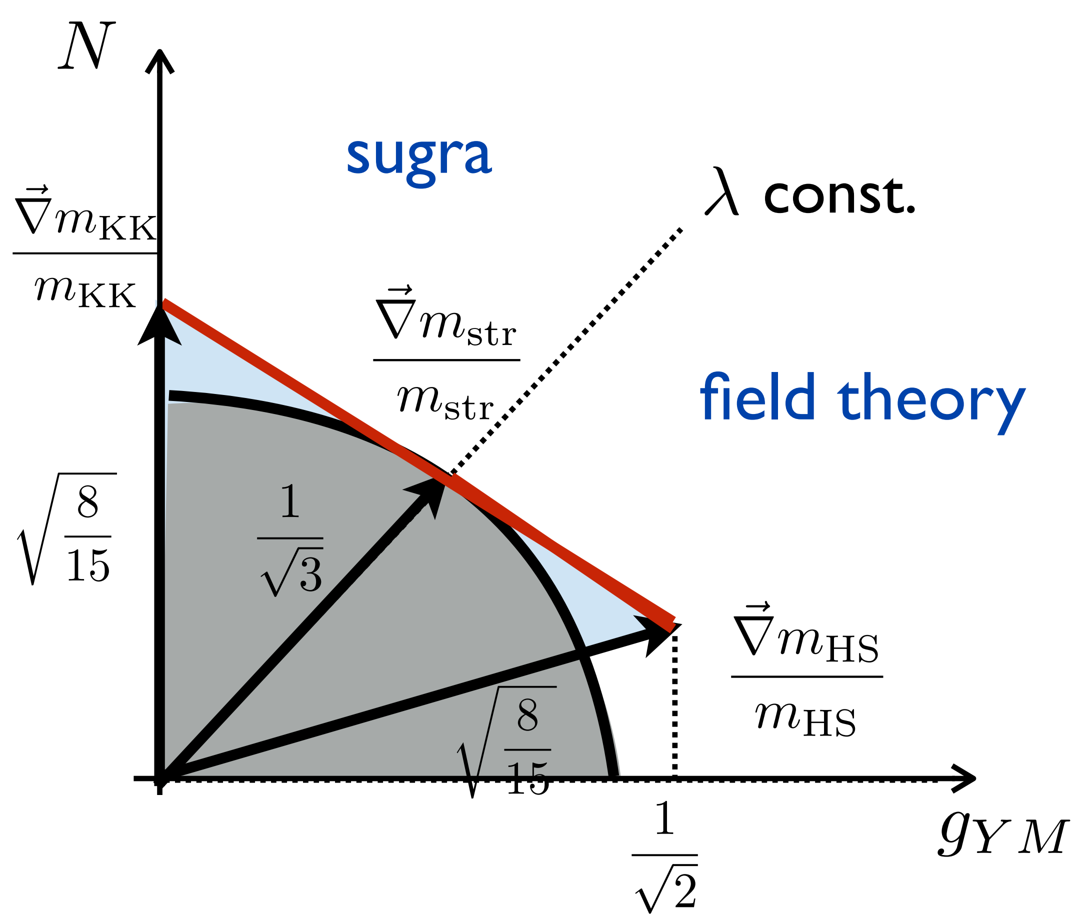
$$m \sim \frac{\sqrt{\gamma_{\text{cusp}}(\lambda)}}{N^{2/3}}$$

see e.g. [Dorigoni, Hatsuda'15]

Convex Hull and Scale Separation

Let us plot the convex hull of all the light towers, including the KK towers

$$m_{KK} R_{AdS} \sim \mathcal{O}(1) \longrightarrow \begin{array}{c} m_{KK} \sim N^{-2/3} \\ R_{AdS} \sim R_{S^5} \end{array} \longrightarrow \frac{\vec{\nabla} m_{\rm KK}}{m_{\rm KK}} = \left(0, \frac{2\sqrt{30}}{15}\right)$$



It satisfies taxonomy rules for the towers

It also satisfies the CH DC (it includes the ball of radius

$$\alpha \geq \frac{1}{\sqrt{d-2}} = \frac{1}{\sqrt{3}} \quad)$$

The sharpened bound for the Distance conjecture $\alpha \geq \frac{1}{\sqrt{d-2}}$ is satisfied for every infinite distance direction

