LIOUVILLE FOR YANG–MILLS

w/ S. Stieberger & B. Zhu

2209.02724   2302.12830 & in PREPARATION

From Amplitudes to Gravitational Waves

Nordita, 27.07.2023

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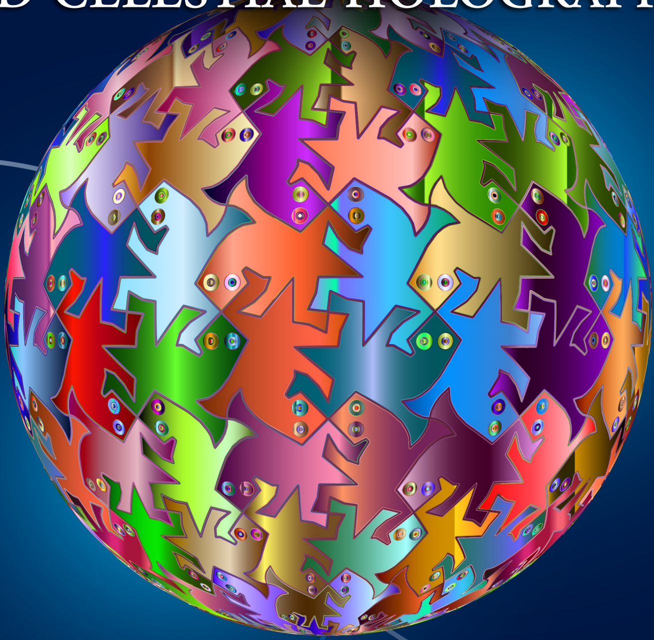
Advanced school on asymptotically flat spacetime and celestial holography



FACULTY OF PHYSICS, UNIVERSITY OF WARSAW
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Liouville Theory : 2D QG theory of conformal factor

Liouville field

$$\text{CFT}: \quad \mathcal{L} = \frac{1}{\pi}\left|\frac{\partial\phi}{\partial z}\right|^2 + \mu e^{2b\phi}$$

coupling constant

cosmological constant

2D $z, \bar{z}$ coordinates

Fundamental work of Dorn, Otto & Zamolodchikov^2 DOZZ (1994-6)

$$b + \frac{1}{b} = Q \qquad\qquad C = 1 + 6Q^2$$

background charge

central charge

$b \to 0$ $[C \to \infty]$ limit is nontrivial ($b \to \frac{1}{b}$ duality)

Primary field operators $V_\alpha(z,\bar{z}) = e^{2\alpha\phi(z,\bar{z})}$

dimension $\Delta = 2\alpha(Q-\alpha)$

$\langle V_{\alpha_1} V_{\alpha_2} V_{\alpha_3} \rangle$ = DOZZ formula, exact!

special case $\alpha = \sigma b$: light operators

$b \to 0$ $(\Delta = 2\sigma)$ studied by Harlow, Maltz, Witten 2011

$$\langle V_{\sigma_1}(x_1)\, V_{\sigma_2}(x_2)\, V_{\sigma_3}(x_3)\rangle = |x_{12}|^{\Delta_3-\Delta_1-\Delta_2}\, |z_{23}|^{\Delta_1-\Delta_2-\Delta_3}\, |z_{13}|^{\Delta_2-\Delta_1-\Delta_3}$$

$$\times\, \tilde{\mu}\, \mu^{1-\sigma_1-\sigma_2-\sigma_3}\, \frac{\Gamma(\sigma_1+\sigma_2+\sigma_3-1)\,\Gamma(\sigma_1+\sigma_2-\sigma_3)\,\Gamma(\sigma_2+\sigma_3-\sigma_1)\,\Gamma(\sigma_3+\sigma_1-\sigma_2)}{\Gamma(2\sigma_1)\,\Gamma(2\sigma_2)\,\Gamma(2\sigma_3)}$$

$$\pi\tilde{\mu}\,\gamma\!\left(\tfrac{1}{b^2}\right) = \left(\pi\mu\,\gamma(b^2)\right)^{\frac{1}{b^2}}$$

In CELESTIAL HOLOGRAPHY, CFT correlators are related to celestial amplitudes

We are interested in YM gluons – so where are gauge charges?

WZW : $\Delta = 1$ currents $J^a(z)$

$\Delta = -1$ $\hat{J}^a(z)$

$$\left\langle \hat{J}^{a_1}(z_1)\, \hat{J}^{a_2}(z_2)\, J^{a_3}(z_3) \right\rangle = \frac{z_{12}^4}{z_{12}\, z_{23}\, z_{31}}\, f^{a_1 a_2 a_3}$$

Costello-Paquette

$$= \mathrm{WZW}$$

Looks more familiar?

Liouville – WZW

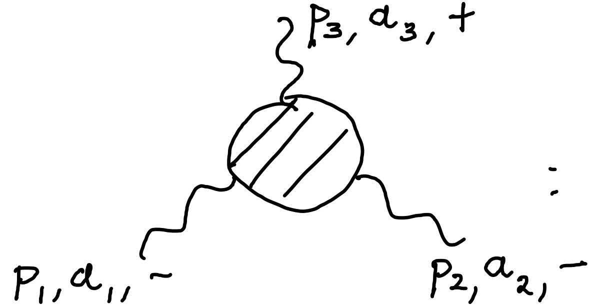
$$\mathcal{O}^{+a}_{\Delta}(z,\bar{z}) = \underset{\sim\sim}{M}^{\Delta-1} c_{+}(\mu,b)\,\Gamma(2\sigma)\, J^{a}(z)\, \underbrace{e^{2\sigma_{+} b\,\phi(z,\bar{z})}}_{\text{dim } \Delta-1}$$

$(b\to 0:\ 2\sigma_{+}=\Delta-1)$

$$\mathcal{O}^{-a}_{\Delta}(z,\bar{z}) = \underset{\sim\sim}{M}^{\Delta-1} c_{-}(\mu,b)\,\Gamma(2\sigma)\, \hat{J}^{a}(z)\, \underbrace{e^{2\sigma_{-} b\,\phi(z,\bar{z})}}_{\text{dim } \Delta+1}$$

$(b\to 0\ ;\ 2\sigma_{-}=\Delta+1)$

looks heterotic 

We want 

: use celestial map

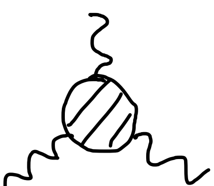
$$p^\mu = \omega\,(1+|z|^2,\, z+\bar{z},\, -i(z-\bar{z}),\, 1-|z|^2)$$

energy, Mellin dual of Δ

position

$O_\Delta(z,\bar{z})$

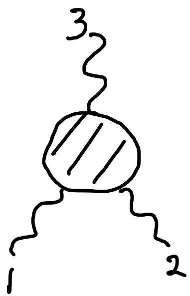
start from $\langle O^-_{\Delta_1}(z_1)\, O^-_{\Delta_2}(z_2)\, O^+_{\Delta_3}(z_3)\rangle$



$$= \underbrace{M^{\Delta_1+\Delta_2+\Delta_3-3}\cdots}_{\text{dim } = -3}\int d\Delta_1\, \omega_1^{-\Delta_1}\int d\Delta_2\, \omega_2^{-\Delta_2}\int d\Delta_3\, \omega_3^{-\Delta_3}\; \mathrm{DOZZ} \times \mathrm{WZW}$$

Inverse Mellin transforms are (sometimes) simple : $\int d\Delta\, \omega^{-\Delta}\, \Gamma(\Delta) \sim e^{-\omega}$