It would be violated if the AdS vacuum was scale-separated $R_{KK} \ll R_{AdS}$

Convex Hull and Scale Separation

The Convex Hull Distance Conjecture with $\alpha \geq \dfrac{1}{\sqrt{d-2}}$

implies that holographic AdS spaces with a conformal manifold

(namely, 5d AdS spaces with 8 or more supercharges)

cannot be scale separated

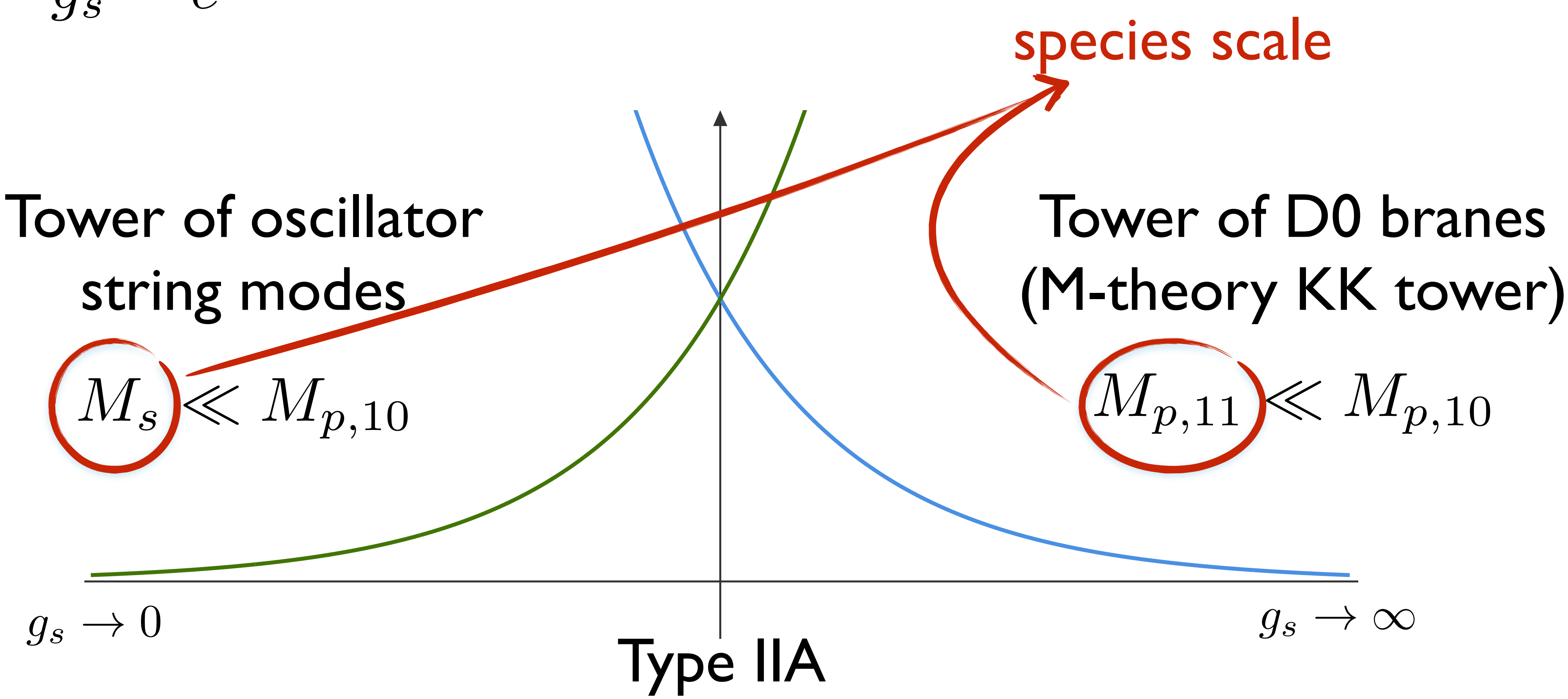
Here we are using the moduli space distance

but can we generalize the notion of distance including a scalar potential?

Example: 10d Type IIA

One dimensional moduli space: $S = M_p^2 \int d^{10}x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{4}(\partial\phi)^2 + \ldots\right)$

$g_s = e^{-\phi}$



As $g_s \to 0$ ➡ $M_s = M_{p,10}\, g_s^{1/4} = M_{p,10} \exp\left(-\frac{1}{\sqrt{8}}\Delta\phi\right)$

As $g_s \to \infty$ ➡ $m_{\rm KK} = \frac{M_s}{g_s} = M_{p,10} \exp\left(-\frac{3\sqrt{2}}{4}\Delta\phi\right)$; $M_{p,11} \sim m_{\rm KK}^{1/8}$