$b \to 0 \qquad \mathcal{O}(b^0)$

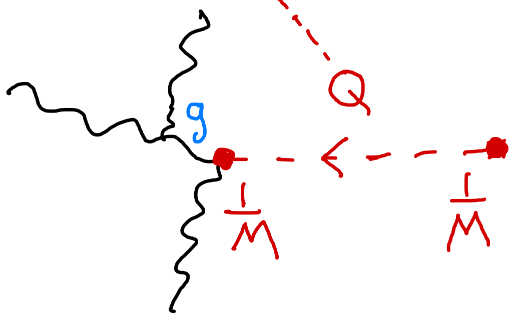


$$= \frac{c(\mu,b)}{M^2} \sqrt{\frac{M^2}{Q^2}}\, K_1\!\left(\sqrt{\frac{Q^2}{M^2}}\right) \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\, f^{123}$$

$$\underset{M^2\to\infty}{\sim} \quad \frac{c(\mu,b)}{M^2 Q^2} \times \mathcal{M}(1^- 2^- 3^+)$$

MHV Feynman matrix element

$$\frac{c(\mu,b)}{M^2 Q^2} \times \mathcal{M}(1^- 2^- 3^+)$$ What is it?



dilaton source ($\delta(x)$ - shockwave)

$Q = p_1 + p_2 + p_3$

Soft dilaton theorem of Ademollo...1975, Di Vecchia...2015:

$$\mathcal{M}(1^- 2^- 3^+) = \lim_{M\to\infty,\, Q\to 0} M^2 Q^2 \times \text{Inverse Mellin of DOZZ} \times \text{WZW}$$

$$\mathcal{M}(1^-2^-3^+) = \lim_{M\to\infty,\, Q\to 0} M^2 Q^2 \times \text{Inverse Mellin of DOZZ} \times \text{WZW}$$

$g \to 0$ $\quad b \to 0$: How are they related?

$\mathcal{O}(b^2)$: $\quad b^2 \log \frac{p_i p_j}{M^2}$ terms

$$b^2 \approx -\frac{g^2}{8\pi^2}\beta_0$$

Exact DOZZ × WZW

→ Exact YM ?

where is my \$ 1 million?

HOLOGRAPHIC PRINCIPLE:

(~~STANDARD~~ VERSION)
MODIFIED

matter/gravity

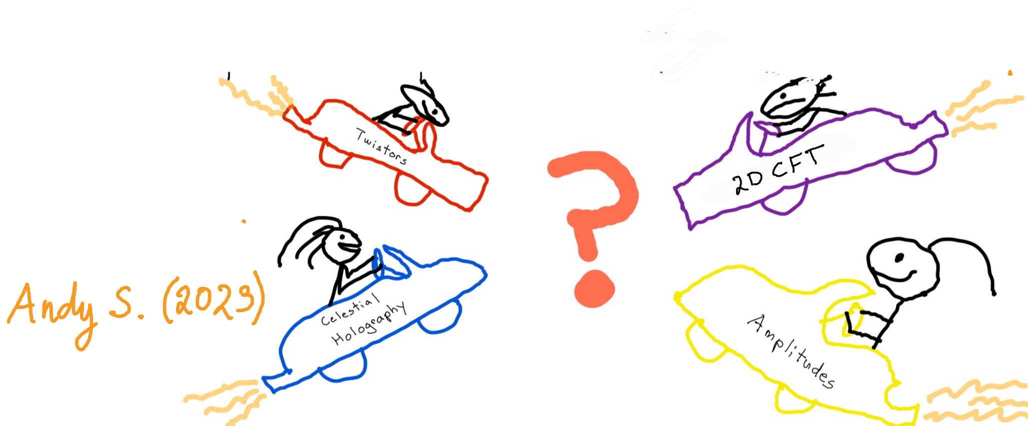
bulk quantum theory of ~~gravity~~

has a dual formulation in terms of

~~non~~-gravitational QFT on the boundary

?

Many things to do...



THANK YOU!